Prices in Networks

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Abstract

When there is strategic complementarity of consumption between neighbors in a social network, we find that certain consumers may have a bigger impact than other consumers on the market demand and therefore the equilibrium price. The influence that a particular consumer has on the market demand depends on the network structure and the consumer's location in the network. This analysis may, for example, shed light on the segment of consumers that should be the target of selective advertisements or promotions.

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1 Introduction

The recent decades have been marked by an increasingly interconnected world due largely to advances in communication and transportation technology. The ubiquitous cellular phone, the television, and the Internet have become almost indispensable to present day living. They have, to an unprecedented extent, enabled interaction among people who are physically separated. Cars and air travel have become much more accessible forms of

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transportation, while mass transit systems have become common features of urban living throughout the world. Political and economic developments in the European Union, Eastern Europe, and China have resulted in an explosion of emigration and travel. As a result of all these technological and social developments, people are becoming increasingly interconnected both within and between countries.

The consumption of fashion products, ostentatious products, gifts, game tickets, guns and other forms of expenditures tend to be influenced by social or cultural norms in a way that displays a high degree of conformism. For example, our decisions to purchase a vacation package may be influence by whether our friends are also going on the same trip, perhaps because the trip would be more enjoyable with some friends. A household might keep up with the purchases of only those other households it comes into social contact with and not the purchases of the rest of the other households. However, because of the overlap of social circles, households that are distantly connected through the network of neighbors can have an indirect, though somewhat diminished, effect on one another's purchases. It is useful to think of this interaction among consumers occurring within a social network since consumers would typically respond to only those other consumers in their social circle rather than to respond to all other consumers. A network structure allows much richer social interactions, rather than distinctly local or global interactions.

This paper investigates how equilibrium prices are affected by consumers interacting strategically with their neighbors in a social network. In order to abstract from issues of market power and to focus on consumer behavior, we consider an exchange economy comprising many agents endowed with two goods. One of the two goods involves strategic complementarity in consumption between neighbors in the social network. However because no agent has market power over any good, the markets for both goods are competitive. We use the vocabulary of graph theory and network games to describe the strategic interactions occurring within the social network. The intention is neither to explain why strategic complementarity in consumption occurs nor to explain the existence of a social network. Instead, we would like to focus on the consequences of such phenomena, that is, to determine how the network structure affects the market demand and the equilibrium price.

To examine the impact of a particular agent's increased demand for a good, we analyze the response of price to a change in the endowment of the agent under various network structures. We find that the increase in supply of a good may in fact raise its relative price. This effect depends on the structure of the network and the source of the supply increase. This result suggests that goods that involve strategic complementarity in consumption between neighbors may become more valuable even as they become more abundant.

The idea that a consumer's demand depends on the demands of other consumers has been explored variedly in the literature. For example, a consumer's demand for a good may depend on the aggregate demand or network externalities, that is, the number of consumers consuming [Duesenberry (1949), Leibenstein (1950), Becker (1991), Karni and Levin (1994), Corneo and Jeanne (1997), Grilo et al. (2001), Amaldoss and Jain (2005)]. Alternatively, a consumer's demand may be affected by the demands of other consumers because his utility depends on how his consumption of the good ranks against that of all other consumers [Frank (1985), Hopkins and Kornienko (2004), Hopkins and Kornienko (2006)]. There are also models that incorporate both local and global interactions but treat the effects distinctly [Glaeser and Scheinkman (2002), Horst and Scheinkman (2005)]. However, these forms of social interactions do not take into consideration how consumers not directly connected can be influenced indirectly and mutually by other consumers via a network of social relations. There is a growing literature on network formation and network games, which show how the structure of networks affects equilibrium outcomes [Galeotti et al. (2006)]. Discrete choice interactions have been analyzed in a network structure but not with reference to the price mechanism [Ioannides (2006)].

The rest of this paper is organized as follows. Section 2 presents the general model, which includes a discussion of a few prominent network structures. Section 3 examines the equilibrium in the minimum consumption model under the various network structures. Section 4 concludes. The appendix contains the proofs.

2 The General Model

Given a set of agents $N = \{1, \ldots, n\}$, an undirected ***network*** g is a set of pairs of agents linked to each other. For any pair of agents i and j, $ij \in g$ indicates that i and j are linked in the network g.

A pair of agents are ***neighbors*** in a network g if and only if they are linked in the network g. The set of agents with at least one neighbor in the network g is $N(g) = \{j \in N : \exists \quad ij \in g\}$. The set of neighbors of agent i in the network g is $N_i(g) = \{j \in N : ij \in g\}$. The ***degree*** of agent i is $n_i(g) = |N_i(g)|$, the number of neighbors that agent i has in network g. Assume that every agent has at least one neighbor in the network g so that $N(g) = N$ and $N_i(g) \neq \{\phi\}$.

A ***path*** in the network g connecting agents i and j is a sequence of distinct neighbors $i_1, \ldots, i_K$ such that $i_k i_{k+1} \in g$ for each $k \in \{1, \ldots, K-1\}$ with $i_1 = i$ and $i_K = j$. The ***length*** of a path connecting agents i and j is the number of links connecting agents i and j on that path.

A network is ***connected*** if there exists a path connecting any agent to any other agent in the network. A network $g' \subset g$ is a ***component*** of network g if it is a maximal connected subnetwork of network g. That is,
(a) if $i \in N(g')$, $j \in N(g')$, and $j \neq i$, then there exists a path in g' connecting i and j, and
(b) if $i \in N(g')$, $j \in N(g)$, $j \neq i$, and $ij \in g$, then $ij \in g'$.
The set of components of network g is $C(g)$, so that $g = \bigcup_{g' \in C(g)} g'$. Since neighbors are in the same component of a network, the set of neighbors of agent i in the component g' of the network g is equivalent to the set of neighbors of agent i in the network g. That is, $N_i(g') = N_i(g)$.

The ***distance*** between any pair of agents i and j in the same component g' is $d_{ij}(g')$, the length of the shortest path between the pair of agents. For any integer $k \geq 1$, the set of all other agents that are connected to agent i by a distance of k is $N_i^k(g') = \{j \in N(g') : j \neq i, d_{ij}(g') = k\}$. Hence, $N_i^1(g') = N_i(g')$. The cardinality of $N_i^k(g)$ is $n_i^k(g) = |N_i^k(g)|$. The ***eccentricity*** of agent i is $\epsilon_i(g') = \max_{j \in N(g')} d_{ij}(g')$, the maximum distance between agent i and any other agent in the same component g'.

The ***radius*** of a component is $\underline{d}(g') = \min_{ij \in g'} d_{ij}(g')$, the minimum eccentricity of any agent in the component. The ***diameter*** of a component is $\bar{d}(g') = \max_{ij \in g'} d_{ij}(g')$, the maximum eccentricity of any agent in the component. The ***closeness*** of an agent i in the component is $c_i(g') = \frac{1}{\sum_{j \in N(g') \setminus \{i\}} d_{ij}(g')}$, the reciprocal of the sum of distances to all other agents in the component. An agent is ***central*** in a component if its eccentricity is equal to the radius of the component. The ***center*** of a component is the set of all central agents. An agent is ***peripheral*** in a component if its eccentricity is equal to the diameter of the component.

There are two goods - 1 and 2. x_ℓ^i denotes agent i's consumption of good ℓ. ω_ℓ^i denotes agent i's endowment of good ℓ. Both goods are traded throughout the economy so no agent has market power over any good. The price of good 1, which is the numeraire, is normalized to one and so the price of good 2, p, is also the price of good 2 relative to good 1. The wealth level of agent i, $m^i = \omega_1^i + p\omega_2^i$, is endogenously determined by the equilibrium price and the pattern of endowments.

Agents have identical, continuous, strictly convex, and strongly monotone preferences over goods 1 and 2. Hence, each agent i's preferences can be represented by a strictly quasiconcave and twice continuously differentiable utility function $u^i(x_1^i, x_2^i, x_2^{N_i(g)})$, where $x_2^{N_i(g)}$ is the vector of good 2 consumptions by each of the agents in the set of agents $N_i(g)$. There is no restriction on whether the good 2 consumption of each neighbor is a positive or negative externality, that is, there is no restriction on the sign of $u^i_{x_2^j}$ per se, where $j \in N_i(g)$. For any given level of good 2 consumption by each of its neighbors, each agent chooses its consumption of goods 1 and 2 to maximize its utility subject to its budget constraint. Formally,

$$\forall \quad i \in N : \quad \max_{\{x_1^i, x_2^i\}} u^i(x_1^i, x_2^i, x_2^{N_i(g)}) \quad \text{s.t.} \qquad x_1^i + px_2^i = m^i = \omega_1^i + p\omega_2^i$$

There is strategic complementarity in the consumption of good 2 between each pair of neighbors in that an agent would increase its consumption of good 2 if its neighbor does so, holding all other factors, including the price, constant. Formally,

$$\forall \quad i \in N, \quad j \in N_i(g) : \quad (\frac{\partial x_2^i}{\partial x_2^j})_{x_2, p, m^i} > 0$$

Since we seek to determine the effect of a particular agent's increased demand from a change in the agent's endowment, we assume that both goods are normal. Formally,

$$\forall \quad i \in N: \quad (\frac{\partial x_\ell^i}{\partial m^i})_{x_2,p} > 0$$

The above two conditions depend on ordinal properties of the utility function because the best response function, which is implicitly determined by the first order conditions, is invariant to a monotonic transformation of the utility function. The best response correspondences are in fact best response functions because the utility functions are strictly quasiconcave.

Definition 1 (Nash-Walrasian Equilibrium). *(x_1^N, x_2^N, p) is a **Nash-Walrasian Equilibrium** if it satisfies every agent's best response function and budget constraint, and the market for good 2 clears. That is,*

$$\begin{cases} \forall \quad i \in N: \quad u^i_{x_2^i} = p u^i_{x_1^i} \quad \textit{which implicitly determines} \quad x_2^i = x_2^i(x_2^{N_i(g)}, p, m^i) \\ \forall \quad i \in N: \quad x_1^i + p x_2^i = m^i = \omega_1^i + p\omega_2^i \\ \sum_{i \in N} x_2^i = \sum_{i \in N} \omega_2^i \end{cases}$$

Lemma 1 (Demand Correspondences). *Given every agent's best response function and budget constraint, each agent's demand for each good is a correspondence of the relative price of the two goods and the wealth levels of every agent in its component of the network. That is,*

$$\forall \quad g' \in C(g), \quad i \in N(g'): \quad x_1^i \in x_1^i(p, m^{N(g')}) \textit{ and } x_2^i \in x_2^i(p, m^{N(g')})$$

As long as there is a path connecting a pair of agents, their demands and therefore their incomes would affect each other's demand. Whether the demand correspondences are in fact demand functions depends on the ordinal curvature properties of the best response functions and their upper and lower bounds [Randon (2004)].

Without loss of generality, we can examine the effect of an increase in the endowment of good 2 held by an agent by a perturbation of ω_2^1, the endowment of good 2 held by agent 1.

Proposition 1 (Equilibrium Price)**.** *Given a downward-sloping aggregate demand for good* 2*, its price is increasing in the endowment of an agent's endowment of the good if any resulting increase in aggregate demand exceeds the increase in endowment.*

This proposition is a generalization of that which would emerge if preferences were independent. It holds for all network structures.

Definition 2 (Influence)**.** *Agent* i*'s* ***influence*** *on aggregate demand is equal to* $\frac{\partial p}{\partial \omega_2^i}$*, the marginal effect of an increase in agent* i*'s endowment of good* 2 *on the price of good* 2*.*

Influence is a measure of the centrality of an agent in the network. We would like to determine which agents in the economy have a higher influence on aggregate demand and how the network structure affects their influence.

Network Structures: Network structures can be categorized into those which are regular and those which are irregular. Since there are very many possible regular and irregular networks, we focus our attention on a few prominent network structures. The aim is to show how aggregate demand and prices are affected by whether the network is regular or not, and by the particular network structure in question. Within the class of regular networks, we consider the complete network and the ring network. Within the class of irregular networks, we consider the star network and the line network.

A network is ***regular*** if all agents have the same number of neighbors. That is,

$$\forall \quad i \in N : \quad n_i(g) = r \quad \text{where } r \text{ is a non-negative integer}$$

A network is ***complete*** if all agents are linked to one another. Hence, for any agent, every other agent is a neighbor of the agent. This network structure in effect describes the case of global interactions. Formally,

$$g = \{ij : i \in N, j \in N, j \neq i\}$$

$$\Rightarrow \begin{cases} \forall \quad i \in N: & \epsilon_i(g) = 1 \\ \forall \quad i \in N: & N_i(g) = \{j \in N : j \neq i\} \\ \forall \quad i \in N: & n_i(g) = n - 1 \end{cases}$$

A network is a ***ring*** if there is a single cycle through all agents. Hence every agent has a pair of neighbors. Without loss of generality, assume that agent 1 is linked to agent 2, which is, in turn, linked to agent 3, and so on until agent n. In addition, agent n is linked to agent 1, thereby completing the single cycle. Formally,

$$g = \{ij : i \in N, j \in N, j = i \pm 1\} \bigcup \{1n\}$$

$$\forall \quad i \in N: \quad \epsilon_i(g) = \begin{cases} \frac{n}{2} & \text{if } n \text{ is even} \\ \frac{n-1}{2} & \text{if } n \text{ is odd} \end{cases}$$

$\forall \quad k \leq \frac{n-1}{2}:$

$$N_i^k(g) = \begin{cases} \{j \in N : j = i \pm k\} \bigcup \{i + (n-k)\} & \text{if} \quad i = 1, \ldots, k \\ \{j \in N : j = i \pm k\} & \text{if} \quad i = k+1, \ldots, n-k \\ \{j \in N : j = i \pm k\} \bigcup \{i - (n-k)\} & \text{if} \quad i = n-k+1, \ldots, n \end{cases}$$

$\forall \quad k = \frac{n}{2}:$

$$N_i^k(g) = \begin{cases} \{j \in N : j = i \pm k\} \bigcup \{i + (n-k)\} & \text{if} \quad i = 1, \ldots, k \\ \{j \in N : j = i \pm k\} \bigcup \{i - (n-k)\} & \text{if} \quad i = n-k+1, \ldots, n \end{cases}$$

$$\forall \quad i \in N: \quad n_i(g) = 2$$

Turning to the class of irregular networks, we first consider the star network, which is a maximally centralized network. A network is a ***star*** if it has one central agent and all other agents are linked only to the central agent. Without loss of generality, assume that the central agent is agent 1. Formally,

$$g = \{ij : i = 1, j \in N \backslash \{1\}\}$$

$$\forall \quad i \in N: \quad \epsilon_i(g) = \begin{cases} 1 & \text{if} \quad i = 1 \\ 2 & \text{if} \quad i \in N \backslash \{1\} \end{cases}$$

$$N_i(g) = \begin{cases} N \backslash \{1\} & \text{if} \quad i = 1 \\ \{1\} & \text{if} \quad i \in N \backslash \{1\} \end{cases}$$

$$N_i^2(g) = \begin{cases} \{\phi\} & \text{if} \quad i = 1 \\ N\backslash\{1, i\} & \text{if} \quad i \in N\backslash\{1\} \end{cases}$$

$$n_i(g) = \begin{cases} n-1 & \text{if} \quad i = 1 \\ 1 & \text{if} \quad i \in N\backslash\{1\} \end{cases}$$

A network is a ***line*** if all agents form a single acyclic path. Without loss of generality, assume that the path connects agent 1 to agent n through all other agents. It would identical to the ring but for the absence of a link between agent 1 and agent n. Formally,

$$g = \{ij : i \in N, j \in N, j = i \pm 1\}$$

$$\forall \quad i \in N : \quad \epsilon_i(g) = \max\{n - i, i - 1\}$$

$$\forall \quad i \in N, \quad \forall \quad k \geq 1 : \quad N_i^k(g) = \{j \in N : j = i \pm k\}$$

$$n_i(g) = \begin{cases} 1 & \text{if} \quad i \in \{1, n\} \\ 2 & \text{if} \quad i \in N\backslash\{1, n\} \end{cases}$$

3 The Minimum Consumption Model

In order to impose more structure on each agent's best response function, we consider a specific model that introduces two sets of assumptions. First, assume that agents have identical Cobb-Douglas preferences symmetric in both goods. This assumption is consistent with the requirement that both goods are normal. Second, assume that every agent needs to consume an amount of good 2 that exceeds the fraction $\alpha \in [0, 1)$ of the average good 2 consumption of its neighbors. This is similar to the Stone-Geary utility [Stone (1954), Geary (1950-1951)], where consumption needs to exceed a certain parameterized minimum level, except that the minimum level of consumption here is not parameterized but is endogenously determined. Formally,

$$\forall \quad i \in N : \quad \max_{\{x_1^i, x_2^i\}} x_1^i (x_2^i - \alpha \frac{1}{n_i(g)} \sum_{j \in N_i(g)} x_2^j) \quad \text{s.t.} \quad x_1^i + p x_2^i = m^i = \omega_1^i + p\omega_2^i$$

As a result, the best response function of each agent is linear in the good 2 consumption of each of its neighbors:

$$\forall \quad i \in N : \quad x_2^i = \frac{1}{2p}(m^i + \alpha p \frac{1}{n_i(g)} \sum_{j \in N_i(g)} x_2^j)$$

$$\Rightarrow \forall \quad i \in N, \quad j \in N_i(g): \quad (\frac{\partial x_2^i}{\partial m^i})_{x_2,p} = \frac{1}{2p}, \quad (\frac{\partial x_2^i}{\partial x_2^j})_{x_2,p,m^i} = \frac{\alpha}{2n_i(g)}$$

This further implies that, holding the price and the demands of other agents constant, an agent would increase its consumption of good 2 by half the increase of its endowment of the good. Since the agent's increase in demand is independent of its own wealth and the consumption levels of the agent's neighbors, this ensures that, before taking into consideration the reaction of other agents, every agent responds symmetrically to an increase in its own endowment. Another implication is that, holding all other factors constant, the response of an agent to the good 2 consumption of one of its neighbors is decreasing in the number of neighbors that the agent has. This also simplifies the situation because the response is hence independent of the price, the agent's wealth, and the consumption levels of the agent's neighbors.

Regular Networks:

Proposition 2 (Regular Networks)**.** *In the minimum consumption model with a regular network, the price of good* 2 *is decreasing in the endowment of good* 2 *held by any agent and independent of the number of neighbors that every agent has.*

In a regular network, since every agent has the same number of neighbors, an agent i's response to the an increased demand by any neighbor $j \in N_i(g)$, $(\frac{\partial x_2^i}{\partial x_2^j})_{x_2,p,m^i}$, is the same for every agent. Hence, the response of aggregate demand to an increase in the endowment of good 2 is independent of the number of agents in the economy because the multiplier $\frac{1}{1-\sum_{j \in N_i(g)}(\frac{\partial x_2^i}{\partial x_2^j})_{x_2,p,m^i}}$ is independent of the number of neighbors that every agent has. The greater the number of neighbors, the greater the number of neighbors responding to the increase in demand for good 2 by agent i. However, at the same time, every agent responds proportionately less to the increase in demand because every agent has more neighbors. The two opposing effects on the multiplier exactly counteract each other.

Corollary 1 (Regular Networks)**.** *In the minimum consumption model with a regular network, every agent has the same level of influence on aggregate* *demand.*

This follows from Proposition 2.

Since no agent has a higher influence than any other agent in a regular network, this suggests that no segment of consumers would be favored over any other in being the target of advertisements or promotions.

Complete Networks: Since complete networks are regular networks, according to Proposition 2, the price of good 2 is decreasing in the endowment of good 2.

From the best response function of every agent, we have:

$$Ax = b$$

where $A = (a_{ij}) \in \mathbb{R}^{n\times n}$ with $a_{ij} = \begin{cases} 2 & \text{if} \quad i = j \\ -\frac{\alpha}{n-1} & \text{if} \quad i \neq j \end{cases}$

$$x = \begin{bmatrix} x_2^1 \\ \vdots \\ x_2^n \end{bmatrix}, \quad b = \begin{bmatrix} \frac{m^1}{p} \\ \vdots \\ \frac{m^n}{p} \end{bmatrix}$$

Solving for x, we obtain the demand function for each agent:

$$\forall \quad i \in N : \quad x_2^i = \frac{1}{p}[\frac{2(n-1)-(n-2)\alpha}{4(n-1)-2(n-2)\alpha-\alpha^2}m^i + \frac{\alpha}{4(n-1)-2(n-2)\alpha-\alpha^2}\sum_{j\in N_i(g)} m^j]$$

In the absence of the strategic complementarity in consumption of good 2 between each pair of neighbors, that is, if $\alpha = 0$, then each agent's demand for each good depends only on its own income. However, with the strategic complementarity between neighbors, each agent's consumption of each good depends not just on its own income but also on the income of all other agents.

From the demand functions, we have:

$$\sum_{i\in N}(\frac{\partial x_2^i}{\partial \omega_2^1})_p = \frac{1}{2-\alpha} < 1$$

Hence, the increase in aggregate demand for good 2 is less than the increase in endowment of the good, and the extent of which is independent of the number of agents.

Ring Networks: Since ring networks are regular networks, according to Proposition 2, the price of good 2 is decreasing in the endowment of good 2.

From the best response function of every agent, we have:

$$Ax = b$$

$$\text{where } A = \begin{bmatrix} 2 & -\frac{\alpha}{2} & 0 & \dots & 0 & -\frac{\alpha}{2} \\ -\frac{\alpha}{2} & 2 & -\frac{\alpha}{2} & 0 & \dots & 0 \\ 0 & -\frac{\alpha}{2} & 2 & -\frac{\alpha}{2} & \ddots & \vdots \\ \vdots & 0 & -\frac{\alpha}{2} & 2 & \ddots & 0 \\ 0 & \vdots & \ddots & \ddots & \ddots & -\frac{\alpha}{2} \\ -\frac{\alpha}{2} & 0 & \dots & 0 & -\frac{\alpha}{2} & 2 \end{bmatrix} \in \mathbb{R}^{n\times n}, \text{ a circulant matrix,}$$

$$x = \begin{bmatrix} x_2^1 \\ \vdots \\ x_2^n \end{bmatrix}, \quad b = \begin{bmatrix} \frac{m^1}{p} \\ \vdots \\ \frac{m^n}{p} \end{bmatrix}$$

Solving for x, we obtain the demand function for each agent:

$$\forall \quad i \in N: \quad x_2^i = \frac{1}{p}[\frac{C_{1,1}}{|A|}m^i + \sum_{k=1}^{\epsilon_i(g)} \frac{C_{k+1,1}}{|A|} \sum_{j\in N_i^k(g)} m^j]$$

where the cofactor $C_{i,j}$ of matrix A is $(-1)^{i+j}$ times the determinant of the matrix formed by removing row i and column j of matrix A.

An agent's sensitivity of demand to another agent's income depends on the distance between the pair of agents. The greater the distance between the pair of agents, the less sensitive an agent's demand is to the other agent.

Star Networks: Turning to irregular networks, we begin with star networks. From the best response function of every agent, we have:

$$Ax = b$$

where $A = \begin{bmatrix} 2 & -\frac{\alpha}{n-1} & \dots & \dots & -\frac{\alpha}{n-1} \\ -\alpha & 2 & 0 & \dots & 0 \\ \vdots & 0 & 2 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ -\alpha & 0 & \dots & 0 & 2 \end{bmatrix} \in \mathbb{R}^{n \times n}$,

$$x = \begin{bmatrix} x_2^1 \\ \vdots \\ x_2^n \end{bmatrix}, \quad b = \begin{bmatrix} \frac{m^1}{p} \\ \vdots \\ \frac{m^n}{p} \end{bmatrix}$$

Solving for x, we obtain the demand function for each agent:

$$x_2^i = \begin{cases} \frac{1}{p}\left(\frac{2}{4-\alpha^2}m^i + \frac{\alpha}{4-\alpha^2}\frac{1}{n-1}\sum_{j \in N \setminus \{1\}} m^j\right) & \text{if} \quad i = 1 \\ \frac{1}{p}\left(\frac{4(n-1)-(n-2)\alpha^2}{4-\alpha^2}\frac{1}{2(n-1)}m^i + \frac{\alpha}{4-\alpha^2}m^1 + \frac{\alpha^2}{4-\alpha^2}\frac{1}{2(n-1)}\sum_{k \in N_i^2(g)} m^k\right) & \text{if} \quad i \in N \setminus \{1\} \end{cases}$$

A peripheral agent's demand for good 2 is more sensitive to the central agent's income than the income of other peripheral agents because the central agent is a neighbor whereas the other peripheral agents are connected by a distance of two.

Proposition 3 (Star Network)**.** *In the minimum consumption model with a star network, the price of good* 2 *is increasing in the endowment of good* 2 *held by the central agent if the fraction* α *is large enough.*

The central agent has all other agents as neighbors and so has all other agents responding to its increase in demand for good 2. In addition, these other agents respond sensitively to the central agent's increased demand for good 2 because the central agent is their only neighbor. As the number of peripheral agents approaches infinity, the number of neighbors the central agent has approaches infinity and so the critical value of α decreases and approaches zero.

In contrast, the price of good 2 is decreasing in the endowment of good 2 held by a peripheral agent. This is because a peripheral agent has only one agent, the central agent, responding to its increase in demand. In addition, the central agent does not respond sensitively to the peripheral agent's demand increase because it has many other neighbors as well. As the number of peripheral agents increases, the number of neighbors for each peripheral agent remains constant but the number of agents connected to each peripheral agent by a distance of two increases.

Corollary 2 (Star Network)**.** *In the minimum consumption model with a star network, the central agent has a higher influence than all other agents.*

This follows from Proposition 3.

Since the central agent also has the highest closeness, the analysis of the minimum consumption model with a star network seems to suggest that an agent's closeness is the key to its relative influence over market demand in the network. However, we shall see, in the analysis of line networks, that closeness does not in fact determine an agent's influence.

Line Networks: From the best response function of every agent, we have:

$$Ax = b$$

where $A = \begin{bmatrix} 2 & -\alpha & 0 & \dots & \dots & 0 \\ -\frac{\alpha}{2} & 2 & -\frac{\alpha}{2} & \ddots & & \vdots \\ 0 & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ \vdots & & \ddots & -\frac{\alpha}{2} & 2 & -\frac{\alpha}{2} \\ 0 & \dots & \dots & 0 & -\alpha & 2 \end{bmatrix} \in \mathbb{R}^{n\times n}$, a tridiagonal matrix,

$$x = \begin{bmatrix} x_2^1 \\ \vdots \\ x_2^n \end{bmatrix}, \quad b = \begin{bmatrix} \frac{m^1}{p} \\ \vdots \\ \frac{m^n}{p} \end{bmatrix}$$

Proposition 4 (Line Network)**.** *In the minimum consumption model with a line network, the price of good* 2 *is increasing in the endowment of good* 2 *held by a non-peripheral agent if the fraction* α *is large enough.*

The critical value of α is increasing in the number of agents because every non-peripheral agent's influence is correspondingly reduced. This is because an additional agent linked to a peripheral agent decreases the hitherto peripheral agent's response to its hitherto only neighbor. This reduced response is translated across the line network to other agents.

Corollary 3 (Line Network)**.** *In the minimum consumption model with a line network, the agents with the k^{th} highest influence are the non-peripheral agents connected to the nearest peripheral agent by a distance of k.*

The critical value of α is increasing in an agent's distance to the nearest peripheral agent but is asymptotic to a value less than one.

Proposition 5 (Influence)**.** *In the minimum consumption model, the influence of an agent on aggregate demand is increasing in the agent's degree but decreasing in the degree of any other agent connected to it.*

For each non-peripheral agent, the critical value of α is increasing in the number of agents linked to a peripheral agent. Equivalently, the influence of each non-peripheral agent is decreasing in the number of agents linked to a peripheral agent. An agent with a higher degree has more other agents responding to the agent's increase in demand. This response is greater if the agents responding have a lower degree. For example, in the six-agent line network, the central agents (3 and 4) have the same degree and in fact a higher measure of closeness than agents 2 and 5. However, agents 2 and 5 each have a neighbor that has only one link and so the neighbor provides a greater response to each of agents 2 and 5. This is because, in the minimum consumption example, an agent responds to the average of its neighbors. Hence, the importance of an agent on market demand does not depend on its centrality per se.

This analysis suggests that sellers of a product should want to focus their advertising and promotional efforts on those consumers who have a relatively high influence on the consumption of others, specifically those with many neighbors who in turn have few neighbors themselves. A highly centralized network, such as a star, offers sellers the opportunity to focus their promotional attempts at the central agent, which has a large impact on market demand.

4 Conclusion

The analysis in this paper shows that where consumers tend to conform with the average purchases of their neighbors in a social network, unless the network structure is fairly regular, certain consumers, by virtue of their location in the network, would tend to have a greater influence on aggregate demand than other consumers. Contrary to what one might expect, it is not an agent's closeness or degree per se that matters for its relative influence on market demand. Instead, the number of neighbors of an agent's neighbors also matters for the agent's influence. Since promotional efforts are costly, producers would be better off focusing their efforts on those consumers who have a relatively large influence on the purchase of others.

We have considered only connected networks in the analysis. Within the model in this paper, we can also analyze how prices are affected when networks are disconnected, comprising a number of components.

We have considered on a general equilibrium model to focus on the effect of consumer behavior on prices. We can also incorporate firm behavior by considering a partial equilibrium model. A variety of industry structures can be considered in this context, including strategic interaction among firms.

It may also be possible to consider directed networks. Directed networks may be especially relevant when considering the impact of celebrities and other prominent figures whose consumption patterns are observed by many in the general public, but who do not in turn observe the consumption patterns of those who observe their consumption patterns.

Appendix

Proof of Lemma 1 (Demand Correspondences):

$$\forall \quad g' \in C(g), \quad i \in N(g'): \quad N_i(g) = N_i(g') \quad \text{and} \quad x_2^i = x_2^i(x_2^{N_i(g)}, p, m^i)$$
$$\Rightarrow x_2^i = x_2^i(x_2^{N_i(g')}, p, m^i)$$
$$\Rightarrow x_2^i \in x_2^i(x_2^{N_i^2(g')}, p, m^{\{i\} \bigcup N_i(g')})$$
$$\Rightarrow x_2^i \in x_2^i(x_2^{N_i^3(g')}, p, m^{\{i\} \bigcup N_i(g') \bigcup N_i^2(g')})$$
$$\vdots$$
$$\Rightarrow x_2^i \in x_2^i(x_2^{N_i^{\epsilon_i}(g')}, p, m^{\{i\} \bigcup N_i(g') \bigcup N_i^2(g') \bigcup \ldots \bigcup N_i^{\epsilon_i - 1}(g')})$$
$$\Rightarrow x_2^i \in x_2^i(p, m^{\{i\} \bigcup N_i(g') \bigcup N_i^2(g') \bigcup \ldots \bigcup N_i^{\epsilon_i}(g')})$$
$$\Rightarrow x_2^i \in x_2^i(p, m^{\{i\} \bigcup (\bigcup_{k=1}^{\epsilon_i(g')} N_i^k(g'))})$$
$$\Rightarrow x_2^i \in x_2^i(p, m^{N(g')})$$
$$\Rightarrow x_1^i \in x_1^i(p, m^{N(g')})$$

□

Proof of Proposition 1 (Equilibrium Price):

$$\Rightarrow \begin{cases} u^i_{x_2^i x_1^i} dx_1^i + u^i_{x_2^i x_2^i} dx_2^i + \sum_{j \in N_i(g)} u^i_{x_2^i x_2^j} dx_2^j = p(u^i_{x_1^i x_1^i} dx_1^i + u^i_{x_1^i x_2^i} dx_2^i + \sum_{j \in N_i(g)} u^i_{x_1^i x_2^j} dx_2^j) + u^i_{x_1^i} dp \\ dx_1^1 + p dx_2^1 + x_2^1 dp = dm^1 = p d\omega_2^1 + \omega_2^1 dp \\ \forall \quad i \backslash 1: \quad dx_1^i + p dx_2^i + x_2^i dp = dm^i = \omega_2^i dp \\ \sum_{i \in N} dx_2^i = d\omega_2^1 \end{cases}$$

$$\Rightarrow \begin{cases} (u^i_{x_2^i x_1^i} - p u^i_{x_1^i x_1^i}) dx_1^i + (u^i_{x_2^i x_2^i} - p u^i_{x_1^i x_2^i}) dx_2^i + \sum_{j \in N_i(g)} (u^i_{x_2^i x_2^j} - p u^i_{x_1^i x_2^j}) dx_2^j - u^i_{x_1^i} dp = 0 \\ dx_1^1 = p d\omega_2^1 + (\omega_2^1 - x_2^1) dp - p dx_2^1 \\ \forall \quad i \backslash 1: \quad dx_1^i = (\omega_2^i - x_2^i) dp - p dx_2^i \\ \sum_{i \in N} dx_2^i = d\omega_2^1 \end{cases}$$

$$\Rightarrow \begin{cases} [u^1_{x^1_2x^1_2} - pu^1_{x^1_1x^1_2} - p(u^1_{x^1_2x^1_1} - pu^1_{x^1_1x^1_1})]dx^1_2 + \sum\limits_{j\in N_1(g)} (u^1_{x^1_2x^j_2} - pu^1_{x^1_1x^j_2})dx^j_2 \\ +[(\omega^1_2 - x^1_2)(u^1_{x^1_2x^1_1} - pu^1_{x^1_1x^1_1}) - u^1_{x^1_1}]dp = -p(u^1_{x^1_2x^1_1} - pu^1_{x^1_1x^1_1})d\omega^1_2 \\ \\ \forall \quad i\backslash 1: \quad [u^i_{x^i_2x^i_2} - pu^i_{x^i_1x^i_2} - p(u^i_{x^i_2x^i_1} - pu^i_{x^i_1x^i_1})]dx^i_2 + \sum\limits_{j\in N_i(g)} (u^i_{x^i_2x^j_2} - pu^i_{x^i_1x^j_2})dx^j_2 \\ +[(\omega^i_2 - x^i_2)(u^i_{x^i_2x^i_1} - pu^i_{x^i_1x^i_1}) - u^i_{x^i_1}]dp = 0 \\ \\ \sum\limits_{i\in N} dx^i_2 = d\omega^1_2 \end{cases}$$

$$\Rightarrow \begin{cases} \frac{dx^1_2}{d\omega^1_2} + \sum\limits_{j\in N_1(g)} \frac{u^1_{x^1_2x^j_2} - pu^1_{x^1_1x^j_2}}{u^1_{x^1_2x^1_2} - pu^1_{x^1_1x^1_2} - p(u^1_{x^1_2x^1_1} - pu^1_{x^1_1x^1_1})} \frac{dx^j_2}{d\omega^1_2} \\ + \frac{(\omega^1_2 - x^1_2)(u^1_{x^1_2x^1_1} - pu^1_{x^1_1x^1_1}) - u^1_{x^1_1}}{u^1_{x^1_2x^1_2} - pu^1_{x^1_1x^1_2} - p(u^1_{x^1_2x^1_1} - pu^1_{x^1_1x^1_1})} \frac{dp}{d\omega^1_2} = -p \frac{u^1_{x^1_2x^1_1} - pu^1_{x^1_1x^1_1}}{u^1_{x^1_2x^1_2} - pu^1_{x^1_1x^1_2} - p(u^1_{x^1_2x^1_1} - pu^1_{x^1_1x^1_1})} \\ \\ \forall \quad i\backslash 1: \quad \frac{dx^i_2}{d\omega^1_2} + \sum\limits_{j\in N_i(g)} \frac{u^i_{x^i_2x^j_2} - pu^i_{x^i_1x^j_2}}{u^i_{x^i_2x^i_2} - pu^i_{x^i_1x^i_2} - p(u^i_{x^i_2x^i_1} - pu^i_{x^i_1x^i_1})} \frac{dx^j_2}{d\omega^1_2} + \frac{(\omega^i_2 - x^i_2)(u^i_{x^i_2x^i_1} - pu^i_{x^i_1x^i_1}) - u^i_{x^i_1}}{u^i_{x^i_2x^i_2} - pu^i_{x^i_1x^i_2} - p(u^i_{x^i_2x^i_1} - pu^i_{x^i_1x^i_1})} \frac{dp}{d\omega^1_2} = 0 \\ \\ \sum\limits_{i\in N} \frac{dx^i_2}{d\omega^1_2} = 1 \end{cases}$$

$$\Rightarrow \begin{cases} \frac{dx^1_2}{d\omega^1_2} - \sum\limits_{j\in N_1(g)} (\frac{\partial x^1_2}{\partial x^j_2})_{x_2,p,m^1} \frac{dx^j_2}{d\omega^1_2} - (\frac{\partial x^1_2}{\partial p})_{x_2} \frac{dp}{d\omega^1_2} = p(\frac{\partial x^1_2}{\partial m^1})_{x_2,p} \\ \\ \forall \quad i\backslash 1: \quad \frac{dx^i_2}{d\omega^1_2} - \sum\limits_{j\in N_i(g)} (\frac{\partial x^i_2}{\partial x^j_2})_{x_2,p,m^i} \frac{dx^j_2}{d\omega^1_2} - (\frac{\partial x^i_2}{\partial p})_{x_2} \frac{dp}{d\omega^1_2} = 0 \\ \\ \sum\limits_{i\in N} \frac{dx^i_2}{d\omega^1_2} = 1 \end{cases}$$

where $\quad (\dfrac{\partial x^i_2}{\partial p})_{x_2} = (\dfrac{\partial x^i_2}{\partial p})_{x_2,u^i} + (\dfrac{\partial x^i_2}{\partial m^i})_{x_2,p}(\omega^i_2 - x^i_2) = (\dfrac{\partial x^i_2}{\partial p})_{x_2,m^i} + (\dfrac{\partial x^i_2}{\partial m^i})_{x_2,p}\omega^i_2$

which decomposes the substitution and income effects

$$\Rightarrow Ax = b$$

$$\text{where} \quad A = \begin{bmatrix} 1 & -(\frac{\partial x_2^1}{\partial x_2^2})_{x_2,p,m^1} & \cdots & -(\frac{\partial x_2^1}{\partial x_2^n})_{x_2,p,m^1} & -(\frac{\partial x_2^1}{\partial p})_{x_2} \\ -(\frac{\partial x_2^2}{\partial x_2^1})_{x_2,p,m^2} & 1 & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & -(\frac{\partial x_2^{n-1}}{\partial x_2^n})_{x_2,p,m^{n-1}} & \vdots \\ -(\frac{\partial x_2^n}{\partial x_2^1})_{x_2,p,m^n} & \cdots & -(\frac{\partial x_2^n}{\partial x_2^{n-1}})_{x_2,p,m^n} & 1 & -(\frac{\partial x_2^n}{\partial p})_{x_2} \\ 1 & \cdots & \cdots & 1 & 0 \end{bmatrix}$$

$$\text{with} \quad \forall \quad j \notin N_i(g): \quad (\frac{\partial x_2^i}{\partial x_2^j})_{x_2,p,m^i} = 0$$

$$\text{and} \quad x = \begin{bmatrix} \frac{dx_2^1}{d\omega_2^1} \\ \vdots \\ \frac{dx_2^n}{d\omega_2^1} \\ \frac{dp}{d\omega_2^1} \end{bmatrix}, \quad b = \begin{bmatrix} p(\frac{\partial x_2^1}{\partial m^1})_{x_2,p} \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

$$\Rightarrow \frac{dp}{d\omega_2^1} = \frac{|A_{n+1}|}{|A|}$$

where A_{n+1} is the matrix formed by replacing the $(n+1)^{th}$ column of matrix A with the column vector b.

$$\Rightarrow \frac{dp}{d\omega_2^1} = \frac{\frac{|A_{n+1}|}{M_{n+1,n+1}}}{\frac{|A|}{M_{n+1,n+1}}}$$

where the minor $M_{n+1,n+1}$ of matrix A is the determinant of the matrix formed by removing row $n+1$ and column $n+1$ of matrix A.

$$\Rightarrow \frac{dp}{d\omega_2^1} = -\frac{\sum_{i \in N} (\frac{\partial x_2^i}{\partial \omega_2^1})_p - 1}{\sum_{i \in N} \frac{dx_2^i}{dp}}$$

□

Proof of Proposition 2 (Regular Networks):

$$\forall \quad i \in N: \quad x_2^i = \frac{1}{2p}(m^i + \alpha p \frac{1}{r} \sum_{j \in N_i(g)} x_2^j)$$

$$\sum_{i\in N} m^i + \alpha p \frac{1}{r} \sum_{i\in N} \sum_{j\in N_i(g)} x_2^j = 2p \sum_{i\in N} \omega_2^i$$

$$\sum_{i\in N} m^i + \alpha p \frac{r}{r} \sum_{i\in N} x_2^i = 2p \sum_{i\in N} \omega_2^i$$

$$\sum_{i\in N} \omega_1^i + p \sum_{i\in N} \omega_2^i + \alpha p \sum_{i\in N} \omega_2^i = 2p \sum_{i\in N} \omega_2^i$$

$$p = \frac{1}{1-\alpha} \frac{\sum_{i\in N} \omega_1^i}{\sum_{i\in N} \omega_2^i}$$

$$\forall \quad i \in N: \quad \frac{dp}{d\omega_2^i} < 0$$

□

Proof of Proposition 3 (Star Network):

Combining every agent's demand function for good 2 and the market clearing condition for good 2, we can solve for the price of good 2:

$$p = \frac{(n-1)[2+(n-1)\alpha]\omega_1^1 + [2(n-1)+\alpha] \sum\limits_{j\in N_1(g)} \omega_1^j}{(n-1)[2-(n-1)\alpha-\alpha^2]\omega_2^1 + [2(n-1)-\alpha-(n-1)\alpha^2] \sum\limits_{j\in N_1(g)} \omega_2^j}$$

$$\frac{dp}{d\omega_2^1} > 0 \quad \text{if} \quad \alpha > \frac{-(n-1)+\sqrt{(n-1)^2+8}}{2}$$

□

Proof of Proposition 4 (Line Network):

From the best response of every agent and the market clearing condition, the equilibrium price is implicitly determined by:

$$-\begin{vmatrix} A & b \\ \iota^T & 0 \end{vmatrix} = |A| \sum_{i\in N} \omega_2^i$$

where ι is an n-vector of ones

Assume n=3

$$p = \frac{(4+\alpha)(\omega_1^1 + \omega_1^3) + (4+4\alpha)\omega_1^2}{(4-\alpha-2\alpha^2)(\omega_2^1 + \omega_2^3) + (4-4\alpha-2\alpha^2)\omega_2^2}$$

$$\text{For} \quad i = 2: \quad \frac{\partial p}{\partial \omega_2^i} > 0 \quad \text{if} \quad 4 - 4\alpha - 2\alpha^2 < 0 \iff \alpha > 0.732$$

Assume n=4

Agent i's demand function for good 2:

$$x_2^i = \begin{cases} \frac{1}{p}\left[\frac{(32-6\alpha^2)m^i + \alpha(16-2\alpha^2)m^{N_i(g)} + 4\alpha^2 m^{N_i^2(g)} + \alpha^3 m^{N_i^3(g)}}{64-20\alpha^2+\alpha^4}\right] & \text{if} \quad i \in \{1, n\} \\ \frac{1}{p}\left[\frac{(32-4\alpha^2)m^i + \alpha(8-\alpha^2)m^{i-1} + 8\alpha m^{i+1} + 2\alpha^2 m^{i+2}}{64-20\alpha^2+\alpha^4}\right] & \text{if} \quad i = 2 \\ \frac{1}{p}\left[\frac{(32-4\alpha^2)m^i + \alpha(8-\alpha^2)m^{i+1} + 8\alpha m^{i-1} + 2\alpha^2 m^{i-2}}{64-20\alpha^2+\alpha^4}\right] & \text{if} \quad i = 3 \end{cases}$$

Although non-peripheral agents 2 and $(n-1)$, which are the agents next to the peripheral agents, each have a pair of neighbors, the income of their peripheral neighbor has a smaller marginal effect on their demand for good 2 than the income of their non-peripheral neighbor.

$$p = \frac{(8+2\alpha-\alpha^2)(\omega_1^1 + \omega_1^4) + (8+6\alpha-\frac{1}{2}\alpha^3)(\omega_1^2 + \omega_1^3)}{(8-2\alpha-4\alpha^2+\frac{1}{4}\alpha^4)(\omega_2^1 + \omega_2^4) + (8-6\alpha-5\alpha^2+\frac{1}{2}\alpha^3+\frac{1}{4}\alpha^4)(\omega_2^2 + \omega_2^3)}$$

$$\forall \quad i \in \{2, 3\}: \quad \frac{\partial p}{\partial \omega_2^i} > 0 \quad \text{if} \quad 8 - 6\alpha - 5\alpha^2 + \frac{1}{2}\alpha^3 + \frac{1}{4}\alpha^4 < 0 \iff \alpha > 0.828$$

Assume n=5

Agent i's demand function for good 2:

$$x_2^i = \begin{cases} \frac{1}{p}\left[\frac{(16-4\alpha^2+\frac{1}{8}\alpha^4)m^i + \alpha(8-\frac{3}{2}\alpha^2)m^{N_i(g)} + \alpha^2(2-\frac{1}{4}\alpha^2)m^{N_i^2(g)} + \frac{1}{2}\alpha^3 m^{N_i^3(g)} + \frac{1}{8}\alpha^4 m^{N_i^4(g)}}{32-18\alpha^2+\alpha^4}\right] & \text{if} \quad i \in \{1, n\} \\ \frac{1}{p}\left[\frac{(16-3\alpha^2)m^i + \alpha(4-\frac{3}{4}\alpha^2)m^{i-1} + \alpha(4-\frac{1}{2}\alpha^2)m^{i+1} + \alpha^2 m^{i+2} + \frac{1}{4}\alpha^3 m^{i+3}}{32-12\alpha^2+\alpha^4}\right] & \text{if} \quad i = 2 \\ \frac{1}{p}\left[\frac{(16-3\alpha^2)m^i + \alpha(4-\frac{3}{4}\alpha^2)m^{i+1} + \alpha(4-\frac{1}{2}\alpha^2)m^{i-1} + \alpha^2 m^{i-2} + \frac{1}{4}\alpha^3 m^{i-3}}{32-12\alpha^2+\alpha^4}\right] & \text{if} \quad i = 4 \\ \frac{1}{p}\left[\frac{(4-\frac{1}{2}\alpha^2)^2 m^i + \alpha(4-\frac{1}{2}\alpha^2)\sum_{j \in N_i(g)} m^j + \alpha^2(1-\frac{1}{8}\alpha^2)\sum_{k \in N_i^2(g)} m^k}{32-12\alpha^2+\alpha^4}\right] & \text{if} \quad i = 3 \end{cases}$$

$$p = \frac{(16+4\alpha-3\alpha^2-\frac{1}{2}\alpha^3+\frac{1}{8}\alpha^4)(\omega_1^1+\omega_1^5) + (16+12\alpha-2\alpha^2-\frac{3}{2}\alpha^3)(\omega_1^2+\omega_1^4) + (16+8\alpha-\alpha^3-\frac{1}{4}\alpha^4)\omega_1^3}{(16-4\alpha-9\alpha^2+\frac{1}{2}\alpha^3+\frac{7}{8}\alpha^4)(\omega_2^1+\omega_2^5) + (16-12\alpha-10\alpha^2+\frac{3}{2}\alpha^3+\alpha^4)(\omega_2^2+\omega_2^4) + (16-8\alpha-12\alpha^2+\alpha^3+\frac{5}{4}\alpha^4)\omega_2^3}$$

$$\forall \quad i \in \{2,4\}: \quad \frac{\partial p}{\partial \omega_2^i} > 0 \quad \text{if} \quad 16-12\alpha-10\alpha^2+\frac{3}{2}\alpha^3+\alpha^4 < 0 \iff \alpha > 0.851$$

$$\text{For} \quad i = 3: \quad \frac{\partial p}{\partial \omega_2^i} > 0 \quad \text{if} \quad 16-8\alpha-12\alpha^2+\alpha^3+\frac{5}{4}\alpha^4 < 0 \iff \alpha > 0.927$$

Assume n=6

$$\forall \quad i \in \{2,5\}: \quad \frac{\partial p}{\partial \omega_2^i} > 0 \quad \text{if} \quad 32-24\alpha-22\alpha^2+5\alpha^3+3\alpha^4-\frac{1}{8}\alpha^5-\frac{1}{16}\alpha^6 < 0 \iff \alpha > 0.856$$

$$\forall \quad i \in \{3,4\}: \quad \frac{\partial p}{\partial \omega_2^i} > 0 \quad \text{if} \quad 32-16\alpha-24\alpha^2+\frac{5}{2}\alpha^3+\frac{7}{2}\alpha^4-\frac{1}{16}\alpha^6 < 0 \iff \alpha > 0.952$$

Assume n=7

$$\forall \quad i \in \{2,6\}: \quad \frac{\partial p}{\partial \omega_2^i} > 0 \quad \text{if} \quad \alpha > 0.857$$

$$\forall \quad i \in \{3,5\}: \quad \frac{\partial p}{\partial \omega_2^i} > 0 \quad \text{if} \quad \alpha > 0.959$$

$$\text{For} \quad i = 4: \quad \frac{\partial p}{\partial \omega_2^i} > 0 \quad \text{if} \quad \alpha > 0.979$$

Assume n=8

$$\forall \quad i \in \{2,7\}: \quad \frac{\partial p}{\partial \omega_2^i} > 0 \quad \text{if} \quad \alpha > 0.857$$

$$\forall \quad i \in \{3,6\}: \quad \frac{\partial p}{\partial \omega_2^i} > 0 \quad \text{if} \quad \alpha > 0.961$$

$$\forall \quad i \in \{4,5\}: \quad \frac{\partial p}{\partial \omega_2^i} > 0 \quad \text{if} \quad \alpha > 0.986$$

Assume n=9

$$\forall \quad i \in \{2,8\}: \quad \frac{\partial p}{\partial \omega_2^i} > 0 \quad \text{if} \quad \alpha > 0.858$$

$$\forall \quad i \in \{3,7\}: \quad \frac{\partial p}{\partial \omega_2^i} > 0 \quad \text{if} \quad \alpha > 0.961$$

$$\forall \quad i \in \{4,6\}: \quad \frac{\partial p}{\partial \omega_2^i} > 0 \quad \text{if} \quad \alpha > 0.988$$

$$\text{For} \quad i = 5: \quad \frac{\partial p}{\partial \omega_2^i} > 0 \quad \text{if} \quad \alpha > 0.994$$

$$\vdots$$

□

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