

## Taxable and Tax-Free Equivalence of Interest Rate Yields: A Brief Note

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1. February 2010

Online at http://mpra.ub.uni-muenchen.de/55263/ MPRA Paper No. 55263, posted 14. April 2014 14:55 UTC

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In converting the yield on tax-free municipals to an equivalent yield on a comparable taxable bond, most textbooks (Ceccchetti, 2006, pp. 159-160; Mishkin, 2010, pp. 128-129; Ross, Westerfield, and Jordan, 2010, p. 208) adopt either of the following two formulations:

$$Rtx_k = Rtf_i/(1-mftr_j)$$
(1)

or

$$Rtfi = Rtx_k X (1-mftr_j)$$
<sup>(2)</sup>

Where:

 $Rtx_k$  = the nominal annualized taxable interest rate yield (as a %) on bond k;  $Rtf_i$  = the nominal annualized tax-free interest rate yield (as a %) on bond i; and  $mftr_j$  = the relevant marginal *federal* income tax rate (as a %) for economic agent j.

The formulations in equations (1) and (2) are a reasonable general guide by which to express either a taxable interest rate yield in terms of a tax-free equivalent yield or a tax-free municipal interest rate yield in term of a taxable equivalent yield.

Naturally, if one is a legal resident of a state (state *m*) that has an income tax on bond interest and endeavors to compare a tax-free yield in state *m* to the relevant taxable yield, the outcome might appear to be (for i = m) either (3) or (4):

$$Rtx_k = Rtf_m / (1 - mftr_j - mstr_{mj})$$
(3)

or

$$Rtf_m = Rtx_k X (1-mftr_j - mstr_{mj})$$
(4)

Where:

 $mstr_{mj}$  = the relevant marginal *state* income tax rate (as a %) for economic agent *j* legally residing in state *m*.

This is the case of the "dual exempt" tax-free municipal, as it is usually represented. The problem with specifications (3) and (4) is the neglect of federal income deductibility of state income taxes, i.e., on Form A of Schedule 1040 of the federal individual personal income tax.

To reflect this tax deductibility, equations (3) and (4) must be rewritten as (5) and (6), respectively:

$$Rtx_k = Rtf_m / [1 - mftr_j - (1 - mftr_j) mstr_{mj}]$$
(5)

 $Rtf_m = Rtx_k X [1-mftr_j - (1-mftr_j) mstr_{mj}]$ (6) Consider an example. Assume that the relevant marginal federal income tax rate is 40%, that the relevant marginal state income tax rate (in state *m*) is 10%, and that the municipal bond interest rate yield is 5%.

According to the formulation in (3), we would have the following:

$$Rtx_k = 5\%/(1-.4-.1) = 5\%/0.5 = 10\%$$
(7)

However, allowing for the federal income tax deductibility of the state income tax levied on bond interest in state m yields a lower taxable interest rate yield equivalence for the 5% tax-free yield, as follows:

$$Rtx_{k} = 5\%/[1 - .4 - (.6 x .1)] = 5\%/[1 - .44] = 9.26\%$$
(8)

Thus, properly allowing for federal tax deductibility of state income taxation of taxable bonds reduces the taxable equivalent yield somewhat since that very deductibility partially offsets the advantages of the tax-free municipal.

In closing, it is clear that similar adjustments would be needed for accurate conversion of "triple exempt" tax-free municipal yields to equivalent taxable yields.

## References

Cecchetti, Stephen M. 2006. *Money, Banking and Financial Markets*. New York: McGraw-Hill/Irwin.

Mishkin, Frederic S. 2010. *The Economics of Money, Banking & Financial Markets*. New York: Addison-Wesley.

Ross, Stephen A., Westerfield, Randolph W. and Jordan, Bradford D. 2010. *Fundamentals of Corporate Finance*. New York: McGraw-Hill/Irwin.

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