The impact of public investment in Sweden: A VAR approach.

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Abstract

The traditional approach towards analyzing the impact of public investment has been through including public capital as a third input factor in a Solow-model production function. Nonetheless, such approach implies several problems both at the theoretical and empirical level. Given such problems, econometric models that require as few theoretic assumptions as possible become important. It is in this framework that the Vector Autoregressive model is introduced and explained. An application to the Swedish economy is consequently conduced as it is interesting to verify the impact of public investment in a country known for its comparably large public sector.

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1. Introduction

Sweden has a comparably large public sector. In fact, about 30% of the labour force works in the public sector. Public investment and tax-financed consumption represent 31% of GDP, as further 24% are redistributed through transfers. It is of a major importance to understand the underlying mechanisms that relate public investment to growth as well as other macroeconomic aggregates such as private investment and employment. Being able to comprehend these interrelationships is the first step towards the development of better and more efficient public investment policies. The main purpose of this essay is to assess the impact of public investment on GDP in Sweden. So, the first question to be addressed is: how does public investment affects GDP according to the existing literature?

The traditional approach has been to assess the impact of public investment on output through the inclusion of the public capital stock in an augmented Solow production function. This raises a second research question: What are the main theoretic and econometric problems facing the estimation of the impact of public investment in an augmented Solow model production function?

In the context of the theoretical and econometric problems raised by the estimation of a production function, and the lack of consensus in theorising and modelling the effects of public investment, models that require as little restrictions to the relationship between the variables as possible become more important. Given this a Vector Autoregressive (VAR) model is used and given its characteristics, the effects of public investment in employment and private investment can also be estimated. It is within this framework that the third research question is introduced: what is the impact of public investment in GDP, employment and private investment in a VAR model framework for the Swedish economy?

1.1 Limitations

Concerning the first and second research questions being made, the literature review, though extensive, is not exhaustive. This essay is limited in the sense that only the most cited and known relationships between public investment and growth are reported. On what the empirical application is concerned, the availability of the data always conditions the analysis. Most econometric tests and statistics reported are asymptotically valid which means that for small samples, as it is the case, their power greatly diminishes. It has to be noted also that this analysis is valid only for the given sample, and as it can be seen, the explanatory power is also limited. Finally, the proxy for public investment found here relates to government investment, not total public investment. This way, investment by local authorities is not taken into account in the empirical analysis.

1.2 Methodology

An extensive literature review was made with a twofold purpose: to find out how public investment has been related to GDP, and to determine the main theoretic and econometric problems of including the public capital stock in an augmented Solow model production function. The empirical application is conducted using the VAR methodology.

1.3 Structure

This paper is divided into six sections. After the introduction, section two introduces the main issues behind the modelling of public investment and its relationship to other macroeconomic aggregates, mainly investment and output. Section three introduces the production function approach where the main theoretical and empirical problems concerning such approach are addressed. Section four introduces the Vector Autoregressive approach. After an introduction to the methodology used, an empirical application to the Swedish economy is made. The fifth part is devoted to some concluding remarks.

2 Theoretic Background

The first reference found, concerns public infrastructure and its relation to output comes from Mera (1973). This paper however concentrated in regional production functions. In fact, as Lakshmanan and Anderson (2004) refer, this area of research was of primarily concern for regional economists and economic geographers. Ten years later Ratner (1983) analyzes the phenomena at the aggregate level. His work relates to aggregated time-series data for the US economy for the period from 1947 until 1973, and finds public investment's output elasticity to lie close to 0.06, again using the production function approach. Nonetheless, it is only after Aschauer (1989) that the number of articles on the matter greatly expands. Following Gloom and Ravikumar (1997), most economists today find that "the productivity of public capital is simply not believed to be larger than the productivity of the capital stock (which is roughly 0.36)". Most of this findings concern the estimation of elasticities in a framework of a Solow-model production function. Actually, most estimates attribute output elasticities between 0.05 and 0.15 to public investment, even though subject to a large degree of variation.

On what the effect on private investment is concerned, Ghura and Goodwin (2000) show that government investment crowds-in private investment in sub-Saharan Africa, considered to be among the least developed regions of the world. As the level of development rises, the crowding-out hypothesis finds confirmation in an increasing number of studies. Nonetheless, evidence is at best mixed. Pereira (2001a), finds evidence for the crowding-in hypothesis for the United States, where Everhart and Sumlinski (2001) find support for the crowding out hypothesis in Europe.

The link between public spending and other macroeconomic aggregates, mainly output and private investment, has attracted considerable interest by many researchers. In an aggregate supply-demand Keynesian framework, public spending is important in correcting short-term business cycle fluctuations as in Singh and Sahni (1984), driving the economy towards full employment, avoiding inflation associated with excessive activity of the economy or fighting unemployment in times of recession. Nonetheless, public spending needs to be financed so it implies taxation, a major distortion of economic incentives leading to inefficient economic decisions, as in Barro (1990).

Economic theory does not provide an immediate answer to the question on whether public spending promotes or stalls economic growth. For the analysis here being made, it matters to establish the difference between public spending and public investment. While public spending encompasses total public expenditure, public investment concerns the share that is spent on infrastructure, in its widest sense. This paper focuses mainly on the effect that public investment has upon the main economic aggregates.

Public and private investments are, in market economies, different in nature, and these differences spur from many sources. While the motivations for private investment concern the maximization of profits, governments try to maximise aggregate welfare, under the framework of an implicit "contract" (the constitution) between them and the people they are supposed to serve. Stiglitz (2000), argues that the state should intervene in the economy in three cases: market failure, macroeconomic stabilisation or under a paternalistic function. Market failures concern situations when a market negotiation does not suffice to achieve efficiency (in Pareto's sense) and the state intervenes under the assumption that such market failure may be corrected through regulation. Examples of market failures are incomplete markets (as financial markets for young students), externalities (as investment in research in development) and under-provision of public goods (as it is the case for national defence). The reason for intervening under the macroeconomic stabilization argument finds its roots on the cyclical characteristics of economic activity and in the Keynesian notion that the government can improve efficiency by intervening in correcting those cycles. Accordingly, during the early 90's economic crisis in Sweden, government investment increased, in counter-cycle with what happened with GDP and Employment. Finally, the paternalistic function relates to the notion that individuals sometimes do not act in a self-maximising way, in for example vaccination and seat-belt usage for example, and introduces legislation to induce more efficient behaviours.

It is obvious that a policy of zero public investment would seriously damage a nation's ability to enforce the rule of law or the creation of infrastructure, critically hampering economic activity. But excessive public investment can also be harmful to growth. While building a two-lane road where there was none, connecting two major points of economic activity, may spur competition, by broadening markets and increasing efficiency, turning it into a sixteen-lane road can hardly be seen as efficiency enhancing. Indeed, while most literature assumes public investment to be productive (in the sense that its output elasticity is positive), there is a trade-off to take into account, and precisely the above mentioned distortionary effect of its most common source of financing: taxes.

The accounting of the effect of public investment is also problematic due to the nature of the goods it produces, as it can be observed from the case of the non-inclusion of non-market values in an output measure such as GDP. As Blinder et al. (1991) observes, relating to the missing effects of infrastructure in GNP, “If my car and my back absorb fewer shocks from potholes, I am surely better-off; but the GNP may even decline as a result of fewer car repairs and doctor’s bills.”

However, it has to be taken into account that there may be circumstances, like in early stages of development, when it is consensually expected for public investment to have a major role, due precisely to the serious hamper to economic development that the lack of basic infrastructures constitutes. Nonetheless, as the level of development rises, and most basic infrastructures are provided, two driving forces become determinant: one favouring the crowding-out hypothesis, when governments use it with the intent of increasing productive capacity; other, favouring crowding-in, when public investment is decided upon the interests of the private sector, acting as a complement, as Pereira (2001b) puts it.

4 The production function approach

Focusing on the production function approach, the stock of public capital may enter the production function in two ways, influencing multifactor productivity and/or directly as a third input:

$$Y_t = A(G_t) f(K_t, L_t, G_t) \quad (1)$$

The functional form most widely used is an aggregated Cobb-Douglas production function:

$$Y_t = A_t K_t^{\alpha} L_t^{\beta} G_t^{\gamma} \quad (2)$$

As in most cases this equation is estimated in log-levels, it does not matter how public investment enters the production function once both alternatives yield similar equations to be estimated, more precisely:

$$\ln Y_t = \ln A_t + \alpha \ln K_t + \beta \ln L_t + \gamma \ln G_t + \varepsilon_t \quad (3)$$

Most studies concentrate on variations of equation (3). Many times, a proxy for capacity utilization rate is included to control for the influence of the business cycle. In nearly all cases, the model estimation concerns the use of time-series or panel-data.

From a theoretic point of view, the inclusion of government capital stock in this way violates standard marginal productivity theory, as in Duggal et al. (1995). It implies that a market determined price per unit of government capital stock is known and paid by the private sector. Generally, those prices are not set by market forces but by the government and only when directly applicable, as it is not the case for public investment in infrastructure with public good charactereristics, making it just not reasonable to assume that it is paid its marginal productivity.

Another problem is that if we assume that G affects A multiplicatively, logging the variables, as stated before, makes it indifferent whether government capital stock is included as a third input factor or as influencing multifactor productivity. As logging the variables is essential for running OLS estimations, one cannot quantify or even verify the existence of each effect in separate.

This methodology also faces several econometric problems. Looking closely at the variables in question, one realizes that they are most of the times non-stationary. Indeed, most economic time series are $I(1)$ i.e. first order integrated meaning that they are $I(0)$ once first differences are taken. Therefore, in the absence of cointegration between the variables, OLS will yield spurious relationships between the variables. Taking first differences of the logs (that approximates growth rates) is the standard approach when there is no cointegration, but then the long-run relationship between the variables cannot be captured in such a static relationship and often implausible output elasticities are the result, as Duggal et al. (1995) refer. Furthermore, Pereira and Andraz

(2004) even find public capital stock in Portugal to be $I(2)$. This is probably due to intensive public investment as a result of the cohesion funds from the European Union. Estimation using such a variable in a regression, together with labour and private capital that are found to be $I(1)$, is not the best approach.

Another problem relates with the fact that it seems implausible that the relationship between GDP and labour, public and private capital stock is static. It is much more likely that this relationship is of a dynamic nature i.e. that growth may be explained not only by the contemporaneous values of other variables but also their lags. Autocorrelation is an endemic problem when estimating static production functions. The estimated output elasticities might be unbiased and consistent in the presence of autocorrelation but are not efficient (minimum variance) and the estimated standard errors will be biased and inconsistent. Nonetheless, if the true relationship is dynamic, strong autocorrelation is expected. As autocorrelation is, as referred in Hendry et al. (1984), often a symptom of bad specification, and the fact that including lags often makes autocorrelation disappear, suggest that indeed the relationship between growth and other variables is of a dynamic nature.

Another issue is the fact that in this approach, private and public capital are exogenous. However, it is more than reasonable to assume that public investment has an effect not only in GDP as a third input factor but also as an externality in the private capital and labor markets. As mentioned before, public investment may *crowd out* private investment and therefore having a substitution effect. The contrary is also reasonable, especially in less developed countries where it is the construction of basic infrastructure that lays the basis for economic development. In this case, public investment is expected to have a complement effect. Either way, private investment is likely to be affected by public investment. If the effect is very strong, i.e. if public and private investments are strongly correlated, multicollinearity might arise. In such case, the estimates are not reliable and tend to be very imprecise i.e. showing high standard errors, which in turn may lead to unexpected results on what magnitude and sign of the variables is concerned.

A significant issue concerns direction of causality. Above is assumed that it is a linear combination of the logged input factors that causes GDP. Nonetheless, it is also reasonable to assume reverse causation. An increase in GDP will increase tax revenues and give the government means to increase public investment. Following Wagner's Law, as in Wagner (1883), causality might also run from GDP to public investment. The referred author defended that as industrial economies develop so would increase the share of public expenditure in GDP. Testing equation (3) by OLS will not take this feedback into account. In fact, in cases where a variable cannot be treated as exclusively exogenous, as it might be the case where causality runs both ways, such variable will be correlated with the error term. Under such circumstances, and following Verbeek (2005), this model will no longer be a best linear approximation and the effect of public capital stock will be overestimated. In this example, where GDP affects public investment and vice-versa, a system containing two equations should be estimated.

Finally, and following Gramlich (1994), even if the long-run aggregate supply side effect of public investment in output is negligible, it is likely that estimations of a static nature capture the increase in output resulting from a boost of aggregate demand in the short-run. This simultaneity bias leads to an improper conclusion of the effects of public investment in output.

Given all these problems, an estimation technique that circumvents at least some of them is needed. It is within this line of reasoning that the VAR approach is introduced and an empirical application conduced to the Swedish economy.

5 The VAR approach

The VAR approach imposes as little economic theory as possible. Proposed by Sims (1980), it is widely referred to as macroeconomics without theory, as in Cooley and LeRoy (1985), since it is mainly data oriented and no functional form is implied. The VAR estimation technique evolved from the traditional setting of a system of simultaneous equations. The problem with this approach is that, in order for the estimation to be made, the variables had to be classified as either exogenous or endogenous. This imposes null restrictions under some parameters of the model, sometimes something hard to justify in light of economic theory. The improvement the VAR methodology brings is that the distinction between endogenous and exogenous variables no longer exists, since every variable is treated as endogenous.

Given this, one chooses to use different variables from the ones used in the production function approach. Calculating capital stocks for the business sector and especially for the government sector is something that implies several assumptions that lack theoretical support. For instance, Kamps (2004) calculates government capital stocks for 22 OECD countries. However, in order to achieve such purpose, Kamps had to resort to very strong assumptions like assuming public investment to have grown by 4% a year between 1860 and 1959 as well as a geometric depreciation rate. Furthermore, there is an absolute lack of consensus on what measuring capital stocks is concerned giving raise to several measures used by different national entities. Following the same author, this was one of the reasons why OECD stopped publishing them in 1997. As a result of the practical difficulties in calculating the capital stock, I decided not to use them. The variables chosen do not rely so heavily on such assumptions and are easier to understand in the estimation framework.

The econometric approach begins with testing the cointegrating properties of the data. I proceed to the VAR specification and estimation. Next, the analysis of the plotted impulse response and accumulated response functions is undertaken in order to show the dynamic relationship between the variables. Finally, accumulated elasticities are reported in order to quantify the effects of public investment in other variables and variance decomposition undertaken.

The econometric software package used was RATS for Windows version 6.10.

5.1 Data Description

Concerning the empirical application, I choose to investigate the relationship between GDP $(SWEGDPV)$, investment of the business sector $(SWEIBV)$, government investment $(SWEIGV)$ and total employment $(SWEET)$. The data is in annual periodicity, at 2000 prices. The first three variables are in millions of SEK and the last one in number of persons employed. This data was taken from the OECD Economic Outlook Statistics and Projections 2005 edition database. The sample covers the period from 1962 to 2003, for a total of 42 observations.

On what GDPV is concerned, it evolved from the initial sample value in 1962 of 835344.05 to 2301975 million SEK in 2003. It grew at the average annual growth rate of 2.44% in this period. Business sector investment started with an initial value of 71567.36 million SEK and reached the amount of 257283 million SEK in 2003, growing at the average annual growth rate of 3.01%. In the case of government investment, it evolved from 22549.394 to 64307 million SEK in the same period, at the average rate of 2.53% per year.

It is worth noting though that the evolution of the variables is not fully characterized by the average growth rates. Indeed, large fluctuations occurred during this time period. GDP growth slowed down due the two international oil crisis, largely as a result of a weakening foreign demand for Swedish products, even though employment was relatively unaffected. In fact, between 1962 and 1974, GDP had grown at 3.36% per year. From 1974 until 1981, GDP slowed down to an annual growth rate of 1.33%. Growth in the 80's kept the same pace as the rest of Western Europe, at around 2%. However, the beginning of the 90's was marked by the worst economic crisis in Sweden since the great depression. Between 1990 and 1993, GDP decreased 5% and employment by about 10%. But it is private investment that has the most startling figure: in 1993 it reached 133321 million SEK, only 63.84% of its 208846.7 million SEK 1990's amount. It was during this period that the role of the government in stimulating the economy was important. In fact, public investment grew in the same period almost 20%. In spite of a severe deterioration in public sector finances, mainly due to unemployment benefits and government sponsored job programs, the fiscal stimulus was successful as the Swedish economy recovered from this crisis. From 1993 to 2003 GDP grew at an annual rate of 2.67%. There was however, the deterioration of the pubic finances to deal with. Given that the economy was performing well after 1993, major cutbacks were made by the government in public sector consumption, as well as in the number of public employees. Government investment was also hampered as it has slowly increased at the 0.79% annual growth rate since 1993, in contrast with 9 % in the 60's or the 1.4% of the 80's.

In 2003, more than 30% of the labour force was employed in the public sector, and general government expenditure added up to more than half the GDP, where public investment accounts only for 2,7%.

5.2 Univariate and Cointegration Analysis

A sufficient condition for the stability of the model is that the variables are stationary, or, if not stationary, that they are cointegrated. In the last case, an error correcting mechanism needs to be included in the original VAR model. It is then necessary then to conduct an analysis of each variable to check their integration order and possible cointegration.

The standard approach is to conduct Dickey-Fuller (1979) i.e. (DF) tests. However, serial correlation of the residuals can seriously bias the estimation of the unit root. One way to get rid of the serial correlation problem is to add a sufficient number of lagged terms to the DF regression. An Augmented Dickey-Fuller (ADF) test is then performed and the T and Z statistics observed, as in Hamilton (1994). It is then needed to calculate the appropriate number of lagged variables to include in order not to incur into autocorrelation. Hence, a Lagrange Multiplier Test $(LM\ Test)$ is conducted, successively adding lags to the ADF equation, until the null hypothesis of no serial correlation in the residuals cannot be rejected at the 5% significance level. Nonetheless, as the null hypothesis of no serial correlation can never be rejected, in practice, only DF tests are conducted. A constant and/or a trend are included when significant at the same significance level. The critical values used are from Dickey and Fuller (1979). Under the ADF test framework, the null hypothesis (H_0) of no stationarity cannot be rejected for the four time series under consideration, as can be depicted from the *table 1* below. The observation of the slow decline of the Auto-Correlation Function (ACF), as well as of the Partial Correlation Function (PCF) both shown in annex I, further confirms such result, once if the series were stationary, the ACF and PCF should not be statistically different from zero after one or two periods.

Table 1 – ADF T and Z tests of the variables in natural logarithms

Variables in Logs	Deterministic components	Lags (Lmtest)	ADF T-Test t statistic	Critical Values 5%	Critical Values 1%	ADF Z-Test z statistic	Critical Values 5%	Critical Values 1%
SWEET	constant	0	-1,60	-2,93	-3,58	-2,85	-13,3	-18,9
SWEGDPV	constant and trend	0	-3,21	-3,50	-4,15	-7,78	-19,8	-25,7
SWEIBV	constant and trend	0	-2,51	-3,50	-4,15	-11,73	-19,8	-25,7
SWEIGV	constant and trend	0	-3,09	-3,50	-4,15	-9,86	-19,8	-25,7

The rejection of the null hypothesis of non-stationarity of the series in first differences would suggest that they are $I(1)$. One proceeds to perform the very same tests to the series after first differentiating them. The analysis of the T and Z statistics both point to the rejection of the null-hypothesis that the series are non-stationary. Furthermore, the observation of the *ACF* and *PCF* gives additional support for the conclusion that the original series are stationary in first differences.

Table 2 – ADF T and Z tests of the variables in first differences of natural logarithms

Variables in First Difference of Logs	Deterministic components	Lags (Lmtest)	ADF T-Test t statistic	Critical Values 5%	Critical Values 1%	ADF Z-Test z statistic	Critical Values 5%	Critical Values 1%
dSWEET	none	0	-3,15	-1,95	-2,62	-16,27	-7,7	-12,9
dSWEGDPV	constant	0	-3,98	-2,93	-3,58	-22,62	-13,3	-18,9
dSWEIBV	constant	0	-4,46	-2,93	-3,58	-27,64	-13,3	-18,9
dSWEIGV	constant	0	-5,42	-2,93	-3,58	-30,23	-13,3	-18,9

Since the series are $I(1)$, it is crucial for any OLS regression including them to be meaningful, that the variables converge to their long-term relationship i.e. they are cointegrated. If that is the case, following Engle and Granger (1987), a regression containing $I(1)$ variables will yield stationary residuals. Since the VAR approach implies regressing all series on each other and their lagged values, stationarity of the residuals is tested in four regressions, each regression having one of the four series as the dependent variable. A constant and/or a trend are included when significant at the 5% level in the cointegration-test regression. Nonetheless, one does not have access to the actual residuals of the regression but only their estimate. Hence, an Engle-Granger test is conducted. Basically it consists in conducting normal ADF tests to the estimated residuals but having more restrictive critical values, here computed for the specific sample size, from the response surface regressions in Mackinnon (1991). As can be observed in *table 3*, one cannot reject the null hypothesis of non-stationarity of the residuals for every regression. Since non-stationarity cannot be rejected one concludes that the series are not cointegrated and OLS will yield spurious relationships between the series.

Table 3 – Engle-Granger tests of cointegration

Variables in Logs dependent var	Deterministic components	Lags (Lmtest)	t statistic	ADF T-Test Critical Values 5%	1%
SWEET	constant and trend	0	-2,86	-4,79	-5,53
SWEGDPV	constant and trend	0	-3,17	-4,79	-5,53
SWEIBV	constant	0	-3,45	-4,37	-5,09
SWEIGV	constant and trend	0	-3,48	-4,79	-5,53

Given this, estimation in first differences is the next step. As seen before, the series are stationary in first differences and therefore OLS estimates will be meaningful. Hence, the stability of the VAR model is assured once, as referred before, stationarity of the variables is a sufficient condition to assure it. One then proceeds with the VAR specification and estimation.

5.3 VAR Specification and Estimates

Due to lack of space, a new notation for the variables is introduced: the variables *dSWEET, dSWEGDPV, dSWEIBV* and *dSWEIGV* are now represented by $\dot{E}$, $\dot{Y}$, $\dot{I}$ and $\dot{G}$ respectively. To choose the lag length, two criteria are mostly used. One proposed by Akaike (1974), namely Akaike's Information Criteria (AIC), and other proposed by Schwarz (1978) and it is know as the Schwarz-Bayesian Information criteria (BIC or SBC). The system in (4) constitutes a second-order VAR since the longest lag length is two. The AIC indicated the inclusion of only one lag. The BIC however indicated for the inclusion of four lags. As the inclusion of only one lag is considered not to properly capture the dynamics of the system, and four to consume too many degrees of freedom given the limited data set available, one chooses to include two lags in the model.

The VAR model is then given by the following system of equations:

$$\begin{cases} \dot{E}_t = c_1 - b_{12}\dot{Y}_t - b_{13}\dot{I}_t - b_{14}\dot{G}_t + \sum_{i=1}^{2}\alpha_{1i}\dot{E}_{t-i} + \sum_{i=1}^{2}\alpha_{2i}\dot{Y}_{t-i} + \sum_{i=1}^{2}\alpha_{3i}\dot{I}_{t-i} + \sum_{i=1}^{2}\alpha_{4i}\dot{G}_{t-i} + \varepsilon_{Et} \\ \dot{Y}_t = c_2 - b_{21}\dot{E}_t - b_{23}\dot{I}_t - b_{24}\dot{G}_t + \sum_{i=1}^{2}\beta_{1i}\dot{E}_{t-i} + \sum_{i=1}^{2}\beta_{2i}\dot{Y}_{t-i} + \sum_{i=1}^{2}\beta_{3i}\dot{I}_{t-i} + \sum_{i=1}^{2}\beta_{4i}\dot{G}_{t-i} + \varepsilon_{Yt} \\ \dot{I}_t = c_3 - b_{32}\dot{Y}_t - b_{31}\dot{E}_t - b_{34}\dot{G}_t + \sum_{i=1}^{2}\delta_{1i}\dot{E}_{t-i} + \sum_{i=1}^{2}\delta_{2i}\dot{Y}_{t-i} + \sum_{i=1}^{2}\delta_{3i}\dot{I}_{t-i} + \sum_{i=1}^{2}\delta_{4i}\dot{G}_{t-i} + \varepsilon_{It} \\ \dot{G}_t = c_4 - b_{42}\dot{Y}_t - b_{43}\dot{I}_t - b_{41}\dot{E}_t + \sum_{i=1}^{2}\varphi_{1i}\dot{E}_{t-i} + \sum_{i=1}^{2}\varphi_{2i}\dot{Y}_{t-i} + \sum_{i=1}^{2}\varphi_{3i}\dot{I}_{t-i} + \sum_{I=1}^{2}\varphi_{4i}\dot{G}_{t-i} + \varepsilon_{Gt} \end{cases} \quad (4)$$

This system of equations is called a primitive VAR. As it can be observed, the problems associated with the lack of feedback between the variables in an Augmented Solow production function no longer exist. As it can be noticed, all variables are affected by each other's contemporaneous value and their lags. In this case, all ε_t terms are pure innovations, or shocks, in each corresponding variable. The problem is that each variable is correlated with the error term in the other equations due to the feedback inherent to the system. These equations cannot be estimated directly. Next, one applies a procedure in order to try to circumvent such problem as in Enders (1995).

Passing the variables in time t to the left hand side of each equation and writing this system in matrix notation yields:

$$\begin{bmatrix} 1 & b_{12} & b_{13} & b_{14} \\ b_{21} & 1 & b_{23} & b_{24} \\ b_{32} & b_{31} & 1 & b_{34} \\ b_{42} & b_{43} & b_{41} & 1 \end{bmatrix} \begin{bmatrix} \dot{E}_t \\ \dot{Y}_t \\ \dot{I}_t \\ \dot{G}_t \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} + \sum_{i=1}^{2} \left[\begin{bmatrix} \alpha_{1i} & \alpha_{2i} & \alpha_{3i} & \alpha_{4i} \\ \beta_{1i} & \beta_{2i} & \beta_{3i} & \beta_{4i} \\ \delta_{1i} & \delta_{2i} & \delta_{3i} & \delta_{4i} \\ \varphi_{1i} & \phi_{2i} & \varphi_{3i} & \varphi_{4i} \end{bmatrix} \begin{bmatrix} \dot{E}_{t-i} \\ \dot{Y}_{t-i} \\ \dot{I}_{t-i} \\ \dot{G}_{t-i} \end{bmatrix} \right] + \begin{bmatrix} \varepsilon_{Et} \\ \varepsilon_{Yt} \\ \varepsilon_{It} \\ \varepsilon_{Gt} \end{bmatrix} \quad (5)$$

Again, for simplicity sake, the system above can be represented by:

$$Bx_t = \Gamma_0 + \Gamma_1 x_{t-1} + \Gamma_2 x_{t-2} + \varepsilon_t \quad (6)$$

where:

$$B = \begin{bmatrix} 1 & b_{12} & b_{13} & b_{14} \\ b_{21} & 1 & b_{23} & b_{24} \\ b_{32} & b_{31} & 1 & b_{34} \\ b_{42} & b_{43} & b_{41} & 1 \end{bmatrix}; \Gamma_0 = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix}; \Gamma_i = \begin{bmatrix} \alpha_{1i} & \alpha_{2i} & \alpha_{3i} & \alpha_{4i} \\ \beta_{1i} & \beta_{2i} & \beta_{3i} & \beta_{4i} \\ \delta_{1i} & \delta_{2i} & \delta_{3i} & \delta_{4i} \\ \varphi_{1i} & \phi_{2i} & \varphi_{3i} & \varphi_{4i} \end{bmatrix} \begin{bmatrix} \dot{E}_{t-i} \\ \dot{Y}_{t-i} \\ \dot{I}_{t-i} \\ \dot{G}_{t-i} \end{bmatrix}, i = 1,2; \varepsilon_t = \begin{bmatrix} \varepsilon_{Et} \\ \varepsilon_{Yt} \\ \varepsilon_{It} \\ \varepsilon_{Gt} \end{bmatrix}$$

Pre-multiplying system (6) by B^{-1} yields the VAR model in what is called across the literature as the standard form:

$$x_t = A_0 + A_1 x_{t-1} + A_2 x_{t-2} + e_t \qquad (7)$$

as:

$$A_0 = B^{-1}\Gamma_0 \; ; \; A_1 = B^{-1}\Gamma_1 \; ; \; A_2 = B^{-1}\Gamma_2 \; ; \; e_t = B^{-1}\varepsilon_t$$

Taking c_i as the i^{th} element of the A_0 vector, a_{ij} and b_{ij} as the elements in row i and column j of the matrixes A_1 and A_2 respectively and e_{it} as the element $i = \left\{ \dot{E}, \dot{Y}, \dot{I}, \dot{G} \right\}$ of the vector e_t, one can present system (7) as the notation one first started with in (4).

$$\begin{cases} \dot{E}_t = c_1 + \sum_{i=1}^{2} \alpha_{1i} \dot{E}_{t-i} + \sum_{i=1}^{2} \alpha_{2i} \dot{Y}_{t-i} + \sum_{i=1}^{2} \alpha_{3i} \dot{I}_{t-i} + \sum_{i=1}^{2} \alpha_{4i} \dot{G}_{t-i} + e_{Et} \\ \dot{Y}_t = c_2 + \sum_{i=1}^{2} \beta_{1i} \dot{E}_{t-i} + \sum_{i=1}^{2} \beta_{2i} \dot{Y}_{t-i} + \sum_{i=1}^{2} \beta_{3i} \dot{I}_{t-i} + \sum_{i=1}^{2} \beta_{4i} \dot{G}_{t-i} + e_{Yt} \\ \dot{I}_t = c_3 + \sum_{i=1}^{2} \delta_{1i} \dot{E}_{t-i} + \sum_{i=1}^{2} \delta_{2i} \dot{Y}_{t-i} + \sum_{i=1}^{2} \delta_{3i} \dot{I}_{t-i} + \sum_{i=1}^{2} \delta_{4i} \dot{G}_{t-i} + e_{It} \\ \dot{G}_t = c_4 + \sum_{i=1}^{2} \varphi_{1i} \dot{E}_{t-i} + \sum_{i=1}^{2} \varphi_{2i} \dot{Y}_{t-i} + \sum_{i=1}^{2} \varphi_{3i} \dot{I}_{t-i} + \sum_{I=1}^{2} \varphi_{4i} \dot{G}_{t-i} + e_{Gt} \end{cases} \qquad (8)$$

Worth noting is the fact that the error terms e_{it} of this equation are composites of the four shocks ε_{it} for $i = \left\{ \dot{E}, \dot{Y}, \dot{I}, \dot{G} \right\}$, once:

$$e_t = B^{-1} \varepsilon_t \qquad (9)$$

Given that, by assumption, ε_{it} are white-noise processes, e_{it} will have zero mean and constant variance. They will also be individually serially uncorrelated even though it is not, in general, the case for ε_{it} when compared across equations. In fact, only when the contemporaneous effects of the variables does not take place, such serial correlation will cease to exist. But, since e_{it} are uncorrelated with the regressors of its respective equation, the system can be estimated using the OLS procedure. The estimated parameters as well as relevant statistics for each equation are in figures three to six in annex III.

5.4 Granger-Causality Analysis

Another issue often referred when conducting VAR analysis concerns causality. Following Granger (1969), a test of causality concerns whether the lags of *x* are statistically different from zero in the equation for *y*. If such is the case, then *x* is said to Granger-cause *y*. In this case, the test is whether the set of lags of each variable should enter each equation. The F-Test shown in annex III, figures three to six, tests the null hypothesis that each set of lagged variables is different from zero. As it can be seen for the case of the growth rate of public investment, no lagged set of variables is expected to be different from zero at the 5% significance level. This result is in line with Pereira and Andraz (2004) for the Portuguese economy. The growth rate of public investment is not Granger-caused by the growth rate of other variables. The interpretation of this result is not straightforward. It suggests that public decision makers do not take into concern macroeconomic aggregates when deciding upon public investment and that possibly other components of government expenditure are used in stabilization policy.

When analyzing the Granger-causality results for the growth rate of private investment, one reaches the conclusion that it is Granger caused by all sets of lagged variables but it self's at the 5% significance level. This result suggests the existence of an indirect effect of the growth rate of public investment in GDP through the growth rate of private investment as well as a feedback mechanism between the growth rates of GDP and private investment (if private investment's growth rate is found to Granger-cause GDP), something that a static production function as the one analyzed before would not properly capture.

On what the growth rate of employment is concerned, the results of the Granger-causality test are somewhat problematic. It seems not to be Granger-caused by any set of lagged variables, not even by itself. This result is hard to explain in light of economic theory. One hypothesis is that, as the variable is originally measured in the number of jobs, changes in productivity of labour are not reflected in the employment levels but in a rather quick adjustment through wages. When productivity varies, companies reflect such variations on wages hence not affecting the number of jobs. In this case, if government policies are effective when aiming at increasing output, one should expect such policies not to promote employment but wage levels instead.

Finally, the observation of the Granger-causality test for the growth rate of GDP is quite disappointing. The F-test's points to the rejection of the null hypothesis that the growth rate of GDP is Granger-caused by the other variables. It seems that GDP's growth rate is mainly caused, in the Granger sense, by itself. A plausible explanation is hard to find.

5.5 Innovation Accounting

After the analysis of Granger-causality, one proceeds to analyze the dynamics of the model. The standard approach is to observe the time path value of each variable as response to a shock in other variable. This methodology is commonly known in the literature as *innovation accounting*. Such time path values are referred to as impulse response functions. To do so, one needs to rewrite the model as a vector moving average (VMA). As in Enders (1995), a VAR of order p can generally be represented by:

$$x_t = \bar{c} + \sum_{i=0}^{\infty} \Phi^i e_{p-i} \tag{10}$$

This procedure generates x_t as an infinite sum of random lagged errors, weighted by increasingly smaller coefficients, where $\bar{c}$ is a vector with the average values of the variables. These coefficients can be interpreted as impact multipliers. As it can be seen from the general equation, the value of a variable subject to a shock is given by its average plus its response to the shock. Following this line of reasoning, the impulse responses can be seen as deviations of the variable around its mean.

The problem is that the model dynamics cannot be assessed inducing shocks to the estimated residuals from (10). In fact, if one wants to observe how the variables react to a typical shock (one standard deviation) to public investment's growth rate for example, it is the standard deviation of the residuals of the primitive VAR (ε_{Gt}) that should be taken into account and not the estimated residuals of the standard VAR (e_{Gt}). From (9) one is able to relate (ε_{Gt}) and (e_{Gt}). The fact remains of whether it is possible to retrieve the information concerning (ε_{Gt}) in the primitive VAR through the OLS estimates of (e_{Gt}) computed from the standard VAR. Unfortunately there is no way of achieving such purpose without imposing certain restrictions to the VAR in the standard form. The problem resides in the fact that there is no unique solution to equation (7).

Given this, the system is under identified. In fact, for a VAR with n variables, $(n^2 - n)/2$ restrictions need to be imposed for it to become exactly identified. That can easily be seen by comparing the number of parameters to be estimated between the VAR in primitive form and in the standard form. As the number of parameters of the latter is less than the former, the primitive VAR will be under identified if only the estimated parameters from the standard VAR are used. This is the point when economic theory comes into play and helps to choose the appropriate restrictions.

One assumes that public investment is not affected contemporaneously by any other variable. The justification for this restriction $(\{b_{42}, b_{43}, b_{41}\} = 0)$ comes from the fact that decisions concerning the amount of public investment are typically made even before the year starts, in the framework of the government yearly budget. Next, one chooses to assume that private investment is contemporaneously affected by public investment but not by the employment or GDP i.e. $(\{b_{31}, b_{32}\} = 0)$. It is reasonable to assume that private agents review their decisions following the announced government budget. Once employment is seen as jobs created by either the government or the private sector, it is also reasonable to assume that it is affected contemporaneously by these two variables but that it doesn't affect any of them in the same period. GDP is seen as a result of the combination of all this input factors so it is reasonable to assume that it is affected contemporaneously by all of them while affecting none in the same period i.e. $(b_{12} = 0)$. Once these restrictions are imposed to the six referred parameters, the equation in (7) can be uniquely solved. The system becomes exactly identified and the information present in the primitive VAR deductible from the estimated parameters for the standard VAR. Imposing such restrictions is described in the literature as ordering the variables. Decomposing the estimated residuals in (8) is now possible through equation (9) and one is now able to trace the time paths of the effects of pure ε_t shocks. Such procedure is called the Cholesky decomposition. The impulse response functions are then shown in annex IV and V. As impulse responses are random variables it is important that the uncertainty around their respective estimates is shown. The confidence intervals were computed using the bootstrap methodology (1000 draws). The chosen confidence interval was 80%. In fact, Sims (1987) says that "there is no scientific justification for testing hypothesis at the 5% significance level in every application". He argues that it is a characteristic of VAR models that most parameters estimated are not statistically different from zero at the 95% confidence level. Following this line of reasoning, Sims and Zha (1999) defend computing 68% confidence intervals for estimated impulse responses. In this case, the use of 80% confidence levels seems a good compromise between standard econometric analysis and what the referred authors defend.

The plotted impulse response yields, for each period, how much a variable deviates from its average as a result from a structural shock in public investment in the first period and all the responses in the other variables as a result of that very same shock.

The first important feature of the plotted impulse response functions is that they converge to zero after ten iterations. This means that the estimated VAR is stable, otherwise the impulse responses would not converge to zero. Many studies plot impulse responses that are not convergent as Kamps (2005). It is hard to understand how a shock to a variable can permanently affect other stationary variable in every period from that moment on or, if in the presence of a VAR with a vector error correcting mechanism (VECM), as it is the case when the variables are $CI(1,1)$, how that error correcting mechanism does not make the shock disappear after a reasonable number of periods.

Second, as it can be noticed from the observation of figure seven, *G* does not have noticeable effects upon the growth rates of *Y* or *E*. It has however some effect upon the *I*. This gives support for the crowding out hypothesis above mentioned. In the case of *I*, the observation of figure nine shows that I has a small but noticeable effect in both Y and E. To assess the accumulated effect of a typical shock of *I* or *G* in the other variables, one adds up the impulse response function value in the first period to its consequent values. The accumulated impulse responses are shown in figures eight and ten.

Calculating elasticities and marginal products requires that all other factors remain constant, i.e. the *ceteris paribus* condition. In this case, such concepts are understood in a different way, once all the dynamic feedback effects are reflected. This way, following Pereira (2001b), elasticities reflect the total accumulated percentage change of the variables as a result of an accumulated variation of 1% of the variable to which a shock was induced (*j*). To calculate Pereira computes the ratios of the sum of the impulse response values until the last period for each variable (*i*) over the sum of the impulse response values of the variable on which the shock was induced (*j*):

$$\xi_{ij} = \frac{\sum_{t=1}^{15} IR_t(i)}{\sum_{t=1}^{15} IR_t(j)} \tag{11}$$

These elasticities can be interpreted as how sensible the evolution of the other variables are to an unexpected accumulated 1% change of the instrument variable's growth rate, i.e. a one standard deviation shock to the primitive system:

Table 4 – Long run elasticities of variable *i* as a result of a shock to variable *j*

Variables i =	Y	E	I	G
Elasticity of i related to G	0,01%	-0,04%	-0,48%	1
Elasticity of i related to I	0,08%	0,17%	1	-0,50%

As it can be seen, $\dot{G}$ hardly has any effect in $\dot{Y}$ (0.01%) and a very small one in $\dot{E}$ (0.04%). These results are not statistically different from zero at the 80% confidence level. This suggests that a policy of increasing $\dot{G}$ would have no effects on either $\dot{E}$ or $\dot{Y}$. When it comes to $\dot{I}$, the effect is significant. It basically means that if the Swedish government would induce a shock to G that would result in an accumulated deviation equal to its average value (an increase of 100%), $\dot{I}$ would have fallen 48% after 15 years. This gives support for the crowding out hypothesis. In the case of the effects in the variables of a shock to $\dot{I}$, the effect is small in the case of $\dot{Y}$ (0.08%) moderate in the case for $\dot{E}$ (-0.17%) and strong in the case of public investment (-0.5%).

Finally, one performs the forecast error variance decomposition. Such procedure allows one to see the long-run (after 15 periods) percentage variation in all variables as a result of a shock to public investment's growth rate. If the share of variation of a variable would be 0%, then that variable would be completely exogenous in the model as in Enders (1995). As it can be observed, the forecast error variance decomposition shows that 9.6% of total variation of $\dot{Y}$ is directly accountable to the shock in $\dot{G}$. The fact that even after 15 years, the variance of $\dot{G}$ is still explained mostly by itself (82.9%), suggest that this variable behaves exogenously, in line with Pereira and Andraz (2004) findings for the Portuguese economy. The results are shown in table 5 below.

Table 5 – Forecast error variance decomposition of each variable as a result of a shock to $\dot{G}$

1 S.D. Shock to G	Y	E	I	G
Long-run Value	8,2%	12,9%	9,6%	82,9%
80% Confidence Interval	[5,1% ; 28,2%]	[6,3% ; 33,6%]	[5,9% ; 29,8%]	[57,8% ; 85,3%]
Variation Interval	[3,6% ; 8,2%]	[5,8% ; 12,9%]	[6,2% ; 9,8%]	[82,9% ; 88,3%]

The variance decomposition is also performed when analyzing the effects of a shock to $\dot{I}$. As it can be depicted from table 6, a shock to $\dot{I}$ explains a significant part of the forecast error variance of $\dot{Y}$ and $\dot{E}$, strengthening the results found with the impulse response analysis.

Table 6 – Variance decomposition of each variable as a result of a shock to $\dot{I}$

1 S.D. Shock to I	Y	E	I	G
Long-run Value	12,5%	12,7%	53,9%	9,3%
80% Confidence Interval	[6,4% ;30,6%]	[4,5% ;32,7%]	[31,7% ;61,7%]	[3,1% ;19,4%]
Variation Interval	[12,5% ; 18,1%]	[12,6% ; 14,9%]	[53,9% ; 98,7%]	[7,2% ; 9,6%]

table 6

5.6 Sensitivity Analysis to Alternative Identification Restrictions

One of the assumptions made in section 5.5 was that the variables were ordered in a particular way. A restriction was imposed in the primitive VAR system to allow for the Cholesky decomposition of the residuals in the standard VAR. Some insights from economic theory were used in order to order the variables in a particular way, more precisely $\left[\dot{G},\dot{I},\dot{E},\dot{Y}\right]$. This ordering was determinant to calculate the impulse response functions. It is important then to test the robustness of the results by testing different ordering schemes, to check if the results are dependent on the ordering. In such a case, the economic theory behind the justification of the particular Cholesky decomposition used would have to be quite solid.

The problem is that a VAR model with N variables can be subject to $N!$ different Cholesky residuals' decomposition i.e. $N!$ different ordering of the variables. In the case analyzed in this paper it would imply testing 4!=24 possible orderings once there are 4 variables under consideration. Given this, one chooses to invert the ordering of the variables and test $\left[\dot{Y},\dot{E},\dot{I},\dot{G}\right]$. This way a comparison between two extreme cases is being made. The first ordering that implies a shock in public investment to affect contemporaneously all other variables against the alternative one where the same shock affects no variables in the same time period.

The results can be observed in annex VI. As can be seen, the plotted impulse response functions hardly differ from the ones that resulted from the initial ordering of the variables. This way, the restriction imposed in order to do the Cholesky decomposition is not determinant for the results observed as they seem to be quite robust to this alternative ordering.

5.7 Comments on the empirical findings

The main conclusions to be drawn from the empirical application is that the accumulated effect of innovations in $\dot{G}$ in $\dot{Y}$ and $\dot{E}$ are not statistically different from zero, even at the 80% confidence level, while the negative effect on $\dot{I}$ is statistically different from zero and apparently high, with an accumulated elasticity of -0.48. The effect of innovations of $\dot{G}$ in itself are also shown to be positive and statistically different from zero. Furthermore, $\dot{I}$ has a significant and positive effect in $\dot{E}$ and a marginally insignificant effect in $\dot{Y}$.

On what the negligible effects of $\dot{G}$ in $\dot{Y}$ is concerned, many explaining hypothesis can be formulated. First it can be a result of inappropriate data. The variable $\dot{G}$ only takes into concern government investment. It rules out investment by municipalities which means that if there are asymmetries in the type, amount or productivity in the two types of investment, which is most likely to be the case, the estimated values will not reflect the effect of all public investment. In such scenario, it would be more suitable to have data on both types of investment, something that it is not available, at least in the OECD database.

A different kind of approach relates to the law of diminishing returns. If, as referred above, Sweden has a comparably large public sector, the fact might be that the Swedish economy is characterized by a comparably large public capital stock. The greater the public capital stock the lesser the effects of extra public investment are expected to be. In this framework, further increases, in excess of the necessary amount to cover the depreciated public capital, will yield lower and lower returns.

Another hypothesis is that efficiency gains in the productivity of public capital might have been compensated by the decreasing ratio of public investment to GDP that has been observed since the 60's. In fact, in this period, public investment was, in average, 3.6% of GDP, decreased to 3.3% in the 70's and to 2.7% in the 80's. In the 90's, the crisis that opened the decade made the ratio go up to 3.2% but since 2000 the average has returned to what had been observed in the 80's, 2.7%. If the total value added to production of public investment remained the same, the referred decrease investment did not have any effect in GDP.

On what $\dot{E}$ is concerned, an important fact that should be referred to is that the size of the labour force has been relatively stable during the period of analysis. In fact, it grew at an annual rate of 0.4% from 1962 to 2003, according to the referred OECD database. It is possible that $\dot{G}$ has had an impact in the labour market through real wages instead of the number of people employed.

One other aspect to take into concern when analyzing the labour market is that employment in the government sector has grown at the average annual rate of 2.4% since 1962. This growth is well above the annual average growth in total employment in the same period: 0.3%. Consequently, the growth of employment in the business sector was negative, about -0.2%. From this perspective, the public sector has created many jobs, but at the cost of the private sector, a sector that saw its employment levels actually decrease in the same period. This might explain the negligible effect of public investment in total employment. Nonetheless, this could only be confirmed or not by looking on the effects of public investment and government expenditure in employment in separate. The fact that the volume of investment has grown faster than total employment in the private sector (3% comparing to the -0.2%), suggests that this sector is becoming more and more capital intensive. On the other hand, the public sector has created more jobs at a pace relatively similar to the increase in public investment, suggesting it has lagged behind in productivity compared to the private sector.

Relating to the significant negative effect of $\dot{G}$ in $\dot{I}$, the literature as stated before, reports mixed conclusions. Since the estimation is made in first differences and in spite of the VAR including two lags, at least some of the long-term relationship between the variables is lost. Given this, one hypothesis is that the strong negative impact may result from the short-term substitution effect between private and public capital. In a Keynesian framework, public investment crowds out private investment by increasing interest rates as a result of an increase in aggregate demand.

Again, under the assumption that Sweden possesses a comparably large public capital stock and following the fact that many goods considered to be private goods are provided by the state out of concern for under-provision of such goods among less favoured classes, as it is the case for tertiary education, it is reasonable to expect that public investment crowds-out private investment. Again, as Pereira (2001b) puts it, public investment is expected to act as substitute if it aims at increasing production instead of acting as a complement to private investment.

The fact that the reported elasticity is big (-0.48) has to be taken carefully. The large confidence intervals inspire some caution in the analysis. Nonetheless, public investment showed to be Granger-caused by public investment. Given this, I conclude that there is strong support for the crowding-out hypothesis even though some caution has to be taken when looking at the magnitude of the calculated elasticity.

The effect of $\dot{G}$ in itself relates to the fact that public investment is often connected with projects that take several years to complete. The same way the investment in flattening large pieces of land in a year, anticipates the investment in highway pavement in the next, public investment tends to show strong autocorrelation over the years.

On what the effect of innovations in $\dot{I}$ is concerned, they demonstrated to positively affect $\dot{E}$ and $\dot{Y}$ even though the accumulated effect on the latter lies in the limit of not being considered statistically different from zero.

It is a stylized fact that investment in general is determinant to growth. I would expect such effect to be more dramatic in the results. An hypothesis is that Sweden, taken as one of the most developed countries in the world might has its economy close to what is called, within an augmented Solow model framework, steady-state. In such conditions, the growth rate of GDP is exogenous and is a function of the progress of technology, or, put in a different way, of increases in multifactor productivity. This hypothesis, would explain the fact that $\dot{Y}$ behaves exogenously in the Granger-causality analysis. It would also explain how typical innovations in investment seem only to marginally have any effect in $\dot{Y}$.

Finally, the positive effect that innovations in $\dot{I}$ seem to exert in $\dot{E}$ is also in line with the literature. Private investment is expected to increase employment. Nonetheless, a consistent increase of private investment, the referred 3% per year over the period under analysis, did not translate into more jobs in the private sector, as the number of jobs in that sector actually diminished at the average annual rate of -0.2%, as shown above. That fall was however more than compensated by increases in contracting by the public sector. This inspires some caution into drawing conclusions from the effect of $\dot{I}$ in $\dot{E}$ and suggests that it would have been more appropriate to disaggregate the variables in the VAR model, in spite of the Granger-causality analysis strongly indicating that $\dot{I}$ Granger-causes $\dot{E}$.

6. Concluding Remarks

Given the analysis made, one concludes that the inclusion of public capital stock in the Augmented Solow model has been the traditional approach towards modelling the long-term impact of public investment in GDP. The problem is that, it cannot be made without violating some standard assumptions, like marginal productivity theory. Among other econometric problems, such inclusion does not take into account the possible feedback effects, and interrelationships that may exist between the variables. Another issue relates to the fact that the necessary estimates of public capital stock imply a number of assumptions hard to justify in theoretical terms. It is within this framework that the VAR methodology has been increasingly used to assess this subject. By imposing as little theory as possible, some theoretic problems that were posed to the estimation of the Augmented Solow production function no longer exist. Given its dynamic nature, several econometric problems are also solved. The results of the VAR estimation show that private investment's growth rate is positively affected by innovations in public investment's growth rate, suggesting that the traditional setting of an augmented Solow production function cannot properly capture the interrelationship between the variables. Looking at the impulse response functions, innovations in the growth rate of public investment has a negligible effect in both employment and GDP growth rates. It suggests however that it has a significant negative impact in private investment's growth rate. On the other hand, innovations in the growth rate of private investment has a small but significant impact in both employment and GDP's growth rates. These results suggest that hampering public investment may have a positive impact in boosting private investment's growth. Policies aiming at promoting private investment's growth are suggested, in this setting, to positively affect employment and GDP growth. However, the variance decomposition results, along with the Granger-causality analysis inspire some caution in drawing any strong policy conclusions from this analysis.

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8. References

Akaike, H. (1974). A new look at the statistical model identification. *IEEE Transactions on Automatic Control* 19 (6): 716–723.

Aschauer, D.A., (1989). Is Public Expenditure Productive? *Journal of Monetary Economics*, vol. 23, pp. 177-200

Barro, R. (1990), Government Spending in a Simple Model of Endogenous Growth", *Journal of Political Economy*, 98, pp. 103-125,

Blinder, Alan S & Maccini, Louis J, (1991). Taking Stock: A Critical Assessment of Recent Research on Inventories, *Journal of Economic Perspectives*, American Economic Association, vol. 5(1), pages 73-96

Cooley and Leroy (1985). Atheoretical Macroeconomics: A Critique. *Journal of Monetary Economics*, 16, pp. 283-308

Dickey D. and W. Fuller (1979). Distribution of the Estimators for Time Series Regressions with a Unit Root, *Journal of the American Statistical Association*, pp 427-431

Duggal, V., C. Saltzman and R. Klein (1995). Infrastructure and Productivity: A Nonlinear Approach, paper presented at the 7th World Congress of the Econometric Society, Tokyo, Japan.

Enders, W. (1995). *Applied Econometric Time Series*, Wiley Series in Probability and Mathematical Statistics

Engle, R. and C. Granger (1987). Co-integration and Error Correction: Representation, Estimation and Testing, *Econometrica*, 55(2), 251-276, 1987

Everhart, S. and M. Sumlinski (2001). Trends in Private Investment in Developing Countries, Statistics for 1970-2000 and the Impact on Private Investment of Corruption and the Quality of Public Investment. Discussion paper No. 44, International Finance Corporation, World Bank, Washington D.C.

Ghura, D. and B. Goodwin (2000), Determinants of private investment: a cross-regional empirical investigation. *Applied Economics*, 32, 1819-1829.

Gloom, G., and B. Ravikumar (1997). Productive Government Expenditures and Long-Run Growth. *Journal of Economic Dynamics and Control* 21, pp. 183-204

Granger, C., (1969). Investigating Causal Relations by Econometric Models and Cross-Spectral Methods, *Econometrica*, vol. 37(3), pages 424-38

Gramlich, E. (1994). Infrastructure Investment: A Review Essay, *Journal of Economic Literature*, 32. 1176-1196.

Hamilton, James D. & Susmel, Raul, 1994. Autoregressive conditional heteroskedasticity and changes in regime. *Journal of Econometrics*, Elsevier, vol. 64(1-2), pages 307-333

Hendry, D, A. Pagan, and J. Sargan (1984). Dynamic specification. In: Griliches, Z. and M.D. Intriligator (Eds.). *Handbook of Econometrics*, Volume 2-3, Chapter 18

Kamps, C. (2004). New Estimates of Government Net Capital Stocks for 22 OECD Countries 1960-2001, IMF Working Paper 04/67.

Kamps, C. (2005), The Dynamic Effects of Public Capital: VAR Evidence for 22 OECD Countries, *International Tax and Public Finance*, Volume 12, Issue 4, Aug, Pages 533 - 558

Lakshmanan, T and W. Anderson (2004). Contextual Determinants of Transport Infrastructure Productivity: The Case for a New Modeling Strategy. Working Paper Series, Center for Transportation Studies, Boston University.

Mackinnon, J. (1991), *Critical Values for Cointegration Tests. In: R. Engle and C. Granger, eds., Long-Run Economic Relationships: Readings in Cointegration*. Oxford University Press, 267-276

Mera, K., (1973). Regional Production Functions and Social Overhead Capital: An Analysis of the Japanese Case. *Regional and Urban Economics*, Vol. 3., pp. 157-186

Pereira, A. (2001a). On the effects of public investment on private investment: What Crowds in What?. *Public Finance Review*, Vol. 29, nº1, 3-25

Pereira, A. (2001b). Public Investment and Private Sector Performance – International Evidence. *Public Finance and Management*, 1(2), 261-277

Pereira, A. and J. Andraz (2004). *O Impacto do Investimento Público na Economia Portuguesa*, Fundação Luso-Americana.

Ratner, J. (1983). Government Capital and the Production function of U.S. Private Output. *Economic Letters* 13 (2/3) pp. 213-217

Sims, C. (1980). Macroeconomics and Reality. *Econometrica,* 48, 1-49

Sims, C. (1987). Comment on "D.E. Runkle, Vector Autoregressions and Reality". *Journal of Business and Economic Statistics* 5 (4): 443-449

Sims, C. and T. Zha (1999) Error Bands for Impulse Responses. *Econometrica* 67 (5): 1113-1155

Singh, B. and B. Sahni (1984). Causality Between Public Expenditure and National Income, *Review of Economics and Statistics*, Vol. 66, Nº 4, pp. 630-644

Stiglitz, J. (2000). *Economics of the Public Sector*, Third Edition, New York: W.W. Norton & Co.

Schwartz, G. (1978). Estimating the Dimension of a Model. *Annals of Statistics*, 6, 461-464

Verbeek, M. (2004). *A Guide to Modern Econometrics*, 2nd Edition John Wiley & Sons,Ltd.

Wagner, A. (1883). Three Essays in Public Finance, in R. A. M. Musgrave and A.T. Peacock (eds) (1958), Classics in Theory of Public Finance. London: Macmillan.

Annex I

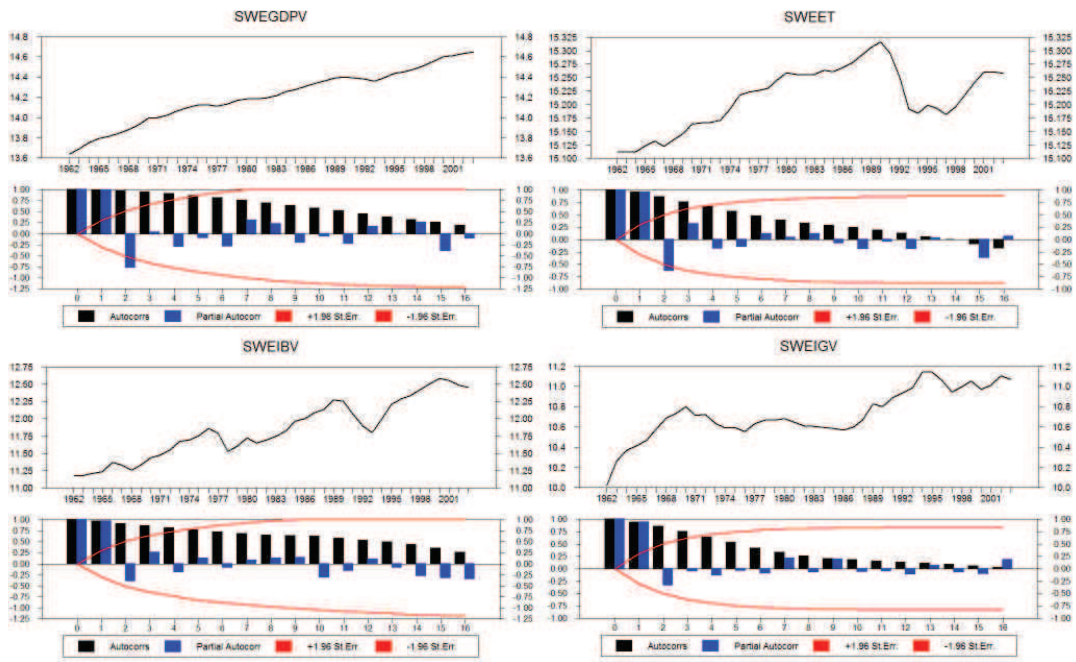

Figure 1

Annex II

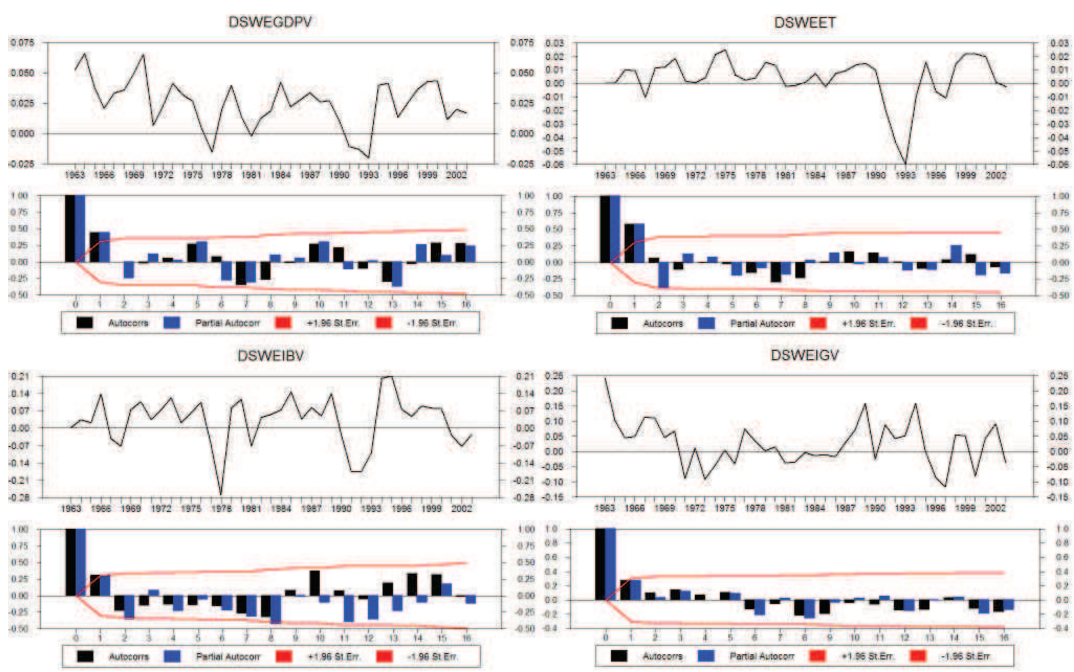

Figure 2

Annex III

VAR/System - Estimation by Least Squares

Dependent Variable DSWEIGV
Annual Data From 1962:01 To 2003:01

Usable Observations	39	Degrees of Freedom	30
Total Observations	42	Skipped/Missing	3
Mean of Dependent Variable		0.0179966261	
Std Error of Dependent Variable		0.0664683855	
Standard Error of Estimate		0.0655780691	
Sum of Squared Residuals		0.1290144943	
Durbin-Watson Statistic		1.963079	

Variable	Coeff	Std Error	T-Stat	Signif
1. DSWEIGV{1}	0.164281592	0.183654572	0.89451	0.37816774
2. DSWEIGV{2}	-0.021183148	0.172815541	-0.12258	0.90325983
3. DSWEIBV{1}	-0.145435062	0.145075063	-1.00248	0.32412894
4. DSWEIBV{2}	-0.139666241	0.135812712	-1.02837	0.31199450
5. DSWEET{1}	-0.183421727	1.198702655	-0.15302	0.87940956
6. DSWEET{2}	-0.838696038	1.087621673	-0.77113	0.44666215
7. DSWEGDPV{1}	-0.906301874	0.874030960	-1.03692	0.30805838
8. DSWEGDPV{2}	1.362457906	0.850856402	1.60128	0.11979447
9. Constant	0.016473446	0.024012687	0.68603	0.49796168

F-Tests, Dependent Variable DSWEIGV

Variable	F-Statistic	Signif
DSWEIGV	0.4072	0.6691365
DSWEIBV	1.2130	0.3114799
DSWEET	0.5670	0.5732028
DSWEGDPV	1.5456	0.2296758

Figure 3

Dependent Variable DSWEET
Annual Data From 1962:01 To 2003:01

Usable Observations	39	Degrees of Freedom	30
Total Observations	42	Skipped/Missing	3
Mean of Dependent Variable		0.0037214028	
Std Error of Dependent Variable		0.0166914511	
Standard Error of Estimate		0.0126331230	
Sum of Squared Residuals		0.0047878739	
Durbin-Watson Statistic		2.054429	

Variable	Coeff	Std Error	T-Stat	Signif
1. DSWEIGV{1}	-0.018771225	0.035379675	-0.53057	0.59962600
2. DSWEIGV{2}	-0.053806433	0.033291617	-1.61622	0.11651675
3. DSWEIBV{1}	-0.009585081	0.027947622	-0.34297	0.73401583
4. DSWEIBV{2}	0.006958024	0.026163300	0.26595	0.79210006
5. DSWEET{1}	0.454639005	0.230921073	1.96881	0.05827423
6. DSWEET{2}	-0.247266789	0.209522155	-1.18015	0.24721544
7. DSWEGDPV{1}	0.402921504	0.168375506	2.39299	0.02317347
8. DSWEGDPV{2}	-0.048901693	0.163911102	-0.29834	0.76749824
9. Constant	-0.003759803	0.004625864	-0.81278	0.42274852

F-Tests, Dependent Variable DSWEET

Variable	F-Statistic	Signif
DSWEIGV	1.8017	0.1824139
DSWEIBV	0.0824	0.9211492
DSWEET	1.9394	0.1614003
DSWEGDPV	2.8818	0.0716457

Figure 4

```
Dependent Variable DSWEIBV
Annual Data From 1962:01 To 2003:01
Usable Observations     39      Degrees of Freedom    30
 Total Observations     42      Skipped/Missing        3
Mean of Dependent Variable      0.0318579411
Std Error of Dependent Variable 0.1019046138
Standard Error of Estimate      0.0786413410
Sum of Squared Residuals        0.1855338154
Durbin-Watson Statistic             2.326914

   Variable                        Coeff       Std Error      T-Stat     Signif
*******************************************************************************
1.  DSWEIGV{1}                  -0.415691229  0.220238900    -1.88746  0.06880737
2.  DSWEIGV{2}                  -0.249970483  0.207240714    -1.20618  0.23717304
3.  DSWEIBV{1}                   0.007024626  0.173974283     0.04038  0.96805979
4.  DSWEIBV{2}                  -0.253850562  0.162866854    -1.55864  0.12957010
5.  DSWEET{1}                    0.157516087  1.437486428     0.10958  0.91347422
6.  DSWEET{2}                   -3.560406320  1.304277909    -2.72979  0.01050169
7.  DSWEGDPV{1}                  2.249834820  1.048139534     2.14650  0.04004000
8.  DSWEGDPV{2}                  1.034928476  1.020348561     1.01429  0.31855575
9.  Constant                    -0.011654255  0.028796059    -0.40472  0.68855683

F-Tests, Dependent Variable DSWEIBV
Variable              F-Statistic       Signif
DSWEIGV                     3.3498     0.0486303
DSWEIBV                     1.2338     0.3055489
DSWEET                      5.3482     0.0103151
DSWEGDPV                    3.4025     0.0465808
```

Figure 5

```
Dependent Variable DSWEGDPV
Annual Data From 1962:01 To 2003:01
Usable Observations     39      Degrees of Freedom    30
 Total Observations     42      Skipped/Missing        3
Mean of Dependent Variable      0.0229688327
Std Error of Dependent Variable 0.0188058973
Standard Error of Estimate      0.0171116986
Sum of Squared Residuals        0.0087843069
Durbin-Watson Statistic             2.163286

   Variable                        Coeff       Std Error      T-Stat     Signif
*******************************************************************************
1.  DSWEIGV{1}                   0.013773864  0.047922144     0.28742  0.77576555
2.  DSWEIGV{2}                  -0.066550907  0.045093847    -1.47583  0.15040900
3.  DSWEIBV{1}                  -0.040164651  0.037855350    -1.06100  0.29715583
4.  DSWEIBV{2}                   0.020895285  0.035438466     0.58962  0.55985919
5.  DSWEET{1}                   -0.080816139  0.312785034    -0.25838  0.79788118
6.  DSWEET{2}                   -0.463148361  0.283799973    -1.63195  0.11314379
7.  DSWEGDPV{1}                  0.561854688  0.228066403     2.46356  0.01971156
8.  DSWEGDPV{2}                  0.078281950  0.222019320     0.35259  0.72686051
9.  Constant                     0.011451478  0.006265782     1.82762  0.07756860

F-Tests, Dependent Variable DSWEGDPV
Variable              F-Statistic       Signif
DSWEIGV                     1.0949     0.3475638
DSWEIBV                     0.6577     0.5253413
DSWEET                      2.4179     0.1062798
DSWEGDPV                    3.4152     0.0461026
```

Figure 6

Annex IV

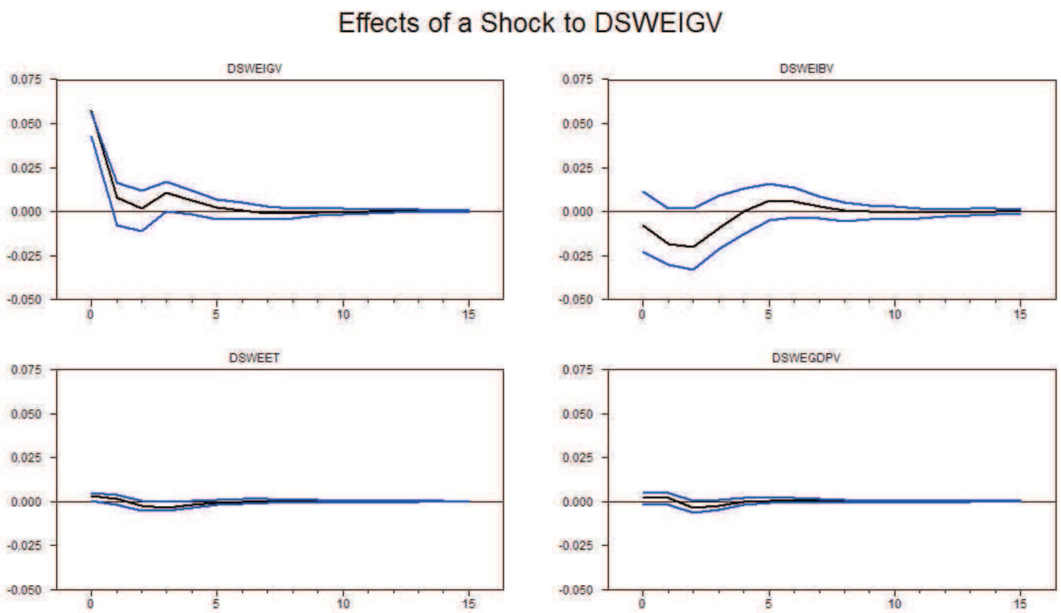

Figure 7

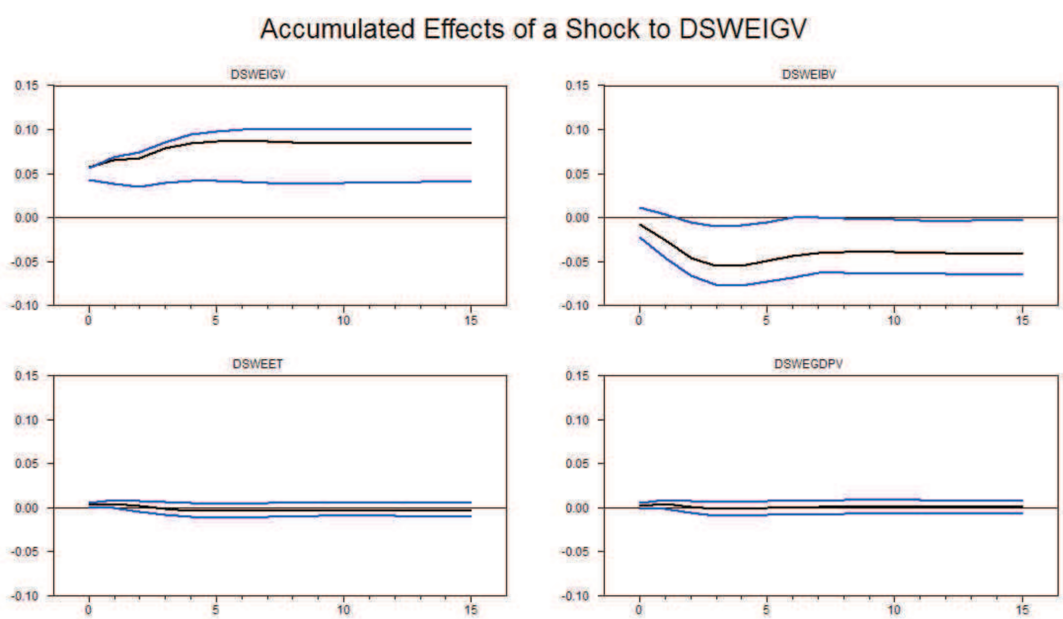

Figure 8

Annex V

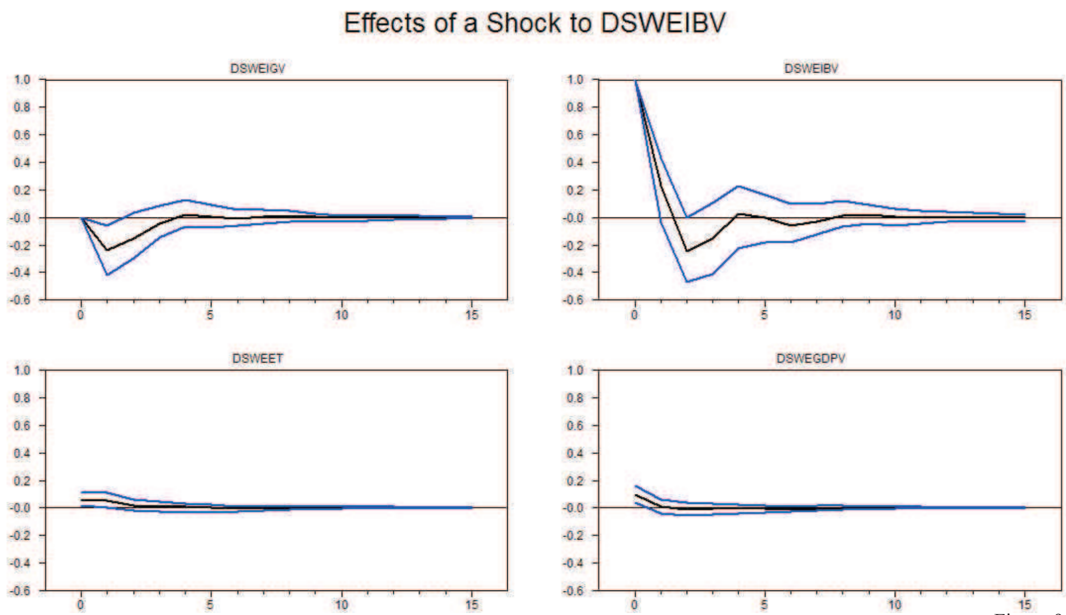

Figure 9

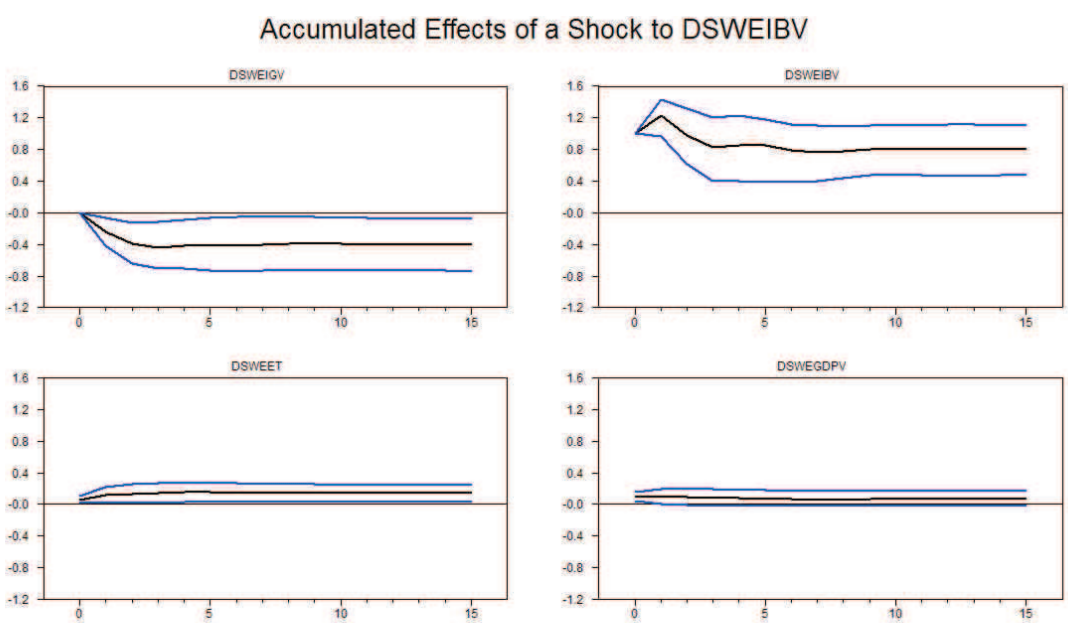

Figure 10

Annex VI

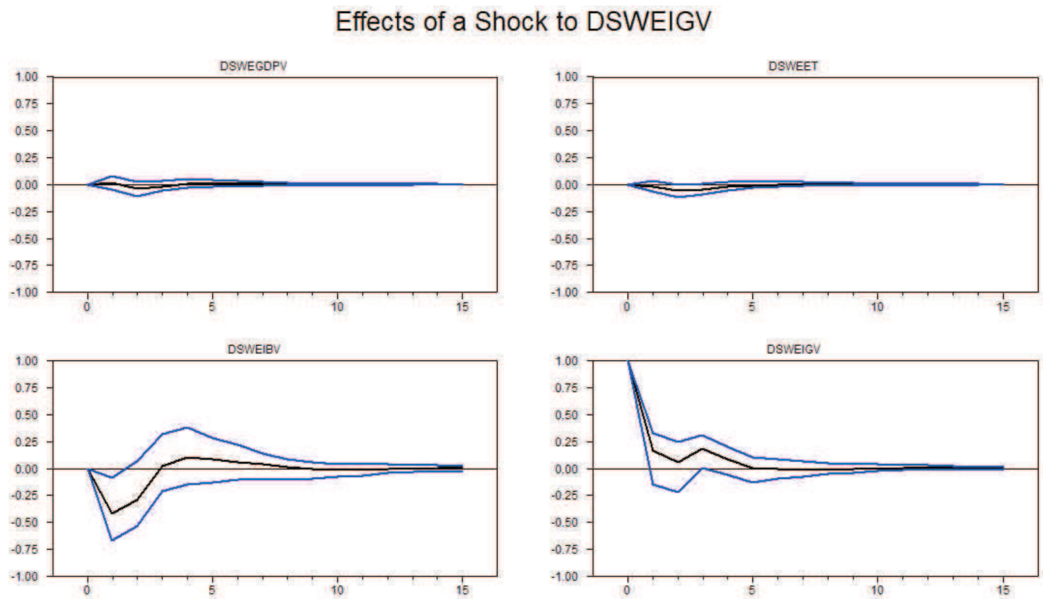

Figure 11