Inequality aversion in long-term contracts

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Abstract

This paper examines a two-period moral hazard model with an inequality-averse agent. We show how the agent's past performance will help the principal to relax incentive compatibility constraints and how the existence of an inequality aversion of the agent affects a level of wage in each period in a long-term contract. In particular, we focus on the performance in period 1 on the level of wage in period 2. We show that the agent's wage in period 2 depends on performance in periods 1 and 2. This implies that the long-term relationship creates the opportunity for intertemporal risk and inequality sharing.

JEL classification numbers: D63, D86, L23, J31.

Keywords: Moral hazard, Inequality aversion, Behavioral contract theory, Distribution

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1 Introduction

In recent years, other-regarding concerns such as fairness and reciprocity have been introduced into strategic environments by many researchers.[1] In particular, Fehr and Schmidt (1999) and Bolton and Ockenfels (2000) propose a class of preferences with inequality aversion and investigate their implications in economic models. It is observed that other-regarding preferences may have a positive role in a moral-hazard situations (Charness, 2004; Fehr, Gächter, and Kirchsteiger, 1997; Hannan, Kagel, and Moser, 2002). The observation has stimulated research into a theoretical analysis of the contract theory.[2] Itoh (2004) examines the fundamental properties of the second-best contract with other-regarding preferences by introducing the Fehr-Schmidt utility function. By using a continuous-effort model of Holmstrom (1979), Englmaier and Wambach (2010) set up a model where an agent has an inequality-averse preference, which is a variant of the Fehr-Schmidt preference, and provide a comprehensive treatment of the moral-hazard problem under inequality aversion.

Most works of behavioral contracts employ a one-period moral hazard model. However, since the works of Lambert (1983) and Rogerson (1985), the importance of long-term contracts are widely recognized. This paper aims to examine how inequality-averse concerns affect long-term labor contracts. In particular, we incorporate inequality aversion into the standard model of repeated moral hazard, developed by Lambert (1983) and Rogerson (1985).[3] Our formulation of inequality aversion follows the work of Englmaier and Wambach (2010) with a variant of Fehr-Schmidt preference. We clarify the properties of the first-best contract and the second-best contract. Since we assume that individuals have separable utility functions, the first-best contract is independent of the history of outcomes: the second period wage function is determined only by the current outcome. In the case of the second-best contract, the second-period wage function is dependent on the past histories. In such a case, the first-period economic performance, which determines the first-period inequality, also

[1] See Fehr and Schmidt (2006).

[2] Many works focus on the effects of other-regarding preferences on optimal contracts. Other-regarding concerns are introduced into relationships between a principal and an agent (Itoh, 2004; Dur and Grazer, 2008; Englmaier and Wambach, 2010), between agents (Demougin, Fluet, and Helm, 2006; Bartling, 2008; Bartling and von Siemens, 2010a; Neilson and Stowe, 2010), between agents in team production (Daido, 2004, 2006; Rey-Biel; 2008; Bartling and von Siemens, 2010b), and between agents in tournaments (Grund and Sliwka, 2005).

[3] For other papers on repeated moral hazard, see Radner (1981) and Chiappori, Macho, Rey, and Salanie (1994).

affects the second-period effort level of the agent. Thus, economic performance and inequality in the second period depend on the first-period outcomes, and the interaction between the past inequality and the current inequality arises in our model.

The rest of this paper is organized as follows. Section 2 formulates our model. Section 3 presents our results. Section 4 concludes the paper. The Appendix contains detailed derivations of our results.

2 The model

A principal hires an agent to work and the relationship between them is repeated for two periods. We basically follow the notation introduced by Englmaier and Wambach (2010). In period $t \in \{1, 2\}$, the agent selects a level of effort $e_t \in [\underline{e}, \overline{e}]$. The outcome (profit) $x_t \in [\underline{x}, \overline{x}]$ is realized at the end of each period depending on the level of the effort. Then, x_1 and x_2 have distribution functions $F(x_1|e_1)$ and $F(x_2|e_2)$, with everywhere-positive density functions in the intervals, $f(x_1|e_1)$ and $f(x_2|e_2)$, respectively. These probability functions are assumed to be differentiable. Note that the outputs, x_1 and x_2, are independently distributed over time, so the past realization, x_1, does not yield any information on the current realization of the profit x_2. The effort exerted in one period has no effect on the profit in any other period.

We impose the monotone likelihood ratio property:

$$\frac{\partial \frac{f_e(x_1|e_1)}{f(x_1|e_1)}}{\partial x_1} > 0, \quad \frac{\partial \frac{f_e(x_2|e_2)}{f(x_2|e_2)}}{\partial x_2} > 0.$$

The principal's strategy is to determine what wage is paid in each period. Let $w_1(x_1)$ and $w_2(x_1, x_2)$ denote the wages paid to the agent in periods 1 and 2, respectively. The contract offered by the principal specifies the pair of wages $\mathbf{w} = \{w_1(x_1), w_2(x_1, x_2)\}$. Note that w_2 depends on x_1 and x_2.

The principal's net expected utility is

$$EU_P = \int_{\underline{x}}^{\overline{x}} f(x_1|e_1) \left[x_1 - w_1(x_1) + \int_{\underline{x}}^{\overline{x}} f(x_2|e_2(x_1))[x_2 - w_2(x_1, x_2)]dx_2 \right] dx_1.$$

Note that the principal is risk-neutral.

Given the contract, the agent decides how much effort e_t to exert in each period t. Let the pair of efforts made by the agent be denoted by $\mathbf{e} = \{e_1, e_2(x_1)\}$. Note that e_2 can depend on x_1.

The agent's payoff consists of three parts: utility from wealth $u(w_t)$, cost incurred from efforts $c(e_t)$, and inequality aversion $G(x_t - 2w_t)$ (explained below). We also assume that $u'(\cdot) > 0$ and $c'(\cdot) > 0$.

$$
\begin{aligned}
EU_A = &\int_{\underline{x}}^{\overline{x}} f(x_1|e_1)\Bigg[u(w_1(x_1)) - G(x_1 - 2w_1(x_1)) \\
&+ \int_{\underline{x}}^{\overline{x}} f(x_2|e_2(x_1))[u(w_2(x_1,x_2)) - G(x_2 - 2w_2(x_1,x_2)) - c(e_2(x_1))]dx_2\Bigg]dx_1 - c(e_1), \\
&\text{with } G'(\cdot)\begin{cases} > 0 \text{ if } [x_1 - w_1(x_1)] > w_1(x_1) \text{ or } [x_2 - w_2(x_1,x_2)] > w_2(x_1,x_2) \\ < 0 \text{ if } [x_1 - w_1(x_1)] < w_1(x_1) \text{ or } [x_2 - w_2(x_1,x_2)] < w_2(x_1,x_2) \end{cases} \\
&G''(\cdot) > 0,\ G(0) = 0,\ G'(0) = 0.
\end{aligned}
$$

G represents the other-regarding concern. Under our formulation, the agent is altruistic toward the principal if the agent's income exceeds the principal's income, while the agent feels envy otherwise.

The timing of the game is organized as follows. The players can precommit to a two-period contract beforehand. At the beginning of period 0, the principal offers the wage contract $\mathbf{w} = \{w_1(x_1), w_2(x_1,x_2)\}$. Then, the agent can decide whether to accept the offer or not. If the agent rejects the offer, the game ends and the agent obtains an identical reservation utility $\bar{U}/2$ in each period. If the agent accepts the offer, he chooses a level of effort, e_1, in period 1. At the end of period 1, both players observe the realization of profit x_1. In period 2, the agent chooses the level of the effort, $e_2(x_1)$. At the end of period 2, x_2 is realized, and both the principal and the agent observe it. Then, the wage is paid to the agent.

3 Analysis

In this section, we begin by solving the first-best problem, and then consider the second-best problem.

3.1 The first-best contract

We now examine the case where the level of the effort is contractible, i.e., there is no moral hazard problem. Then, the principal's maximization problem is given by

$$\max_{\{\mathbf{e},\mathbf{w}\}} EU_P = \int_{\underline{x}}^{\overline{x}} f(x_1|e_1)[x_1 - w_1(x_1) + \int_{\underline{x}}^{\overline{x}} f(x_2|e_2(x_1))[x_2 - w_2(x_1,x_2)]dx_2]dx_1,$$

$$\text{subject to (PC)} \int_{\underline{x}}^{\overline{x}} f(x_1|e_1)[u(w_1(x_1)) - G(x_1 - 2w_1(x_1)) +$$

$$\int_{\underline{x}}^{\overline{x}} f(x_2|e_2(x_1))[u(w_2(x_1,x_2)) - G(x_2 - 2w_2(x_1,x_2))]dx_2 - c(e_2(x_1))]dx_1 - c(e_1) \geq \bar{U}.$$

The constraint in this problem is called a participation constraint (PC). It requires that working for the principal is at least as good as other work opportunities for the agent.

The Lagrangian of the problem, $\mathcal{L}_F$, is as follows:

$$\begin{aligned}\mathcal{L}_F = &\int_{\underline{x}}^{\overline{x}} f(x_1|e_1)[x_1 - w_1(x_1) + \int_{\underline{x}}^{\overline{x}} f(x_2|e_2(x_1))[x_2 - w_2(x_1,x_2)]dx_2]dx_1 \\ &+ \lambda\Big\{ \int_{\underline{x}}^{\overline{x}} f(x_1|e_1)[u(w_1(x_1)) - G(x_1 - 2w_1(x_1)) + \\ &\int_{\underline{x}}^{\overline{x}} f(x_2|e_2(x_1))[u(w_2(x_1,x_2)) - G(x_2 - 2w_2(x_1,x_2))]dx_2 - c(e_2(x_1))]dx_1 - c(e_1) - \bar{U}\Big\},\end{aligned}$$

where λ is the Lagrange multiplier of (PC). Differentiating $\mathcal{L}_F$ with respect to $w_1(x_1)$ and $w_2(x_1,x_2)$, we obtain

$$\lambda[u'(w_1(x_1)) + 2G'(x_1 - 2w_1(x_1))] = 1, \tag{1}$$

$$\lambda[u'(w_2(x_1,x_2)) + 2G'(x_2 - 2w_2(x_1,x_2))] = 1. \tag{2}$$

Then $\lambda[u' + 2G'] = 1$ holds under the first-best contract.

Differentiating (1) with respect to x_1 and (2) with respect to x_2 and rearranging the expressions yield

$$\begin{aligned}w_1'(x_1) &= \frac{1}{2} + \frac{u''(w_1(x_1))}{8G''(x_1 - 2w_1(x_1)) - 2u''(w_1(x_1))}, \\ \frac{\partial w_2(x_1,x_2)}{\partial x_2} &= \frac{1}{2} + \frac{u''(w_2(x_1,x_2))}{8G''(x_2 - 2w_2(x_1,x_2)) - 2u''(w_2(x_1,x_2))}.\end{aligned}$$

The following result states the slope of the first-period wage function.

Proposition 1. *Under the first-best contract,*

(i) if the agent is risk-neutral with respect to wealth, then $w_1'(x_1) = 1/2$*;*

(ii) if the agent is risk-averse with respect to wealth, then $w_1'(x_1) \in (0, 1/2)$*.*

Proof. See the Appendix. ■

According to Proposition 1, the slope of the wage in period 1 is 1/2 when the agent is risk-neutral, while that of the wage in period 1 is greater than 0 and less than 1/2 when the agent is risk-averse. This result is similar to the result of Englmaier and Wambach (2010), which incorporates inequality aversion into a one-period moral-hazard model.

The intuition behind Proposition 1 is as follows. When the agent is risk-neutral, only inequality matters. In the presence of inequality aversion, the agent dislikes the income difference between the two. Thus, both get a more equitable share, yielding $w_1'(x_1) = 1/2$. When the agent is risk-averse, he prefers the slope of the wage to be more flat, yielding $w_1'(x_1) < 1/2$.

Next, consider the wage function in period 2. We have the following proposition.

Proposition 2. *Under the first-best contract,*

(i) if the agent is risk-neutral with respect to wealth, then $\partial w_2(x_1, x_2)/\partial x_2 = 1/2$*;*

(ii) if the agent is risk-averse with respect to wealth, then $\partial w_2(x_1, x_2)/\partial x_2 \in (0, 1/2)$*;*

(iii) $\partial w_2(x_1, x_2)/\partial x_1 = 0$*.*

Proof. See the Appendix. ■

The intuitions behind Proposition 2 (i) and (ii) are the same as those behind Proposition 1 (i) and (ii), respectively. Proposition 2 (iii) states that the realization of the outcome in period 1 has no effect on the wage of period 2. This is because the principal chooses the optimal level of effort in each period by the assumption that the effort can be contracted. Thus, we can confirm that the cross effect $\partial w_2(x_1, x_2)/\partial x_1$ is zero.

3.2 The second-best contract

Subsequently, we examine the case where the level of effort cannot be contracted. The offered contract cannot be made contingent on the agent's choice, and thus, there exists a moral-hazard problem. The first and second constraints are incentive constraints in periods 1 (IC1) and 2 (IC2), which require that the agent's strategy is incentive-compatible in both periods:

$$\text{(IC1)}\ e_1 \in \arg\max_{\hat{e}} \int_{\underline{x}}^{\overline{x}} f(x_1|\hat{e})[u(w_1(x_1)) - G(x_1 - 2w_1(x_1))$$

$$+ \int_{\underline{x}}^{\overline{x}} f(x_2|e_2(x_1))[u(w_2(x_1,x_2)) - G(x_2 - 2w_2(x_1,x_2))]dx_2 - c(e_2(x_1))]dx_1 - c(\hat{e}), \tag{3}$$

$$\text{(IC2)}\ e_2(x_1) \in \arg\max_{\hat{e}} \int_{\underline{x}}^{\overline{x}} f(x_2|\hat{e})[u(w_2(x_1,x_2)) - G(x_2 - 2w_2(x_1,x_2))]dx_2 - c(\hat{e}). \tag{4}$$

Following the first-order approach, equations (3) and (4) are replaced by the first-order conditions of the agent, (IC1') and (IC2'), respectively (see below). The second-best contract is then obtained as a solution to the next problem:

$$\max\ EU_P = \int_{\underline{x}}^{\overline{x}} f(x_1|e_1)[x_1 - w_1(x_1) + \int_{\underline{x}}^{\overline{x}} f(x_2|e_2(x_1))[x_2 - w_2(x_1,x_2)]dx_2]dx_1,$$

subject to

$$\text{(PC)} \int_{\underline{x}}^{\overline{x}} f(x_1|e_1)[u(w_1(x_1)) - G(x_1 - 2w_1(x_1)) +$$

$$\int_{\underline{x}}^{\overline{x}} f(x_2|e_2(x_1))[u(w_2(x_1,x_2)) - G(x_2 - 2w_2(x_1,x_2))]dx_2 - c(e_2(x_1))]dx_1 - c(e_1) \geq \bar{U},$$

$$\text{(IC1')} \int_{\underline{x}}^{\overline{x}} f_e(x_1|e_1)[u(w_1(x_1)) - G(x_1 - 2w_1(x_1)) +$$

$$\int_{\underline{x}}^{\overline{x}} f(x_2|e_2(x_1))[u(w_2(x_1,x_2)) - G(x_2 - 2w_2(x_1,x_2))]dx_2 - c(e_2(x_1))]dx_1 - c'(e_1) \geq 0,$$

$$\text{(IC2')} \int_{\underline{x}}^{\overline{x}} f_e(x_2|e_2(x_1))[u(w_2(x_1,x_2)) - G(x_2 - 2w_2(x_1,x_2))]dx_2 - c'(e_2(x_1)) \geq 0.$$

The Lagrangian of the maximization problem is defined as follows:

$$
\begin{aligned}
\mathcal{L} = &\int_{\underline{x}}^{\overline{x}} f(x_1|e_1)[x_1 - w_1(x_1) + \int_{\underline{x}}^{\overline{x}} f(x_2|e_2(x_1))[x_2 - w_2(x_1,x_2)]dx_2]dx_1 \\
&+ \lambda\{ \int f(x_1|e_1)[u(w_1(x_1)) - G(x_1 - 2w_1(x_1))+ \\
&\int_{\underline{x}}^{\overline{x}} f(x_2|e_2(x_1))[u(w_2(x_1,x_2)) - G(x_2 - 2w_2(x_1,x_2))]dx_2 - c(e_2(x_1))]dx_1 - c(e_1) - \bar{U}\} \\
&+ \mu_1\{ \int_{\underline{x}}^{\overline{x}} f_e(x_1|e_1)[u(w_1(x_1)) - G(x_1 - 2w_1(x_1))+ \\
&\int_{\underline{x}}^{\overline{x}} f(x_2|e_2(x_1))[u(w_2(x_1,x_2)) - G(x_2 - 2w_2(x_1,x_2))]dx_2 - c(e_2(x_1))]dx_1 - c'(e_1)\} \\
&+ \int_{\underline{x}}^{\overline{x}} \{\mu_2(x_1) \int_{\underline{x}}^{\overline{x}} f_e(x_2|e_2(x_1))[u(w(x_1,x_2)) - G(x_2 - 2w_2(x_1,x_2))]dx_2 - c'(e_2(x_1))\}dx_1.
\end{aligned}
$$

Differentiating the Lagrangian $\mathcal{L}$ with respect to $w_1(x_1)$ and $w_2(x_1,x_2)$, we have

$$
(\lambda + D_1)[u'(w_1(x_1)) + 2G'(x_1 - 2w_1(x_1))] = 1, \tag{5}
$$

$$
(\lambda + D_1 + D_2)\big[u'(w_2(x_1,x_2)) + 2G'(x_2 - 2w_2(x_1,x_2))\big] = 1, \tag{6}
$$

where

$$
D_1 = \mu_1 \frac{f_e(x_1|e_1)}{f(x_1|e_1)}, \; D_2 = \frac{1}{f(x_1|e_1)} \mu_2(x_1) \frac{f_e(x_2|e_2(x_1))}{f(x_2|e_2(x_1))}.
$$

Differentiating (5) with respect to x_1 and (6) with respect to x_2, we have

$$
w_1'(x_1) = \frac{1}{2} + \frac{\frac{1}{2}u''(w_1(x_1)) + \mu_1 \frac{\partial\left(\frac{f_e(x_1|e_1)}{f(x_1|e_1)}\right)}{\partial x_1}[u'(w_1(x_1)) + 2G'(x_1 - 2w_1(x_1))]\frac{1}{\lambda+D_1}}{4G''(x_1 - 2w_1(x_1)) - u''(w_1(x_1))},
$$

$$
\begin{aligned}
\frac{\partial w_2(x_1,x_2)}{\partial x_2} = &\frac{1}{2} + \frac{1}{4G''(x_2 - 2w_2(x_1,x_2)) - u''(w_2(x_1,x_2))} \\
&\times \left\{ \frac{1}{2}u''(w_2(x_1,x_2)) + \frac{u'(w_2(x_1,x_2)) + 2G'(x_2 - 2w_2(x_1,x_2))}{\lambda + D_1 + D_2} \frac{\mu_2(x_1)}{f(x_1|e_1)} \frac{\partial \frac{f_e(x_2|e_2(x_1))}{f(x_2|e_2(x_1))}}{\partial x_2} \right\}.
\end{aligned}
$$

Before presenting our results, we discuss how the incentive constraints distort the contracts by comparing (1) with (5) and (2) with (6).

Let us consider period 1. D_1 is the difference between the first-order conditions of the first-best and second-best contracts. This represents a distortion arising from the incentive constraint. We provide an interpretation of this term. Note that μ_1 is the Lagrange multiplier of (IC1'). It then represents the marginal change of the principal's expected utility arising from a marginal relaxation of (IC1'). D_1 can be interpreted as a marginal value of deviation from optimal risk sharing.[4]

Subsequently, we consider period 2. We focus on the difference of first-order conditions between the first-best and second-best contracts. The distortion of period 2 between the two contracts is decomposed into two terms, D_1 and D_2. D_1 is the first-period distortion. Then, the second-period wage function is directly affected by the first-period incentive constraint.

We explain D_2. Note that $\mu_2(x_1)$ is the Lagrange multiplier of (IC1'). It represents the marginal change of the principal's expected utility arising from a marginal relaxation of (IC2'). Then, $\mu_2(x_1)/f(x_1|e_1)$ represents a *conditional* value of the sensitivity of (IC2') on the realization of the outcome of period 1. Moreover, an interpretation of $f_e(x_2|e_2(x_1))/f(x_2|e_2(x_1))$ is as a benefit-cost ratio for deviation from optimal risk sharing in period 2 depending on the realization of the outcome of period 1. As a whole, D_2 can be interpreted as a marginal value of deviation conditional on x_1. It is noteworthy that this term depends on both the first-period and second-period outcomes.

We now present our results on the wage function in period 1.

Proposition 3. *Under the second-best contract,*
(i) if the agent is risk-neutral with respect to wealth, then $w_1'(x_1) > 1/2$*;*
(ii) if the agent is risk-averse with respect to wealth, then $w_1'(x_1) > 0$*.*

Proof. See the Appendix. ■

In the absence of inequality aversion, if the agent is risk-neutral, then the first-best effort is achieved by a simple contract. The solution is an upfront "sale" of the profit to the agent before the profit is realized since the delegation is costless to the principal. Thus, the agent can obtain residuals of all profit, and only providing incentives matters. In this case, similar to the standard moral-hazard problem, the principal offers the wage with slope 1 in each period, that is, $w_1'(x_1) = 1$

[4]See Holmstrom (1979, p.79).

and $\partial w_2(x_1, x_2)/\partial x_2 = 1$.

Proposition 3 (i) states that in the presence of inequality aversion, the slope of the wage in period 1 is greater than 1/2 and not equal to 1. Thus, considering the case where the agent does not care for the whole risk of the profit but cares for the difference in income distribution, the slope is greater than 1/2 and less than 1.[5] Proposition 3 (i) and (ii) can be interpreted as extensions of Englmaier and Wambach's (2010) study of one-period contracts.

We next present our results on the wage function in period 2.

Proposition 4. *Under the second-best contract,*
(i) if the agent is risk-neutral with respect to wealth, then $\partial w_2(x_1, x_2)/\partial x_2 > 1/2$*;*
(ii) if the agent is risk-averse with respect to wealth, then $\partial w_2(x_1, x_2)/\partial x_2 > 0$*;*
(iii) $\partial w_2(x_1, x_2)/\partial x_1 > 0$ *if* $\partial[\mu_2(x_1)/f(x_2|(e_2(x_1)) \times f_e(x_2|e_2(x_1))/f(x_2|e_2(x_1))]/\partial x_1 > 0$.

Proof. See the Appendix. ■

Proposition 4 (i) and (ii) explain how the outcome in period 2 affects the wage in period 2. These results have the same implications to Proposition 3 (i) and (ii). Proposition 4 (iii) states a sufficient condition for the positive responsiveness of w_2 to x_1.

We now explain why Proposition 4 (iii) holds. In general, the sign of the slope of the wage in period 2 with respect to x_1 can be positive or negative. As shown in the Appendix, we have the following equation:

$$\frac{\partial w_2(x_1, x_2)}{\partial x_1} = \underbrace{\frac{1}{4G''(x_2 - 2w_2(x_1, x_2)) - u''(w_2(x_1, x_2))}}_{(+)} \times \underbrace{\frac{u'(w_2(x_1, x_2)) + 2G'(x_2 - 2w_2(x_1, x_2))}{\lambda + D_1 + D_2}}_{(+)}$$
$$\times \left\{ \underbrace{\mu_1 \frac{\partial \frac{f_e(x_1|e_1)}{f(x_1|e_1)}}{\partial x_1}}_{(+)} + \frac{\partial \left(\frac{\mu_2(x_1)}{f(x_2|e_2(x_1))} \frac{f_e(x_2|e_2(x_1))}{f(x_2|e_2(x_1))} \right)}{\partial x_1} \right\}. \tag{7}$$

The expression in the braces of (7) is the key to determine the sign of $\partial w_2(x_1, x_2)/\partial x_1$. As such, we focus on two terms in this brace, $\partial D_1/\partial x_1 = \mu_1 \partial[f_e(x_1|e_1)/f(x_1|e_1)]/\partial x_1$ and $\partial D_2/\partial x_1 =$

[5] If the limited liability constraint exists, then this ensures that the slope of the wage is bounded between 0 and 1. For more detail, see Innes (1990).

$\partial[\mu_2(x_1)/f(x_2|(e_2(x_1))\times f_e(x_2|e_2(x_1))/f(x_2|e_2(x_1))]/\partial x_1$. The two terms represent the second-period distortion effect of deviating from optimal risk sharing as x_1 changes. We call the first term the *first-period-distortion effect*. Since the second-period wage function directly depends on the first-period distortion, the first-period outcome affects the second-period wage through the first-period distortion. It is clear that the first-period distortion effect yields a positive impact.

The second term represents an indirect effect of the first-period outcome. Since the effort is state-contingent, a change in the first-period outcome induces a change in the second-period effort. This leads to a change in the distortion related to the second-period incentive constraint. We call the second term in the brace the *commitment effect*.

As stated in Proposition 4 (iii), if the commitment effect is positive, then the wage in period 2 is increasing in x_1. Thus, the higher the effort level that is exerted in period 1, the higher the wage in period 2 is. On the other hand, if the commitment effect is negative, then the sign of (7) is ambiguous: it is positive when the first-period-distortion effect dominates the commitment effect, but is negative when the first-period distortion effect is dominated by the commitment effect.

Finally, we explain a persistent feature of economic inequality. Note that the Lagrange multiplier μ_1 in the first period is affected by the inequality aversion G of the agent. Then, μ_1 partially reflects the degree of inequality in the first period. The Lagrange multiplier μ_1 is included in the second-period first-order condition (6). Therefore, the second-period outcome, which includes economic performance and inequality, depends on both the first-period Lagrange multiplier μ_1 and the second-period Lagrange multiplier μ_2. This implies that the degree of inequality in the second period correlates with that in the first period.

4 Concluding remarks

In this paper, we investigated the repeated moral hazard under inequality aversion. We clarified the properties of the second-best contracts. In particular, it was shown that the wage is dependent of the past history under the second-best contract. Moreover, we characterized the basic mechanism of the historical dependency of the worker's wage.

Throughout this paper, we considered the two-period case. Our results can be extended to the T-period model ($T \geq 3$). As in the two-period model, past histories affect the optimal contract, but the mechanism is more complicated. The fundamental suggestion is, however, the same: the principal must take both current-period and future-period incentives and inequality aversion into account to exert the agent's effort. To clarify this point, consider the three-period case. We explain how the first-period outcome x_1 affects the wage in the third-period. First, the third-period effort directly depends on the first-period outcome x_1. Second, there exists another indirect effect. The second-period effort, which determines the distribution over the second-period outcome, depends on the first-period outcome x_1. Thus, the first-period outcome x_1 affects the third-period effort through the second-period outcome.

The persistence of past histories is observed in various experiments. For example, the results of multiple trials of the public-goods experiment contrast with that of a single trial. In the case of a single trial, most subjects cooperate with each other: they contribute 40-60% of their allotments (Camerer, 2003). When the game is repeated, the cooperative behavior of subjects declines sharply (Kim and Walker, 1984; Isaac, Walker, and Thomas, 1984; Andreoni, 1988). What is the cause of this decline? Researchers have provided various explanations by experimental and theoretical studies. One reasonable hypothesis is a reciprocity consideration among subjects (Dawes and Thaler, 1988). In this case, past history matters for subjects. However, it is not obvious how past history relates to a reciprocity consideration. In a general class of game with a long-term relationship, the relationship is significant but complicated when players have other-regarding preferences. Our analysis clarified the mechanism for a simplified but important class of games.

Appendix

Proof of Proposition 1: Differentiating $\mathcal{L}_F$ with respect to $w_1(x_1)$, we have

$$\lambda[u'(w_1(x_1)) + 2G'(x_1 - 2w_1(x_1))] = 1. \tag{1}$$

Differentiating (1) with respect to x_1 yields

$$\begin{aligned}
& u'(w_1(x_1))w_1'(x_1) + 2G''(x_1 - 2w_1(x_1))(1 - 2w_1'(x_1)) = 0 \\
\Rightarrow\ & w_1'(x_1) = \frac{2G''(x_1 - 2w_1(x_1))}{4G''(x_1 - 2w_1(x_1)) - u''(w_1(x_1))} \\
\Rightarrow\ & w_1'(x_1) = \frac{2G''(x_1 - 2w_1(x_1)) - \frac{1}{2}u''(w_1(x_1)) + \frac{1}{2}u''(w_1(x_1))}{4G''(x_1 - 2w_1(x_1)) - u''(w_1(x_1))} \\
\Rightarrow\ & w_1'(x_1) = \frac{1}{2} + \frac{u''(w_1(x_1))}{8G''(x_1 - 2w_1(x_1)) - 2u''(w_1(x_1))}. \qquad (8)
\end{aligned}$$

(i) Consider the case where the agent is risk-neutral. Note that $u''(\cdot) = 0$ holds. Substituting $u''(\cdot) = 0$ into (8), we have $w_1'(x_1) = 1/2$.

(ii) Next, consider the case where the agent is risk-averse. Since $u''(\cdot) < 0$ and $G''(\cdot) > 0$ hold, the second term of (8) is negative. Thus, we have $w_1'(x_1) < 1/2$. Since $u''(\cdot) < 0$ and $G''(\cdot) > 0$ hold, we have

$$w_1'(x_1) = \frac{1}{2} + \frac{u''(w_1(x_1))}{8G''(x_1 - 2w_1(x_1)) - 2u''(w_1(x_1))} > \frac{1}{2} - \frac{u''(w_1(x_1))}{2u''(w_1(x_1))} = 0.$$

Thus, we have $w_1'(x_1) \in (0, 1/2)$. ■

Proof of Proposition 2: Differentiating $\mathcal{L}_F$ with respect to $w_2(x_1, x_2)$, we have

$$\lambda[u'(w_2(x_1, x_2)) + 2G'(x_2 - 2w_2(x_1, x_2))] = 1. \qquad (9)$$

Differentiating (9) with respect to x_2 and rearranging the above expression, we have the following:

$$\begin{aligned}
& u''(w_2(x_1, x_2))\frac{\partial w_2(x_1, x_2)}{\partial x_2} + 2G''(x_2 - 2w_2(x_1, x_2))\left[1 - 2\frac{\partial w_2(x_1, x_2)}{\partial x_2}\right] = 0 \\
\Rightarrow\ & \frac{\partial w_2(x_1, x_2)}{\partial x_2} = \frac{1}{2} + \frac{u''(w_2(x_1, x_2))}{8G''(x_2 - 2w_2(x_1, x_2)) - 2u''(w_2(x_1, x_2))}. \qquad (10)
\end{aligned}$$

(i) If the agent is risk-neutral with respect to wealth, $u''(w_2(x_1, x_2)) = 0$ holds. By (10), we have $\partial w_2(x_1, x_2)/\partial x_2 = 1/2$.

(ii) If the agent is risk-averse, $u''(w_2(x_1, x_2)) < 0$ holds. Since the second term of (10) is negative, we have $\partial w_2(x_1, x_2)/\partial x_2 < 1/2$.

(iii) Differentiating (9) with respect to x_1, we have

$$u''(w_2(x_1,x_2))\frac{\partial w_2(x_1,x_2)}{\partial x_1}+2G''(x_2-2w_2(x_1,x_2))\left[-2\frac{\partial w_2(x_1,x_2)}{\partial x_1}\right]=0$$
$$\Rightarrow \frac{\partial w_2(x_1,x_2)}{\partial x_1}=0.$$

Thus, we have the desired results. ■

Proof of Proposition 3: (i) Differentiating the Lagrangian with respect to $w_1(x_1)$ and rearranging this, we have

$$\left[\lambda+\mu_1\frac{f_e(x_1|e_1)}{f(x_1|e_1)}\right][u'(w_1(x_1))+2G'(x_1-2w_1(x_1))]=1. \tag{5}$$

Differentiating (5) with respect to x_1, we have

$$\begin{aligned}0=&\mu_1\frac{\partial\left(\frac{f_e(x_1|e_1)}{f(x_1|e_1)}\right)}{\partial x_1}\left[u'(w_1(x_1))+2G'(x_1-2w_1(x_1))\right]\\&+\left[\lambda+\mu_1\frac{f_e(x_1|e_1)}{f(x_1|e_1)}\right][u''(w_1(x_1))w_1'(x_1)+(1-2w_1'(x_1))2G''(x_1-2w_1(x_1))],\end{aligned}$$

$$w_1'(x_1)=\frac{1}{2}+\frac{\frac{1}{2}u''(w_1(x_1))+\mu_1\frac{\partial\left(\frac{f_e(x_1|e_1)}{f(x_1|e_1)}\right)}{\partial x_1}[u'(w_1(x_1))+2G'(x_1-2w_1(x_1))]\frac{1}{\lambda+\mu\frac{f_e(x_1|e_1)}{f(x_1|e_1)}}}{4G''(x_1-2w_1(x_1))-u''(w_1(x_1))}. \tag{11}$$

Since the agent is risk-neutral, $u''(\cdot)=0$ holds, yielding the following equation:

$$w_1'(x_1)=\frac{1}{2}+\frac{\mu_1\frac{\partial\left(\frac{f_e(x_1|e_1)}{f(x_1|e_1)}\right)}{\partial x_1}[u'(w_1(x_1))+2G'(x_1-2w_1(x_1))]\frac{1}{\lambda+\mu\frac{f_e(x_1|e_1)}{f(x_1|e_1)}}}{4G''(x_1-2w_1(x_1))}. \tag{12}$$

Since the monotone likelihood ratio property is assumed, $\partial(f_e(x_1|e_1)/f(x_1|e_1))/\partial x_1>0$ is implied. Then, the signs of the second term are all positive except

$$[u'(w_1(x_1))+2G'(x_1-2w_1(x_1))]\frac{1}{\lambda+\mu\frac{f_e(x_1|e_1)}{f(x_1|e_1)}}. \tag{13}$$

To examine the sign of the above expression, we focus on the first-order condition (5), again. This condition ensures that both terms $[u'(w_1(x_1))+2G'(x_1-2w_1(x_1))]$ and $[\lambda+\mu(f_e(x_1|e_1)/f(x_1|e_1))]$ have the same sign. Thus, since the second term of (12) is positive, we have $w_1'(x_1)>1/2$.

(ii) Arranging (11) yields

$$
\begin{aligned}
w_1'(x_1) =& \frac{2G''(x_1 - 2w_1(x_1))}{4G''(x_1 - 2w_1(x_1)) - u''(w_1(x_1))} \\
&+ \frac{\mu_1 \dfrac{\partial \left(\frac{f_e(x_1|e_1)}{f(x_1|e_1)} \right)}{\partial x_1} [u'(w_1(x_1)) + 2G'(x_1 - 2w_1(x_1))]}{\left[\lambda + \mu_1 \dfrac{f_e(x_1|e_1)}{f(x_1|e_1)} \right] [4G''(x_1 - 2w_1(x_1)) - u''(w_1(x_1))]}.
\end{aligned}
$$

The first term is positive since $G''(x_1 - 2w_1(x_1)) > 0$ and $u''(w_1(x_1)) < 0$ hold. The second term is positive by (13) > 0. Thus, we have the desired result, $w_1'(x_1) > 0$. ■

Proof of Proposition 4: (i) Differentiating the Lagrangian with respect to $w_2(x_1, x_2)$, we have

$$
(\lambda + D_1 + D_2)\big[u'(w_2(x_1, x_2)) + 2G'(x_2 - 2w_2(x_1, x_2))\big] = 1, \tag{6}
$$

where

$$
D_1 = \mu_1 \frac{f_e(x_1|e_1)}{f(x_1|e_1)}, \;\; D_2 = \frac{1}{f(x_1|e_1)} \mu_2(x_1) \frac{f_e(x_2|e_2(x_1))}{f(x_2|e_2(x_1))}.
$$

Differentiating (6) with respect to x_2, we have

$$
\begin{aligned}
&\left[u''(w_2(x_1, x_2)) \frac{\partial w_2}{\partial x_2} + 2(1 - 2\frac{\partial w_2}{\partial x_2}) G''(x_2 - 2w_2(x_1, x_2)) \right] \\
&+ \frac{1}{E} \times \frac{\mu_2(x_1)}{f(x_1|e_1)} \frac{\partial \frac{f_e(x_2|e_2(x_1))}{f(x_2|e_2(x_1))}}{\partial x_2} [u'(w_2(x_1, x_2)) - 2G'(x_2 - 2w_2(x_1, x_2))] = 0,
\end{aligned}
$$

where

$$
E = \left[\lambda + \mu_1 \frac{f_e(x_1|e_1)}{f(x_1|e_1)} + \frac{\mu_2(x_1)}{f(x_1|e_1)} \frac{f_e(x_2|e_2(x_1))}{f(x_2|e_2(x_1))} \right].
$$

Similar to the above cases, differentiating this expression with respect to x_2, we have the following:

$$
\begin{aligned}
\frac{\partial w_2(x_1, x_2)}{\partial x_2} =& \frac{1}{2} + \frac{1}{4G''(x_2 - 2w_2(x_1, x_2)) - u''(w_2(x_1, x_2))} \\
\times& \left\{ \frac{1}{2} u''(w_2(x_1, x_2)) + \frac{u'(w_2(x_1, x_2)) + 2G'(x_2 - 2w_2(x_1, x_2))}{\left[\lambda + \mu_1 \frac{f_e(x_1|e_1)}{f(x_1|e_1)} + \frac{\mu_2(x_1)}{f(x_1|e_1)} \frac{f_e(x_2|e_2(x_1))}{f(x_2|e_2(x_1))} \right]} \frac{\mu_2'(x_1)}{f(x_1|e_1)} \frac{\partial \left(\frac{f_e(x_2|e_2(x_1))}{f(x_2|e_2(x_1))} \right)}{\partial x_2} \right\}.
\end{aligned} \tag{14}
$$

Since the agent is risk-neutral, substituting $u''(w_2(x_1,x_2)) = 0$ into (14) yields

$$\frac{\partial w_2(x_1,x_2)}{\partial x_2} = \frac{1}{2} + \frac{1}{4G''(x_2 - 2w_2(x_1,x_2))} \times \left\{ \frac{u'(x_2|w_2(x_1,x_2)) + 2G'(x_2 - 2w_2(x_1,x_2))}{\left[\lambda + \mu_1 \frac{f_e(x_1|e_1)}{f(x_1|e_1)} + \frac{\mu_2(x_1)}{f(x_1|e_1)} \frac{f_e(x_2|e_2(x_1))}{f(x_2|e_2(x_1))}\right]} \frac{\mu_2'(x_1)}{f(x_1|e_1)} \frac{\partial \left(\frac{f_e(x_2|e_2(x_1))}{f(x_2|e_2(x_1))}\right)}{\partial x_2} \right\}. \tag{15}$$

The second term of (15) is positive by the first-order condition, the shape of function G, and the monotone likelihood ratio property. Thus, we have $\partial w_2(x_1,x_2)/\partial x_2 > 1/2$.

(ii) Rearranging (14) yields

$$\frac{\partial w_2(x_1,x_2)}{\partial x_2} = \frac{2G''(x_2 - 2w_2(x_1,x_2))}{4G''(x_2 - 2w_2(x_1,x_2)) - u''(w_2(x_1,x_2))} + \frac{1}{4G''(x_2 - 2w_2(x_1,x_2)) - u''(w_2(x_1,x_2))} \times \left\{ \frac{u'(w_2(x_1,x_2)) + 2G'(x_2 - 2w_2(x_1,x_2))}{\left[\lambda + \mu_1 \frac{f_e(x_1|e_1)}{f(x_1|e_1)} + \frac{\mu_2(x_1)}{f(x_1|e_1)} \frac{f_e(x_2|e_2(x_1))}{f(x_2|e_2(x_1))}\right]} \frac{\mu_2'(x_1)}{f(x_1|e_1)} \frac{\partial \left(\frac{f_e(x_2|e_2(x_1))}{f(x_2|e_2(x_1))}\right)}{\partial x_2} \right\}. \tag{16}$$

The first and the second terms of (16) are positive, similar to Proposition 3. Thus, we have $\partial w_2(x_1,x_2)/\partial x_2 > 0$.

(iii) Totally differentiating the first-order condition (6) with respect to x_1, we have

$$\frac{\partial w_2(x_1,x_2)}{\partial x_1} = \underbrace{\frac{1}{4G''(x_2 - 2w_2(x_1,x_2)) - u''(w_2(x_1,x_2))}}_{(+)} \times \underbrace{\frac{u'(w_2(x_1,x_2)) + 2G'(x_2 - 2w_2(x_1,x_2))}{\left[\lambda + \mu_1 \frac{f_e(x_1|e_1)}{f(x_1|e_1)} + \frac{\mu_2(x_1)}{f(x_1|e_1)} \frac{f_e(x_2|e_2(x_1))}{f(x_2|e_2(x_1))}\right]}}_{(+)} \times \left\{ \underbrace{\mu_1 \frac{\partial \left(\frac{f_e(x_1|e_1)}{f(x_1|e_1)}\right)}{\partial x_1}}_{(+)} + \frac{\partial \left(\frac{\mu_2(x_1)}{f(x_2|e_2(x_1))} \frac{f_e(x_2|e_2(x_1))}{f(x_2|e_2(x_1))}\right)}{\partial x_1} \right\}.$$

The sign of the derivative, $\partial w_2(x_1,x_2)/\partial x_1$, depends on the sign of the last term in the brace. Thus, if $\partial[\mu_2(x_1)/f(x_2(e_2(x_1))) \times f_e(x_2(e_2(x_1)))/f(x_2(e_2(x_1)))]/\partial x_1 > 0$ holds, we always have $\partial w_2(x_1,x_2)/\partial x_1 > 0$. ■

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