Central Banks' Asset Purchase Programs, Asset Distributions, and Endogenous Market Segmentation

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Abstract

This paper investigates the effects of open-market operations on the distributions of assets and prices. It offers a theoretical framework to incorporate multiple asset holdings in a tractable heterogeneous-agent model. This model features competitive search, which produces distributions of money and bond holdings as well as price dispersion among submarkets. At a high enough bond supply, the equilibrium shows segmentation in the asset market; only households with good income shocks participate in the bond market. Segmentation in the asset market is generated endogenously without assuming any rigidities or frictions in the asset market. Numerical exercises show that when the asset market is segmented, the central bank can improve welfare by purchasing bonds and supplying money.

JEL Classification Numbers: E0, E4, E5

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1 Introduction

What are the long-run effects of open-market operations on the distribution of assets and prices in the economy? Why do some people participate in the market for interest-bearing assets, while others do not participate in the asset market and only hold money? In order to answer these questions, I construct a model of a monetary economy with heterogeneous agents, in which the central bank implements policies by changing the supply of nominal bonds and money. I show that a segmented asset market arises under a specific parameters set; households with high income participate in the bond market and hold positive portfolios of bond and money, while low income households do not participate in the bond market and only keep money for transaction purposes. When deciding whether to participate in the asset market, households compare liquidity services provided by money with returns on bond. In an equilibrium with a segmented asset market, open-market operations affect the participation decisions of the households and, therefore, have real effects on the distribution of assets and prices in the economy.

The distribution of asset holding and segmentation in the asset market is well documented. Some agents participate in the market for interest bearing assets and hold positive portfolios of different assets, while others do not participate in the asset market.[1] Explaining these facts requires a heterogeneous agent model in which households choose to hold different portfolios of asset holdings. In this paper, households have different preferences towards labor supply, and they experience different matching shocks. Heterogeneity in preferences towards labor supply and idiosyncratic matching shocks allows me to generate an equilibrium distribution of asset holding among different households and a distribution of prices among different markets.

A branch of literature uses asset market segmentation to explain persistence responses to monetary shocks observed in the data[2]. In these models with segmented asset markets, only the fraction of agents who are active in the asset markets immediately receive the monetary shocks. As a result, it would take time for the monetary shocks to affect other agents in the economy. This literature explains the real effects of money injection and open-market operations using the generated segmentation in the asset market. These models use two ways to generate the segmented asset market: limited participation models that assume

[1] In 2009, 7.7% of the surveyed U.S. households did not have access to banking products and services, and at least 71% of these unbanked U.S. households earned less than $30,000 in a year. Also, 26.5% of U.S. households did not have any savings in a bank account; moreover, they did not hold any financial assets similar to a bank account. Source: FDIC (2009) and The Panel Study of Income Dynamics (PSID). According to the Survey of Consumer Finances (2010), 92.5% of households had access to transaction accounts and 12% held saving bonds.

[2] For an overview of this literature see Edmond and Weill (2008)

only certain agents attend the asset market and models that assume agents must pay a fixed cost to enter the asset market or to transfer assets between the asset market and the goods market. In a cash-in-advance framework, Grossman and Weiss (1983) assume that only a fixed fraction of the population can withdraw funds from banks each period.[3] In Alvarez et al. (2000), agents must pay a fixed cost to transfer money between the asset market and the goods market. In a similar fashion, Khan and Thomas (2010) assume agents pay idiosyncratic fixed costs to transfer wealth between interest-bearing assets and money. Chiu (2007) assumes that agents pay a fixed cost to attend the asset market, and they choose the timing of money transfers. In a micro-founded monetary framework, Williamson (2008) links the the asset market segmentation to the goods market segmentation.

In this paper, I generate segmentation in the asset market without assuming any rigidities and frictions in the asset market. All of the agents can participate in the asset market every period, and there is no transaction cost or any other frictions that prohibit agents from trading in the asset market. Segmentation in the asset market is generated endogenously. When deciding whether to participate in the asset market, households compare liquidity services provided by money with return on bond. Agents hold different amounts of assets, and some agents choose to hold no bond in their asset portfolio. In a segmented asset market, the return on bond is not high enough to attract all of the households to the asset market. Here, the real and welfare effects of open-market operations and money injections are not caused by the assumed segmentation in the asset market. However, open-market operations have real effects on the participation decisions of agents in the asset market, and as a result, on the distributions of assets and prices when the markets are segmented. In a segmented asset market, agents at the participation margin of trading assets may change their decision with a marginal change in the bond supply. Numerical exercises show that the central bank can improve welfare by purchasing bonds and supplying money. The policy of open-market purchase of bond is most effective when the asset market is segmented. This policy would increase the participation rate in the asset market and help households smooth consumption. By participating in the asset market, households are able to better insure themselves against idiosyncratic income shocks. Moreover, in a segmented asset market, open-market purchase of bond decreases both the intensive and extensive margins of trade in the decentralized market. The results are robust to exogenous segmentation in the asset market.

Shi (2008) and Williamson (2012) assume that assets other than money provide partial liquidity services. In these models, open-market operations change the overall liquidity in the economy. Because of partial liquidity, government bonds are not perfect substitutes for money and a Modigliani-Miller argument does not hold. The same logic is applied in this

[3] Similarly Alvarez et al. (2001), assumes only a fixed fraction of agents attend the asset market.

paper. Agents can only trade with money, and government bond is completely illiquid in the market for goods. Government bond is an imperfect substitute for money, thus open-market operations can have real effects on the economy. Here, households with good shocks use nominal bond to smooth their consumption over time, and the illiquidity of bond is important for this purpose. In a model with liquid bonds, households are indifferent between holding bond and money, since bond and money are perfectly substitutable. They cannot use bonds to smooth consumption. Kocherlakota (2003) uses a similar argument and shows that in a centralized market, agents use illiquid bonds to smooth consumption intertemporally.

This paper is related to the literature on the distribution of money and assets in the economy. In a search model of monetary economy with bargaining, after each round of trading there would be agents that have been matched and have succeeded in trade and agents that have not traded. This would generate an evolving distribution of asset holdings among agents, which is a state variable and makes the model intractable. Camera and Corbae (1999) generate distribution of asset holdings among agents and price dispersion in equilibrium in a framework based on Kiyotaki and Wright (1989).[4] The evolving distribution of asset holdings makes their model highly intractable for policy analysis. A large section of the monetary literature avoids the distribution of asset holdings by simplifying assumptions. Lucas (1990) and Shi (1995) assume a large household structure, and with this insurance mechanism, agents within a household share consumption and asset holdings after each round of trade. The sharing mechanism collapses the distribution of asset holdings to a single point. Lagos and Wright (2005) assume a quasi-linear preference structure for the agents along with one round of centralized trading. These assumptions make the distribution of money holdings degenerate and the model highly tractable. By using competitive search in the decentralized market for goods, Menzio et al. (2011) are able to make the distribution of money holdings non-degenerate. Sun (2012) puts Menzio et al. (2011) in a Lagos-Wright framework and, by using a household structure, the model becomes more tractable for studying the effects of different fiscal and monetary policies. Models in Sun (2012) and Menzio et al. (2011) are block recursive; the household's problem can be solved without involving the endogenous distribution of asset holdings.

My paper closely follows Sun (2012) in using competitive search in the decentralized market together with a centralized market to adjust asset balances. Households and firms trade goods in markets with and without frictions. The frictional markets are characterized by competitive search, where households face a trade-off between higher matching probability and better terms of trade. Competitive search in the goods market makes the model highly

[4] Zhu (2003) and Green and Zhou (1998) use a similar approach and have distribution of asset holdings and price dispersion as an equilibrium object

tractable.[5] Households with different idiosyncratic labor cost shocks choose to hold different amounts of assets. Search is directed in the sense that households with different portfolios of asset holdings have different preferences towards matching probability and terms of trade and choose different submarkets. Agents with a high income shock choose a submarket with high price and higher matching probability. Despite a nontrivial distribution of money and bond across agents, the competitive search in the frictional markets makes this model highly tractable.

Unlike models with bargaining and due to the competitive nature of the frictional goods market, the distribution of households across asset holdings does not directly affect the firms' cost/benefit of opening a shop in a submarket. Households' decisions do not affect matching probabilities and terms of trade in the frictional goods market. Households take the specification of the submarkets as given and choose which submarket to participate in. Households only need to know the prices in the economy, and these prices contain all of the information about the distributions in the economy. Hence, the equilibrium is *partially block recursive*.[6] Households' decisions do not directly depend on the distribution of asset holding in the economy.

The rest of the paper is organized as follows. In Section 2, I develop the model environment and characterize value and policy functions. Section 3 defines and characterizes the stationary equilibrium. Section 4 presents the computational algorithm and the results of a numerical example. In Section 5, I introduce exogenously segmented asset markets to the model. Section 6 concludes the paper.

2 Model environment

Time is discrete and has an infinite horizon. Each period consists of four subperiods: labor market, asset market, frictionless goods market and frictional goods market. They operate sequentially, one in each subperiod. The economy is populated by measure one of *ex-ante* homogeneous households. Each household consists of a worker and a buyer. There is a general good that can be produced and consumed by all of the households. There are also at least three types of special goods. Each household is specialized in the production

[5] Aside from tractability, comparing to random search, competitive search is closer to the real world. As Howitt (2005) states: "In contrast to what happens in search models, exchanges in actual market economies are organized by specialist traders, who mitigate search costs by providing facilities that are easy to locate. Thus, when people wish to buy shoes they go to a shoe store; when hungry they go to a grocer; when desiring to sell their labor services they go to firms known to offer employment. Few people would think of planning their economic lives on the basis of random encounters with nonspecialists...".

[6] Unlike a block recursive equilibrium (Shi (2009), Menzio and Shi (2010) in labor, and Menzio et al. (2011) and Sun (2012) in monetary economics), here distributions affect households' decision through prices.

and consumption of one of the special goods, and there is no double coincidence of wants. Because of the specialized structure of households and the no-double-coincidence-of-wants assumption, a medium of exchange is necessary in this environment. The utility function of the household is

$$U_h(y, q, l) = U(y) + u(q) - \theta l$$

where y is the consumption of the general good, q is the consumption of the special goods, and l is the labor supply in a period of time. The parameter $\theta \in [\underline{\theta}, \overline{\theta}]$ is the random disutility of labor. It is *iid* across households and time, and it is drawn from the probability distribution $F(\theta)$ at the beginning of each period. θ captures the heterogeneity of households. $U()$ and $u()$ are continuous and twice differentiable. $u' > 0$, $U' > 0$; $u'' < 0$, $U'' < 0$; $u(0) = U(0) = u'(\infty) = U'(\infty) = 0$; and $u'(0)$ and $U'(0)$ are large and finite. Goods are divisible and perishable. There are two fiat objects in the economy: money and nominal bonds. Both are supplied by the central bank. Nominal bonds are supplied in a centralized market after the utility shocks have been realized. Agents redeem each unit of bonds from the last period for one unit of money at the beginning of each period. Bonds can be costlessly counterfeited by all households, and they cannot differentiate between the original bond that is printed by the central bank and fake bonds that are printed by other households[7]. As a result, nobody accepts bonds as a medium of exchange and, households cannot trade with bonds.

Agents can trade the general good in a perfectly competitive market. There are search frictions in the market for special goods. There is a measure one of competitive firms, who hire workers from the households at the beginning of a period in a competitive labor market. Firms pay hired workers by issuing IOUs. These IOUs can be used to trade goods for money and are redeemed at the end of the period[8]. Households own equal shares in these firms. Firms need labor for production of the general good and one type of special goods. These firms are destroyed at the end of each period, and new firms are formed in the second subperiod of each period[9].

Following Sun (2012), I assume a competitive search environment where agents choose to search in submarkets indexed by terms of trade and matching probability. Agents are randomly matched, and only matched agents can trade goods. Firms choose the measure of shops to operate in each submarket. There is free entry in these submarkets. The fixed cost

[7]Bonds liquidity services have been discussed in the literature, e.g., Shi (2008), Mahmoudi (2014), and Kocherlakota (2003).

[8]Because of the large structure of firms, they do not face unpredictable matching shocks and there is no commitment problem in redeeming IOUs.

[9]With this assumption, there is no need to keep track of firms' asset holdings.

of operating a shop in a submarket is $k > 0$ units of labor. In producing q units of special goods, firms incur $\psi(q)$ units of labor in production costs where, $\psi()$ is twice continuously differentiable and $\psi' > 0$, $\psi'' > 0$ and $\psi(0) = 0$.

Each submarket is a particular set of terms of trade (q: amount of special goods and x: money to be paid) and matching probabilities (b: matching probability for buyers and e: matching probability for sellers/shops). Firms and households take terms of trade and matching frictions as given and decide which submarket to participate in. In each submarket, buyers and shops randomly match according to the respective matching probability. Households and firms decide which submarket to enter, therefore matching probabilities are a function of terms of trade (x, q). Each submarket can be indexed by the respective terms of trade. Matching probability is characterized by a constant return to scale matching function ($e = \mu(b)$), which has the standard characteristics of a matching function[10].

In the asset market, central bank prints money at rate γ, redeems last period nominal bonds (A_{-1}) for one unit of money, issues and sells bonds (A) for the current period at nominal price s, and balances budget by a lump sum tax/transfers (T). The asset market is a competitive market, and households take bonds price (s) as given.

I study the steady state equilibrium, and I will use labor as the numeraire of the model. Figure 1 shows the timing of events.

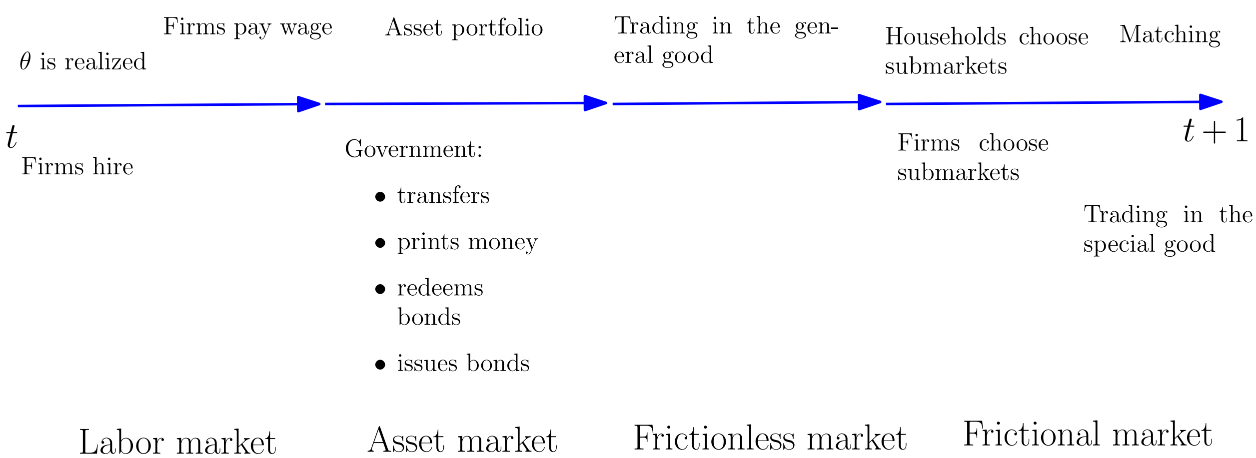

Figure 1: Timing

2.1 Firms' decision

Firms have access to a linear production technology. For each unit of labor input, they produce one unit of output. Firms decide how much to produce in the frictionless market (Y)

[10] For a survey of literature on the properties of matching functions, see Petrongolo and Pissarides (2001)

and determine the measure of shops in each submarket ($dN(x,q)$). They sell the produced general good at the given market price P. In each submarket the matching probability for each shop is $e(x,q)$. Shops sell the produced special goods to matched buyers at price x. In the production process, firms incur k units of labor in fixed cost and $\psi(q)$ units of labor in variable costs. Firms maximize the following profit function:

$$\begin{aligned} \pi = \quad & \max_{Y}\{PY - Y\} + \\ & \max_{dN(x,q)} \int \{e(x,q)x - [k + e(x,q)\psi(q)]\} dN(x,q). \end{aligned} \tag{1}$$

The term $e(x,q)x - [k + e(x,q)\psi(q)]$ is the expected profit of a shop. If the expected profit in a submarket is strictly positive, firms will choose $dN(x,q) = \infty$. If the expected profit is strictly negative, firms will choose $dN(x,q) = 0$. Therefore, the optimal $dN(x,q)$ satisfies the following inequalities with complementary slackness.

$$e(x,q)[x - \psi(q)] \leq k, \quad dN(x,q) \geq 0. \tag{2}$$

As is standard in the competitive search literature, I assume that the profit maximizing condition holds for the submarkets that are not visited by any buyers and firms. For all submarkets where $k < x - \psi(q)$, we have:

$$e(x,q)[x - \psi(q)] = k$$

$$dN(x,q) = 0.$$

For the submarkets where $k \geq x - \psi(q)$ we have $dN(x,q) = 0$, and I assume $e = 1$ and $b = 0$. I can write these two cases as:

$$e(x,q) = \begin{cases} \frac{k}{x-\psi(q)} & k \leq x - \psi(q) \\ 1 & k > x - \psi(q) \end{cases} \tag{3}$$

Note that the matching probabilities do not depend on the distributions in the economy. This property of the frictional market simplifies the households' problems, and we can write households' matching probabilities as a function of terms of trade (x,q).

2.2 Households' decision

2.2.1 Decision in the frictionless goods market

In the asset market, households redeem each unit of their nominal bonds from the previous period for one unit of money. Government prints and injects money at rate γ. Government supplies one period nominal bonds in a centralized market at the competitive price s. Then the asset market closes until the next period.

Let $W(m, a_{-1}, \theta)$ be the value function of a household at the beginning of a period. The household holds a portfolio composed of m units of money and a_{-1} units of nominal bonds in units of labor at the beginning of the period. Let w be the normalized wage rate, which is the nominal wage rate divided by the money stock (M). The nominal wage rate associated with real balance m is wMm.

Given the prices (p, s) and transfers (T), the household decides how much to consume in the frictionless market ($y \geq 0$), labor supply ($l \geq 0$), money balances for transaction purposes (z), money balances for precautionary saving (h), and bond holdings at the beginning of the following period (a). Let $V(z, h, a)$ be the value function of the household at the start of the next subperiod (frictional market). The household chooses an asset portfolio consisting of the money needed for transaction purposes (z), precautionary savings (h), and bond holdings (a) for the frictional market. In order to purchase nominal bonds, one has to pay the nominal price s this period to receive the nominal return of one in the following period. In order to have a real return a in the following period, one needs to pay $s\gamma a$ in terms of labor units. The households solve the following optimization problem subject to a standard budget constraint:

$$W(m, a_{-1}, \theta) = \max_{y,l,z,h,a} U(y) - \theta l + V(z, h, a)$$

$$st. \quad py + z + h + s\gamma a \leq m + a_{-1} + l + T.$$

Let us assume that $V(z, h, a)$ is differentiable[11] and the choice of l is an interior solution. As $U()$ is positively sloped the budget constraint is binding. I use the binding budget constraint to eliminate l from the optimization problem. Using the equilibrium condition $p = 1$, the value function of the representative household can be written as:

$$W(m, a_{-1}, \theta) = \theta(m + T + a_{-1}) + \max_{y \geq 0} \{U(y) - \theta y\} + \max_{z,a,h} \{-\theta(z + s\gamma a + h) + V(z, h, a)\}.$$

The above expression is linear in the households' portfolio of asset holding at the beginning of the period (m, a_{-1}). As I will show later, this linearity will simplify the problem of

[11] I will prove this later

the household in the decentralized market for goods. Furthermore, the households' choice of asset holding for the following subperiod (z, h, a) is independent of the asset holding of the current subperiods (m, a_{-1}).[12]

The optimal choices of y must satisfy:

$$U'(y) = \theta. \tag{4}$$

In the above equation, I have used the equilibrium condition $p = 1$. Similarly z, h and a satisfy:

$$V_z(z, h, a) \begin{cases} \leq \theta & z \geq 0 \\ \geq \theta & z \leq \overline{m} - s\gamma a - h \end{cases} \tag{5}$$

$$V_h(z, h, a) \begin{cases} \leq \theta & h \geq 0 \\ \geq \theta & h \leq \overline{m} - s\gamma a - z \end{cases} \tag{6}$$

$$V_a(z, h, a) \begin{cases} \leq \theta s\gamma & a \geq 0 \\ \geq \theta s\gamma & sa\gamma \leq \overline{m} - z - h \end{cases} \tag{7}$$

where the inequalities hold with complimentary slackness. $\overline{m}$ is the maximum amount of money that households can hold in terms of labor units. Clearly, households' money balance (m) and bond holdings (a_{-1}) does not affect the choices of y, z, h, and a. This is an important property of households' policy function. Households' decisions are independent of their current portfolio of asset holdings. As a result, I can write policy functions as functions of household type (θ). Using the optimization problem of the household, I can write the value function as a linear function of m and a_{-1}:

$$W(m, a_{-1}, \theta) = W(0, 0, \theta) + \theta m + \theta a_{-1}, \tag{8}$$

where

$$W(0, 0, \theta) = U(y(\theta)) - \theta y(\theta) + V(z(\theta), h(\theta), a(\theta)) - \theta(z(\theta) + h(\theta) + s\gamma a(\theta)). \tag{9}$$

It is clear that the value function is continuous and differentiable. The following lemma summarizes these findings.

Lemma 1 *The value function $W(m, a_{-1}, \theta)$ is continuous and differentiable in (m, a_{-1}, θ). It is also affine in m and a_{-1}.*

[12] The quasi-linear preference structure allows me to remove wealth effects.

Lemma 1 shows the standard linearity property that is shared by frameworks based on Lagos and Wright (2005). I can use this property to simplify households' decision in the frictional market.

2.2.2 Decision in the frictional market

A household's decision in the frictional market is similar to Sun (2012). The household chooses which submarket to participate in. As I can index the submarkets by the respective terms of trade, the household chooses the terms of trade (x and q) to maximize the expected value of attending the respective submarket. In choosing which submarket to participate in, households are constrained by their amount of money holding ($x \leq z$).[13] In a submarket, the household matches with probability $b(x,q)$ and trades according to the stated terms of trade. The matching probability comes from the firms' decision problems. In each match, the representative household spends x amount of money and consumes q amount of special good. With probability $1 - b(x,q)$ there is no match and the representative household exits the frictional market with the starting portfolio of assets. As is standard in the search and matching literature, I assume $b(x,q)$ is nonincreasing. The representative household solves the following optimization problem:

$$
\begin{aligned}
v(z,h,a) = \\
&\max_{x \leq z, q} \; b(x,q)\left[u(q) + \beta E[W(\frac{z-x+h}{\gamma}, a, \theta)]\right] + \\
&[1-b(x,q)]\beta E\left[W(\frac{z+h}{\gamma}, a, \theta)\right]. \qquad (10)
\end{aligned}
$$

Using the linearity of $W(.)$ (8) and firms' optimization problem (3), I can eliminate q from the above expression. The household's problem becomes:

$$
\begin{aligned}
v(z,h,a) = \\
&\max_{x \leq z, b}\left\{ b\left[u(\psi^{-1}(x - \frac{k}{\mu(b)})) - \beta E(\theta)\frac{x}{\gamma}\right] + \beta E\left[W(\frac{z+h}{\gamma}, a, \theta)\right]\right\} \qquad (11)
\end{aligned}
$$

The optimal choices of x and b satisfy the following first-order conditions:

$$
\frac{u'\left(\psi^{-1}(x - \frac{k}{\mu(b)})\right)}{\psi'\left(\psi^{-1}(x - \frac{k}{\mu(b)})\right)} - \frac{\beta E(\theta)}{\gamma} \geq 0, \quad x \leq z \qquad (12)
$$

[13] Because households are committed to posted terms of trade they cannot choose a submarket in which they cannot afford to trade.

$$u\left(\psi^{-1}(x-\frac{k}{\mu(b)})\right)-\frac{\beta E(\theta)x}{\gamma}+\frac{u'\left(\psi^{-1}(x-\frac{k}{\mu(b)})\right)}{\psi'\left(\psi^{-1}(x-\frac{k}{\mu(b)})\right)}\frac{kb\mu'(b)}{[\mu(b)]^2}\leq 0, \quad b\geq 0. \tag{13}$$

where the two sets of inequalities hold with complementary slackness. Note that $b = 1$ cannot be an equilibrium outcome[14].

For $b(z) = 0$, I assume $x(z) = z$. Define $\Phi(q) = u'(q)/\psi'(q)$. As is shown in Sun (2012), without loss of generality, I can focus on the case $x(z) = z$. Similar to Sun (2012), households do not need to hold more money than they want to spend. If the following condition holds: [15]

$$u\left(\phi^{-1}\left[\frac{\beta E(\theta)}{\gamma}\right]\right)-\frac{\beta E(\theta)}{\gamma}\left(\psi\left(\phi^{-1}\left[\frac{\beta E(\theta)}{\gamma}\right]\right)+k\right)>0. \tag{14}$$

the household's problem becomes:

$$B(z)+\beta E\left[W(\frac{z+h}{\gamma},a,\theta)\right], \tag{15}$$

where

$$B(z)=\max_{b\in[0,1]} b\left[u(\psi^{-1}(z-\frac{k}{\mu(b)}))-\beta\frac{z}{\gamma}E(\theta)\right]. \tag{16}$$

The value function $B(z)$ may not be concave in z. Furthermore, equation 16 is the product of the choice variable b, and a function of b and this product may not be concave. Following Menzio et al. (2011) and Sun (2012), I introduce lotteries to make the households' value function concave[16]. A lottery is a choice of probabilities (π_1, π_2) and respective payments

[14] $b = 1$ implies $e = 0$, $dN(z, q) = \infty$, and positive profits for the firms. This violates free firms' entry condition.

[15] Let us assume for $b(z) > 0$, $x(z) < z$. Then 11 is independent of z. 13 holds with equality and can be written as:

$$q^* = \Phi^{-1}[\frac{\beta E(\theta)}{\gamma}]$$

Given q^*, 13 can be written as:

$$u(q^*)-\frac{\beta E(\theta)}{\gamma}\left[\psi(q^*)+\frac{k}{\mu b^*}\right]+\left[\frac{u'(q^*)}{\psi'(q^*)}\right]\frac{kb^*\mu'(b^*)}{[\mu(b^*)]^2}=0.$$

The left-hand side of the above equation is strictly increasing in b^*, and b^* exists and is unique if $\frac{E(\theta)}{\gamma}$ satisfies:

$$u(q^*)-\beta\frac{E(\theta)}{\gamma}[\psi(q^*)+k]>0.$$

For all $z < x^* = \psi(q^*) + \frac{k}{\mu(b^*)}$, $x(z) = z$. For $z \geq x^*$, $x(z) = x^*$.

[16] The numerical exercise in Section 4 shows that households play lotteries only when they have very low real balances and this does not happen in equilibrium.

(L_1, L_2) that solves the following problem:

$$\widetilde{V}(z) = \max_{L_1, L_2, \pi_1, \pi_2} \left[\pi_1 B(L_1) + \pi_2 B(L_2)\right], \tag{17}$$

subject to:

$$\pi_1 L_1 + \pi_2 L_2 = z; \quad L_2 \geq L_1 \geq 0$$

$$\pi_1 + \pi_2 = 1; \quad \pi_i \in [0, 1].$$

Note that the agent's policy functions for the lottery choices are: $L_{i\in\{1,2\}}(z)$ and $\pi_{i\in\{1,2\}}(z)$. $\widetilde{V}(z)$ is the household's value function after playing the lottery. After playing this lottery, the value function of the household becomes concave.

2.3 Government

Government imposes policies by either changing the inflation rate (γ) or changing the relative supply of bond (λ). I assume that the government runs a balanced budget at each period. Let us define

$$\lambda = \frac{A_{-1}}{M}$$

as the ratio of stock of bond to stock of money in the economy. λ shows the composition of the central bank's balance sheet. A temporary jump in λ indicates that the central bank has issued more short-term debt and the composition of its balance sheet has shifted to short-term financing of the government transfers. The total real transfer that a household receives (T) is the sum of transfers from printing money (seigniorage) and the transfers received from the bond market.[17]

$$T = \frac{\gamma - 1}{w\gamma} + \frac{sA - A_{-1}}{wM'}. \tag{18}$$

2.4 Properties of value and policy functions

As shown in the previous section, the choice of bonds holdings and bonds prices do not directly affect households' decisions in the frictional market. The solution to firms' problem (3) shows that the matching probabilities do not depend on the distributions in the economy. Sun (2012) discusses fiscal policy in a framework similar to the one used here. In that environment, fiscal policy variables do not directly affect households' decisions in the frictional market. As a result, the properties of value functions and the households' choice of which submarket to search (x, q) are the same as in Sun (2012). Let us define $\widehat{z}$ as the

[17]Note that because of the quasi-linear structure of households' utility function, government transfers can be interpreted as a public good.

maximum value of real balance (z) that equation 14 holds. The following lemma shows the properties of the value functions and policy functions:

Lemma 2 *The following statements about the value functions and policy functions are true*

1. *The value function $B(z)$ is continuous and increasing in $z \in [0, \widehat{z}]$.*
2. *The value function $\widetilde{V}(z)$ is continuous, differentiable, increasing, and concave in $z \in [0, \widehat{z}]$.*
3. *For z such that $b(z) = 0$, the value function $B(z) = 0$ and the choice of q is irrelevant.*
4. *If and only if there exists a $q > 0$ that satisfies*

$$u(q) - \frac{\beta E(\theta)}{\gamma}[\psi(q) + k] > 0$$

   there exists a $z > 0$ such that $b(z) > 0$.

5. *For z such that $b(z) > 0$, the value function $B(z)$ is differentiable, $B(z) > 0$, and $B'(z) > 0$.*
6. *$b(z)$ and $q(z)$ are unique and*

$$b'(z) > 0$$

$$q'(z) > 0$$

7. *$b(z)$ solves*

$$\max_{b \in [0,1]} \left\{ u\left(q(z)\right) - \frac{\beta E(\theta) z}{\gamma} + \frac{u'\left(q(z)\right)}{\psi'\left(q(z)\right)} \frac{k b \mu'(b)}{\left[\mu(b)\right]^2} \right\} \tag{19}$$

   where

$$q(z) = \psi^{-1}\left(z - \frac{k}{\mu(b(z))}\right) \tag{20}$$

8. *$b(z)$ strictly decreases with $E(\theta)$, and $q(z)$ strictly increases in $E(\theta)$.*
9. *There exists $z_1 > k$ such that $b(z) = 0$ for all $z \in [0, z_1]$ and $b(z) > 0$ for all $z \in (z_1, \widehat{z}]$.*
10. *There exists $z_0 > z_1$ such that a household with $z < z_0$ will play the lottery with the prize z_0.*

Since the choices of bond holdings and bond prices do not directly affect households' decisions in the frictional market, the proof of lemma 2 is exactly similar to lemma 2 in Sun

(2012). Lemma 2 summarizes the characteristics of the value functions. According to part 6, households with higher money balances choose to trade in submarkets with higher matching probabilities and higher terms of trade. They sort themselves in to different submarkets according to their money holdings. A household with a higher money balance has lower marginal value for money. This household wants to get rid of a high amount of money in a short period of time and therefore chooses a submarket with high price and high matching probability.

Equations 8, 10, 16, and 17 give:

$$\begin{aligned} V(z,h,a) \quad &= \widetilde{V}(z) + \beta E\left[W(\frac{z+h}{\gamma}, a, \theta)\right] \\ &= \widetilde{V}(z) + \beta E\left[W(0,0,\theta)\right] + \frac{\beta E(\theta) z}{\gamma} + \frac{\beta E(\theta) h}{\gamma} + \beta E(\theta) a. \end{aligned} \tag{21}$$

Equation 21 shows that $V(z,h,a)$ is linear in a and h, and the slopes are

$$V_a(z,h,a) = \beta E(\theta) \tag{22}$$

$$V_h(z,h,a) = \frac{\beta E(\theta)}{\gamma}. \tag{23}$$

Using lemma 2, equations 22, and 23 and policy functions 26 and 25, I can conclude the following lemma.

Lemma 3 *The value function V is continuous and differentiable in (z,h,a). $V(z,h,a)$ is increasing and concave in $z \in [0,\widehat{z}]$. $V(z,h,a) \geq \beta E[W(0,0,\theta)] > 0$ for all z.*

Continuity and differentiability of V with respect to precautionary savings (h) and bond holdings (a) is trivially concluded from the linearity condition (21, 22, 23). V is increasing and concave in $z \in [0,\widehat{z}]$, because equation 21 can be differentiated as:

$$\frac{\partial V(z,h,a)}{\partial z} = \frac{d\widetilde{V}(z)}{dz} + \frac{\beta E(\theta)}{\gamma} \tag{24}$$

and lemma 2 shows that $\widetilde{V}(z)$ is increasing and concave in z.

Using conditions 6, 7, 22, and 23, I can write the household's choice of bond holdings and precautionary savings in money as follows:

$$\begin{cases} h(\theta) \geq 0 & \theta \geq \frac{\beta E(\theta)}{\gamma} \\ h(\theta) \leq \overline{m} - z(\theta) - s\gamma a(\theta) & \theta \leq \frac{\beta E(\theta)}{\gamma} \end{cases} \tag{25}$$

$$\begin{cases} a(\theta) \geq 0 & \theta \geq \frac{\beta E(\theta)}{s\gamma} \\ s\gamma a(\theta) \leq \overline{m} - z(\theta) - h(\theta) & \theta \leq \frac{\beta E(\theta)}{s\gamma} \end{cases} \tag{26}$$

where the inequalities hold with complementary slackness. Using equations 21, 26, 25, and 24, I can characterize the policy functions of the households with respect to asset holdings and labor supply in Lemma 4.

Lemma 4 *$a(\theta)$, $h(\theta)$, $z(\theta)$, and $l(m, a_{-1}, \theta)$ follow the following rules:*

I: Negative nominal interest rate ($s \geq 1$)

$$\begin{cases} \theta < \frac{\beta E(\theta)}{\gamma} & \begin{cases} h(\theta) = \overline{m} - z(\theta) \\ a(\theta) = 0 \\ l(m, a_{-1}, \theta) = py(\theta) + \overline{m} - a_{-1} - T \\ V_z = \widetilde{V}_z(z) \end{cases} \\ \theta \geq \frac{\beta E(\theta)}{\gamma} & \begin{cases} h(\theta) = 0 \\ a(\theta) = 0 \\ l(m, a_{-1}, \theta) = py(\theta) - m - a_{-1} - T \\ V_z = \widetilde{V}_z(z) + \frac{\beta E(\theta)}{\gamma} \end{cases} \end{cases} \tag{27}$$

II: Positive nominal interest rate ($s < 1$)

$$\begin{cases} \theta < \frac{\beta E(\theta)}{s\gamma} & \begin{cases} h(\theta) = 0 \\ a(\theta) = \frac{\overline{m} - z(\theta)}{s\gamma} \\ l(m, a_{-1}, \theta) = py(\theta) + z(\theta)(1 - s\gamma) + s\gamma\overline{m} - a_{-1} - T \\ V_z = \widetilde{V}_z(z) + \beta E(\theta)(\frac{1}{\gamma} - 1) \end{cases} \\ \theta \geq \frac{\beta E(\theta)}{s\gamma} & \begin{cases} h(\theta) = 0 \\ a(\theta) = 0 \\ l(m, a_{-1}, \theta) = py(\theta) + z(\theta) - m - a_{-1} - T \\ V_z = \widetilde{V}_z(z) + \frac{\beta E(\theta)}{\gamma} \end{cases} \end{cases} \tag{28}$$

Lemma 4 and equation 5 show the characteristics of the policy functions in two cases: when the nominal interest rate is negative (27) and when it is positive (28). In an equilibrium with a negative nominal interest rate, households choose to hold all of their portfolio in terms of money. Money has a greater return compared to bonds in this case and households decide to hold all of their precautionary savings in terms of money. Higher amount of portfolio from the previous period (m, a_{-1}) reduces the labor supply $(l(m, a_{-1}, \theta))$, and households choose to work less because of the higher value of their asset portfolio.

Lemma 4 shows that the equilibrium is *partially block recursive*. Households do not need to know the distribution of the asset holding for their decision problems, and prices (p, s, and w) contain all the information they need about the distributions in the aggregate economy. In the next section, I show that we cannot have a negative nominal interest rate in the stationary equilibrium and households' policy functions can be described by 28.

3 Stationary Equilibrium

Here, I characterize the stationary equilibrium.

Definition 1 *A stationary equilibrium is the set of household value functions $(W, B, V, \widetilde{V})$; household choices $(y, l, z, a, h, q, b, L_1, L_2, \pi_1, \pi_2)$; firm choices $(Y, dN(q, b))$; and prices (p, s, w) that satisfy the following conditions:*

1. *Given the prices (p, s, w), realization of shocks (θ), asset balances, and terms of trade in all submarkets (q, x), household choices solve households' optimality conditions (28 and 27).*
2. *Given prices and the terms of trade in all submarkets, firms maximize profit (1).*
3. *Free entry condition (3)*
4. *Stationarity: quantities, distributions and prices are constant over time.*
5. *Symmetry: Households with the same shock values and the same asset portfolios make the same decisions.*
6. *Bond market clears (29), labor market clears (30), and general goods market clears $(p = 1)$.*

In the bonds market, the total amount of bonds supplied equals the sum of demanded bonds by households of different type. The nominal amount of supplied bonds by the central bank is As, and therefore the real supply of bonds is $\frac{As}{wM}$. The market clearing for bonds gives:

$$\frac{As}{wM} = \int\int\int_{\underline{\theta}}^{\overline{\theta}} a \; dF(\theta)dG(m)dH(a_{-1}) = \int_{\underline{\theta}}^{\overline{\theta}} a(\theta)dF(\theta), \tag{29}$$

where in the second equality, I use the fact that households' decisions on their asset holdings is only based on their labor supply shock.

Lemma 5 *No positive bond supply ($\lambda > 0$) can support an equilibrium with negative nominal interest rate ($s \geq 1$). Households choose to hold bonds as precautionary saving, and they only choose money for transaction purposes:*

- $h(\theta) = 0$.
- $z(\theta) \geq 0$.

From condition 27 and bond market clearing condition 29, it is straightforward to show that positive amounts of bond supply would not clear the market when $s \geq 1$. With a positive real interest rate, households never use money for precautionary motives.

There are two cases for the equilibrium: First, when $\underline{\theta} < \frac{\beta E(\theta)}{s\gamma} < \overline{\theta}$, households with low enough θ choose to hold a positive amount of bond (traders in the asset market), while households with high θ only hold money for transaction purposes (non-traders in the asset market). Figure 2 shows that the threshold $\frac{\beta E(\theta)}{s\gamma}$ determines who participates in the asset market.

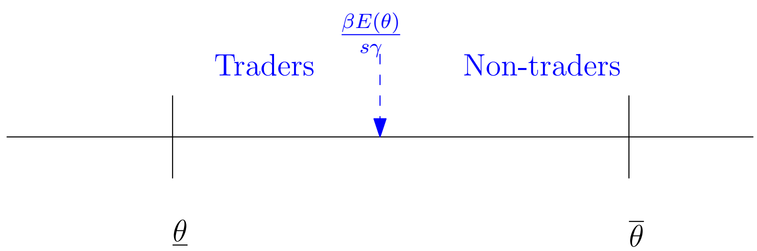

Figure 2: Segmented asset market

Second, in the case where $\overline{\theta} < \frac{\beta E(\theta)}{s\gamma}$ all of the households hold a portfolio of bonds and money[18]. In figure 3, $\frac{\beta E(\theta)}{s\gamma}$ is very high and everyone participates in the market for bond. In deciding whether to participate in the asset market, households compare two scenarios: 1-working this period and buying bonds and redeeming purchased bonds for money next period, and 2-not working this period and working next period.

From lemma 5 and equations 28, 17, and 5, I can show that changes in bond supply (λ) would change the threshold ($\frac{\beta E(\theta)}{s\gamma}$). Figure 2 shows that an equilibrium with a segmented asset market arises when $\underline{\theta} < \frac{\beta E(\theta)}{s\gamma} < \overline{\theta}$. In an equilibrium with a segmented asset market, open-market operations affect the decision of the households regarding the composition of their real portfolio of assets for households at the participation margin and therefore have real effects on the distributions of asset portfolios in the economy.

Lemma 4 shows that when we have an equilibrium with no segmentation in the asset market, money holding from the previous period does not affect agents' labor supply. In

[18]Note that with positive bonds supply, we cannot have the case in which $\frac{\beta E(\theta)}{s\gamma} < \underline{\theta}$.

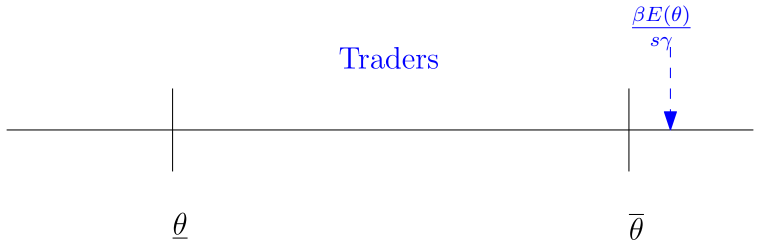

Figure 3: Asset market with no segmentation

the same type of equilibrium, households' bond holding has a negative effect on their labor supply. This property of the equilibrium is due to the fact that households in an asset market with no segmentation and households with good shocks in a segmented asset market ($\theta < \frac{\beta E(\theta)}{s\gamma}$) hold money balances only for transaction purposes. Note that this is different from the pure precautionary motive ($h(\theta) > 0$), and bonds always dominate money because of positive real interest rate ($s < 1$). In a segmented asset market, households with bad labor supply shocks ($\theta \geq \frac{\beta E(\theta)}{s\gamma}$) consider unmatched buyers' expected money balances as a precautionary motive for saving.

As shown in appendix 4.0.1, the labor market clearing condition is

$$\frac{1}{w\gamma}[\gamma - 1 + \lambda(s-1)] =$$
$$(2 - s\gamma)\int_{\underline{\theta}}^{\overline{\theta}} a(\theta)dF(\theta) + (1 - \frac{1}{\gamma})\int_{\underline{\theta}}^{\overline{\theta}} \pi_1(z(\theta))(1 - b(L_1(z(\theta))))L_1(z(\theta))dF(\theta)$$
$$+(1 - \frac{1}{\gamma})\int_{\underline{\theta}}^{\overline{\theta}} \pi_2(z(\theta))(1 - b(L_2(z(\theta))))L_2(z(\theta))dF(\theta) - \frac{1}{\gamma}\int_{\underline{\theta}}^{\overline{\theta}} h(\theta)dF(\theta). \tag{30}$$

Using lemmas 4 and 5, market clearing condition for the asset market and the fact that households do not save money for precautionary motives, I can summarize the market clearing conditions to a single equation that could be solved for bond price (s).

$$[\frac{1}{\gamma^2 s} + (1 - \gamma - \lambda(s-1)(2 - s\gamma))]\int_{\underline{\theta}}^{\frac{\beta E(\theta)}{s\gamma}} a(\theta)dF(\theta) =$$
$$\gamma(1-\gamma)\int_{\underline{\theta}}^{\overline{\theta}} \pi_1(z(\theta))(1 - b(L_1(z(\theta))))L_1(z(\theta))dF(\theta)$$
$$+\gamma(1-\gamma)\int_{\underline{\theta}}^{\overline{\theta}} \pi_2(z(\theta))(1 - b(L_2(z(\theta))))L_2(z(\theta))dF(\theta). \tag{31}$$

For $\gamma < 2$, the left-hand side of the above expression does not increase with bond price (s). Note that equation 31 cannot solely be used for numerical computations, and we need to compute wage (30) and check for positive wages. From equations 28, 29, and 31, I can characterize the set of prices in equilibrium. Let us define $\overline{s}$ and $\underline{s}$ as:

$$\overline{s} = \frac{\beta E(\theta)}{\underline{\theta}\gamma}$$

$$\underline{s} = \frac{\beta E(\theta)}{\overline{\theta}\gamma}.$$

Let bond price be in the range: $\overline{s} \leq s$. The left-hand side of 31 is 0, while the right-hand side is a positive number. In this case, there is no equilibrium. Previously, I have shown that $s < 1$ in equilibrium. Therefore, the market clears at a price in the range $s < \min\{1, \overline{s}\}$.

Let us define $\zeta(\gamma, \lambda)$ as:

$$\zeta(\gamma, \lambda) = \sum_{i=1,2} \int_{\underline{\theta}}^{\overline{\theta}} \pi_i(z(\theta))(1 - b(L_i(z(\theta))))L_i(z(\theta))dF(\theta). \tag{32}$$

For the case where $s < \underline{s}$, figure 3 shows the policy functions. In this case, $\zeta(\gamma, \lambda)$ is independent of λ, and I show it by $\zeta_1(\gamma)$. The only policy variable on the right-hand side of 31 is γ. For a constant positive rate of inflation ($\gamma > 1$), the left-hand side shows a positive relationship between bond price and bond supply[19]. From figures 2 and 3 and the positive relationship between bond price and bond supply, I can summarize the demand for bond in figure 4.

For low bond price, the return on the bond is high enough to attract all of the households to the asset market. They hold a positive portfolio of money and bond according to figure 2. With higher bond price (lower interest rate), we have a segmented asset market, and higher price in this type of equilibrium leads to low asset market participation.

From 31 and 32 proposition 1 follows:

Proposition 1 *Let us define $\widetilde{s} = \min(1, \overline{s})$. Then for a positive rate of inflation ($\gamma > 1$), there exists the following thresholds: s_{min} and λ_u, which solves the following equations:*

[19]We can rewrite the expression on the left hand side of 31 as

$$[\frac{1}{\gamma^2 s} + (2 - s\gamma)(1 - \gamma) + \lambda(s - 1)(s\gamma - 2)] \int_{\underline{\theta}}^{\frac{\beta E(\theta)}{s\gamma}} a(\theta)dF(\theta)$$

The above expression shows a positive relationship between bond price (s) and bond supply (λ) for positive real interest rates ($s < 1$) and positive inflation rate ($\gamma > 1$).

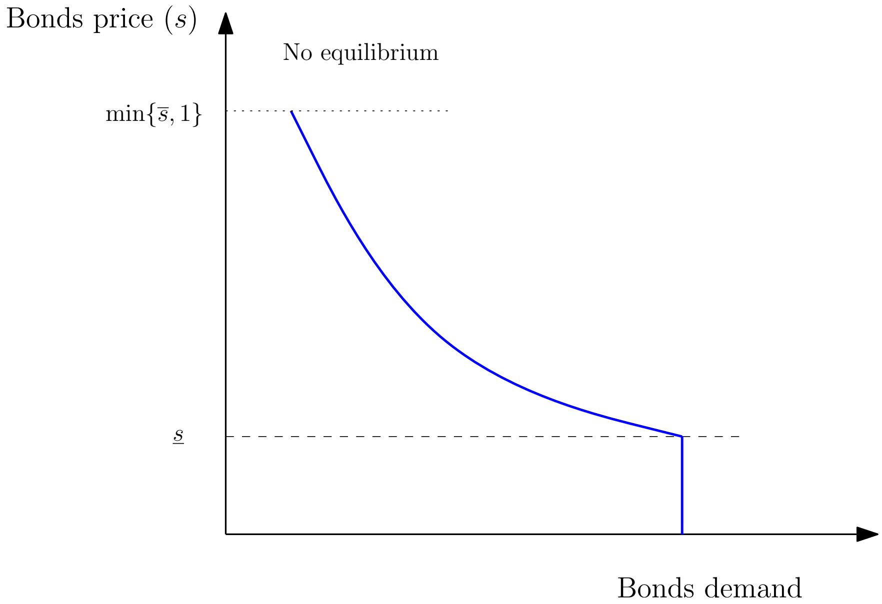

Figure 4: Demand for bonds ($\gamma > 1$)

$$[\frac{1}{\gamma^2 \widetilde{s}} + (1 - \gamma - \lambda(\widetilde{s} - 1)(2 - \widetilde{s}\gamma))] \int_{\underline{\theta}}^{\frac{\beta E(\theta)}{\widetilde{s}\gamma}} a(\theta) dF(\theta) = \gamma(1-\gamma)\zeta(\gamma, \lambda_u). \tag{33}$$

$$[\frac{1}{\gamma^2 s_{min}} + 1 - \gamma] \int_{\underline{\theta}}^{\frac{\beta E(\theta)}{s_{min}\gamma}} a(\theta) dF(\theta) = \gamma(1-\gamma)\zeta(\gamma, 0), \tag{34}$$

where

1. *Bond price is in the range:* $s_{min} < s < \widetilde{s}$.

2. *Bond supply is in the range:* $0 < \lambda < \lambda_u$.

In the above proposition, I have used the positive relationship between bond price (s) and bond supply (λ) from equation 31 to derive the equilibrium limits for bond price and bond supply.

3.1 Welfare analysis

I have shown in the appendix 4.0.1 that the steady state welfare can be calculated using the following expression:

$$
\begin{aligned}
\varpi =& \int [U(y(\theta)) - \theta y(\theta) + u(q(z(\theta))) - \theta z(\theta) - \theta h(\theta) - s\gamma\theta a(\theta)]dF(\theta) \\
&+ \left[\int_{\underline{\theta}}^{\bar{\theta}} \pi_1(z(\theta))[1 - b(L_1(z(\theta)))]\frac{L_1(z(\theta))}{\gamma}dF(\theta)\right]\int \theta dF(\theta) \\
&+ \left[\int_{\underline{\theta}}^{\bar{\theta}} \pi_2(z(\theta))[1 - b(L_2(z(\theta)))]\frac{L_2(z(\theta))}{\gamma}dF(\theta)\right]\int \theta dF(\theta) \\
&+ (1 + \frac{1}{\gamma})\left[\int a_{-1}dH_{a_{-1}}\right]\int \theta dF(\theta) + \frac{1}{\gamma}\left[\int h_{-1}dJ_{h_{-1}}\right]\int \theta dF(\theta) \\
&+ \frac{1}{w\gamma}[\gamma - 1 - \lambda + s\lambda]\int \theta dF(\theta).
\end{aligned}
$$

Using lemma 5 and equations 30 and 32 the measure of welfare can be simplified as:

$$
\begin{aligned}
\varpi =& \int [U(y(\theta)) - \theta y(\theta)]dF(\theta) + \int [u(q(z(\theta))) - \theta z(\theta) - s\gamma\theta a(\theta)]dF(\theta) \\
&+ \left[\zeta(\gamma, \lambda) + (2 - s + 1 + \frac{1}{\gamma})\int a(\theta)dF(\theta)\right]\int \theta dF(\theta).
\end{aligned} \tag{35}
$$

4 Numerical Example

In order to simulate the economy, I use the partial block recursivity of the equilibrium. I use the following algorithm:

1. For given supply of bonds (λ) and inflation (γ), and an arbitrary bond price (s) calculate policy functions $(a(\theta), h(\theta), z(\theta), y(\theta), l(\theta), b(z(\theta)), q(z(\theta)))$ (4) and lottery choices $(\pi_1(z(\theta)), \pi_2(z(\theta)), L_1(z(\theta)), L_2(z(\theta)))$ (17)

2. Calculate the value functions $(B(z(\theta)), \widetilde{V}(z(\theta)))$

3. Calculate wage (w) using labor market clearing condition 30

4. If $w < 0$ change s and start from 1.

5. Check bonds market clearing condition 29, adjust bond price and start from 1. until bond market clears.

I simulate the economy using the following functional forms:

$$u(c) = u_0 \frac{(c+a)^{1-\sigma} - a^{1-\sigma}}{1-\sigma}; U(c) = U_0 \frac{(c+a)^{1-\sigma_u} - a^{1-\sigma_u}}{1-\sigma_u}$$

$$\psi(q) = \psi_0 q^{\psi}; \mu(b) = 1 - b; F(\theta) \text{ is continuous uniform on } [\underline{\theta}, \overline{\theta}].$$

I use the following parameter values:

$\beta = 0.96$	$u_0 = 1$	$U_0 = 100$	$a = 0.01$
$\sigma = 2$	$\sigma_u = 2$	$\phi = 2$	$\psi_0 = 1$
$k = 1$	$\overline{m} = 20$	$\theta \in [0.25, 1.75]$	$F(\theta)$ uniform

Figure 5 shows the policy functions regarding households' portfolio and the effects of open-market operations. In a segmented asset market, high income households choose a positive portfolio of bond and money. The threshold that determines who participates in the asset market is affected by open-market operations. As it can be seen in figure 5 an open-market purchase of bond would increase the real interest rate and shift the threshold to the right. More households decide to participate in the bond market as a result of an open-market purchase of assets.

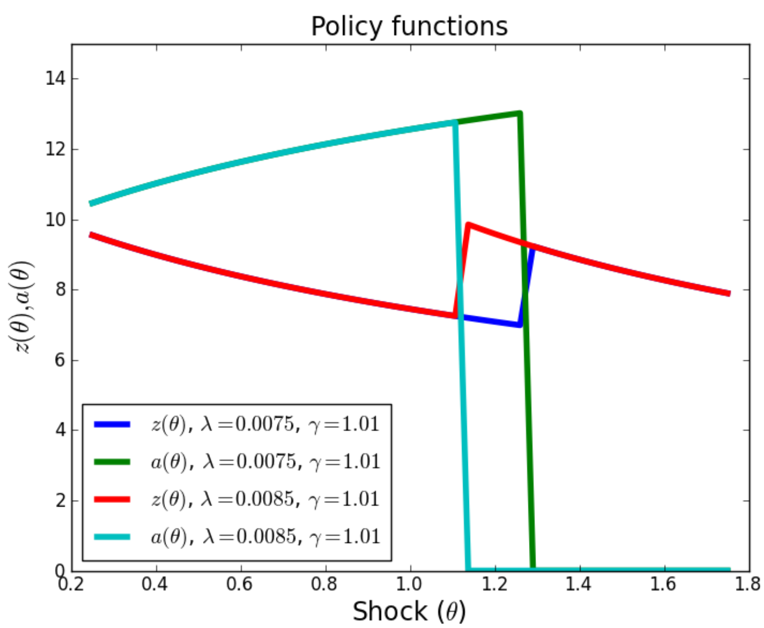

Figure 5: Choice of asset holding in a segmented asset market

Figure 6 shows the asset portfolio choice of households in an equilibrium with no segmentation. Comparing to figure 5, here bond supply is so low that high real interest rates attract all of the households to the asset market and they hold a positive portfolio composing of money and bond. A marginal policy of pure open-market operation would not affect the decision of households regarding their real asset holding, and would not have real effects on the distributions in the economy.

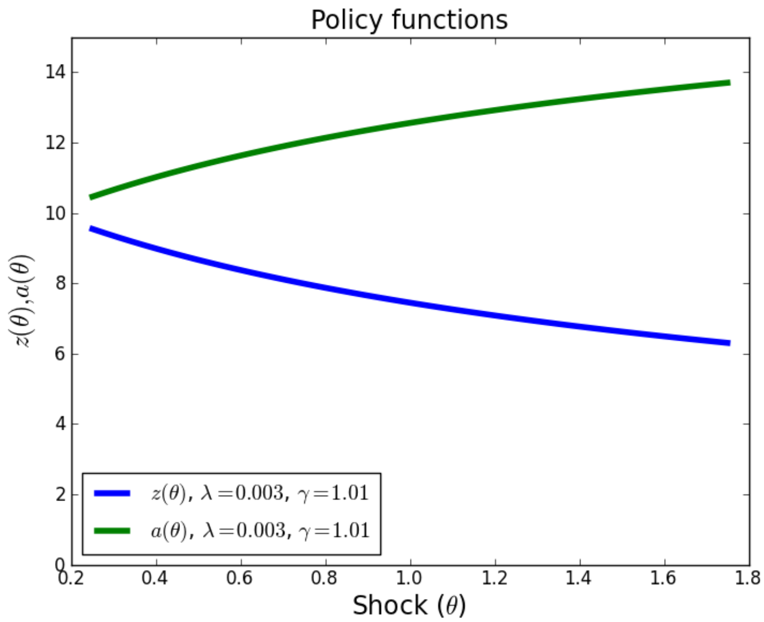

Figure 6: Choice of asset holding in an equilibrium with no segmentation

Figure 7 shows labor supply in an equilibrium with no segmentation in the asset market. Households with bad shocks (high θ) and high asset balance work less. Households with good shocks (low θ) work more and hold more money for transaction purposes. A pure policy of open-market operations (marginal change in λ) would shift the labor supply. Higher real interest rate would change labor supply of households but households' real asset holding would not change (Figure 6).

Figure 8 shows labor supply in an equilibrium with segmented asset market. Households with bad shocks ($\theta \geq \frac{\beta E(\theta)}{s\gamma}$) supply labor only to fund their money holding for the next subperiod ($z(\theta)$). Households who received better shocks than the threshold for asset market participation ($\theta < \frac{\beta E(\theta)}{s\gamma}$) provide high labor supply to buy bonds ($a(\theta)$) as a precautionary saving for the next period. A pure policy of open-market operations (marginal changes in λ) has two effects: First, it has a level effect on the labor supply. This is similar to the case with no segmentation. Higher real interest rates requires higher labor supply for the same real asset holding. Second, open-market operations changes the threshold ($\frac{\beta E(\theta)}{s\gamma}$), and therefore affects the participation decision of households in the market for bonds. Higher real interest rate, attracts some of households who were not participating in the asset market, and these households supply more labor.

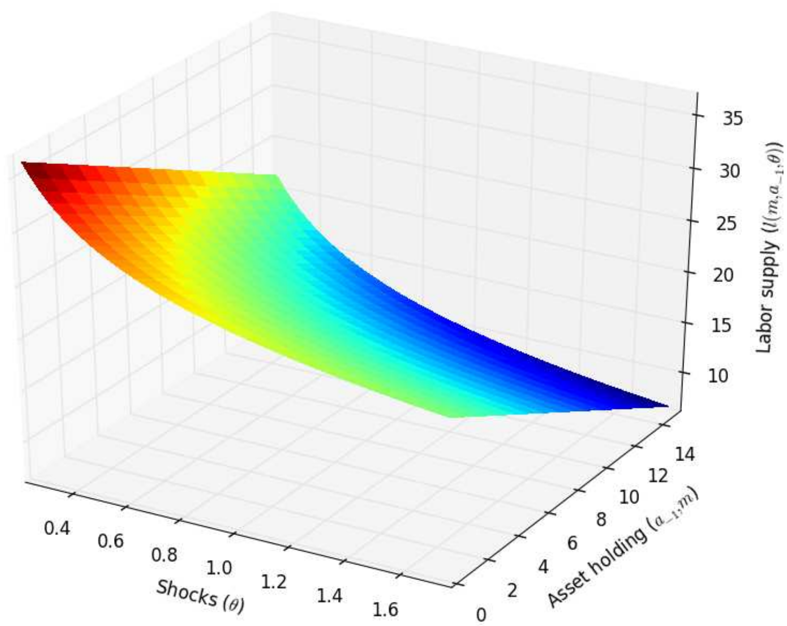

Figure 7: Labor supply in an equilibrium with no segmentation ($\gamma = 1.01, \lambda = 0.003$)

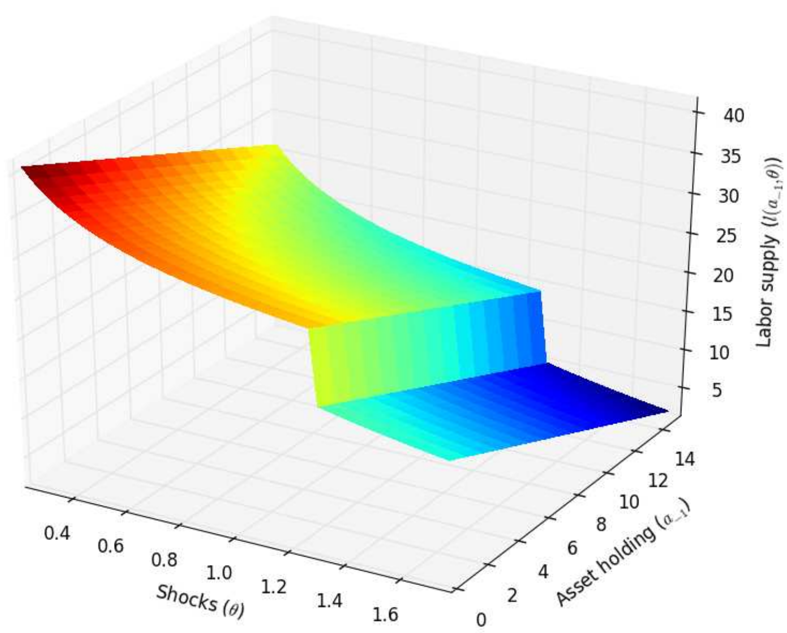

Figure 8: Labor supply in an equilibrium with segmented asset market ($\gamma = 1.01, \lambda = 0.0085$)

Figure 9 shows the characteristics of submarkets in the decentralized market. Agents with higher money holdings search in submarkets with higher price, output and matching probabilities. This property of the equilibrium is shared with many competitive search models[20]. Agents sort themselves according to their money holdings. Households with higher money balances have low marginal value for money. As shown in lemma 2, they decide to get rid of a high amount of money as soon as they can and choose submarket with higher price and higher matching probability compared to households with low money balances. Unlike models of bargaining, buyers and sellers know the marginal value of money holdings of all of the households in the economy and they commit to posted terms of trade.

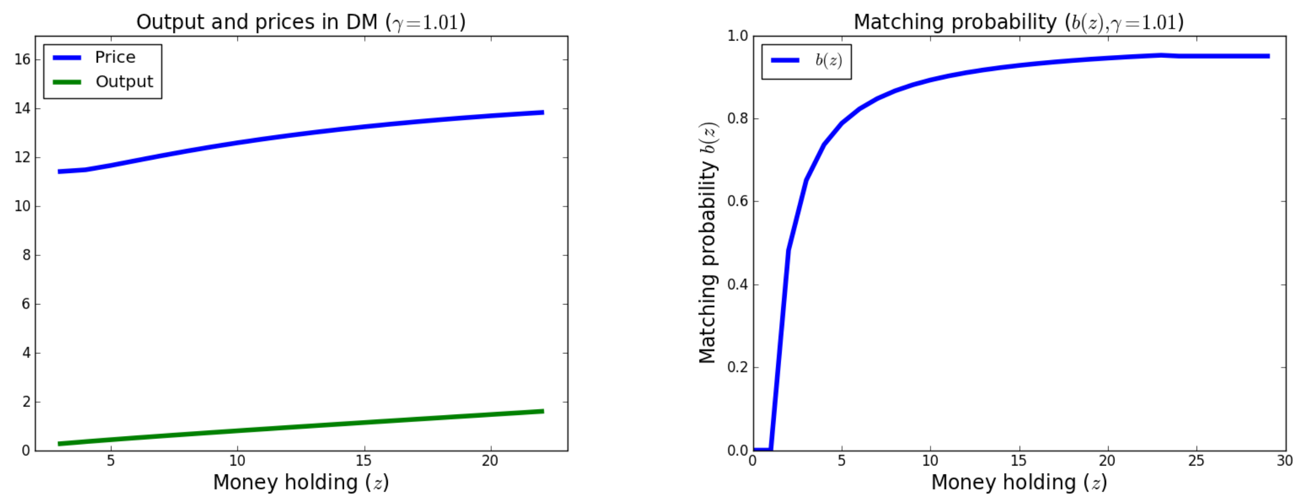

Figure 9: Policy function in decentralized market

Figures 10 shows the output choice of households in the decentralized market. Generally, households with better shocks participate in submarkets with higher output. As shown in figures 5 and 6, conditional on participation(/not participation) in the asset market, households with better shocks choose higher amounts of money balances. Figure 9 shows that households with higher money balances choose higher output. Therefore, we can see that conditional on participating (/not participating) in the asset market households with better shocks choose submarkets with higher output and figure 10 confirms this. In the case with a segmented asset market (figure on the right) a marginal open-market purchase of bond would increase asset market participation and reduce real output choice of households on the participation margin to lower values.

Figure 11 shows the matching probability choice of households in he decentralized market. As shown in figures 5 and 6, conditional on participation(/not participation) in the asset

[20]e.g. equilibrium in Menzio et al. (2011) and Sun (2012) shows similar properties.

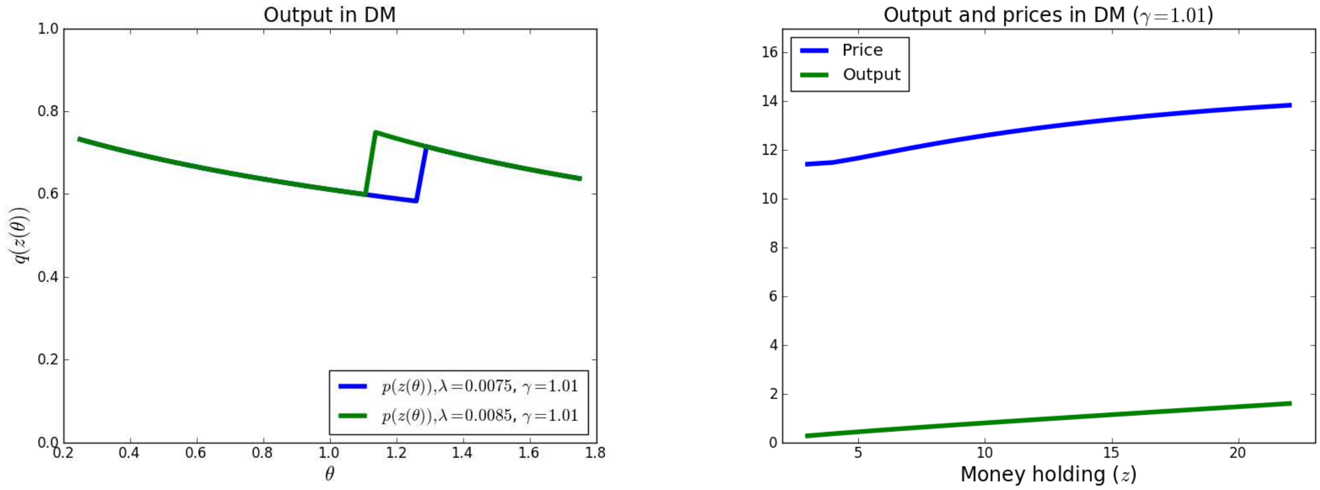

Figure 10: Output choice of households in decentralized market

market, households with better shocks choose higher amounts of money balances. Figure 9 shows that households with higher money balances choose submarkets with higher matching probabilities. As a result, conditional on participating (/not participating) in the asset market households with better shocks choose submarkets with higher matching probability and figure 11 confirms this. In the case with a segmented asset market (figure on the right) a marginal open-market purchase of bond would increase asset market participation and reduce real output choice of households on the participation margin to lower values.

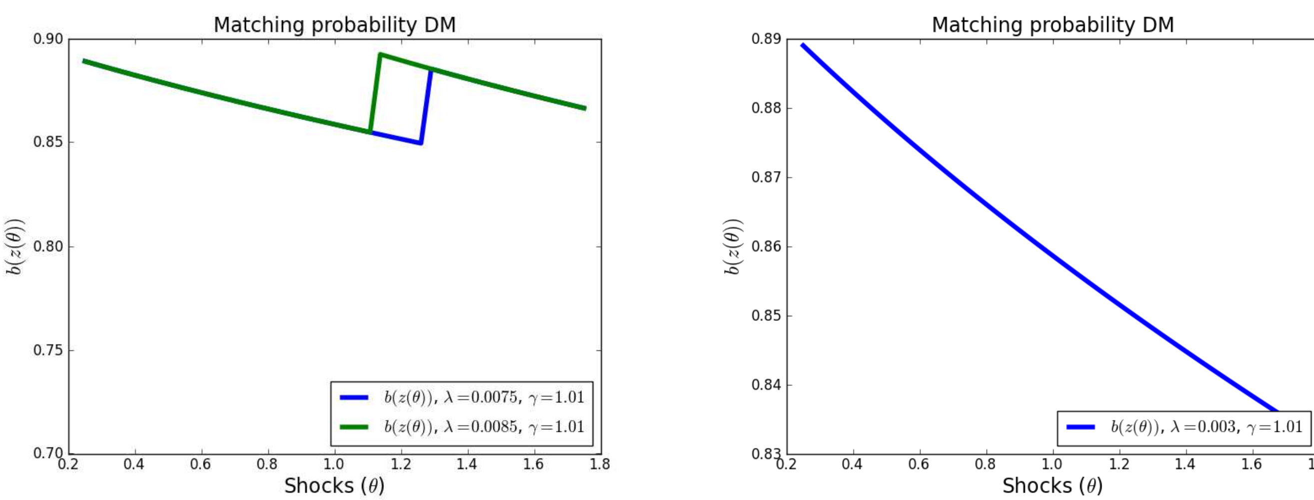

Figure 11: Matching probability choice of households in decentralized market

I can discuss the effects of open-market operations on the extensive and intensive margins using figures 10 and 11. A marginal open-market purchase of bond would decrease λ

and bond price (s). This policy will have no effects on the intensive margin (right graph in 10) and extensive margin (right graph in 11) when we have an asset market with no segmentation. As shown in figures 10 and 11 In a segmented asset market, open market purchase of bonds would shift the threshold for asset market participation to the right. This will decrease both the intensive and the extensive margins of trade for households in the participation margin. Higher real interest rate attracts a subset of households to the bond market. In the decentralized market these households choose to apply to submarkets with lower matching probability and lower output and this will decrease both the extensive margin and the intensive margin of trade.

Figure 12 shows that conditional on participating (/not participating) in the asset market, household with better income shock pay lower price per unit of output in the decentralized market.

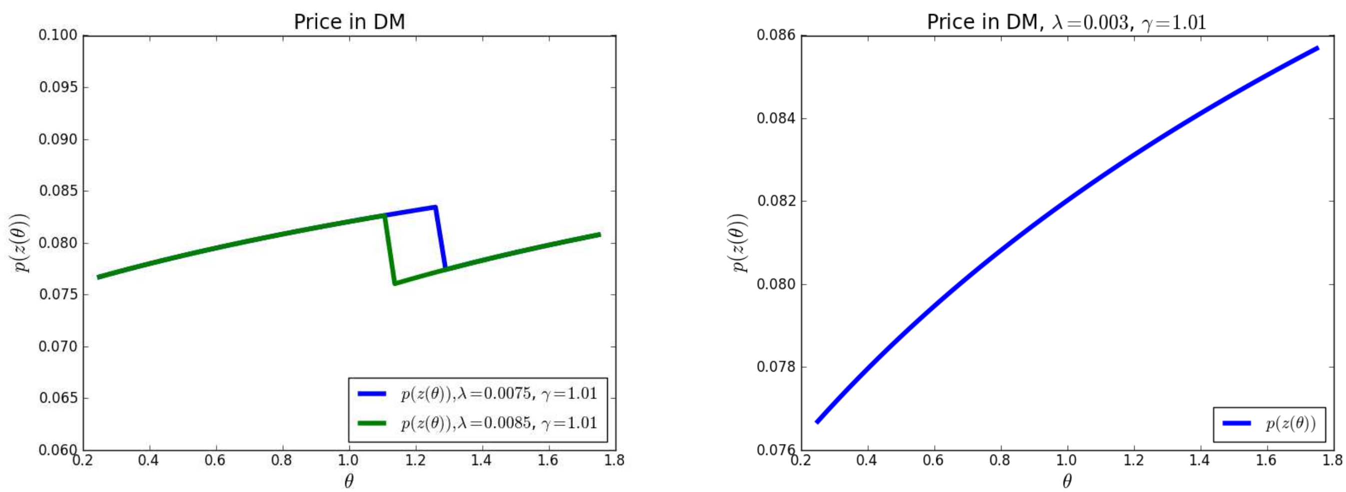

Figure 12: Price per unit choice of households in decentralized market

Figures 13 shows welfare for different values of bond supply and inflation rate. The central bank can generally affect overal welfare by purchasing bonds and supplying money. The policy of open-market purchase of bond is most effective when the asset market is segmented. This policy would increase the participation rate in the asset market and help households smooth consumption. By participating in the asset market, households are able to better insure themselves against idiosyncratic income shocks. When the asset market is not segmented, marginal open-market purchase/sale of bond would only change the real interest rate.

Figure 14 shows equilibrium bond price (s) for different amounts of bond supply (λ) and different inflation rates (γ). At each level of inflation bond price increases with higher supply

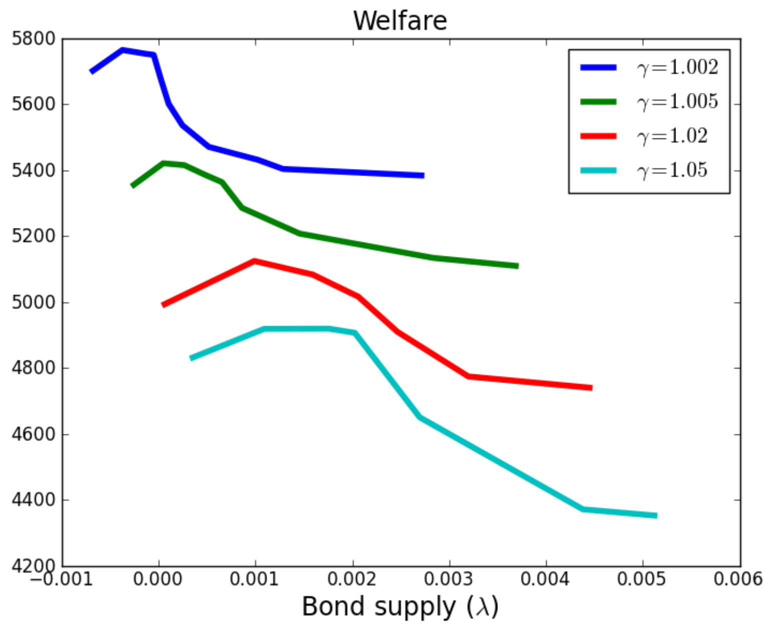

Figure 13: Welfare

of bonds[21].

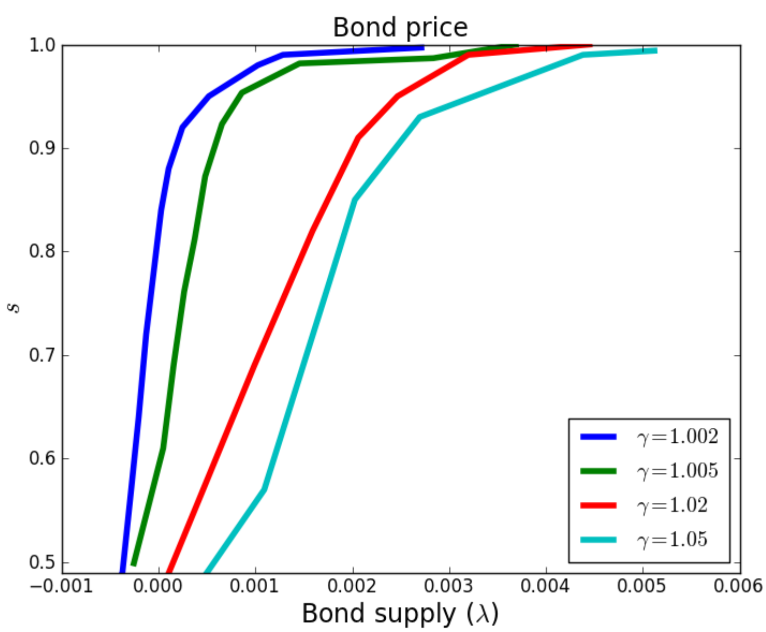

Figure 14: Bond prices

[21] Note that the price of bond is the inverse of nominal return on bonds

Figure 15 shows equilibrium wage (w) for different amounts of bond supply (λ). For a fixed rate of inflation, wage increases with bond supply.

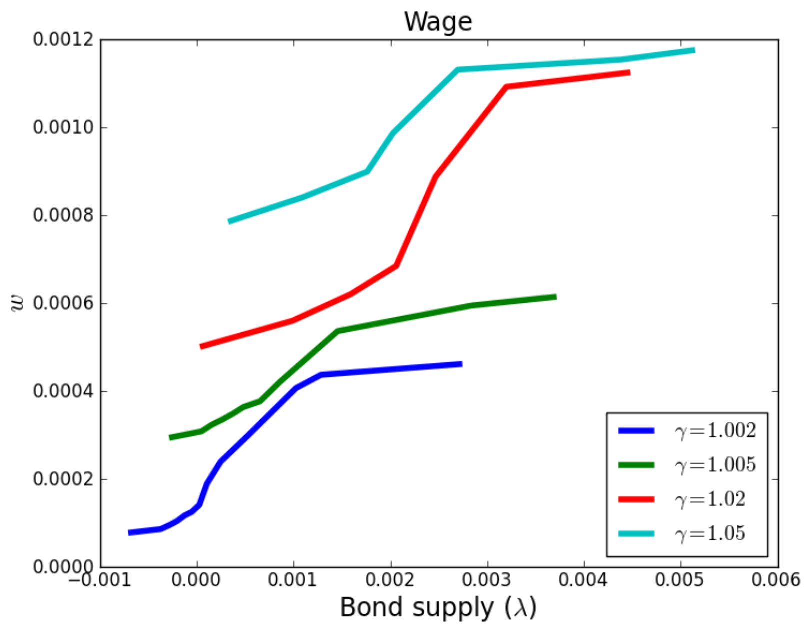

Figure 15: Wage

4.0.1 Exogenously segmented asset market

In this section I introduce another source of heterogeneity to the model. Following Alvarez et al. (2001), I assume only a fixed fraction of households attend the asset markets (traders), and the remaining never has access to the asset market (non-traders). This extension allows me to compare the results of this paper to the literature that assumes the asset markets are exogenously segmented[22]. Proposition 2 shows that the same logic from the case with endogenous asset market segmentation applies and asset market traders and non-traders solve optimization problems similar to the problem in the previous sections. The households' decisions are only linked through the market clearing conditions and prices. Households do not take in to account the distribution of asset holdings among traders and non-traders. The following proposition shows that the main results in the previous sections are robust to adding exogenously segmented asset market.

Proposition 2 *With exogenously segmented asset markets, value functions, policy functions and labor choices of traders in the asset market have the same properties as the case without exogenously segmented asset market.*

[22]e.g. Alvarez et al. (2001)

The formal proof is in the appendix 4.0.1.

Appendix

A Market clearing conditions

I can find the cumulative distribution of money before lotteries by:

$$G(m) = \int \int_{z^{-1}(m)}^{\overline{\theta}} dF(\theta)dH \tag{36}$$

and similarly the distribution of bond before lotteries follows:

$$H(a_{-1}) = \int \int_{a^{-1}(a_{-1})}^{\overline{\theta}} dF(\theta)dG \tag{37}$$

I assume a balanced budget for government at each period of time. The total real transfer that a household receives is the sum of transfers from printing money and the transfers received from bond market:

$$T = \frac{\gamma - 1}{w\gamma} + \frac{sA - A_{-1}}{wM'} \tag{38}$$

In the bond market the total amount of bonds supplied equals the sum of demanded bonds by households of different type. Thus, the market clearing for bonds gives:

$$\frac{As}{wM} = \int \int \int_{\underline{\theta}}^{\overline{\theta}} a(\theta)dF(\theta)dG(m)dH(a_{-1}) \tag{39}$$

In the general-good market, the market clearing condition is:

$$Y = \int_{\underline{\theta}}^{\overline{\theta}} y(\theta)dF(\theta) \tag{40}$$

LD is the same as Sun (2012):

$$\begin{aligned} LD = \quad & Y + \int_{\underline{\theta}}^{\overline{\theta}} \frac{\pi_1(z(\theta))b(L_1(z(\theta)))}{\mu(b(L_1(z(\theta))))} \left[k + \psi(q(L_1(z(\theta))))\mu(b(L_1(z(\theta))))\right] dF(\theta) \\ & + \int_{\underline{\theta}}^{\overline{\theta}} \frac{\pi_2(z(\theta))b(L_2(z(\theta)))}{\mu(b(L_2(z(\theta))))} \left[k + \psi(q(L_2(z(\theta))))\mu(b(L_2(z(\theta))))\right] dF(\theta) \end{aligned} \tag{41}$$

The firms zero-profit condition gives:

$$k + \psi\left(q(L_i(z(\theta)))\right)\mu(b(L_i(z(\theta)))) = L_i(z(\theta))$$

Then LD becomes:

$$\begin{aligned} LD = \quad & \int_{\underline{\theta}}^{\overline{\theta}} y(\theta)dF(\theta) + \int_{\underline{\theta}}^{\overline{\theta}} \pi_1(z(\theta))b\left(L_1(z(\theta))\right)L_1(z(\theta))dF(\theta) \\ & + \int_{\underline{\theta}}^{\overline{\theta}} \pi_2(z(\theta))b\left(L_2(z(\theta))\right)L_2(z(\theta))dF(\theta) \end{aligned} \tag{42}$$

Aggregate labor supply is the sum of households labor supply:

$$LS = \int_{\underline{\theta}}^{\overline{\theta}} \int \int l(m, a, \theta)dF(\theta)dG_a(m)dH(a_{-1})$$

in which dG_m is the distribution of money holdings at the beginning of the period. Substituting for l in the above equation we get

$$LS = \int_{\underline{\theta}}^{\overline{\theta}} \int \int \left[py(\theta) + z(\theta) + s\gamma a(\theta) - m - a_{-1} - T\right] dF(\theta)dG_a(m)dH(a_{-1}) \tag{43}$$

Substituting for T:

$$\begin{aligned} LS = \quad & \int_{\underline{\theta}}^{\overline{\theta}} \int \int [py(\theta) + z(\theta) + s\gamma a(\theta) - m - a_{-1} - \frac{\gamma - 1}{w\gamma} \\ & - \frac{sA}{wM'} + \frac{A_{-1}}{w\gamma M}]dF(\theta)dG_a(m)dH(a_{-1}) \end{aligned} \tag{44}$$

LS becomes:

$$\begin{aligned} LS = \quad & \frac{A_{-1}}{w\gamma M} - \frac{sA}{wM'} - \frac{\gamma - 1}{w\gamma} - \int mdG_a(m) - \int a_{-1}dH(a_{-1}) \\ & + \int_{\underline{\theta}}^{\overline{\theta}} [y(\theta) + z(\theta) + s\gamma a(\theta)]dF(\theta) \end{aligned} \tag{45}$$

Labor market clearing condition gives:

$$\int_{\underline{\theta}}^{\overline{\theta}} s\gamma a(\theta)dF(\theta) + \int_{\underline{\theta}}^{\overline{\theta}} \pi_1(z(\theta))(1-b(L_1(z(\theta))))L_1(z(\theta))dF(\theta)$$
$$+\int_{\underline{\theta}}^{\overline{\theta}} \pi_2(z(\theta))(1-b(L_2(z(\theta))))L_2(z(\theta))dF(\theta)$$
$$= \frac{sA}{wM'} + \frac{\gamma-1}{w\gamma} - \frac{A_{-1}}{w\gamma M} + \int mdG_a(m) + \int a_{-1}dH(a_{-1})$$

m is the distribution of money at the beginning of the period. Therefor, it consists of balances that are not spent plus the payments on nominal bonds:

$$\int mdG_a(m) =$$
$$\int_{\underline{\theta}}^{\overline{\theta}} \pi_1(z(\theta))[1-b(L_1(z(\theta)))]\frac{L_1(z(\theta))}{\gamma}dF(\theta)$$
$$+\int_{\underline{\theta}}^{\overline{\theta}} \pi_2(z(\theta))[1-b(L_2(z(\theta)))]\frac{L_2(z(\theta))}{\gamma}dF(\theta) + \int \frac{a_{-1}}{\gamma}dH(a_{-1}) + \int \frac{h_{-1}}{\gamma}dJ_{h_{-1}}$$

Plug in the labor market clearing condition:

$$\frac{sA}{wM'} + \frac{\gamma-1}{w\gamma} - \frac{A_{-1}}{w\gamma M} =$$
$$\int_{\underline{\theta}}^{\overline{\theta}} s\gamma a(\theta)dF(\theta) + (1-\frac{1}{\gamma})\int_{\underline{\theta}}^{\overline{\theta}} \pi_1(z(\theta))(1-b(L_1(z(\theta))))L_1(z(\theta))dF(\theta)$$
$$+(1-\frac{1}{\gamma})\int_{\underline{\theta}}^{\overline{\theta}} \pi_2(z(\theta))(1-b(L_2(z(\theta))))L_2(z(\theta))dF(\theta) - \int_{\underline{\theta}}^{\overline{\theta}} a_{-1}(1+\frac{1}{\gamma})dH(a_{-1}) - \int \frac{h_{-1}}{\gamma}dJ_{h_{-1}}$$

The labor-market-clearing can be written as:

$$\frac{1}{w\gamma}[\gamma-1-\lambda+s\lambda] =$$
$$(2-s\gamma)\int_{\underline{\theta}}^{\overline{\theta}} a(\theta)dF(\theta) + (1-\frac{1}{\gamma})\int_{\underline{\theta}}^{\overline{\theta}} \pi_1(z(\theta))(1-b(L_1(z(\theta))))L_1(z(\theta))dF(\theta)$$
$$+(1-\frac{1}{\gamma})\int_{\underline{\theta}}^{\overline{\theta}} \pi_2(z(\theta))(1-b(L_2(z(\theta))))L_2(z(\theta))dF(\theta) - \int \frac{h(\theta)}{\gamma}dF(\theta) \tag{46}$$

B Welfare analysis

I use the household's utility function to calculate welfare:

$$
\begin{aligned}
\varpi =& \int \int \int \{U(y) + u(q) - \theta l\} dF(\theta) dG(m) dH(a_{-1}) \\
=& \int U(y(\theta)) dF(\theta) + \int u(q(z(\theta))) dF(\theta) - \int \int \int \{\theta l\} dF(\theta) dG(m) dH(a_{-1})
\end{aligned}
$$

I can write the last integral as:

$$
\begin{aligned}
\int \int \int & (\theta l) dF(\theta) dG(m) dH(a_{-1}) = \\
& \int \left[\theta(y(\theta) + z(\theta) + h(\theta) + s\gamma a(\theta))\right] dF(\theta) \\
& - \int \theta \left(\int m dG_a \right) dF(\theta) - \int \theta \left(\int a_{-1} dH(a_{-1}) \right) dF(\theta) - T \int \theta dF(\theta)
\end{aligned}
$$

I have shown in the market clearing appendix the distribution of money before the lotteries is:

$$
\begin{aligned}
\int & m dG_a(m) = \\
& \int_{\underline{\theta}}^{\overline{\theta}} \pi_1(z(\theta))[1 - b(L_1(z(\theta)))] \frac{L_1(z(\theta))}{\gamma} dF(\theta) \\
& + \int_{\underline{\theta}}^{\overline{\theta}} \pi_2(z(\theta))[1 - b(L_2(z(\theta)))] \frac{L_2(z(\theta))}{\gamma} dF(\theta) + \int \frac{a_{-1}}{\gamma} dH(a_{-1}) + \int \frac{h_{-1}}{\gamma} dJ_{h_{-1}}
\end{aligned}
$$

I can substitute for the distribution of money (mdG_a) and labor supply (l) from the above equations, and for government transfers (T) from the the market clearing appendix to simplify the equation for welfare:

$$
\begin{aligned}
\varpi =& \int [U(y(\theta)) - \theta y(\theta) + u(q(z(\theta))) - \theta z(\theta) - \theta h(\theta) - s\gamma\theta a(\theta)]dF(\theta) \\
&+ \left[\int_{\underline{\theta}}^{\bar{\theta}} \pi_1(z(\theta))[1 - b(L_1(z(\theta)))]\frac{L_1(z(\theta))}{\gamma}dF(\theta)\right]\int \theta dF(\theta) \\
&+ \left[\int_{\underline{\theta}}^{\bar{\theta}} \pi_2(z(\theta))[1 - b(L_2(z(\theta)))]\frac{L_2(z(\theta))}{\gamma}dF(\theta)\right]\int \theta dF(\theta) \\
&+(1 + \frac{1}{\gamma})\left[\int a_{-1}dH_{a_{-1}}\right]\int \theta dF(\theta) + \frac{1}{\gamma}\left[\int h_{-1}dJ_{h_{-1}}\right]\int \theta dF(\theta) \\
&+\frac{1}{w\gamma}\left[\gamma - 1 - \lambda + s\lambda\right]\int \theta dF(\theta)
\end{aligned}
$$

C Proof of proposition 2

Lets assume there are two types of agents in the economy, traders in the asset market (denoted by subscript T) and non-traders (denoted by subscript N).

Value function of a trader:

$$W_T(m_T, a_{-1}, \theta) = \max_{y_T, l_T, z_T, a} U(y_T) - \theta l_T + V_T(z_T, h_T, a)$$

$$st. \;\; py_T + z_T + s\gamma a \leq m_T + a_{-1} + l_T + T$$

Value function of a non-trader:

$$W_N(m_N, \theta) = \max_{y_N, l_N, z_N} U(y_N) - \theta l_N + V_N(z_N, h_N)$$

$$st. \;\; py_N + z_N \leq m_N + l_N + T$$

Using the budget constraint to eliminate $l_{i=N,T}$:

$$W_T(m_T, a_{-1}, \theta) = \theta(m_T + T + a_{-1}) + \max_{y_T \geq 0} \{U(y_T) - \theta p y_T\} + \max_{z_T, a, h_T} \{-\theta(z_T + s\gamma a + h_T) + V_T(z_T, h_T, a)\}$$

$$W_N(m_N, \theta) = \theta(m_N + T) + \max_{y_N \geq 0} \{U(y_N) - \theta p y_N\} + \max_{z_N, h_N} \{-\theta(z_N + h_N) + V_N(z_N, h_N)\}$$

The optimal choices of $y_{i\in\{T,N\}}$, $z_{i\in\{T,N\}}$ and a must satisfy:

$$U'(y_T) = U'(y_N) = \theta \tag{47}$$

The above expression shows that a trader and a non-trader choose the same amount of consumption in the frictionless market:

$$y_T(\theta) = y_N(\theta) = y(\theta)$$

$$\frac{\partial V_T(z_T, h_T, a)}{\partial z_T} \begin{cases} \leq \theta & z_T \geq 0 \\ \geq \theta & z_T \leq \overline{m} - s\gamma a - h_T \end{cases} \tag{48}$$

$$\frac{\partial V_T(z_T, h_T, a)}{\partial h_T} \begin{cases} \leq \theta & h_T \geq 0 \\ \geq \theta & h_T \leq \overline{m} - s\gamma a - z_T \end{cases} \tag{49}$$

$$\frac{\partial V_T(z_T,h_T,a)}{\partial a}\begin{cases} \leq \theta s\gamma & a \geq 0 \\ \geq \theta s\gamma & sa \leq \overline{m} - z_T - h_T \end{cases} \tag{50}$$

$$\frac{\partial V_N(z_N,h_N)}{\partial z_N}\begin{cases} \leq \theta & z_N \geq 0 \\ \geq \theta & z_N \leq \overline{m} - h_N \end{cases} \tag{51}$$

$$\frac{\partial V_N(z_N,h_N)}{\partial h_N}\begin{cases} \leq \theta & h_N \geq 0 \\ \geq \theta & h_N \leq \overline{m} - z_N \end{cases} \tag{52}$$

The value functions can be written as:

$$W_T(m_T, a_{-1}, \theta) = W_T(0,0,\theta) + \theta m_T + \theta a_{-1} \tag{53}$$

Where:

$$W_T(0,0,\theta) = U(y(\theta)) - \theta y(\theta) + V_T(z_T(\theta), h_T(\theta), a(\theta)) - \theta(z_T(\theta) + h_T(\theta) + s\gamma a(\theta)) \tag{54}$$

$$W_N(m_T, \theta) = W_T(0,\theta) + \theta m_N \tag{55}$$

Where:

$$W_N(0,\theta) = U(y(\theta)) - \theta y(\theta) + V_N(z_N(\theta), h_N(\theta)) - \theta(z_N(\theta) + h_N(\theta)) \tag{56}$$

We can see that the value function $W()$ is linear in household's asset holdings for both traders and non-traders. Agents problem in the frictional market for traders and non-traders are similar. The difference comes from their value function which has 3 state variables for traders and 2 state variables for non-traders. After simplification and applying the lotteries as the previous section I can write agents value function as:

$$\begin{aligned} V_T(z_T,h_T,a) \quad &= \widetilde{V_T}(z) + \beta E\left[W_T(\frac{z_T+h_T}{\gamma}, a, \theta)\right] \\ &= \widetilde{V_T}(z_T) + \beta E\left[W_T(0,0,\theta)\right] + \frac{\beta E(\theta) z_T}{\gamma} + \frac{\beta E(\theta) h_T}{\gamma} + \beta E(\theta) a \end{aligned} \tag{57}$$

$$
\begin{aligned}
V_N(z_N, h_N) \quad &= \widetilde{V_N}(z) + \beta E\left[W_N(\frac{z_N + h_N}{\gamma}, \theta)\right] \\
&= \widetilde{V_N}(z_N) + \beta E\left[W_N(0,\theta)\right] + \frac{\beta E(\theta) z_N}{\gamma} + \frac{\beta E(\theta) h_N}{\gamma}
\end{aligned} \tag{58}
$$

Trader's and non-trader's choice of bond holding follows the following condition with complementary slackness:

$$
\begin{cases}
a(\theta) \geq 0 & \theta \geq \frac{\beta E(\theta)}{s\gamma} \\
a(\theta) \leq \overline{m_T} - z_T(\theta) - h_T(\theta) & \theta \leq \frac{\beta E(\theta)}{s\gamma}
\end{cases} \tag{59}
$$

$$
\begin{cases}
h_T(\theta) \geq 0 & \theta \geq \frac{\beta E(\theta)}{\gamma} \\
h_T(\theta) \leq \overline{m_T} - z_T(\theta) - a'(\theta) & \theta \leq \frac{\beta E(\theta)}{\gamma}
\end{cases} \tag{60}
$$

$$
\begin{cases}
h_N(\theta) \geq 0 & \theta \geq \frac{\beta E(\theta)}{\gamma} \\
h_N(\theta) \leq \overline{m_N} - z_N(\theta) & \theta \leq \frac{\beta E(\theta)}{\gamma}
\end{cases} \tag{61}
$$

The labor choices of traders are the same as the labor choices in equations 28 and 27. Labor choices of non-traders are as 62:

$$
l_N(m,\theta) = \begin{cases}
py(\theta) + z_N(\theta) - m_N - T_N & \theta > \frac{\beta E(\theta)}{s\gamma} \\
py(\theta) + z_N(\theta) - m_N - T_N & \theta = \frac{\beta E(\theta)}{s\gamma} \\
py(\theta) + \overline{m} - m_N - T_N & \theta < \frac{\beta E(\theta)}{s\gamma}
\end{cases} \tag{62}
$$

C.1 Market clearing condition and welfare measure

Similar to the case where all of the agents trade in the asset market the real transfer is:

$$
T = \frac{\gamma - 1}{w\gamma} + \frac{sA - A_{-1}}{wM'} \tag{63}
$$

The market clearing condition for the bond market and the general good market is the same as 29 and 40.

Similar to the case with only one type of agent, the labor demand can be written as:

$$
\begin{aligned}
LD = \quad & \int_{\underline{\theta}}^{\overline{\theta}} y(\theta) dF(\theta) + \int_{\underline{\theta}}^{\overline{\theta}} \pi_1(z_T(\theta)) b\left(L_1(z_T(\theta))\right) L_1(z_T(\theta)) dF_T(\theta) \\
& + \int_{\underline{\theta}}^{\overline{\theta}} \pi_1(z_N(\theta)) b\left(L_1(z_N(\theta))\right) L_1(z_N(\theta)) dF_N(\theta) \\
& + \int_{\underline{\theta}}^{\overline{\theta}} \pi_2(z_T(\theta)) b\left(L_2(z_T(\theta))\right) L_2(z_T(\theta)) dF_T(\theta) \\
& + \int_{\underline{\theta}}^{\overline{\theta}} \pi_2(z_N(\theta)) b\left(L_2(z_N(\theta))\right) L_2(z_N(\theta)) dF_N(\theta) \qquad (64)
\end{aligned}
$$

Labor supply is the sum of households labor supply:

$$
LS = \int_{\underline{\theta}}^{\overline{\theta}} \int \int l_T(m_T, a, \theta) dF_T(\theta) dG_a(m) dH(a_{-1}) + \int_{\underline{\theta}}^{\overline{\theta}} \int l_N(m_N, \theta) dF_N(\theta) dG_a(m)
$$

Substituting for labor choices and transfers:

$$
\begin{aligned}
Ls = \quad & \int_{\underline{\theta}}^{\overline{\theta}} \int \int \left[py(\theta) + z_T(\theta) + s\gamma a(\theta) - m_T - a_{-1}\right] dF_T(\theta) dG_a(m_T) dH(a_{-1}) \\
& + \int_{\underline{\theta}}^{\overline{\theta}} \int \left[py(\theta) + z_N(\theta) - m_N\right] dF_N(\theta) dG_a(m_N) \\
& - \frac{\gamma - 1}{w\gamma} - \frac{sA}{wM'} + \frac{A_{-1}}{w\gamma M} \qquad (65)
\end{aligned}
$$

$$
\begin{aligned}
LS = \quad & \frac{A_{-1}}{w\gamma M} - \frac{sA}{wM'} - \frac{\gamma - 1}{w\gamma} - \int m_T dG_a(m_T) - - \int m_N dG_a(m_N) - \int a_{-1} dH(a_{-1}) \\
& + \int_{\underline{\theta}}^{\overline{\theta}} [y(\theta) + z_T(\theta) + s\gamma a(\theta)] dF_T(\theta) + \int_{\underline{\theta}}^{\overline{\theta}} [y(\theta) + z_N(\theta)] dF_N(\theta) \qquad (66)
\end{aligned}
$$

Labor market clearing condition gives:

$$\frac{A_{-1}}{w\gamma M} - \frac{sA}{wM'} - \frac{\gamma-1}{w\gamma} - \int m_T dG_a(m_T) - \int m_N dG_a(m_N)$$
$$- \int a_{-1} dH(a_{-1}) + \int_{\underline{\theta}}^{\overline{\theta}} [z_T(\theta) + sa(\theta)] dF_T(\theta) + \int_{\underline{\theta}}^{\overline{\theta}} [z_N(\theta)] dF_N(\theta)$$
$$= \int_{\underline{\theta}}^{\overline{\theta}} \pi_1(z_T(\theta)) b\left(L_1(z_T(\theta))\right) L_1(z_T(\theta)) dF_T(\theta) + \int_{\underline{\theta}}^{\overline{\theta}} \pi_1(z_N(\theta)) b\left(L_1(z_N(\theta))\right) L_1(z_N(\theta)) dF_N(\theta)$$
$$+ \int_{\underline{\theta}}^{\overline{\theta}} \pi_2(z_T(\theta)) b\left(L_2(z_T(\theta))\right) L_2(z_T(\theta)) dF_T(\theta) + \int_{\underline{\theta}}^{\overline{\theta}} \pi_2(z_N(\theta)) b\left(L_2(z_N(\theta))\right) L_2(z_N(\theta)) dF_N(\theta)$$

Similar to the appendix A:

$$\int m_T dG_a(m_T) =$$
$$\int_{\underline{\theta}}^{\overline{\theta}} \pi_1(z_T(\theta))[1 - b(L_1(z_T(\theta)))] \frac{L_1(z_T(\theta))}{\gamma} dF_T(\theta)$$
$$+ \int_{\underline{\theta}}^{\overline{\theta}} \pi_2(z_T(\theta))[1 - b(L_2(z_T(\theta)))] \frac{L_2(z_T(\theta))}{\gamma} dF_T(\theta) + \int \frac{a_{-1}}{\gamma} dH(a_{-1}) + \int \frac{h_{-1T}}{\gamma} dJ_{h_{-1T}}$$

where $dG_a(m_T)$ is the traders' distribution of money holdings at the beginning of the period. Similarly, I can state the same for distribution of money holding among non-traders

$$\int m_N dG_a(m_N) =$$
$$\int_{\underline{\theta}}^{\overline{\theta}} \pi_1(z_N(\theta))[1 - b(L_1(z_N(\theta)))] \frac{L_1(z_N(\theta))}{\gamma} dF_N(\theta)$$
$$+ \int_{\underline{\theta}}^{\overline{\theta}} \pi_2(z_N(\theta))[1 - b(L_2(z_N(\theta)))] \frac{L_2(z_N(\theta))}{\gamma} dF_N(\theta) + \int \frac{h_{-1N}}{\gamma} dJ_{h_{-1N}}$$

Plug in the labor market clearing condition:

$$
\begin{aligned}
&\frac{1}{w\gamma}[\gamma-1-\lambda+s\lambda]= \\
&\quad(2-s\gamma)\int_{\underline{\theta}}^{\overline{\theta}} a(\theta)dF_T(\theta)+(1-\frac{1}{\gamma})\int_{\underline{\theta}}^{\overline{\theta}}\pi_1(z_T(\theta))(1-b(L_1(z_T(\theta))))L_1(z_T(\theta))dF_T(\theta) \\
&\quad+(1-\frac{1}{\gamma})\int_{\underline{\theta}}^{\overline{\theta}}\pi_2(z_T(\theta))(1-b(L_2(z_T(\theta))))L_2(z_T(\theta))dF_T(\theta)-\int\frac{h_T(\theta)}{\gamma}dF_T(\theta) \\
&\quad+(1-\frac{1}{\gamma})\int_{\underline{\theta}}^{\overline{\theta}}\pi_1(z_N(\theta))(1-b(L_1(z_N(\theta))))L_1(z_N(\theta))dF_N(\theta)-\int\frac{h_N(\theta)}{\gamma}dF_N(\theta) \\
&\quad+(1-\frac{1}{\gamma})\int_{\underline{\theta}}^{\overline{\theta}}\pi_2(z_N(\theta))(1-b(L_2(z_N(\theta))))L_2(z_N(\theta))dF_N(\theta)
\end{aligned}
$$

It can be shown as in appendix for the benchmark model that the measure of welfare is:

$$
\begin{aligned}
\varpi =&\int[U(y(\theta))-\theta y(\theta)-u(q(z_T(\theta)))-\theta z_T(\theta)-s\gamma\theta a(\theta)]dF_T(\theta) \\
&+\left[\int_{\underline{\theta}}^{\overline{\theta}}\pi_1(z_T(\theta))[1-b(L_1(z_T(\theta)))]\frac{L_1(z_T(\theta))}{\gamma}dF_T(\theta)\right]\int\theta dF_T(\theta) \\
&+\left[\int_{\underline{\theta}}^{\overline{\theta}}\pi_2(z_T(\theta))[1-b(L_2(z_T(\theta)))]\frac{L_2(z_T(\theta))}{\gamma}dF_T(\theta)\right]\int\theta dF_T(\theta) \\
&+(1+\frac{1}{\gamma})\left[\int a_{-1}dH_{a_{-1}}\right]\int\theta dF_T(\theta)+\frac{1}{\gamma}\left[\int h_{-1T}dJ_{h_{-1T}}\right]\int\theta dF_T(\theta) \\
&-\frac{1}{w\gamma}[\gamma-1-\lambda+s\lambda]\int\theta dF_T(\theta) \\
&+\int[U(y(\theta))-\theta y(\theta)+u(q(z_N(\theta)))-\theta z_N(\theta)]dF_N(\theta) \\
&+\left[\int_{\underline{\theta}}^{\overline{\theta}}\pi_1(z_N(\theta))[1-b(L_1(z_N(\theta)))]\frac{L_1(z_N(\theta))}{\gamma}dF_N(\theta)\right]\int\theta dF_N(\theta) \\
&+\left[\int_{\underline{\theta}}^{\overline{\theta}}\pi_2(z_N(\theta))[1-b(L_2(z_N(\theta)))]\frac{L_2(z_N(\theta))}{\gamma}dF_N(\theta)\right]\int\theta dF_N(\theta) \\
&+\frac{1}{\gamma}\left[\int h_{-1N}dJ_{h_{-1N}}\right]\int\theta dF_N(\theta)+\frac{1}{w\gamma}[\gamma-1-\lambda+s\lambda]\int\theta dF_N(\theta)
\end{aligned}
$$

As I have shown above, the problem of traders and non-traders in the bond market are very similar to the case with no exogenous segmentation in the asset market. The same logic from the case with endogenous asset market segmentation applies and asset market traders and non-traders solve optimization problems similar to the problem in the previous sections. The households' decisions are only linked through the market clearing conditions and prices. We have a partial block recursive equilibrium in which the distributions in the economy affects households' decision through prices. Households do not take in to account the distribution of asset holdings among traders and non-traders. The main results in the previous sections are robust to adding exogenously segmented asset market.

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