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Redefining the Economical Power of Nations: An international macroeconomic model that introduces a future aspect into the GDP

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Redefining the Economical Power of Nations

**An international macroeconomic model that introduces a future aspect
into the GDP**

**Economic Modelling/ Macroeconomics/ Investment/ GDP/ Econo-
metrics/**

JEL Classification: E010, E10, O11, C02, C01, E01, F02, C10, C50

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1 Abstract

The paper challenges the current common rankings of economies when measured over the GDP, and introduces a set of new Indices based on the factors GDP, Gini Coefficient and Inflation. This *Social factors relativized GDP* cuts out that part of the GDP, which is structurally long term frozen up by social transfers. Furthermore, the paper introduces an economic model that implements the *future aspect into the GDP* over the factors of change in capital expenditures and change in private investments. The first part of the paper explains the κ _Index models, the factors that were applied, as well as the reasons for the choice. The second part of the paper shows variations of the κ _Index including a few ideal-typical examples. The final part asks some critical questions and concludes the paper.

Basic social factors relativized GDP: $\kappa_Index = GDP - GDP \times Gini$ is $(1 - GINI) \times GDP$

Inflation indexed Version: $\kappa_Index_Infl. = (1 - Gini - Inflation) \times GDP$

Future aspect of GDP

With Private Investment variable: $\kappa p = (B + B \times \Delta P) - (B \times G)$

With Private Investment variable and inflation: $\kappa ip = (B + B \times \Delta P) - [(B \times G) + (B \times In)]$

Productivity Index: $\kappa_PROD = \kappa_Index / Labor\ Force = \kappa_PROD$

Inflation indexed Productivity Index: $\kappa_PROD_Infl. = \kappa_Index_Infl. / Labor\ Force$

Debt-to- κ_Index $\kappa_Debt = National\ debt / \kappa_Index$

Debt-to- $\kappa_Index_Inflation$ $\kappa_Debt_Infl. = National\ debt / \kappa_Index_Infl.$

JEL Classification: E010, E10, O11, C02, C01, E01, F02, C10, C50

Keywords: Econometric Modelling, Economic Indicator, Macro Models, International Economic Order, GDP, GNP, GINI, Productivity, Capital Expenditure, Private Investment, Inequality, Income Distribution, Poverty Growth, Poverty Measurement, International Industrial Order, Econometrics, Econometric Methods, , Macroeconometrics, Mathematical Methods, Mathematical Models,

2 The Problem

Questioning the measurement of economies by GDP alone is not new. Economists, from Nobel Prize winning Joseph E. Stiglitz to less well-known scientists, were internationally looking for a way to get away from the “GDP fetishism”¹, or to deal with the weaknesses² of the GDP³. These efforts went so far, that many considered replacing it with alternatives⁴ like the Gross National Happiness⁵. Graham for instance quit the attempt to define the GNH Index with an open-ended book⁶, while Stiglitz for instance attempted to create a broader version of the GDP⁷ for a more comprehensive approach⁸.

If necessary social transfer payments permanently freeze a part of the GDP, why not make this permanently visible and directly comparable in the GDP? It does relativize the factually available economic means.

How can economies be measured more realistically while keeping the complexity of the index *practical*?⁹ To solve this difficult task, which contains a target conflict in itself, only highly aggregated factors can be considered. This paper logically and coherently models the GDP with the introduction of the factors of the Gini coefficient and the factor of Inflation. For dealing with the *future aspect of the GDP*¹⁰, the factors of change in capital expenditure and in the factor of change of private investments have been introduced.

¹ The Guardian (2009): Sarkozy attacks focus on economic growth, under <http://www.theguardian.com/business/2009/sep/14/sarkozy-attacks-gdp-focus>

² Stiglitz proposes five fixes for the “deficiencies of the GDP (as an indicator for living standards)”, Stiglitz J.E./ Sen A./ Fitoussi J.-P. (2009): Report by the Commission on the Measurement of Economic Performance and Social Progress,p.22

³ Stiglitz J.E./ Sen A./ Fitoussi J.-P. (2009): Report by the Commission on the Measurement of Economic Performance and Social Progress,p.22

⁴ Beyond-GDP (2013): Indicators, Enlarged GDP, Social Indicators, under http://www.beyond-gdp.eu/indicatorList.html?indicator=Enlarged_GDP . Countless further statistical indicators try in principle the same. For instance the Human Development Index HDI (with a strong emphasis on social factors), as well as indices from the World Economic Forum.

⁵ Especially in the recent economical crisis

⁶ Mainly due to problems of measuring the multi-dimensionality of well-being, Graham C. (2011):The Pursuit of Happiness. An economy of Well-being, p.125-126. Also described in Stiglitz proposes five fixes for the “deficiencies of the GDP (as an indicator for living standards)”, Stiglitz J.E./ Sen A./ Fitoussi J.-P. (2009): Report by the Commission on the Measurement of Economic Performance and Social Progress,p.14-15 and Fleurbaey M. (2008):Individual well-being and social welfare: Notes on the theory, p.23-24

⁷ With fixing the GDP issues + Quality of Life measurement + Environment and Sustainability

⁸ Stiglitz J.E./ Sen A./ Fitoussi J.-P. (2009): Report by the Commission on the Measurement of Economic Performance and Social Progress, p.21-85

⁹ Which index has the right balance between measuring economical strength while being more comprehensive and statistically adequate than the GDP alone?

¹⁰ Which to my knowledge has not been introduced before

This leads to revised rankings of nations *based on the real factual means*, as shown in ideal-typical examples.

3 The Index and its Versions

3.1 Why is the GDP not enough?

The GDP mainly measures market production – expressed in monetary units¹¹. It says nothing about the household income, and nothing about the distribution of that income nationwide¹². It does not include unemployment or the price burden¹³ on the consumers, which can smalen disposable income significantly¹⁴.

Still the GDP is treated as if it were a measure of general economic well-being¹⁵. This leads to a perceived reality-gap between GDP Data and the experienced well-being of the people. This happens especially when focusing on growth (of the GDP). Figure 1 defines growth as *additional economic activity*¹⁶. It shows the reduced economical activities and its possible reasons as cutouts.

To get away from this “growth fetishism”¹⁷ Stiglitz argues that disposable income¹⁸ of nations and households is more suited to measure economic well-being. *“Material living standards are more closely associated with measures of net national income, real household income and consumption – production can expand while income decreases or vice versa when account is taken of depreciation, income flows into and out of a country, and differences between the prices of output and the prices of consumer products....citizens’ material living standards are better followed through measures of household income and consumption”*¹⁹. This predestines income indices for expansion of the GDP. This turns the Gini coefficient into a valid choice.

¹¹ Stiglitz J.E./ Sen A./ Fitoussi J.-P. (2009): Report by the Commission on the Measurement of Economic Performance and Social Progress,p.12-13

¹² “If inequality increases enough relative to the increase in average per Capital GDP, most people can be worse off even though average income is increasing”, Stiglitz J.E./ Sen A./ Fitoussi J.-P. (2009): Report by the Commission on the Measurement of Economic Performance and Social Progress,p.8

¹³ Inflation

¹⁴ Stiglitz J.E./ Sen A./ Fitoussi J.-P. (2009): Report by the Commission on the Measurement of Economic Performance and Social Progress,p.12-13

¹⁵ Stiglitz J.E./ Sen A./ Fitoussi J.-P. (2009): Report by the Commission on the Measurement of Economic Performance and Social Progress,p.21

¹⁶ Similar to Stiglitz’s definition of economical activity. Stiglitz J.E./ Sen A./ Fitoussi J.-P. (2009): Report by the Commission on the Measurement of Economic Performance and Social Progress,p.11

¹⁷ The Guardian (2009): Sarkozy attacks focus on economic growth, under <http://www.theguardian.com/business/2009/sep/14/sarkozy-attacks-gdp-focus>

¹⁸ Available for expenses

¹⁹ Stiglitz J.E./ Sen A./ Fitoussi J.-P. (2009): Report by the Commission on the Measurement of Economic Performance and Social Progress,p.13

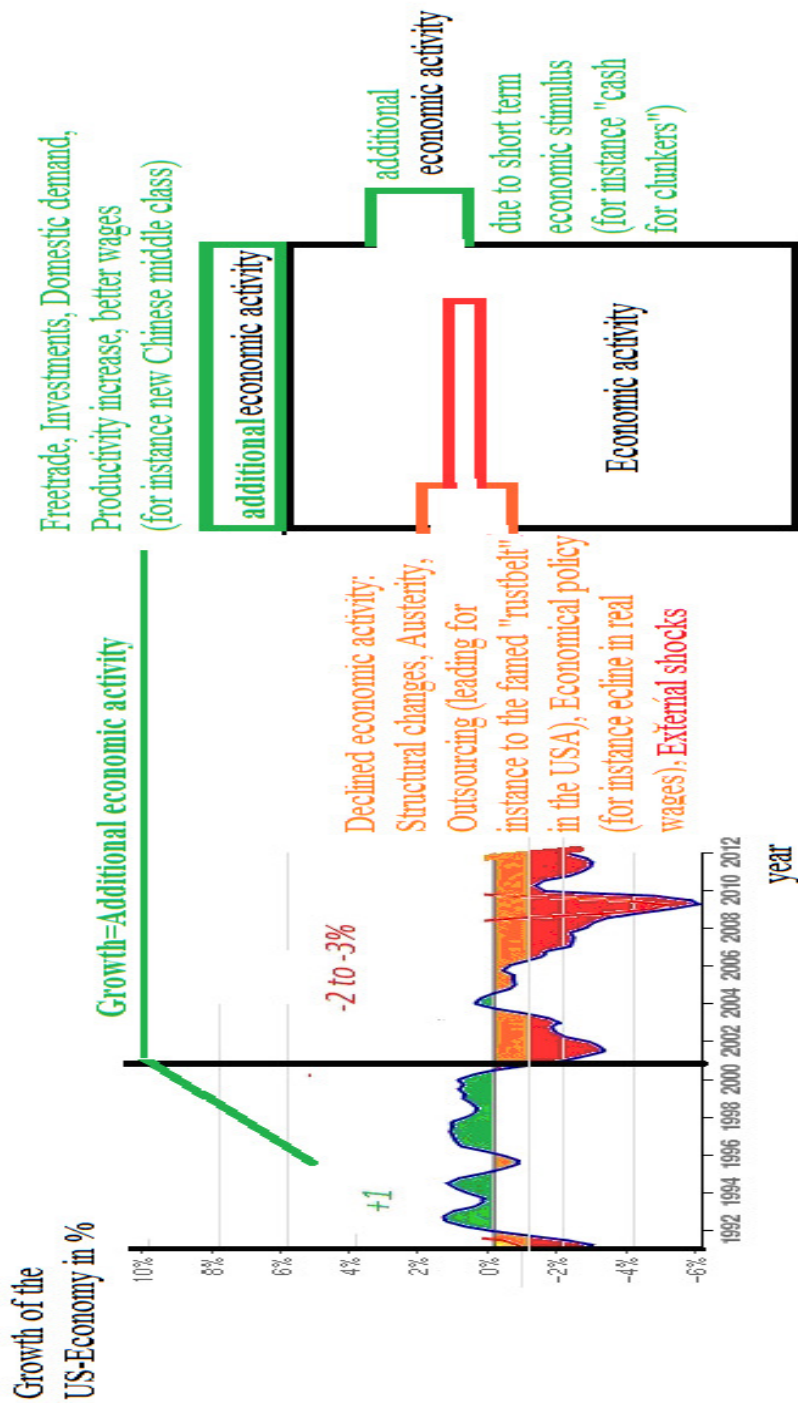


Figure 1 Growth visualized with its influencing factors²⁰

²⁰ Own creation with a table from Shadowstats.com (2012): under http://www.shadowstats.com/alternate_data/gross-domestic-product-charts

3.2 GDP - GDP × Gini

$$\kappa_Index = (1 - GINI) \times GDP$$

GINI²¹ measuring the long term cause for social transfers, which freeze a part of the GDP already.

The GINI-coefficient is a highly aggregated statistical measure for income inequality. The incomes of a Nation are put in relation with an absolutely even distribution of all incomes²². Figure two shows this graphically with the squared surface between the Lorenz curve and the 45° even.

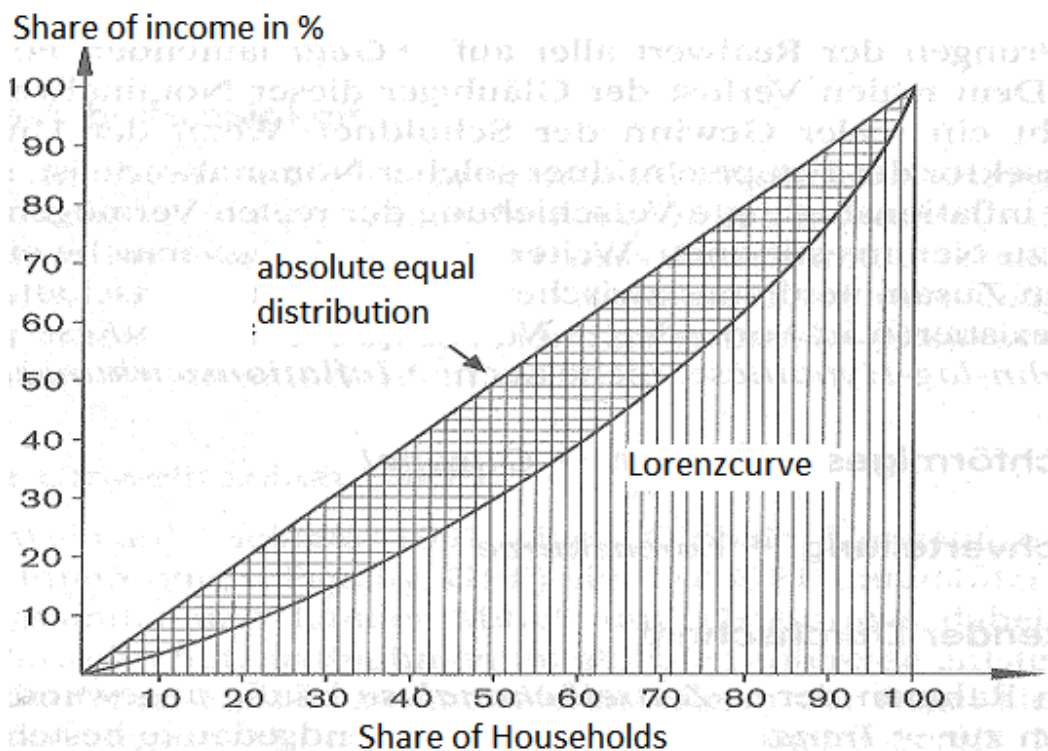


Figure 2 The GINI²³

As described in the chapter before it makes sense to expand the GDP with an income measure. However, why doing so with an inequality measure that shows²⁴ the income

²¹ Why would you want to mix this coefficient into the “holye” GDP?

²² Hohlstein, Michael (2003): Lexikon der Volkswirtschaft, p.317

²³ Hohlstein, Michael (2003): Lexikon der Volkswirtschaft, p.317

²⁴ highly aggregated

distribution of a nation? “Over the past two decades, the dominant pattern in OECD countries is one of a fairly widespread increase in income inequality, with strong rises in Finland, Norway, Sweden (from a low base) and Germany, Italy, New Zealand, and the United States (from a high base)”²⁵.

Main Axiom: Higher income inequality = higher long-term pressure to increase public social spending, especially in developed nations.

Social expenses have to occur²⁶ due to problems²⁷ caused by inequality. *The Gini coefficient measures indirectly the long-term pressures on the economy, which require social transfer payments²⁸*. Since economical policy²⁹ is often *short term* and not truly comparable in detail³⁰, the Gini coefficient is. Most countries spend about 20% - 30% of their income for public social spending³¹. *These social expenses³² are more or less³³ fixed costs in any GDP. The true GDP is therefore “de facto” relativized already downwards by this fixed-spending-factuality on the ground.* Figure 3 shows public social spending in percent of the GDP by OECD nations³⁴. The public pension expenditures have to be added to these numbers.

²⁵ Stiglitz J.E./ Sen A./ Fitoussi J.-P. (2009): Report by the Commission on the Measurement of Economic Performance and Social Progress, p.33

²⁶ At least partially

²⁷ Multiplier effects. For instance well paying jobs with positive multiplier effects, while unemployment creates negative multiplier effects (i.e. not just opportunity costs like lost taxes but external effects as well like dealing with crime)

²⁸ These may be structurally underfunded or overfunded compared to the needs of the society based on the inequality pressures, making the society (and economy) structurally more or less prone to misery (visible or covert).

²⁹ If for instance (when designing a new indicator) simply choosing the percentage of GDP paid for social matters, those countries which pay less to social transfers would keep a higher GDP. You would need an inverse ranking logic when you want to measure economic strength. This and the **short term timeframes for economical policy** makes a social expenses based index problematic.

³⁰ Democracies may have more difficulties in cutting social expenses than less democratic systems. Philosophical or historical aspects (like Calvinism in Anglo-American societies) decide as well how high these expenses should be. The living standards these payments create may be also not easily comparable- besides all the problems with statistical adequation (i.e. what you measured -and how vs. what you wanted to measure).

³¹ OECD (2013): Social Expenditure Database (SOCX), under <http://www.oecd.org/els/soc/socialspendingdatabasesocx.htm>

³² Including public pensions

³³ Depends on the grade of democratization and tradition of protest (for instance very common in France, but not very well liked in Germany)

³⁴ OECD (2013): Social Expenditure Database (SOCX), under <http://www.oecd.org/els/soc/socialspendingdatabasesocx.htm>

Income inequality can be distinguished and measured by many ways. The methods vary besides the Gini Coefficient from Theil Indices, to Pareto Distributions to Big-Mac Indices. Generally, inequality can be a massive expansion of income of a few households or persons at the very top, while keeping most household income steady or it can be a relative decrease of household wealth for the broad population. Arguably latter is economically worse to catastrophic for any economy.

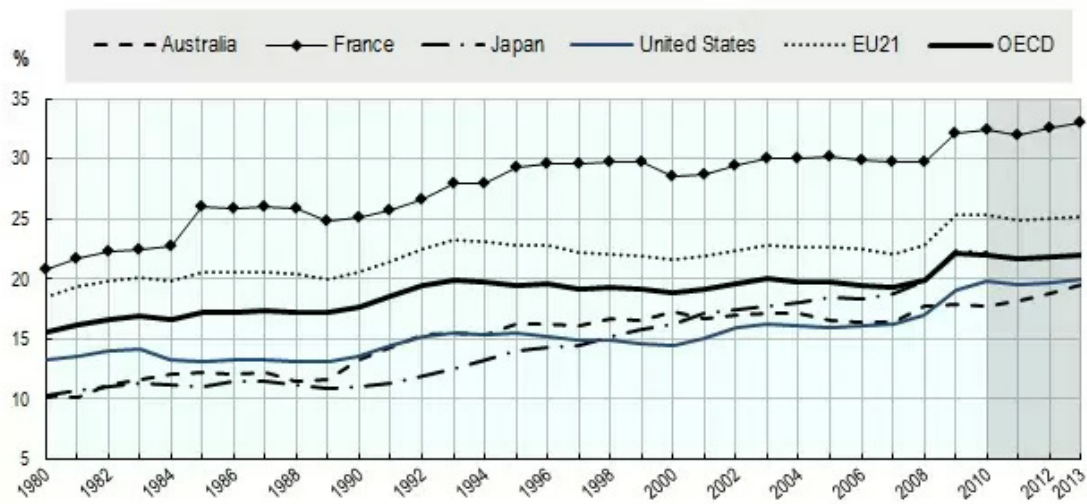


Figure 3 Public social spending based on detailed data for 1960-2009; national aggregates for 2010-2012 and estimates for 2013, in percentage of GDP³⁵

“Income flows are an important gauge for the standard of living, but in the end it is Consumption and consumption possibilities over time that matter. The time dimension brings in wealth”³⁶. The very same time dimension brings long-term necessities to fund inequality-based problems, is therefore a *negative income flow/ decreased consumption possibility* (i.e. costs wealth long term).

If necessary social transfer payments permanently use up a part of the GDP, why not make this permanently visible and directly comparable in the GDP? It does relativize the factually available economic means downwards. Countries tend to cover these expenditure pressures with additional debt, that is why it does not seem to matter. However, these debts must be paid later on, despite the remaining underlying pressures. (The details of downgrading a currency to deflate the real debts are not in this model.)

³⁵ OECD (2013): Social Expenditure Database (SOCX), under <http://www.oecd.org/els/soc/socialexpendituredatabasesocx.htm>

³⁶ Stiglitz J.E./ Sen A./ Fitoussi J.-P. (2009): Report by the Commission on the Measurement of Economic Performance and Social Progress, p.29

The κ -Index ranks with the introduction of the Gini coefficient the less developed nation *a lot* lower, and the developed nation moderately lower, depends on how well the income is distributed. Structurally underfunding the necessary public social payments will not help the States in these rankings.

This index is vaguely related to the national accounts measure “Net national disposable income (as percentage of gross domestic product)” that Stiglitz proposes³⁷, but is more practical due to the highly aggregated but internationally accepted factors.

Less developed economies tend to have a higher inequality index.

Economists tend to view those Economies as healthier, which have a population and workforce, which can afford goods and services³⁸. This is also known as domestic demand driven economy³⁹. Import- and export-balance is a further relevant factor for long-term stability, since deficit equals debt or additional taxation.

Economies that are mostly exporting raw materials have often a high GDP, while their people are poor, cannot afford goods or services, and an often corrupt elite finds ways to cut its share off the income from the exports. The country is looted, very few get very rich, but the people generally are chanceless and miserable.

When it comes to GDP, both nations are equal.

If they shouldn't be equal, because it does not represent the factual reality on the ground, how do we sort these nations scientifically, logically consistent while staying practical and relevant enough?

Ideal-typical example: What oil price does an energy exporting country⁴⁰ need, to finance its social programs? *Moreover, why not ask this question from the metaphoric “tail of the horse” over the inequality Index, with the axiom that income inequality decides more or less directly and long term⁴¹ about the amount of necessary transfers?*

“For a poor developing country to be told that its GDP has gone up may be of little relevance. It wants to know whether its citizens are better-off, and national income measures are more relevant to this question than GDP”⁴².

³⁷ Stiglitz J.E./ Sen A./ Fitoussi J.-P. (2009): Report by the Commission on the Measurement of Economic Performance and Social Progress,p.24, 30

³⁸ The choice of Stiglitz to choose disposable income is an approach that builds on this fact.

³⁹ For instance USA, OECD (2013): 1. Gross domestic product, under http://stats.oecd.org/Index.aspx?DatasetCode=SNA_TABLE1

⁴⁰ For instance Russia

⁴¹ More long term than most economic policy timescale changes at least (usually based on election cycles)

⁴² Stiglitz J.E./ Sen A./ Fitoussi J.-P. (2009): Report by the Commission on the Measurement of Economic Performance and Social Progress,p.24

For the formulas $\kappa_{Index} = GDP - GDP \times Gini$, also writeable as $\kappa_{Index} = (1 - GINI) \times GDP$ we use the following variables:

GDP = B
GINI = G
 κ_{Index} = κ

The formulas are mathematically represented as:

$$\kappa^{43} = B - B \times G$$

restated as

$$\kappa = (1-G)B$$

3.3 $GDP - [(GDP \times GINI) + (GDP \times Inflation)]$

$$\kappa_{Index_Infl.} = (1 - Gini - Inflation\ nominal) \times GDP$$

This κ_{Index} shows good governance over the social factors relativized GDP with the interdependence between Gini and Inflation.

The relativization of less healthy or underdeveloped economies⁴⁴ over income distribution can, as explained above, show a more realistic ranking of nations. However, there are further pressures on the economy that could also help rate economies clearer. One of these pressures is inflation.

“The Consumer Price Index (CPI) is a measure of the average change over time in the prices paid by urban consumers for a market basket of consumer goods and services. The CPI affects nearly all Americans because of the many ways it is used. Following are major uses: As an economic indicator, as a deflator of other economic series and as a means of adjusting dollar values”⁴⁵.

⁴³ If there is such a thing as *mathematical beauty* then this is one of them. $B - B * G$ or $(1-G)B$ despite all its depth.

⁴⁴ If the complete economy relies on exporting oil or ore, the whole nation becomes *very* dependant on pricing and vulnerable to external shocks. Reagan pressured the Soviet Union with lowering the oil price, and while the oil price was high in the 70s, the Soviets invaded Afghanistan, while once the oil price was down in the 80s, they had to retreat. The complete raw materials exports of the whole empire basically barely topped the years earning of just one of many American multi nationals (like GE) at the time.

⁴⁵ Bureau of Labor Statistics (2013): Consumer Price Index, under http://stats.bls.gov/cpi/cpifaq.htm#Question_1

Ideal typical examples (based on homo oeconomicus): It is viewed by the “tiny homo Oeconomicus” (the individual) as “everything got so expensive” or as “why is so much month left over at the end of the paycheck, it was enough earlier...” or “...what? Grandpa? Cinema for 50 cents in 1950?”. For the “large homo Oeconomicus” (the Investors, Companies) it means the investment made does not pay off anymore, the profit is eaten up by inflation, or the savings need an investment method *at least* above this inflation level. For the “supersized homo Oeconomicus” (the States) it means on one hand cheaper exports, and on the other hand more expensive imports. So it depends if you have a balanced trade budget⁴⁶, or if you are an export driven nation⁴⁷, or if you are an importer (for instance of energy), the effects mean something good or bad to you⁴⁸.

Generally, inflation is viewed as *not* good by most economical actors and economists, certainly *not in the long run*.

Including the factor of inflation in the relativization of the GDP is meant to downgrade nations that undermine the spending power/buying power of its people or gambles with its fiscal stability, while ranking those that act responsibly above the irresponsible. In any case it *is too important (on too many levels) to be simply ignored if the GDP should be more realistic. The more broken the economy, the more relevant this factor becomes*, since Inflation that is out of control is typically not a simple external shock, but is often caused by longstanding structural problems⁴⁹. Those hit hardest are not so much investors that can relocate, but the weaker social layers like elderly, which cannot⁵⁰. These persons get a share of the GDP over income re-distribution from public social payments.

High inflation relativizes the social transfer payments⁵¹ (the receiving persons get) downwards, but amplifies countless negative external effects that are caused by misery in an economy (and society), while causing the necessity for even more substantive transfer payments.

Second hypothesis: The Gini coefficient and Inflation are interdependent if the relativization of the GDP towards realism is the goal.

⁴⁶ Italy for instance

⁴⁷ Japans famed Abenomics devalued the Japanese currency Yen to make the exports cheaper and to leave a decade long stagnation over export surpluses.

⁴⁸ The mechanism of artificially downgrading the currency to make the nation and its products cheaper is important for “staying in the game” of export, attract investments etc. There are furthermore anticipation games, when all expect things to get cheaper tomorrow you don’t spend today, but when you know its more expensive tomorrow and the savings are less valuable then you’re likely to invest today.

⁴⁹ It can prevent a recovery as well

⁵⁰ Example Euro crisis: Wealthy Greeks bought the UK and German housing market empty (causing a real estate bubble “by the book”), while in Greece old people died on lack of affordability of medical care.

⁵¹ Typically a third of GDP

The relativization in the κ _Index is less grave than the relativization over the GINI. It can only serve as a pinnacle in tight rankings in cases when the countries are well governed⁵².

For the Formulas $\kappa_{Index_Infl.} = GDP - [(GDP \times GINI) + (GDP \times Inflation)]$ and $\kappa_{Index_Infl.} = (1 - Gini - Inflation\ nominal) \times GDP$ we introduce the additional variables of:

$$\begin{array}{lcl} \text{INFLATION nominal} & = & \text{In} \\ \kappa_{Index_Inflation} & = & \kappa_i \end{array}$$

The formulas are mathematically represented therefore as

$$\kappa_i = B - [(B \times G) + (B \times In)]$$

or also writeable as

$$\kappa_i = (1 - G - In)B$$

3.4 The future aspect of the GDP

The main index with introduction of the future aspect into the GDP

The GDP is arguably based on past performance data. The introduction of the Gini coefficient puts a basic measure of the pressures of social expenditures into the GDP. It has therefore a *long-term* future aspect in itself. Given the Axiom that the Gini does represent long-term pressures for social security expenditures, this represents the future aspect of the expenditures relativization side only, i.e. pressures for *future long-term expenditures*.

For the inclusion of *future income-side based pressures*, a further factor is required. The factor of Capital Expenditures⁵³ or the factor of Private Investment Rates. Chinas economy grew for instance mostly through FDI's, while current Investment rates changed somewhat, and are to a sizeable part no longer private investments any longer. Using *private Capital as measure, excludes government distortions causing investment bubbles*. These are often passive and non-profit-generating investments, like "ghost cities" in China or EU Highways into nowhere. This assumes that the factors of Capital expenditure and private investment rate are generally targeted towards generating profit, and have a lower misallocation rate of capital.

⁵² Now we have a beautiful way to measure the nations with a more realistic view of the economy. (I admit it, I just single handedly destroyed the beauty of GDP –GDP x Gini with the technical and quirky additional inflation relativization). However there are variants doable

⁵³ Economist, The (2014): Corporate spending / If they build it, from July, 5th, 2014, under http://www.economist.com/news/finance-and-economics/21606329-Capital-expenditure-around-world-has-been-disappointing-if-they-build-it?fsrc=scn/tw_ec/if_they_build_it

Investments do not have only a limited effect on the GDP, since they have a very broad effect on welfare generation. Therefore, the Investment Rates must be adapted *before* other factors in the economic model alter the GDP.

The future aspect of the GDP is the anticipated growth or shrinking effect of the Economy, that comes with the change in private investment or Capital Expenditures in the recent timeframe i.e. a past to current change of Investment will lead to shrinking or growth of the Economy in the future..

The private investments occur on an already existing level of investment. This means that *only the change of investment matters for limited prediction time frames, such as for the GDP*. The difference or variation of these rates is called delta.

It depends at which position of the Economic Model the change of these Factors (Capital Expenditures or Private Investments) is applied. This leads to two different model types. In one model the Investment Factors are applied on the GDP *before* the other varying model Factors, and in the second model, the Investment Factors are applied on the GDP *after* the varying Model Factors.

The application ex-ante is more relevant. *Investments affect the GDP on a broad scale, and not just a part of it in a relativized GDP* (as would be in the second ex-post Model). Therefore, the second model lacks of relevance in reality.

This leads to a GDP that boosts or lowers its nominal value by change of Capital Expenditures or private investments, but relativizes this with the frozen up means of Social Expenses as well as with the element Inflation. Latter is only relevant in unstable economies or bad governance economies.

The mathematical modelling of this requires the introduction of further factors:

Delta Private Investment	=	ΔP
Delta Capital Expenditures	=	ΔC
$\kappa_{\text{Index_Priave_Investment}}$	=	κp
$\kappa_{\text{Index_Capital_Expenditures}}$	=	κc
$\kappa_{\text{Index_Inflation_Private_Invest.}}$	=	κip
$\kappa_{\text{Index_Inflation_Capital_Exp.}}$	=	κic

Ex-ante Model for delta Private Investment: applied on the GDP *before* all other factors, leading to the mathematical formula stated as:

$$\kappa p = (B + B \times \Delta P) - (B \times G)$$

With inflation factor:

$$\kappa p = (B + B \times \Delta P) - [(B \times G) + (B \times In)]$$

Ex-post Model for delta Private Investment: applied on the GDP after all other factors, leading to the mathematical formula stated as:

$$\kappa p = B - [(B \times G) + (B \times \Delta P)]$$

With inflation factor:

$$\kappa p = B - [(B \times G) + (B \times In)] + \{[(B \times G) + (B \times In)] \times \Delta P\}$$

Ex-ante Model for delta Capital Expenditures: applied on the GDP before all other factors, leading to the mathematical formula stated as:

$$\kappa c = (B + B \times \Delta C) - (B \times G)$$

With inflation factor:

$$\kappa c = (B + B \times \Delta C) - [(B \times G) + (B \times In)]$$

Ex-post Model for delta Capital Expenditures: applied on the GDP after all other factors, leading to the mathematical formula stated as:

$$\kappa c = B - [(B \times G) + (B \times \Delta C)]$$

With inflation factor:

$$\kappa c = B - [(B \times G) + (B \times In)] + \{[(B \times G) + (B \times In)] \times \Delta C\}$$

3.5 GDP_per_Capita, and GNP variants

$$(1 - GINI) \times GDP_per_Capita$$

$$(1 - Gini - Inflation\ nominal) \times GDP_per_Capita$$

$$(1 - Gini - Inflation\ nominal - Unemployment\ rate\ nominal) \times GDP = K_Index_Infl._Un.$$

$$(1 - GINI) \times GNP$$

$$(1 - GINI) \times GNP_per_Capita$$

Transformation of an economical power index into an inequality measurement index.

The advantage of this formula construction is it works as a variety of Indices. GDP_per_Capita, and as a retired Professor⁵⁴ suggested, Gross National Product GNP⁵⁵ (or GNP_per_Capita). The advantage of the GNP is the removal of inflation with using real numbers. However, this modification transforms the κ _Index to an *income distribution index*, away from a *reality adjusted national economic power index*. Variants with the unemployment rate further downgrades nations that do not show responsible good governance⁵⁶.

This requires the Introduction of the Variables

GDP_per_Capita	=	Bc
Gross National Product	=	N
GNP_per_Capita	=	Nc
Inflation nominal	=	In
Unemployment rate nominal	=	Un
κ _Index_GDP_per_Capita	=	κ gc
κ _Index_Infl._GDP_per_Capita	=	κ igc
κ _Index_GNP	=	κ gn
κ _Index_GNP_per_Capita	=	κ gp
κ _Index_inflation_unempl.	=	κ iu

Mathematically the Formulas for GDP_per_Capita are described as:

$$\begin{aligned}\kappa gc &= (1-G)Bc \\ \kappa igc &= (1-G-In)Bc \\ \kappa iu &= (1-G-In-Un)G \\ \kappa n &= (1-G)N \\ \kappa nc &= (1-G)Nc\end{aligned}$$

3.6 Productivity Index (κ _Index_Infl. / Labor Force)

$$\kappa_{PROD} = \kappa_{Index} / \text{Labor Force}$$

$$\kappa_{PROD_Infl.} = \kappa_{Index_Infl.} / \text{Labor Force}$$

Theoretical value (how much) a worker creates when the GDP takes income inequality in account.

⁵⁴ Prof. Dr. Enke (retired) from University of applied Sciences Nuertingen and University Marburg, who I would love to thank for the support and encouraging me to pursue this Index further

⁵⁵ In German it's the bnE = Bruttonationaleinkommen

⁵⁶ This downgrading might go too far, besides impractical complexity since there are different levels of unemployment. (For instance frictional unemployment or the statistical values themselves, like broadness definitions U1 U2 etc that might not be comparable internationally).

Each economy is in a different state of development. The less developed an economy is, the less value the working force creates per work hour or per worker. A lower developed economy that has half of its labor force in agriculture, can barely earn a sufficient GDP, that is equal to that of a developed economy, where for instance the same workers create turbines instead of tomatoes in the same working timeframe.

When the κ _Index is applied as the basis of this input-output question, the distortions caused by “workforce assumption” are reduced. A nation that exports energy, has rarely substantial numbers of workers in that industry, but gets over the high GDP a good productivity Index. *This does not reflect realistically the situation of workforce productivity.*

Ideal typical example: Poor people watching a pipeline pumping billions of petro-dollars aside their village out of their country, until they become terrorists or sabotage it out of economical motives (to get some fuel), *is not valid productivity* (in economical sense).

Since these less developed economies tend to have a higher Gini coefficient than a balanced developed high-wage economy, picking the κ _Index (with its GINI) can benefit a more realistic assessment.

3.7 Debt ratios K_Debt or $K_Debt_Infl.$

$\kappa_Debt = National\ Debt / \kappa_Index$

$\kappa_Debt_Infl. = National\ Debt / \kappa_Index_Infl.$

Ability (of a nation) to pay its debts without cutting structurally into the social transfers.

Debt-to-GDP ratios are very common in economics. Using these κ _Index and κ _Index_Infl. values as basis for a debt-ratio makes only sense, when there is an interest in a *broader scope about the economy*. If only the *strict financial ability* of a nation to pay its debts counts, this index is less relevant. Early warning systems that analyze the development path of nations might get sooner warning signs with the κ _Debt_Infl. than with classical debt-to-GDP ratios. *This is especially the case when a nation has to cut deeper into the public social payments*⁵⁷. All values lead to technically higher debt percentages than with the classical debt-to-GDP ratios, since the κ _Index and κ _Index_Infl. generally lowers the GDP⁵⁸.

⁵⁷ These measures are perceived as a positive measure in the financial sector, which means the markets value financial stability higher than the living standards of those at the receiving end of public social transfers.

⁵⁸ Disposable income is always lower than the original income value

3.8 Digression A: Four-/ Five-Sector-Model

Identifying government overheads or social sector overheads (in the economic structure)- with or without shadow economy.

Redefining the Economical Power of Nation requires also a closer look at the structure itself. This section contains a small addition to the model, the separation of the Economic Structures into a new insight model. Analyzing the *sectoral structure* of economies shows the level of development of economies, and to a certain extent good or bad governance. The “three sector hypothesis” segments the economy into three sectors. Primary, Secondary and Tertiary Sector. Economies tend to grow towards the tertiary sector, the higher developed the economy is. Table 1 shows the definitions of these three sectors and gives a brief example about the size of each sector for a developed or undeveloped economy.

Restructuring the third sector and adding the fourth sector of Government and Nonprofit helps to identify government overheads⁵⁹. If taking demographic changes of the population in account, analyzing the economies with a separated nonprofit sector can be a useful modification of the three-sector-model⁶⁰. Including the fifth sector of shadow economy is problematic due to lack of data (quantity of data). Many problems occur also in the quality of the data due to significant statistical adequation problems when measuring these additional sectors⁶¹.

Table 2 shows the proposed sectoral structure with a brief description of each sector.

Table 1 Three sector hypothesis (common)

Sector I	Extraction of raw materials + fishing (developed Nation 1-5%/ undeveloped 50% or more)
Sector II	Industry, manufacturing, construction (developed Nation 20 to 30%/ undeveloped 20%)
Sector III	Services (developed Nation 50% or more/ undeveloped 10%)

⁵⁹ These oversizes are typical for “inflexible/ incrustrated” economies or bad governance economies. For instance was this characteristic for Argentina before its crisis in the 90s or Greece in the current Euro crisis.

⁶⁰ Analyzing the Greek economy or the German economy over this model could be interesting.

⁶¹ Stiglitz J.E./ Sen A./ Fitoussi J.-P. (2009): Report by the Commission on the Measurement of Economic Performance and Social Progress,p.26

Table 2 Modified three sector hypothesis with two additional sectors and readjusted third sector

Sector I	Extraction of raw materials + fishing
Sector II	Industry, manufacturing, construction
Sector III	Services + I.T. (with communications industry)
Sector IV	State sector ⁶² and Nonprofit (Govt. Jobs, churches, red cross etc.)
Sector V (option)	Shadow economy (problem to get correct numbers, usually 15-20%)

3.9 Digression B: Who should join a single currency system?

Expenditures pressures caused debts, which depended in value on currency values. With the united currency system like the euro, the trade benefits should outweigh the negative effects of the inability to downgrade the currency.

Governments tend to cover up the social expenses with debts. These must be paid back despite the still remaining economic pressures. Governments use many methods to still be able to pay the debts; one of these methods is currency downgrading. What happens when the country is in a currency alliance and cannot downgrade the currency? The economy suffers, that is consensus. So who should be therefore in a united currency system? A Hypothesis of mine is that those should be in a currency system whose benefit from free Trade and joint Currency are greater or equal to the negative effects of inability to downgrade the currency. This is a fluid process and not a static occasion. Sometimes Trade benefits and single currency benefits will be higher than without a united currency, but sometimes the economy will suffer and the negative effects of inability to downgrade the currency become more relevant.

When nations do not have enough free trade ongoing, to make it profitable for a nation to join a single currency system (i.e. NOT greater or equal negative effects) at year 1, it still does not mean it won't have enough trade benefits in year 10.

This means the solution for the inability to downgrade a currency problem is intensified free trade, i.e. that the trade benefits outweigh the negative effects of inability to downgrade the currency.

This leads to the formula

Trade benefits + other positive income effects and externalities of being in a single currency \geq negative effects and externalities of the inability to downgrade own currency⁶³

⁶² "Government output represents around 20% of GDP in many OECD countries and total government expenditure more than 40% for the OECD countries", Stiglitz J.E./ Sen A./ Fitoussi J.-P. (2009): Report by the Commission on the Measurement of Economic Performance and Social Progress, p.12

⁶³ for instance FDI effects on reduced currency values, opportunity costs and other externalities

Trade benefits	=	X
Positive external effects	=	Y
Positive externalities	=	EZ
Negative external effects	=	Ny
Negative external effects	=	En

$$X + Y + EZ \geq Ny + En$$

4 Ideal-Typical Examples

4.1 Example κ _Index and κ _Index_Infl.

Two ideal typical countries that seem equal. Country **A** (less developed) and Country **B** (developed) have a GDP of 2.5 Trillion Euro each.

Country A (Less developed):

Gini	0.40
Inflation	7% = 0.07 nominal
Relativization	0.47
GDP	2,500 Billion Euro
Labor Force	42 Million people

$$\begin{aligned} \kappa_Index \text{ (without Inflation): } & (1 - 0.4) \times 2,500\text{bn} = 1,500\text{bn} \\ \kappa_Index_Infl. : & (1 - 0.4 - 0.07) \times 2,500\text{bn} = \mathbf{1,325bn} \end{aligned}$$

Country B: (Higher developed)

Gini	0.28
Inflation	2% = 0.02 nominal
Relativization	0.3
BIP	2,500 Billion Euro
Labor Force	42 Million people

$$\begin{aligned} \kappa_Index \text{ (without Inflation): } & (1 - 0.28) \times 2,500\text{bn} = 1,800\text{bn} \\ \kappa_Index_Infl. : & (1 - 0.28 - 0.02) \times 2,500\text{bn} = \mathbf{1,725bn} \end{aligned}$$

Comparison of the results:

Even if both nations have a similar GDP, the picture looks very different when only two further factors are included (Gini and Inflation). The difference between κ _Index and κ _Index_Infl. is small, but will be important in tight rankings.

Country A (less developed) relativized GDP	:	1,325bn
Country B (higher developed) relativized GDP	:	1,725bn

$$\kappa p = (B + B \times \Delta P) - (B \times G)$$

With inflation factor:

$$\kappa ip = (B + B \times \Delta P) - [(B \times G) + (B \times In)]$$

The shrinking of private investment rates at 23% representing a significant shock. This is an extreme ideal-typical example that would severely harm any economy. The real worth of the economy after the shock will be:

Country A (less developed)

$$\kappa p = [2,500bn + 2,500bn \times (-0.23)] - (2,500bn \times 0.4) = \mathbf{925bn}$$

With the relatively high inflation of 7%

$$\kappa ip = [2,500bn + 2,500bn \times (-0.23)] - [(2,500bn \times 0.4) + (2,500bn \times 0.07)] = \mathbf{750Bn}$$

Country B (higher developed)

$$\kappa p = [2,500bn + 2,500bn \times (-0.23)] - (2,500bn \times 0.28) = \mathbf{1,225bn}$$

With the relatively high inflation of 2%

$$\kappa ip = [2,500bn + 2,500bn \times (-0.23)] - [(2,500bn \times 0.28) + (2,500bn \times 0.02)] = \mathbf{1,175bn}$$

With a 10 % growth of investment rates the worth of the economy will be:

Country A (less developed)

$$\kappa p = [2,500bn + 2,500bn \times 0.1] - (2,500bn \times 0.4) = \mathbf{1,750bn}$$

With the relatively high inflation of 7%

$$\kappa ip = [2,500bn + 2,500bn \times 0.1] - [(2,500bn \times 0.4) + (2,500bn \times 0.07)] = \mathbf{1,575bn}$$

Country B (higher developed)

$$\kappa p = [2,500bn + 2,500bn \times 0.1] - (2,500bn \times 0.28) = \mathbf{2,050bn}$$

With the relatively high inflation of 2%

$$\kappa ip = [2,500bn + 2,500bn \times 0.1] - [(2,500bn \times 0.28) + (2,500bn \times 0.02)] = \mathbf{2,000bn}$$

The comparison shows clearly the advantage, since both economies would be equal at 2,500bn GDP, at any inflation, any investment rates, and any inequality in the society.

With the differentiation with social expenditure pressures, a relativization is already noticeable.

(1-G)B = 1,500bn undeveloped Economy

(1-G)B= 1,800bn developed Economy

It does still lack of the important future aspect though. With Capital Investment Factor and Inflation, the undeveloped nation would be worth:

750bn in economic shock / 1,575bn in prosperity

And the highly developed nation would be worth

1,225bn in economic shock / 2000bn in prosperity.

This is a significant advantage compared to the current measurements of the economic rankings.

4.2 Example Productivity κ_{PROD} or κ_{PROD_Infl} .

$\kappa_{Index} / \text{Labor Force} = \kappa_{PROD}$

$\kappa_{Index_Infl.} / \text{Labor Force} = \kappa_{PROD_Infl.}$

Productivity county A (with inflation)

$\kappa_{PROD_Infl.} = 1,325\text{bn Euro} / 42\text{m} = \mathbf{31,547}$ Euro per Capita

Productivity country B (with inflation)

$\kappa_{PROD_Infl.} = 1,725\text{bn Euro} / 42\text{m} = \mathbf{41,071}$ Euro per Capita

Productivity country USA level (with inflation)

$\kappa_{Index_Infl.} = (1 - 0.49 - 0.032) \times 11,363\text{bn Euro} = 5,431\text{bn Euro}$

$\kappa_{PROD_Infl.} = 5,431\text{bn Euro} / 142\text{m Labor Force} = \mathbf{38,250}$ Euro per Capita

Productivity country China level (with inflation)

$\kappa_{Index_Infl.} = (1 - 0.47 - 0.054) \times 6,230\text{bn Euro} = 2,965\text{bn Euro}$

$\kappa_{PROD_Infl.} = 2,965\text{bn Euro} / 802\text{m Labor Force} = \mathbf{3,697}$ Euro per Capita

The new numbers of productivity are generally lower.

5 Criticism

5.1 Generalized Gini coefficients

The argument that there could be overlappings with the Generalized Ginis⁶⁴ or K-Theory from the Axiomatic Index-Theory⁶⁵ was critically checked. Generalized Ginis try for instance to include the multidimensionality of wellbeing measurement into aggregated re-

⁶⁴ Weymark J.A. (1981): Generalized Gini inequality indices,/ Index verified after a recommendation from a German Professor, Prof. Dr. Kuhn from TU-Chemnitz

⁶⁵ Brown, L.G./ Pedersen, G.K. (2014): Non-stable K-theory and extremally rich C*-algebras, P. 262-298 / Pennig, U.(2014): Twisted K-theory and obstructions against positive scalar curvature metrics,Journal of K-Theory,

defined Gini coefficients⁶⁶. *The models in this paper are not modified Gini coefficients.* There are no resemblances to Generalized Ginis or mathematical K-Theory despite the letter K or κ (*Kappa*) in the Formulas. The Economic models of this paper show a GDP that deducts long-term frozen up assets over the social pressures axiom and introduce a future aspect into the GDP. The S-Ginis or other variants of modernized Generalized Gini coefficients could be implemented later.

5.2 *Relevance in the reality of the Economies*

Does it matter how realistically you relativize the GDP downwards, and how adequate the new picture is, when all that matters is the de-facto nominal GDP (and therefore spending power) that has been generated? Does it matter when the Governments still can make debts and simply cover the new economic model ranking with debt spending? Therefore, does it matter when you downgrade for instance a raw materials exporting economy, when all that counts is the *nominal cash* the economy generates, i.e. *the ability to pay off its debts (including reserves in foreign currency)*?

A debt-to- κ _Index_Infl. ratio will always be worse (higher) than the classical debt-to-GDP ratio. How relevant can the Index therefore be, when used to measure *debt* ratios? A sinking κ _Index_Infl. (maybe due to worsening Gini or Inflation) just warns the economists sooner about internal problems in the economy/ society (that might or might not be addressed/quelled). *The less easily a democracy can cut into social transfers, the more sense this index makes.*

Is a relativization of 1 Trillion Euro in a 2.5 Trillion Euro economy, which has a high Gini coefficient value, or a relativization of 700 Billion Euro in a similar sized economy with a low Gini coefficient value “overkill-downgrading”? This means a relativization of 28-40% depends on whether the Gini coefficient is 0.28 or 0.40. Chapter 2.2 and 2.3 explain the rational behind the pick⁶⁷, but it might be controversial if the *height* of these transfer payments justifies the height of the GDP relativizations, even if these amounts are factually frozen up.

5.3 *The Factors chosen*

As for the Gini coefficient, a lower Gini coefficient means only a more equal income distribution, but it says nothing about the *level* of income. When all are equally poor, the Gini is low. This however would only be likely in a low GDP nation, a high GDP with a good income distribution is a very good sign in contrary. Inequality might not be a static

⁶⁶ Weymark J.A. / Gaidos T. (2005): Multidimensional generalized Gini indices, p.471-496

⁶⁷ Public social spending + pensions freeze up factually a part of the GDP already

variable. This means that Inequality caused by massive increase of income of a few, does not necessarily mean that the other households have a reduced income.

As for the productivity Index κ_{PROD} or $\kappa_{\text{PROD_Infl.}}$, a lowered Gini coefficient (inequality gets better) is *technically not a productivity increase*, the same with a lowered Inflation rate. On the other hand, the classical productivity index is also “just a theoretical value”, which includes distortions from the upper 10% of income.

A professor⁶⁸ mentioned that not all nations might see inequality as a bad thing. I agree that for instance Calvinistic societies⁶⁹ might tolerate inequality more than European economies, but the *general acceptance of the Gini coefficient* makes this worry irrelevant. The formula uses the Gini coefficient, and a high Gini coefficient value is not accepted as a *positive* factor in an economy generally. A very abstract professor⁷⁰ argued that the pick of the factors could be seen as arbitrary, even if mixing indicators is not uncommon in economical sciences. The indicator sets a very clear goal, namely increased realism or reality based adjustment of the GDP while *still staying practical enough*. This predestined only the highly aggregated factors. Chapter 2.2 and 2.3 describes with references to Stiglitz⁷¹ why the pick is not as arbitrary⁷² as it might seem.

No one can predict the future, and this is a valid criticism. Including the future aspect into the GDP over the change in the variables of Capex or Priv. Investments could be seen as anticipation. However, anticipation is common in economics over a large variety of indices, though it could be seen as blunt betting how the GDP would develop if for instance Investment Rates fall.

6 Concluding Remarks

The Social factors relativized GDP is the basis for a variety of *more realistic economical indicators*⁷³, including productivity and debt-ratios. *Cutting out that part of the GDP, which is financially long term reserved for social transfers*⁷⁴, is the basis of these models. For this purpose, the Gini coefficient is most suitable, since it tries to measure⁷⁵ the basis of the transfer payments, the income inequality. Out of practicability, reasons of the new Indices only highly aggregated factors were suitable.

⁶⁸ Prof. Dr. Hayo from University Marburg

⁶⁹ USA/ UK and Anglo-American influenced

⁷⁰ Prof. Dr. Koerber-Weik (retired) from University of applied Sciences Nuertingen

⁷¹ Stiglitz J.E./ Sen A./ Fitoussi J.-P. (2009): Report by the Commission on the Measurement of Economic Performance and Social Progress,p.29

⁷² As it might seem

⁷³ Scientific gain

⁷⁴ “financially long term frozen up part of the GDP”

⁷⁵ Statistical adequacy in mind

The Gini coefficient and the factor of Inflation are interdependent if realism adjustment of GDP is the goal, especially in relation to good governance measurement. Countries with a high-income inequality and a high inflation rate are downgraded strongly, while countries with a low Gini coefficient and a low inflation rate are relativized down only by a small margin. The new models allow sooner identification of instabilities and problems in economies, and revise economic rankings of nations.

The κ _Index or κ _Index_infl. “repairs” some of the weaknesses of the classical GDP, while relying on established and commonly acknowledged factors and methods of measurement. The κ _Index or κ _Index_Infl. is axiomatic, but it cannot be questioned by its coherent logical construction, only by its relevance when debt making covers rankings of economies and when used in debt-ratios (where only the *nominal* financial credit generated counts, with or without possible foreign currency reserves).

The comparisons in the ideal-typical examples of chapter 4 show clearly the advantage of the new model. Two economies with different basic traits have the same level of GDP, and would be perceived as equal by current GDP modelling (at the level of 2,500bn GDP). With the differentiation just over the social expenditure pressures, a significant relativization is already noticeable.

(1-G)B = 1,500bn undeveloped Economy

(1-G)B= 1,800bn developed Economy

With the introduction of the future aspects over the change of the factors of Capital Investments or Private Investments (with or without Inflation) the less developed nation would be worth (instead 2,500bn):

750bn in economic shock / 1,575bn in prosperity, and the highly developed nation would be worth 1,225bn in economic shock / 2000bn in prosperity.

Both nations would be equal without this model, and the negative or positive externalities following in the near term (due to investment rates) would not be present. This marks a significant advantage over the current measurements of the economic rankings.

One of the two digression models suggest that intensified free trade is the solution for the problems, which occur when a nation cannot downgrade its currency (for instance having joined a single currency system like the EURO). The second digression model helps identifying Government overheads, a source for multiple crises in the past and in the future (like the Argentina crisis of the recent decades).

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