Forecasting Inflation in Tunisia Using Dynamic Factors Model

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Introduction

Inflation defined as the sustainable price increase. It translates into a decrease in the purchasing power attributed to excess liquidity, resulting in an imbalance between supply and demand. Inflation could also lead to a reduction in the external competitiveness of an economy, making its exports relatively more expensive, which can have a negative impact on the balance of trade transactions. Thus, the inflation forecast is a primary concern for policy makers and has been the subject of numerous studies whose objective is to provide a better view of the future evolution of inflation.

However, the current inflation forecast in Tunisia is essentially based on the use of VAR and SARIMA models aggregating monthly data, which anticipate price changes in a relatively short horizon. These models do not consider the international changes in world prices and they are sensitive to the problems of over parameterization (modeling involves too much parameter in an equation system). In attempt to remedy these problems, predictive models were implemented, integrating a large number of detailed and diversified information for reliable forecasts. Among these models, the most famous is the dynamic factor model. The use of this model for the modeling of inflation in Tunisia can improves the quality of short-term forecasts of this phenomenon.

In this work, we base our work on the model of **(Stock et Watson 2002b)** used for the US economy. Their model is characterized by its performance and its operation, using a large number of data and endless observations taking into account the heteroscedasticity and autocorrelation problems.

1. Literature review

Economic forecasting is an estimate for future values of economic variables. Forecasting techniques are used to predict the future evolution of the macroeconomic indices and guide the economic policies towards the desired goal.

Economic forecasting appeared for a century with the outbreak of business cycle theory. The forecast has been cultivated in "Havard School". The economic crises in the world before and after World War enriched the studies in this field **(Fisher 1925)**, **(Slutzky 1937)**, **(Keynes 1936)** and the implementation of the first econometric model by **(Tinbergen 1939)**.

The inflation forecast occupies an important place in the literature. However, there are various prediction models, the difference between them reflected in the information used and the level of over parameterization.

(Stock et Watson 1999) combined the forecasting models of inflation into four families: the forecast based on past inflation; the forecast based on the Philips curve; the forecast with the advanced indicator and the forecast with diffusion index.

(Phillips 1958) proposed an estimate between the change in nominal wage rate and the unemployment rate in the UK over the period 1861-1957. He obtained a nonlinear decreasing empirical relationship between the growth rate of nominal wages and the unemployment rate of the form $\Delta w_t = a - cU_t + \varepsilon_t$. This theory is known by its changes through the empirical simplicity. It formed the basis for the dynamic analysis of modern macroeconomics. Phillips, through his theory has generated an important instrument for predicting the short-term inflation and analysis of monetary policy.

Phillips theory generates incentives to theoreticians to develop this theory. **(Samuelson et Solow 1960)** have shown that the Phillips curve (1958) implies an unemployment dilemma inflation both the short and long term.

The Dilemma of **(Samuelson et Solow 1960)** is based on the Keynesian theory of the labor market and price rigidity. This hypothesis has been criticized by monetarists. **(Phelps 1967)** and **(Friedman 1968)** have shown that in the long term, there is no arbitration between inflation and unemployment, and the inflation is a purely monetary phenomenon. The monetary policy of regulation of long-term demand generates only the inflation. This new concept is known under name "the Phillips curve increased". where Friedman and Phelps have classified the anticipation of the inflation in the short-term analysis. This relationship will be deformed by the evolution of inflation expectations which induces the appearance of a new 'adaptive anticipation theory' explaining inflation by an autoregressive process.

In the augmented Phillips curve, wage growth is considered as a function of price. Thus, the causality between prices and wages is represented as a wage fixing phenomenon. This limit has been fixed in the work proposed by **(R. J. Gordon 1997)** and some economists of the US Federal Reserve such as **(Fuhrer et Moore 1995)**. **(R. J. Gordon 1997)** accorded its delay increased supply shocks and an output gap to the inflation model by an autoregressive process **(Le-Bihan 2009)**. The posterior studies have substituted the unemployment through by the output gap affecting prices.

The market failure is an outcome essentially of the monopolistic competition. The development of economic theory and different schools of thought, participated in the appearance of the new Keynesian school. The Keynesian Phillips curve Version proposed by **(Gali et Gertler 1999)**, is based on the assumption of the following price of revision rule **(Calvo 1983)**. This theory escapes the price fixing assumptions by companies and the presence of rational expectation induced by the monopolistic competition.

Taking account of the macroeconomic forecasting literature, who announces the performance of autoregressive models, **(Atkeson et Ohanian 2001)** have stated that for the last 15 years, economists failed to produce a version of the Phillips curve that makes a better accurate forecasts of inflation than a naive model (autoregressive) which assume that the inflation during the next four quarters will be equal to the inflation during the last four quarters.

(Stock et Watson 1999) have studied the stability of the Phillips curve in the United States, and the possibility of using other measures of economic activity that are potentially useful for inflation forecasting. They forecast the inflation in the United States during a period of thirty years spread over the period 1959:1 to 1997:9. They found the similar conclusions that were found in the most recent studies of the breaking Phillips curve between 1997and 1998, such as **(Gordon et Filardo 1998)** and **(Stock et Watson 1998)**. They have proposed an improvement of traditional forecasting methods by the Phillips curve using different economic indicators, in actual fact they have considered 189 indicators. However, the forecast based on these informers cannot improve the Phillips curve forecasting at least in the one-year horizon. The models who add money supply indices to the Phillips curve have made marginal improvements for some sampling periods and measure of inflation driving to a serious deterioration in the accuracy of inflation forecasting based on the CPI during the 70s and the early eighties. The product price does not improve the inflation forecasting over a horizon of 12 months. The measures of overall activity improve the forecasting of the Phillips curve, and the combination of these indices forecasts with the Phillips curve forecasts, produce forecast gains that are statistically and economically significant.

Recent progress in information technology has provided access to thousands of economic time series. This raises the prospect of a new frontier in terms of macroeconomic forecasting using many time series to forecast some indicators of economic conjuncture. The macroeconomic forecasting models currently used are multiple such as the vector autoregressive who combines dozens of variables.

The groupings of these variables, as well as identifying common factors are the subject of the work of **(Stock et Watson 2002a)**.

(Stock et Watson 2002a) use an approximate factors model in the aim of replacing the information in the large number of predictors factors by a less factors forecasting. This idea is based on the economic cycle theory cited by**(Burns 1950)** and the indices of advanced indicators that have been modeled by **(Tomas 1977)** in their dynamic generalization of the factorial analysis classical model. Their model is used to study the dynamics covariance between sets of variables, **(Geweke 1977)**, **(Watson et Engel 1983)**, **(Stock et Watson 1989)**, **(Stock et Watson 1993)** and **(Forni et Reichlin 1998)**.

(Stock et Watson 2002a) have used 215 time series to build six factors contributed a large part of series variance. They have successfully conducted to some factors that are needed to foresee real activity. This suggests that a very low state vector can be required for forecasts of macroeconomic series. They have faced limits such as the use of calculated indices in the basis of a linear transformation of the data, the use of monthly data (data homogeneity frequency), the use of data from the United States only, the factors estimates are based on simple estimators (without considering heteroscedasticity and serial correlation in the data) and the use of finite data (215).

These limits are solved in the Working Paper of **(Stock et Watson 2002b)** by considering a larger number than the number of time-series observation. This dimension of the problem is simplified by modeling the co-variability of the series in terms of a small number of unobserved latent factors. The forecast is realized in two steps; First, the time series of factors will be estimated from the preachers. Second, the relationship between the variable foresees and the factors, will be estimated by a linear regression analysis using the principal components. They concluded that the principal components of the variables compose the estimators of the latent factors.

2. modeling

2.1. static modeling

The static factors model can represent the information provided by a large number of variables. In the static part we find the exact factors model (explain any correlation between variables) and approximate factor models (largely explains the correlations between variables).

The model is written in the form of sum of two terms, the common is written as a linear combination of latent variables (factors) and another specific component or heteroscedastic residue.

$$x_{it} = \mu_i + \lambda_{i1} f_{1t} + \lambda_{i2} f_{2t} + \cdots + \lambda_{iq} f_{qt} + \varepsilon_{it}$$

For $i = 1, \dots, n$ qnd $t = 1, \dots, T$, $q < n$, we assume that the ε_{it} are independent of one another and the factors $E(\varepsilon_t \varepsilon_\tau^{'}) = 0 \quad \forall (t, \tau)$, $t \neq \tau$ (represents the part of the variable $x_{i,t}$ which is not explained by the common factors). Similarly f_q common factors are not perfectly uncorrelated as the specific components.

The matrix form for an observation t is:

$$\begin{bmatrix} x_{1,t} \\ \vdots \\ x_{i,t} \\ \vdots \\ x_{n,T} \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \vdots \\ \mu_i \\ \vdots \\ \mu_n \end{bmatrix} + \begin{bmatrix} \lambda_{1,1} & \cdots & \cdots & \lambda_{1,q} \\ \vdots & \cdots & \cdots & \vdots \\ \vdots & \cdots & \cdots & \vdots \\ \vdots & \cdots & \cdots & \vdots \\ \lambda_{n,1} & \cdots & \cdots & \lambda_{n,q} \end{bmatrix} \times \begin{bmatrix} f_{1,t} \\ \vdots \\ f_{j,t} \\ \vdots \\ f_{q,t} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \vdots \\ \varepsilon_{i,t} \\ \vdots \\ \varepsilon_{n,T} \end{bmatrix}$$

$$x_t = \mu + \Lambda f_t + \varepsilon_t$$

Our variables used are centered, That's why $\mu_i = 0$.

The $\lambda_{i,q}$ are called the factor weights (factor loading). They measure the covariance between the observed variable i and the common factor j.

If we consider the case where the factors are not correlated with each other and have unit variance, the interpretation of the model in terms of variance-covariance of the variables is as follows:

$$V(x_t) = \Lambda\Lambda' + D$$

With $D = E(\varepsilon\varepsilon')$ (the variance of ε_t) is the part of the variance of x_i which cannot be explained by common factors. $\Lambda\Lambda'$ represents the total contribution of factors to the variance of variance x_i. Common shocks and factorial coefficients constitute the common component. The estimated common component requires a linear combination of the series that explains most of the total variance, which amounts to minimize the specific component. So we show a link with OLS, but the problem suppose at the level of common shocks that are not observable, and the number q of common shocks that explain the evolution of the studied variables.

2.2. Choice of factors number

The number of factors can be estimated using the method of principal components (nonparametric).

(Onatski 2010) has formed a number of static factors test based on the values of $X'X$ number of nonzero Eigen values.

(Ng et Bai 2002) proposed a criteria for the choice of the factors in the static frame.

PC : Panel C_p criteria

$$\begin{cases} PC_{p1}(k) = V(k, \hat{F}^k) + k\hat{\sigma}^2\left(\frac{N+T}{NT}\right)\ln\left(\frac{NT}{N+T}\right) \\ PC_{p2}(k) = V(k, \hat{F}^k) + k\hat{\sigma}^2\left(\frac{N+T}{NT}\right)\ln C_{NT}^2 \\ PC_{p3}(k) = V(k, \hat{F}^k) + k\hat{\sigma}^2\left(\frac{\ln C_{NT}^2}{C_{NT}^2}\right) \end{cases}$$

IC : panel information criteria

$$\begin{cases} IC_{p1}(k) = \ln\left(V(k, \hat{F}^k)\right) + k\left(\frac{N+T}{NT}\right)\ln\left(\frac{NT}{N+T}\right) \\ IC_{p2}(k) = \ln\left(V(k, \hat{F}^k)\right) + k\left(\frac{N+T}{NT}\right)\ln C_{NT}^2 \\ IC_{p3}(k) = \ln\left(V(k, \hat{F}^k)\right) + k\left(\frac{\ln C_{NT}^2}{C_{NT}^2}\right) \end{cases}$$

2.3. Dynamic factors model

The focus of the dynamic model is the dynamic representation of the observed variables. Generalizes static models one hand by the VAR or VARMA modeling of the common factors, on the other hand, explains the observed variables across contemporary values and delay factors.

The dynamic factor model has a similar view to the static model with dynamic factor coefficients **(Forni, Hallin, et al. 2000)** :

In this context we assume a VAR (p) to present the common factors, we have:

$$f_t = \sum_{i=1}^{p} A_i^0 f_{t-i} + e_t$$

Where the process e_t can have a dynamic (their components are pairwise uncorrelated and uncorrelated with the factors).

$$f_t = A_1^0 f_{t-1} + A_2^0 f_{t-2} + \cdots + A_p^0 f_{t-p} + e_t$$

$$f_t - A_1^0 f_{t-1} - A_2^0 f_{t-2} - \cdots - A_p^0 f_{t-p} = e_t$$

$$f_t - A_1(L)^1 f_t - A_2(L)^2 f_t - \cdots - A_p(L)^p f_t = e_t$$

$$A(L) f_t = f_t - A_1^0 f_{t-1} - A_2^0 f_{t-2} - \cdots - A_p^0 f_{t-p} = e_t$$

We suppose for $\forall\, t$:

$$F_t = \begin{bmatrix} f_t \\ f_{t-1} \\ \vdots \\ \vdots \\ \vdots \\ f_{t-(p-1)} \end{bmatrix};\ \xi_t = \begin{bmatrix} e_t \\ 0_n \\ \vdots \\ \vdots \\ \vdots \\ 0_n \end{bmatrix};\ A = \begin{bmatrix} A_1 & A_2 & \cdots & \cdots & A_{p-1} & A_p \\ I_n & 0_n & \vdots & \vdots & 0_n & 0_n \\ 0_n & I_n & \cdots & \cdots & 0_n & 0_n \\ 0_n & 0_n & I_n & 0_n & 0_n & 0_n \\ 0_n & 0_n & 0_n & I_n & 0_n & 0_n \\ 0_n & 0_n & 0_n & 0_n & I_n & 0_n \end{bmatrix}$$

Then the $VAR(p)\ f_t$ process can be rewritten in the form of a transformed process F_t satisfying a $VAR(1)$ representation, such as:

$$F_t = AF_{t-1} + \xi_t$$

The estimated parameters of the equation (which represents the dynamic factors) will be obtained through the implementation of an OLS estimator factors (VAR model).

2.4. Dynamic factors number

(Giannone, Reichlin et Sala 2006) have formed a heuristic method based on the inspection of the Eigen values of residual VAR static factors. **(Amengual et Watson 2007)** have widened the Bai-Ng studies to estimate the number of dynamic factors (q) by applying the information criterion to the covariance matrix residues of the VAR.

2.5. State-space and Kalman filter

$$\begin{cases} x_t = \Lambda\, F_t + \varepsilon_t \\ F_t = A\, F_{t-i} + B\, e_t \end{cases}$$

The above model admits a state-space representation in which the general form is represented as follows:

$$\begin{cases} x_t = \Lambda\, F_t + \varepsilon_t & DFM(1) \\ F_t = A\, F_{t-i} + \xi_t & DFM(2) \end{cases}$$

With :

$$\begin{cases} V(\varepsilon_t) = \Phi = diag(\phi_1, \dots, \phi_n) \\ V(\xi) = \Sigma_\xi \quad (où\ \xi = B\ e_t) \end{cases}$$

The two equations DFM(1) and DFM(2) constitute a state-space model. The first, is a measurement equation that describes the relationship between the observed variable (x_t) and the unobserved state variable (F_t). The second is an equation of state (transition) that describes the process of latent variables (dynamic state vector across the transition matrix A).

$$\begin{cases} x_t = \Lambda\ F_t + \varepsilon_t & (M) \\ F_t = A\ F_{t-1} + \xi_t & (T) \end{cases}$$

The process ε_t and ξ_t are the vector of measurement errors at time t and the innovation vector at time t, respectively. They satisfy the following conditions:

$$\begin{pmatrix} \varepsilon_t \\ \xi_t \end{pmatrix} \approx N\left(0, \begin{pmatrix} R & 0 \\ 0 & Q \end{pmatrix}\right)$$

To obtain the estimates of the factor model parameters, factors will be re-estimated $\ddot{F}_t = Proj[F_t | x_1', \dots, x_{144+h}']$ by applying the Kalman filter on the state-space model as we define it.

We decompose this estimation step in three stages. But before we start, we must initialize the state at time $t = 0$ by F_0 factors are equal to zero and variances of ω_0 tending to infinity.

At time $t = 0$, we have $F_0\ et\ \omega_0$.

At time $t = t - 1$,

The first step: forecast: we calculate the conditional expectation at time t, knowing that

we have the date (t-1).

$$\begin{cases} \hat{F}_{t|t-1} = E(F_{t|t-1}) = E(A_{t-1} F_{t-1}) \\ \hat{\omega}_{t|t-1} = E(\omega_{t|t-1}) = E(\Lambda_{t-1}^2 \omega_{t-1} + Q_{t-1}) \end{cases}$$

The second step: revision

At the time (t = t)

We set $x = x_t$,

$$\vartheta_t = x_t - \Lambda_{t-1} F_{t|t-1}$$

$$V(\vartheta_t) = \Gamma_t = \Lambda_{t-1}^2 \omega_{t|t-1} + R_{t-1}$$

ϑ_t: Error (specific innovation to each variable x)

We use ϑ_t and Γ_t to update F_t and ω_t.

$$\begin{cases} F_t = F_{t|t-1} + \dfrac{\Lambda_{t-1}\omega_{t|t-1}\vartheta_t}{\Gamma_t} = F_{t|t-1} + K_t(x_t - \Lambda F_{t|t-1}) \\ K_t = \omega_{t|t-1}\Lambda'(\Lambda\omega_{t|t-1}\Lambda' + R)^{-1}\ \text{(gain matrix)} \end{cases}$$

$$\omega_t = \omega_{t|t-1} + \frac{\Lambda_{t-1}^2 \omega_{t|t-1}^2}{\Gamma_t} = (I - K_t\Lambda)\omega_{t|t-1}$$

Subsequently:

$$\begin{cases} F_{t+1|t} = AF_{t|t} \\ \omega_{t+1|t} = A\omega_{t|t}A' + Q \end{cases}$$

The third step: parameter estimation.

$$L_j = -\frac{1}{2}\sum' \vartheta'\Gamma\vartheta - \ln(2\pi)^{\frac{n}{2}}(|\Gamma|)^{\frac{1}{2}} = -\frac{1}{2}\ln\Gamma_t - \frac{1}{2}\frac{\vartheta_t^2}{\Gamma_t}$$

$$Loglik = \sum L_j = -\frac{1}{2}\sum \ln \Gamma_t - \frac{1}{2}\sum \frac{\vartheta_t^2}{\Gamma_t}$$

We move to the time (t = t + 1) and we repeat this three-step procedure until the period T. To refine the estimate of the states, we use the smoothing algorithm. We iterate calculates of the backward for (t = T-1 to 1).

$$\begin{cases} F_{t|T} = F_{t|t} + \omega_{t|t} A' \omega_{t+1|t}^{-1} \left(F_{t+1|T} - F_{t+1|t}\right) \\ \omega_{t|T} = \omega_{t|t} + \omega_{t|t} A' \omega_{t+1|t}^{-1} \left(\omega_{t+1|T} - \omega_{t+1|t}\right)\left(\omega_{t|t} A' \omega_{t+1|t}^{-1}\right)' \end{cases}$$

Factors estimated by the Kalman filter are as follows:

$$F_{Kal} = F_{t|T}$$

3. Empirical Work

3.1. The Data

Gross data cover the full period extending from 2000 to 2014. They are collected from general publications, statistical yearbooks of Tunisia, the Tunisian product classification, the nation's accounts, the foreign trade balance Tunisia, annual reports on the characteristics of the agents of the public service and their wages and household consumption statistics available from the National Institute of statistics. Concerning the monetary aggregates data, they are collected from the Central Bank of Tunisia. Finally, data on prices of international commodities are downloaded from the base of the index data Mundi (see **Appendix 1**).

Presentation of gross data: Data collected cover details on consumer prices, industrial selling prices, industrial production, wages, monetary aggregates, the exchange market, foreign trade, energy consumption, public finance, tourism, interest rates, stock market data of Tunisia, global demand, the international prices of some products including beverages, seafood, oils, agricultural commodities, metals, cereals, energy, and fruit. The data also cover the international stock market and international trade (see **Appendix 1** exhaustively detailing the data). So we build a database gathering 234 quantitative variables. This number is consistent with most of the empirical work using this type of model for the prediction of the price index. Seventy-eight percent of these variables relate to the Tunisian economy. The rest is a set of data on international prices.

Seasonal adjustment: Given the specificities of the Tunisian economy as the moving holidays, the Muslim calendar. This process concerns the two hundred and thirty four series from groupings agreed in the previous step.

Stationary of the series: In relation with the assumptions of the factor model **(Stock et Watson 1998)** all explanatory variables must be stationary. To process the stationary of two hundred forty variables, who studies stationary throughout the Dickey-Fuller test, which is based on the assumption of the body:

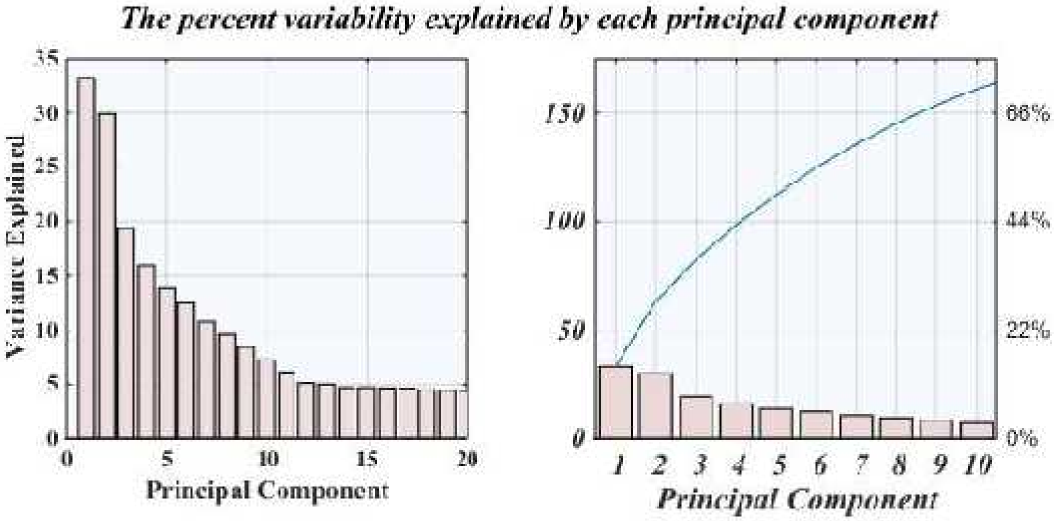
$$\begin{cases} H_0 : \ \psi = 1 & (stationary\ proces) \\ H_1 : \ \psi \neq 1 & (no\ stationary\ proces) \end{cases}$$

Data transformations: Most of the data used are index some rates. Before the treatment of stationarity and seasonality of the data, we transform them into natural logarithm without the rates such as TMM and the foreign exchange market.

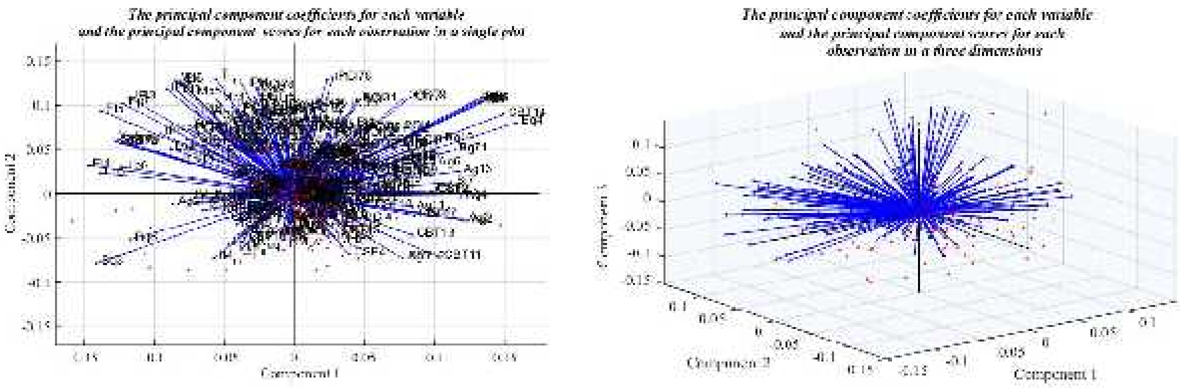
Centering and reducing of the series: Before starting the principal component analysis, it is important to respect the principles of the PCA. His hypothesis is data normalization. To overcome the effects of scale due to the possible heterogeneity of variables, they are standardized in general, i.e. Each series is divided by its standard deviation; All of them are therefore expressed in the same standard scale.

3.2. Estimation and Interpretation of Factors

According to the charts below we see that 16% of all information is represented by the first factor. The first three principal axes represent a cumulative percentage of the total information which reached 37%. Despite this low percentage cumulative we choose the first three factorial axes (the landing between the fourth and fifth own value is lower than the second and third eigenvalue(5%)).



With the representation of the variables on the factor plane, we can see a high correlation between the studied variables and the factorial axes as shows the graph below.

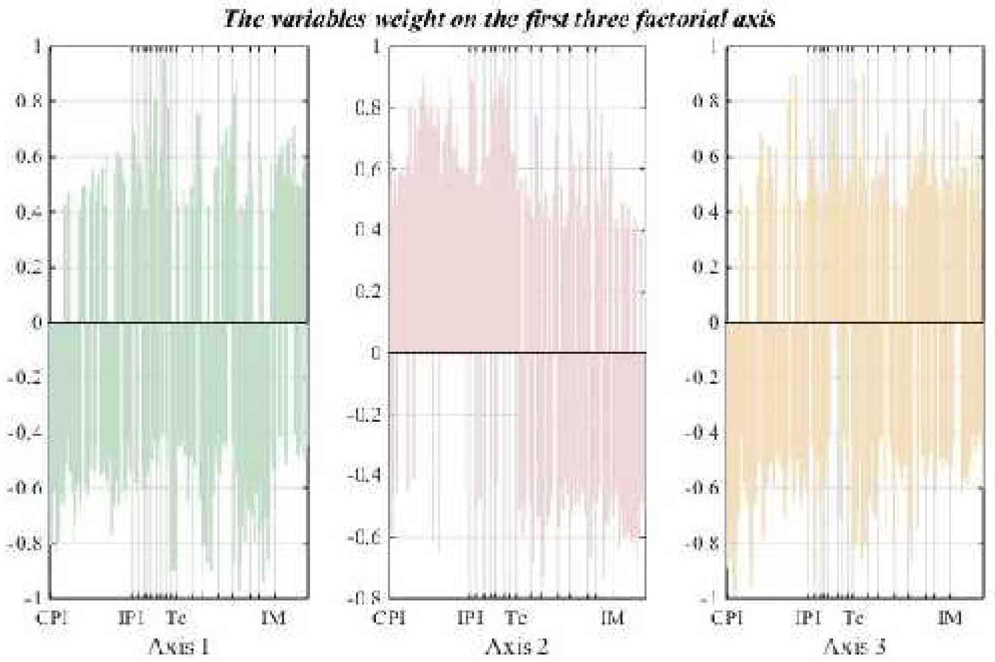


According to the graph below, we can rename the factors based on the coordinates of the variables by "the factorial axes".

Axis 1: The analysis allows us to distinguish a coordinated exceeding 0.9 for some variables such as energy consumption by up trying, direct and indirect taxes, indexes leather, rubber, wool and newspapers, productions cook in industrial, automotive industrial equipment, mining, rubber, ceramic, and exports and imports and general offshore textile, mechanical, electronic, transport and cooking. So the first factor informs about the industrial sector.

Axis 2: Imports are not well correlated to the first factor axis and are general imports and global indices of cereals, drinks and energy are strongly correlated with the second factor axis. The second factor axis shows the foodstuffs.

Axis 3: The offshore imports and administered price index for consumption are not related to the first factor axis but are highly correlated to the third factor axis. So this factor axis gives information on administered prices for consumption.



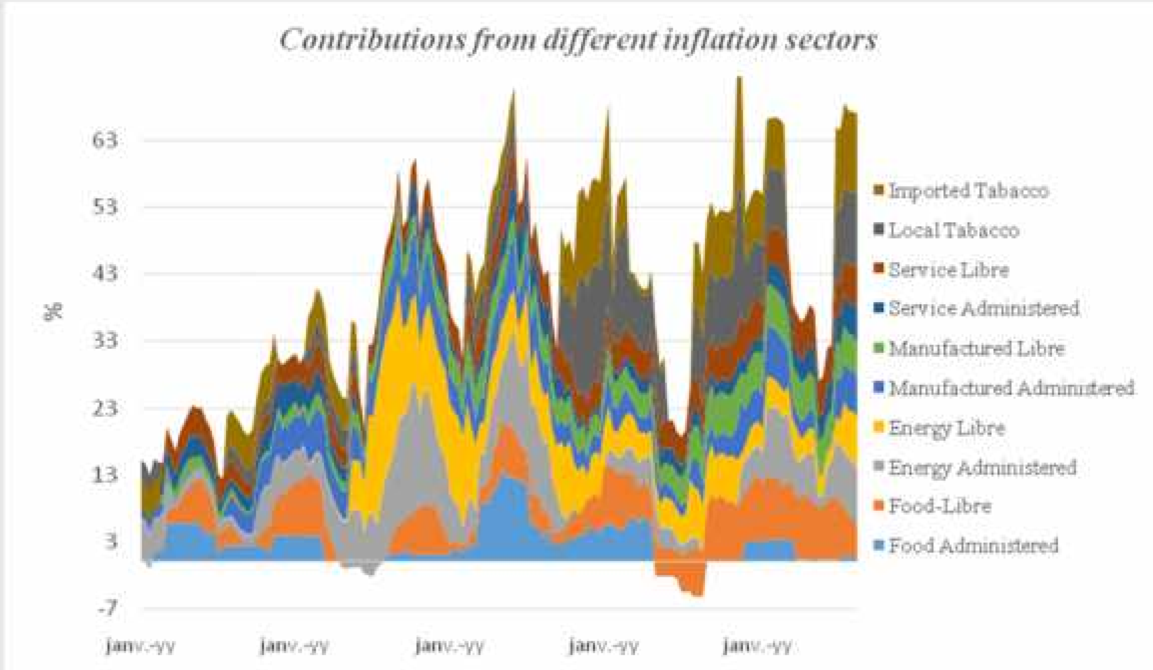
4. Forecasting Framework

4.1. Forecasting Design and Forecasting Models

In this section we evaluate the applicability of the dynamic factor model in predicting future values of inflation. Prediction is applied to a variable built in compliance with the following formula:

$$infl_t = \frac{IPC_{t-12} - IPC_t}{IPC_{t-12}}$$

The decomposition of the global year inflation showed a significant effect of the contribution of both the price of fresh food and those of manufactured products. However, the party managed inflation experienced a sharp contraction since January 2011, related to the reduction in prices of commodities at the beginning of the year and maintaining these almost prices unchanged, despite the fiscal burden. In 2013, we see some stabilization of the contribution of fresh food while those of processed and manufactured food products continued to increase as shows the figure below:



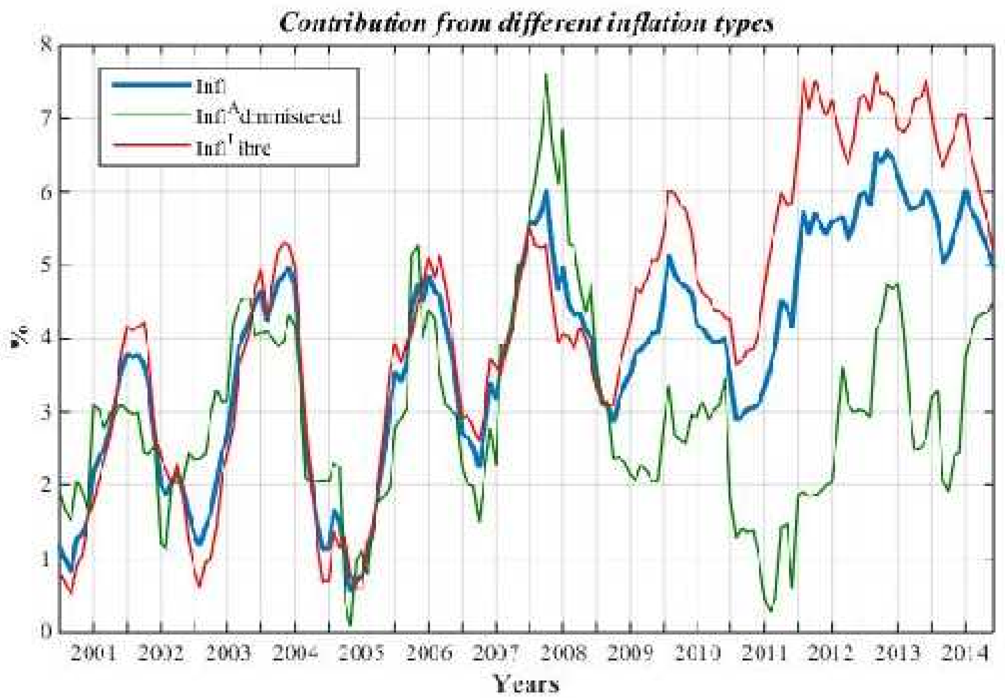
The PCI consists of 247 products divided on twelve groups (NATIONAL INSTITUTE OF STATISTICS - TUNISIA). They are: Food and drinks; Tobacco; Clothing and footwear; Housing, water, gas, electricity and other; Furniture, household equipment and routine home maintenance; Health; Transport ; Communications ; Leisure and culture ; Education ; Restaurants and hotels ; Miscellaneous goods and services.

We will extract the indices of 247 products and the weight of the NIS. Then we will classify these products, according to their regime (administered or free) in groups shown in the table below. (for more details see **Appendix 1**).

Products classification to the regime

Household consumer price index	Household consumer price index Libre	Household consumer price index Administered
Food-Fresh	Food Fresh Libre	Food Fresh Adminisered
Food	Food-Libre	Food Administered
Food-Transformed	Food Transformed Libre	Food Transformed Administered
Manufacturer Construction	Manufacturer Construction Libre	
Manufacturer Maintenance	Manufacturer Maintenance Libre	
Manufacturer Clothing	Manufacturer Clothing Libre	Manufacturer Maintenance Administered
Manufacturer Household	Manufacturer Household Libre	
Other Manufactured	Other Manufacturer Libre	Other Manufacturer Administered
Service Loyer	Service Loyer Libre	Service Loyer Administered
Service Health	Service Health Libre	Service Health Administered
Service Tarif		Service Tarif Administered
Other Service	Other Service Libre	Other Service Administered
Energy	Energy Libre	Energy Administered
Local Tabacco		
Imported Tabacco		

The figure below illustrates the difference between inflation, administered and free inflations. We notice divergence and change in the slope of the curves since middle 2008. This turn is due through structural change in the construction of the consumer price index, (some food products become free). From 2008 the state adopted a market liberalization policy. She begins to remove the compensation on certain products in the CPI, citing for example the tomato, cement, electricity, etc ....



Further to our objective in this paper, we propose three types of model. The first is to evaluate the performance of forecasting total inflation, while the second deals inflation administered and the third about free inflation. The models proposed are as following:

Model 1: (Total Inflation)

$$PCI_t = \alpha_0 + \sum_{i=1}^{10} \alpha_i \, D\_F_t^i + \varepsilon_t$$

Model 2: (Administered Inflation)

$$PCI\_A_t = \alpha_0 + \sum_{i=1}^{10} \alpha_i \, D\_F_t^i + \varepsilon_t$$

Model 3: (Free Inflation)

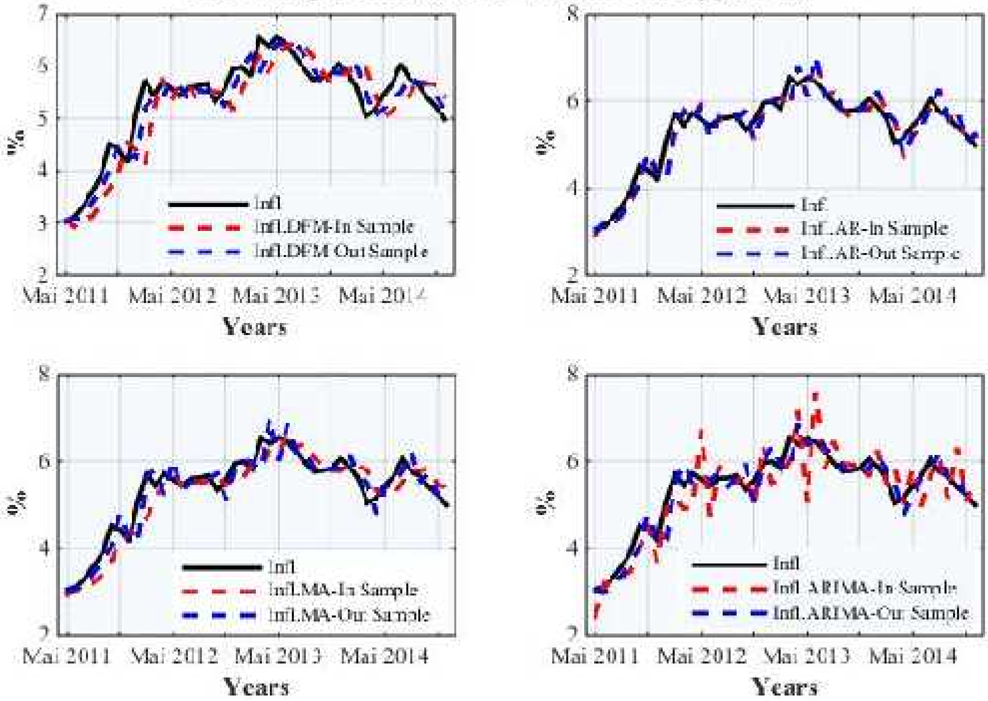
$$PCI\_L_t = \alpha_0 + \sum_{i=1}^{10} \alpha_i \, D\_F_t^i + \varepsilon_t$$

Results of the estimation are presented in the **Appendix 3**. For the first and the third model the approach of the DFM presents the performance of the variability of the predictor variable, who reaches 75%, against a benchmark the models (AR, MA, and ARIMA) are strictly lower variability 75%. The estimated three models with the four approaches are globally significant at the 1% level, and the test on the residue shows the normality of residuals.

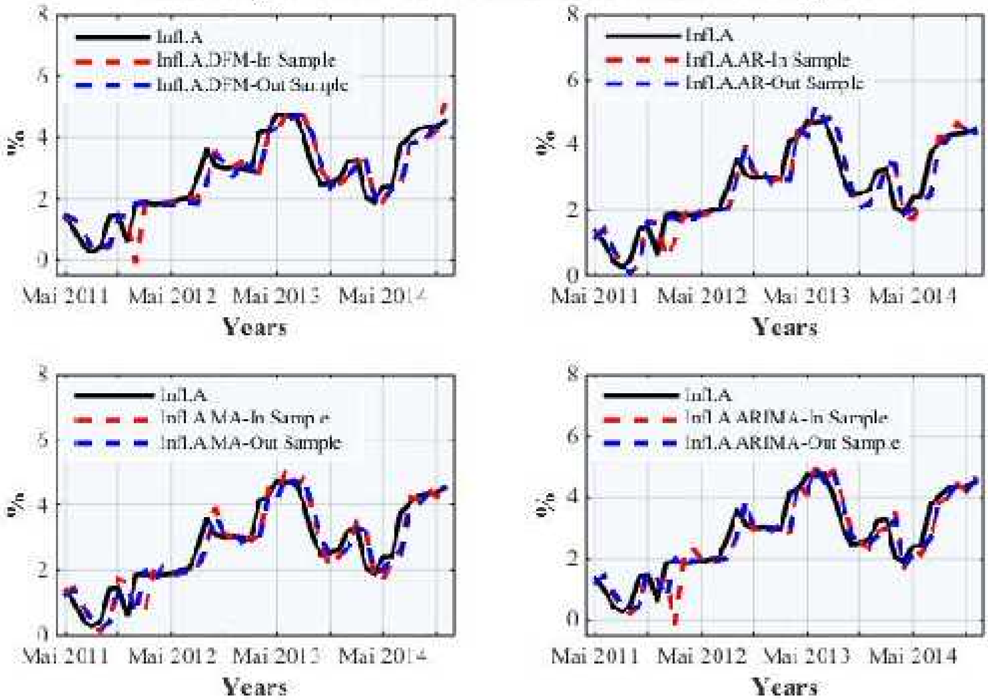
4.2. Forecast Performance

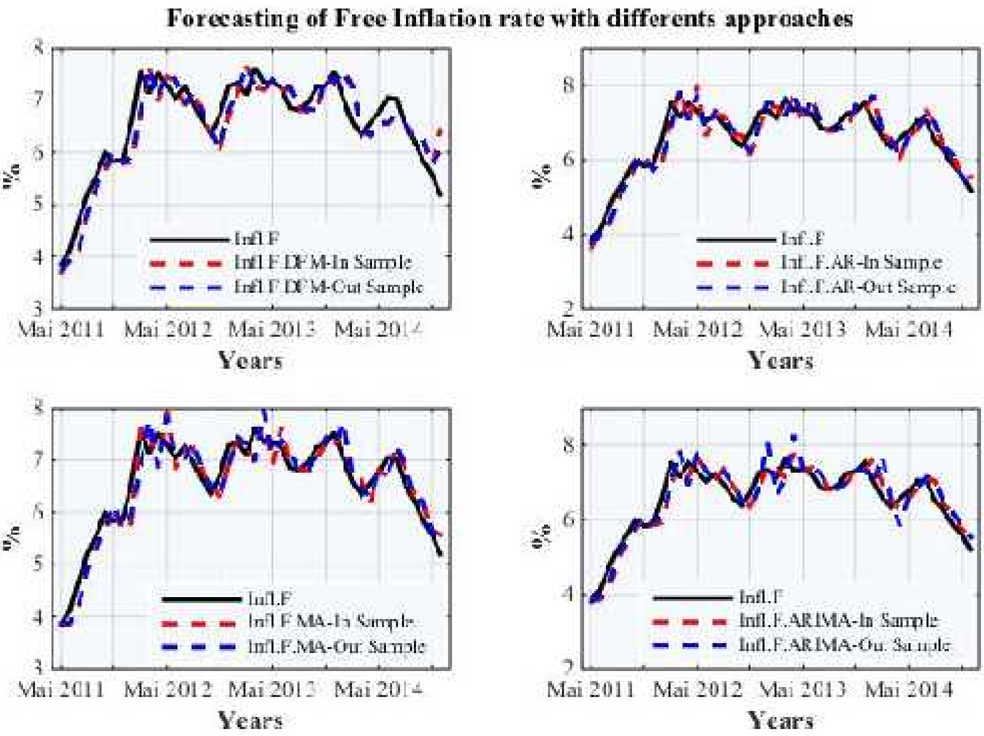
Our sample is spread between 2000 and 2014. We divide this in two samples, a period for learning and the other for testing. The period of the test covers the period from May 2010 until end-2014. We use two types of forecasting namely In-Sample and Out-Sample. The figures below illustrate the difference between the two types of prediction adopted by the three models.

Forecasting of Inflation rate with differents approaches



Forecasting of Administred Inflation rate with differents approaches





5. Forecasting Results

The forecasts calculated from the three models were constructed to better simulate the forecasting process. The Bayesian Information Criterion BIC was used to determine the dynamic structure of the models.
The criterion that was used to evaluate the performance of forecasting models examined is the Root Mean Square Error (RMSE).

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(inf_i - \widehat{inf}_i)^2}$$

The second criterion is based on the results of a test proposed by **(Chong et Hendry 1986)**. This test is founded on the estimation of parameters of the following equation:

$$infl_{t+h} = \alpha\, \widehat{infl}_{t+h}^{MFD} + (1-\alpha)\, \widehat{infl}_{t+h}^{BENCH} + v_{t+h}$$

If $\alpha = 1$ then we conclude that the inflation forecast based on the factors in the forecast model encompasses competitive models. However, if $\alpha = 0$, the forecast based on the competitive model includes forecasts obtained from the factor model. The audit will allow us to test the following hypotheses:

$$\begin{cases} S_1: \begin{cases} H_0: \alpha = 1 \\ H_1: \alpha \neq 1 \end{cases} \\ S_2: \begin{cases} H_0: \alpha = 0 \\ H_1: \alpha \neq 0 \end{cases} \end{cases}$$

The acceptance of the first assumption 'zero' ($\alpha = 1$) and the rejection of the second ($\alpha = 0$) lead us to conclude that the forecast values obtained from the factor model are closer to the realizations than those obtained from competitive models.
The evaluation of the performance of the DFM by RMSE and C-H Test criterions shows the performance of DFM model over the auto-regressive model (AR, MA, and ARIMA) in both type In-sample and Out-sample forecasting, for total inflation and inflation free revolving around 0.3. As against, according to RMSE the DFM is inefficacies, for the inflation forecast administered as part of the Out-Sample.

RMSE of the estimates equation

		RMSE In-Sample inflation	RMSE Out-Sample inflation
Model 1	DFM	**0,2945**	**0,2989**
	AR	0,3227	0,3199
	MA	0,3657	0,3311
	ARMA	0,5325	0,3152
Model 2	DFM	**0,5306**	0,5426
	AR	0,6059	0,5348
	MA	0,5843	0,5308
	ARIMA	0,6639	**0,5205**
Model 3	DFM	**0,3299**	**0,3361**
	AE	0,3389	0,3487
	MA	0,3222	0,3577
	ARIMA	0,3405	0,4192

Model performance according to the test Chong-Hendry

		S_1		S_2		**Result**
		H_0	H_1	H_0	H_1	
AR	Model 1	x			x	x
	Model 2		x		x	
	Model 3	x			x	x
MA	Model 1	x			x	x
	Model 2		x		x	
	Model 3	x			x	x
ARIMA	Model 1	x			x	x
	Model 2		x		x	
	Model 3	x			x	x

6. Conclusion

This paper has investigated the use of Dynamic Factor Model methods for forecasting inflation in Tunisia. This research proves the importance of Dynamic Factors Model for forecasting total and free inflation over benchmark model regression. In our empirical work we expose this importance through the use of RMSE criterion and that to compare our model with the benchmark model. The RMSE criterion has shown the performance of DFM in free and total inflation.

Appendix –1

Label	Type	Source	Duration	ADF Test
National				
Household consumer price index	Index (base 100=2005)	Authors	01-2000 To 12-2014	
Administered Food				1
Adminisered Fresh				1
Administered Excluding Energy				1
Food Fresh Libre 1				1
Food Fresh Libre 2				2
Food Fresh Libre 3				1
Food Administered				1
Food-Energy				1
Food-Fresh				1
Food-Libre				1
Food-Tabacco				1
Food-Transformed				1
Food-Transformed Administered				1
Food-Transformed Libre				1
Food-Transformed-Service Libre				1
Food Transformed Administered 1				1
Food Transformed Administered 2				1
Food Transformed Administered 3				1
Food Transformed Administered 4				1
Food Transformed Administered 5				1
Food Transformed Administered 5				1
Food Transformed Libre 1				1
Food Transformed Libre 2				1
Food Transformed Libre 3				1
Food Transformed Libre 4				1
Food Transformed Libre 5				1
Energy Administered				1
Energy Libre				1
Fresh-Energy				1
Fresh-Tabacco				1
Household consumer price index Administered				1
Household consumer price index Libre				1
Other Manufacturer Administered				1
Other Manufacturer Administered				
Other Manufacturer Administered				1
Other Manufacturer Administered				1
Other Manufacturer Libre				1
Other Manufacturer Libre				1
Other Manufacturer Libre				1
Other Manufacturer Libre				1

Other Manufacturer Libre	1
Manufacturer Food Libre	1
Manufactured Service Excluding Food Libre	1
Manufactured Service Libre	1
Manufactured Administered	1
Other Manufactured	1
Other Manufactured Administered	1
Other Manufactured Libre	1
Manufacturer Construction	1
Manufacturer Maintenance	1
Manufacturer Maintenance Administered	1
Manufacturer Maintenance Libre	1
Manufacturer Clothing	1
Manufactured Libre	1
Manufacturer Household	1
Manufacturer Construction Libre 1	1
Manufacturer Construction Libre 2	1
Manufacturer Maintenance Administered 1	1
Manufacturer Maintenance Libre 1	1
Manufacturer Maintenance Libre 2	1
Manufacturer Clothing Libre 1	1
Manufacturer Clothing Libre 2	1
Manufacturer Household Libre 1	1
Manufacturer Household Libre 2	1
Other Service Administered 1	1
Other Service Administered 2	1
Other Service Administered 3	1
Other Service Libre 1	1
Other Service Libre 2	1
Other Service Libre 3	1
Other Service Libre 4	1
Other Service Libre 5	1
Service Administered	1
Other Service	1
Other Service Administered	1
Other Service Libre	1
Service Excluding Food Libre	1
Service Libre	1
Service Loyer	1
Service Loyer Administered	
Service Loyer Libre	1
Service Health	1
Service Health Administered	1
Service Health Libre	1
Service Tarif	1
Service-Food Libre	1

Service Loyer Administered				
Service Loyer Libre				1
Service Health Administered				1
Service Health Administered				1
Service Health Administered				1
Service Health Libre				1
Service Health Libre				1
Service Health Libre				1
Service Tarif Administered				1
Service Tarif Administered				1
Service Tarif Administered				1
Service Tarif Administered				1
Service Tarif Administered				1
Service Tarif Administered				1
Local Tabacco				1
Imported Tabacco				1
Industrial production index (per branch)	Index (base 100=2000)	NIST	01-2000 To 09-2014	
Agri-food ind				1
Construction materials Ceramic and Glass				1
Mechanical and electrical industries				1
Chemical industries				1
Textile, apparel and leather				1
Miscellaneous manufacturing industries				1
MINING				1
ENERGY				1
Industrial selling price index (per branch)	Index (base 100=2000)	NIST	01-2000 To 10-2014	
Products of Agri-Food Industries				1
Construction Materials, Ceramic and Glass				2
Products of Mechanical and Electrical Industries				1
Chemical Products				1
TEXTILE, APPAREL AND LEATHER				1
Miscellaneous Products of Manufacturing Industries				1
Mining				1
Energy				1
Tourism				
Entries		CBT	01-2000 To 06-2012	1
Non-residents Monthly hotel nights	Thousand	NIST	01-2000 To 12-2014	
Residents Monthly hotel nights	Thousand	NIST	01-2000 To 12-2014	
Accommodation Days		CBT	01-2000 To 06-2012	1
Touristic Revenues		CBT	01-2000 To 06-2012	1
Labor Market		CBT	01-2000 To 01-2012	
Guaranteed Minimum Agricultural Wage				1
Guaranteed minimum interoccupational wage _40H				1
Guaranteed minimum interoccupational wage _48H				1
Global demand for employment				1

Global offer for employment				1
Placements in Tunisia				1
Exports per group of sectors	Million dinars	NIST	01-2000 To 1162014	
Agriculture and agri-food industries				1
Energy and lubricants				1
Mining, phosphates and derivatives				1
Textile, apparel and leather				1
Mechanical and electrical industries				1
Other manufacturing industries				1
Imports per group of sectors	Million dinars	NIST	01-2000 To 1162014	
Agriculture and agri-food industries				1
Energy and lubricants				2
Mining, phosphates and derivatives				2
Textile, apparel and leather				1
Mechanical and electrical industries				1
Other manufacturing industries				1
Eléctricity consumption	10^6kw/h	CBT		
Eléctricity consumption Medium high voltage			01-2000 To 03-2012	1
Eléctricity consumption high voltage			01-2000 To 03-2012	1
Eléctricity consumption Medium voltage			01-2000 To 03-2012	1
Eléctricity consumption Low voltage			01-2000 To 01-2012	1
Eléctricity consumption			01-2000 To 01-2012	1
Energy Production		CBT		
Petrol Production SM3			01-2004 To 10-2011	
Gaz Production SM3			01-2004 To 10-2011	
National Electricity Production			01-2000 To 06-2012	1
Mouvement of the Earth			01-2000 To 07-2012	1
Extraction			01-2000 To 07-2012	1
Phosfat Production			01-2000 To 07-2012	1
Local cement Sales			01-2000 To 10-2011	1
Transport	Enteger	CBT		
Maritime Transport Number			01-2000 To 10-2011	1
Arien Transport Number			01-2000 To 06-2012	1
Public Finance		CBT	06-2000 To 05-2012	
Income Taxes				1
Company Taxes				1
Customs Duties				1
Value Added Tax				1
Consumer Taxes				1
Other Indirect Taxes				1
Non-Tax Revenues				1
INTERBANK MARKET EXCHANGE RATE AVERAGES FOR MONTH			01-2000 To 11-2014	
ALGERIAN DINAR (DZD)	Rates (U 10)			1
SAUDI RIYAL (SAR)	Rates (U 10)			1
CANADIAN DOLLAR (CAD)	Rates (U 1)			1
DANISH KRONE (DKK)	Rates (U 10)			1

U.S DOLLAR (USD)	Rates (U 1)			1
STERLING POUND (GBP)	Rates (U 1)			1
JAPANESE YEN (JPY)	Rates (U 1000)			1
MOROCCAN DIRHAM (MAD)	Rates (U 10)			1
NORWEGIAN KRONE (NOK)	Rates (U 100)			2
SWEDISH KRONE (SEK)	Rates (U 10)			1
SWISS FRANC (CHF)	Rates (U 10)			1
KUWAITI DINAR (KWD)	Rates (U 1)			1
UAE DIRHAM (AED)	Rates (U 10)			1
EURO (EUR)	Rates (U 1)			1
LIBYAN DINAR (LYD)	Rates (U 1)			1
MAURITANIAN OUGUIYA (MRO)	Rates (U 100)			1
Interest rates	Rates			
Money market average			01-2000 To 11-2014	1
Savings Remuneration Rate			01-2000 To 12-2014	1
Key rate of the BCT			01-2000 To 11-2014	1
RESIDENT FINANCIAL SYSTEM COUNTERPARTS	MTD	CBT	01-2000 To 10-2014	
Net claims on abroad*				1
Net foreign assets				1
Domestic loans				1
Net claims on the state				1
Treasury bonds (in portofolio)				1
Treasury current account				2
Financing of the economy				1
Credit to the economy				1
Loans/ordinary resources				1
Loans/special resources				1
Commercial papers				1
Securities portfolio				1
RESIDENT FINANCIAL SYSTEM RESOURCES		CBT	01-2000 To 11-2014	
M4 aggregate				1
Money supply M3				1
Money supply M2				1
Money M1				1
Fiduciary money				1
Bank money				1
Companies and individual deposits				1
Deposits ay the CCP				2
Quasi money				1
Companies '.deposits				1
Certificates of deposits				2
Special savings accounts				1
Deposits at the CEP				2
M3 -M2				1
Home savings				2
Debenture loans				1

M4 -M3				1
Commercial paper				1
Special resources				0
Capital stock equity				1
Other resources				1
Offshore banks' statistics	(in thousand of TD)	CBT	01-2000 To 06-2014	
Loans to th economy				1
Loans on ordinary resources				1
Discount portfolio on Tunisia				1
Syndicated loans				2
Frozen claims and others				1
Loans on special resources				1
Securities Portofolio				1
Othes items of assets				2
Fixed assets				1
Sight deposits				1
Forward dep. fin.products				1
Savings accounts				1
Non residents'deposits				1
Special resources				
Capital				1
Provisions				1
Reserves				1
Banks' statistics	(in thousand of TD)	CBT	01-2000 To 06-2014	
Loans to th economy				1
Loans /ordinary resources				1
Current accounts receivable(R)				1
Discount portfolio on Tunisia				2
Leasing transactions				0
Frozen claims and others				1
Adv/forwd.accnts. voucher				1
Loans /special resources				0
Sécurities Portofolio				2
Fixed assets				2
Sight deposits				1
Forward.depositsfin.prdcts				1
Certificates of deposits				0
Special savings accounts				1
Home savings				2
Debenture loans				1
Dep.in currency or in conv.TND				1
Deposits of non residents				1
Special resources				2
Capital				1
Reserves				1

Provisions				1
Leasing companies' statistics	(in thousand of TD)	CBT	01-2000 To 06-2014	
Loans to the economy				1
Loans/ordinary resources				1
Leasing transactions				1
Frozen claims and others				1
Loans/special resources				
Sécurities Portofolio				1
Real estates				1
Debenture loans				1
Special resources				1
Capital				1
Reserves				1
CBT's monthly statement	(In MTD)	CBT	01-2000 To 11-2014	
Foreign assets				1
Foreign currency assets				1
Claims on the state				1
Claims purchased firm				1
Claims on bank				1
Monetary market				1
Claims/BDEV				
Sécurities Portofolio				
Othes items of assets				1
Total Assets=Total Liabilities				2
Central bank money				1
Fiduciary money				1
Claims of banks				1
Curreny money market				1
Ordinary current accounts				1
Claims of investment banks				1
Deposits of companies and individuals				1
Ordinary current accounts of non residents				2
Claims of the state				2
Treasury current account				2
Counterparts' funds				0
Commitments on abroad				1
International			01-2000 To 12-2011	
Beverages				
Coffee, Arabica Monthly price	Cent USD pound			1
Coffee, Robusta Monthly Price	Cent USD pound			2
Tea Monthly prices	cents US par kilogramme			1
Cocoa beans Monthly Price	Dollars US par tonne métrique			1
shellfish				
Fish (salmon) Monthly Price	USD kilogram			2
Shrimp Monthly Price	Cent USD pound			1

Vegetable oil and protein meal	Dollars US par tonne métrique	
Soya oil Monthly Price		2
Olive oil extra virgin Monthly Price		2
Sunflower oil Monthly prices		1
Agricultural commodities		
Coarse Wool Monthly Price	Cent USD kilogram	1
Wood Pulp Monthly Price	USD metric ton	1
Cotton Monthly Price	Cent USD pound	2
Harsh Journals Monthly Price	USD cubic meter	2
Harsh Sawnood Monthly Price	USD cubic meter	1
Plywood Monthly Price	Cent USD leaves	2
Rubber Monthly Price	Cent USD pound	1
Journals flexible, pleasant Monthly Price	USD cubic meter	0
Skins Monthly Price	Cent USD poung	1
Fine Wool Monthly Price	Cent USD kilogram	1
Doux Sawnwood Monthly Price	USD cubic meter	2
Metals	USD metric ton	
Tin Monthly Price		1
Steel Wire Stem Monthly Price		1
Rebar Monthly Price		1
Iron Ore Monthly Price		1
Hot Rolled Steel Monthly Price		1
Copper Grade A Monthly Price		1
Aluminum Monthly Price		1
Cold Rolled Steel Monthly Price		1
Cereals	USD metric ton	
Orge Monthly Price		1
Wheat Monthly Price		2
Sorghum Monthly Price		1
Soft red winter wheat Monthly Price		1
Rice Monthly Price		1
Maize Monthly Price		1
Energy		
Crude Petroleum Monthly Price	USD barrel	1
Natural Gaz Monthly Price	USD thousand cubic meters of gas	1
Gasoline Monthly Price	USD gallon	2
Crude Petroleum; West Texas Intermediate Monthly Price	USD barrel	2
Crude Petroleum, Dubaï Fateh Monthly Price	USD barrel	1
Fruits	Dollars US par tonne métrique	
Bananas Monthly Price		2
Oranges Monthly Price		0

Appendix –2

	ADF				PP			
	level		first diff		level		first diff	
	trend constant	no constant	trend constant	no constant	trend constant	no constant	trend constant	no constant
D_F_1	-20.283***	-10.724***	-	-	-19.631***	-19.447***	-	-
D_F_2	-13.731***	-9.361***	-	-	-13.622***	-13.552***	-	-
D_F_3	-13.657***	-4.466***	-	-	-13.373***	-12.239***	-	-
D_F_4	-20.660***	-6.266***	-	-	-20.790***	-20.678***	-	-
D_F_5	-7.210***	-0.975	-	-	-5.6544***	-7.1810***	-	-
D_F_6	-23.133***	-10.343***	-	-	-20.453***	-20.125***	-	-
D_F_7	-14.654***	-4.567***	-	-	-13.980***	-12.436***	-	-
D_F_8	-14.243***	-4.142***	-	-	-17.823***	-16.398***	-	-
D_F_9	-12.767***	-2.98	-	-	-22.435***	-21.329***	-	-
D_F_10	-7.545***	-1.324	-	-	-9.348***	-8.431***	-	-
PCI	0.591	15.083	-7.676***	-3.167**	0.439	5.942	-10.393***	-8.910***
PCI_A	-2.167	8.610	-12.737***	-12.602***	-2.234	1.320	-12.737***	-12.60***
PCI_L	0.763	13.032	-3.600**	-1.824	1.013	7.609	-11.013***	-9.278***

*Indicates significance at the 10% level.
**Indicates significance at the 5% level.
***Indicates significance at the 1% level.

1% Critical Value	-4.014
5% Critical Value	-3.440
10% Critical Value	-3.140

Variabls	**D_F_ 1**	**D_F_ 2**	**D_F_ 3**	**D_F_ 4**	**D_F_ 5**	**D_F_ 6**	**D_F_ 7**	**D_F_ 8**	**D_F_ 9**	**D_F_ 10**	**PCI**	**PCI_A**	**PCI_L**
ADF Test	d(0)	d(0)	d(0)	d(0)	d(0)	d(0)	d(0)	d(0)	d(0)	d(0)	d(1)	d(1)	d(1)
PP Test	d(0)	d(0)	d(0)	d(0)	d(0)	d(0)	d(0)	d(0)	d(0)	d(0)	d(1)	d(1)	d(1)

Appendix –3

Output estimation Model 1 (Total Inflation				
	DFM	AR(4)	MA(1)	ARIMA(3,1,3)
F1	-.0726158 [-3.63]***	-	-	-
F5	-.1988746 [-6.82]***	-	-	-
F6	.0992874 [3.03]***	-	-	-
F7	-	-	-	-
F8	.1424654 [2.55]***	-	-	-
F9	-.1768236 [-4.28]***	-	-	-
Ar(1)	-	.4693492 [7.36]***	-	-.6779146 [-5.80]***
AR(2)	-	-.1712221 [-2.61]***	-	-.7018971 [-6.14]***
AR(3)	-	.426575 [5.43]***	-	.2971289 [2.47]**
AR(4)	-	-.2369226 [-3.05]***	-	-
MA(1)	-	-	.5680302 [10.55]***	1.283462 [16.60]***
MA(3)	-	-	-	.3772386 [4.11]***
C	-	.364232 [7.16]***	.3629342 [9.42]***	.3649177 [9.42]***
R-squared	0.73	0.65	0.40	0.74
Prob F	0.0000	0.0000	0.0000	0.0000
Normality test	6.24**	19.32***	20.31***	13.40***
N	178	178	178	178

Output estimation Model 2 (Administered Inflation)				
	DFM	AR(3)	MA(3)	ARIMA(3,1,3)
F2	.1255367 [2.87]***	-	-	-
F3	-.2120448 [-2.56]**	-	-	-
Ar(1)	-	.2742415 [3.49]***	-	-.7817199 [-9.97]***
AR(2)	-	.1102053 [2.01]**	-	.9407532 [39.64]***
AR(3)	-	.2223761 [2.83]***	-	.8398824 [9.79]***
MA(1)	-	-	.2710806 [3.17]***	.9319869 [18.55]***
MA(2)	-	-	.1652658 [2.59]***	-.909915 [-14.00]***
MA(3)	-	-	.2066471 [2.37]**	-.9611703 [-13.43]***
C	.387564 [7.09]***	-	-	-
R-squared	0.39	0.56	0.40	0.58
Prob F	0.0076	0.00000	0.0016	0.0000
Normality test	31.82***	30.10****	30.67***	26.03***
N	178	178	178	178

Output estimation Model 3 (Free Inflation)				
	DFM	AR(4)	MA(2)	ARIMA(2,1,1)
F1	-.1180782 [-3.60]***	-	-	-
F5	-.3438172 [-7.78]***	-	-	-
F6	.1709512 [3.71]***	-	-	-
F8	.2290492 [3.04]***	-	-	-
F9	-.2476073 [-4.46]***	-	-	-
Ar(1)	-	.4845917 [-4.46]***	-	1.180456 [13.91]***
AR(2)	-	-.2244253 [-4.46]***	-	-.1816939 [-2.18]
AR(3)	-	.4387682 [-4.46]***	-	-
AR(4)	-	-.2671732 [-4.46]***	-	-
MA(1)	-	-	.5411173 [7.51]***	-.9462663 [-25.15]***
MA(2)	-	-	-.1269618 [-1.91]**	-
C	-.351957 [-2.15]**	.4132754 [-4.46]***	.4105896 [8.77]***	-
R-squared	0.75	0.60	0.53	0.74
Prob F	0.0000	0.0000	0.0000	0.0000
Normality test	9.56***	18.25***	19.35***	22.72***
N	178	178	178	178

*Indicates significance at the 10% level.

**Indicates significance at the 5% level.

***Indicates significance at the 1% level.