

# Health Care Quality vs Health Care Quantity: A General Equilibrium Analysis

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## Health Care Quality vs Health Care Quantity: A General Equilibrium Analysis

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**Abstract:** This paper attempts to relate the issues of health care quality with international trade. For this purpose we have mixed both flavours of Heckscher-Ohlin-Samuelson and Neo- Heckscher-Ohlin frameworks and developed a hybrid type of trade theoretic general equilibrium model. In such a set up we have shown that a movement from a regime of international health capital immobility to a regime of international health capital mobility may lead to an expansion of the health quality exporting sector. Apart from quality aspect of health services, the quantity aspect of health care has been also considered in this study. Moreover, from that hybrid model we have illustrated that the sizes of health care and composite export sector expand, where as import sector of our small open economy contracts.

**Key words:** Health Care quality, International trade, International health capital mobility and General equilibrium.

JELClassification:I10,I15,F11,F21

### Health Care Quality vs Health Care Quantity: A General Equilibrium Analysis

#### 1. Introduction

Trade in health services have put on the shelves an increasingly importance and interest among policymakers and economists those who are engaged with health trade related issues. According to General Agreement on Trade in Services (GATS) there are four modes through which trade in health services will occur. In this paper we are more emphasizing on mode 2 and mode 3 among these four modes<sup>1</sup>. Consumption of health services abroad implies a movement of consumers to the country that supplying high quality health services for treatment. Mode 2 gets more importance due to the fact that exports of health services (from the producers of South) with high quality will be always preferred by the importers (consumers of North) of these products. For instance patients of north countries (USA, UK etc,) and south nations (Bangladesh, Nepal, Bhutan etc,) come to India for surgery, neurology, cardiology etc. There is enough evidence that foreign patients come here to get traditional treatments like Ayurbedic and Unani also. Now a question may arise that why patients of north prefer south for their treatment? Though it has been historically observed that south has always produced low quality products. Then low price will be the only reason for such type of high demand for exports of health services. But low price is a good indicator of bad quality products and patients of north will not compromise with their health. So there exists a straight forward contradiction between the evidences and the above arguments. The only way out from that contradiction is south will supply better quality health services along with relatively lower price compared to the northern counterpart.

<sup>&</sup>lt;sup>1</sup> These are namely, cross border delivery of trade, consumption of health services abroad, commercial presence and movement of health personnel. Consumption of health services abroad implies a movement of consumers to the country that supplying high quality health services for treatment.

However, mode 3, that is, commercial presence in health sector<sup>2</sup> may improvise the pattern of trade in health services through mode 2 with the help of foreign capital mobility. Hence historically observed facts (production with low quality) will be eliminated due to trade liberalization and south will produce and export high quality health services with the implicit effects of foreign capital mobility through skill formation, technology transfer and it will persist with their spillover effects<sup>3</sup>.

Though there exists quite a few empirical works related to health care and FDI<sup>4</sup>, even at the theoretical level there are few papers which relate health care quality with international trade<sup>5</sup> but there exists almost no work that relates trade in health service quality with mode 3 of health trade in a general equilibrium trade model. In this paper we are trying to fill up this lacuna of trade literature. In this study we are also trying to show that how quality of health care in a small open economy affects rest of the economy (export (other that health sector) and import sectors of the small open economy). As we are focusing to integrate health quality exporting sector along with rest of the economy, model based on general equilibrium structure will provide more informative as well as generalized results compare to partial equilibrium analysis.

<sup>&</sup>lt;sup>2</sup> Feedback Ventures expects private equity funds to invest at least US\$ 1 billion during 2009-2013. 12 percent of the US\$ 77 million venture capital investments in July-September 2009 were in the healthcare sector. GE plans to invest over US\$ 3 billion on R&D, US\$ 2 billion to drive healthcare information technology and health in rural and underserved areas, US\$ 1 billion in partnerships, content and services, over the next six years. International clinic chain Asklepios International plans to invest US\$ 100 -200 million in the Indian healthcare market. Gulf-based group Dr Moopen is planning to invest US\$ 200 million for setting up hospitals and eye-care centres across India. Fortis is planning to invest US\$ 55 million to expand its pan-India operations. In the recent decade the medical devices and equipments industry has been successful attracting foreign direct investment too though this sector is importing 50%-60% till now. From merely US\$2.3 million in 2000 it reached US\$ 147.69 million in 2009. Some of big foreign firms in the sector invested in India either directly or through collaborations and joint ventures. Some to mention are GE (USA), Isoft (Australia), Proton Healthcare (USA) and Seimens (Germany) etc.

<sup>&</sup>lt;sup>3</sup> Though, we have not considered these facts (skill formation, technology transfer and spillover effects, etc.) explicitly in our analysis, rather we have tried to show the composite effect of theses factors in terms of foreign health capital mobility.

<sup>&</sup>lt;sup>4</sup> Interested readers may look at Deaton (2003), Herzer and Nunnenkamp (2012), Stevens, Urbach and Wills (2013) and Outreville (2007) etc,.

<sup>&</sup>lt;sup>5</sup> For details see Alonso and O'Donnel (2001) and Acharyya and Alonso (2008) etc.

#### 2. The Model

To capture the above mention problem of quality exporting health sector of a small open economy we develop a general equilibrium trade model. We assume a Small open economy and it consists of three sectors in a Heckscher-Ohlin-Samuelson (HOS) framework. Here we also consider that all the three goods that have produced in our small open economy are traded goods. One is an exportable composite good (X<sub>1</sub>) (other than health) that has been produced by sector A with unskilled labour (F<sub>1</sub>) and capital (F<sub>2</sub>). One is an import competing good (X<sub>2</sub>) that are produced by sector 2 with skilled labour (F<sub>3</sub>) and capital (F<sub>2</sub>) and finally the third is the health product (X<sub>3</sub>) (produced by the health sector H). It is to be noted that the Health sector is a quality –differentiated service producing sector and hence the quality ( $\Omega$ ) of health services can be indexed in a closed interval ranging from zero to one. Markets are competitive, technology is neoclassical and resources are fully employed. Note that there is no open unemployment as workers cannot survive without jobs and hence both the unskilled and skilled labour markets always clear.

We use following notations to describe the set of equations of our model.

 $P_1^* =$  world price of commodity 1;  $P_1$  = domestic price of commodity 1, we assume  $P_1 = P_1^* = 1$ ;  $P_2^* = P_2$  = world price of good 2;  $F_{4d}$  = domestic health capital stock of the economy;  $F_{4f}$  = foreign health capital stock of the economy;  $F_{2f}$  = foreign capital stock;  $F_{2d}$  = domestic capital stock;  $a_{ji}$  = quantity of the jth factor for producing one unit of output in the ith sector,  $j=F_1,F_2,F_3$ ,  $F_4$  and i=1,2,3;  $\theta_{ji}$  = distributive share of the jth input in the ith sector;  $\lambda_{ji}$  = proportion of the jth factor used in the production of the ith sector;  $R_1$  = competitive unskilled wage rate;  $R_3$  = competitive skilled wage rate;  $R_2$  = rate of return to capital;  $R_4$  = rate of return to health capital;  $\sigma_i$  = elasticity of factor substitution in sector i, i = 1, 2, 3.

The competitive price equations are:

$$a_{F11} (R_1, R_2) R_1 + a_{F21} (R_1, R_2) R_2 = 1$$
(1)

 $a_{F32} (R_3, R_2) R_3 + a_{F22} (R_3, R_2) R_2 = P_2$ (2)

$$a_{F33}(\Omega) R_3 + a_{F43}R_4 = P_3(\Omega)$$
(3)

Here, we assume $a_{F43}$ is fixed.	
Full-employment conditions are:	
$a_{F43}X_3 = F_{4d} + F_{4f} = F_4$	(4)
$a_{F21}(R_1,R_2) X_1 + a_{F22}(R_3,R_2) X_2 = F_{2d} + F_{2f}$	(5)
$a_{F11}(R_1,R_2)X_1 = F_1 g(X_3)$	(6)
$a_{F32}(R_3,R_2) X_2 + a_{F33}(\Omega) X_3 = F_3 h(\Omega)$	(7)

#### 2.1 International Health Capital Immobility

We consider the above mention small open economy where we assume international health capital is immobile<sup>6</sup>. In such a set up we have eight endogenous variables with seven independent equations. Thus the system can not be solved. To complete the working of that model we have to consider the marginal condition of health quality – differentiated service producing sector along with following axioms.

**Axiom 1:** Input-output skilled labour coefficient of health quality exporting sector and price of health product are increasing function of differentiated levels of quality, i.e.,  $\partial a_{F33} / \partial \Omega > 0$  and

 $\partial P_{_H} \, / \, \partial \Omega > 0 \,. \, Here, \, \, a_{_{F33}} = a_{_{F33}}(R_{_3}, R_{_4}, \Omega) \,, \, where, \, \, a_{_{F33}}^1 = a_{_{F33}}^2 = 0 \,, \, \, a_{_{F33}}^3 > 0 \, and \, \, \Omega \in (0, 1) \,.$ 

**Axiom 2:** In a small open economy where quality of health care services lies between zero and one, i.e.,  $\Omega \in (0,1)$ ,  $a_{F33}^1 = a_{F33}^2 = a_{F43}^1 = a_{F43}^2 = a_{F43}^3 = 0$  and  $a_{F33}^3 > 0$ ,  $P_H^1 > 0$ , changes in  $a_{F33}^3$  due to changes in quality of health care multiplied by the per unit return of skilled labour will dominates over the changes in  $P_H^1$  due to changes in  $\Omega$ , i.e.,  $R_3 \partial^2 a_{F33} / \partial \Omega^2 > \partial^2 P_H / \partial \Omega^2 > 0$ .

Using Axiom 1 and Axiom 2

$$P_{3}(\Omega) = R_{3} a_{F33}(\Omega)$$
 (8)

Where,  $P_{1_3}(\Omega) > 0$ .

<sup>&</sup>lt;sup>6</sup> International health capital immobility is a situation where domestic rate of return on foreign health capital ( $R_4$ ) is greater than the rate of return on foreign health capital in the international market ( $R_4^*$ ) and there is restriction on the entry of foreign health capital to the domestic economy.

The working of the model is as follows. There are eight endogenous variables in the system:  $R_1$ ,  $R_2$ ,  $R_4$ ,  $R_3$ ,  $X_1$ ,  $X_2$ ,  $X_3$ , and  $\Omega$ . From equations (1) and (2) we can express  $R_1$  and  $R_2$  as a function of  $R_3$ . Again from equation (3) we can express  $R_4$  in terms of  $R_3$  and  $\Omega$ . To complete the working of this model we have to consider following lemmas.

**Lemma 1:** An increase in the level of quality of heath services,  $\Omega$ , has a negative effect on the rate of return of skilled labour (R<sub>3</sub>) if competitive equilibrium conditions along with the equality of marginal condition of health quality exporting sector have been satisfied and  $\hat{F}_{4f} = 0$ .

*Proof of Lemma 1:* Simply by differentiation of equation (8) we can derive (see Appendix A for detail derivation)

$$\frac{d\Omega}{dR_3}\Big|_{H-\Omega} = a_{F33}^{33} / \{P_H^{11}(\Omega) - R_3 a_{F33}^{33}(\Omega)\} < 0$$

Let us start with a rise in  $\Omega$ . From equation (8) we can say that a rise in  $\Omega$  implies rise in both P/<sub>3</sub>( $\Omega$ ) and a<sub>F33</sub>/( $\Omega$ ). Using Axiom 2 from the profit maximizing condition of health sector one can argue that R<sub>3</sub> must go down for maintaining the equality between marginal conditions. Thus the locus of different combinations of  $\Omega$  and R<sub>3</sub> will be negatively sloped and profit maximizing condition of health sector will be maintained along this locus. It is called by H- $\Omega$  locus.

**Lemma 2:** An increase in the level of quality of heath services,  $\Omega$ , has a positive effect on the rate of return of skilled labour (R<sub>3</sub>) if full employment condition of skilled labour market has been hold and  $\hat{F}_{4f} = 0$ .

*Proof of Lemma 2:* Simply by differentiation of equation (7) we can illustrate (see Appendix A for details)

$$\frac{d\Omega}{dR_3}\Big|_{S-\Omega} = \left[(\phi_1 - \lambda_{F32}\sigma_2)/\{\varepsilon_h - \psi(\Omega)\}\right](R_3/\Omega)$$

Similarly, we can get another schedule of  $\Omega$  and  $R_3$  for which skilled labour market will be in equilibrium. It is called by S- $\Omega$  schedule and it is positively sloped. Intuition behind positively sloped S- $\Omega$  locus is given below. From equations (1) and (2) we can say that a rise in  $R_3$  implies an increase  $R_1$  and a reduction in  $R_2$ . An increase in  $R_1$  and a fall in R<sub>2</sub> will lead to reduction in a<sub>F11</sub>, a<sub>F32</sub> and rise in a<sub>F21</sub> and a<sub>F22</sub>. Thus from equation (6) we can find an increase in X<sub>1</sub> due to fall in a<sub>F11</sub>. Using above arguments, from (5) we can show that X<sub>2</sub> will go down. So, fall in both of a<sub>F32</sub> and X<sub>2</sub> will lead to a reduction in left hand side of equation (7). Hence an increase in  $\Omega$  becomes necessary for maintaining the full employment condition of skilled labour market, if an increase in a<sub>F33</sub> ( $\Omega$ ) X<sub>3</sub> due to an increase in R<sub>3</sub> dominates over an increase in F<sub>3</sub>h( $\Omega$ ).

**Corollary 1:** In case of autarky rate of return of health quality augmented skilled labour and health quality will be positively if  $\psi(\Omega) > \varepsilon_h$  and negatively related if  $P_H^{11}(\Omega) > R_3 a_{F33}^{33}(\Omega) > 0$ .



The intersection of H- $\Omega$  and S- $\Omega$  locus gives us the equilibrium values of R<sub>3</sub> and  $\Omega$ . Once R<sub>3</sub> and  $\Omega$  are known, R<sub>1</sub>, R<sub>2</sub>, R<sub>4</sub>, X<sub>1</sub> and X<sub>2</sub> are also known. It is to be noted that X<sub>3</sub> can be determined from equation (4) as a<sub>F43</sub> and F<sub>4</sub> are given.

#### 2.2 International Health Capital Mobility

So far we assume the case of international health capital immobility, where we have  $R_4 > R_4^*$ , where  $R_4^*$  is the given return on foreign health capital in the international market.

In such a situation we have no foreign health capital inflow. If R<sub>4</sub> falls to  $\tilde{R}_4$ , where, R<sub>4</sub>>  $\tilde{R}_4$  > R<sub>4</sub>\*, we find that there is some amount of inflow of foreign health capital (F<sub>4</sub>) and at last we will reach at the equilibrium level<sup>7</sup> of F<sub>4</sub> where, R<sub>4</sub> = R<sub>4</sub>\*.

Here, we assume that  $F_{4d}$  is exogenous whereas  $F_{4f}$  is assumed to be an endogenous variable and we use  $R_4 = R_4^*$  in our basic model. By using equations (1) and (2) we can express  $R_1$  and  $R_2$  in terms of  $R_3$ . Using  $R_4 = R_4^*$  in equation (3) we can express Ws as a function of  $\Omega$ . Similarly from equation (8) we can also express  $R_3$  in terms of  $\Omega$ . Thus the values of  $R_3$  and  $\Omega$  can be determined from equations (3) and (8) simultaneously. Once  $R_3$  is known implies  $R_1$  and  $R_2$  are also known. Since all factor prices are determined,  $a_{ij}$ s are calculated from CRS assumption. Hence  $X_2$ ,  $X_2$ ,  $X_3$  and  $F_4$  are solved from (4-7). This completes the working of the model with international health capital mobility.

#### 2.2.A Quality of Health Services and Mobility of International Health Capital

An increase in  $F_{4f}$  implies a fall in  $R_4$ . From equation (3) we can say that a fall in  $R_4$  implies an increase in  $R_3$ , for given  $\Omega$ . Again from equation (8) one can find that for given  $\Omega$  an increase in  $R_3$  implies rightward shift of H- $\Omega$  locus. It is to be noted that the shift in H- $\Omega$  locus is mainly due to the effect of fall in  $R_4$  and we refer it as factor price effect.

<sup>&</sup>lt;sup>7</sup> At  $R_4=R_4^*$ , we have the equilibrium level of foreign health capital inflow due to equilibrium in the international health capital market



**Corollary 2:** *Statement of Corollary 1 is still valid in the presence of international health capital mobility.* 

**Proposition 1**: A shift from international health capital immobility regime to an international health capital mobility regime leads to under some reasonable conditions: i) increase in the levels of both health quality and health quantity of the health quality exporting sector.

**Proof of proposition 1**: An increase in  $F_{4f}$  implies a fall in  $R_4$ . From equation (3) we can say that a fall in  $R_4$  implies an increase in  $R_3$ , for given  $\Omega$ . From equations (1) and (2) we can say that a rise in  $R_3$  implies an increase  $R_1$  and a reduction in  $R_2$ . An increase in  $R_1$  and a fall in  $R_2$  will lead to reduction in  $a_{F11}$ ,  $a_{F32}$  and rise in  $a_{F21}$  and  $a_{F22}$ . Thus from equation (6) we can find an increase in  $X_1$  due to fall in  $a_{F11}$ . Using above arguments, from (5) we can show that  $X_2$  will go down. So, fall in both of  $a_{F32}$  and  $X_2$  will lead to a reduction in left hand side of equation (7). Hence an increase in  $\Omega$  becomes necessary for maintaining the full employment condition of skilled labour market. But the increase in  $\Omega$  at equation (7) should be in such a way that an increase in  $a_{F33}(\Omega) X_3$  due to an increase in  $\Omega$  dominates over an increase in  $F_{3h}(\Omega)$ . Again from equation (4) we can easily prove that an increase in  $F_4$  implies a rise in  $X_3$ , for given  $a_{F43}$ .

**Corollary 3:** Trade in health services in the form of international health capital mobility leads to an expansion of composite export sector and contraction of import sector of the small open economy.

#### **Concluding Remarks**

The present study explored the impact of trade in health services of mode 3, that is, trade through international health capital mobility on the quality as well as quantity aspects of a health care. To capture such types of issue we have developed a three sector general equilibrium trade model that mixes both flavors of Neo-Heckscher-Ohlin and Heckscher-Ohlin-Samuelson (HOS) framework. In such framework we have shown that the quality exporting health service sector will expand duo to finite change in foreign health capital. Moreover from this study we have also captured the directions of movement of output levels of all the three sectors. For instance here we have found that all the sectors except import sector have moved towards their desirable directions.

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## Appendix

Differentiation of equation (8)

$$\begin{aligned} a_{F33}^{33} dR_3 &= \{ P_H^{11}(\Omega) - R_3 a_{F33}^{33}(\Omega) \} d\Omega \\ \frac{\hat{\Omega}}{\hat{R}_3} \bigg|_{H-\Omega} &= R_3 \; a_{F33}^{33} / \Omega \{ P_H^{11}(\Omega) - R_3 a_{F33}^{33}(\Omega) \} \\ (A.8) \end{aligned}$$

Differentiating equation (2) we get

$$\hat{R}_{2} = -(\theta_{F32} / \theta_{F22})\hat{R}_{3}$$
(9)

Similarly from equation (1) one obtain

$$\hat{R}_{1} = -(\theta_{F21} / \theta_{F11})\hat{R}_{2}$$
(10)

Equation (7) gives us

$$\lambda_{F32}\hat{X}_{2} + \lambda_{F32}\hat{a}_{F32} + (a'_{F33}(\Omega)X_{3} / F_{3})\hat{\Omega} + \lambda_{F33}\hat{X}_{3} = \varepsilon_{h}\hat{\Omega}$$
$$\lambda_{F32}\hat{X}_{2} + \lambda_{F32}\hat{a}_{F32} + \lambda_{F33}\hat{X}_{3} = [\varepsilon_{h} - \psi(\Omega)]\hat{\Omega}$$
(11)

Where,  $\psi(\Omega) = (a'_{F33}(\Omega)X_3 / F_3)$  and  $\varepsilon_h = (\partial h / \partial \Omega)(\Omega / h)$ .

Differentiation of equation (5) gives us

$$\lambda_{F22} \hat{X}_{2} = \lambda_{F21} \hat{a}_{F21} - \lambda_{F22} \hat{a}_{F22} - \lambda_{F21} \hat{X}_{1}$$
(5.1)

Differentiation of equation (6) gives us

$$\lambda_{F11} \hat{a}_{F11} + \lambda_{F11} \hat{X}_1 = \varepsilon_g \hat{X}_3$$

$$\lambda_{F11} \hat{a}_{F11} + \lambda_{F11} \hat{X}_1 = \varepsilon_g \hat{X}_3$$
Here,  $\varepsilon_g = (\partial g / \partial \Omega) (\Omega / g)$ .
(6.A)

We know,  $\sigma_1 = (\hat{a}_{F21} - \hat{a}_{F11} / \hat{R}_1 - \hat{R}_2)$ 

$$\hat{a}_{F11} = \hat{a}_{F21} - \sigma_1 \left( \hat{R}_1 - \hat{R}_2 \right) \tag{A.10}$$

Using envelop condition we get

$$\hat{a}_{F21} = -(\theta_{F11} / \theta_{F21})\hat{a}_{F11}$$

Inserting the value of  $\hat{a}_{F21}$  in the above equation

$$\hat{a}_{F11} = -\sigma_1 (\theta_{F11} / \theta_{F22}) \hat{R}_3 \tag{A.11}$$

Using it in equation (6.A) we get

$$\hat{X}_{1} = \varepsilon_{g} \hat{X}_{3} + \sigma_{1} (\theta_{F11} / \theta_{F22}) \hat{R}_{3}$$
(a.12)

Similarly, using (A.11) in (A.10)

$$\hat{a}_{F21} = \sigma_1 (\theta_{F11}^2 / \theta_{F21} \theta_{F22}) \hat{R}_3$$
(a.13)

Differentiation of equation (2) and envelop condition gives us

$$\hat{a}_{F22} = \sigma_2 (\theta_{F32} / \theta_{F22}) \hat{R}_3 \tag{a.14}$$

Using (a.12), (a.13) and (a.14) in (5.1)

$$\hat{X}_{2} = -(1/\lambda_{F22})[\sigma_{1}\lambda_{F21}(\theta_{F11}^{2}/\theta_{F21}\theta_{F22}) + \sigma_{2}\lambda_{F22}(\theta_{F32}/\theta_{F22}) + \sigma_{1}\lambda_{F21}(\theta_{F11}/\theta_{F22})]\hat{R}_{3} - [\lambda_{F21}\varepsilon_{g}/\lambda_{F22}]\hat{X}_{3}$$
Using (9), (10) and inserting the value of  $\hat{a}_{F22}$  in the expression of  $\sigma_{2}$ 

$$\hat{a}_{F32} = -\sigma_2 \hat{R}_3$$
 (a.15)

Putting the values of  $\hat{X}_2$  and  $\hat{a}_{F32}$  in equation (11)

$$\frac{\hat{\Omega}}{\hat{R}_3}\Big|_{S-\Omega} = (\phi_1 - \lambda_{F32}\sigma_2) / \{\varepsilon_h - \psi(\Omega)\}$$
(a.16)

Where,  $\phi_1 = -(1/\lambda_{F22})[\sigma_1\lambda_{F21}(\theta_{F11}^2/\theta_{F21}\theta_{F22}) + \sigma_2\lambda_{F22}(\theta_{F32}/\theta_{F22}) + \sigma_1\lambda_{F21}(\theta_{F11}/\theta_{F22})]$ 

$$d\Omega/dR_3 = [(\phi_1 - \lambda_{F32}\sigma_2)/\{\varepsilon_h - \psi(\Omega)\}](R_3/\Omega)$$