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Abstract. We consider consumers with the same reservation price, who desire to buy at most one unit of a good. Firms compete only in prices but there are other features firms cannot control that would eventually lead an agent to buy in one firm or another. We introduce such uncertainty in a model of a price competition game with incomplete information. This competition takes place under stability and we provide equilibrium existence results. We analyze different specifications of residual demands which yield further interpretations that deepen the phenomenon of price dispersion, Bertrand's paradox and market power.

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1 Introduction

This work adds to the literature on price competition which goes back to the classical Bertrand model that has originated numerous studies with alternative assumptions on economic primitives. In this scenario of oligopolistic price competition, the formulations of the demands faced by firms play a key role. Different ways of defining these residual demands lead to different games with a variety of equilibrium notions and a wide range of results. For instance, different works considering that the demand faced by each firm depends on the consumers' information, lack stability in competition due to the discontinuity of such demands (see Salop and Stiglitz, 1977, and Varian, 1980). However, it is not easy to argue that arbitrarily small variations in the prices lead to significant changes in the demand functions. This was already pointed out by Hotelling (1929); if a seller gradually increases the price of a good while her rivals keep their prices fixed, sales will diminish continuously, rather than fall in an abrupt way.

In this paper, we consider a continuum of consumers with the same reservation price, who desire to buy at most one unit of a good. There is a finite set of firms that produce and sell such a commodity and compete in prices but the demand each firm faces does not depend only on prices. There are other features, collected in a variable that we call "type", that have influence on determining the mass of consumers that buy in a firm or another. This type variable is not a strategy for any firm but represents certain attributes of the stores which may be assessed differently by consumers and may encompass many different specifications, like reputation, kind sellers, crowding effects, or even something as simple as having heating in winter time, or air conditioning in summer.[1]

Thus, given a vector of prices, the measure of the set of consumers that each firm absorbs (residual demands) is defined by taking into account the type of every firm. We assume that these residual demands are continuous and we elaborate on consumers' behavior that ensures such a continuity property, allowing for gradual shiftings of consumers from one firm to another when they perceive differences in the price of the good. Thus, sometimes consumers choose a certain shop even though they are aware that the price of the good is slightly more

[1]Indeed, we can find somehow the idea of the type variable in Hotelling's work, when he mentions some reasons for which a costumer would prefer to buy a good in one shop than in another even if she pays more for the good, such as location, way of doing business, family relationship or friendship with the owner, etc.

expensive than in another because of the effect that the type variable has in the distribution of demand.

On the other hand, the types of the firms are not necessarily perfectly known. Therefore, we introduce a game with incomplete information where firms compete in prices, and where such competition takes place in a stable way. Thus, there is an exogenous information structure which is defined by a probability distribution on the set of type vectors which is common knowledge for the firms. When this probability distribution is degenerated we are in the complete information scenario.

In this way, we adapt the incomplete information game theoretical setting proposed by Milgrom and Weber (1985) to a game where firms compete in prices and the lack of information is associated to the type variable that jointly with the prices determine the residual demands. Therefore, the incomplete information our game presents is symmetric. There are papers that also analyze price competition with incomplete information but differ from our approach since they consider information asymmetries that mainly refer to the fact that firms are privately informed about their respective cost functions (see, for instance, Spulber,1995, and Bagwell and Lee, 2014).

Under standard assumptions, we show existence of equilibrium in distributional strategies.[2] We also prove that there exists an approximate equilibrium in pure strategies. We emphasize that, in the incomplete information setting, a pure strategy of a firm is a function that assigns a price to each type. A game with demand functions which are linear on prices illustrates this point.

The model we consider opens up the possibility of testing it with different specifications of the information structure and the demands, which lead to many different games, explaining a variety of concerns in the light of a price competition analysis under stability.

First, taking residual demand functions which are linear in prices, and in a context of incomplete information, price dispersion arises as an equilibrium in pure strategies, in contrast with other approaches that explain this phenomenon by means of a mixed strategy equilibrium (see, for instance, Varian, 1980). Then, we also explore whether the incomplete information framework might give advantage to some firms over the complete information situation. For it, using the

[2] We remark that Milgrom and Weber (1985) showed that distributional strategies are simply another way of representing mixed and/or behavioral strategies.

same game, we show that in the equilibrium, the expected payoffs of a firm are higher under uncertainty on its type than in a complete information scenario. Therefore, an analysis of the expected payoffs under complete or incomplete information would be an interesting exercise for the firms in order to apply it to their advertising policy: depending on the result, firms may prefer to advertise in such a way their type becomes public information, or on the contrary, to do it trying to keep their type "hidden".

Next, we consider a particular formulation of the residual demands by separating the effects of prices and types. Within this setting, we state a game which not only provides a different way to overcome the Bertrand paradox, but also shows a situation where arbitrarily small firms have market power. This is basically due to the shape of the residual demands and not to the cost functions.

Finally, types may become a relevant variable only when the difference in prices is small enough. Thus, when the differences among prices are sufficiently large, the effect of types becomes negligible and the firm charging the lowest price faces all the demand. We state specific demands highlighting this fact. In this case, we point out that our analysis allows for a better explanation of the degree of price dispersion.

The remainder of the paper is organized as follows. In Section 2, we present an incomplete information game where a finite number of firms compete in prices and there is stability in competition. We also show existence results for different notions of equilibrium. In Section 3, we analyze a more particular situation where the demands faced by the firms depend separately on types and prices. In Section 4, we study a game in which the type becomes the relevant variable only when prices do not differ too much. Each section includes different specifications of the residual demands which illustrate our general approach and give rise to a variety of further results and interpretations. The demands that we consider in each example can be explained by a behavior of consumers taking into account perturbed prices as we state in a final appendix.

2 The game

Let us consider a continuum of consumers[3] represented by the interval $[0,1]$, who desire to buy, at most, one unit of a commodity. Every consumer has the same reservation price r, which is the maximum price they are willing to pay for the good.

There are n firms or stores that produce the commodity. Let $N = \{1, \ldots, n\}$. Each firm $i \in N$ has a continuous cost function $C_i : [0,1] \to \mathbb{R}_+$ defined on the mass of customers. Firms have market power and compete in prices. Let K_i be a closed real subinterval of $[0,r]$ where firm i selects a price[4], and let $\mathcal{K} = K_1 \times \ldots \times K_n$. We consider a scenario with stability in competition in the sense that small changes in prices do not lead to abrupt modifications in the residual demands. This relies on the fact that the demand each firm faces does not depend only on prices; there are other features that are collected in a variable that takes values in a real compact set Θ and that we refer to as "type".[5]

The type variable is not necessarily perfectly known by firms. Thus, the information structure is given by a probability measure η on Θ^n which is assumed to be common knowledge. Let $\eta_i,\ i = 1, \ldots, n$, be the marginal distributions of η.

In this incomplete information setting, a strategy is a complete plan of actions that covers every contingency of the game. That is, a pure strategy for firm i is a measurable function from Θ to K_i. Moreover, a distributional strategy[6] for the firm i is given by a probability measure on $\Theta \times K_i$ for which the marginal distribution on Θ is η_i, that is, the one specified by the information structure. Note that pure strategies are in one-to-one correspondence with distributional strategies whose conditional distributions are Dirac measures for each type.

[3]Note that the consideration of a continuum of consumers allows us to provide reasons for their non-strategic behavior. We might also consider a large number of consumers as in Varian (1980).

[4]For instance, if we consider a technology resulting in strictly decreasing average costs, we may consider $K_i = [\delta_i, r]$, where δ_i is the minimum average cost $C_i(1)$.

[5]The type variable encompasses several features. For instance, it may be interpreted in terms of reputation, transmission of the degree of satisfaction by previous clients, skills of each firm's employees or any other characteristic of the firm itself which affects the number of customers it is able to get.

[6]See Milgrom and Weber (1985) for a discussion on distributional, mixed and behavioral strategies.

To define the payoff functions of the game, let $d_i(\theta, p)$ denote the demand that firm i faces whenever the vector of types is θ and firms choose prices p. We point out that, given a strategy profile of prices p and the vector of types θ, the aggregate demand equals 1, that is, $\sum_{i=1}^{n} d_i(\theta, p) = 1$ for every $(\theta, p) \in \Theta^n \times \mathcal{K}$. We also assume the following continuity property:

(A.1) $d_i : \Theta^n \times \mathcal{K} \to [0, 1]$ is a continuous function for every firm $i = 1, \ldots, n$.

A consumers' behavior leading to continuous demands. We stress that different behaviors of consumers may ensure that the above assumption holds since when the firms charge prices below r, the consumers choose one firm or another depending on the vector of prices and types. For instance, consider a situation where consumers decide in which firm to buy taking into account perturbed prices, that is, prices that are adjusted by the types of the firms. Let $\hat{\rho} : \Theta^n \times \mathcal{K} \to \mathbb{R}_+^n$ be the price adjustment function. Indexing these prices in a non-decreasing order we obtain $\rho(\theta, p) = \left(\rho^{(1)}(\theta, p), \ldots, \rho^{(n)}(\theta, p)\right)$, where $\rho^{(i)}(\theta, p) \leq \rho^{(i+1)}(\theta, p)$. Therefore, when (θ, p) is the vector of types and prices, $\rho^{(1)}(\theta, p)$ is the minimum perturbed price.[7] Then, consumers present diverse degree of tolerance regarding the significant differences of the modified prices with respect to the minimum perturbed price. To be precise, given (θ, p), the differences $\Delta_i = \rho^{(i)}(\theta, p) - \rho^{(1)}(\theta, p)$ define a partition of the interval $[0, 1]$ given by $I_i(\theta, p) = [\Delta_i, \Delta_{i+1})$ if $i < n(\theta, p) = \max\{j \in N | \Delta_j < 1\}$ and $I_{n(\theta,p)}(\theta, p) = \left[\Delta_{n(\theta,p)}, 1\right]$. Consumers in I_i are indifferent to buy in the set of shops $\{(j) | j \leq i\}$. In other words, consumer t selects with the same probability any firm whose perturbed price belongs to $\left[\rho^{(1)}(\theta, p), \rho^{(1)}(\theta, p) + t\right)$ since she dismisses the firms whose modified prices differ from the minimum more than t. Thus, consumers are heterogeneous about the significant differences of perturbed prices as they are perceived. In this case, we have

$$d_{(i)}(\theta, p) = \begin{cases} \sum_{j=i}^{n(\theta,p)} \frac{\mu(I_j(\theta,p))}{j} & \text{if } (i) \leq n(\theta, p) \\ 0 & \text{for every } (i) > n(\theta, p) \end{cases}$$

We remark that the continuity of $\hat{\rho}$ implies the continuity of the residuals demands. However, this continuity of the perturbed prices is not a necessary condition to get continuous residual demands. Note also that different perturbed

[7]Note that it is possible to have several perturbed prices equal to the minimum.

prices could lead to the same demand each firm faces. The first example stated in Section 4 shows these points.

Despite the consumers' behavior we have explained is not the only one that guarantees the continuity of the residual demands, for all the examples we state along this paper, we specify in an appendix the perturbed prices that eventually lead to the corresponding demand each firm faces, and we use the same notation previously stated.

Now we state the following regularity assumptions on the information structure of the game which allow us to state the players' expected payoff (firms' expected profits) in a convenient manner and, moreover, to get existence results for both distributional strategy equilibria and pure strategy approximate equilibria, which we define later on.

(A.2) The informational variable θ_i belongs to a compact set $\Theta \subset \mathbb{R}_+$ for every firm $i = 1, ..., n$.

(A.3) The measure η is absolutely continuous with respect to $\hat{\eta} = \eta_1 \times \ldots \times \eta_n$.

Given a profile $(\nu_1, \ldots, \nu_n)$ of distributional strategies, assumption (A.3) allows us to write the expected payoff Π_i for firm i as follows:

$$\Pi_i(\nu_1, \ldots, \nu_n) = \int_{\Theta^n \times \mathcal{K}} \pi_i(\theta, p)\ f(\theta)\ d\nu_1 \ldots d\nu_1$$

where f is the density of η with respect to $\hat{\eta}$ and $\pi_i(\theta, p) = d_i(\theta, p)p_i - C_i(d_i(\theta, p))$. Note that the continuity of demands guarantees that the functions π_i, $i = 1, \ldots, n$, are continuous, which is a standard condition in games with incomplete information. Observe also that assumption (A.2) guarantees the equicontinuity property of the functions $(\theta, p) \to \pi_i(\theta, p)$ which is the usual hypothesis to obtain purification results.

The price competition game $\mathcal{G}$ with incomplete information is defined by the informational structure η, the strategy set K_i for each firm $i = 1, ..., n$ and the payoff functions Π_i, $i = 1, ..., n$.

A profile $(\nu_1, \ldots, \nu_n)$ of distributional strategies is an equilibrium of the game $\mathcal{G}$ if for every firm i one has $\Pi_i(\nu_1, \ldots, \nu_n) \geq \Pi_i(\nu_1, \ldots, \nu_i', \ldots, \nu_n)$ for every alternative distributional strategy ν_i'.

Theorem 2.1 *The set of equilibrium points in distributional strategies for the game $\mathcal{G}$ is non empty.*

Proof. First, assumption (A.1) guarantees that the functions $\pi_i, 1 = 1, \ldots, n$ are continuous. By assumption (A.2) the type set is compact and therefore the continuity of $\pi_i(\theta, \cdot)$ is uniform over types.

Second, by assumption (A.3) the game $\mathcal{G}$ has absolutely continuous information. Therefore, we conclude that there exists a distributional strategy equilibrium for the game $\mathcal{G}$ (see Theorem 1 in Milgrom and Weber, 1985).[8]

Q.E.D.

An ε-equilibrium point of the game $\mathcal{G}$ is an n-tuple $(\nu_1, \ldots, \nu_n)$ such that $\Pi_i(\nu_1, \ldots, \nu_n) + \varepsilon \geq \Pi_i(\nu_1, \ldots, \nu_i', \ldots, \nu_n)$ for every firm i and every alternative strategy ν_i'. That is, we have an approximate equilibrium (i.e., ε-equilibrium) whenever every firm is not able to increase its expected profit more than ε by deviating unilaterally.

Theorem 2.2 *If each η_i is atomless, then for every $\varepsilon > 0$ there exists a pure strategy ε-equilibrium point for the game $\mathcal{G}$.*

Proof. Since each η_i is atomless, it follows that for every player, the set of degenerated distributional strategies (those which are in one-to-one correspondence with pure strategies) is dense in her set of distributional strategies (see Theorem 3 in Milgrom and Weber, 1985). This denseness property together with the equicontinuity properties of the functions π_i allow us to conclude, as in Milgrom and Weber (1985), that for any mixed strategy equilibrium we can find an ε-equilibrium in pure strategies which is actually arbitrarily close to the former (for the weak* topology).

Q.E.D.

For the special situation where the payoff function for each firm depends only on her own type and each firm's strategy set is restricted to a finite subset, Theorem 4 in Milgrom and Weber (1985) allows us to conclude that the game with incomplete information has an equilibrium point in pure strategies.

[8] We remark that this existence result by Milgrom and Weber (1985) was extended by Balder (1988) to a setting with abstract type spaces under weaker assumptions in the payoff functions and the proofs are based in the theory of weak convergence for transition probabilities. See also Balder (2004) for more recent developments.

Our model allows us to specify different residual demands, which lead to different games, each of them shedding light on a precise issue in economic theory within an industrial economics framework. This is the aim in the reminder of the paper.

Example 1: Price dispersion as pure strategy equilibrium. Consider two firms competing in prices and facing residual demands that depend continuously on prices and types. Both have just fixed costs and choose prices in $[0, r]$, where r is the reservation price of their customers represented by the unit interval $[0, 1]$. The type of each firm takes values in the closed interval $[1, 2]$.[9] The demands associated to firms are given by

$$d_1(p_1, p_2, \theta_1, \theta_2) = \frac{1}{2} - \frac{\theta_2}{\theta_1 + \theta_2} p_1 + \frac{\theta_1}{\theta_1 + \theta_2} p_2 \quad \text{and}$$
$$d_2(p_1, p_2, \theta_1, \theta_2) = \frac{1}{2} - \frac{\theta_1}{\theta_1 + \theta_2} p_2 + \frac{\theta_2}{\theta_1 + \theta_2} p_1$$

if these values are both positive, otherwise one of the firms absorbs all the demand and the other sells nothing.

Consider the informational structure such that the type of firm 1 is known, namely, $\theta_1 = c$, while the type of firm 2 is uniformly distributed in $[1, 2]$. Then, some calculations show that there is an equilibrium in pure strategies given by

$$\bar{p} = \frac{1}{\int_1^2 \frac{\theta}{\theta + c} d\mu(\theta)} = \frac{1}{1 + c \ln \frac{1+c}{2+c}} \quad \text{and } p(\theta) = \frac{2\theta p_1 + \theta + c}{4c},$$

where $\bar{p}$ is the price charged by firm 1 and $p : [1, 2] \to [0, r]$ is the pure strategy for firm 2 defining the equilibrium. Note that since there is incomplete information regarding the type of firm 2, a pure strategy for this firm assigns a price to each type. We remark that, in this case, price dispersion arises as pure strategy equilibrium of a game with incomplete information.

Observe that when the informational structure η is given by a Dirac measure on the set of vector of types Θ^n, there exists just one possible profile of types $\theta \in \Theta^n$. Therefore, in this situation, the uncertainty disappears and then we recast a complete information game, where the payoff functions are $\pi_i(\theta, \cdot) : \mathcal{K} \to \mathbb{R}$, $i = 1, \ldots, n$. The continuity of these functions ensures existence of Nash equilibrium in mixed strategies.[10] Furthermore, if for every i, the profit

[9] We consider $\Theta = [1, 2]$ for simplicity.

[10] We recall that, in this normal form game, a mixed strategy for firm i is a probability measure on the set of pure strategies K_i which is compact and convex.

$\pi_i(\theta, \cdot)$ is also quasi-concave in the strategy (price) selected by firm i, there is Nash equilibrium in pure strategies. Therefore, our framework paves the way to compare equilibria with complete and incomplete information.

Complete vs. incomplete information. Consider again the previous game but with complete information. For every vector of types (θ_1, θ_2), there is an equilibrium in pure strategies, given by $p_1^* = \frac{\theta_1+\theta_2}{2\theta_2}$ and $p_2^* = \frac{\theta_1+\theta_2}{2\theta_1}$, which leads to profits $\pi_1^* = \frac{\theta_1+\theta_2}{4\theta_2}$ and $\pi_2^* = \frac{\theta_1+\theta_2}{4\theta_1}$ for firms 1 and 2, respectively. Then, we find price dispersion provided that firms with different attributes charge different prices, whereas in the incomplete information setting price dispersion appears as an equilibrium in pure strategies (where each firm sets a price for every type in Θ). We remark that, in this case, the ratio of prices is given by the ratio of types, which determines the degree of dispersion of prices.

To compare, let us return to this example with incomplete information. Computing the expected payoffs at the equilibrium, we have $\Pi_1^* = \frac{\bar{p}}{4} = \frac{1}{4\left(1+c\ln\frac{1+c}{2+c}\right)}$ and $\Pi_2^* = \frac{3\bar{p}^2}{8c} + \frac{\bar{p}(3-2c)}{8c} + \frac{3}{32c} + \frac{1}{16}$ for firms 1 and 2, respectively. It is not hard to show that the equilibrium payoff Π_1^* of firm 1 is increasing in its own type, whereas the equilibrium profits Π_2^* of firm 2 decreases as the type of their opponent increases. Moreover, the minimum equilibrium expected payoff of firm 2, that is attained when $c = 2$, is higher than the maximum payoff that firm 2 can obtain at equilibrium with complete information, which is attained when $\theta_1 = 1$ and $\theta_2 = 2$. A conclusion from this fact is that firms may prefer a situation with incomplete information and this preference may have implications regarding their advertising policy.

3 Separating the effects of types and prices

Let us consider a particular game where the demand that each firm gets depends separately on the two kinds of variables that we have considered, namely, prices and types. The way in which the prices affect the demands is captured by a function denoted by α while the implication of the types is expressed by a function β. The overall effect is then stated by a convex combination of these functions, α and β, depending on the profiles of prices and types, respectively.

To be precise, α is a continuous function from $\mathcal{K}$ to $[0,1]^n$, where the parameter $\alpha_i(p)$ defines the part of demand faced by firm i that is determined by the profile of prices selected by the stores. On the other hand, given a vector of types θ, $\beta_i(\theta)$ reflects the proportion of consumers that firm i gets coming from such types. That is, a realization of types θ determines $\beta(\theta) \in [0,1]^n$ which, joint with the previous function α, affects the residual demands.

Finally, the demands' dependence on prices and types is given by a convex combination of the aforementioned functions α and β. The balance of the corresponding effects is given by the weights defining such a convex combination which may depend on the prevailing prices and types. Thus, for each $(\theta, p) \in \Theta^n \times \mathcal{K}$, let us consider the parameter $\lambda(\theta, p) \in [0,1]$. Then, the residual demand for each store i is

$$d_i(\theta, p) = \lambda(\theta, p)\alpha_i(p) + (1 - \lambda(\theta, p))\beta_i(\theta).$$

Let us consider this particular formulation of the residual demands in the game with incomplete information described in the previous section. Applying Theorems 2.1 and 2.2 we have existence of equilibrium in distributional strategies and also of approximate equilibrium in pure strategies.

Note that taking $\lambda(\theta, p) = 1$ for every (θ, p) there is no effect of types and therefore we have a game with no incomplete information. In this case, if we consider a duopoly, we can obtain as a particular situation the classical Bertrand's model resulting in the Bertrand paradox. In addition, as the next example points out, this way of separating the effects of types and prices gives light to some other interesting features of price competition even when one considers complete information. Actually, the following price competition game highlights a way of overcoming the Bertrand paradox that differs from those that have already been considered in the literature. Furthermore, the same game allows us to illustrate a situation where arbitrarily small firms have market power as it happens in monopolistic competition situations à la Chamberlin (1933, 1937).

Example 2. Take $n > 1$ firms with the same technology and consumers in $[0,1]$. For a set of consumers of measure m, the cost function for every firm is $C(m) = cm + F$, where F denotes the fixed costs and $c < r$. Firms choose prices in the compact set $[c, r]$.

Let $\theta = (\theta_1, \ldots, \theta_n)$ be a vector of types of firms. For each profile of prices $p = (p_1, \ldots, p_n)$, the demand faced by firm i is $d_i(\theta, p) = \lambda\, \alpha_i(p) + (1 - \lambda)\, \beta_i(\theta)$, where $\alpha_i(p) = \frac{1}{n-1}\left(1 - \frac{p_i}{\sum_{j \in N} p_j}\right)$ and $\beta_i(\theta) = \frac{\theta_i}{\sum_{j \in N} \theta_j}$. Note that $\lambda(\cdot)$ is constant and equals $\lambda \in (0,1)$. Then, the profit function for each firm $i \in N = \{1, \ldots, n\}$ is given by

$$\pi_i(\theta,p) = p_i\,\left(\frac{\lambda}{n-1}\left(1-\frac{p_i}{\hat{p}}\right)+\frac{(1-\lambda)\theta_i}{\hat{\theta}}\right) - C\left(\frac{\lambda}{n-1}\left(1-\frac{p_i}{\hat{p}}\right)+\frac{(1-\lambda)\theta_i}{\hat{\theta}}\right),$$

where $\hat{p} = \sum_{j\in N} p_j$ and $\hat{\theta} = \sum_{j\in N} \theta_j$.

Some calculations show that the payoff function π_i of firm i is strictly increasing in the price selected by firm i. That is, $\frac{\partial \pi_i}{\partial p_i}$ is positive for every i. Therefore, independently of the number n of firms and their types, the unique Nash equilibrium in pure strategies is the profile where all the firms charge the reservation price r.

In this situation, for the case of a duopoly, Bertrand paradox is overcome. Moreover, when the number of firms is enlarged, the equilibrium is also attained when all the firms choose the reservation price $r > c$, although the profits of every firm i tend to zero when n increases. Therefore, this example also illustrates a situation of monopolistic competition in the sense that arbitrarily small firms have market power.[11] Moreover, the result is the same when there are no fixed costs and then we have constant returns to scale. Thus, our remark is in accordance with Chamberlin (1933, 1937), who pointed out, in contrast to Kaldor (1935), that what marks the contrast between monopolistic competition and perfect competition is the shape of demand curve and not the shape of the cost curve.

4 Types affect demand only if prices are similar

In many markets the type variable becomes really effective or shapes the demand functions only when prices belong to a certain threshold. In other words, when prices differ too much the type is not a relevant variable and the firms that offer the lowest price face all the demand. However, when prices are close, individuals are more vulnerable to the abilities of firms to attract costumers.

To address this feature, let us consider again the game presented in Section 2 where demand d_i that firm i faces is a continuous function on types and prices (θ, p). Given a vector of prices $p \in \mathcal{K}$ that firms select, let $m(p)$ denote the minimum price. Now, let us consider a threshold $\varepsilon > 0$ and define the new

[11] The definition of the residual demands that underlies this imperfect competition situation is not an explicit commodity differentiation approach, even though the variables defining these demands may be interpreted as a degree of "differentiation of firms".

residual demands as follows:

$$d_i^{\varepsilon}(\theta, p) = \begin{cases} d_i(\theta, p) & \text{if } p_i \leq m(p) + \varepsilon \\ 0 & \text{if } p_i > m(p) + \varepsilon \end{cases}$$

If the above demands are continuous, the additional assumptions stated in Section 2 allow us to apply Theorems 2.1 and 2.2 obtaining existence of equilibrium.

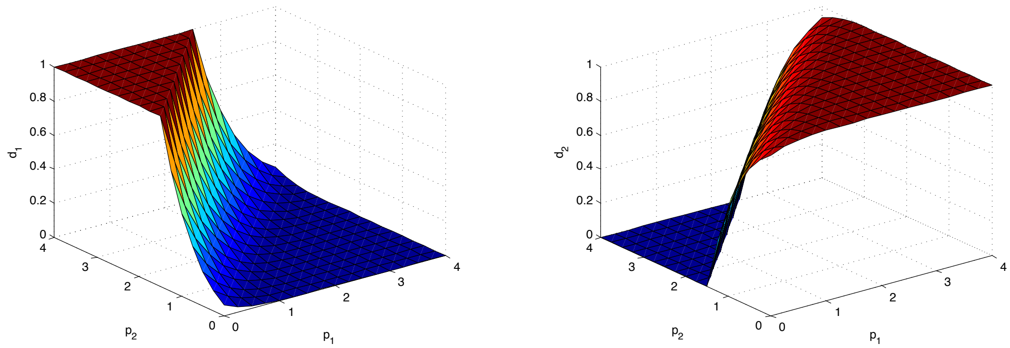
To illustrate how types affect demands only if prices are similar, we consider the next two different scenarios where the demands satisfy the aforementioned continuity property.

Example 3. Consider two firms with types θ_1 and θ_2 for which the demand functions are given as follows: If the difference between the prices is large (i.e., above a given threshold) then the firm charging the lowest price gets all the demand, but when this difference is small (i.e., below the given threshold) then the demand is shared by both firms in such a way that the firm with the better type will face a higher demand. To be precise, without loss of generality, let $\theta_i \geq \theta_j$. Let us fix a threshold $\varepsilon > 0$ and consider the following residual demands:

$$d_i(\theta_1, \theta_2, p_1, p_2) = \begin{cases} 1 & \text{if} \quad p_i < p_j - \varepsilon \\ 1 - \left(\frac{p_i - p_j + \varepsilon}{2\varepsilon}\right)^{1+\theta_i-\theta_j} & \text{if} \quad \mid p_j - p_i \mid \leq \varepsilon \\ 0 & \text{if} \quad p_i > p_j + \varepsilon \end{cases}$$

$d_j(\theta_1, \theta_2, p_1, p_2) = 1 - d_i(\theta_1, \theta_2, p_1, p_2)$.

We observe that when $\theta_i < \theta_j$ and $p_i < p_j - \varepsilon$ firm i gets all the demand (i.e., firms j sells nothing) but once it increases the price so that $\mid p_i - p_j \mid \leq \varepsilon$, its residual demand decreases very quickly, equivalently, the demand faced by firm j increases promptly. Thus, the type variable not only provides stability in the demands but also determines the shape of such demands. (See next figures).



In the figure on the left (resp. right) d_1 (resp. d_2) is represented taking $\varepsilon = 3/2, \theta_1 = 1$ and $\theta_2 = 4$.

When $\theta_1 - \theta_2 = 1$, we obtain the following Nash equilibrium: $p_1 = \left(\frac{3\sqrt{17}-5}{8}\right)\varepsilon = 0.92\varepsilon$ and $p_2 = \frac{p_1+\varepsilon}{3} = 0.64\varepsilon$. However, when both firms have the same type, we have that the equilibrium is $p_1 = p_2 = \varepsilon$. Note that when types are different $(\theta_1 - \theta_2 = 1)$ we obtain price dispersion. Moreover, if we understand price dispersion as the difference between both prices, such a dispersion is increasing with ε. Note also that when types are equal, we find no dispersion of prices at equilibrium.

We conclude that at equilibrium the difference between prices is less than the threshold. Moreover, when types are distinct, then the prices can actually be different and, as we have obtained in the example, the difference depends on the threshold. Thus, the type variable can explain not only price dispersion in the sense that at equilibrium different prices can arise, but also the degree of price dispersion, that is, how different the equilibrium prices can be.

Let us state a final specification of the residual demands that separates the effect of prices and types (as in the previous section) and, in addition, types affect demands only if prices are similar.

Example 4. Consider two firms producing with null variable costs. Given types $\theta = (\theta_1, \theta_2)$ and prices $p = (p_1, p_2)$, the residual demands are $d_i(\theta, p) = \lambda(p)\alpha_i(p) + (1 - \lambda(p))\beta_i(\theta),\ i = 1, 2,$ where

$$\lambda(p_1, p_2) = \begin{cases} 1 & \text{if} \quad |p_1 - p_2| > \varepsilon \\ \frac{|p_1 - p_2|}{\varepsilon} & \text{if} \quad |p_1 - p_2| \leq \varepsilon \end{cases}$$

$$\alpha_i(p_1,p_2)=\begin{cases}1 & \text{if} \quad p_i<p_j\\ 1/2 & \text{if} \quad p_i=p_j\\ 0 & if \quad p_i>p_j\end{cases} \qquad \text{and} \qquad \beta_i(\theta_1,\theta_2)=\frac{\theta_i}{\theta_1+\theta_2}$$

That is, the demands faced by firm 1 and 2 are

$$d_1(\theta_1,\theta_2,p_1,p_2)=\begin{cases}1 & \text{if} \quad p_1-p_2<-\varepsilon\\ \left(1-\frac{|p_2-p_1|}{\varepsilon}\right)\frac{\theta_1}{\theta_1+\theta_2}+\max\{0,\frac{p_2-p_1}{\varepsilon}\} & \text{if} \quad |p_1-p_2|\le\varepsilon\\ 0 & \text{if} \quad p_1-p_2>\varepsilon\end{cases}$$

$$d_2(\theta_1,\theta_2,p_1,p_2)=1-d_1(\theta_1,\theta_2,p_1,p_2).$$

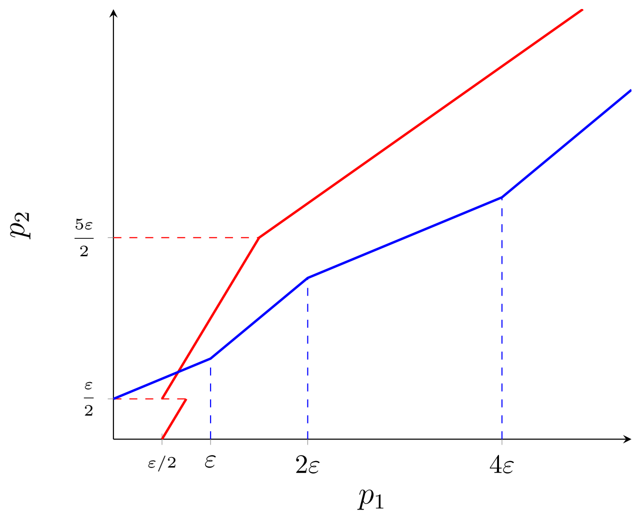
Note that when prices differ more than ε the demands do not depend on types. However, as we show below, at the equilibrium the prices differ less than ε and they depend basically on the ratio $\tau=\theta_1/\theta_2$. Thus, first we calculate the equilibrium when types are different and then when they are equal.

To obtain the best response functions when types differ, two cases are considered: and $\theta_2>\theta_1$ and $\theta_1>\theta_2$, respectively.

For the case $\theta_2>\theta_1$, some calculations allow us to write the best response functions as follows:

$$\text{Reaction function for firm 1:} \quad \begin{cases}p_1=\frac{p_2}{2}+\frac{\varepsilon}{2} & if \quad p_2\in\ (0,\varepsilon\tau)\\ p_1=\frac{p_2}{2}+\frac{\varepsilon}{2}\tau & if \quad p_2\in[\varepsilon\tau,\varepsilon(2+\tau))\\ p_1=p_2-\varepsilon & if \quad p_2\ge\varepsilon(2+\tau)\end{cases}$$

$$\text{Reaction function for firm 2:} \quad \begin{cases}p_2=\frac{p_1}{2}+\frac{\varepsilon}{2} & if \quad p_1\in\ (0,\varepsilon)\\ p_2=p_1 & if \quad p_1\in[\varepsilon,\varepsilon\tau^{-1})\\ p_2=\frac{p_1}{2}+\frac{\varepsilon}{2}\tau^{-1} & if \quad p_1\in\ [\varepsilon\tau^{-1},\varepsilon(2+\tau^{-1}))\\ p_2=p_1-\varepsilon & if \quad p_1\ge\varepsilon(2+\tau^{-1})\end{cases}$$



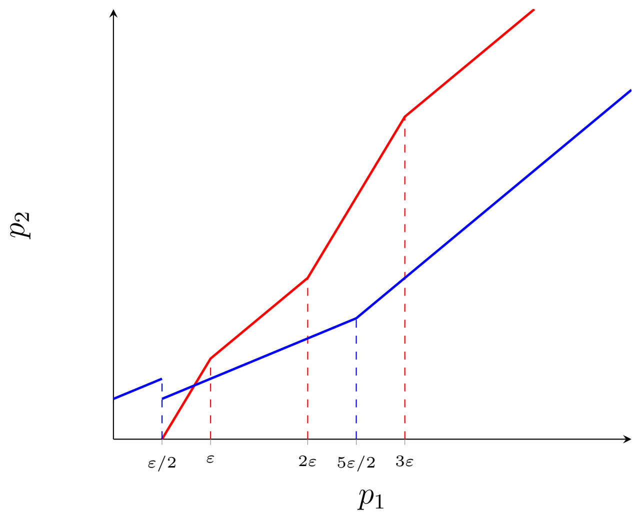
The figure above represents the reaction functions for firm 1 (red line) and for firm 2 (blue line), taking θ_1=1, $\theta_2 = 2$, and $\varepsilon = 1.5$

For the case $\theta_1 > \theta_2$, the best response functions are as follows:

Reaction function for firm 1: $$\begin{cases} p_2 = 2p_1 - \varepsilon & if \quad p_1 \in \left[\frac{\varepsilon}{2}, \varepsilon\right) \\ p_2 = p_1 & if \quad p_1 \in [\varepsilon, \varepsilon\tau) \\ p_2 = 2p_1 - \varepsilon\tau & if \quad p_1 \in [\varepsilon\tau, \varepsilon(1+\tau)) \\ p_2 = p_1 + \varepsilon & if \quad p_1 \geq \varepsilon(1+\tau) \end{cases}$$

Reaction function for firm 2: $$\begin{cases} p_2 = \frac{p_1}{2} + \frac{\varepsilon}{2} & if \quad p_1 \in \ (0, \varepsilon\tau^{-1}) \\ p_2 = \frac{p_1}{2} + \frac{\varepsilon}{2}\tau^{-1} & if \quad p_1 \in \ [\varepsilon\tau^{-1}, \varepsilon(2+\tau^{-1})) \\ p_2 = p_1 - \varepsilon & if \quad p_1 \geq \varepsilon(2+\tau^{-1}) \end{cases}$$

Note that in this second situation, to simplify the expressions, the reaction function of firm 1 is written with p_2 in terms of p_1.



Reaction functions for firm 1 (red line) and for firm 2 (blue line), taking in this case θ_2=1, $\theta_2 = 1$, and $\varepsilon = 1.5$

When $\tau = \theta_1/\theta_2 > 1$, the equilibrium of this game is $p_1 = \frac{\varepsilon}{3}(1+2\tau)$ and $p_2 = \frac{\varepsilon}{3}(2+\tau)$. However, when $\tau < 1$, the equilibrium is $p_1 = \frac{\varepsilon}{3}\left(1+\frac{2}{\tau}\right)$ and $p_2 = \frac{\varepsilon}{3}\left(2+\frac{1}{\tau}\right)$. Then, at equilibrium, the firm with higher type selects a higher price. Moreover, the ratio of prices $\gamma = p_1/p_2$ does not depend on the threshold ε and just depends on the ratio of types τ. To be precise, $\gamma = \frac{1+2\tau}{2+\tau}$ if $\tau > 1$ whereas $\gamma = \frac{2+\tau}{1+2\tau}$ if $\tau < 1$ instead. Therefore τ, and in turn, the dispersion of prices is uniformly bounded on types. As in the example 3, when both firms are of the same type (i.e., $\tau = 1$), they charge the same price equal to ε.

In the setting addressed in this section, when $\varepsilon = 0$ we have the classical Bertrand's price competition model. Moreover, when ε goes to zero, the continuous demands d_i^{ε} converge to the discontinuous residual demands, leading to the Bertrand paradox. Thus, our approach provides a way of solving smoothly such a paradox. This is due to the presence of stability in competition, jointly with the consideration that types matter only when prices are similar.

5 Concluding remarks

We have provided a game with incomplete information, where firms compete in prices, for which we have shown existence of different kinds of equilibrium. We have also specified residual demand functions, as particular cases of our model, that lead to different games which are used to explain several topics in economic theory. We have considered the case when the effects of types and prices enter separately in the residual demands, and also the situation where the types matter only if the prices belong to a certain threshold. We have also drawn conclusions regarding each case. For instance, we have provided alternative explanations to those already present in the literature for the phenomenon of price dispersion and also for Bertrand's paradox. In addition, our approach gives room to compare the equilibria with complete and incomplete information.

We remark that the argument we have stated on perturbed prices has allowed us to describe a behavior of consumers which results in continuous residual demands (i.e. assumption (A.1) holds); however other different approaches could also guarantee stability in competition.

The different specifications of the residual demands that we have considered lead to a decrease in the competition in the sense that the equilibrium of the game deviates from the competitive equilibrium and therefore allows for the analysis of several issues where market power is strengthened through the type variable.

Appendix

In this appendix we specify the perturbed prices that lead to the different continuous residual demands of the examples we have addressed along this paper. That is, adapting the consumers' behavior described in Section 2 to the perturbed prices we state below, we obtain the corresponding demands we have considered as particular cases in each of the four examples we have studied.

Example 1. Consider the perturbed prices $\hat{\rho}(p_1, p_2, \theta_1, \theta_2) = \left(\frac{2\theta_2}{\theta_1+\theta_2}p_1,\ \frac{2\theta_1}{\theta_1+\theta_2}p_2\right)$. We observe that whenever $|\ \Delta_2\ |=|\ \rho^{(2)}(p,\theta) - \rho^{(1)}(p,\theta)\ |< 1$, both firms face the positive demand that we have analysed in the first example. Note that the perturbed price $\hat{\rho}_i$ is increasing in the price selected by firm i and in the type of the other firm whereas is decreasing in its own type. Note also that when both types are equal there is no adjustment of prices, that is, the perturbed prices are those that each firm chooses.

Example 2. Let $\lambda \in [0,1]$. For each vector of types $\theta = (\theta_1, \ldots, \theta_n)$ and profile of prices $p = (p_1, \ldots, p_n)$, let us define, for each firm i, the signals $s_i(\theta, p) = \frac{\lambda}{n-1}\frac{p_i}{\hat{p}} - (1-\lambda)\frac{\theta_i}{\hat{\theta}}$, where $\hat{p} = \sum_{j \in N} p_j$ and $\hat{\theta} = \sum_{j \in N} \theta_j$. These signals define a ranking for the perturbed prices where the effect of prices and types is separated. To be precise, we index the vector of signals in a non-decreasing order, i.e., $s_{(1)}(\theta, p) \leq s_{(2)}(\theta, p) \leq \ldots \leq s_{(n)}(\theta, p)$ and the signal indicates an order for the perturbed prices which are given by:

$$\rho^{(i)}(\theta, p) = \begin{cases} 0 & \text{if } i = 1 \\ \left(1 - \frac{\lambda n}{n-1}\right) + n s_{(n)}(\theta, p) & \text{if } i = n \\ \rho^{(i+1)}(\theta, p) - i\left(s_{(i+1)} - s_{(i)}\right) & \text{otherwise} \end{cases}$$

The above perturbed prices conduct to the continuous residual demands that we have studied in the second example.

Example 3. Consider the perturbed prices given by

$$\hat{\rho}(p_1, p_2, \theta_1, \theta_2) = \begin{cases} (0,1) & \text{if } p_1 < p_2 - \varepsilon \\ \left(2\left(\frac{p_1 - p_2 + \varepsilon}{2\varepsilon}\right)^{1+\theta_1-\theta_2},\ 1\right) & \text{if } \theta_1 \geq \theta_2 \text{ and } \mid p_2 - p_1 \mid \leq \varepsilon \\ \left(2 - 2\left(\frac{p_2 - p_1 + \varepsilon}{2\varepsilon}\right)^{1+\theta_2-\theta_1},\ 1\right) & \text{if } \theta_1 < \theta_2 \text{ and } \mid p_2 - p_1 \mid \leq \varepsilon \\ (2,1) & \text{if } p_1 > p_2 + \varepsilon \end{cases}$$

where $\varepsilon > 0$ is the threshold considered in our example 3.

Note that when the prices selected by both firms coincide, the perturbed prices are adjusted by the vector of types in a continuous way avoiding instability. In addition, when either $p_1 < p_2 - \varepsilon$ or $p_1 > p_2 + \varepsilon$, the perturbed prices are constant and the difference is 1 implying that one of the firms has all the demand. However, when the difference between the prices charged by the firms is small (i.e., $\mid p_2 - p_1 \mid \leq \varepsilon$), the firm with a higher type is associated with a lower perturbed price and therefore will face a larger demand. These perturbed prices lead to the continuous residual demands considered in the third example. Moreover, we remark that we obtain the same demands if we consider the same perturbed prices except when $\theta_1 < \theta_2$ and $\mid p_2 - p_1 \mid \leq \varepsilon$ that are $\left(1, 2\left(\frac{p_2 - p_1 + \varepsilon}{2\varepsilon}\right)^{1+\theta_2-\theta_1}\right)$. Therefore, we show that to obtain continuous residual demands we do not need continuity of the perturbed price and, moreover, different perturbed prices can result in the same residual demand.

Example 4. The residual demands in the Example 4 can be obtained by defining the following perturbed prices:

$$\hat{\rho}(\theta_1,\theta_2,p_1,p_2) = \begin{cases} (1,2) & \text{if } p_1 - p_2 < -\varepsilon \\ \left(1, 2\left(\left(1 - \frac{|p_2-p_1|}{\varepsilon}\right)\frac{\theta_1}{\theta_1+\theta_2} + \max\{0, \frac{p_2-p_1}{\varepsilon}\}\right)\right) & \text{if } |p_1 - p_2| \leq \varepsilon \\ (1,0) & \text{if } p_1 - p_2 > \varepsilon \end{cases}$$

References

Balder, E.J. (1988): Generalized equilibrium results for games with incomplete information. Mathematics of Operations Research 13, 265-276.

Balder, E.J. (2004): An equilibrium existence result for games with incomplete information and indeterminate outcomes. Journal of Mathematical Economics 40, 297-320.

Bagwell, K., Lee, G. (2014): Number of firms and price competition. Working-Paper.

Chamberlin, E. (1933): The theory of monopolistic competition. Harvard University Press, Cambridge.

Chamberlin, E. (1937): Monopolistic imperfect competition?. Quarterly Journal of Economics 51, 557-580.

Hotelling, H. (1929): Stability in competition. The Economic Journal 39, 41-57.

Kaldor, N. (1935): Market imperfection and excess capacity. Economica 2, 33-50.

Milgrom, P., Weber, R. (1985): Distributional strategies for games with incomplete information. Mathematics of Operations Research 10, 619-632.

Salop, S., Stiglitz, J. (1977): Bargains and ripoffs: A model of monopolistically competitive price dispersion. The Review of Economic Studies 44, 493-510.

Spulber, D.F. (1995): Bertrand competition when rivals' costs are unknown. Journal of Industrial Economics 43(1), 1-11.

Varian, H.R. (1980): A model of sales. American Economic Review 70, 651-659.