Is Harrod-neutrality Needed for Balanced Growth? Uzawa's Theorem Revisited

Defu Li and Jiuli Huang and Ying Zhou

School of Economics and management, Tongji University, TEDA College, Nankai University, Department of Economics, Auckland University of Technology

2014

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Defu Li

School of Economics and management, Tongji University

Jiuli Huang

TEDA College, Nankai University

Ying Zhou

Department of Economics, Auckland University of Technology

Acknowledgments We thank Benjamin Bental, Charles I. Jones, Dean Scrimgeour, Ekkehart Schlicht and seminar participants at Wuhan University for helpful comments. Li gratefully acknowledges support from the Social Science Foundation of China (Grant No.: 10CJL012), Philosophy Social Science Planning of Shanghai (GrantNo.: 2008BJL003) and Liberal Arts Research Fund of Tongji University. Huang acknowledges the financial support from National Social Science Foundation (13BJL050) and Program for New Century Excellent Talents in University of Ministry of Education of China. Zhou thanks Auckland University of Technology for research support and Xinsheng Lu for stimulating discussions.

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Abstract: Taking into account the adjustment costs of investment, this paper proves that it is not the neoclassical growth model itself but the specific form of capital accumulation function that requires technical change to exclusively be Harrod neutral in steady state. Uzawa's(1961)steady-state growth theorem holds only when *the marginal efficiency of capital accumulation* is constant, which implies that the capital supply is infinitely elastic. Therefore, it is unnecessary to make strong assumptions about the shape of the production function and the direction of technical change for neoclassical growth model to exhibit steady-state growth.

Keywords: Neoclassical Growth Model; Uzawa's Steady-state Growth Theorem; Direction of Technical Change;Adjustment Cost

JEL Classifications: E13, O33, O40

1 Introduction

Uzawa's (1961) steady-state growth theorem (Uzawa theorem, hereafter) says that for a neoclassical growth model to exhibit steady-state growth, either the production function must be Cobb-Douglas or technical change must be Harrod neutral in the long run.Because of this theorem, much of macroeconomics—and an even larger fraction of the growth literature—makes strong assumptions about the shape of the production function and the direction of technical change (see Jones, 2005). However, are these assumptions really necessary? Considering the fact that technical change can also be Hicks neutral and Solow neutral in reality, it seems that there are no compelling reasons for us to think so. We try to argue in this paper that the Uzawa theorem can be only derived from specific prerequisites (which are not pointed out clearly in existing literature) and will not hold under general circumstances.

Over the last decades, researchers have delved into Uzawa theorem by either providing more simplified proofs (see Barro and Sala-i-Martin,2004, chapter 1; Schlicht, 2006; Acemoglu, 2009,chapter 2) or seeking for more satisfactory justifications(see Fellner, 1961; Kennedy, 1964; Samuelson, 1965; Drandakis and Phelps, 1966; Acemoglu, 2003; Jones, 2005; Jones and Scrimgeour, 2008).However, these endeavors do not clarify the prerequisites of the theorem. Specifically, it is not clear whether it is the neoclassical growth model itself or its particular assumptions that compel technical change to be Harrod neutral in the steady-state equilibrium.

In his newly-published book, Acemoglu(2009, Chapter 2) specifies the critical condition of the Uzawa theorem, but fails to highlight how special these conditions are. Based on the work of Schlicht (2006), Acemoglu proves that balanced growth rates of capital and output result immediately from the assumed capital accumulation process, $\dot{K} = Y - C - \delta K$,[1] and directly lead to the derivation of Uzawa theorem. This is to say that the assumed capital accumulation process is a necessary condition for the Uzawa theorem. However, this commonly-used capital accumulation process ignores adjustment costs that are typically associated with the replacement for worn-out equipment, the installation of new machines, the cost of learning, and sometimes the cost related to the purchase of machines from capital goods producers(Eisner and Strotz, 1963;Lucas, 1967;Foley and Sidrauski, 1970;Mussa, 1977;Bailey and Scarth, 1980, 1983). When such adjustment costs are taken into account, it takes more than one unit of the final product to get one additional unit of capital. While Abel and Blanchard (1983) have developed a neoclassical growth model with adjustment costs, they have not considered the implications for the direction of balanced-growth technical change. Our findings are related to Sato's conclusions (see Sato, 1996, 1999, 2000) which were ignored by Acemoglu (2003), Jones (2005) and Jones and Scrimgeour (2008). However, unlike Sato, we obtain our

[1]The variables of the equation have the standard definition and will be specifically defined in the second section.

conclusions by considering the adjustment costs of investment. Furthermore, we prove that the constant marginal efficiency of capital accumulation implies that the capital supply is infinitely elastic and provide a clear revision of the Uzawa theorem.

In this paper, we prove that frequently-cited Uzawa theorem does not hold in more general cases, and specify its prerequisites explicitly. We consider the steady-state equilibrium of a neoclassical growth model with several specific functions of adjustment cost. These examples show that the Uzawa theorem holds only when the marginal efficiency of capital accumulation is constant. We believe that this requirement is unrealistic as it implies that the capital supply is infinitely elastic. Thus the Uzawa theorem should be revised in light of this condition. By clarifiying the special requirements associated with the Uzawa theorem it becomes more compatible with our economic intuition and relaxes the conditions under which a neoclassical growth model can exhibit balanced growth.

The rest of the paper is organized as follows. Section 2 examines the steady-state equilibrium of a neoclassical growth model with adjustment costs; Section 3 presents several examples of adjustment cost functions and analyzes their specific requirements; Section 4 specifies the prerequisite of the Uzawa theorem and its economic implications, and Section 5 concludes.

2 A Neoclassical Growth Model with Adjustment Costs

2.1 Formulation of the Model

Consider a representative consumer in the economy with the usual constant relative risk aversion (CRRA) preferences. The lifetime utility of the representative consumer can be expressed as

$$\int_{t=0}^{\infty} \frac{C(t)^{1-\theta}}{1-\theta} e^{-\rho t} dt, \tag{1}$$

where $C(t)$ is the consumption at the period t, θ is the coefficient of relative risk aversion, and ρ is the rate of time preferences.

The production function satisfies the standard neoclassical properties,[2] and allows for both capital-augmenting and labor-augmenting technologies. That is,

$$Y(t) = F[B(t)K(t), A(t)L(t)], \tag{2}$$

[2]That is, constant returns to scale (CRS), positive but diminishing marginal products, Inada conditions, and essentiality of each input (Barro and Sala-i-Martin, 2004, chapter 1).

where $Y(t), K(t), L(t)$ denote output, capital stock and labor at the time t, $B(t)$ and $A(t)$ refer to the capital-augmenting and labor-augmenting technologies. Thus, the interaction terms $B(t)K(t)$ and $A(t)L(t)$ represent, respectively, the effective capital and effective labor at time t. Further, the initial endowment of technology and labor is no less than one, i.e. $A(0), B(0), L(0) \geq 1$.In addition, the growth rates of labor L and both technologies are given exogenously, that is,$\dot{A}(t)/A(t) = a \geq 0$,$\dot{B}(t)/B(t) = b \geq 0$, and $\dot{L}(t)/L(t) = n \geq 0$.

The budget constraint of the representative consumer is given by

$$Y(t) = C(t) + I(t), \text{where } C(t), I(t) > 0. \quad (3)$$

The investment function $I(t)$ has two parts, including the purchase of new capital goods $I_k(t)$ and the additional adjustment cost incurred $h[I_k(t), Z]$,

$$I(t) = I_k(t) + h[I_k(t), Z], \quad (4)$$

where $h[0, Z] = 0, \partial h/\partial I_k > 0, \partial^2 h/\partial I_K^2 \geq 0$. Z represents factors that affect adjustment cost other than $I_k(t)$.It may include the capital stock K, factor-augmenting technology A or B,or other factors.

The net increase in the stock of capital at a point in time t is the difference between the amount of investment $I_k(t)$ (rather than $I(t)$) and the depreciation $\delta K(t)$. To be more accurate, our capital accumulation function can be formulated as follows:

$$\dot{K}(t) = I_k(t) - \delta K(t), \quad (5)$$

where $K(0) > 0, \delta \geq 0$, and $I_k(t) > 0$.

By equation (4), the investment $I(t)$ is surely a monotonically increasing function of $I_k(t)$ as $\partial I(t)/\partial I_k(t) = 1 + \partial h/\partial I_k(t) \geq 1$ .Solving for the inverse function of equation (4) yields:

$$I_k(t) = G[I(t), Z] \leq I(t), \quad (6)$$

where $G[I(t), Z]$ is the efficiency function of capital accumulation,which reflects the degree to which investment is converted to new capital goods.

By simply inserting formula (6) into (5), we obtain the capital accumulation equation with investment adjustment costs:

$$\dot{K}(t) = G[I(t), Z] - \delta K(t). \quad (7)$$

It is evident from equations (6) and (7) that $\dot{K}(t) = G[I(t), Z] - \delta K(t) \leq I(t) - \delta K(t)$, which shows that the speed of capital accumulation depends not only on the level of investment $I(t)$, but also on the conversion efficiency from investment to capital. By the property of the inverse function, we obtain the following relations:

$$\begin{cases} G_I \equiv \dfrac{\partial G}{\partial I(t)} = \dfrac{1}{\partial I(t)/\partial I_k(t)} = \dfrac{1}{1+\partial h/\partial I_k(t)} > 0 \\ G_{II} \equiv \dfrac{\partial^2 G}{\partial I(t)^2} = \dfrac{\partial[1+\partial h/\partial I_k(t)]^{-1}}{\partial I(t)} = -\dfrac{\partial^2 h/\partial I_k(t)^2}{[1+\partial h/\partial I_k(t)]^3} \leq 0 \end{cases}, \quad (8)$$

where G_I and G_{II} refer to the *marginal efficiency of capital accumulation* and its first-order derivative respectively. Equation group (8) shows that the *marginal efficiency of capital accumulation* diminishes with additional investment incurring adjustment costs.

2.2 Steady-state Equilibrium

We can analyze this optimization problem by setting up the Hamiltonian

$$H(C,K,\lambda) = \frac{C(t)^{1-\theta}}{1-\theta}e^{-\rho t} + \lambda(t)\{G[Y(t)-C(t),Z] - \delta K(t)\}. \quad (9)$$

$\lambda(t)$ is a costate variable. The usual transversality condition is expressed as:

$$\lim_{t\to\infty}\lambda(t)K(t) = 0. \quad (10)$$

The first-order conditions thus are:

$$\begin{cases} \dfrac{\partial H}{\partial C} = C^{-\theta}e^{-\rho t} - \lambda G_I = 0 \\ \dot{\lambda} = -\dfrac{\partial H}{\partial K} = -\lambda\left(G_I\dfrac{\partial Y}{\partial K} + G_Z\dfrac{\partial Z}{\partial K} - \delta\right) \end{cases}. \quad (11)$$

After some mathematical manipulation of the first-order conditions, we obtain the Euler equation:

$$\theta\frac{\dot{C}}{C} = G_I\frac{\partial Y}{\partial K} + G_Z\frac{\partial Z}{\partial K} - \frac{\dot{G}_I}{G_I} - \rho - \delta. \quad (12)$$

Substituting $\partial Y/\partial K = B[\partial Y/\partial(BK)]$ derived from the production function (2) into equation (12), we can further arrive at the following necessary condition for consumers to achieve dynamic optimality:

$$\theta\frac{\dot{C}}{C} = G_I B\frac{\partial Y}{\partial(BK)} + G_Z\frac{\partial Z}{\partial K} - \frac{\dot{G}_I}{G_I} - \rho - \delta. \quad (13)$$

Let k be the ratio of effective capital to effective labor (i.e. $k \equiv BK/AL$), the intensive form of the production function can be rewritten as $f(k) = F(BK/AL, 1)$. This implies that the marginal product of effective capital is $f'(k) = \partial Y/\partial(BK)$.

Define $c \equiv C/AL$ as the consumption per effective labor. After using equations (7) and (13), we get:

$$\begin{cases} \frac{\dot{k}}{k} = b + \frac{G[I,Z]}{K} - \delta - a - n \\ \frac{\dot{c}}{c} = \frac{1}{\theta}\left[G_I B f'(k) + G_Z \frac{\partial Z}{\partial K} - \frac{\dot{G}_I}{G_I} - \rho - \delta\right] - a - n \end{cases}. \tag{14}$$

Suppose that at some point t_0 $\dot{c}(t)/c(t) = 0$ and $\dot{k}(t)/k(t) = 0$, which corresponds to the steady-state equilibrium path. Then we have:

$$\begin{cases} \frac{G[I,Z]}{K} = a + n + \delta - b \\ G_I B f'(k) + G_Z \frac{\partial Z}{\partial K} - \frac{\dot{G}_I}{G_I} = \rho + \delta + \theta(a+n) \end{cases}. \tag{15}$$

Let $G_I(t)B(t) = G_I(t_0)B(t_0)\exp\left\{\int_{t_0}^{t}\left[\dot{G}_I(\tau)/G_I(\tau) + \dot{B}(\tau)/B(\tau)\right]d\tau\right\}$. Using this in equation (15), we obtain:

$$f'(k^*) = \frac{\rho + \delta + \theta(a+n) + \dot{G}_I/G_I - G_Z\frac{\partial Z}{\partial K}}{G_I(t_0)B(t_0)\exp\left\{\int_{t_0}^{t}\left[\dot{G}_I(\tau)/G_I(\tau) + \dot{B}(\tau)/B(\tau)\right]d\tau\right\}}. \tag{16}$$

Since by assumption $\dot{k}(t)/k(t) = 0$ the left-hand side of equation (16) is a positive constant. Since the right-hand side of equation (16) must be a constant too, it requires $G_Z\frac{\partial Z}{\partial K}$ being a constant and,

$$\dot{G}_I/G_I = -\dot{B}/B \tag{17}$$

When all the conditions (i.e. $G_Z\frac{\partial Z}{\partial K}$ being a constant and $\dot{G}_I/G_I = -\dot{B}/B$) are satisfied the ratio of effective capital to effective labor k is a constant. Then we have

$$f'(k^*) = \left[\rho + \delta + \theta(a+n) - b - G_Z\frac{\partial Z}{\partial K}\right]/G_I(t_0)B(t_0). \tag{18}$$

By equation (7), we obtain the steady-state growth rate of capital

$$\dot{K}^*/K^* = G(I,Z)/K - \delta = a + n - b. \tag{19}$$

Similarly, by equations (2) and (3) and $c = C/AL$, we obtain the steady-state growth rate of the other three endogenous variables as follows:

$$\dot{Y}^*/Y^* = \dot{I}^*/I^* = \dot{C}^*/C^* = a + n. \tag{20}$$

So, it is evident that the neoclassical growth model exhibits steady-state growth path when these conditions (i.e. $\dot{G}_I/G_I = -\dot{B}/B$ and $G_Z\frac{\partial Z}{\partial K}$ being a constant) are satisfied. This is independent of either Harrod-neutral technical change or the Cobb-Douglas production function.

Further, if we take the first-order Taylor expansionof equations (14) around the steady-state (c^*, k^*), we get

$$\begin{pmatrix} \frac{\dot{k}(t)}{k(t)} \\ \frac{\dot{c}(t)}{c(t)} \end{pmatrix} \approx \begin{pmatrix} \frac{\partial[\dot{k}(t)/k(t)]}{\partial k}_{|k=k*,\, c=c*}, & -\frac{G_I}{k^*} \\ \frac{1}{\theta}G_I(t_0)B(t_0)f''(k^*), & 0 \end{pmatrix} \binom{k}{c}. \tag{21}$$

With the coefficient determinant being clearly negative:

$$\det\begin{bmatrix} \frac{\partial[\dot{k}(t)/k(t)]}{\partial k}_{|k=k*,\, c=c*}, & -\frac{G_I}{k^*} \\ \frac{1}{\theta}G_I(t_0)B(t_0)f''(k^*), & 0 \end{bmatrix} = \frac{1}{\theta}G_I(t_0)B(t_0)f''(k^*)\frac{G_I}{k^*} < 0. \tag{22}$$

As can be seen from (22), steady-state growth of this model actually implies the stable saddle path when $\dot{G}_I/G_I = -\dot{B}/B$ and $G_Z\frac{\partial Z}{\partial K}$ is a constant.

3 Examples of Adjustment Cost Function

In this section, we try to examine what conditions on technical change would yield a steady-state equilibrium in the neoclassical growth model under some specific functions of adjustment cost.

Example 1: Adjustment cost is given by

$$h[I_k(t), K] = I_k(t)\emptyset[I_k(t)/K(t)],\ \emptyset[0]=0, \emptyset' > 0, \emptyset'' \geq 0. \tag{23}$$

This adjustment cost function is commonly usually used in the existing literature (see Abel and Blanchard,1983; Barro and Sala-i-Martin,2004,chapter3). Total investment is then

$$I(t) = I_k(t) + I_k(t)\emptyset\left[\frac{I_k(t)}{K(t)}\right]. \tag{24}$$

Dividing both sides by K we can obtain：

$$\frac{I(t)}{K(t)} = \frac{I_k(t)}{K(t)} + \frac{I_k(t)}{K(t)}\emptyset\left[\frac{I_k(t)}{K(t)}\right] \tag{25}$$

Therefore, as formula (25) indicates that $\frac{I(t)}{K(t)}$ is monotonically increasing in $\frac{I_k(t)}{K(t)}$, its inverse function is well-defined. Let that inverse function be $\omega(.)$, where $\omega' > 0, \omega'' \le 0$. By construction we obtain $I_k(t) = K(t)\omega\left[\frac{I(t)}{K(t)}\right]$. Inserting it into the capital accumulation function (7), we can further get

$$\dot{K}(t) = K(t)\omega\left[\frac{I(t)}{K(t)}\right] - \delta K(t). \tag{26}$$

Equation (26) indicates $G[(I(t), K(t)] = K(t)\omega\left[\frac{I(t)}{K(t)}\right]$. From it we can obtain $G_I = \omega'$, $\frac{\dot{G}_I}{G_I} = \frac{\omega''}{\omega'}\left[\frac{\dot{I}}{I} - \frac{\dot{K}}{K}\right]\frac{I}{K}$, $G_Z\frac{\partial Z}{\partial K} = G_K = \omega - \omega'\frac{I(t)}{K(t)}$. According to the result of section 2,G_K must be constant in steady-state equilibrium, namely $\frac{dG_K}{dt} = -\omega''\left[\frac{\dot{I}}{I} - \frac{\dot{K}}{K}\right]\left(\frac{I}{K}\right)^2 = 0$. Since $\frac{I}{K} > 0$, it must be the case that $\omega''\left[\frac{\dot{I}}{I} - \frac{\dot{K}}{K}\right] = 0$, yielding also $\frac{\dot{G}_I}{G_I} = 0$. The condition $\dot{G}_I/G_I = -\dot{B}/B$ implies that $\dot{B}/B = 0$. Therefore, for steady-state equilibrium under the above adjustment cost to exist, technical change must be Harrod neutral.

Example 2: Adjustment cost is given by

$$h[I_k(t), Z(t)] = \emptyset[I_K(t)] = I_K(t)\left[I_K(t)^{(1-\beta)/\beta} - 1\right], 0 < \beta \le 1. \tag{27}$$

This function is a special case of the adjustment cost function used in Acemoglu(2009,chapter7).Inserting it in the investment function (4) yields $I_k(t) = I(t)^{\beta}$. Combining it with the capital accumulation function, we obtain:

$$\dot{K}(t) = I(t)^{\beta} - \delta K(t). \tag{28}$$

Obviously, when $\beta = 1$, there are no adjustment costs and equation (28) is just the usual capital accumulation function applied in the discussions of Uzawa's theorem (see Acemoglu, 2003; Barro and Sala-i-Martin,2004, chapter 1;Jones, 2005; Schlicht, 2006; Jones and Scrimgeour, 2008).Since $G_I = 1$ and $\dot{G}_I/G_I = 0$, steady-state equilibrium requires $\dot{B}/B = 0$, ie, technical change must be Harrod neutral.

However, when $0 < \beta < 1$, equation (28) indicates that the marginal efficiency of capital accumulation is diminishing in investment. From equation (28) we get $G_I = \beta I(t)^{\beta-1}$, $\frac{\dot{G}_I}{G_I} = (\beta - 1)\frac{\dot{I}}{I}$, $G_Z\frac{\partial Z}{\partial K} = 0$. Therefore, when $0 < \beta < 1$, capital-augmenting technical change is possible along the steady-state path, with the rate of technical change satisfying $\frac{\dot{B}}{B} = (1-\beta)\frac{\dot{I}}{I}$.Since at the steady-state $\frac{\dot{I}}{I} = n + a$

as given by equation (20), we obtain $\frac{\dot{B}}{B} = (1-\beta)(n+a)$.[3] Thus, β can take any positive value less than one and technical change need not be Harrod neutral for a neoclassical growth model to possess a steady-state equilibrium.

Example 3: Adjustment cost is given by

$$h[I_k(t), B(t)] = I_K(t)[B(t) - 1]. \qquad (29)$$

According to this adjustment cost function, we can rewrite the capital accumulation function as:[4]

$$\dot{K}(t) = I(t)/B(t) - \delta K(t) \qquad (30)$$

The above equation implies $G[(I(t), B(t)] = \frac{I(t)}{B(t)}$, from which we can obtain $G_I = \frac{1}{B(t)}$, $\frac{\dot{G}_I}{G_I} = -\frac{\dot{B}}{B} = -b \leq 0$, $G_Z \frac{\partial Z}{\partial K} = G_B \frac{\partial B}{\partial K} = 0$. Thus for any rate of capital-augmenting technological progress $b \geq 0$ there exists a steady-state equilibrium. The steady-state ratio of effective capital to effective labor, k^*, is determined by the equation $f'(k^*) = [\rho + \delta + \theta(a+n) - b]$, the growth rate of capital is $\frac{\dot{K}^*}{K^*} = a + n - b$,the growth rates of output, investment and consumption are $\frac{\dot{Y}^*}{Y^*} = \frac{\dot{I}^*}{I^*} = \frac{\dot{C}^*}{C^*} = a + n$. Since the rates of labor- and capital-augmenting technical change *a* and *b* can both be greater than zero, there is no need for technical change to be necessarily Harold neutral.

4 Prerequisite for Uzawa theorem and its economic implications

From the above we have seen that the steady-state equilibrium of the neoclassical growth model with adjustment costs requires $\dot{G}_I/G_I = -\dot{B}/B$. Therefore, when and

[3] This conclusion may be also obtained by a method similar to Schlicht's (2006).

[4] Note that B(t) is capital-augmenting technology rather than investment-embodied technology. These two types of technological improvement differ in that the productivity of capital increases steadily for capital-augmenting technology but decreases steadily for investment-embodied one.

only when $\dot{G}_I/G_I = 0$, (that is, the marginal efficiency of capital accumulation is constant)the existence of a steady-state equilibrium requires that the rate of capital-augmenting technical change be zero, namely$\dot{B}/B = 0$ (that is, technical change must be Harrod-neutral). However, some examples of adjustment cost functions presented above have shown that $\dot{G}_I/G_I$ may not be equal to zero. Therefore, an amended version of the Uzawa theorem is as follows:

Under constant marginal efficiency of capital accumulation technical change must be Harrod neutral for a steady-state equilibrium of the neoclassical growth model to exist.

This formulation makes it clear that the Uzawa theorem, typically viewed as a surprising and troublesome result (see Jones and Scrimgeour, 2008; Acemoglu,2009,chapter 2),holds only under special circumstances. The revised theorem not only indicates that steady-state equilibrium may exist for other directions of technical change, but also allows us to put forward intuitive explanations more conveniently. Specifically, under our formulation one needs to answer why the existence of steady-state equilibrium requires that technical change be Harrod neutral only for the case in which the marginal efficiency of capital accumulation is constant.

We argue here that the result is due to the fact that the constant marginal efficiency of capital accumulation implies that capital supply is infinitely elastic in steady-state. Namely, if $\dot{G}_I/G_I = 0$, then $\varepsilon_K = \frac{\dot{K}/K}{\dot{r}/r} = \infty$.

Proof: Let r denote the price of capital, α denote the output elasticity of capital.By the neoclassical production function we have $r = \alpha Y/K$ in a competitive market. Since α is constant in steady-state equilibrium, we get:

$$\dot{r}/r = \dot{Y}/Y - \dot{K}/K. \qquad (31)$$

Substitute equation (31) into $\varepsilon_K = \frac{\dot{K}/K}{\dot{r}/r}$, we obtain:

$$\varepsilon_K = \frac{\dot{K}/K}{\dot{Y}/Y - \dot{K}/K} \qquad (32)$$

From equation (7) we get $\dot{K}/K - \delta = G(I, Z)/K$. When the growth rate of capital is constant, it must be the case that:

$$\frac{\dot{G}}{G} = \frac{G_I I}{G}\frac{\dot{I}}{I} + \frac{G_Z Z}{G}\frac{\dot{Z}}{Z} = \frac{\dot{K}}{K} \qquad (33)$$

Using again the fact that all growth rates are constant and $\frac{G_I I}{G} > 0, \frac{\dot{I}}{I} > 0$, $\frac{G_I I}{G}$must be a constant along the steady-state path. Therefore we get

$$\frac{\dot{G}_I}{G_I} + \frac{\dot{I}}{I} - \frac{\dot{G}}{G} = 0. \tag{34}$$

Thus, under the maintained assumption that $\frac{\dot{G}_I}{G_I} = 0$, together with equation (20) implying that $\frac{\dot{I}}{I} = \frac{\dot{Y}}{Y}$ in steady-state, equations (33) and (34) yield $\frac{\dot{Y}}{Y} = \frac{\dot{K}}{K}$ thereby implying $\varepsilon_K = \infty$.[5]

QED .

Since the constant marginal efficiency of capital accumulation implies that capital supply is infinitely elastic, the revised version of the Uzawa theorem can be also expressed as:

If capital supply is perfectly elastic with respect to the interest rate then the steady-state equilibrium of the neoclassical growth model requires technical change to be Harrod neutral.

5 Conclusions

The frequently-cited Uzawa theorem says that for a neoclassical growth model to possess a steady-state growth path, either the production function must be Cobb-Douglas or technical change must be purely labor-augmenting in the steady-state equilibrium. However, by taking into account the adjustment costs of investment and using several specific functions of adjustment cost, this paper proves that this requirement is necessary only when the marginal efficiency of capital accumulation is constant, which implies that the capital supply is perfectly elastic.That is, the puzzling requirement that technical change must be Harrod neutral along a steady-state equilibrium path does not derive from the neoclassical growth model itself but from the special assumption about the shape of the capital accumulation function. Our revised version of the Uzawa theorem clarifies this issue and removes a misunderstanding that has affected growth theory for quite a long time.

[5]Similarly, Schlicht's (2006) crucial step in his proof of Uzawa's theorem is to get the equation $\dot{Y}/Y = \dot{K}/K$. However, equations (34) and (20) show that this condition is obtained only when $\dot{G}_I/G_I = 0$. If $\dot{G}_I/G_I \neq 0$, then $\dot{Y}/Y \neq \dot{K}/K$, and the production function cannot be expressed as Y=F(K,AL) and Schlicht's method can only prove the revised version of Uzawa's theorem.

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