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January 2014

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MPRA Paper No. 61823, posted 4 February 2015 14:40 UTC

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Abstract

Starting from some of the most recent literature developed after the world financial crisis, it has been developed a model with heterogeneous agents and an active interbank market, characterized by an endogenous default probability. The key feature of the analysis is that the probability of default evolves endogenously and is taken into account by banks in their investment decisions. In each period banks, that are heterogeneous, decide to invest only a part, or even none, of their surplus funds on loans to other financial institutions, if the probability of default is high enough, preferring to use that funds to purchase riskless assets. This decision effects the total supply of credit to firms and, through it, the total level of investments, output and employment.

When a financial crisis occurs, banks reduce their supply of interbank funds replicating, to some extent, the behaviour of the interbank market during the last crisis. Through the definition of an endogenous default probability and the analysis of how it effects the credit supply, it is possible to understand the connections between the behaviour of financial markets and the real economy. The model, at last, is calibrated in order to test the response of the system to exogenous shocks and to conventional and unconventional economic policies.

1 Introduction

The last financial crisis has shown how even financial markets are far from the “perfect competition” paradigm and how they are influenced by friction just as any other type of market. The economic theory was, in some case, unable to predict the real extent of the crisis and, at the beginning, to formulate effective policy measures to offset its effect on the real economy (Bernanke (2013) [9], Guillén (2011) [48], Thornton (2012) [75]). These facts call for the formulation of new theories and models that try to describe more realistically the behaviour of agents on the markets in order to understand some of the dynamics that were not considered so far. The final objective of this struggle is to understand why some of the policy measures carried out by policy makers were so ineffective and to develop new tools to counterbalance negative shocks (Claessens et al. (2010) [23]).

Economics needs, today, to develop new theories and paradigms in order to complete its understanding of the more complex reality that it tries to describe¹. In this context, one of the key issue to analyze is the relationship between financial markets and the real economy, trying to understand, from a macroeconomic point of view, which is the role played by the probability of default and by financial frictions. In the past, this topic was more popular in the literature on developing economies, for example in Ordoñez (2009)[67], but now we know how it is important to study those issues also in more advanced economies.

To answer to all those question it is necessary to understand the relationships that exists between financial agents, to overcome the paradigm of the “representative agent” and to develop models that endogenize the default probability.

Regarding the first point, in the last years has been developed a robust literature that exploits the proprieties of networks to understand the connection between economic agents and, in particular, financial agents. In those kind of works the properties of networks are exploited to analyze the mechanisms behind the trasmission of shocks and the effects of monetary policies and bankruptcies on the economic system. Some examples are given by Allen and Gale (2000) [4] and Delli Gatti et al. 2006 [31]². What I have tried to do is something different. Instead of building a formal network of relationships between financial agents, I created an eviornment of heterogeneous agents, partially following the set up of Gertler and Kiyotaki (2010) [44], in which banks are linked together by borrowing relationships that evolves and changes trough time³.

In order to develop my analysis, than, I had to introduce some form of heteogeneity between agents. In the recent years this field of research increased its importance as economists have tried to overcome some limits connected to the rapresentative agent framework. In particular, it has been shown how even keeping standard assumption, with the introduction of some heterogeneity between agents it is possible to derive significantly different results, than in the case of the “representative agent”⁴. Basing on these results, I believe that it is necessary to allow at least for some forms of heterogeneity if our models what to describe effectively the economic system. Moreover, as it is now clear that the introduction of heterogeneity has a significant impact on the equilibrium, allowing for that is necessary in order to avoid possible misleading conclusions.

At last, it is not possible, in the wake of the last financial crisis, to not consider the effects of financial frictions on the equilibrium. The last crisis has shown us how they can considerably effect the economy, and economics researchers have started to incorporate them in their models. The very first attempt was made by Bernanke, Gertler and Gilchrist in (1999) [11] and gave birth to the research vein of “financial accelerator”

¹Friedman (2012) [41], Krugman (2012) [58], Stiglitz (2010) [74], Turner (2012) [78].

²Additional references may be given by Beltratti et al. (1996) [8], Gale and Kariv (2007) [42], Elliott et al. (2014) [35].

³In particular, I allowed for banks to become lenders or borrowers on the interbank market conditioned on the realization of an exogenous shock.

⁴Branch and Evans (2005) [17], Delli Gatti and Longaretti (2006) [30], Hommes (2006) [52], Krusell and Smith (2006) [59], de Graeve et al. (2010) [34], Motta and Tirelli (2010) [66].

models⁵. Those models focused their attention on the relationship between lender and borrowers, in particular removing the assumption on the base of the Modigliani-Miller Theorem⁶. But over the years economist have focused their attention on other sources of friction. In particular after the world financial crisis, this particular field of research experienced a consistent boost⁷. The topics that I believe are more interesting on this field are the introduction of some elements of game theory in macroeconomic models, that allow to incorporate some participation and incentive constraints to the agents decision problems and the presence of agency problems⁸. To be more specific, I believe that it is important to incorporate in our model the possibility that agents answers to private incentives and may have convenience to cheat the opponent. This is, of course, an assumption that does not give much credit people's integrity, but we all know how, especially in the finance sector, immediate gains are preferred to the establishment of long-run trust relationship⁹. For this reason I decided to explicitly include in my model the presence of an incentive constraint in the financial sector.

But all these elements are not sufficient, in my opinion, to completely understand what happened in 2007, when our economy was hit by the greatest crisis since the Great Depression.

With respect to what was observed in the 30s, the present crisis has followed a different path and has displayed different dynamics, that have enlighten how tight now is the connection between the financial sector and the real economy. What started as a crisis on one single financial market, even not so important compared to the whole financial transactions of the U.S. economy, rapidly spread through the financial system and arrived, with a speed never seen before, to the real economy¹⁰. More importantly, the tools I have previously presented, are not able to explain alone the sudden fall of interbank loans we have observed after the initial shock. The role of the interbank market was crucial. Suddenly, financial intermediaries stopped to lend money each other, tightening the liquidity constraint that each of them faced and reducing even more the credit supplied to firms¹¹. It is clear that an important role was played by the increased default probability¹². For this reason, I decided to incorporate that variable into my model and to endogenize it. This is, I believe, an important aspect of the research. The role of the default probability was analyzed in the past years by several empirical works¹³, but has received moderate attention in theoretical researches.

⁵Just to make a couple of example, it is possible to see Gilchrist and Zakrajsek (2012) [45], Hammersland and Trae (2012) [49].

⁶Modigliani and Miller (1958) [64] and (1963) [65].

⁷For example, it is possible to see Bernanke and Gertle (1989) [10], Krishnamurthy (2003) [57], Angeloni and Faia (2009) [6], Gertler and Kiyotaki (2010) [44], Brunnermeier et al. (2012) [19], Brunnermeier and Sannikov (2012) [20].

⁸On this topic, it is possible to see the already cited Gertler and Kiyotaki (2010) [44], and also Phelan and Townsend (1991) [69], Dib (2010) [33], Meeks et al. (2013) [62].

⁹[5].

¹⁰Delli Gatti (2012) [29], Guillèn (2011) [48], Allen, Babus, Carletti (2009) [3].

¹¹On this topic, can be seen Brunnermaier (2009) [18] and Gorton (2010) [46].

¹²De Socio (2011) [21].

¹³For example by Hong et al. (2009) [53], Heider et al. (2009) [51], Bracke (2010) [15].

On this topic, the larger part of the studies comes from the microeconomic finance literature¹⁴, although in the recent years there have been some attempts to incorporate it into macroeconomic models¹⁵. Beside those attempts, I think that there is still the necessity to develop a more comprehensive and complete framework that could allow us to define the default probability as an endogenous variable. This is what I will try to do in the following pages.

The last topic I want to cover in this paper is the effect of public policies during financial crisis. In particular, I will start from some consideration about conventional and unconventional policies¹⁶, in order to evaluate their effectiveness in the framework I developed.

The paper will be developed as follows: section 2 presents the model, section 3 presents some preliminary conclusions, section 4 discusses some unconventional monetary policies that can be used by the public sector, section 5 presents the calibration of the model, section 6 discusses the results of the simulations carried out and in section 7 we have the final conclusions.

2 The Model

The economy of the model is populated by households, entrepreneurs, banks, Government and a Central Bank.

Households, that are the only owners of firms, may be workers or bankers, with the total number of bankers and workers fixed. Workers supply labor to firms and earn wages; bankers run the banks and return their profits¹⁷ to families as dividends that are paid only when a bank ends its activity. To avoid that banks accumulate enough capital to overcome any kind of financial constraint, I assume that in each period there is a probability that a bank ends its activity. In that case, the banker transfer all his remaining assets to households and becomes a worker while households provide to new bankers an initial endowment of capital. In each family there is perfect consumption insurance between members. Households use their incomes to finance consumption, to pay taxes and to save, in the form of the purchase of deposits.

Firms use labor and capital to produce final goods that are sold on a national market and operate under perfect competition. They do not accumulate capital, so in each period they have to borrow funds from the banking system in order to finance their investments. Firms are located on different areas of the economy and can borrow money only from banks located in the same area. In each period, in addition, there is an exogenous probability that in an area emerge new investments opportunities. I am following what proposed by Gertler and Kiyotaki (2010), and I call this probability π^i and the opposite probability $\pi^n = 1 - \pi^i$.

¹⁴For example Leland and Toft (1996) [60], Diamond and Rajan (2001) [22].

¹⁵For example in Stiglitz and Greenwald (2003) [73], Angeloni and Faia (2009) [6], Dib (2010) [33].

¹⁶Brunnermeier (2009) [18], Curdia and Woodford (2010) [26], Claessens et al. (2010) [23], Gertler and Karadi (2011) [43], Thornton (2012) [75].

¹⁷As we are in perfect competition, profits are defined as the returns on the invested capital.

The financial sector is composed by banks that raise liquidity from deposits and interbank loans on national markets. Each bank may use its funds to finance interbank loans, loans to firms or to acquire riskless assets. Banks are run by bankers, who optimize the capital structure in order to maximize the value of the bank's capital at the end of the period. Because they run the bank, they can divert a fraction of the total volume of funds intermediated and transfer it to households. If they do so the bank defaults¹⁸. Each bank, at last, is located on a specific area and while it can borrow on national markets, it can lend money only to firms located in its same area. Bankers know if in their area there are new investment possibilities only after that the period is already started, but they have to decide the level of deposit at the beginning of each period, before that the shock is revealed. For this reason, some banks will have excess funds while others will have a deficit of liquidity. Following Greenwald and Stiglitz (2003) I assume that only banks on areas without new investments acquire riskless assets, this assumption will be proved endogenously later on. What is crucial for the development of the model is the way in which the credit cycle works: bankers choose the level of deposits before the beginning of the period; than they know if there are new investment possibility on their area and, than, they make their investment choices. I assume also that there is some degree of uncertainty on the outcome of new investments projects, for the reason that they are new and, therefore, banks have not already developed the competences to precisely evaluate them.

For what concerns the last two agents of the model, the Central Bank and the Government, I assume that the Central Bank sets the riskless rate and the Government chooses the level of its consumption of final goods, that is treated as an exogenous variable. We will see in the next section how we can model a more active role of both these agents.

2.1 Households

Households may be either workers or bankers. Workers supply labor to firms in exchange for wages, while each banker runs a bank. Wages and profits from the banking activity are transferred to households and within each family there is perfect consumption insurance. The fraction(f) of workers and $(1 - f)$ of bankers is constant through all the periods.

Because banks are finitely lived¹⁹, in each period a fraction $1 - \sigma$ of bankers becomes workers and, on the opposite, the same number of workers becomes bankers²⁰. When a bank is closed, the banker transfers all the assets to families that will provide the starting capital for new bankers. Following standard assumptions, than, firms are entirely owned by households.

¹⁸This dynamic leads to the definition of an agency problem between lenders and borrowers, following, between the others, Kiyotaki and More (1997) [55], Krishnamurthy (2003) [57] and Fostel and Geanakoplos (2009) [38].

¹⁹In each period there is a probability σ that a bank closes.

²⁰The workers becoming bankers are, following the assumption, $(1 - \sigma) f$.

Households get utility from consumption and leisure, can save acquiring bank's deposits and pays lump sum taxes.

Their objective function, therefore, is defined as:

$$E_t \sum_{t=0}^{\infty} \beta^t [\ln C_t + \nu \ln (1 - L_t)] \quad (1)$$

with C the consumption, and L the fraction of time devoted to work. ν the weight of labor disutility equal to the elasticity of leasure and β^t is a discount factor.

If we define D_h the deposits in one type of bank, R^D the interest rate granted on them between t and $t+1$ ²¹, T the taxes paid²², W the nominal wage, Π the profits of firms and banks transferred to families²³ and P_t the price level, it is possible to define the budget constraint as:

$$C_t + D_{h,t+1} + \frac{T_t}{P_t} \leq \frac{W_t}{P_t} L_t + \frac{\Pi_t}{P_t} + \frac{R_{t-1}^D D_{h,t}}{P_t} \quad (2)$$

from the solution of the households decision problem, it is possible to derive the following equilibrium conditions:

$$\frac{1}{C_t} = \lambda_t^C \quad (3)$$

$$\frac{\nu}{1 - L_t} = \frac{W_t}{P_t} \lambda_t^C \quad (4)$$

$$E_t \beta^t \frac{\lambda_{t+1}^C}{\lambda_t^C} \frac{R_t^D}{P_t} = 1 \quad (5)$$

$$C_t + \frac{D_{h,t+1}}{P_t} + \frac{T_t}{P_t} = \frac{W_t}{P_t} L_t + \frac{\Pi_t}{P_t} + \frac{R_{t-1}^D D_{h,t}}{P_t} \quad (6)$$

with $\{\lambda_t^C\}_{t=0}^{\infty}$ the sequence of Lagrangian multipliers associated to the optimization problem, that also define the real marginal utility.

Equations (3) and (4) describe the optimal choice of consumption and leisure, and link together the choice of consumption and hours devoted to work. They also describe the demand of goods and the supply of labor. Equation (5) describes the Euler condition on deposits and says that the riskless rate must be such that it equalities the present

²¹It will be proved that the interest rate on deposits is equal to the riskless interest rate in the equilibrium.

²²Following the well established literature on this topic, started with Ramsey (1927) [71], taxes are assumed to be a lump-sum.

²³ $\Pi_t = \Pi_t^B + \Pi_t^F$, with Π^B the dividends of banks and Π^F the profits of firms.

discounted marginal utility of future consumption with the marginal utility of present consumption. The last equation allows us to determine the value of deposits in $t+1$ given the optimal choice of all the other variables and the constants.

2.2 Good producers

Firms operate in perfect competition and produce homogeneous goods. They combine labor and capital with Cobb-Douglass production function:

$$Y_t = A_t K_t^a L_t^{(1-a)} \quad 0 < a < 1 \quad (7)$$

with A , the productivity of factors, that evolves following a Markov process²⁴. The total cost function is given by:

$$TC = \frac{W_t}{P_t} L_t + Z_t K_t \quad (8)$$

with Z the gross profits for unit of capital. From the firm's optimization problem it is possible to derive the following first order conditions:

$$\frac{\frac{W_t}{P_t} L_t}{1 - \alpha} = \frac{Z_t K_t}{\alpha} \quad (9)$$

$$Z_t = \frac{Y_t - \frac{W_t}{P_t} L_t}{K_t} = \alpha A_t \left(\frac{L_t}{K_t} \right)^{1-\alpha} \quad (10)$$

equation (9) defines the demand for labor, while equation (10) defines the gross profits per unit of capital. Firms do not accumulate capital, so in each period they have to finance their investment with loans acquired from the banking system. As long as they are able to obtain that funds, they do not face any other friction and commit to pay to the creditor bank the gross profits per unit of capital. Each unit of equity, in other words, is a claim to the future returns on one unit of investments:

$$\psi_{t+1} Z_{t+1}, (1 - \delta_K) \psi_{t+1} \psi_{t+2} Z_{t+2}, (1 - \delta_K)^2 \psi_{t+1} \psi_{t+2} \psi_{t+3} Z_{t+3} \dots$$

δ_K describes the rate of depreciation of capital while ψ_{t+1} describes the effect a shock on the quality of capital. This last term, following the finance literature²⁵, has been

²⁴See Bharucha-Reid (1960) [12]. Among recent papers that use this assumption there are Gertler and Kiyotaki (2010) [44], Brunnermeier and Sannikov (2012) [20] and Gourio (2009) [47].

²⁵For example in Merton (1973) [63].

introduced to describe in a simple way an exogenous source of variation in the value of capital and evolves following a Markov process²⁶. Firms use loans from the banking system to acquire capital goods from capital goods producers. Because they operate in perfect competition, they earn 0 profits in the equilibrium. Given this assumption, it is possible to define the rate of returns on each unit of financed equity, that is given by:

$$R_t^{h,K} = \frac{[Z_t + (1 - \delta_K) Q_t^h]}{Q_t^h} \quad (11)$$

with $h = i, n$. The rate of returns on each unit of capital is given by the gross profits plus capital gains or losses.

In each period, as we know, there is a probability π^i that firms on an area have new investment opportunities and, on the contrary, a probability π^n that they have not. It is possible, than, to define a law of motion of capital as follows:

$$K_{t+1} = \psi_{t+1} \left\{ [I_t + \pi^i (1 - \delta_K) K_t] + \pi^n (1 - \delta_K) K_t \right\} = \psi_{t+1} \{ I_t + (1 - \delta_K) K_t \} \quad (12)$$

with I the value of new investments. Therefore, the aggregate demand for loans on each type of area is given by:

$$S_t^h = \begin{cases} \pi^n (1 - \delta_K) K_t & \text{for } h = n \\ \pi^i (1 - \delta_K) K_t + I_t & \text{for } h = i \end{cases} \quad (13)$$

2.3 Capital goods producers

Capital goods producers operate on a national market and make new capital using input of final output, subject to an adjustment cost function. This function takes the form of:

$$f \left(\frac{I_t}{I_{t-1}} \right) I_t$$

it is used to describe physical adjustments cost in the production process²⁷, with $f(1) = f'(1) = 0$ and $f'' \left(\frac{I_t}{I_{t-1}} \right) > 0$. In this way the production function has decreasing returns to scale in the short run and constant returns to scale in the long run. The price

²⁶Among recent papers that use this assumption there are Gertler and Kiyotaki (2010) [44], Brunnermeier and Sannikov (2012) [20].

²⁷The presence of physical adjustment costs is widespread in the literature, for example can be seen Friedman and Woodford (2011) [40], Alid Dib (2010) [33] and Angeloni and Faia (2009) [6].

at which these agents sell the new capital is driven to Q_t^{i28} by the perfect competition assumption. Capital goods producers, than, choose I_t in order to maximize the expected profits. Their problem is given by:

$$\max E_t \sum_{t=0}^{\infty} \beta^t \Lambda_t^C \left\{ Q_t^i I_t - \left[1 + f \left(\frac{I_t}{I_{t-1}} \right) \right] I_t \right\} \quad (14)$$

Because households are the only owners of firms, the discount factor is given by $\beta^t \Lambda_t^C$ ²⁹.

From the solution of this problem, it is possible to derive the following equilibrium condition:

$$Q_t^i = 1 + f \left(\frac{I_t}{I_{t-1}} \right) + \frac{I_t}{I_{t-1}} f' \left(\frac{I_t}{I_{t-1}} \right) - E_t \beta^t \lambda_t^C \left(\frac{I_{t+1}}{I_t} \right)^2 f' \left(\frac{I_{t+1}}{I_t} \right) \quad (15)$$

the above equation has the role of a Tobin's Q equation³⁰ for the model and endogenizes the price of equity.

2.4 Financial system

The financial system is populated by finitely lived banks each of them run by a banker. Each bank receives funds from capitalists and depositors and can access to a national interbank market, where banks emit loans one to another. With the funds raised each bank can acquire riskless assets, finance interbank loans or emit loans to firms. Following Gertler e Kiyotaki (2010) [44], I assume that banks receive funds on national market but that they can lend only on a local area. In other words, each bank can emit loans only to firms that operate in his same area. The balance sheet of a representative bank may be described as follows:

Assets	Liabilities
Loans to firms	Capital
Interbank loans	Deposits
riskless assets	Interbank debts

we also know that in each area of the economy there is an exogenous probability π^i that new investment opportunities arises for firms. Because the type of area in which

²⁸New capital can be sold only on areas with new investments and the price of capital, due to the perfect competition assumption is driven to its market value.

²⁹ $\Lambda_t^C = \frac{\lambda_{t+1}^C}{\lambda_t^C}$ defines the households discount factor and is a function of the real marginal utility of consumption. For the economic meaning of the lagrangian multiplier, see Cowell (2006) [24].

³⁰Brainard and Tobin (1968) [16], Tobin (1969) [76].

the bank will operate is revealed after the beginning of each period, when bankers have already decided the level of deposits, the banks on areas with new investments do not have sufficient funds to finance all the investments projects of firms³¹. For this reason they ask interbank loans to banks on areas without new investment opportunities, that can only refinance the investments of the previous period and, thus, have a surplus of funds. A key aspect of the model is the choice of this second group of banks. As we will see later, lending banks on the interbank market can decide to invest their excess of liquidity on interbank lendings or on riskless assets; when the economy faces some types of crisis, lending banks may decide to invest all their excess liquidity on riskless assets, not financing any interbank loan and causing a reduction of credit to firms.

What is important to understand now, is that only banks on areas without new investment decide to invest part of their funds on the interbank market or on riskless assets, because, as long as there are investment projects to be financed, it is more convenient to invest on them³². The last assumption that I make is that there is some degree of uncertainty on the returns of loans issued to firms on new investment island. What I am saying is that banks are not able to forecast exactly the returns on the investment projects on areas with new investment possibilities and may pay additional cost of monitoring. To take this into account, I assume that there is an exogenous shock on the returns of loans to firms for banks on “i” areas. This shock takes the form of a stochastic variable $x_{i,t}$ with $E(x_{i,t}) = 0$, $Var(x_{i,t}) = \sigma_x > 0$ that spans on an interval $[-h; h]$ with a uniform distribution and probability $\frac{1}{2h}$ ³³.

We know that banks pay dividends to households only when they end their activity, that can occur in each period with a probability σ ³⁴. When a bank exits the market, the banker transfers all the accumulated net worth to households; in all the previous periods, on the contrary, he accumulates the profits³⁵ to increase the bank’s net worth. The objective of the banker, than, is to maximize the sum that he can transfer to households at the end of each period. Because he runs the bank, I assume that he can divert to households a part of the total funds intermediated by the bank ($A_t^{h,B}$) except for those acquired on the interbank market³⁶, leading to the default of the bank. He decides to do so only if the expected value of the assets he can transfer is bigger than the expected value of the net worth at the end of the period (n_{t+1}^h). We can simply define their objective function as:

³¹Banks choose their level of deposits before of the realization of the shock. Therefore, they choose an average level between the two optimum in each of the possible state of the world.

³²This follows what discussed by Greenwald and Stiglitz (2003) [73] and will be endogenously proved in the next pages.

³³For the modelization of the shock I followed Angeloni e Faia (2010) [6] and Diamond and Raja (2001) [32].

³⁴This assumption is necessary to avoid that banks accumulate enough capital to become financially unconstrained.

³⁵The banking system operates in perfect competition, for profits I intend the returns on the invested capital.

³⁶I am assuming that private banks are more efficient in monitoring their counterparts than depositors.

$$V_t^h = \max \left\{ E_t \left(n_{t+1}^h \right); \theta \left(A_t^{h,B} \right) \right\} \quad (16)$$

with

$$A_t^{h,B} = \begin{cases} Q_t^i s_t^i - b_t & \text{for } h = i \\ Q_t^n s_t^n + b_t + f_t & \text{for } h = n \end{cases}$$

with s_t^h the volume of loans to firms emitted by each type of bank at the price Q_t^h , b_t the value of interbank loans, f_t the value of riskless assets and θ the fraction of assets that bankers can divert³⁷. The net worth of each type of bank, at the end of the period, is given by the sum of the net worth at the beginning of the period plus the returns on the invested capital.

From equation (16) it is straightforward to see how arises an agency problem. In fact, the bank's creditors will not lend to the banker an amount of funds high enough to make $E_t \left(n_{t+1}^h \right) < \theta \left(A_t^{h,B} \right)$, because, in this case, the banker will divert a fraction θ of the active, the bank will default, and they will lost the investment. For this reason, we end up having an additional constraint on the bank's activity, that states that in each period the volume of funds "divertible" by the banker must be smaller than the expected value of the bank's net worth at the end of the period. As long as it holds, bankers will not divert any part of the funds they are intermediating. It is possible, than, to define two incentive constraints, one for each type of bank:

$$E_t \left(n_{t+1}^i \right) \geq \theta \left(Q_t^i s_t^i - b_t \right) \quad (17)$$

$$E_t \left(n_{t+1}^n \right) \geq \theta \left(Q_t^n s_t^n + b_t + f_t \right) \quad (18)$$

the last thing to do before formally analyze the decision problem of banks in each of the two possible state of the world is to formally define the bank's budget constraint, that is given by:

$$Q_t^i s_t^i = n_t^i + d_t + b_t \quad (19)$$

$$Q_t^n s_t^n + b_t + f_t = n_t^n + d_t \quad (20)$$

³⁷Banks on innovative areas invest only in loans to firms, so their active is constituted only by them. On the contrary, banks on areas with no innovation invest also in interbank loans and riskless assets.

2.4.1 Banks on areas with new investment possibilities

Banks on these areas do not have enough funds to supply all the credit demanded by firms, and so they have to ask for interbank loans. We also know that there is a stochastic variables that effects the effective returns on loans to firms. As long as the constraint given by equation (17) is satisfied, bankers that operate on this type of areas will have the objective to maximize the expected net worth at the end of each period. We can, finally, formally define the net-worth at the end of each period for this type of banks as:

$$E_t \left(n_{t+1}^i \right) = E_t \left[\left(R_t^{i,K} + x_{i,t} \right) \psi_t Q_t^i s_t^i - R_t^B b_t - R_t^D d_t + \chi^i n_t^i \right] \quad (21)$$

with $R_t^{i,K}$ the expected returns on loans to firms between t and $t+1$ defined as $\frac{[Z_t + (1 - \delta_K) Q_t^i]}{Q_t^i}$, d_t the deposits acquired by each bank and R_t^B the interbank's interest rate between t and $t+1$. The objective function of this type of banks, than, can be defined as:

$$E_t \sum_{t=0}^{\infty} \Lambda_t^B n_{t+1}^i \quad (22)$$

$$\Lambda_t^B \equiv (1 - \sigma) \sigma^t \beta^t \Lambda_t^C$$

$$E_t \left(n_{t+1}^i \right) = R_t^{i,K} \psi_t Q_t^i s_t^i - R_t^B b_t - R_t^D d_t + n_t^i$$

because banks are owned by households, the discount factor of bank's profits is given by $\beta^t \Lambda_t^C$.

The decision problem of the representative bank, than, can be defined as the maximization of equation (22) under the constraints given by equations (17) and (19).

The solution to that problem leads to the following equilibrium conditions³⁸:

$$R_t^{i,K} \psi_t - R_t^B = 0 \quad (23)$$

$$R_t^B - R_t^D = \frac{\lambda_t^i \theta}{(\Lambda_t^B + \lambda_t^i)} \quad (24)$$

$$R_t^{i,K} \psi_t Q_t^i s_t^i - R_t^B b_t - R_t^D d_t + n_t^i \geq \theta \left(Q_t^i s_t^i - b_t \right) \quad (25)$$

³⁸If we take the expectation it is true that $E_t \left(R_t^{i,K} + x_{i,t} \right) = R_t^{i,K}$.

with $\{\lambda_t^i\}_{t=0}^\infty$ the sequence of Lagrangian multipliers associated to the problem. It is trivial to show how the solution to the problem is a corner solution. Equation (23) defines an Euler condition for banks on areas with new investments, and defines the interest rate on the interbank market. As long as it is satisfied, banks on this type of areas will acquire all the interbank credit supplied. Equation (24) defines the riskless rate as a mark down on the interbank market rate. Combined with the previous equation it is also defined as a mark down on the real returns on investments. The last relationship is the incentive constraint that holds with the equality if $\lambda_t^i > 0$. We can plug into it the budget constraint to get to:

$$\tau_t^i n_t^i + \varpi_t^i b_t \leq \phi_t^i Q_t^i s_t^i \quad (26)$$

with $\tau \equiv (R_t^D + \chi^i)$, $\varpi \equiv (R_t^D + \theta - R_t^B)$, $\phi \equiv (R_t^D + \theta - R_t^{i,K})$. The previous equation defines the supply of loans to firms on areas with new investments. As it can be seen, it depends positively from the total amount of interbank borrowing received by banks on this type of areas. If $b_t = 0$, it is just defined as a mark-up on the bank's net worth at the beginning of each period.

2.4.2 Banks on areas without new investment possibilities

The analysis of the choice for banks that end up operating on areas without new investments is a bit more complicated. In fact they can invest in loans to firms and banks or in riskless assets. In addition, there is the effect of the stochastic shock that influences, as it will be explained later, the returns on interbank loans. If the realization of the shock is sufficiently small, the debtor bank is not able to refund the total value of the loans received and it defaults. In that case, of course, the creditor will receive less or nothing of the original value of the loan. We can start analyzing how the variable $x_{i,t}$ influences the choice of lending banks. As long as the realized value of the shock is positive, so it falls in the interval $[0; h]$, banks on investment areas have no problem to pay back their debts. On the contrary, if $x_{i,t} \in [-h; 0)$, banks may not be able to refund all their creditors. If the shock falls in that interval, banks have to use part of the net worth accumulated in the previous periods to refund their debts. As always, at first are refunded the depositors and only after them, with what is left, are refunded interbank loans. From this assumption we can define two critical values of the shock. One that defines the lowest value of $x_{i,t}$ that still allows the debtor bank to refund all its creditors using all the net worth accumulated in the previous periods. The second defines that realization of $x_{i,t}$ for which the bank is able to totally refund only its depositors. What is left after that they are served is insufficient to fully refund interbank loans; in this case the debtor bank defaults and the lending banks, after having paid some default costs, acquire what is left. We can define the first critical value of the shock as that value that drives to zero the bank's assets after that the creditors are paid:

$$0 = \left(R_t^{i,K} + x_{i,t} \right) \psi_t Q_t^i s_t^i + n_t^i - R_t^D d_t - R_t^B b_t$$

we can solve for $x_{i,t}$ and call that value a :

$$a = \frac{b_t^i R_t^B - n_t^i + R_t^D d_t}{\psi_t Q_t^i s_t^i} - R_t^{i,K} \quad (27)$$

it is possible to see how this value grows as the cost of borrowing grows and decreases in the value of the assets of banks on investment areas and in the returns on the firm's capital. In the second case, the creditor bank pays some default costs and acquires what is left of the debtor bank assets after that its depositors are served. The lending bank will make this choice only as long as what it gets is higher than the default costs. We can derive the critical value of the shock, under which the creditor bank will get 0, as the solution of:

$$0 = \left(R_{t+1}^{i,K} + x_{i,t} \right) \psi_t Q_t^i s_t^i + n_t^i - R_{t+1}^D d_t - \bar{C}$$

it is possible to solve for $x_{i,t}$ and call that value a' :

$$a' = \frac{-n_t^i + R_{t+1}^D d_t + \bar{C}}{\psi_t Q_t^i s_t^i} - R_{t+1}^{i,K} \quad (28)$$

with \bar{C} defines the default costs. It is straightforward to see how this value increases as the default costs and the cost of deposits decrease and the net worth, the value of assets and the returns on firm's capital increase.

We can now identify three areas of the p.d.f. of $x_{i,t}$. In the first if $x_{i,t} \in [-h; a']$ the lending bank does not receive anything, in the second, if $x_{i,t} \in (a; a')$ the lending bank obtains what is left after that are paid the debtors and the default costs and, in the last one, if $x_{i,t} \in [a; h]$ the lending bank is fully repaid. It can be noticed how if the value of the shock is larger than a but smaller than 0, the debtor bank does not completely default, but uses its net worth to pay back its debts. But that influences the critical levels for the next period, according to equations (27) and (28).

It is straightforward to define a payment distribution function for the lending bank, that describes the expected returns on interbank loans:

$$\mathcal{F}_t = \frac{1}{2h} \int_{-h}^{a'} 0 dx_{i,t} + \frac{1}{2h} \int_{a'}^a \left[\left(R_t^{i,K} + x_{i,t} \right) \psi_t Q_t^i s_t^i + n_t^i - R_t^D d_t - \bar{C} \right] dx_{i,t} + \frac{1}{2h} \int_a^h R_t^B b_t dx_{i,t} \quad (29)$$

if the shock falls in the interval between $[-h; \alpha]$ the debtor bank defaults and its creditor is able to salvage only a part of the investment.

We can now define the value of the net worth at the end of each period, for a representative bank on an area without new investment possibilities that is given by:

$$E_t n_{t+1}^n = n_t^n + R_t^{n,K} \psi_t Q_t^n s_t^n + \mathcal{F}_t + R_t^F f_t - R_t^D d_t \quad (30)$$

with R_t^F the riskless interest rate that is equal to the interest rate on deposits in the equilibrium.

The objective function of this type of banks, then, is given by:

$$E_t \sum_{t=0}^{\infty} \Lambda_t^B n_{t+1}^n \quad (31)$$

$$\Lambda_t^B \equiv (1 - \sigma) \sigma^t \beta^t \Lambda_t^C$$

this function should be maximized under the constraints given by equations (18) and (20). The problem appears quite complicated at a first look, but it is possible to simplify it with some easy passages. Consider the incentive constraint. It states that:

$$n_t^n + R_t^{n,K} \psi_t Q_t^n s_t^n + \mathcal{F}_t + R_t^F f_t - R_t^D d_t \geq \theta (Q_t^n s_t^n + b_t + f_t)$$

if we consider the worst case, when the left part of the equation is minor, it is possible to show that this constraint is inactive for banks on areas without new investments. First of all, the left part of the equation is minimum when banks do not invest on the interbank market. This happens when the possibility of default is so high that the expected returns on interbank loans is inferior to expected returns on riskless assets. In this case, the previous constraint collapses to:

$$n_t^n + R_t^{n,K} \psi_t Q_t^n s_t^n + R_t^F f_t - R_t^D d_t \geq \theta (Q_t^n s_t^n + f_t)$$

if we plug in the budget constraint, $f_t = n_t^n + d_t - Q_t^n s_t^n$, we get to:

$$n_t^n + R_t^{n,K} \psi_t Q_t^n s_t^n + R_t^F (n_t^n + d_t - Q_t^n s_t^n) - R_t^D d_t \geq \theta (Q_t^n s_t^n + n_t^n + d_t - Q_t^n s_t^n)$$

$$(1 + R_t^F) n_t^n + (R_t^{n,K} - R_t^F) \psi_t Q_t^n s_t^n \geq \theta (n_t^n + d_t)$$

as it can be seen $n_t^n > \theta n_t^n$ and $R_t^F n_t^n + (R_t^{n,K} - R_t^F) \psi_t Q_t^n s_t^n + R_t^F n_t > \theta d_t$; therefore, the constraint is always satisfied and it is not active.

The problem now is reduced to the maximization of equation (31) under the budget constraint and the non negativity constraint for b_t . The first constraint is always binding, so we can plug it in the objective function and simplify the problem to an unconstrained optimization. From the solution of that problem, we can derive the following first order conditions:

$$R_t^{n,K} \psi_t - R_t^F = 0 \quad (32)$$

this equation states that as long as the real interest rates paid on loans to firms on areas without new investments is higher than the riskless rate, banks will supply all the credit demanded from the private sector. This means that as long as there is demand for loans, banks will use their liquidity to finance them and not interbank loans. The optimum condition on deposits is given by:

$$R_t^F = R_t^D \quad (33)$$

this equation proves the assumption we made at the beginning, deposits are remunerated at the riskless interest rate in the equilibrium. As long as this condition holds, banks will acquire all the deposits supplied by households. If we combine this relationships with the previous one, it is possible to get to a more interesting conclusion. In fact, comparing equation (32) with equation (23) we can now see how the price of assets on areas without new investments is lower than the price of assets on areas with new investments. This result is obtained simply plugging into that relationships the definitions of the riskless interest rate and of the interbank rate. The economic reason behind this conclusion is very simple, and is connected to the fact that on innovative areas there is an excess of demand for loans, that increases the price of capital.

Until now we have derived the supply of loans, that is equal to the demand, for areas without new investments and the condition at which banks acquire deposits from households. It is necessary now to find what use banks on this type of areas make of their excess liquidity, that remains them after that they have served all firms. It is possible to derive the optimum condition for the supply of interbank loans, for a generic bank, that is given by:

$$b_t^* = \left[-R_t^B \frac{\bar{C}}{Q_t^i s_t^i} + R_t^B h - \left(\frac{-n_t^i + R_t^D d_t}{\psi_t Q_t^i s_t^i} - R_t^{i,K} \right) - (R_t^F - \mu_t) 2h \right] \frac{\psi_t Q_t^i s_t^i}{(R_t^B)^2} \quad (34)$$

where μ_t is the Lagrangian multiplier associated to the non negativity constraint for b_t . The previous relationship defines the supply of interbank loans from banks with a surplus of liquidity to banks with a deficit of liquidity. If $\mu_t > 0$ the bank will not invest any fraction of its surplus liquidity on the interbank market. The supply of interbank loans increases as the cost of default decreases, as the net worth, the value of the assets of the debtor banks and the returns on investments on innovative islands increase³⁹ and decreases as the riskless rate, that is the opportunity cost of every euro invested on the interbank market, increases. At last, the dispersion of revenues, given by the parameter h decreases the supply of interbank loans. We can now compute the equilibrium value of riskless assets held by each bank of this type from the budget constraint, that is given by:

$$f_t^* = n_t^n + d_t - \psi_t Q_t^n s_t^n - b_t^* \quad (35)$$

at last, it is also possible to define the probability of a default(δ) to occur⁴⁰, simply as the probability of a shock to fall in the interval $[-h; \alpha]$:

$$\delta = \frac{1}{2h} \int_{-h}^{\alpha} 1 dx_{i,t} = \frac{1}{2h} (\alpha + h) \quad (36)$$

as it can be seen, because α is a negative number, the probability of default increases as the dispersion of revenues increases. In addition, we can see how in each period the probability of default is defined by the financial conditions of debtor banks. It is also possible to compute the expected payoff for interbank lenders in case of default, that is given by:

$$\frac{1}{2h} \int_{-h}^{\alpha'} 0 dx_{i,t} + \frac{1}{2h} \int_{\alpha'}^{\alpha} [(R_t^{i,K} + x_{j,t}) \psi_t Q_t^i s_t^i + n_t^i - R_t^D d_t - \bar{C}] \quad (37)$$

if it is divided for the total amount of interbank loans of the period, it defines the fraction of the value of loans that the banks are able to save after that a default occurs.

³⁹This is quite intuitive, all these variables increase the active of the debtor bank and. So, on one hand they increase the pay off of the interbank lender in case of default, on the other they reduce the default probability.

⁴⁰For a default I intend a default on a loan, so also the case that a bank defaults on a loan but uses its net worth to refund the creditor.

2.4.3 Aggregation

Because the condition of banking and the choices of banks are different depending on the type of area on which they operate in each period, it is not possible to aggregate through all the areas, but we can do so only between banks on the same area.

We can start from banks on areas without new investments (capital letters define aggregate variables). As they are all equal between them, it is possible to aggregate through them, to get to the supply function of loans to firms, that is given by:

$$S_t^n = \frac{1}{Q_t^n} (\pi^n N_t + \pi^n D_t - B_t - F_t) \quad (38)$$

with, according to equation (13), $S_t^n = \pi^n (1 - \delta_K) K_t$.

The total supply of interbank loans is given by:

$$B_t^* = \left[-R_t^B \frac{\bar{C}}{Q_t^i S_t^i} + R_t^B h - \left(\frac{-\pi^i N_t + \pi^i R_t^D D_t}{\psi_t Q_t^i S_t^i} - R_t^{i,K} \right) - (R_t^F - \mu_t) 2h \right] \frac{\psi_t Q_t^i S_t^i}{(R_t^B)^2} \quad (39)$$

and the total value of riskless assets by:

$$F_t^* = N_t^n + D_t - \psi_t Q_t^n S_t^n - B_t^* \quad (40)$$

on the other type of areas, the total supply of loans to firms is given by:

$$\tau_t^i \pi^i N_t + \varpi_t^i B_t^* \leq \phi_t^i Q_t^i S_t^i \quad (41)$$

with, according to equation (13), $S_t^i = \pi^i (1 - \delta_K) K_t + I_t$. It is easy to see how the value of the previous relationship is influenced by equation (39). In this way, the choices of lending banks directly influences the supply of credit to firms and, because firms rely completely on bank's loans to finance their activity, the level of output, employment and consumption of the system.

Before moving on to the next section, we must define a law of motion for bank's capital. In each period, on each type of areas, the bank's net worth at the end of the period is equal to the sum of the capital accumulated in the previous periods plus the gain or losses from the banking activity. At the end of the period, however, a fraction of banks quits and, therefore, the value of their capital is transferred to households who supply the initial capital for new banks. Therefore, at the beginning of the new period, the aggregate bank's net worth is given by the sum of the bank's capital at the end

of the period, minus the capital of the banks that have quit plus the transfers from households to new banks. We can than set up the equality:

$$N_t = N_{t-1}^o + N_{t-1}^y \quad (42)$$

with N_t the bank's capital at the beginning of each period, N_{t-1}^o the bank's capital of surviving banks and N_{t-1}^y the bank's capital of new banks, that is equal to the transfers received from households. It is straightforward to define the transfer as a fraction of total assets:

$$N_t^y = \xi \left\{ \pi^i \left[Z_t^i + (1 - \delta_Q) Q_t^i \right] \psi_t S_t^i + \pi^n \left[Z_t^n + (1 - \delta_Q) Q_t^n \right] \psi_t S_t^n \right\} \quad (43)$$

with ξ a positive parameter smaller than 1. The capital of surviving banks is equal to the value of the capital of banks that survives from t-1 to t, that is equal to $N_t^o = \sigma (N_t^i + N_t^n)$. I call N_t^i the value of capital of banks on areas with new investments at the end of each period. It is simply equal to the value of the bank's active after that are repaid all its creditors. This value, of course, is influenced by the effective returns on investments, and so by the realization of the stochastic shock. It is, than, defined by:

$$N_t^i = \frac{1}{2h} \int_{-h}^a 0 dx_{i,t} + \frac{1}{2h} \int_a^h \left[R_t^{i,K} + x_{i,t} \right] Q_t^i \psi_t S_t^i + \pi^i N_t - \pi^i R_t^F D_t - R_t^B B_t dx_{i,t} \quad (44)$$

the first integer describes the case in which the bank defaults as a consequence of the realization of the shock. Similarly, it is possible to define N_t^n the aggregate value of capital the end of period t-1 for banks on areas without new investments as the sum of the capital at the beginning of the period plus the returns on the investments. Therefore, we can set up the equality:

$$N_t^n = R_t^{n,k} \psi_t Q_t^n S_t^n - \pi^n R_t^F D_t + \Pi_t^B + R_t^F F_t \quad (45)$$

with Π_{t-1}^B the profits on interbank loans that are defined as:

$$\frac{1}{2h} \int_{-h}^{a'} 0 dx_{i,t} + \frac{1}{2h} \int_{a'}^a \left[\left(R_t^{i,K} + x_{i,t} \right) \psi_t Q_t^i S_t^i + N_t^i - \pi^i R_t^F D_t - \bar{C} \right] dx_{i,t} + \frac{1}{2h} \int_a^h R_t^B B_t \quad (46)$$

the value of bank's capital at the beginning of each period on each type of area is given by $\pi^i N_t = N_t^i$ for areas with new investment opportunities and $\pi^n N_t = N_t^n$ for areas without new investment opportunities.

2.5 Equilibrium

The equilibrium on the goods market is given by the well-known relationship:

$$Y_t = C_t + I_t \left[1 + f \left(\frac{I_t}{I_{t-1}} \right) \right] + G_t \quad (47)$$

if we plug in the optimal values of consumption and investments, we obtain an IS curve augmented for the adjustment costs. On the supply side of the good market, the supply is defined by the firms production function, given by equation (7), after having plugged in the optimal values of labor and capital.

The Central Bank sets the riskless interest rate on Government bonds.

The equilibrium on the credit market is granted by equations (13), (15), (39), (40), (38) and (41); while on the labor market by equations (4) and (9). The total volume of riskless assets in the economy is given by the sum of deposits and government securities acquired by banks:

$$D_t^T = D_t + F_t \quad (48)$$

3 Preliminary Conclusions

This model develops an economic system in which banks interact on the interbank market. This fact, combined with the presence of the stochastic shock has some interesting consequences for the economy.

Independently by the presence of the incentive constraint, banks in surplus of liquidity will never invest all their excess funds on interbank loans. In a world where banks are able to forecast with perfect precision the outcome of new investments, equation (29) would be reduced to:

$$R_t^B b_t$$

and thus, remembering the definition of the riskless interest rate, the problem of banks on areas without new investments would have a corner solution.

This result is critical for the economy, because firms are financially constrained in their investments by the amount of loans emitted by banks. Only if banks invest all

their excess liquidity on the interbank market it is possible to supply the maximum amount of credit demanded by firms in areas with new investment possibilities. For this reason, if the shock on the returns is present, the price of capital will be different between the areas of the economy, due to the excess demand of loans in some of them. But this is only the first consequence. The introduction of the shock prevents also the economy to reach its potential output, because firms do not receive all the credit they need. As a consequence, consumption and employment are lower in this framework than in the case in which the shock is not present.

This is a first interesting conclusion. Even if in a standard model is introduced some uncertainty, banks will react shifting part of their funds on less risky assets. This leads to a reduction of the total credit supplied by the banking sector.

We can see now what happens when the financial system faces a financial crisis. In this framework, a financial crisis is related to a shock in the quality of capital (ψ_t). three variables: a change in the dispersion of $x_{i,t}$, a shock on the bank capital or on the quality of capital.

The first shock is the more linked with the dynamics of the financial sector. If h , the dispersion of the shock, increases, than the probability of default increases, according to equation (36), and therefore the total amount of interbank loans decreases, according to equation (39)⁴¹.

In that case, the effect is that the two critical levels a and a' increase and, therefore, the amount of loans supplied on the interbank market is reduced. This triggers a reduction of the supply of the total supply of loans, that reduces the capital used by firms in the production process and, as a consequence, employment and output. The final outcome is that production and consumption decrease while unemployment rises. Also in this case the effect of a shock is persistent in the next periods, because the bank's capital does not rise as much as it could have done without the shock. In is critical to remember that after any default, as we will see soon, the total bank's net worth is reduced and, therefore, it effects the values of a and a' in the following periods.

The last thing to do is to analyze what happens when a default occurs. At first, the capital of debtor banks go to zero and the creditor banks get only a fraction of what originally lended. This leads to a permanent reduction in the bank's capital. This will have the consequence, in the next period, to permanently increase the value of a and a' , to increase the default probability and to reduce the the total supply of loans by any type of bank. The result on the real economy will be a reduction of output and consumption and an increase of unemployment. This outcome is more severe the lower is the realized value of $x_{i,t}$.

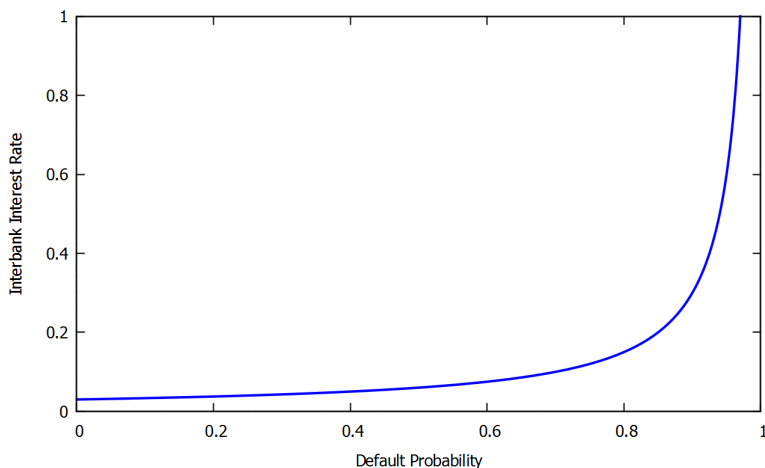
The interesting things is that we assist to a similar effect also without a default on the banking system. In fact, if the realized value of $x_{i,t}$ falls in the interval $[a; 0]$ the debtor bank does not defaults, but has to use part of its accumulated capital to pay back its debts. But doing so, the aggregate value of bank's capital at the end of the period is reduced and, therefore, in the next period the values of a and a' will be higher.

⁴¹As long as $b_t > 0$ $\mu_t = 0$. Therefore, for the definition of R_t^B and R_t^F , the negative effect is greater than the positive effect, because the mark-up is define as smaller than one, according to equation (24).

As seen before, this leads to a reduction of interbank loans and loans to firm.

What is interesting to notice, is that the endogenization of the default probability has lead to the definition of some of the components that influence it. In addition, the model answers to an increase in the default probability with a reduction of the supply of interbank loans, allowing it to replicate some of the dynamics of the interbank market during the crisis⁴². What we have observed in the markets is a drastic reduction of the supply of interbank loans, following the subprime crisis, even if the interest rates on the interbank market felt drastically. In this model, that dynamic is explained through the connection between the expected returns on interbank loans and the value of bank's assets. In addition, the changes on the interbank market effect the real economy through the total supply of loans that is directly connected to the amount of interbank credit supplied.

But given this framework, what can the public sector do to offset a crisis? The classic response of central banks is the reduction of the riskless rate, but in the last years we have seen how this sometimes was ineffective and how central banks had to use unconventional monetary policies⁴³. To analyze what the Central Bank can do in this framework, we can start to look of how the interbank interest rate must change given a change in the default probability to maintain the same level of supply on the interbank market, *ceteris paribus*:

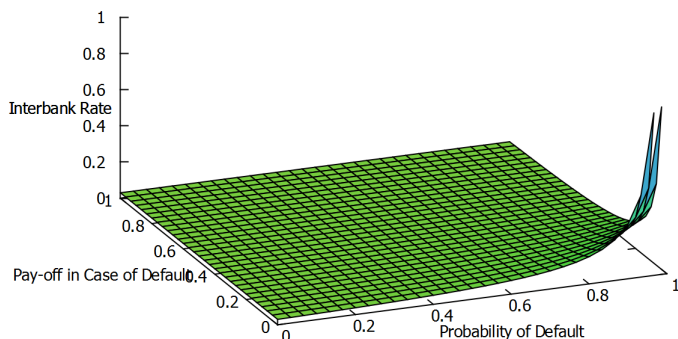


the central bank can change the riskless rate in order to influence the interbank rate. In any case, we know from equation (23) that the interbank rate can not be higher than $R_t^{i,k}$ in order to have an active market. The graph shows how even for a rather small increase in the default probability, there may not be a policy rate that offsets its effect on the interbank market. Even if we allow for an increase in the percentage of the

⁴²On this topic can be seen, between others: Heider et al. (2009) [51], Gorton (2009) [46], Brunnermeier (2009) [20], Afonso, Kovner and Schoa (2010) [2], De Socio (2011) [21], Delli Gatti (2012) [29] and Guillèn (2011) [48].

⁴³On this topic, between others, can be seen Curdia (2007) [25], Reis (2009) [72], Claessens et al. (2010) [23], Curdia and Woodford (2010) [26], Thornton (2012) [75] and Bernanke (2013) [9].

original loan that the lending bank can obtain in case of default (a variation in the fix costs of default) the conclusions do not change:



this means that a (negative) change in the riskless interest rate can have a scarce effect on the supply of interbank loans, after a riskiness crisis. This results replicate what was observed on the interbank market during the last financial crisis and what is still going on. In fact, despite the low rates, the interest rate on the interbank market is still low while the total volume of funds exchanged on the market is now about one quarter of the pre-crisis levels.

This evidences calls for other types of policies to offset crisis. Following what emerges from the analysis of the choice of lending bank, the main feature that leads to a reduction of the supply of interbank lendings (and as a consequence of output and employment), is the increase in the default probability. To offset this increase, the only way is to operate on the causes that have lead to an increase in a and a' , that are the only variables that defines the default probability and, at the same time, can be influenced by policy makers. One way to do that, for example, would be to restore part of the loss in the bank's net worth or to counterbalance the decrease in the assets price. In the next section will be presented some types of unconventional monetary policies.

4 Active Government and Central Bank

Until now we have supposed that the Central Banks just sets the riskless interest rate, while the Government sets the exogenous level of public expenditure. In this section I want to expand the model to allow for a more active role of the Government and the Central Bank during "extraordinary times".

For extraordinary times, I think to measure that the central bank may take during a crisis, that in this model is associated to an increase of the value of a and a' , by the effect of the exogenous shocks ψ and A_t or in case of a default on the interbank market. This section will try to incorporate in the model some of the main measures adopted

by the FED during the last financial crisis, to see how the model adapts to describe some types of unconventional monetary policies⁴⁴. I will discuss two different types of unconventional monetary policies (liquidity facilities and equity injections), that the FED has used according to Section 13.3 of the Federal Reserve Acts that allows the central bank, during “unusual and exigent circumstances”, to use also unconventional policies to fulfill its role of lender of last resort.

4.1 Direct Lendings

With direct lendings the Central Bank acquires directly private securities from high quality firms. In this way, the Central Bank expands directly the amount of investments financed to the private sector substituting the banking system. This type of policy, as it can be easily understood, can counterbalance the effect of a inactive interbank market. In fact, if the supply of interbank loans falls to a value close to zero because of an increase of a and a' , a simple reduction of the interest rate may not be sufficient to increase the value of B_t . In this case, to expand the credit supply, the Central Bank may find more convenient to directly finance non-financial corporations.

To incorporate this kind of policy in the model, we may think that the Central Bank decides to increase the supply of loans on investment areas. The total loan supply in that areas now becomes:

$$Q_t^i S_t^i = Q_t^i (S_{p,t}^i + S_{g,t}^i) \quad (49)$$

where $S_{p,t}^i$ is defined by equation (41) and $S_{g,t}^i$ is the total amount of loans emitted by the Central Banks. It is straightforward to see that direct lendings increase the supply of loans in a proportion of one to one. If we assume that the Central Banks decides to finance a fraction ζ_t^i of the total loans emitted by private banks, than it is easy to show how the total supply of loans on investment areas becomes:

$$Q_s^i S_t^i = \frac{1}{1 - \zeta_t^i} [\phi_t^i N_t^i + \tau_t^i B_t] \quad (50)$$

I assume also that the Central Bank faces additional cost for each unit of credit emitted to the private sector, and I call that costs Θ_t . The central bank finances its loan emission activity by emitting riskless assets that are a substitute to those emitted by the Government. For this reason, each dollar of riskless assets emitted by the Central Bank reduces the ammount of funds lendted to the Government by one. As the Central Bank can not default, we can apply to it the conclusions of the Modigliani-Miller theorem.

⁴⁴To the already mentioned Bernanke (2013) [9], Curdia and Woodford (2010) [26] and Thornton (2012) [75]. I make explicit reference also to Gertler and Kiyotaki (2010) [44].

4.2 Equity Injections

In this last case the Government, through the action of the Central Bank, directly invests in the capital of banks. It is a recapitalization of the banking system made by the Government.

Doing so, the Government sustains additional costs equal to Υ_t to evaluate its investments and monitor bankers. In addition we assume that the Government pays each capital share $Q_t^G > Q_t$ and detains its shares until the bank closes, when it receives the value of the capital plus its share of the returns on the bank's net worth.

In this way, at the beginning of each period, the total bank's capital is given by:

$$N_{t+1} = (N_t^o + N_t^y) + N_t^g \quad (51)$$

after the beginning of each period, than, a fraction π^i of N_t^g will increase the capital on areas with new investments and a fraction π^n on areas without new investments.

Thus, the new level of capital on each type of area is:

$$N_{t+1}^i = \pi^i (N_t^o + N_t^y + N_t^g) \quad (52)$$

$$N_{t+1}^n = \pi^n (N_t^o + N_t^y + N_t^g) \quad (53)$$

again, on one hand this increases the amount of credit that banks on areas with new investments can supply directly to firms and, on the other, it increases the volume of interbank loans. This increase is due to two effects: the first is given by the increased capital of banks without new investments and the second is due to the increase of the capital of debtors banks that drives down the value of a and a' .

Also in this case, the efficiency of the policy depends on the parametrization of the model. To compare this policy with the previous two we know that each unit of capital injections has a cost equal to Υ_t but it increases the total credit supply by $\pi^i + \pi^n \frac{\partial B_t}{\partial N_t}$.

A different way to implement this policy may be to supply funds to banks that operate on investment areas. In this way the government would be sure that banks will use the additional capital provided to finance firms. In this case the new values of net worth will be:

$$N_{t+1}^i = \pi^i (N_t^o + N_t^y) + N_t^g \quad (54)$$

$$N_{t+1}^n = \pi^n (N_t^o + N_t^y) \quad (55)$$

4.3 New Government Budget

Now we have defined a more active public sector than the one described in the original model. As a consequence, it is necessary to define new equilibrium equations that describes the public budget and the public demand for goods.

At first, the component that describes the public expense, now called \overline{G}_t , in the demand equation is defined by

$$\overline{G}_t = G_t + \Theta_t \sum_{h=i,n} Q_t^g S_{g,t}^h + \Upsilon_t \sum_{h=i,n} \pi^h N_t \quad (56)$$

with $S_{g,t}$ the total volume of direct lendings to firms and $\pi^h N_t$ the value of capital injections on each type of island.

The total demand of public goods is equal to the sum of the “canonical” public expense plus the additional costs connected to each kind of policy developed.

The second significant change must be made to the public budget equation, that now must take into account also the liquidity flows connected to each policy. The new public budget equation now is described by:

$$\begin{aligned} G_t + Q_t^G \sum_{h=i,n} \pi^h [N_t - (1 - \delta_K) \psi_t N_{t-1}] + \sum_{h=i,n} Q_t^h [S_{g,t}^h - (1 - \delta_K) \psi_t S_{g,t-1}^h] + \quad (57) \\ + Z_t \psi_t (N_{t-1} + S_{g,t-1}) = T_t + D_{g,t} \end{aligned}$$

With, of course, $S_{g,t} = \sum_{h=i,n} S_{g,t}^h$.

The new budget equations, as always, equalies the Government’s incomes with the Government’s expenses; in this case, however, Government’s incomes and expenses are augmented by the revenues on the economic policies developed.

5 Parametrization

The model is parametrized following the U.S. economy; values assigned to standard preference and technology parameters are reported in Table 1. For those parameters I used conventional values well established in the macroeconomic literature.

The definition of the steady state rates between the variables of the model was made using the data provided by the Federal Reserve Database.

The value of the financial-sensitive parameters (π^i , π^n , σ and ξ) is define following what proposed by Gertler and Kiyotaky (2010); those values are reported in Table 2.

Parameter h describes the dispersion on the returns of corporate bonds. As it has been proved in the empirical literature on this subject, its connection with the output

depends on several variables such as the functional form of the production function or the preferences of households. Some examples of empirical works on these themes are given by Bloom et al. (2012) [13], Knotek and Khan (2011) [56] and Panousi and Papanikolaou (2011) [68]⁴⁵. For this model, I estimated h following what proposed by Angeloni and Faia (2010) [6] using the data from Bloom et al (2012) [13]. The benchmark value of h is reported in Table 2.

Default costs are estimated following James (1991) [54] and Davydenko et al. (2012) [28]. Therefore, the cost of default has been parametrized as a percentage of total assets in the equilibrium.

Shock processes are modeled as Markov processes with standard error equal to 1. I simulated 7 types of shocks: a productivity shock(A_t); a financial shock on the quality of capital(ψ_t); an interest rate shock(given by a reduction of the riskless rate R_t^F); a positive fiscal shock(given by an increase of public consumption G_t); a direct lending shock(given by the reduction of 1% of government spending to finance direct lendings); two equity injection shocks one that hits the whole banking system the other that hits only banks on investment areas.

As we know from the previous paragraphs, there are costs associated to each of these three policies. I setted those costs as equal to 1% of the total ammount of resources invested in the policy. Computational results shows that the conclusions derived from this parametrization are robust to different choices parameters⁴⁶.

6 Shock Response

The economy of the model is effected by 7 types of shock, two exogenous shocks and 5 policy shocks. The exogenous shocks are given by either a productivity shock or a financial shocks. Policy shocks are given by the different types of public policies seen in the previous paragraphs. In particular the public sector may intervene in the economy either with classical policies, such as an increase of public consumption or a reduction of the target rate by the Central Bank , or with unconventional policies. Unconventional policies are defined by direct lendings or equity injections. In this last case, the Government can decide if intervene on the whole banking sector or if acquire equity of banks that operate only on investment islands.

Impulse response functions can be found in the appendix, results are showed as percentage deviation from the steady state.

6.1 Aggregate productivity shock

The shock is simulated as an increase of 1% over the steady state value of the parameter A_t . Impulse response functions are reported in figure 1.

⁴⁵Further references are Arellano et al. (2010) [7], Fernandez-Villaverde et al. (2009) [37], Hassler (1996) [50]

⁴⁶I simulated the model for costs equal to 5%, 10% and 15% of the total resources invested without significant changes in the results.

A productivity shock leads to an immediate increase of the output, that is immediately reduced to small values but persists in the long run due to the persistent increase in the use of both inputs of production. On the other hand, it has neglectible effects on consumption, while it reduces the interest rates. Those effects, however, dies out rapidly and the variables returns quickly to the steady state. On the contrary, there are more persistent results on employment and capital, with small but persistent deviations from the steady state after a positive shock. Those deviation leads to a persistent increase in the economy's output, that persists as long as those variables are over the steady state level.

The financial sector, in this case, plays a crucial role. As it can be seen from figure 1, after the shock we have a persistent increase in the loans supply on both types of areas. This increase allows firms to use more capital and, therefore, more labor and sustains the increase in the output. If we want to understand the reasons behind this increase in the loans supply we have to look at the second IRF of figure 1. We can see how, just after the shock, banks increase their acquisition of riskless assets and, therefore, reduces their interbank loans. We will explain this behaviour later on, for now it is important to notice how this effect immediately dies out. After two periods, in fact, the banking system changes its portfolio choice shifting resources from riskless assets to interbank borrowing. When this happens, we observe the largest increase in the loans supply and in the banks net worth. The cumulative effect of these last two passages (the increase in bank's net worth and in interbank loans) leads to the persistent increase of loans supply and capital we have previously observed. In addition, the changes in bank's net worth and interbank loans are persistent and sustain the higher level of final loans and production of the economy after the shock.

As it can be observed, the total supply of loans, of course, depends both from the aggregate level of bank's capital and from the supply of interbank loans. Therefore, we observed that final loans to firms increase immediately after the shock (as a result of the higher values of bank's capital) but reach the maximum only after that we observe a positive change also in the interbank supply.

The last question at which we need to answer is why immediately after the shock, we assist to a decline of interbank loans and to an increase of the acquisition of riskless assets. The reason is that immediately after the shock, we have an increase of the value of the parameters α and α' . This leads to higher probability of default and, therefore, banks change their portfolio choice increasing the acquisition of riskless assets. This is a behaviour we have witnessed during the past financial crisis and that the model wanted to incorporate. After some periods, however, the higher values of net worth (I remember that the net worth at the end of period t gives us the values of the net worth available in period $t+1$) and the higher values of the interest rates, lead to a reduction of α and α' allowing for a reduction of the default probability and an increase in the interbank supply. It is interesting to notice how the system stabilizes around a new, and slightly lower, value of the default probability.

At last the change in α dies out almost immediately, after that banks are able to benefit from the increase of their net worth. This leads to a significant reduction of the default probability, that benefits both from the reduction of α and α' and the increase

in the interest rates, and, therefore, to a increase of the interbank supply.

6.2 Aggregate shock on the quality of capital

In this case, I simulated an exogenous increase of the variable ψ_t . The impulse reponse functions can be found in figure 2. As it can be imaged from the analysis of the equations of the model, it is clear that this particular shock will have a significant impact on each variable of the model, because of the central role of the financial sector in this economy.

Again, we assist to a significant and persistent increase in consumption and output, with the increase in consumption, again, far below the increase in output. This result is given by the higher use of the production's inputs(capital and labor). As we have seen in the previous chapter, this fact is due to the higher supply of loans, that boost firm's investments and production.

Along with the increase in the real variable, we can observe also an increase in the interest rates.

If we analyse the behaviour of the finance sector, we notice how we observe a positive effect on all its main variables(F_t , B_t and N_t).As we noticed in the previous paragraph, we observe an initial decline in interbank loans along with a drastic increase of the acquisition of riskless assets. This is due to the variable α tha increases as a consequence of the shock, leading to an increase of the default probability. However, this effect dies out slowly, leading to a persistent lower probability of default. As a consequence, the acquisition of riskless assets declines and increases the supply of interbank loans. We can see again how when the default probability increases, we have a significant increase in the acquisition of riskless assets and a decrease in the total interbank borrowings. Another important role is played by the bank's net worth that increases significantly after the shock and leads to an increase of all the other variables.

It is important, I think, to notice how this effects are persistent and dies out very slowly. In case of a negative, shock, than, it may take a lot of time for the economy to recover without any external help.

6.3 Fiscal shock

We can start now to analyse four different types of policies that the Government and the Central Bank may use to offset crisis. The first is given by a classical positive fiscal shock, given by an increase of the government spending. Outputs can be found in figure 3.

According to the DSGE theory, we observe a positive effect on the variables after an increase of Governemtn spending. Similarly for the previous cases, the effect is persistent, higher for the output than for consumption, but significantly smaller than what observed in the previous cases.

The reson behind this is that the effect of a positive fiscal shock on the financial sector is limited. In particular we observe an increase in the bank's net worth, in interbank loans and a decline of the acquisition of riskless assets. Those effects, however, are far smaller than in the previous cases and, therefore, leads to a lower increase in

the total supply of loans and of investments. This policy, however, has positive effects on the variable and, in addition has the advantages of reducing the interest rates and, at the same time, reducing the values of a and a' leading to a lower default probability that persists in time.

6.4 Monetary policy shock

I simulated a 1% reduction of the steady state interest rate. This leads the system out of the equilibrium, with equation (33) not satisfied anymore. Outputs can be found in figure 4.

We can observe as a reduction of the riskless interest rate has positive effects on output and consumption. Again, the increase in the output is due to the more intense use of the inputs. Similarly to the previous paragraphs, the increase in the use of the inputs is given by the higher supply of interbank loans. To investigate the reasons behind that increase, we have to observe the dynamics of the banking sector. An decrease of the policy rates makes acquiring riskless assets less convenient and, therefore, banks shift their liquidity towards interbank loans. Doing so, they increase their net worth and this leads to lower default risk. As it can be seen by the graphs, the positive effect on the probability of default dies out rather quickly, while the increase in net worth is persistent. Similarly to the previous cases, it's the increased value of the net worth that sustains the persistency of higher-than steady state values of output, consumption, investments and employment. Because that increase is persistent through time, we continue to observe out of the equilibrium values of those variables for prolonged periods of time.

6.5 Unconventional policies: direct lendings

In this last two paragraphs I will analyse two different types of unconventional monetary policies: direct lendings and equity injections. In the first case the Government, through the Central Bank, lends directly to private firms, increasing in this way the total supply of loans on investment areas. I recall that in this case the Government faces additional costs and may benefit, in the following periods, of the returns on its investments

I simulated an increase of loans equal to 1% of the steady state Government's total expenditure, setting the additional cost at 1% of the total fund used⁴⁷. The outputs are showned in figure 5.

This policy has a significant impact on the main economic variables. Output, consumption, employment and capital increase, driven by an increase in the total loans supply. As it can be easily understood, the increase of the loans supply on areas with new investments is larger than on areas without new investments. At the same time, however, we assist to an increase of the interest rates and of the default probability. The banking system answers to this policy with a first decrease in net worth and interbank loans and with an increase in the riskless assets acquisition. This can be explained by

⁴⁷As I stated in paragraph 5, the results are robust to different choice of the cost parameter.

the fact that the public sector substitutes private banks in supplying loans to firms. This drives down the demand for interbank loans, decreases total bank's net worth and, of course, increases the acquisition of riskless assets. After the initial shock, however, private banks take advantage of the higher interest rates, that allow them to consolidate their net worth and boost the supply of interbank loans. As interbank loans become more convenient, they substitute riskless assets in banks balance sheets. At the same time, the increase in bank's net worth drives down the default probability as less leveraged banks have lower probability of default. To conclude, this policy has a higher impact on the real variables than a classic expansion of the government spending but it also has the relevant disadvantage of increasing the systemic risk for several periods.

6.6 Unconventional policies: equity injections

In this case, the Government finances banks through the acquisition of new shares. I simulated, again, the use of 1% of the steady state total Government expenditure to finance this type of policy. In addition, I allowed for two different types of implementation of this particular unconventional policy. In the first case the government finances all the types of bank, independently by the type of region in which they operate. The second implementation of the policy, on the contrary, gives funds only to banks on areas with new investments. The output of the first case is shown in figure 6 and those of the second implementation in figure 7.

The most interesting part of this simulation is that the outcomes change drastically depending on how the policy is implemented.

In the first case, when the government spending is used to finance the whole banking system, the results are surprisingly negative for the economy. We assist to a reduction of all the key real variables (output, employment and consumption), an increase in the interest rates and a decrease in the total supply of loans. At the same time, the bank's net worth decreases leading to a decrease of interbank loans. In addition to this, we can see also an increase in the default probability and in the parameters α and α' .

The reasons behind these particular results are complex and related to the particular structure of the finance sector. The first thing to recall is that in this framework only a fraction π^i of the initial shock hits banks on investing areas. The remaining part, that is also the major part of the shock, is assigned to areas without new investments. In this way it does not increase the supply of loans to firms and, at the same time, it does not affect the supply of interbank loans. In this way, we can explain the initial increase in the purchase of riskless assets.

Now, as the demand for riskless assets increases, their price must go down and their interest rate increase. Given the equations of the model, this leads to the increase in the rates. As we do not observe an increase in bank's net worth high enough to boost the supply of interbank loans, the total supply of loans goes down, along with the use of capital and, therefore, labor. These changes affect total output and consumption. A generalized public help, then, appears to be inefficient.

We shall now move on to figure 7, in order to discuss the case of a policy that targets

only areas where new investments are available.

As it immediately appears, the results with this type of implementation are far better. Output, employment and capital increase following an increase of loans to firms. The increase in the supply of loans is due to the increase in total net worth and in interbank loans as long as to the decrease in the acquisitions of riskless assets. All these result are persistent through time but decreases to neglectible values between 20 and 50 periods. Along with these results we see a slight decrease in the interest rates, that dies out after less than 20 periods.

An additional interesting effect of this specification of the policy is that we have a significant increase in the default probability, but it becomes negative in few periods and than stabilizes below the previous steady state level. At the end, than, we have a new steady state level of the default probability, lower than the original steady state value.

6.7 Policies Experiments

From the analysis developed so far, it appears clear how, apparently, the best performing policies are a monetary expansion and direct lendings. However, it could be interesting to see how these policies actually behave in counterballancing a negative shock.

To do so, I simulated a 1% negative shock on the quality of capital, the same type of shock that determined the world financial crisis. As an answer to that shock by the private sector, I simulated a six periods stimulous policy, determined by a positive shock of 1% on each of the previous policy variables repeated for 6 consecutive period by the public sector. As the Government is able to intervene only after that the shock hits the economy, I set the beginning of the policy package with one period of lag with respect to the original shock. Outputs are showned on figures from 8 to 12. As it can be seen from the figures, a monetary expansion and direct lendings are the policies that appears to perform better. Also in this case we can see how direct lendings have positive effects on the real variables but increase the default probability, while the effect on the real variables of a monetary expansion is smaller but has the advantage of positively effect bank's net worth, interbank loans and the default probability. Of course, expecially in the present, it is very difficult to image that a Central Bank could sustain a significant reduction of the riskless rate for so many periods⁴⁸. It appears better, than, to use a mix of policies, instead of only one tool, to counterballance the effects of a crisis. Simulations show, mixing appropriately the previous policies, how the economy can be stabilized also without the intervention of the central bank.

Repeating the same experiment with a negative productivity shock, it is possible to come to the result that all the previous policies are efficient to stabilize the economy, with the only exception of equity injections to the whole banking sector, that leads to unstable outcomes.

⁴⁸Pollin (2012) [70].

7 Final Conclusions

The present model describes an economy in which heterogeneous banks interact between them on the interbank market, making their choices of investment taking into account the probability that their counterpart defaults. As it has been shown in the previous paragraphs, the probability of default plays a crucial role in the economy of the model. If it increases, banks shift their investments to riskless assets reducing therefore the supply of interbank loans. This triggers a reduction of the total supply of loans to firms and, therefore, we assist to a reduction in output, employment and consumption.

Analyzing the equations of the model is possible to identify the components that defines the default probability to comprehend the effect that each of them has on the economy. A preliminary result was that influencing those components would have had positive effects on the economy reducing the systemic risk and, therefore, boosting loans and investments. This preliminary speculation is confirmed by the simulations carried out. In the previous section I have discussed how positive intervention on bank's net worth on areas with new investments positively influences the economy. However, simulations proved also other important and significant results. On one hand, they proved how a general intervention on the bank's net worth may have negative effects on the economy. This is caused by the fact that only a fraction of that intervention would effect directly bank's on investment areas and, therefore, reduce the default probability. The larger part, on the contrary, would go to banks on areas without new investment that, because the default probability is still high, will invest in riskless asset. The other interesting result is the positive effect of direct lendings policies. Simulations proved that this policy tool is extremely effective, in this model, to counterbalance negative shocks. The reason behind this is that it effects directly firms, boosting investments and output and generating the resources that are necessary to sustain the bank's net worth⁴⁹ and counterbalancing the negative effect of the shock. A last result is the particular effective results of a positive monetary shock to the economy. This is due to the fact that by reducing the riskless interest rate on one hand the Central Bank reduces the cost of financing of banks in the next periods and, on the other, it makes less convenient to acquire riskless assets, leading banks to shift part of their surplus funds on the interbank market. Each policy, at last, was evaluated also making reference to its effect on the systemic risk, given by the probability of default.

To conclude, I have presented a model with heterogeneous agents and endogenous default on the interbank market, that is able, to some extent, to describe some dynamics observed during the last crisis. This model is also useful to present some arguments to evaluate public policies and to compare them. A possible direction for future works would be to allow for heterogeneity also between firms and consumers, to introduce the possibility of credit to households and to evaluate the model in an open economy.

⁴⁹I recall the results of equations (10) and (11); the output level influences the returns on investments for banks.

Appendix A: Parameters Values

α	Techology	0,33
β	Discount Factor	0,99
δ	Depreciation Rate	0,025
ε	Preferences	0,1
ν	Weight of Labor Disutility	3
C	Default Costs	$0,271S^{i*}$
C/Y	Cosumption to Output	0,67
I/Y	Investment to Output	0,16
G/Y	Government Spending to Output	0,17
T/G	Taxes to Government Speding	0,59

π^i	Probability of new investments	0,25
π^n	Probability of absence of new investments	0,75
σ	Probability of bank's exit	0,972
ϑ	Inefficiency of the Interbank Market	0,383
h	Dispersion of Returns	0,5

Appendix B: Impulse Response

Figure 1: 1% productivity shock

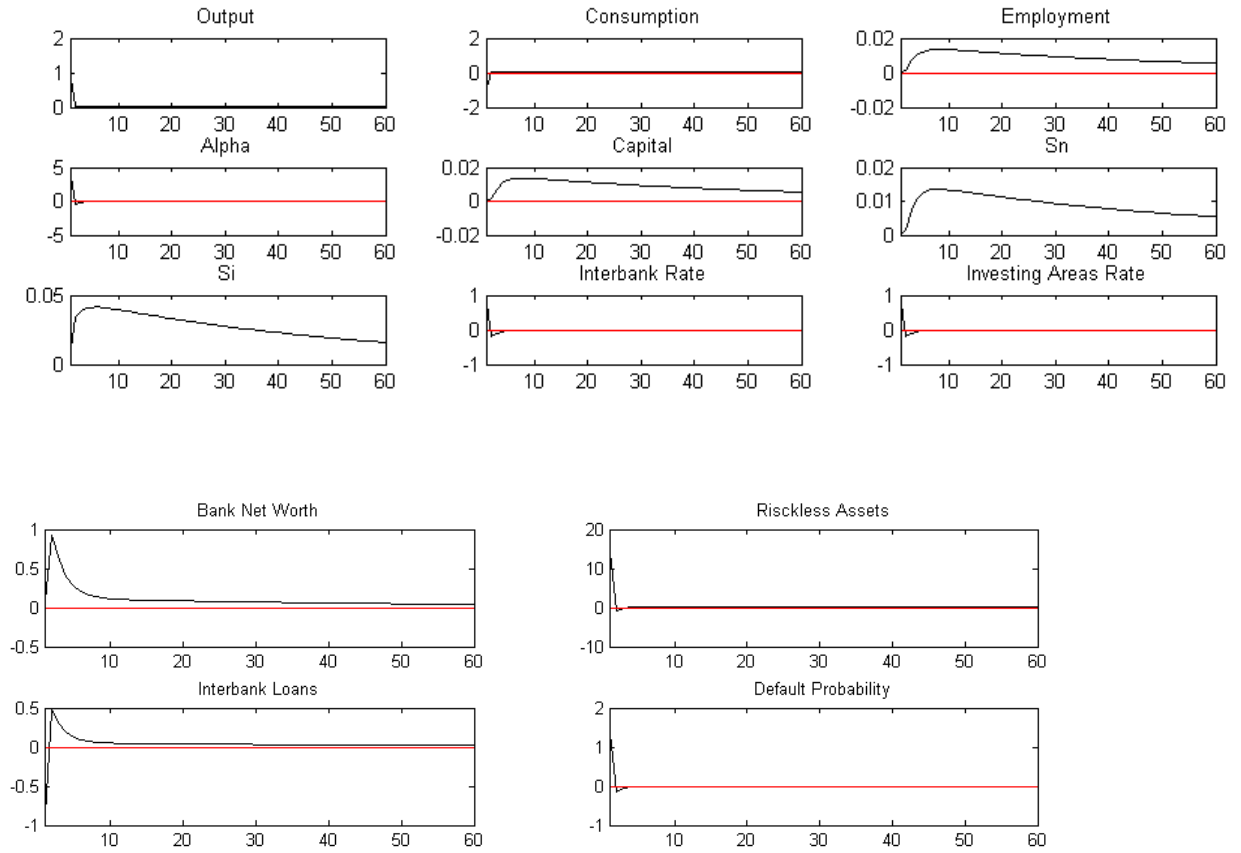
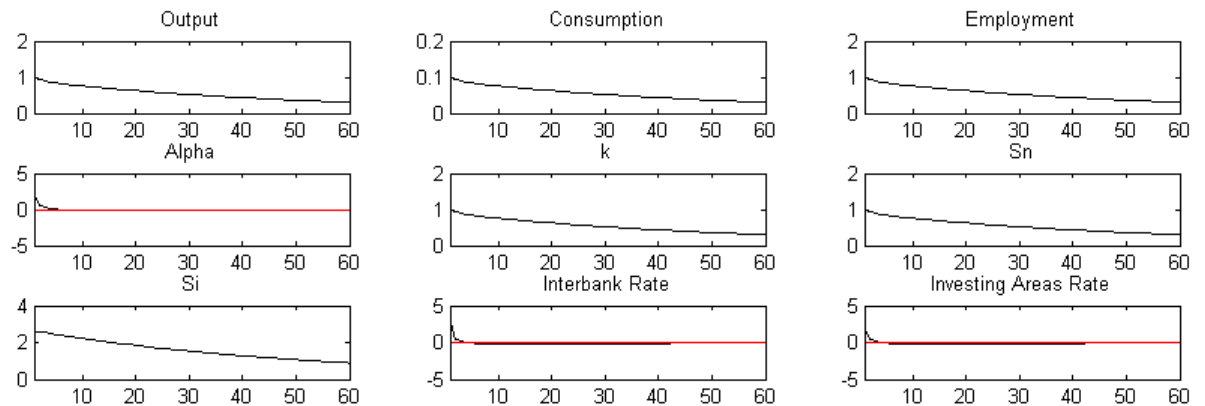


Figure 2: 1% shock on the quality of capital



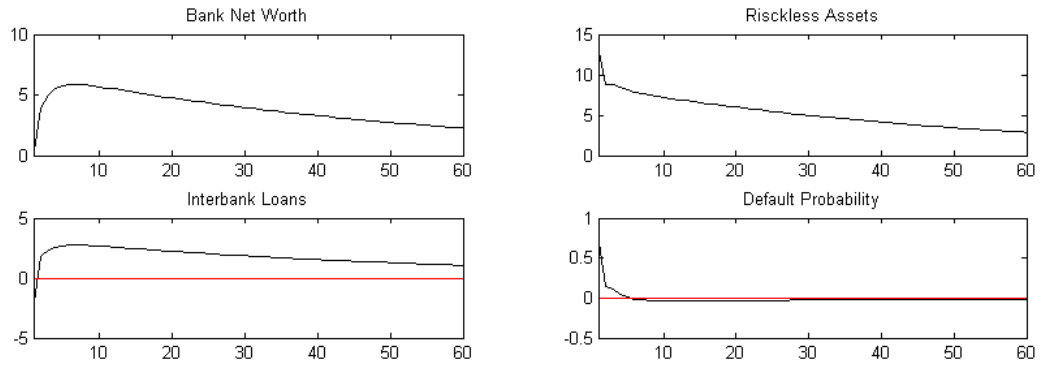


Figure 3: 1% fiscal policy shock

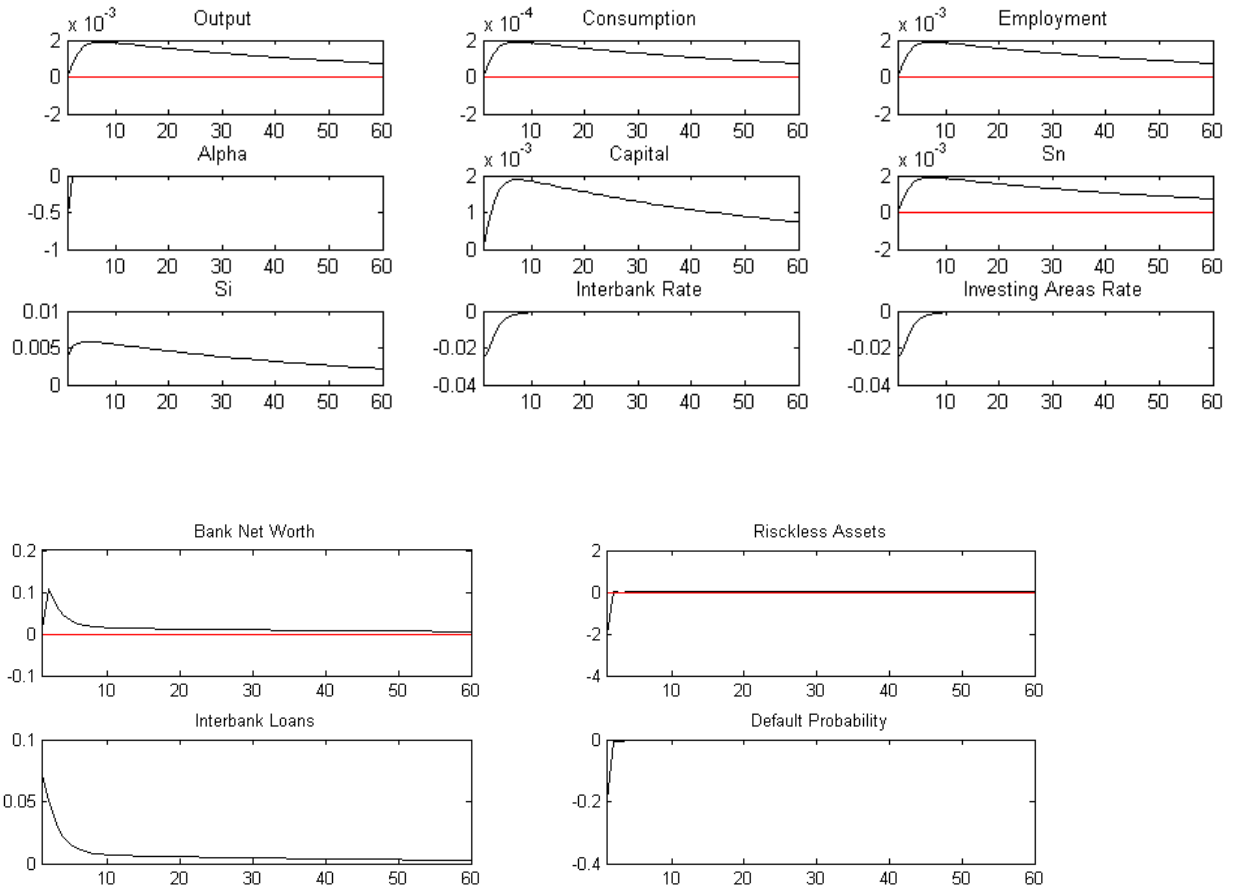


Figure 4: 1% monetary policy shock

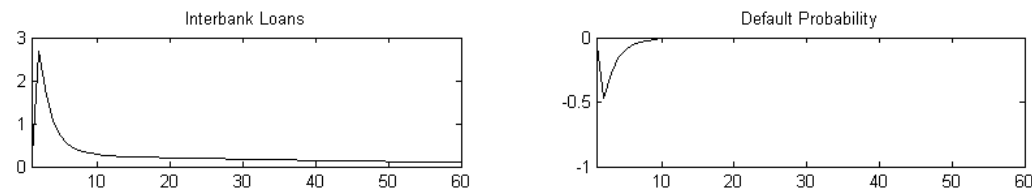
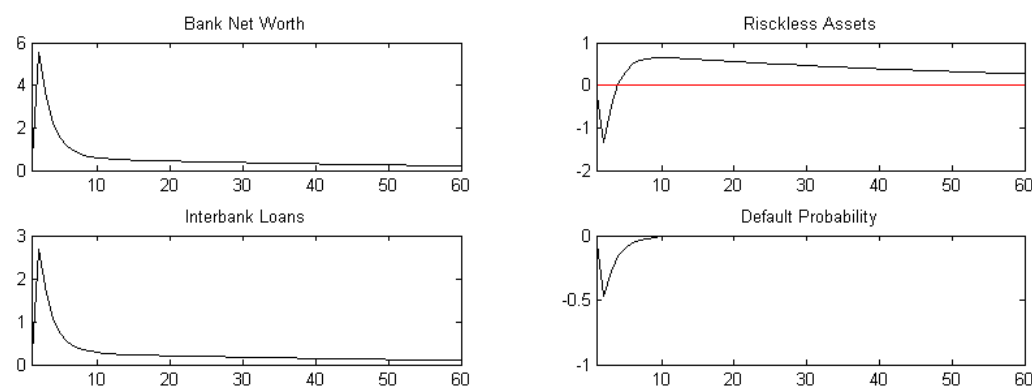
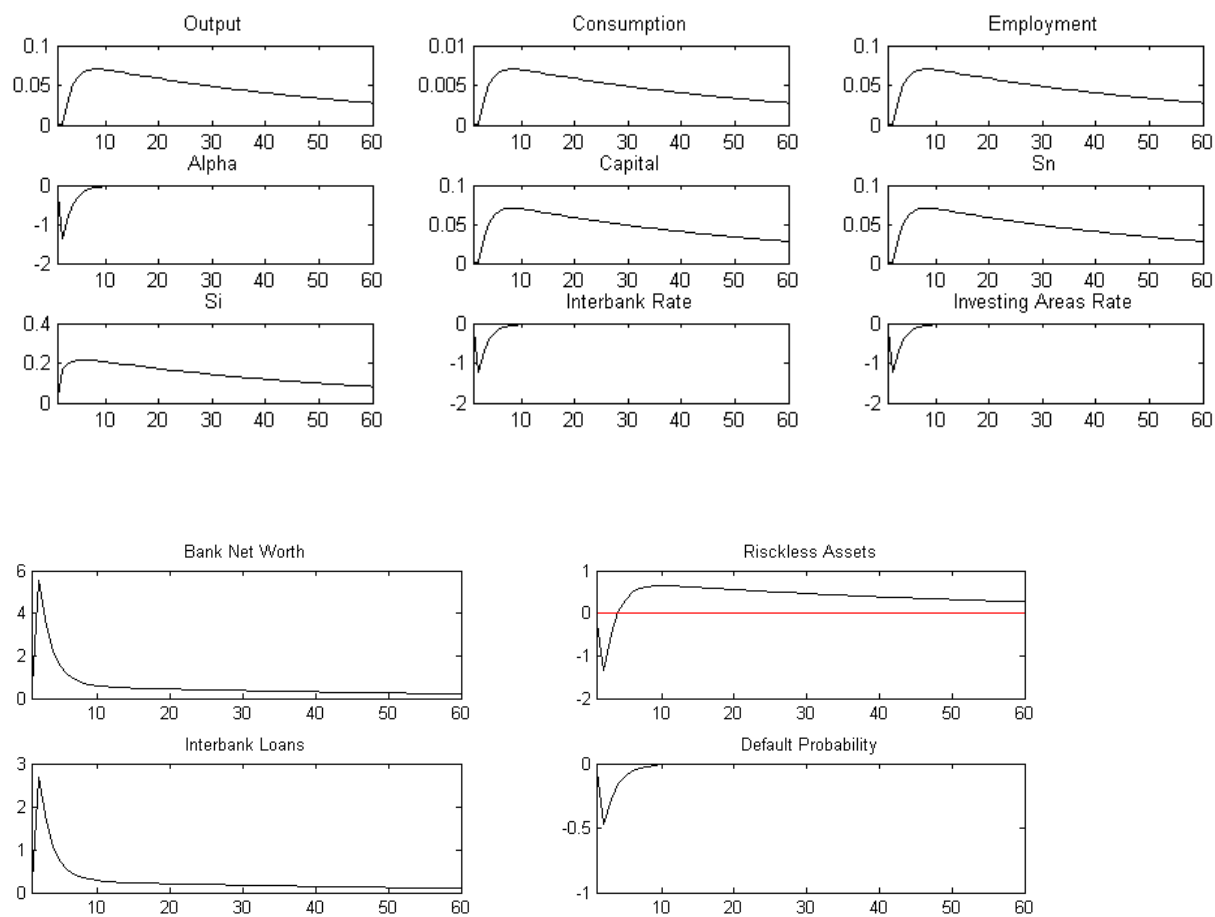
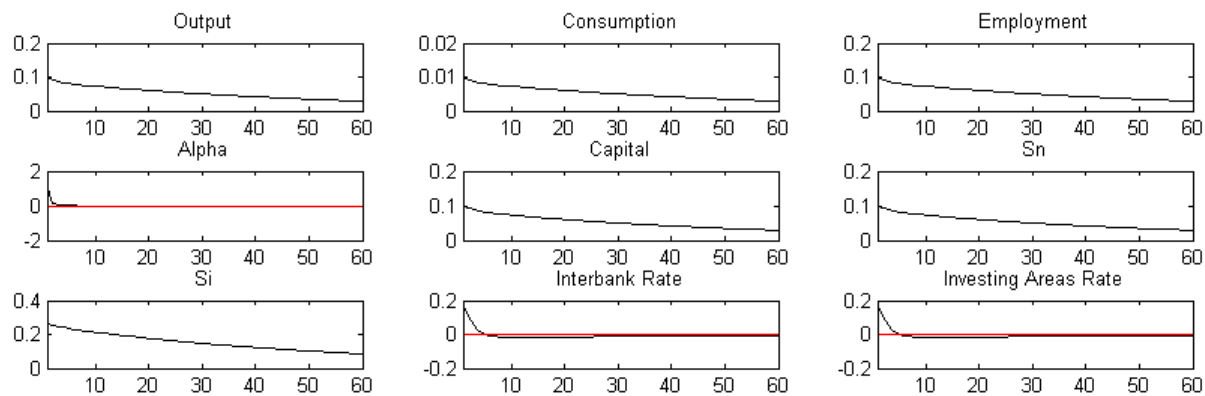


Figure 5: 1% direct lending shock



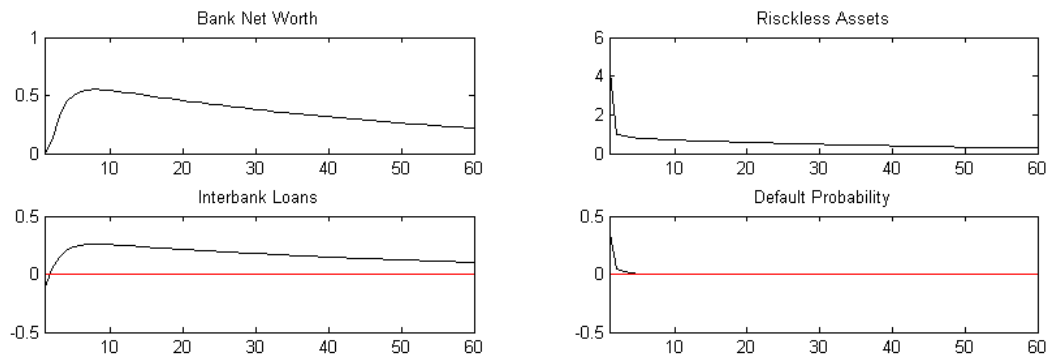


Figure 6: 1% equity injection to all bank

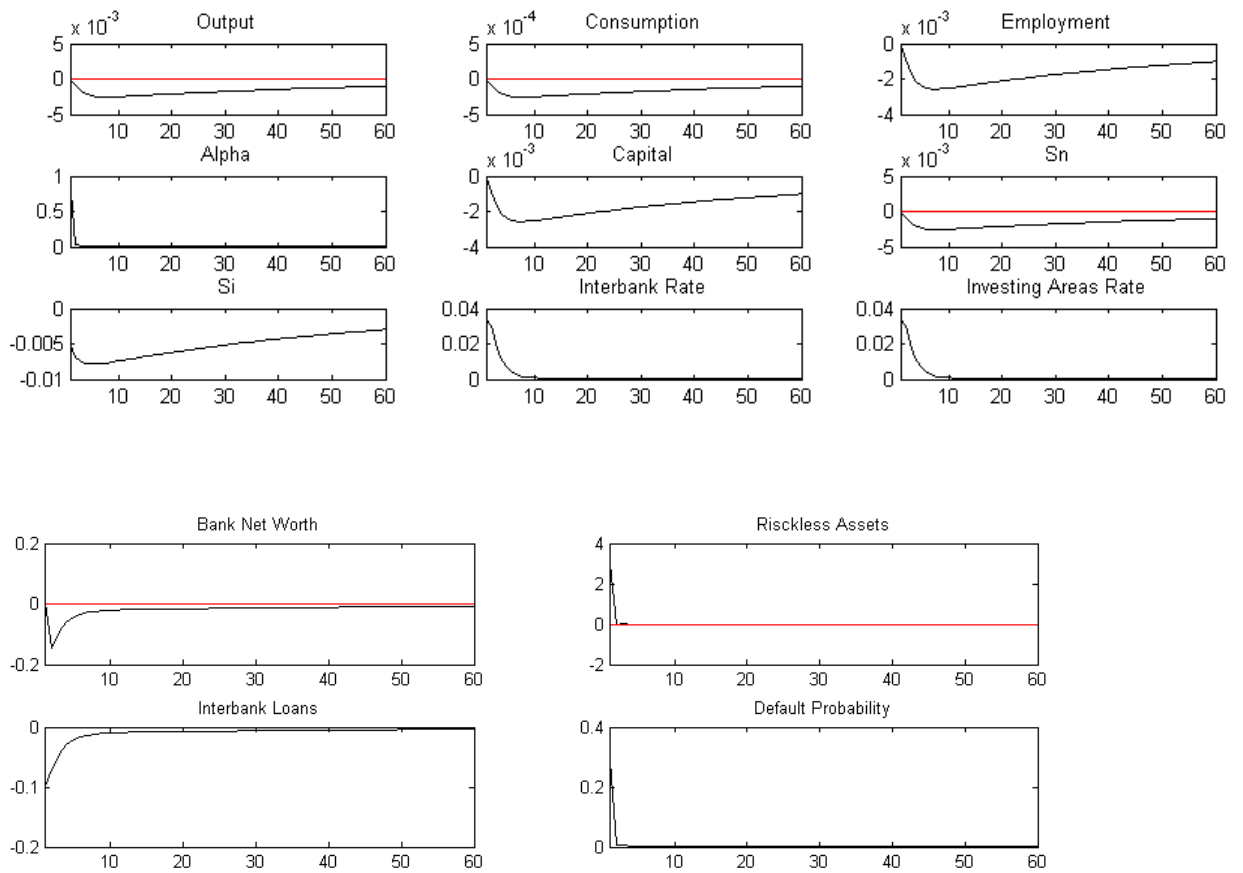


Figure 7: 1% equity injection to banks on investing areas

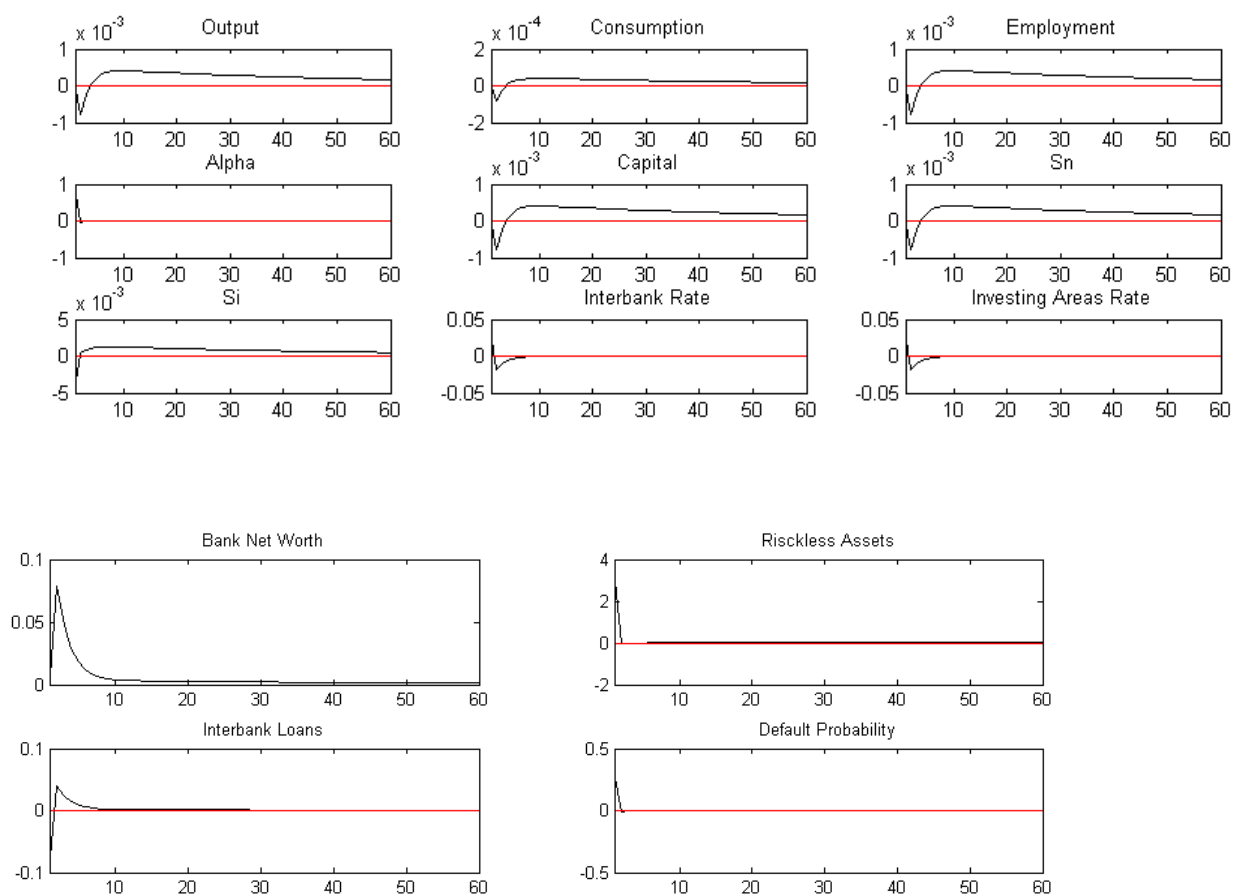
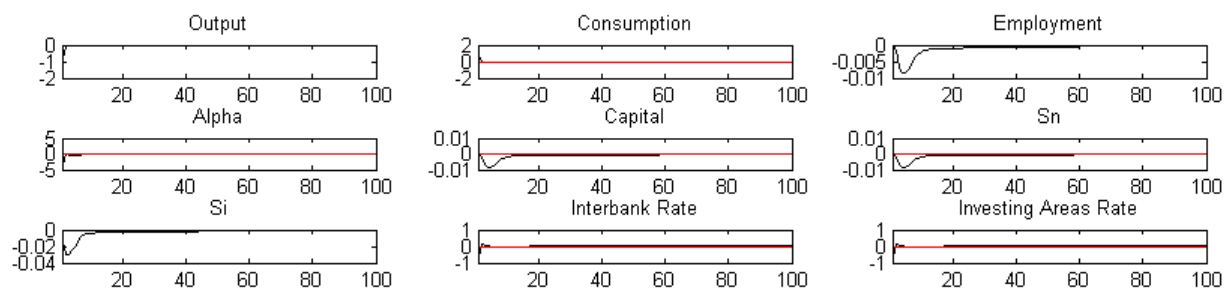


Figure 8: Government spending policy



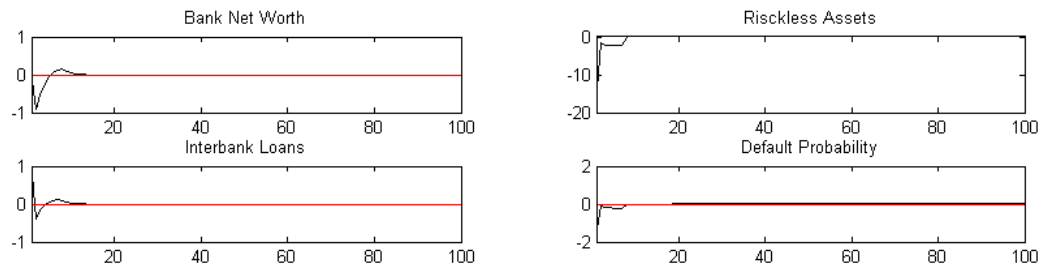


Figure 9: interest rate policy

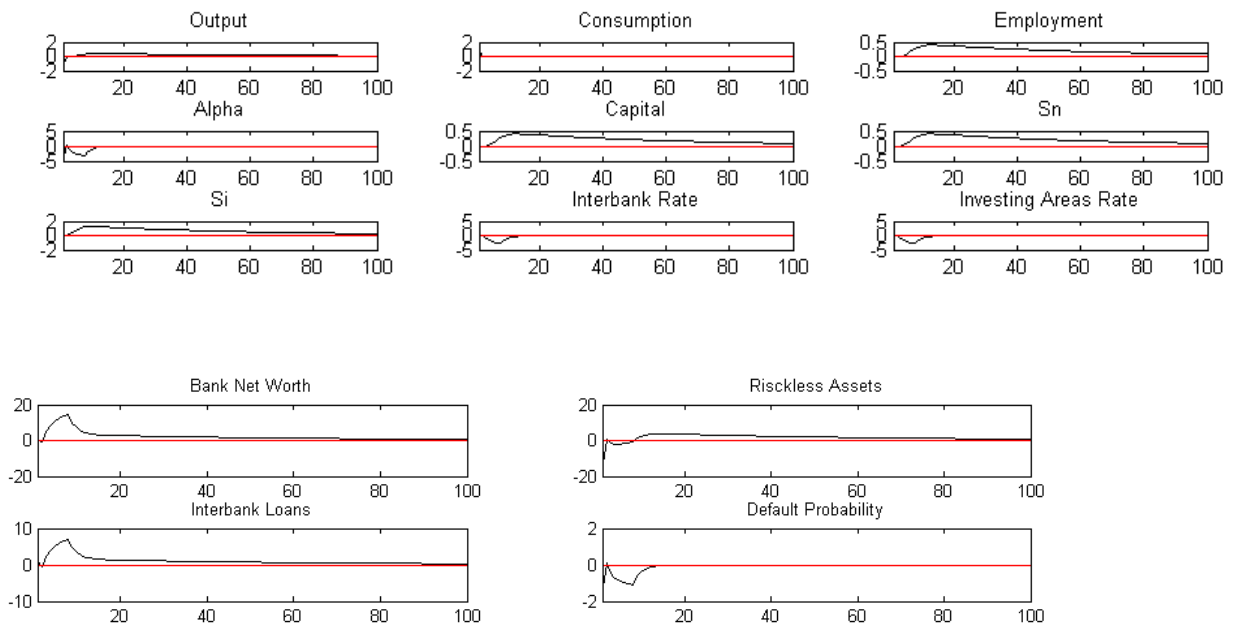
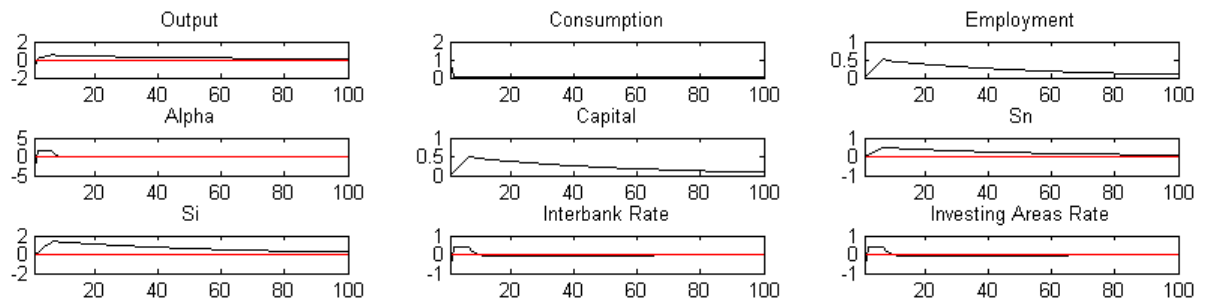


Figure 10: unconventional policy, direct lendings



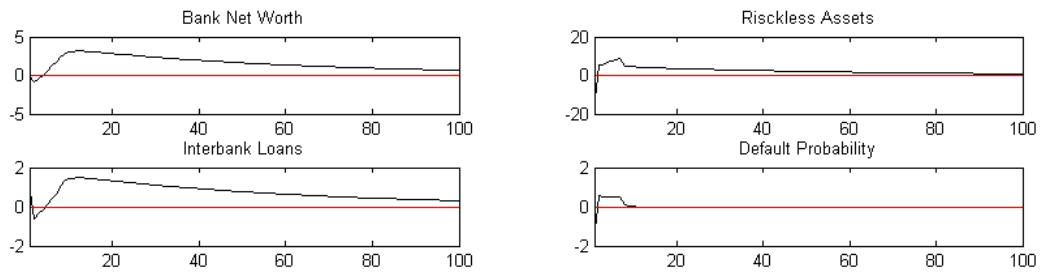


Figure 11: unconventional policy, equity injection to all banks

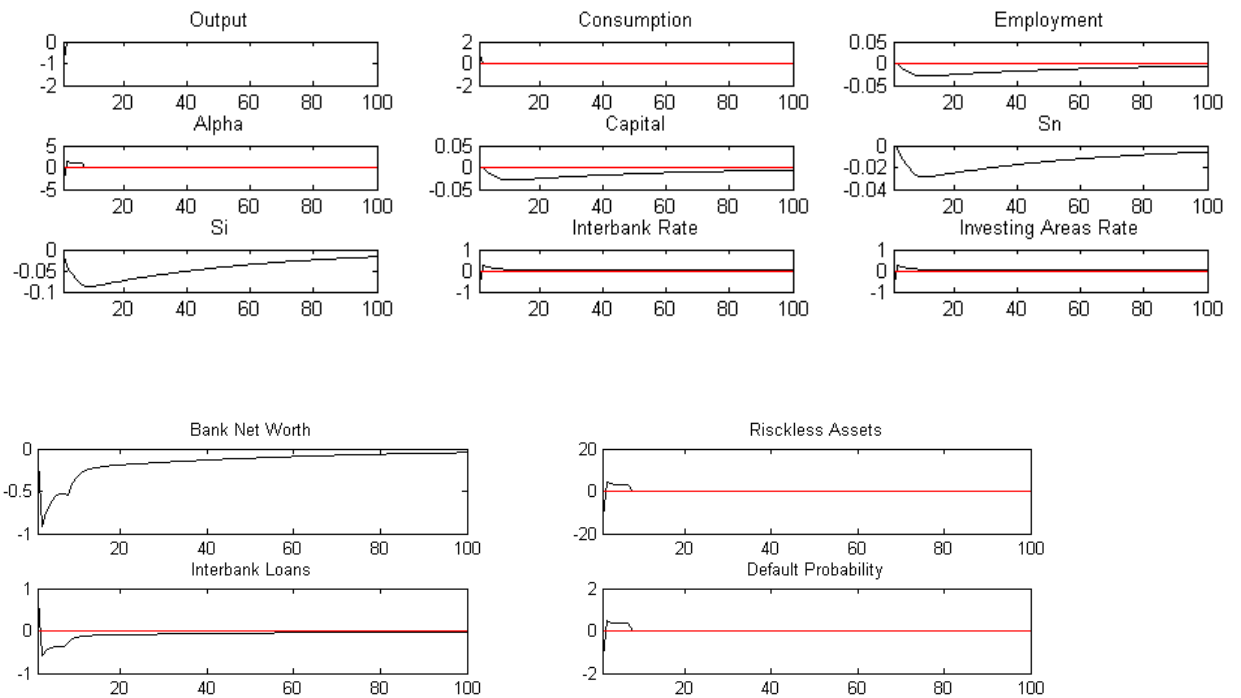
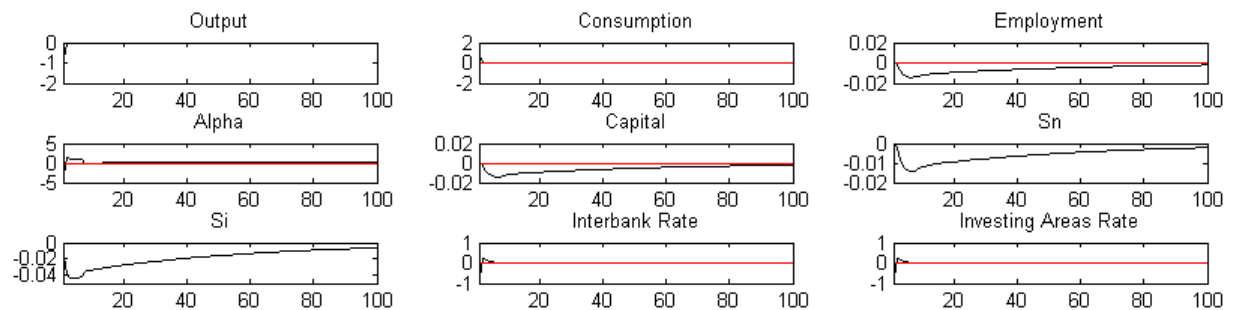
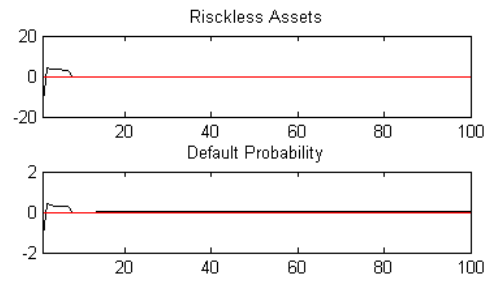
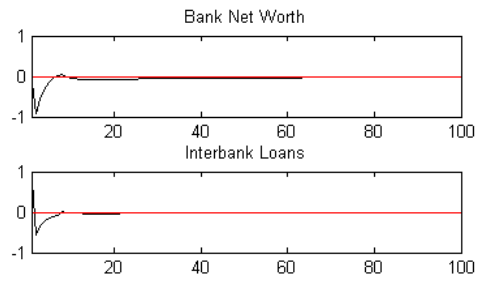


Figure 12: unconventional policy, equity injection to banks on investing areas





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