Structural Break, Nonlinearity, and Asymmetry: A re-examination of PPP proposition

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Abstract

In this study, we propose a new unit root test procedure that allows for both gradual structural break and asymmetric nonlinear adjustment towards the equilibrium level. Small-sample properties of the new test are examined through Monte-Carlo simulations. The simulation results suggest that the new test has satisfactory size and power properties. We then apply this new test along with other unit root tests to examine stationarity properties of real exchange rate series of the sample countries. Our test rejects the null of unit root in more cases when compared to alternative tests. Overall, we find that the PPP proposition holds in majority of the European countries examined in this paper.

Keywords: Smooth Structural Break; Nonlinear Unit Root test; PPP

1. Introduction

In this paper, we propose a novel unit root test procedure. The distinguishing feature of the proposed test is that it allows for simultaneous structural change and asymmetric nonlinear adjustment towards the equilibrium level. We employ logistic transition function to model gradual structural breaks. Logistic functions have widely been used in the empirical literature and been proved to capture structural breaks in the series quite well. See, for example, Granger and Terasvirta (1993), Lin and Terasvirta (1994), and Lundbergh et al. (2003), among others. Leybourne et al. (1998), Sollis (2004) and Omay and Yildirim (2014) also used logistic functions to model structural breaks within unit root testing framework. While Leybourne et al. (1998) considered only linear adjustment, Sollis (2004) modelled adjustment towards the equilibrium level using threshold regression models and Omay and Yildirim (2014) suggest ESTAR nonlinearity adjustment towards the equilibrium level. Threshold models allow for asymmetric adjustment depending on the sign on deviations from equilibrium irrespective of the size of the disequilibrium.

Christopoulos and Leon-Ledesma (2010), on the other hand, combined unit root tests of Kapetanios et al. (2003) and Becker et al. (2006). In particular, following Becker et al. (2006), they used trigonometric functions and Fourier series expansion to model structural breaks. Adjustment towards the trend was modelled using symmetric exponential smooth transition autoregressive (ESTAR) model as in Kapetanios et al. (2003). ESTAR-type nonlinearity assumes that adjustment to equilibrium depends on the size of deviation irrespective of size.

Therefore, we consider asymmetric ESTAR-type adjustment as proposed by Sollis (2009). We examine small-sample properties of the proposed tests using Monte-Carlo simulations. The results of the simulation studies show that the newly proposed test has reasonable good power and outperform alternative unit root tests that also allow for structural break and nonlinear adjustment.

Using the newly proposed test we examine stationarity properties of the real exchange rate series of the sample European countries. We also apply conventional ADF test as well as nonlinear unit root test procedures of Leybourne et al. (1998), Kapetanios et al. (2003) and Sollis (2004; 2009). We find that the newly proposed test rejects the null hypothesis of unit root in many cases when compared to the mentioned test procedures. This shows empirical superiority of our tests against existing unit root test. All in all, the results of this study suggest that the real exchange rate series of 24 countries are stationary, thus providing support for the PPP proposition in these countries.

The remaining of the paper is organized as follows. In the next section we present the test procedure and drive critical values. Small-sample properties of the proposed tests are presented in Section 3. Section 4 present empirical applications, and section 5 concludes.

2. The model and testing framework

2.1. Gradual structural change model

Consider following smooth transition models for the time series y_t for $t = 1,2,\dots,T$:

Case 1 $$y_t = \alpha_1 + \alpha_2 S_t(\gamma,\tau) + u_t \quad (1)$$

Case 2 $$y_t = \alpha_1 + \beta_1 t + \alpha_2 S_t(\gamma,\tau) + u_t \quad (2)$$

Case 3 $$y_t = \alpha_1 + \beta_1 t + \alpha_2 S_t(\gamma,\tau) + \beta_2 t S_t(\gamma,\tau) + u_t \quad (3)$$

where $S_t(\gamma,\tau)$ is the logistic smooth transition function over sample of T :

$$S_t(\gamma,\tau) = [1 + \exp\{-\gamma(t - \tau T)\}]^{-1}, \ \gamma > 0 \quad (4)$$

The function $S_t(\gamma, \tau)$ is a continuous function bounded between zero and one. The parameters γ and τ determine the smoothness and location, respectively, of the transition from one regime to the other. For small values of γ , the transition between two regimes occur very slowly. In the limiting case when $\gamma = 0$, $S_t(\gamma, \tau) = 0.5$ for all values of t . As the smoothness parameter γ becomes very large, the transition function approaches a Heaviside step function, and consequently, the change from one regime to the other becomes almost instantaneous at time $t = \tau T$. Thus, the transition function $S_t(\gamma, \tau)$ nests the no-break and the instantaneous break models as special cases. In particular, if $\gamma = 0$, then the transition function $S_t(\gamma, \tau)$ collapses to constant, and hence, equations (1)-(3) reduce to a conventional linear regression models. On the other extreme, as γ approaches infinity, the model allows for an instantaneous break at time $t = \tau T,$ as analysed by Perron (1989). If it is assumed that u_t is a mean-zero $I(0)$ process, then y_t will be stationary process around the mean that changes from an initial value α_1 to the final value $\alpha_1 + \alpha_2$ in model (1). Note that the specification given in equation (2) allows for a break only in the mean of the series but the slope is assumed to be constant. Similarly, the process given in equation (3) allows the mean of the nonlinear attractor to change gradually from α_1 to t $\alpha_1 + \alpha_2$ whereas its slope changes from β_1 to $\beta_1 + \beta_2$ at the time $t = \tau T$. See also Leybourne et al. (1998).

Leybourne et al. (1998) and Sollis (2004) also used the above gradual structural break specification. Leybourne et al. (1998) modelled adjustment towards the nonlinear attractor using a conventional linear model. On the other hand, Sollis (2004) allowed for threshold-regression type asymmetry whereas speed of adjustments towards the equilibrium depends on the sign of the disequilibrium. Here, we model adjustment towards the equilibrium using asymmetric ESTAR (AESTAR) nonlinearity as in Sollis (2009).

Consider the following AESTAR model for the deviations from the equilibrium level:

$$\Delta u_t = G_t(\theta_1, u_{t-1})\{F_t(\theta_2, u_{t-1})\rho_1 + (1 - F_t(\theta_2, u_{t-1}))\rho_2\}u_{t-1} + \epsilon_t \quad (5)$$

$$G_t(\theta_1, u_{t-1}) = 1 - \exp(-\theta_1(u_{t-1}^2)), \ \theta_1 > 0 \quad (6)$$

$$F_t(\theta_2, u_{t-1}) = [1 + \exp(-\theta_2 u_{t-1})]^{-1}, \ \theta_2 > 0 \quad (7)$$

where $\epsilon_t \sim i.i.d(0, \sigma^2)$.

The logistic transition function $F_t(\theta_2, u_{t-1})$ is similar to the $S_t(\gamma, \tau)$ function that governs the gradual break in the mean and/or trend of the series. As the u_t is a zero-mean variable, the two regimes associated with the $F_t(\theta_2, u_{t-1})$ function are determined by positive and negative realizations of the disequilibrium u_t. The exponential transition function $G_t(\theta_1, u_{t-1})$ is a symmetrically U-shaped function bounded between zero and one. The regimes associated with the $G_t(\theta_1, u_{t-1})$ function are determined by small and large absolute values of the disequilibrium u_t, irrespective of the sign of deviation from the equilibrium.

In order to depict the nonlinear dynamics implied by the AESTAR model, first consider the case when u_{t-1} moves from zero to minus infinity. In this case, the logistic transition function $F_t(\theta_2, u_{t-1}) \to 0$ as $u_{t-1} \to -\infty$, and hence, transition will be from the inner regime

$$\Delta u_t = \epsilon_t \quad (8)$$

to the outer regime

$$\Delta u_t = \rho_2 u_{t-1} + \epsilon_t \quad (9)$$

since the exponential function $G_t(\theta_1, u_{t-1})$ moves from zero to one as $u_{t-1} \to -\infty$. If the disequilibrium goes from zero to positive infinity, on the other hand, the logistic transition function $F_t(\theta_2, u_{t-1}) \to 1$ as $u_{t-1} \to +\infty$. In this case, the transition will be from the inner regime

$$\Delta u_t = \epsilon_t$$

to the outer regime

$$\Delta u_t = \rho_1 u_{t-1} + \epsilon_t \qquad (10)$$

as the exponential function $G_t(\theta_1, u_{t-1})$ also moves from zero to one when $u_{t-1} \to +\infty$. It is evident that global stationarity of the AESTAR process given above requires $\theta_1 > 0$, $\rho_1 < 0$, $\rho_2 < 0$ (see also, Sollis, 2009). Naturally, it might be the case that $\rho_1 \neq \rho_2$, which implies that the adjustment towards the nonlinear attractor depends also on the size, but not only on the sign of the deviation. Notice also that $\rho_1 = \rho_2$ gives symmetric ESTAR adjustment towards equilibrium, first considered by Kapetanios et al. (2003).

2.2. Unit Root Tests against AESTAR stationarity

The unit root hypothesis can be tested against the globally stationary AESTAR nonlinearity formally by testing the null hypothesis:

$$H_0: \theta_1 = 0 \qquad (11)$$

against the alternative:

$$H_1: \theta_1 > 0 \qquad (12)$$

However, testing the null hypothesis (11) directly is not feasible because of the presence of unidentified nuisance parameters under the null. In particular, the parameters θ_2, ρ_1 and ρ_2 are not identified under this null. Kapetanios et al. (2003) and Sollis (2009) solved this problem by replacing the transition functions by appropriate Taylor series approximation following Luukkonen et al. (1988). Replacing the transition functions by their first-order Taylor series approximation in equation (5) we obtain:

$$\Delta u_t = \varphi_1 u_{t-1}^3 + \varphi_2 u_{t-1}^4 + \vartheta_t \tag{13}$$

where $\varphi_1 = \theta_1 \rho_2$, $\varphi_2 = \frac{1}{4}\theta_1 \theta_2 (\rho_1 - \rho_2)$, and the ϑ_t term comprises the original disturbances ϵ_t as well as the error term arising from the Taylor approximation. Now, the null hypothesis $H_0 : \theta_1 = 0$ becomes equivalent to:

$$H_0 : \varphi_1 = \varphi_2 = 0 \tag{14}$$

In equation (5) it was assumed that the error term ϵ_t is serially uncorrelated. In order to allow for serial correlation, we augment the regression equation as follows assuming that these serially correlated errors enter in a linear fashion[1]:

$$\Delta u_t = G_t(\theta_1, u_{t-1})\{F_t(\theta_2, u_{t-1})\rho_1 + (1 - F_t(\theta_2, u_{t-1}))\rho_2\} u_{t-1} + \sum_{j=1}^{p} \delta_j \Delta u_{t-j} + \epsilon_t \tag{15}$$

with $\epsilon_t \sim i.i.d(0, \sigma^2)$.

After replacing transition functions by their appropriate Taylor series expansions, we obtain the following auxiliary regression equation:

[1] Alternatively, as Sollis (2009) argues, if higher order dynamics are nonlinear, then the augmentation terms can be interpreted as a first-order approximation.

$$\Delta u_t = \varphi_1 u_{t-1}^3 + \varphi_2 u_{t-1}^4 + \sum_{j=1}^{p} \delta_j \Delta u_{t-j} + \vartheta_t \qquad (16)$$

Now, one may test the null hypothesis $H_0: \varphi_1 = \varphi_2 = 0$ using auxiliary regression (16) instead.

In practice, this null hypothesis can be tested in two steps. First, using any appropriate nonlinear least squares (NLS) algorithm, one must estimate the preferred deterministic component given in equations (1)-(3) and collect residuals $\hat{u}_t$. Then, using these residuals one may estimate the regression equation (16) by ordinary least squares and test the null hypothesis H_0 using conventional F -test. Such a two-step procedure to test unit root has a nice property in that it allows for a possibly nonlinear deterministic trend function under both the null and the alternative hypotheses, without introducing any parameters that are irrelevant under either. We denote the test statistic as F_{SBAE} statistic.

The NLS estimation of transition parameters does not admit closed-from solutions. Hence, it is extremely difficult to establish any analytical relationship between $\hat{u}_t$ and y_t. This renders determination of the asymptotic distribution of the test statistics by analytical means almost impracticable (see also Leybourne et al., 1998). Therefore, we approximate the distribution of the test statistics via stochastic simulations. Computed critical values of the test statistics are presented in Table (1) below.

(Table 1)

3. Finite sample performance

3.1. Small-sample size analysis

In this section we analyze small sample properties of the proposed tests. We first analyze small-sample size properties of the F_{SBAE} statistic. To evaluate the size of the test statistics, we consider the following data generating process (DGP):

$$y_t = y_{t-1} + \varepsilon_t \text{ for } t = 1,2,\dots,T$$
$$\text{with } \varepsilon_t = \rho\varepsilon_{t-1} + v_t,\ \varepsilon_0 = 0\text{, and} \tag{17}$$

We set the residual autocorrelation parameter $\rho = \{0, 0.5\}$, the sample size $T = \{50,100,200,500\}$ and use 2000 replications to compute empirical size of the test. In the case with no serial correlation ($\rho = 0$), we do not include any augmentation terms in the test regressions. With serially correlated errors ($\rho = 0.5$), we employ one augmentation to avoid substantial size distortions. Simulation results are presented below in Table 2 and suggest no serious size distortions for all three test statistics.

(Table 2)

3.2. Small-sample power analysis

Now, we turn to small-sample power properties of the proposed test. For brevity we consider only the Case 1, where data has no deterministic trend. For comparison purposes, we also compute power properties of several alternative tests that allow for gradual structural breaks and/or nonlinear adjustment towards equilibrium. In particular, we use nonlinear unit root tests of Kapetanios et al. (2003) and Sollis (2009) which use ESTAR-type nonlinear adjustment as well as unit root tests of Leybourne et al. (1998) and Sollis (2004), which use logistic transition functions to model gradual structural changes[2]. A gradually changing mean resembles a straight line rather than a fixed mean. Therefore, we included a linear trend in the test regression in unit root tests of Kapetanios et al. (2003) and Sollis (2009), who do not allow a break in the slope of the series (see also discussions in Leybourne et al., 1998).

We consider the following DGP for power comparisons.

$$y_t = \alpha_1 + \alpha_2 S_t(\gamma, \tau) + u_t \tag{18}$$

$$\Delta u_t = G_t(\theta_1, u_{t-1})\{F_t(\theta_2, u_{t-1})\rho_1 + (1 - F_t(\theta_2, u_{t-1}))\rho_2\}u_{t-1} + \varepsilon_t \tag{19}$$

where $\varepsilon_t \sim iid\ N(0,1)$, and transition functions $G_t(\theta_1, u_{t-1})$ and $F_t(\theta_2, u_{t-1})$ are as defined in equations (6) and (7). We choose a wide range of parameter values for power comparisons. In particular, we consider all combinations of the following parameter values: $\alpha_2 = (5.0, 10.0)$, $\theta_1 = (0.1, 1.0)$, $\theta_2 = (0.1, 0.5, 1.0)$, and $\rho_2 = (-0.05, -0.5, -1.0)$. In all cases we set $\gamma = 1.0$, $\alpha_1 = 1.0$, $\tau = 0.5$, and $\rho_1 = -0.05$, and sample size at $T = 100$. Power comparisons of alternative tests are presented below in Table 3.

(Table 3)

As can readily be seen from the table, our test outperforms all remaining tests when $\alpha_2 = 10.0$. This particular value of the α_2 parameter corresponds to relatively large breaks. It is also noteworthy that our test provides substantial power gains over the unit root test procedure of Sollis (2004) which also takes account of both structural breaks and asymmetric adjustment towards the equilibrium. The F_{SBAE} test preserves relatively good power properties for $\alpha_2 = 5.0$ as well, which corresponds to relatively small breaks in the mean of

[2] We do not include the Omay and Yildirim (2014) OY test due to the reason that it is the symmetric counter part of our test and our test covers this test as a special case.

the series. Other tests outperform the F_{SBAE} test only when $\rho_2 = -0.05$ in the case of small breaks $\alpha_2 = 5.0$. Recall also that we set $\rho_1 = -0.05$ as well. When $\rho_1 = \rho_2 = \rho$, the equation (19) reduces to:

$$\Delta u_t = G_t(\theta_1, u_{t-1})\rho u_{t-1} + \varepsilon_t \tag{20}$$

This is a symmetric ESTAR model considered in Kapetanios et al. (2003). Therefore it could reasonably be expected that the F_{SBAE} test will lose power as this test requires estimation of a redundant parameter. Note also that all tests suffer from substantial power losses when $\rho_1 = \rho_2 = -0.05$ as well. In this case, the coefficient on the lagged level variable u_{t-1} in the equation (20) changes gradually from -0.05 to 0.0, implying that the series under investigation are near unit root process. Hence, power of all tests drop significantly for these particular values of ρ_1 and ρ_2. In passing note that the ts_α test of Sollis (2004) and the s_α test of Leybourne et al. (1998) have relatively good power over the t_{NL} test of Kapetanios et al. (2003) and the $F_{AE,t}$ test of Sollis (2009) as the latter tests do not allow for structural breaks whereas the former tests do.

We also compared powers of these tests using different smoothness parameters, which gave qualitatively the same results. In particular, the simulation results (not reported here, available upon request) suggested that the proposed F_{SBAE} test has better power properties for larger breaks. Other tests had marginally better power only for relatively small breaks and near unit root processes.

4. An Empirical Application. Testing validity of the PPP

The basis of the PPP proposition is the law of one price, which states that the price of a commodity (or a bundle of commodities) must be the same across all countries when expressed in a single currency. According to the PPP proposition, nominal exchange rates move one-for-one with relative prices in the long run. Therefore, tests of the PPP have usually been based on testing stochastic properties of the real exchange rate series.

In this paper, we consider 28 EU member countries. Although most of the previous researchers have used bilateral real exchange rate series, in this study we use trade-weighted real effective exchange rate (REER) series following Bahmani-Oskooee et al. (2007) and Telatar and Hasanov (2009). As they point out, stationarity of the REER series implies that PPP holds not only with respect to a certain trading partner, but with respect to country's many trading partners as well. In addition, movements in the REER series are more important for studies of international trade flows. We use quarterly data on REER (vis-à-vis 42 trading partners, deflated using consumer price indices) covering the period 1994:Q1-2014:Q1. Data were obtained from Eurostat.

In addition to the F_{SBAE} test proposed in this paper, we also employ other tests that allow for possible nonlinear adjustment and gradual structural breaks. In particular, we use conventional ADF test, the nonlinear unit root tests of Kapetanios et al. (2003) and Sollis (2009), the unit root test of Leybourne et al. (1998) that allow gradual structural break, and the test of Sollis (2004) that allow for both gradual structural break and asymmetric adjustment towards the trend. We carried out these tests with and without time trend. Structural reforms and accession to the EU brought about rapid productivity gains especially in the transition countries. Hence, we include a time trend to account for possible differences in productivity growth across countries and their main trading partners. If exchange rates are found to be stationary around linear and/or nonlinear trend, this can be considered as an evidence of the Balassa-Samuelson effect.

The results of these tests are presented in Table 4 below.

(Table 4)

As can be seen from the Table 4, the conventional ADF test rejects the null hypothesis of unit root only in four out of 28 cases, namely for Denmark, Slovenia, Finland and Slovenia. Allowing for nonlinearity and/or structural breaks result in more frequent rejection of the null hypothesis. In particular, the unit root test of Kapetanios et al. (2003) rejects the null hypothesis of unit root in nine cases, i.e. for Bulgaria, Estonia, Italy, Cyprus, the Netherlands, Poland, Romania, Slovenia and the UK. The unit root test of Sollis (2009) rejects the null of unit root in 10 cases, including Bulgaria, Estonia, Italy, Cyprus, the Netherlands, Poland, Romania, Slovenia, Slovakia and Finland. The test proposed by Leybourne et al. (1998) that allows for only structural break but not nonlinear adjustment, rejects the unit root null hypothesis in seven cases only, i.e., in cases of the Czech Republic, Ireland, Hungary, Malta, the Netherlands, Slovakia and Finland. The unit root test of Sollis (2004) that modify the Leybourne et al. (1998) test to allow for asymmetric adjustment towards the changing mean/trend, rejects the null hypothesis of unit root in three more cases, including Italy, Cyprus and Slovenia.

The F_{SBAE} test proposed in this paper, on the other hand, rejects the null hypothesis of unit root in more cases than the existing alternatives in the literature. In particular, our test suggest that real exchange rate series of 14 countries, namely, those of Belgium, the Czech Republic, Germany, Ireland, Spain, France, Croatia, Italy, Luxembourg, Hungary, Portugal, Romania, Slovenia and Slovakia are stationary, consistent with the PPP proposition.

The results of the unit root tests that are reported in Table 4 suggest that allowing for more complex dynamics of real exchange rate series result in more frequent rejection of the null hypothesis of unit root in compliance with the PPP proposition. Note that the ADF test rejects the null hypothesis only in four cases. Modelling structural breaks and nonlinear adjustment towards the equilibrium brought about rejection of the null hypothesis in 10 cases. However, our test which allow for both structural changes and asymmetric nonlinear adjustment towards the attractor produced evidence in favour of the PPP proposition in 14 countries. These findings imply that the real exchange rate dynamics of the sample countries are indeed highly nonlinear, and clearly confirm empirical superiority of our tests in such cases.

Here, we must once remind that none of the above test procedures has absolute power over the other procedures in all cases. In fact, the test procedures proposed by Kapetanios et al. (2003) and Sollis (2009), for example, have relatively better power properties if the true data generating process follows symmetric or asymmetric STAR-type nonlinearity. On the other hand, the unit root tests of Leybourne et al. (1998) and Sollis (2004) have better powers if there are significant structural changes in the series. Similarly, the F_{SBAE} test proposed in this paper have better small-sample properties over the alternative tests if there is a gradual break in the mean and/or slope of the series and adjustment towards the attractor have asymmetric ESTAR-type nonlinearity. If the true data generating process is almost linear or if there is no structural break, introduction of redundant parameters will seriously reduce power of the test. Therefore, the results of these tests must be construed and compared very carefully.

All in all, the results of this study suggest that the PPP proposition holds in majority of the sample countries. Out of the 28 countries, we were not able to reject the null hypothesis of unit root only in cases of Greece, Latvia, Lithuania, and Austria using any of the tests considered above.

5. Conclusions

In this paper, we examined the validity of the PPP proposition for 28 EU-member countries. In order to model real exchange rate dynamics more properly, we developed a new unit root test that allows for gradual structural break and asymmetric nonlinear adjustment towards the attractor. We examined small-sample properties of the proposed test via simulation exercises. The results of these exercises suggest that the proposed test have satisfactory finite sample properties.

We also applied other unit root tests to examine stationarity properties of the trade-weighted real effective exchange rate series of the sample countries. The results suggest that allowing for more complex dynamics in real exchange rates results in more frequent rejection of the null hypothesis of unit root in accordance with the PPP proposition. In particular, our test that allows for both gradual structural break and nonlinear adjustment towards the attractor produced evidence supporting the PPP proposition in more countries. Our results also suggest that adjustment towards the equilibrium might be inherently nonlinear in most of the sample countries. Such nonlinearities imply that the speed of adjustment of deviations of real exchange rates from the equilibrium depends on both the sign and magnitude of the deviation. This also suggests that small and big negative and positive shocks to the equilibrium real exchange rates will have varying effects on trade flows of the sample countries.

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Table 1. Critical values of the F_{SB} statistic

	Case 1			Case 2			Case 3		
T	10%	5%	1%	10%	5%	1%	10%	5%	1%
25	7.814	9.698	14.343	10.245	12.498	17.546	12.125	14.518	19.845
50	7.115	8.374	11.609	8.645	10.198	13.898	9.828	11.584	15.537
100	7.101	8.110	10.756	8.339	9.642	12.681	9.209	10.617	13.621
200	7.010	8.105	10.535	8.394	9.671	12.406	9.129	10.488	13.286
500	6.950	8.086	10.001	8.387	9.595	12.155	9.132	10.520	13.219

Note: Case 1, Case 2 and Case 3 refer to the underlying model with break in mean without a trend (eq. 1), break only in mean but not in trend (eq. 2), and break both in mean and trend (eq. 3), respectively.

Table 2. Empirical Sizes of the Test

	Case 1		Case 2		Case 3	
	$\rho = 0$	$\rho = 0.5$	$\rho = 0$	$\rho = 0.5$	$\rho = 0$	$\rho = 0.5$
$\mathbf{T = 50}$	5.12	6.28	5.04	5.80	5.02	5.08
$T = 100$	5.34	6.60	5.60	6.90	6.10	5.10
$T = 200$	4.74	6.26	4.82	6.28	6.34	4.68
$T = 500$	5.26	5.80	5.64	5.92	5.56	5.62

Table 3. Power analysis of alternative test

α_2	θ_1	θ_2	ρ_2	F_{SBAE}	t_{NL}	$F_{AE,t}$	s_α	ts_α	F_α
10.0	0.1	0.1	-1.0	**0.281**	0.010	0.026	0.141	0.157	0.141
10.0	0.1	0.1	-0.5	**0.122**	0.004	0.009	0.080	0.082	0.074
10.0	0.1	0.1	-0.05	**0.036**	0.004	0.012	0.026	0.033	0.024
10.0	0.1	0.5	-1.0	**0.280**	0.005	0.020	0.142	0.176	0.147
10.0	0.1	0.5	-0.5	**0.135**	0.007	0.012	0.085	0.086	0.081
10.0	0.1	0.5	-0.05	**0.033**	0.006	0.009	0.019	0.025	0.018
10.0	0.1	1.0	-1.0	**0.255**	0.011	0.022	0.140	0.160	0.143
10.0	0.1	1.0	-0.5	**0.130**	0.005	0.011	0.080	0.080	0.073

10.0	0.1	1.0	-0.05	**0.041**	0.008	0.017	0.028	0.031	0.026
10.0	1.0	0.1	-1.0	**0.621**	0.021	0.045	0.503	0.620	0.543
10.0	1.0	0.1	-0.5	**0.259**	0.003	0.008	0.220	0.242	0.225
10.0	1.0	0.1	-0.05	**0.042**	0.003	0.004	0.033	0.029	0.029
10.0	1.0	0.5	-1.0	**0.617**	0.013	0.041	0.496	0.597	0.537
10.0	1.0	0.5	-0.5	**0.253**	0.006	0.015	0.228	0.249	0.232
10.0	1.0	0.5	-0.05	**0.033**	0.003	0.006	0.031	0.027	0.027
10.0	1.0	1.0	-1.0	**0.626**	0.019	0.040	0.512	0.602	0.557
10.0	1.0	1.0	-0.5	**0.239**	0.006	0.013	0.201	0.223	0.207
10.0	1.0	1.0	-0.05	**0.038**	0.008	0.011	0.027	0.028	0.024
5.0	0.1	0.1	-1.0	**0.296**	0.116	0.207	0.175	0.201	0.180
5.0	0.1	0.1	-0.5	**0.141**	0.053	0.106	0.109	0.096	0.103
5.0	0.1	0.1	-0.05	0.042	0.022	**0.047**	0.043	0.045	0.041
5.0	0.1	0.5	-1.0	**0.320**	0.105	0.194	0.163	0.187	0.167
5.0	0.1	0.5	-0.5	**0.148**	0.054	0.097	0.109	0.102	0.105
5.0	0.1	0.5	-0.05	0.046	0.030	**0.052**	0.045	0.047	0.042
5.0	0.1	1.0	-1.0	**0.268**	0.110	0.191	0.163	0.184	0.166
5.0	0.1	1.0	-0.5	**0.143**	0.047	0.096	0.104	0.091	0.102
5.0	0.1	1.0	-0.05	**0.048**	0.023	0.035	0.038	0.046	0.036
5.0	1.0	0.1	-1.0	**0.621**	0.230	0.378	0.534	**0.621**	0.567
5.0	1.0	0.1	-0.5	**0.267**	0.092	0.156	0.242	0.259	0.246
5.0	1.0	0.1	-0.05	0.047	0.020	0.042	**0.048**	0.045	0.043
5.0	1.0	0.5	-1.0	**0.640**	0.231	0.381	0.532	0.626	0.565
5.0	1.0	0.5	-0.5	**0.268**	0.085	0.154	0.253	0.240	0.251
5.0	1.0	0.5	-0.05	0.048	0.026	**0.056**	0.038	0.036	0.032
5.0	1.0	1.0	-1.0	**0.624**	0.232	0.373	0.517	0.620	0.557
5.0	1.0	1.0	-0.5	**0.242**	0.077	0.150	0.219	0.236	0.223
5.0	1.0	1.0	-0.05	0.047	0.025	0.044	**0.051**	0.041	0.044

Notes: t_{NL} denotes t-statistic of Kapetanios et al. (2003), $F_{AE,t}$ is F-statistic of Sollis (2009), s_{α} is the t-statistic of Leybourne et al. (1998), and ts_{α} and F_{α} are t-max and F-statistics of Sollis (2004), respectively. Power analysis is based on 2000 replications. Boldface figures are the highest values and therefore indicate preferred test statistics.