Bilateral and multilateral exchange rate and purchasing power parity indexes: the aggregation problem

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Bilateral and Multilateral Exchange Rate and Purchasing-Power-Parity Indexes: The Aggregation Problem

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I. Introduction and Summary

With the generalization of high variability in exchange rates after the collapse of the Bretton-Wood System, policymakers have increasingly relied on weighted average of exchange rate indexes (nominal effective exchange rate index) or indexes of competitiveness (real effective exchange rate or purchasing-power-parity index)in order to assess the need for and the magnitude of exchange rate adjustments. However, these indexes are plagued by many methodological problems that limit the scope for their implementation as reliable indicators and consequently as targets for policymakers.

In this paper, we will address only one of the main limitations of these indexes. namely, the ad-hoc character of the determination of the weighting system used in their construction. To solve this problem, a new kind of index is proposed, which is based on a complete system of export demand functions. 1/ The paper is organized as follows: Section II briefly reviews the methodology of available effective exchange rate indexes and emphasizes their discording views and the related difficulties in interpreting their evolutions. In Section III, a new concept of effective exchange rate is defined, which takes advantage of the aggregative

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1/ A review of complete systems of consumer demand functions is presented in Barten (1977).

properties of demand functions in the modern theory of consumer demand. Section IV stresses the properties of the new index and its clear and simple meaning: the real effective exchange rate of a given country is equal to its normalized export market share in volume, and the ratio of the effective exchange rates of two countries is equal to their bilateral exchange rate index. The restrictive assumptions underlying this result are pointed out. In Section V, an empirical analysis is developed to test the reliability of these indexes as economic indicators. In connection with this, a comparison is made with the Fund's Merm index, which gives credence to the new indexes. Finally, Section VI presents the conclusion and offers some suggestions concerning the practical implementation of these indexes.

II. A Review of Available Effective Exchange Rates Indexes and the Need for Consistency

With the suppression in 1971 of a stable link between gold and the U.S. dollar, central bankers were deprived of a common standard able to measure the relative position of their domestic currency toward all other currencies. Furthermore, for the monetary authorities who were striving to maintain a seemingly orderly exchange rates system, the need for adjustments was blurred by the increasing volatility in exchange rates of the major currencies, and by the uncertainties surrounding the effects of these multiple exchange rate variations on the economy competitiveness and presumably on its external account position. Moreover, the empirical value of the most popular theory of exchange rate determination among policymakers, that is, the relative version of the "purchasing power parity"

(PPP), has been questioned 1/ at least when considered at a bilateral level. In this connection, governments, international institutions, as well as private banks, found it more convenient to rely on aggregate indexes from a relevant basket of nominal exchange rates (nominal effective exchange rate) and of real exchange rates (real effective exchange rate). The latter index is obtained by deflating the former by a corresponding index of relative prices; in other words, this indicator is merely a weighted average of PPP bilateral indexes.

However, the economic interpretation of the fluctuations of these weighted indicators is hampered by the fact that indexes produced by various institutions 2/ for the same currency, provide rather discording views on its actual position. The reasons for these divergences can be traced back to differences in the implicit or explicit theories underlying the construction of these indexes. With the exception of the Fund's MERM index, they all rely on various aspects of the PPP theory. That is, the appraisal of a currency position is determined by comparison of a nominal effective exchange rate index with a consistent indicator of weighted relative prices. 3/ The Fund's MERM index is based on a more

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1/ See Officer (1976) and Isard (1978).

2/ IMF, OECD, BIS, U.S., Treasury, Federal Reserve of New York, Banque de France, Morgan Guaranty Trust Company etc.... .

3/ It is worth noting here that in such a framework, the real and nominal effective exchange rate indexes play only a role of indicators on the adequacy of the present bilateral exchange rate set. The amplitude of a needed change in the level of bilateral exchange rates is determined in an unique way by bilateral inflation differentials. All other solutions would be inconsistent at a multilateral level if PPP is to be maintained. In other terms, for practical implementation of a currency realignment effective exchange rate indexes do not matter.

complex theoretical framework 1/ which provides an estimation of a currency position by reference to the trade balance effects of its effective exchange rate variations.

These various approaches have resulted in important methodological differences which are reflected in:

(i) The weighting system that is determined by three factors: the countries whose currencies are included into the basket, the methodology and economic basis used to measure the relative share of each country 2/ (these weights originate most often from the bilateral or multilateral international trade and exceptionally from a highly complex model as in the Fund's MERM), and the base period for the weights' calculation.

(ii) The prices that are chosen according to the objectives that the authorities are aiming at (competitiveness of the export sector, and of the import-subsitute sector, current account or basic balance position). 3/

(iii) The mathematical formulations that are either arithmetic, geometric or harmonic averages of bilateral exchange rate or PPP indexes. 4/

However, explaining the reasons for these distortions does not help very much policymakers who have to figure out what are the relevant objectives for the bilateral exchange rates of their domestic currency. In

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1/ See Artus and Rhomberg (1973) and Rhomberg (1976) for the theoretical foundations of the MERM index.

2/ In Rhomberg (1976) for instance, it is shown that the Netherlands guilder was revaluated on the period 1971/75 either by 10.8 per cent or by 26.7 per cent depending on the weighting system applied to the effective exchange rate calculation.

3/ On the relevant price indexes, for implementation of the PPP theory see, for instance, Balassa (1964), Kravis and Lipsey (1971), Officer (1976).

4/ See Pincon (1979) who demonstrates that the geometric average is the most suitable mathematical concept for construction of an effective exchange rate index.

other terms, there is a need for an instrument of measurement providing a clear and simple economic interpretation of the external value of the domestic currency, in the aggregate and at the bilateral level as well.

III. A Theoretical Framework for Consistent Bilateral and Multilateral Indices

In this section, we lay down what the effective exchange rate index is intended to measure and we assess the theoretical framework that is associated with this measurement.

Our objective is twofold. First, we want to build a set of aggregate indicators devoted to the evaluation of the impact of simultaneous variations in prices and exchange rates on the traded goods sector of a given economy with respect to the traded goods sectors of all other countries. Second, we want to infer from this set of aggregate indicators a consistent set of bilateral indexes.

The theoretical framework employed for construction of these indexes relies on the following stringent assumptions:

(i) Conditions for arbitrage prevail in markets for goods and assets denominated in various currencies. If not, only the analysis of bilateral exchange rate effects would have been relevant.

(ii) The analysis is restricted to traded goods which can be defined as "specific substitutes" in the sense of Theil (1980). This assumption comes to focus mainly on industrial goods, since raw materials and industrial goods are likely to be "specific complements." For instance, one can expect for oil and cars to be complements rather

than substitutes; that is, an increase in the relative price of either goods will reduce the demand for the other.

(iii) There are n countries and each country produces only one composite industrial good, the price of which is denominated in the domestic currency of the reporting country.

(iv) The consumer allocates his expenditures between traded and nontraded good sectors according to a group of utility functions endowed with the property of block independence as defined by Theil (1980). This assumption allows us to deal only with the utility function of the traded goods sector.

(v) All consumers are rational and parameters of their demand function are stable. They are endowed with the same system of preferences. Furthermore, goods have income elasticities identically equal to 1. So we can aggregate all consumers in only one agent who allocates its expenditures between the composite industrial goods according to the budget constraint:

$$(5) \quad M_i = \sum_j \frac{P_j}{l_{ij}} Q_j \quad , \qquad j = 1, \ldots n$$

The notations have the following meaning:

M_i is the world total export of industrial goods expressed in currency i, Q_j is the volume of composite goods exported by country j at the domestic price P_j and l_{ij} is the bilateral exchange rate between i and j currencies. 1/

In this context, the world consumer is assumed to distribute his expenditures between the n composite goods, so that he maximizes a certain preference function which can be expressed either in terms of quantities

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1/ l_{ij} is the price of one unity of currency i in terms of currency j.

(direct utility function) or in terms of prices and income (indirect utility function). 1/ The latter approach makes it straightforward to derive the direct consumer demand functions. 2/

The indirect utility function V_i consistent with this set of restrictive assumptions has the following form:

$$(6)\quad V_i(M_i, Pw_i) = \frac{\prod_j Pw_{ij}^{\alpha j}}{M_i} \quad \text{with} \quad 0 < \alpha_j < 1 \ \forall j, \ \sum_J \alpha_j = 1$$

$$(7)\quad Pw_{ij} = \frac{P_j}{1_{ij}} \qquad i \neq j \text{ with } Pw_{ii} = P_i$$

$$(8)\quad Pw_i = \prod_j Pw_{ij}^{\alpha j}$$

Relations (7) and (8) define respectively the price of good j and the aggregate world price expressed in currency i. Note that function (6) is a degenerate form of the two most commonly used specifications, that is, the additive indirect utility function and the translog indirect utility function. 3/

From relation (6) a consistent set of ordinary demand equations

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1/ The duality between direct and indirect utility function is discussed in Lau (1969).

2/ For an analysis of advantages offered by the indirect utility approach see Gorman (1976).

3/ The additive and Translog indirect utility functions proposed respectively by Stone (1954) and Christensen and Alii (1975) have the following forms:

- additive indirect utility: $V = EXP[\sum_i \alpha_i \ Ln \ P_i - Ln(M - \sum_i P_i \ \beta_i) + k]$
- translog indirect utility: $V = EXP[\sum_i \alpha_i \ Ln \frac{P_i}{M} + \frac{1}{2} \sum_i \sum_j \beta_{ij} \ Ln \frac{P_i}{M} \ Ln \frac{P_j}{M}]$

where α's and β's are parameters. When β's are identically null, these two functions reduce to the relation (6) hereabove.

for the n composite goods, can be obtained by applying the Roy's theorem: 1/

$$(9)\quad Q_i = -\frac{\partial V_i}{\partial Pw_{ii}} \Big/ \frac{\partial V_i}{\partial M_i} \qquad \text{(Roy's Identity)}$$

so that we get:

$$(10)\quad \frac{\partial V_i}{\partial Pw_{ii}} = \alpha_i \, Pw_{ii}^{(\alpha_i - 1)} \prod_{\substack{j \\ j \neq i}} Pw_{ij}^{\alpha_j} \, M_i^{-1}$$

$$(11)\quad \frac{\partial V_i}{\partial M_i} = -\frac{1}{M_i^2} \prod_j Pw_{ij}^{\alpha_j} = -\frac{Pw_i}{M^2}$$

Finally, combining relation (9), (10) and (11) yields the demand equation for good i:

$$(12)\quad Q_i = \alpha_i \, Pw_{ii}^{(\alpha_i - 1)} \prod_{\substack{j \\ j \neq i}} Pw_{ij}^{\alpha_j} \, \frac{M_i}{Pw_i} \quad \text{with} \quad \sum_{\substack{J \\ j \neq i}} \alpha_J = 1 - \alpha_i$$

in which, according to assumption (v), income elasticity is equal to one. Note also, that this relation is a simplified form of the classical demand function for export, which includes at least three explanatory variables, i.e., the world demand in volume $(\frac{M_i}{Pw_i})$, the export price of the reporting country Pw_{ii} with a negative elasticity $(\alpha_i - 1 < 0)$, and the aggregate prices of foreign competing goods $(\prod_{\substack{j \\ J \neq i}} Pw_{ij}^{\alpha_j})$ with a positive elasticity $(\sum_{J \neq i} \alpha_j = 1 - \alpha_i)$.

The budget shares w_i of the world consumer can be obtained by applying (12) to (5) which yields the following relation:

1/ See Roy (1942).

$$(13) \quad w_i = \frac{Pw_{ii}Q_i}{M_i} = \alpha_i$$

These results mean that in the specification of the indirect utility function (6), two strong hypotheses are embedded:

(i) The price elasticity of substitution of each good is strictly equal to its budget share in the world total export;

(ii) Equation (12) defines an expenditure system with constant budget shares, if stability in consumers' behavior is assumed. These conclusions are not surprising, for it can be shown that the dual direct utility function of relation (6) has a simple Cobb-Douglas form: $\prod_J Q_J^{\alpha_j}$. Therefore, the effective exchange rate index stemming from this utility function is named hereafter Cobb-Douglas index.

IV. Derivation of the Nominal and Real Effective Exchange Rate Indexes

Having spelled out our objectives as well as the theoretical assumptions which have permitted us to construct a consistent world system of export demand equations, we are now able to derive from equation (12) a corresponding set of effective exchange rate indexes. In order to do that, it is necessary to single out the variables in relation (12) which account for the effects of competitiveness on the export volume of country i. Dividing both sides of (12) by $\frac{M_i}{\alpha_i Pw_i}$ yields:

$$(14) \quad Q_i \frac{Pw_i}{\alpha_i M_i} = Pw_{ii}^{\alpha_i - 1} \prod_{\substack{j \\ J \neq i}} Pw_{ij}^{\alpha_j} \quad \text{with} \quad \sum_{\substack{J \\ J \neq i}} \alpha_j = 1 - \alpha_i$$

Relation (14) states that the normalized market share in volume of good i depends only on the price substitution effects. Second, one notices that relation (14) is not altered if all variables are specified in terms of indexes having a unitary base. 1/

1/ Except that the parameter α_i will vanish from the left hand side of (14).

Following this convention, the real effective exchange rate index PPP_i of currency i can be defined from (14) as follows:

$$(15)\quad PPP_i = PW_{ii}^{\alpha_i - 1} \prod_{\substack{j \\ J \neq i}} Pw_{ij}^{\alpha_j} = \frac{\prod_j Pw_{ij}^{\alpha_j}}{Pw_{ii}} = \frac{Pw_i}{P_i} \quad .$$

Relation (15) points out that the real effective exchange rate of a given country can be expressed by the ratio of a world price index - comprehending all countries - to the domestic price index of the reporting country. This index is equal to the normalized export market share of the country.

To obtain the nominal effective exchange rate index of currency i, we can use relations (7) and (8) to rewrite equation (15) as follows:

$$(16)\quad PPP_i = \frac{\prod_J P_J^{\alpha_j}}{\prod_{\substack{J \\ J \neq i}} l_{ij}^{\alpha_j} \cdot P_i} = \frac{\bar{P}}{\bar{l}_i \cdot P_i} \quad ,$$

so that the nominal effective exchange rate ($\bar{l}_i$) and the undeflated world price ($\bar{P}$) indexes are defined respectively as:

$$(17)\quad \bar{l}_i = \prod_{\substack{J \\ J \neq i}} l_{ij}^{\alpha_j} \quad \text{with} \quad \sum_{\substack{J \\ J \neq i}} \alpha_j = 1 - \alpha_i \quad ,$$

$$(18)\quad \bar{P} = \prod_J P_j^{\alpha_j} \quad \text{with} \quad \sum_J \alpha_j = 1.$$

Besides, it is worth noting that bilateral indexes (PPP_{ij}, l_{ij}) can be inferred in a straightforward manner from effective indexes (PPP_i, $\bar{l}_i$).

Let PPP_i and PPP_j be the real effective exchange rate indexes for countries i and j and PPP_{ij} their bilateral index, so that:

$$(19) \quad PPP_{ij} = \frac{P_j}{I_{ij} P_i} .$$

Application of (16) to the ratio PPP_i/PPP_j yields:

$$(20) \quad \frac{PPP_i}{PPP_j} = \frac{P_j}{P_i} \cdot \frac{\bar{I}_j}{\bar{I}_i} ,$$

and since definition (17) implies:

$$(21) \quad \frac{\bar{I}_i}{\bar{I}_j} = I_{ij}^{(\alpha_i+\alpha_j)} \cdot \prod_{\substack{k=1 \\ k\neq i \\ k\neq j}}^{n} \left[\frac{I_{ik}}{I_{jk}}\right]^{\alpha_k} = I_{ij}^{(\alpha_i+\alpha_j)} \cdot I_{ij}^{(1-\alpha_i-\alpha_j)} = I_{ij} ,$$

we get the result:

$$(22) \quad \frac{PPP_i}{PPP_j} = \frac{P_j}{I_{ij} P_i} = PPP_{ij}$$

That is, the bilateral nominal or real exchange rate index between countries i and j must be equal to the ratio of their effective indexes.

Turning now to the statistical properties of the effective exchange rate indexes and those of their components, we notice that Pw_i and $\bar{P}$ are in fact divisia indexes 1/ which therefore are endowed of all the relevant properties of index numbers. 2/ However, this is not totally the case for PPP_i and $\bar{I}_i$ which satisfy the proportionality rule only under particular conditions. This assertion can be demonstrated by way of comparison between the index PPP_i and the usual form of the real effective exchange rate index PPP_i^* which is defined as:

1/ For a definition of this type of index see Divisia (1925).

2/ The main properties of index numbers are circularity, time-reversibility, factor-reversibility, identity, transitivity, homogeneity and proportionality.

$$(23)\quad PPP_i^* = \frac{\prod_{\substack{J \\ J\neq i}} Pw_J^{\alpha_{ij}}}{P_i} \quad \text{with} \quad \sum_{\substack{J \\ J\neq i}} \alpha_{ij} = 1 \ ,$$

where the vector $|\alpha_i'|$ is different for each country:

$$(24)\quad \alpha_{ij} = \frac{Pw_{ij}\ Q_j}{\sum_{\substack{J \\ J\neq i}} Pw_{ij}\ Q_j}$$

PPP_i^* and PPP_i specification can be compared easily only in two cases:

(i) The purchasing power parity is strictly verified; under this assumption, the two indexes provide obviously similar results.

(ii) Bilateral deviations from PPP for country i are similar all over the countries. In that case, the two definitions are linked together by the following relation:

$$(25)\quad PPP_i = PPP_i^{*(1-\alpha_i)} \ ,$$

that is, the higher the export market share of country i (α_i), the lower will be the weight attached to a given deviation from PPP.

To prove it, suppose that in equation (15) all the Pw_{ij} bilateral indexes increase by the same amount so that we can write:

$$(26)\quad Pw_{ij} = P_f \quad \forall_j \quad \text{with } i \neq j \ .$$

Equation (15) becomes:

$$(27)\quad PPP_i = Pw_{ii}^{\alpha_i - 1} \prod_{\substack{J \\ J\neq i}} P_f^{\alpha_j} = \frac{P_f}{P_i}^{1-\alpha_i} \quad \text{since} \quad \sum_{\substack{J \\ j\neq i}} \alpha_j = 1-\alpha_i \ .$$

Applying now the usual PPP_i^* specification (23) under the same hypothesis, we obtain the following result:

$$(28)\quad PPP_i^* = \frac{\prod_{J\neq i} Pw_{ij}^{\alpha_{ij}}}{P_i} = \frac{P_f}{P_i} \quad \text{since} \quad \sum_{J\neq i} \alpha_{ij} = 1 .$$

It is then readily verified that relations (27) and (28) yield identity (25).

It is worth noting that this particular property of PPP_i is equally shared by its component $\bar{I}_i$ the nominal effective exchange rate index. It means that the magnitude of its fluctuations depends on the export market share of the reporting economy, or to word it differently, on the price elasticities of substitution in the export sector of the world economy, if assumptions integrated in relation (6) are satisfied.

V. Empirical Analysis: Test for Consistency

In this section, the empirical validity of the theoretical framework presented in Sections III and IV is put to test. Since we are concerned only by the problem of consistency between multilateral and bilateral indexes, the analysis could be focused either on the real (PPP) or the nominal ($\bar{I}$) concepts of effective exchange rate. The latter alternative is chosen here for practical reasons of data availability.

Two issues are to be considered. First, it is necessary to test the strong assumptions which underlie the indirect utility function, i.e., the stability of export market shares in value and the fact that by definition the price elasticities of substitution are inferior to unity. Second, it is crucial to see whether the $\bar{I}$ specification will or will not stand the test of empirical analysis. For this purpose, a comparison will be made with the MERM index on the period 1970/1980.

The stability test intends to determine whether during the period 1970/1980 the evolution of the export market share of a given country followed or not a stationary random walk process. To implement this test, recourse is made to the quarterly total exports of the main 18 industrial countries. During the period under study, a great deal of structural deformations might have affected the relative weights of these countries in international trade, namely: extension of the common market, and the rapid growth in oil and gas exports by the Netherlands, Norway and United Kingdom following the huge price increases in 1973 and 1978. Allowance is made for these systematic alterations, by assuming that market share deformations can be represented by an exponential trend having the following usual specification:

$$(29) \quad \ln(w_i) = \alpha_i \, \text{TREND} + C_i + \varepsilon_i \ ,$$

where w_i is the export market share of country i in the group of 18 countries, TREND has its obvious meaning and ε_i is the vector of the regression residuals. If α_i is not significantly different from zero, it means that the average value of the market share of the reporting country is equal to the constant of the regression (expressed in exponential). As the results in Table 1 point out, the degree of significance of the coefficients is considerably higher for the constants (column 5) than for the trends (column 3). Moreover, the standard error (column 6) indicates that the spreading of the market share values, once the trend is removed, is

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1/ United States, Canada, Japan, Belgium, Denmark, France, Germany, Italy, Netherlands, Switzerland, Austria, Norway, Sweden, Finland, Spain, United Kingdom, Australia, Ireland. These countries will be noted hereafter by the following symbols: US, CA, JA, BE, DK, FR, GE, IT, NE, SZ, AU, NO, SW, FI, SP, UK, AS, IR.

Table 1. Structural Deformations in Export Market Shares of the Main Industrial Countries

(Quarterly data 1-1970/4-1980)

Countries	Average 1975 market share (1)	Exponential trend coeffi-cient * (2)	Student test (3)	Constant coeffi-cient * (4)	Student test (5)	Standard error of estimate (6)	R^2 (7)	Box-Pierce white-noise test on residuals (Lag=12) CHI^2 (8) **
US	19.6 %	-.00196 *	-3.1	-1.674 *	-103.3	5.2 %	.19	120.6
CA	6.0	-.00905 *	-16.3	-2.54 *	-177.6	4.6	.86	34.7
JA	10.2	.00391 *	4.8	-2.37 *	-112.5	6.9	.35	66.3
BE	5.0	-.00059	-1.2	-2.92 *	-240.2	3.9	.04	59.3
DK	1.5	-.00031 *	-7.7	-4.15 *	-392.4	3.4	.59	19.0
FR	9.1	.00283 *	7.3	-2.467 *	-246.8	3.2	.56	46.8
GE	15.5	.00027	0.6	-1.824 *	-168.2	3.5	.01	63.8
IT	6.4	.00226 *	3.1	-2.842 *	-150.9	6.1	.19	33.3
NE	6.2	.00201 *	3.6	-2.865 *	-198.3	4.7	.23	49.3
SZ	2.3	.00241 *	3.8	-3.786 *	-233.8	5.3	.26	104.7
AU	1.3	.00273 *	6.75	-4.361 *	-416.8	3.4	.52	15.6 **
NO	1.2	.00641 *	8.2	-4.549 *	-225.4	6.5	.62	16.7 **
SW	3.0	-.00561 *	-9.1	-3.429 *	-213.9	5.2	.67	34.3
FI	0.9	.0022 *	3.2	-4.6311*	-249.5	6.1	.19	24.1 **
SP	1.3	.00974 *	13.4	-4.499 *	-240.1	6.1	.817	19.9 **
UK	7.9	-.00000	0	-2.488 *	-91.2	8.9	.0	81.7
AS	2.2	-.00707 *	-6.5	-3.736 *	-133.8	9.1	.51	45.2
IR	0.4	.00743 *	9.0	-5.3271*	-250.1	6.9	.66	69.6

* One asterisk indicates that coefficient are significant at 1 per cent level.

** Two asterisks indicate that residuals of the regression are following a white-noise pattern. Level of significance is 1 per cent.

generally small; in the case of France, for instance, 95 per cent of the values of its market share ranges between 8.3 and 8.65 per cent. These results are not sufficient, however, to accept the validity of the assumption of constant market shares. It is also necessary to domonstrate that residuals are white noise. The outcome of the Box-Pierce Test (Table 1, Column 8), indicates that this is true for 4 countries out of 18.

These results intend to prove that the very stringent Hypothesis of constant market shares is not deprived of empirical relevance. Furthermore, one would expect this hypothesis to perform better when the analysis is restricted to industrial goods, as it is advocated in Section II.

Turning now to the problem of the magnitude of the price elasticities of substitution, we notice that for a given country i, the aggregate elasticity is equal to $(\alpha_i - 1)$. 1/ Since α_i is never greater than 20 per cent, it means that price elasticities will range between -1 and -.8 in the aggregate and between a -0.0 and -0.2 at a bilateral level. These values seem rather sensible according to previous empirical studies. 2/ Moreover, as Rhomberg (1976) stated in the case of the MERM index, what is important in the determination of the weight given to a country in a currency basket, is not the absolute magnitude of price elasticities but their relative level.

Having shown that the assumptions underlying the Cobb-Douglas utility function are not so unrealistic when international trade is considered

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1/ See equation (12).

2/ In Fair (1981) a sustained 1 per cent depreciation in the German exchange rate during 6 quarters entails the following volume export variations in percentage for: US = -.008, CA = .034, JA = .015, BE = -.096, DK = -.065, FR = -.071, GD = -.052, IT = -.064, NE = -.104, SZ = -.061, AU = -.074, NO = -.064, SW = -.038, UK = -.032, FI = -.05.

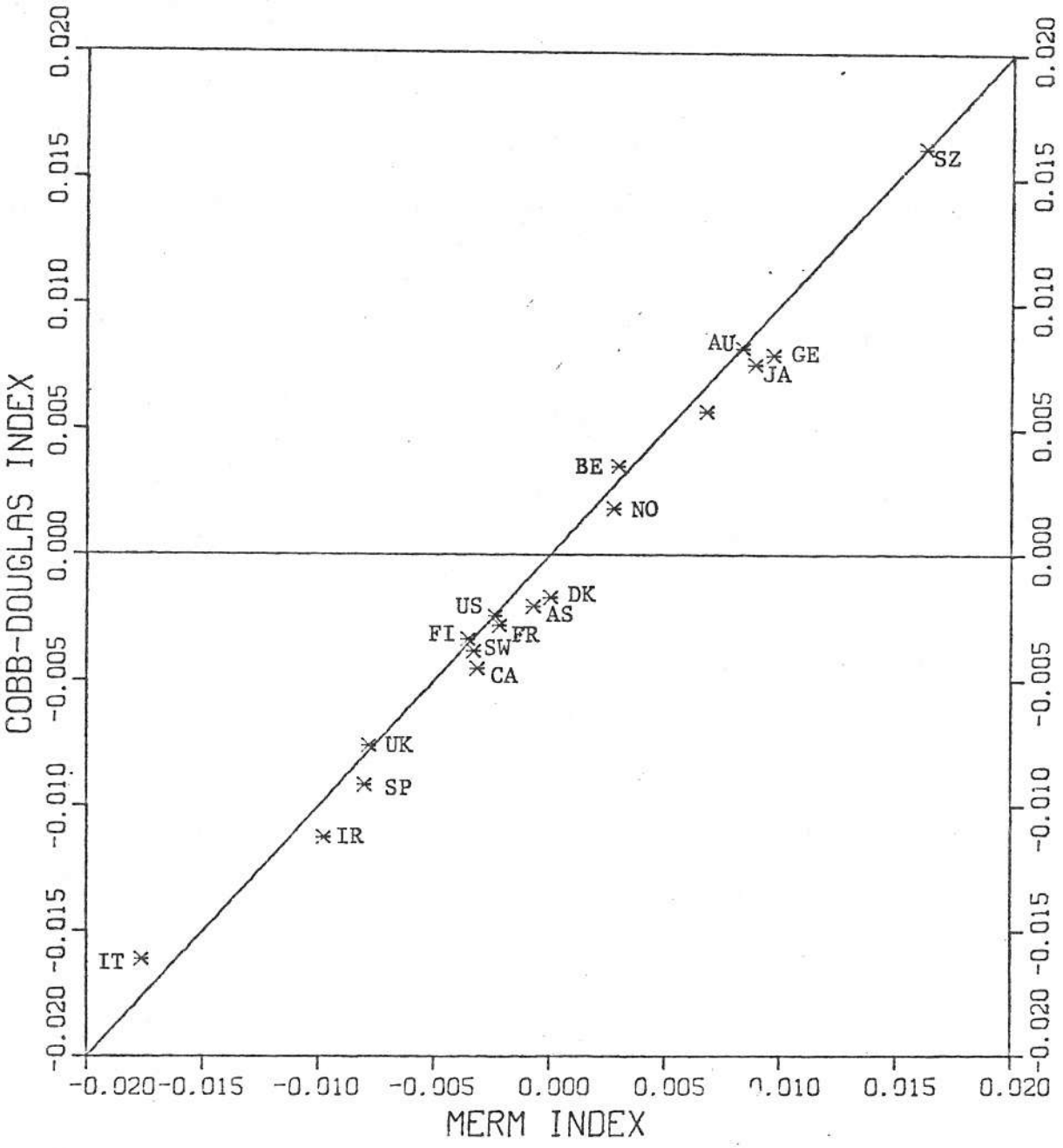
at a highly aggregated level, it is tempting to compare the index $\bar{I}$ with other published effective exchange rates.

The idea of a weighted exchange rate index taking into account the impact of price elasticities on international trade took form for the first time in the Fund's MERM index. 1/ Also, as already mentioned, the MERM is the only published index which relies on an explicit theoretical framework. For these two reasons, a comparison will be made with the MERM index in order to evaluate to what extent the restrictive assumptions implied by the consistency property affect the representation of $\bar{I}$ as a reliable economic indicator of the effects of exchange rate variations on competitiveness.

This exercise has been completed for the period 1970/1980 on the average quarterly exchange rate of the 18 industrial countries previously cited. The base period for the MERM index published in I.F.S. has been modified and is 1 for the first quarter of 1970. The $\bar{I}$ index is calculated on the same basis using average exchange rates published in I.F.S. and export market shares for 1975 (see Table 1, column 1). Charts of MERM and $\bar{I}$ are depicted in appendix. In a more synthetic graph given hereinafter, the average quarterly growth rates of MERM and $\bar{I}$ are plotted against each other for every country. If the two sets of indexes were alike, all intersecting points would have been put on the diagonal. As it could have been expected, this is not the case.

1/ See Rhomberg 1976.

EFFECTIVE EXCHANGE RATE INDEXES
AVERAGE QUATERLY GROWTH RATES
(Q1 1970/Q4 1981)



One notices that 12 intersecting points out of 18 are located on the right-hand side of the diagonal. This fact points to the existence of a systematic bias between the two sets of indexes; with the MERM index, the appreciation and depreciation intend to be respectively larger and smaller than with the Cobb-Douglas index. However, in spite of this distorting aspect, it is worth noting that the intersecting points are close to the straight line particularly in the case of 8 countries that are: Switzerland, Austria, Belgium, United States, Finland, United Kingdom, France and Sweden. It means that the two indexes are giving rather similar results in terms of evolution. A perusal of charts presented in appendix confirms this assertion.

From this comparison we can draw the following conclusions: derivation of an effective exchange rate from a system of demand functions endowed with exact aggregative properties does not prevent this index from being a reliable economic indicator of exchange rate variations on competitiveness.

VI. Conclusion

In this paper a new concept of weighted real and nominal exchange rate indexes has been proposed that provide directly consistent bilateral and multilateral values.

Under the assumption that the export demand for industrial goods is stemming from a Cobb-Douglas utility function, it is demonstrated that this real effective exchange rate index is equal to the normalized market share in volume of the reporting country. Finally, the results of a brief statistical analysis seem to prove that the

restrictive assumptions imposed by the aggregative properties of the demand functions cannot be easily dismissed on an empirical ground. Therefore, it turns out that this sort of weighted exchange rate index might be deemed on a valuable tool whether policymakers are negotiating a currency realignment in a regional fixed exchange rate system, or whether they are trying to target a given external value for their domestic currency.

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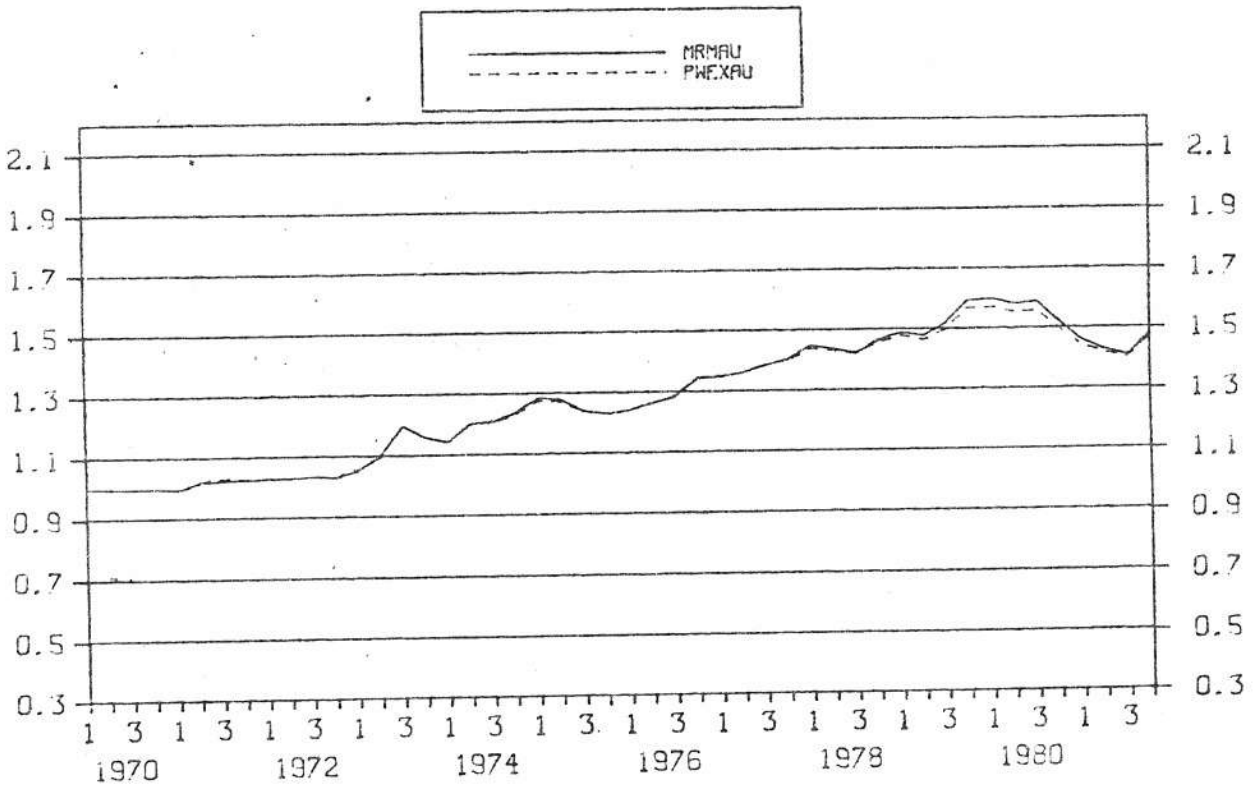
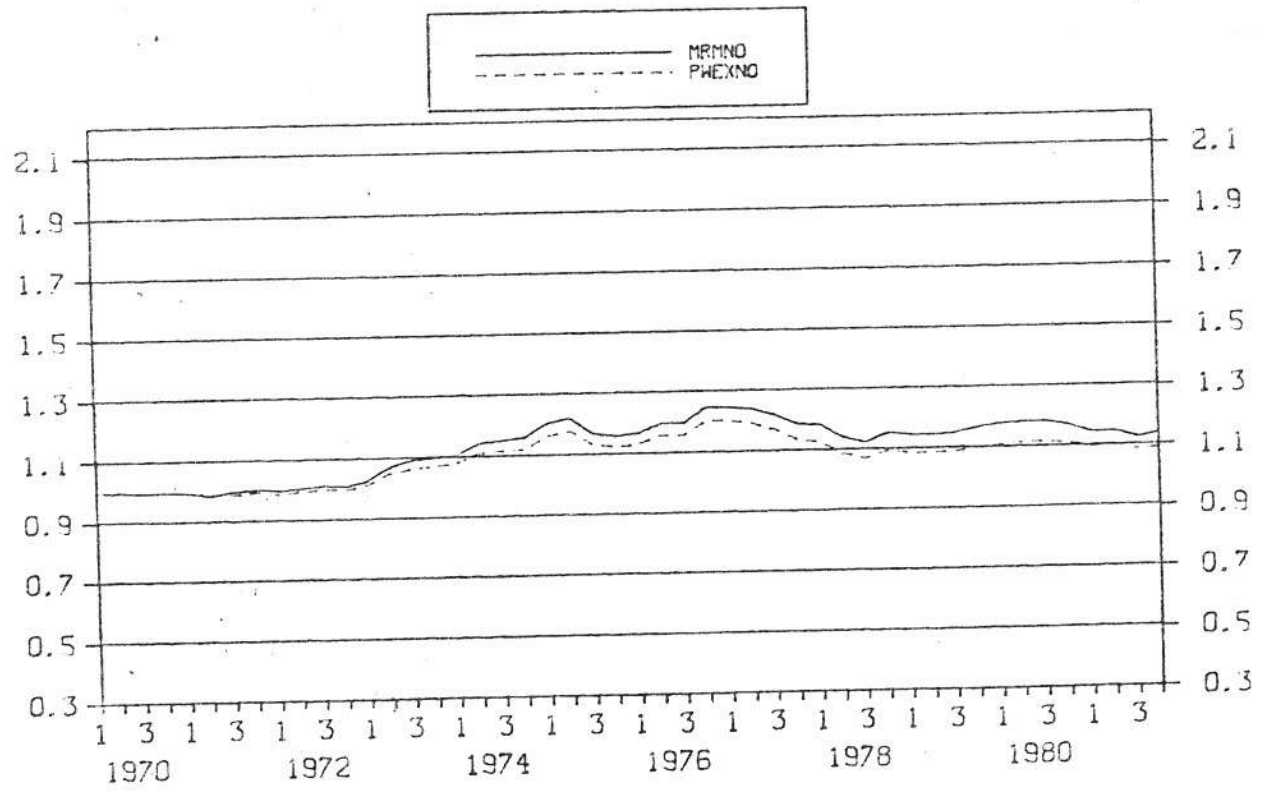
APPENDIX

EFFECTIVE EXCHANGE RATES (*)

01 1970/ 04 1981

(INDEXES BASE 01 1970=1.)





MRMA IS THE FRANCE-GERMANY INDEX

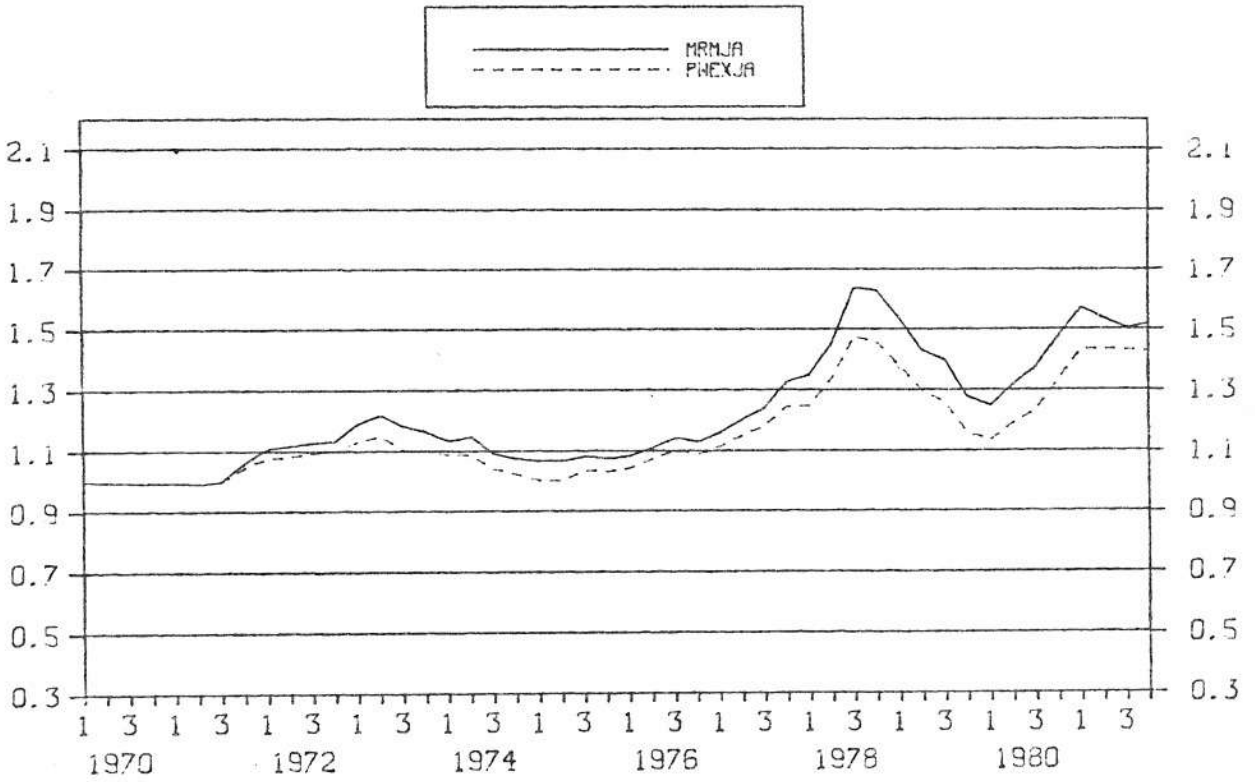
MPWEX IS THE SMITHSONIAN INDEX

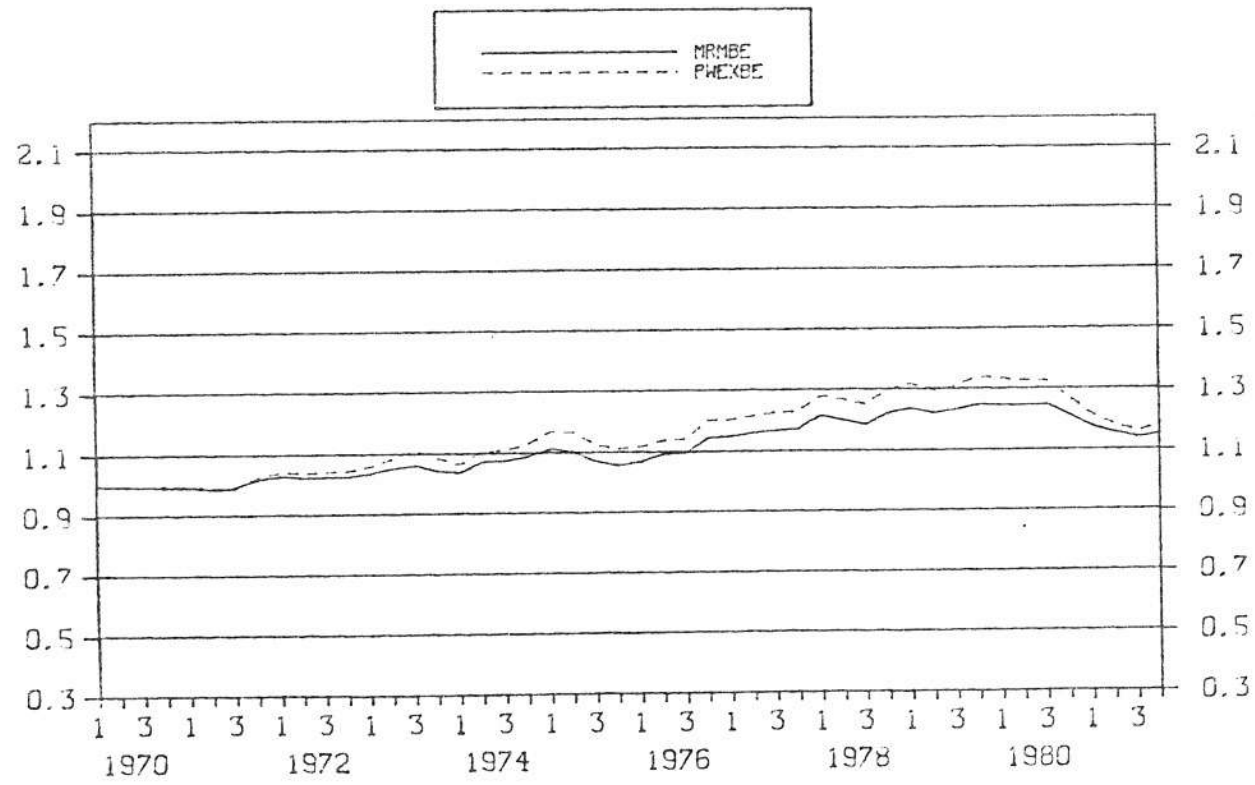
APPENDIX

EFFECTIVE EXCHANGE RATES (*)

Q1 1970/ Q4 1981

(INDEXES BASE Q1 1970=1.)





MRMJ IS THE FUND'S MERM INDEX

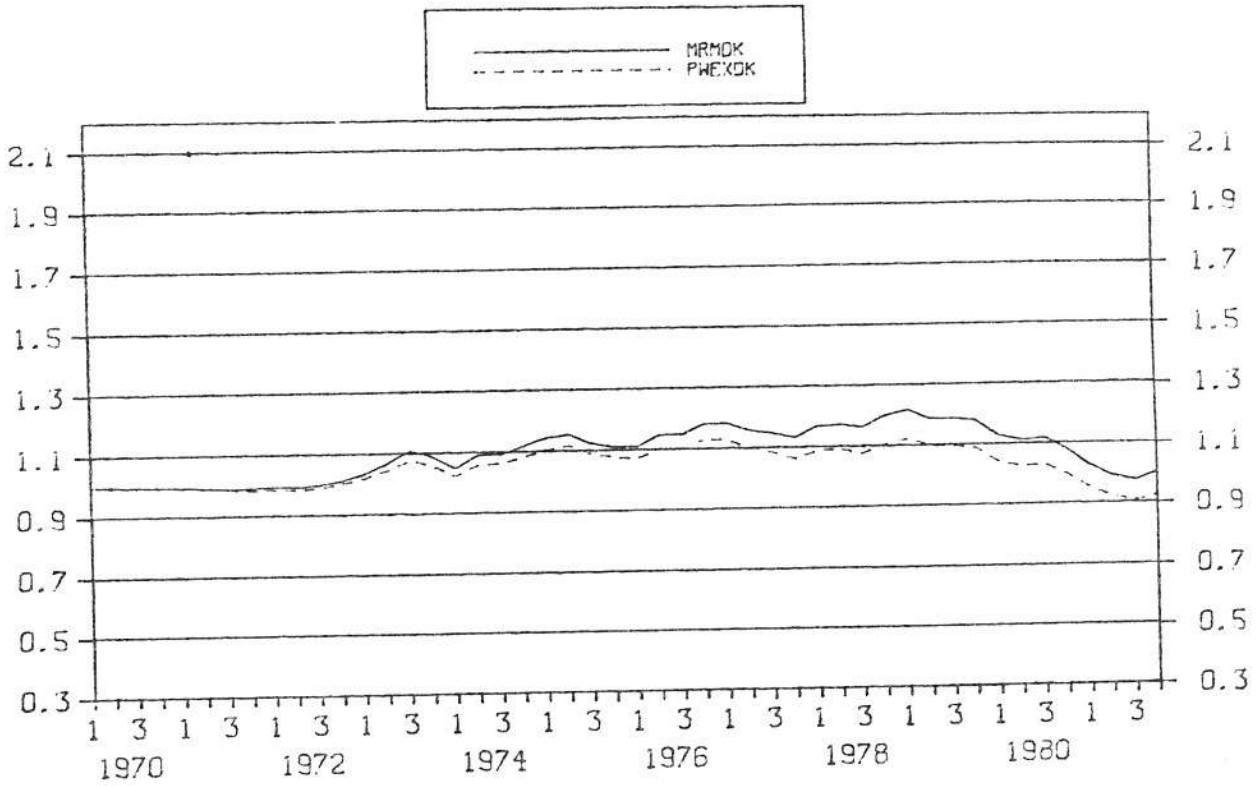
WPWEX IS THE COBB DOUGLAS INDEX

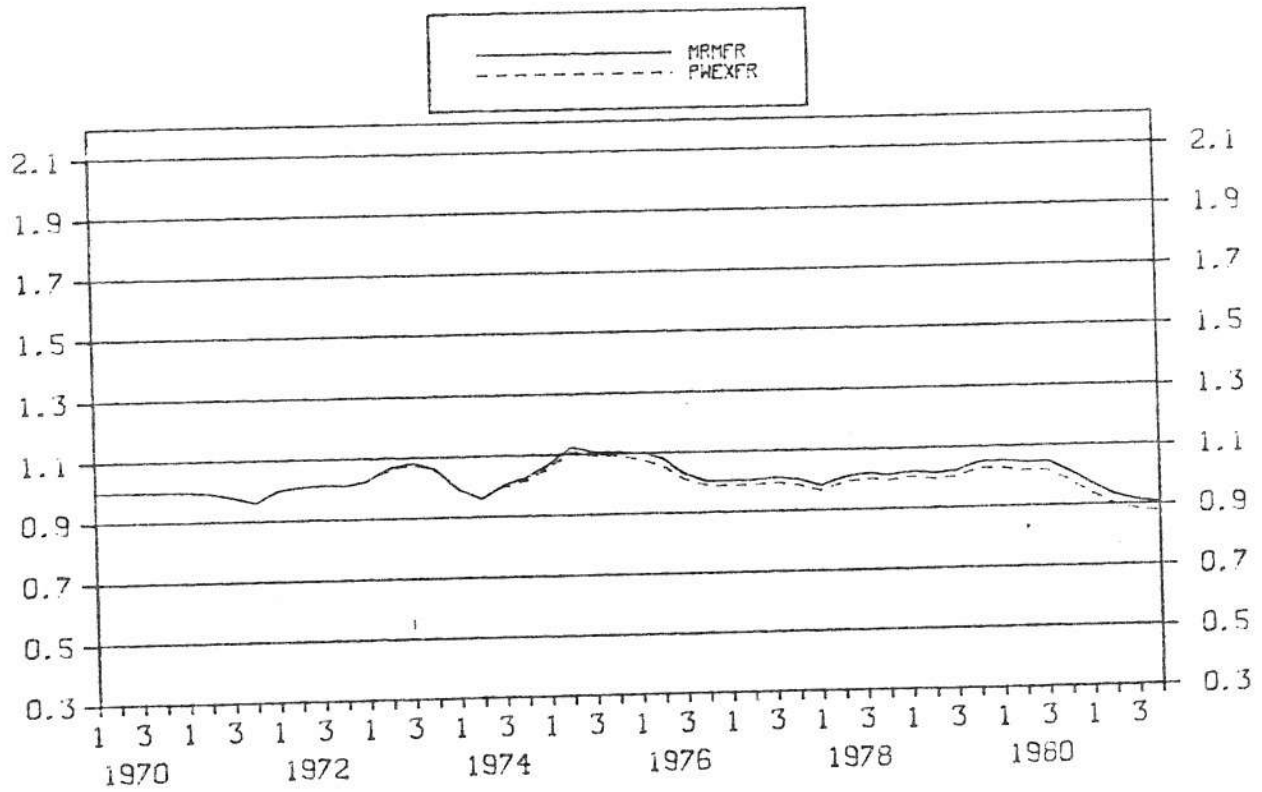
APPENDIX

EFFECTIVE EXCHANGE RATES (*)

01 1970/ 04 1981

(INDEXES BASE 01 1970=1.)





*MRMT IS THE FUND'S MERM INDEX

*PWMT IS THE COBB-DOUGLAS INDEX

APPENDIX

EFFECTIVE EXCHANGE RATES (*)

01 1970/ 04 1981

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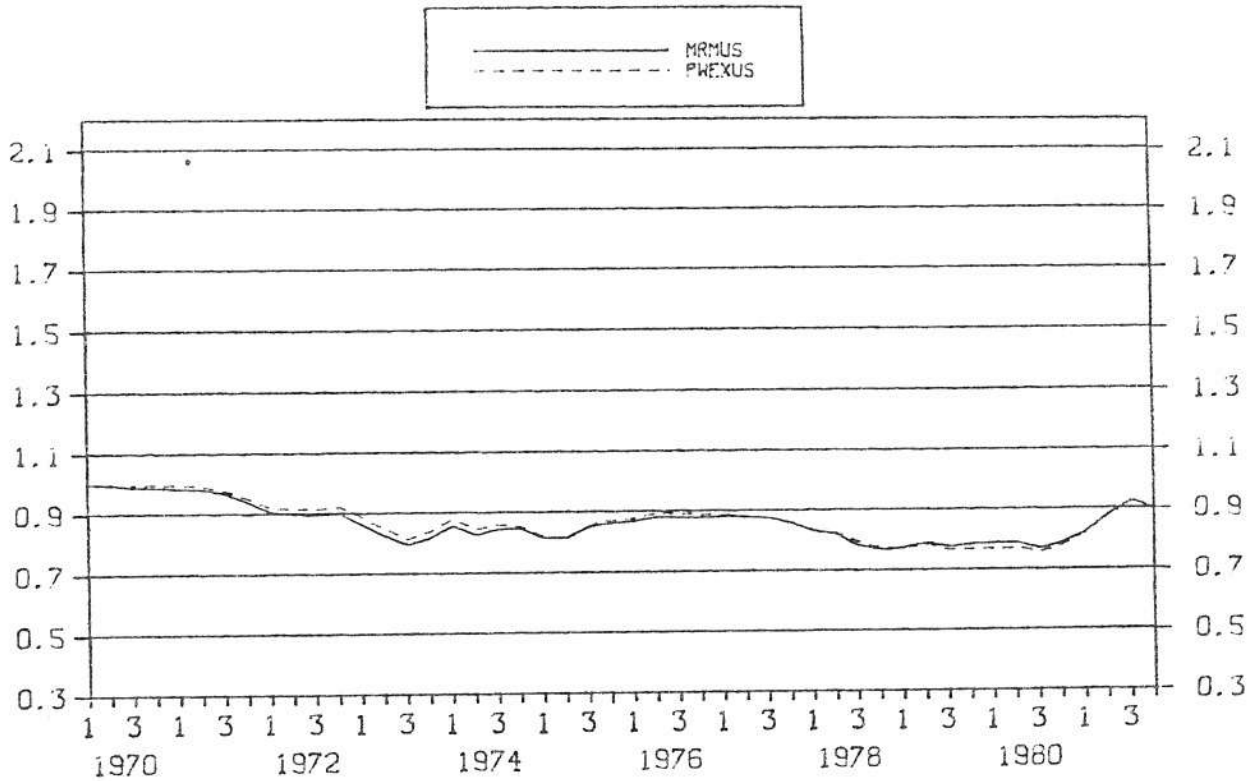
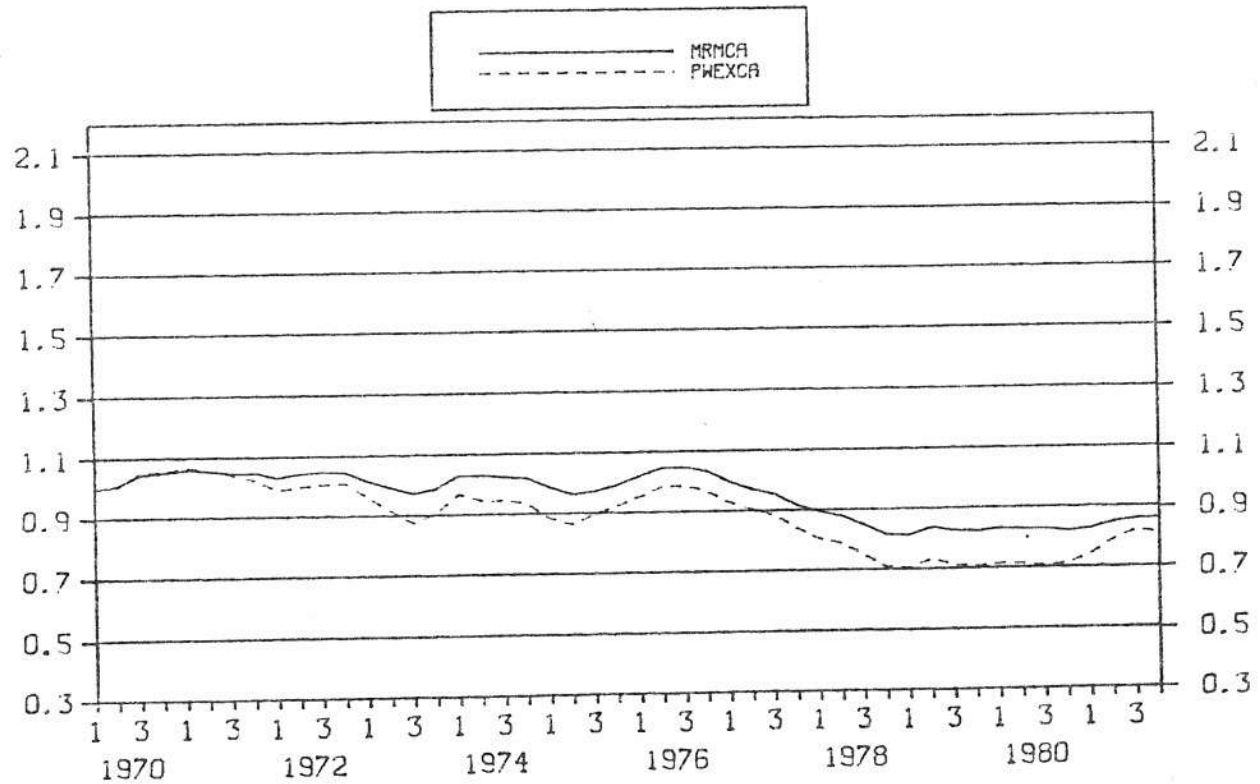




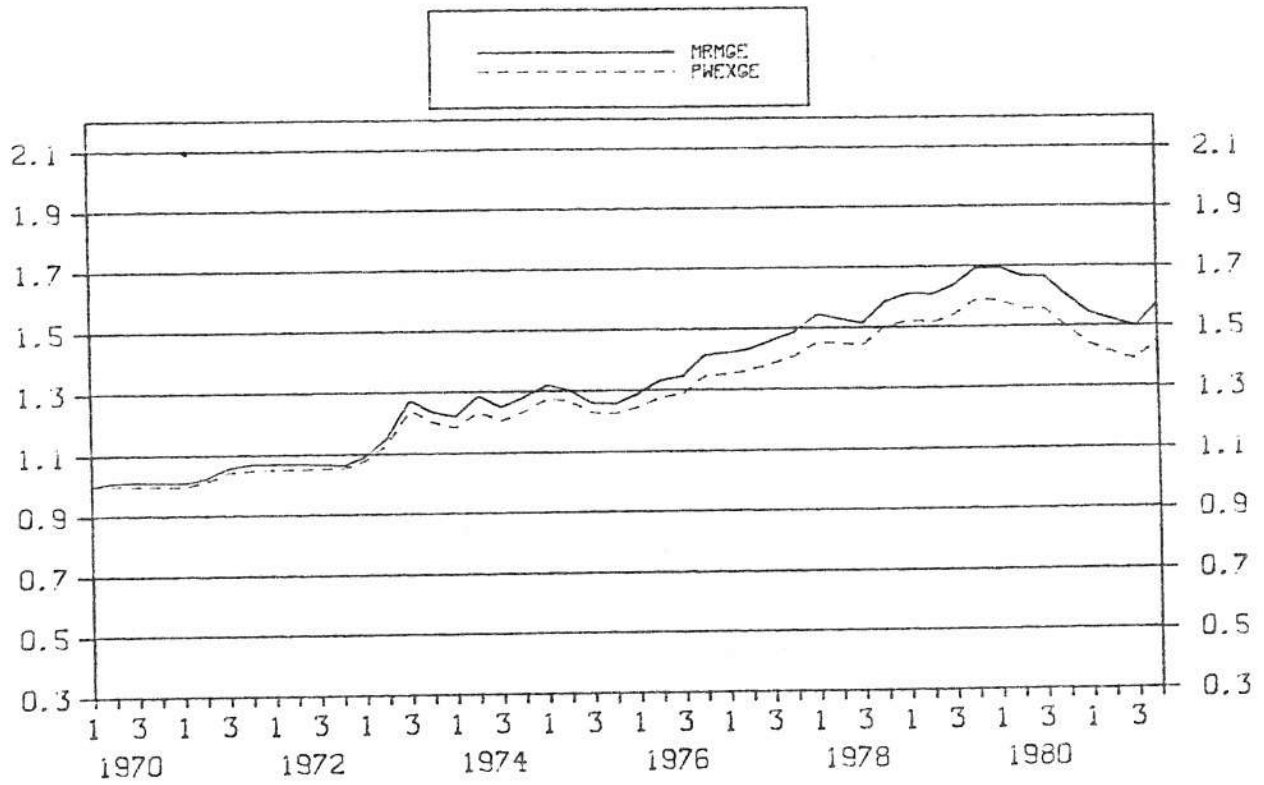
*MRMU IS THE FUND'S MERM INDEX
*PWEX IS THE COBB-DOUGLAS INDEX

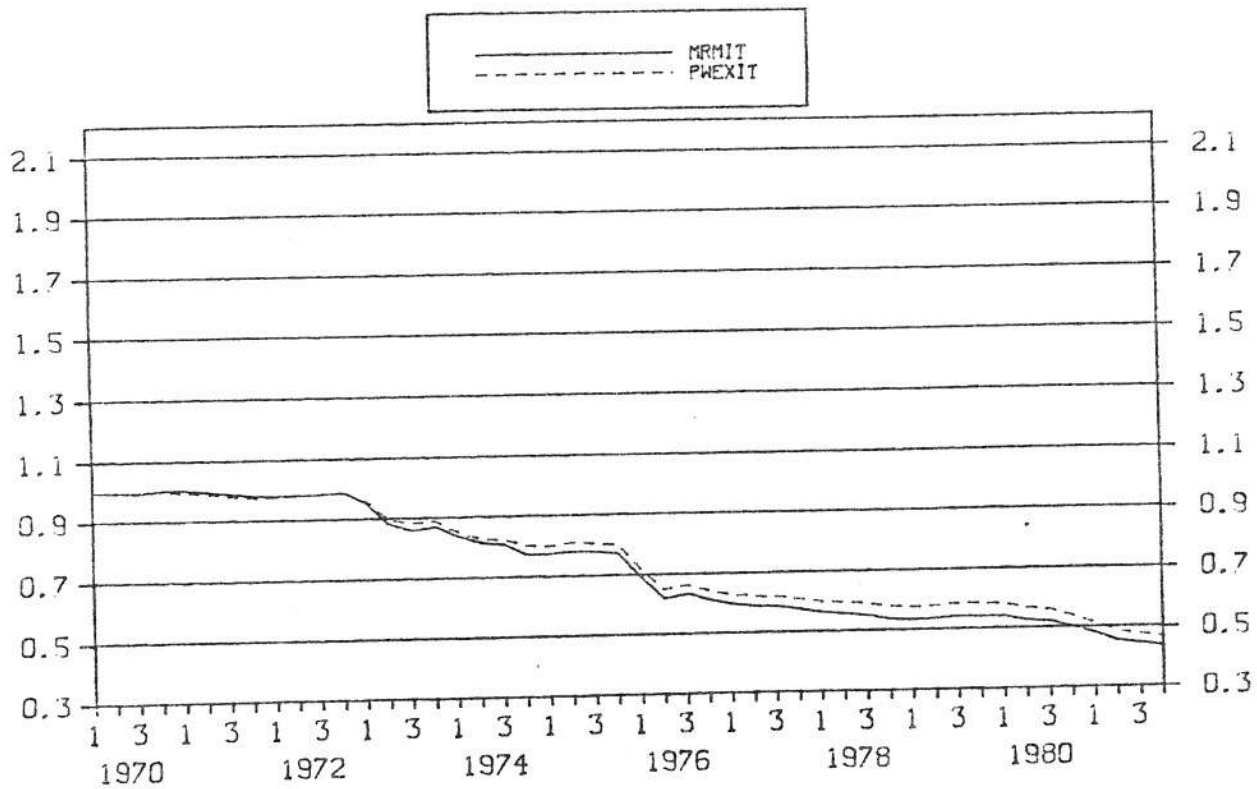
APPENDIX

EFFECTIVE EXCHANGE RATES (*)

01 1970/ 04 1981

(INDEXES BASE 01 1970=1.)





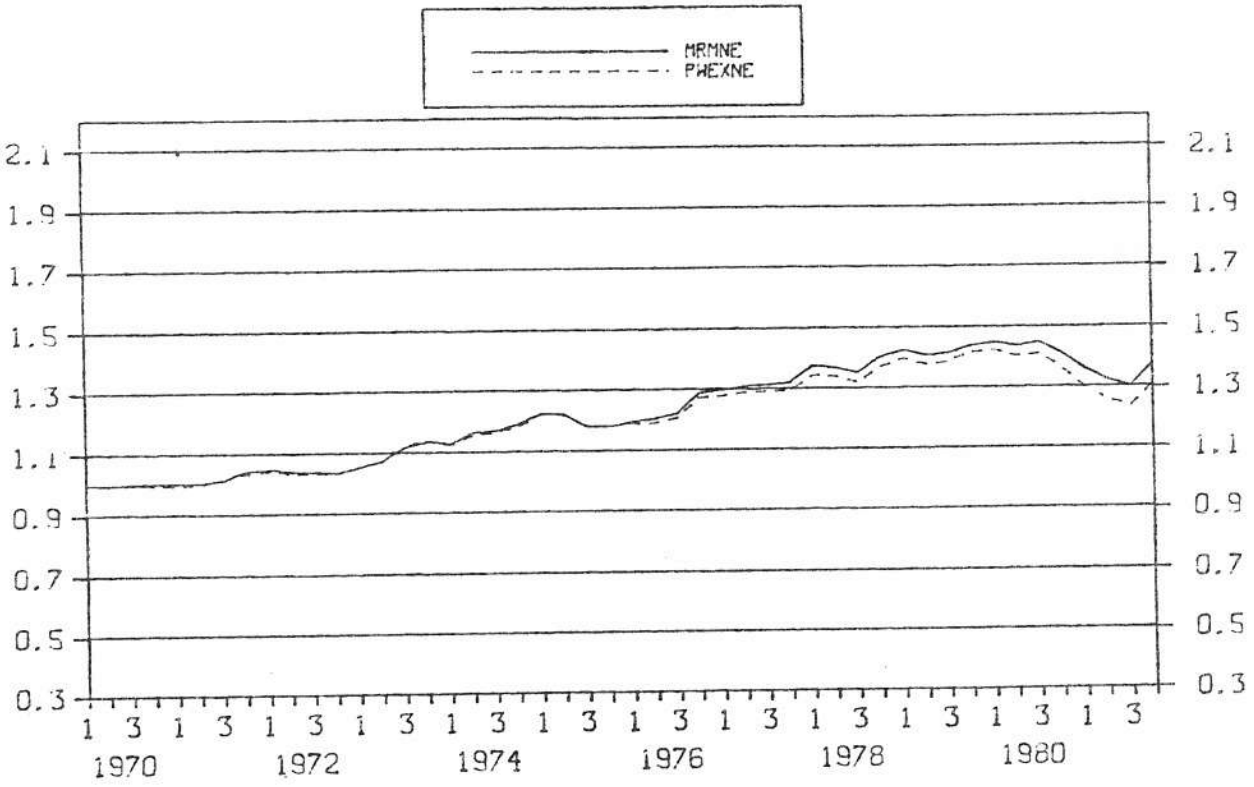
MRMN IS THE FUND'S MERM INDEX
MPWEX IS THE COBB-DOUGLAS INDEX

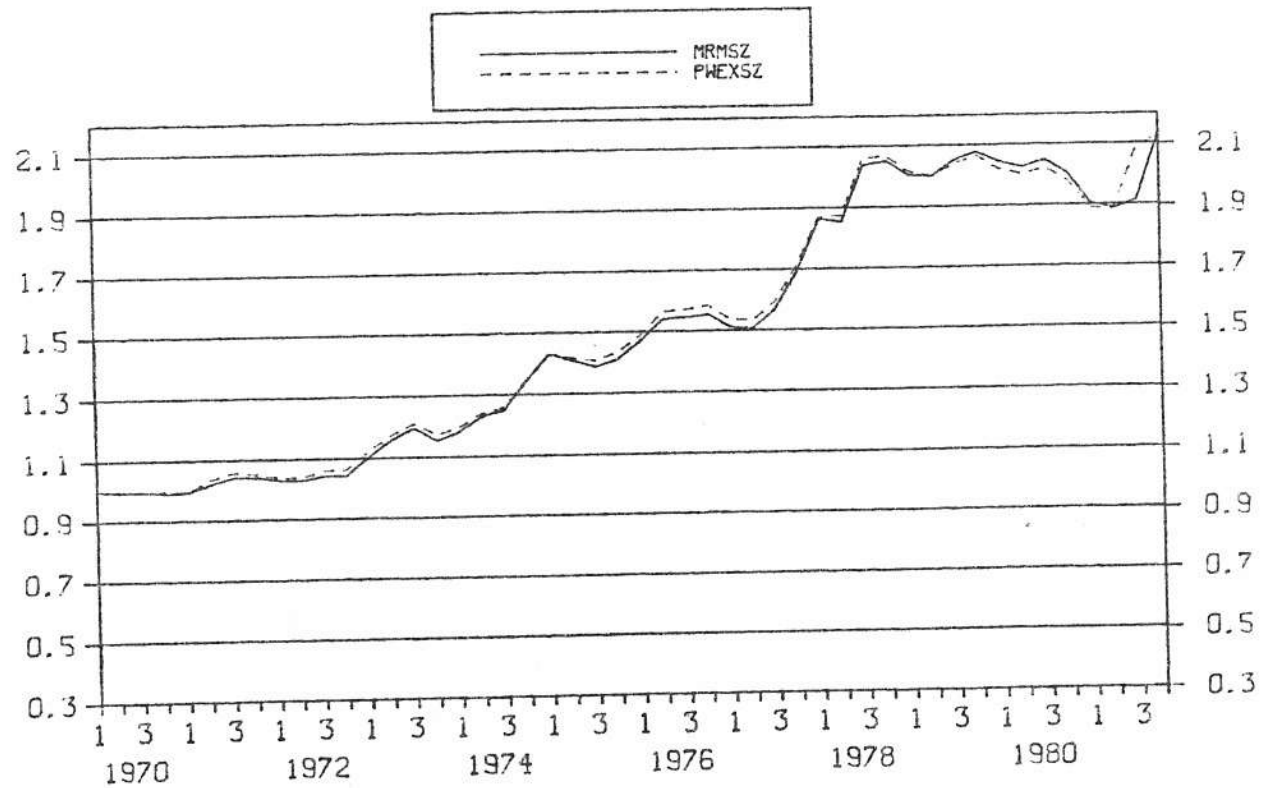
APPENDIX

EFFECTIVE EXCHANGE RATES (*)

Q1 1970/ Q4 1981

(INDEXES BASE Q1 1970=1.)





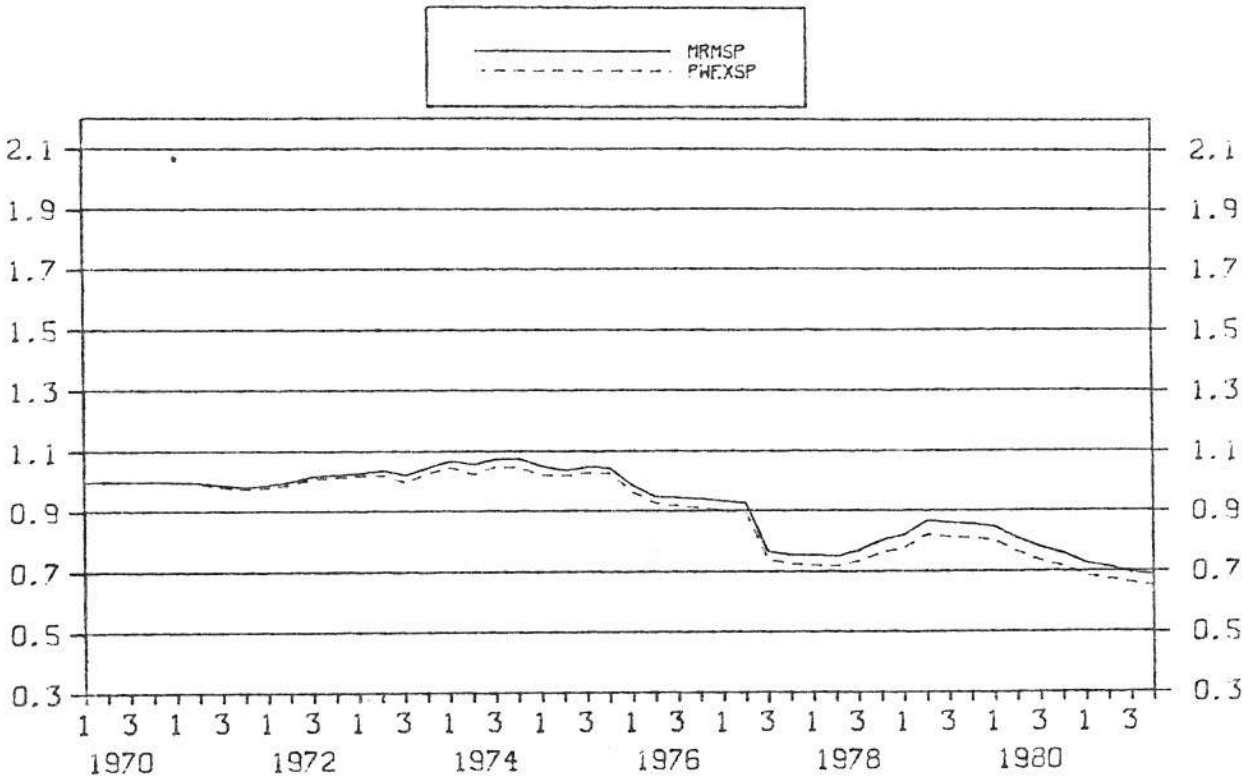
*MRMI IS THE FUND'S MERM INDEX
*PWEX IS THE COBB-DOUGLAS INDEX

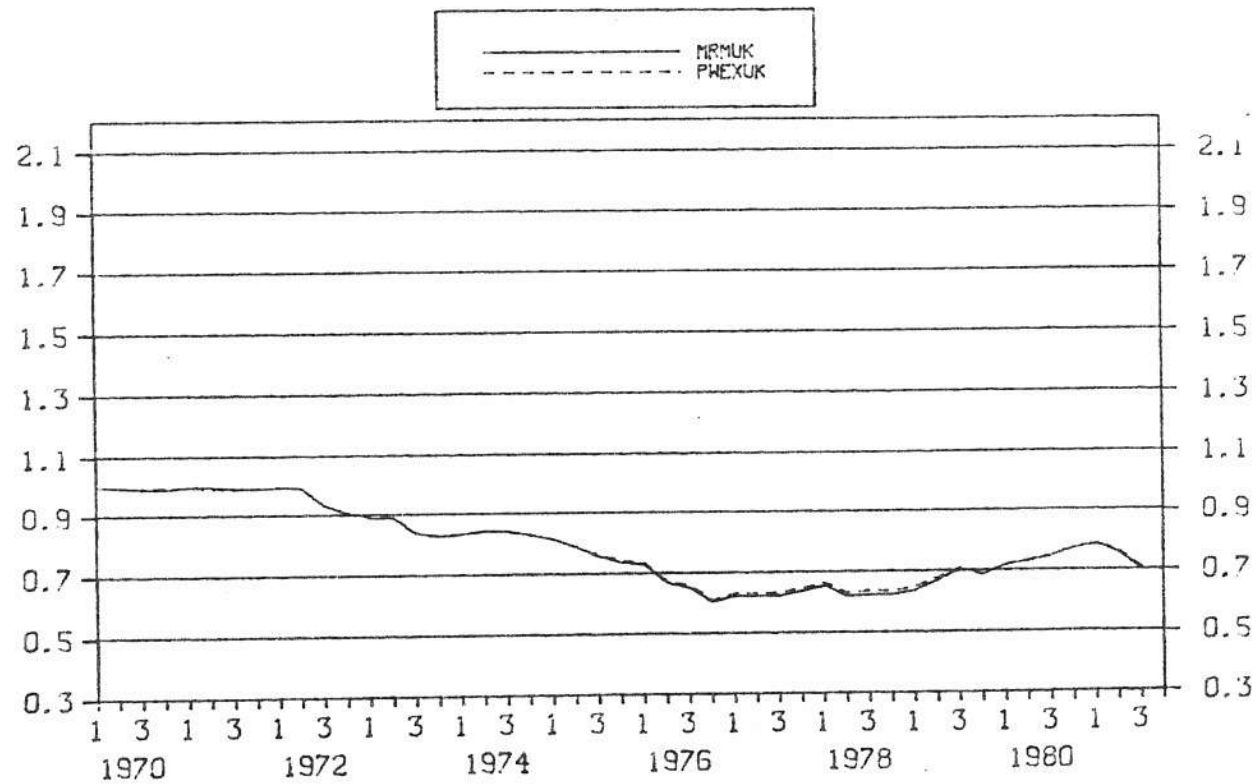
APPENDIX

EFFECTIVE EXCHANGE RATES (*)

01 1970/ 04 1981

(INDEXES BASE 01 1970=1.)





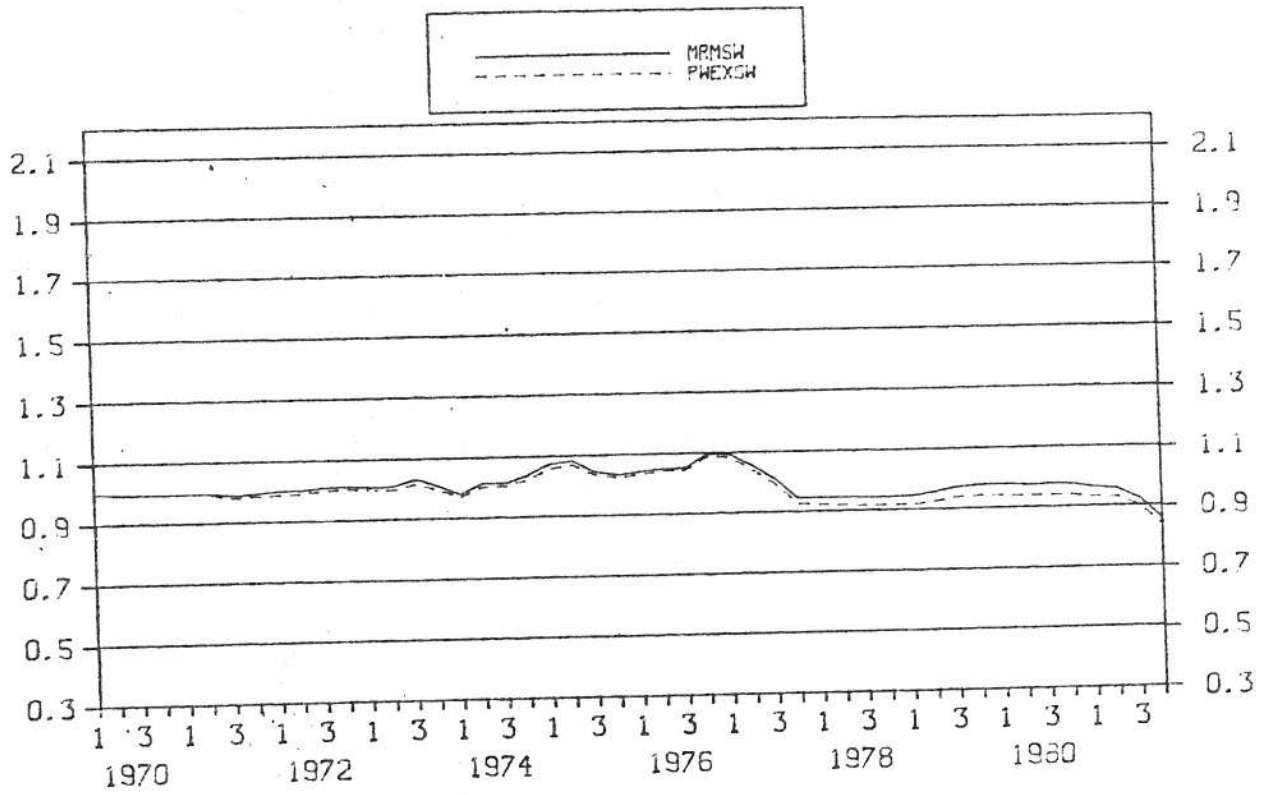
MRMM IS THE FUND'S MERM INDEX
MPMEX IS THE COBB-DOUGLAS INDEX

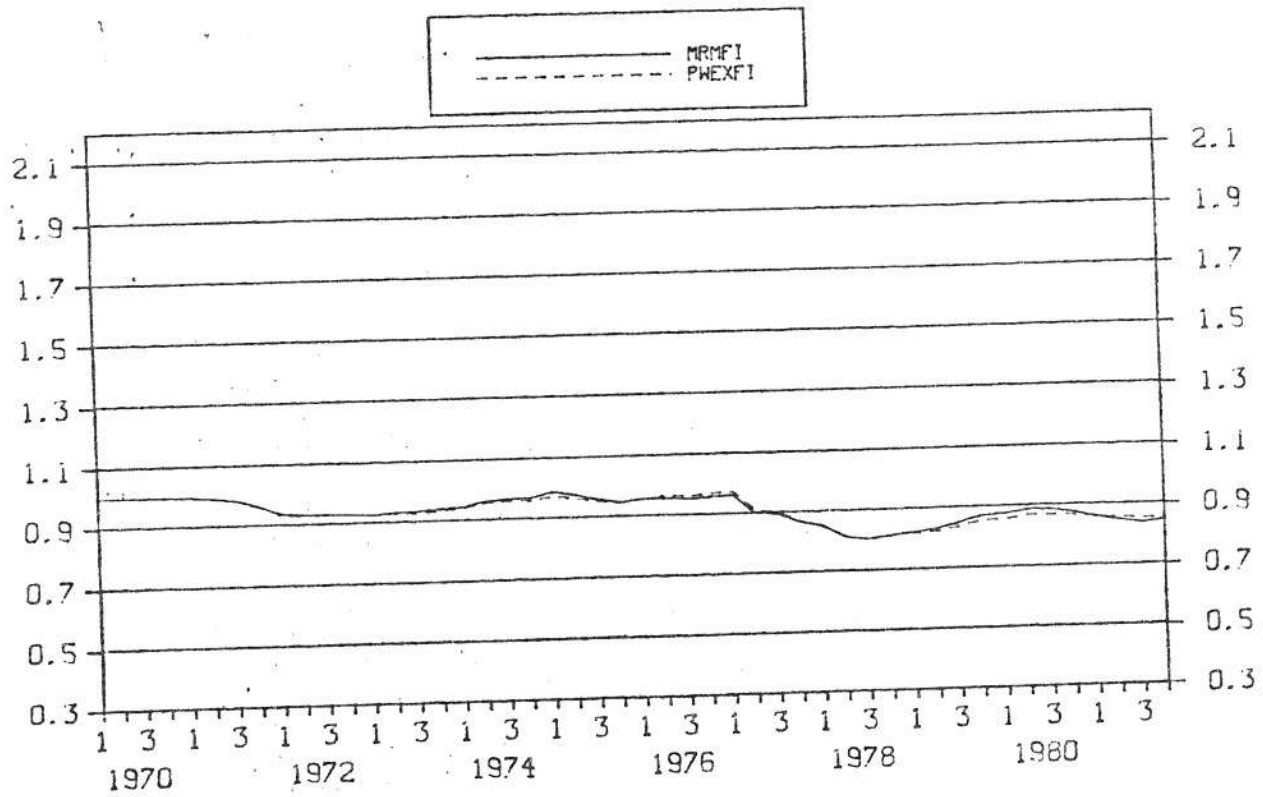
APPENDIX

EFFECTIVE EXCHANGE RATES (*)

01 1970/ 04 1981

(INDEXES BASE 01 1970=1.)





*MRM IS THE FUND'S MERM INDEX
*PWEX IS THE COBB-DOUGLAS INDEX