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Abstract

In Saltari et al. (2012, 2013) we estimated a dynamic model of the Italian economy. The main result of those papers is that the weakness of the Italian economy in the last two decades is due to the total factor productivity slowdown.

In those models the information and communication technology (ICT) capital stock plays a key role in boosting the efficiency of the traditional capital, and hence of the whole economy. The ICT contribution is captured in a multiplicative way through a weighting factor. The other key parameter to explain the Italian productivity decline is the elasticity of substitution.

Recent literature provides estimates of the elasticity of substitution well below 1 – thus rejecting the traditional Cobb-Douglas production function – though there is no particular value on which consensus converges. In our opinion, however, these estimates are affected by a theoretical specification problem. More generally, the technological parameters are long run in nature but the estimates are based on short-run data.

Our aim is to look more deeply into the estimation procedure of the technological parameters. The standard estimation results present a common fundamental problem of serially correlated residuals so that the standard errors will be underestimated (i.e. biased downwards). We think that at the root of this problem there are two theoretical issues: the estimated models are static in nature and do not incorporate frictions and rigidities.

Our modelling strategy takes into account, though implicitly, adjustment costs without leaving out the optimization hypothesis. Although we cannot in general say that this framework gets rid of the serial correlation problem, the correlation statistics for our model do show that residuals are not serially correlated.

JEL classification: C30; E 22; E23; O33.

Keywords: CES production function; Elasticity of substitution; Income distribution; Factor-augmenting technical progress and ICT technical change.

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1 Introduction

In Saltari *et al.* (2012, 2013) we estimated a dynamic disequilibrium model of the Italian economy. The main result of those papers is that the weakness of the Italian economy in the last two decades has been the total factor productivity slowdown. To investigate the roots of this productivity decline, we draw attention to the reducing pace of capital accumulation. The model in both papers is based on the distinction between traditional and innovative capital. In a nutshell, our main finding shows that there exists a structural and persistent gap between "optimal" and observed output which, moreover, increased in the latter part of the sample period.

In those model the ICT capital stock plays a key role in boosting the efficiency of the traditional capital, and hence of the whole economy.[1] Formally, the ICT contribution is captured in a multiplicative way through a weighting factor.[2] The other key parameter to explain the Italian productivity decline is the elasticity of substitution. Since the introduction in the economic analysis by Hicks (1932) and its reformulation by Robinson (1933), the elasticity of substitution has attracted interest by both theoretical and empirical researchers for its central role in many fields such as economic growth, fiscal policy and development accounting. This renewed interest provides estimates consistently below 1 – thus rejecting the traditional Cobb-Douglas production function – though there is no particular value on which the consensus converged. In our opinion, however, these estimates are affected by a specification problem. We will see that this problem has theoretical roots. The technological parameters are long run in nature but are the result of an estimation based on short-run data. In our opinion, the "real" issue is to bridge this gap.

Economic literature has addressed this problem substantially in two ways. The first is based on statistical tools (such as cointegration, filtering, or simply assuming away the existence of the divergence) to recover long run technological parameters from the short run data. The second is to recognize the existence of short run adjustment problems and to model them either explicitly, e.g. as in the Tobin's q framework (see Chirinko 2008 for a comprehensive survey of both lines of research) or implicitly using ad hoc distributed lag processes not motivated by any form of optimization behavior. However, both methods are in some sense inappropriate in that they do not explicitly incorporate the dynamic effect of these costs on the factor inputs in estimating the elasticity of substitution.

Our aim in this paper is to look more deeply into the estimation procedure of the two technological parameters, the elasticity of substitution and the weight of ICT as a factor augmenting the efficiency of traditional capital. We proceed in two steps.

In the first we stay within the standard framework and run a number of estimates,

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[1]The model also assumes that the market environment is one of imperfect competition where firms have similar production functions but different endowments and their products are sufficiently differentiated that they are monopolistic competitors in the short run, setting their own prices. Thus they may set prices according to their marginal costs plus some mark-up or margin. As a consequence, each firm is assumed to be a "quantity-taker" and aims to supply the amount demanded.

[2]Our purpose in the next future is to extend this specification to a nested CES function to allow a better representation of the effect of ICT in the production function.

using both single- and system-equation approaches. The estimation procedure employs normalization as an instrument which allows us to properly identify the deep technological parameters through a suitable choice of baseline point.

We begin with the single-equation approach in that we directly estimate the non-linear CES to recover the elasticity of substitution which better approximate the dynamics of the observed output, calibrating the weight ICT parameter. Since, as we see below, this gives an "unrealistic" value of the elasticity of substitution, we also estimate the weight of the ICT capital. Our results show that single-equation approaches are largely unsuitable for jointly uncovering the elasticity of substitution and the weight of ICT. We then build a system of two equations, the production function and the income share ratio derived from the two first-order conditions of the factor inputs.

This is the most popular estimation method. It is an approach based on two assumptions: there is an instantaneous adjustment of the marginal products to their user costs; it does not consider interactions with other markets. Within this framework, we get estimates for the elasticity of substitution and the weight of ICT. However, the estimation results of these exercises present a common fundamental problem in that the error term is serially correlated so the standard errors will be under-estimated (i.e. biased downwards). At the root of this problem there is a theoretical issue: the estimated models are static in nature and do not incorporate frictions and rigidities. Thus, for instance, the production function is estimated without any correction for the costs of rigidities. The same holds for the estimation of income share ratio since it implicitly hypothesizes instantaneous adjustment between marginal products and input prices. Our model overcomes these difficulties by explicitly incorporating these costs.

The second step compares the results of our specification with those obtained from the standard estimation procedure

This comparison suggests that the more popular approach of using a system with instantaneous adjustment is biased: for example, the weight of ICT appears to be underestimated. Our model is based on the idea that firms optimize their intertemporal profits subject to the production function but taking account of rigidities, adjustment costs and other frictions. This produces a model which, at least to an approximation, enables the true parameters of the production function to be separated from the costs of adjustment, thus eliminating the autocorrelation in the residuals. The parameters then are not biased by those costs. When we take account of these costs, we find an estimated elasticity well below unity, of about two-thirds.

The organization of the paper is as follows. The next section provides a brief literature review. In sections 3 we discuss the theoretical origins of the misspecification problem while section 4 contains a short description of two issues related to the estimation of technological parameters. Section 5 gives the main empirical findings of our model. Section 6 "normalizes" the model and section 7 reports the results of the traditional approach to the estimation of the technological parameters. Section 8 compares our estimation procedure with the standard one, offering some insights for the solution of the misspecification issue. Section 9 concludes.

2 Related Literature

The paper is related to the modern growth literature (e.g. Acemoglu 2008, La Grandville 2008, Aghion and Howitt, 2009) that emphasizes the power of the CES production function. In recent years, the CES production technology has returned to the center of growth theory and increasingly empirical evidence shows that the non-unity elasticity of substitution allows recognizing the existence of biased technical change (see Chirinko *et al.* 1999, Klump *et al.* 2008, León-Ledesma *et al.* 2010). The wider use of CES technologies opens the door to a deeper understanding of the effects of variation in the elasticity of substitution on economic growth (Turnovsky, 2002).

As pointed out by Nelson (1965), the elasticity of substitution can be interpreted as an index of the rate at which diminishing marginal returns set in as one factor is increased with respect to the other. If the elasticity of substitution is large, then it is easy to substitute one factor for the other. Therefore, the greater the elasticity of substitution the smaller the drag caused by diminishing returns. From this interpretation, it is straightforward to notice that the elasticity of substitution will affect the growth rate of output when factors of production are increasing at different rates so that their ratio is changing. The use of a Cobb-Douglas production function, as in most cases in the literature, is a misleading approximation for the behavior of the aggregate economy and hides the role of the elasticity of substitution not only as a source of increase in output but also as a source of technical change. If the elasticity of substitution in production is a measure of how easy it is to shift between factor inputs, typically labor and capital, it provide a powerful tool to answer questions about the distribution of national income between capital and labor.

The relevance of the elasticity of substitution and its relationship with economic growth and technical change has been established since Hicks (1932) and Solow (1957). However, it was after Arrow et al. (1961) that here was a boost on the theoretical and empirical issues involving the elasticity of substitution. More recently, La Grandville (1989) gives proof of the positive relationship between the elasticity of substitution and the output level. On the discussion about the theoretical and empirical role of the CES in the dynamic macroeconomics, see also Klump and Preissler 2000, Klump and La Grandville 2000, Klump et al. 2008 and La Grandville 2009.

Although the CES production technology seems relatively straightforward, its mathematical simplicity can be misleading. La Grandville (1989), Klump and La Grandville (2000), Klump and Preissler (2000) and Klump et al. (2008) have emphasized that the economic interpretation of the CES production technology requires attention and they advocate the use of normalized production function when analyzing the consequences of variation in the elasticity of substitution. Normalization increases the usefulness of CES production functions for growth theorists, and this has led to its use in subsequent work such as Miyagiwa and Papageorgiou (2007) and Papageorgiou and Saam (2008). Normalization starts from the observation that a family of CES functions whose members are distinguished only by different elasticities of substitution need a common benchmark point. Since the elasticity of substitution is originally defined as point elasticity, one

needs to fix benchmark values for the level of production, factor inputs and for the marginal rate of substitution, or equivalently for per-capita production, capital deepening and factor income shares.[3]

3 Two relevant issues

Before addressing the technical aspect, we deem necessary to bring to attention two far-reaching features of the recent evolution of the economic environment of the main industrialised countries in the last decades, which are not only relevant by themselves but also because they affect the estimation robustness.

3.1 ICT role

Several recent studies have stressed the importance of ICT[4] as a key factor behind the upsurge in the USA productivity after 1995 (see among others, Colecchia and Schreyer, 2001; Stiroh, 2002; Jorgenson, 2002). With regards to Europe, EU countries fall well below the United States in terms of ICT penetration (Timmer and van Ark, 2005). Whereas there exist a huge literature for the US economy, the literature is relatively scarce for Italy (see European Commission 2013). By now, it is an accepted fact that the setback of the Italian labour productivity in the last twenty years is explained by two factors: a marked slowdown of capital deepening accompanied by a striking negative contribution of TFP.

To go a step further, notice that these two phenomena go hand-in-hand and are both relevant in explaining the standstill of labour productivity. Capital accumulation is important because, as is well known at least since Solow (1957), most of technical progress is embodied in new capital goods. In fact, what the data about capital deepening show is that in the Italian economy during the last 15 years there occurred a shift towards less capital intensive techniques, thus reducing the efficiency of employment. This shift and the lack of adoption of new technologies, especially of the ICT variety, have been favoured by the particular structure of the Italian specialization, skewed towards the traditional sectors with low technological content and less skilled workers. That is, not only the investment pace decreased in the last 15 years but it was also redirected toward traditional sectors rather than the innovative ones. Such a change in capital accumulation mix explains why both TFP and capital intensity rates decreased at the same time.

To confirm this last point, it is enough to have a look at the Figure 1, where the capital input growth rates for the total economy, the ICT and the non-ICT sectors are depicted.

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[3]See the recent survey by Klump et al. (2011) for a discussion on the normalization issue.

[4]For ICT we refer to ICT producing sectors. That is, hardware, software, and office equipment.

Figure 1 Capital accumulation in Italy (growth rates percent)
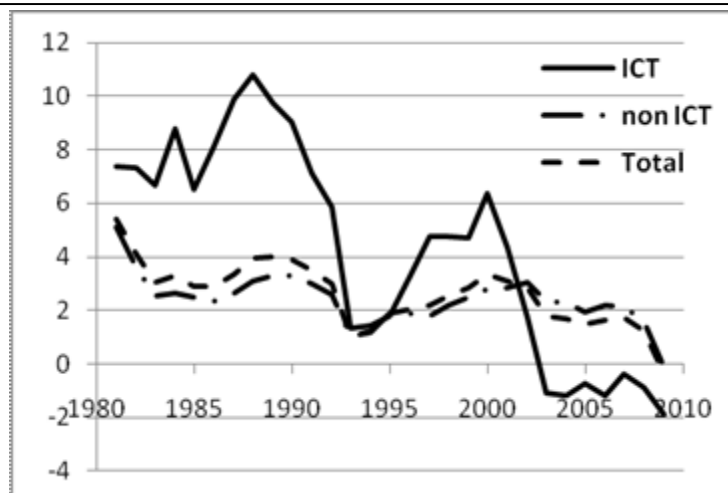
The figure makes clear two aspects of the trend of capital accumulation in Italy. The first is that the dynamics of total capital accumulation mostly follows that of capital accumulation in the traditional (non ICT) sector: the two lines essentially go hand-in-hand. The second is that the investment rate in the *ICT* sector accelerates up to the end of 1980s, and then slows down, albeit with a recovery in the mid-1990s. Notice that it becomes negative in the most recent years.

The contribution of the *ICT* sector to the productivity dynamics has not been modelled. The bulk of the literature assumes that technical progress grows at a constant rate without giving a specific structure within which the *ICT* does play any role (a partial exception is Klump et al. 2008). In our model we take a stance about how *ICT* impacts on technical progress: particularly, we assume that the productivity of the traditional capital stock is augmented by the *ICT* capital stock. This makes a difference with respect to the traditional approach in that the effect of *ICT* is not constant but reflects the pace of investment in innovative technologies.

3.2 The decline of labour share

Evidence shows that since the 80s the labor share has dramatically changed its behavior. Differently from the "stylised fact" of aggregate factor shares constancy, the last three decades have viewed a continuous decline of the labor share, thus casting doubt on the shares invariance. The decline of labor share is not limited to Italy but occurred in the large majority of industrialized countries (see Karabarbounis and Neiman, 2013; Elsby et al. 2013). Empirically, this is a problem for it implies non-stationarity in the income shares, an issue difficult to deal with. The stability of the labor share of income is a key foundation in most macroeconomic models taken for granted until very recently. The figure (2) shows the dynamics of aggregate labor share in Italy, together with those of France, Germany and USA starting from 1970. Actually, until the 70s the labor share

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was approximately constant in almost all the countries thus confirming one of the stylized fact highlighted by Kaldor (1961).

Figure 2 Labour share dynamics
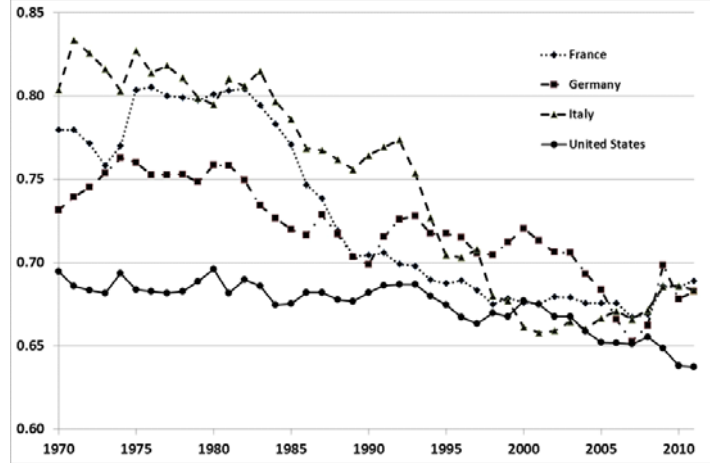
The figure (2) shows that up to the 70s the stationarity of factor share is more or less confirmed. However, starting from the following decade the decline of labor share becomes evident: for the period 1980-2011 the reduction is 11 per cent for Italy and France, 8 per cent in Germany, and 6 per cent in the USA. Obviously, this downward trend will not last forever. It seems that in the past thirty years the income shares dynamics has been (at least locally) nonstationary; in other words, it is likely that this process will come to a halt. The local nonstationarity will create problems since it is an independent source of serial correlation. As far as we know, this is a critical issue which is not taken into account in the estimation of the technological parameters of the production function, and especially in that of the elasticity of substitution. Though this is a relevant question, it is not clear which kind of way out can be adopted.

4   Our model

The core of the model is given by the aggregate production function

$$Y_t = \beta_3 \left[\left(C_t{}^{\gamma_1} K_t \right)^{-\beta_1} + \left(\beta_2 \, e^{\mu_K t} \, Ls_t{}^{\gamma_s} \, Lu_t{}^{\gamma_u} \right)^{-\beta_1} \right]^{-\frac{1}{\beta_1}} \tag{1}$$

In equation (1) $\mu_K = \lambda_K + \gamma_1 \lambda_C$ is the growth rate of labor efficiency and λ_K and λ_C are the rates of technical progress in the use of traditional capital stock K and innovative capital, C. These terms may be interpreted as an indication of the expected long-run term rates of growth, providing the system is stable. The coefficient β_2 is the labor augmenting technical progress, while β_3 is a measure of the total factor productivity. The efficiency of traditional fixed capital stock is augmented by ICT capital, C, with a weighting factor

equal to γ_1; the elasticity of substitution is given by $e_1 = \frac{1}{1+\beta_1}$. Defining L as a Cobb-Douglas function of the skilled and unskilled labor components, $Ls^{\gamma_s} Lu^{\gamma_u}$, .we get:

$$Y_t = \beta_3 \left[\left(C_t^{\gamma_1} K_t \right)^{-\beta_1} + \left(\beta_2 e^{\mu_K t} L_t \right)^{-\beta_1} \right]^{-\frac{1}{\beta_1}} \qquad (2)$$

The estimated dynamics of the Italian national domestic product (NDP) is reproduced, together with the actual one, in figure 3. A visual inspection of the figure reveals that the model replicates pretty well what happened in Italy in the period under observation (the correlation coefficient is 0.99). However, a persistent gap exists between the estimated and observed dynamics of the Italian NDP which, moreover, tends to widen towards the end of the sample period. On average over the sample period, the gap between the estimated and observed NDP is 8 per cent.

Figure 3 The dynamics of estimated and observed NDP
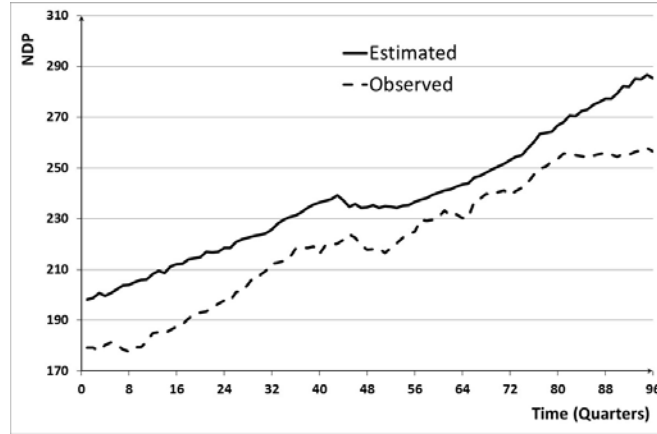
This production function can be easily transformed in the well know form introduced in the literature by Arrow et al.(1961):

$$Y_t = q \left[w\left(C_t^{\gamma_1} K_t \right)^{-\beta_1} + (1-w) \left(\beta_2 e^{\mu_K t} L_t \right)^{-\beta_1} \right]^{-\frac{1}{\beta_1}} \qquad (3)$$

where the "efficiency" parameter is defined as $q = \dfrac{\beta_3}{\left(1+\beta_2^{-\beta_1}\right)^{\frac{1}{\beta_1}}}$ and the "distribution" parameter as $w = \frac{1}{1+\beta_2^{-\beta_1}}$.

This equation leads to the specification of a structural dynamic model of general and ICT investment functions, skilled and unskilled labour sectors, and price determination under imperfect competition (see Appendix for details) which allows us to estimate, among others, the parameters of the production function (2). The key element of the model is played by the adjustment costs and frictions which hamper the instantaneous equality between factor marginal products and their prices. Differently from the traditional approach, the capital stocks adjust more slowly to their marginal products. These

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rates of adjustment reflect the costs and risks of firms changing their capital stock. Analogously, it is not assumed that labor market instantaneously clear but rather that there are imperfections and frictions – such as those measured by Employment Protection Legislation index.

Moreover, and again differently from the traditional approach to economic growth, the model does not assume that capital- and labor-augmenting efficiency grow at constant rates. For instance, as is well known the traditional growth model adopts the Harrod-type technical progress in order for the model to have a steady state with constant growth rates. While such a steady state requirement greatly simplifies the analysis of the model, the constraints it imposes are stringent and preclude behaviour that may well be a crucial feature of economic systems.

Rather, our model assumes that the efficiency of traditional fixed capital stock is augmented by ICT capital according to a weighting factor equal to γ_1. As a consequence, it does not necessarily have a steady state; notwithstanding it might have a local attractor. In other words, the model does not impose the condition that the economy necessarily converges to a steady state, as does much of the existing literature. We do not introduce from the start such a condition but let the estimates tell us what kind of dynamics the economy has. Even if the model economy may not have a steady state and not be stable in a classical sense, it may still have an attractor and be stable.

Inserting the exogenous values of C_t, K_t and L_t. We estimate the dynamics of Y_t for the period 1980:Q2–2005:Q1, a total of 100 quarters.[5] For the reader convenience, the model estimates are reported in table 1.

Table 1 Parameter Estimates
(asymptotic standard errors in parenthesis)

β_1	σ_1	β_2	β_3	β_4	σ_4	β_5	β_6	β_8	β_{10}
0.519	0.658	27.075	0.869	0.76	0.568	36.863	1.198	0.008	0.110
(0.0045)	(0.020)	(3.598)	(0.031)	(0.179)	(0.058)	(12.931)	(0.043)	(0.002)	(0.018)

β_{12}	β_{13}	γ_1	γ_S	γ_U	λ_K	λ_C	λ_S	λ_U	$\ln\left(L_{U0}\right)$
0.850	1.092	0.048	0.027	0.971	0.001	0.036	0.012	0.003	1.386
(0.137)	(0.022)	(0.013)	(0.005)	(0.010)	(0.001)	(0.005)	(0.004)	(0.002)	(0.243)

Our estimation of the elasticity of substitution ($\sigma_1 = 0.658$) is confirmed by recent econometric studies.

These contributions find values of σ_1 that are consistently below unity, but a great deal of variation in the results persists. Pereira (2003) surveyed major papers in the field from the past 40 years and found that,in general, elasticity values were below unity. A recent survey by Chirinko (2008) looked at modern studies of the elasticity parameter and found considerable variation in cross-study results. However, the weight of the evidence suggested a range of σ_1 that is between 0.4 and 0.6, with the assumption of Cobb-Douglas being strongly rejected. Klump et al. (2008) estimated a long-run supply model for the euro area over the period 1970-2005 and they found an aggregate elasticity of

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[5]In our estimation period there are 100 quarters but 4 have been discarded for estimation reasons.

substitution below unity (around 0.7). Mallick (2012) obtained the elasticity parameters for 90 countries by estimating the CES production function for each country separately using respective country time series spanning for the period 1950–2000. The mean value for all 90 countries is 0.338. The mean values for the East Asia and Sub-Saharan African countries are 0.737 and 0.275, respectively. For the $OECD$ countries the mean is 0.340. A clear pattern is evident, he concludes, that, on average, the value of elasticity increases secularly with the growth rate of per capita GDP. One problem with interpreting these cross-study results is that the various analyses are not all measuring the same thing: the results found are generally sensitive to sample size and estimation techniques. La Grandville (1989), Klump and La Grandville (2000) emphasize the role of normalization of the CES production function because it makes more consistent cross-study estimates of the elasticity parameter.

In the following sections we compare the estimates of the technological parameters σ_1 and γ_1 reported in Table 1 with those obtained employing the most frequently approaches: single equation, two- and three-equation system. Single equation estimates concentrate either on the production function or on the first-order conditions, while the system approach combines them exploiting cross-equation restrictions.

It is important to note that the parameters of the CES occur throughout the model in the various marginal product conditions that arise from cost minimisation. The way in which they occur varies with the specific marginal functions. The FIML estimator used ensures that all of the cross-equation constraints implicit in these functions are imposed in the estimation and hence the parameter estimates are consistent across the model.[6]

5 Normalization

Following Klump and La Grandville (2000) and Klump and Preissler (2000), we "normalize" the production function so that allocations and factor income shares are held constant as the input substitution elasticity is changed. The normalization procedure identifies a family of CES production functions that are distinguished only by the elasticity parameter.[7] Normalization is a way to represent the production function so that the variables are independent of the unit of measure, i.e. in an index number form. This makes the parameter estimation easier.[8]

To begin with, we set the base period used for the normalization at the middle of the sample, $t = 48$, corresponding to 1993:Q3. To simplify notation, we denote this period by

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[6]This increases the (statistical) efficiency of the estimates, i.e. they have a lower asymptotic standard error.

[7]Klump and Saam (2008) emphasize that normalization is necessary to avoid "arbitrary and inconsistent results."

[8]It should be emphasized that while the normalization issue is useful in an analysis of the properties of the production function and of importance in some estimation, it does not affect the estimates of our model. In this model, the specification of the equations being estimated are such that models with different normalizations are stochastically equivalent. Once one has consistent estimates of the parameters (as in the FIML case), the functions may be viewed in other ways for analysis. It does not affect their properties.

the index 0. Normalization implies that all the variables are expressed in terms of their baseline values, that is K_0, L_0 and Y_0.

To normalize the production function, we start with the production function:

$$Y_t = \beta_3 \left[\left(KIT_t \right)^{-\beta_1} + \left(\beta_2 \, e^{\mu_K \, (t-t_0)} L_t \right)^{-\beta_1} \right]^{-\frac{1}{\beta_1}} \tag{4}$$

where t_0 is the base period and, to simplify notation, we set $KIT = K \, C^{\gamma_1}$.

Under imperfect competition, factor compensation is subject to a mark-up, by hypothesis constant and denoted by β_{13},[9] so that in any period t the following relation holds:

$$\left(i_t KIT_t + w_t L_t \right) \beta_{13} = Y_t$$

where i_t is the real interest rate and w_t is the wage rate.[10]

In the reference period capital compensation is:

$$i_0 = \frac{1}{\beta_{13}} \frac{\partial Y_0}{\partial KIT_0} = \frac{(\beta_3)^{-\beta_1}}{\beta_{13}} \left(\frac{Y_0}{KIT_0} \right)^{1+\beta_1}$$

so that total capital compensation over total factor income, or the capital share, in the base period is

$$\pi_0 = \frac{i_0 KIT_0}{Y_0} \beta_{13} = (\beta_3)^{-\beta_1} \left(\frac{Y_0}{KIT_0} \right)^{\beta_1} \tag{5}$$

Likewise, the labor compensation in the base period is

$$w_0 = \frac{1}{\beta_{13}} \frac{\partial Y_0}{\partial L_0} = \frac{(\beta_3 \beta_2)^{-\beta_1}}{\beta_{13}} \left(\frac{Y_0}{L_0} \right)^{1+\beta_1}$$

so the labour share is

$$1 - \pi_0 = \frac{w_0 L_0}{Y_0} \beta_{13} = (\beta_3)^{-\beta_1} \left(\frac{Y_0}{\beta_2 L_0} \right)^{\beta_1} \tag{6}$$

Notice that labour share expressed in efficiency units is simply $\beta_2 L$ since in the base period the time-dependent efficiency factor disappears.

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[9] A margin over and above the input marginal products is the traditional way to include the markup. An alternative is Rowthorn (1999), which adds the extraprofit from market power into the capital income share. We choose the former since formally it is the easiest way to take into account the existence of imperfect competition.

[10] The wage rate w is given by:

$$\frac{\partial (wL)}{\partial L} = w_s \frac{L_s}{\gamma_s L} + w_u \frac{L_u}{\gamma_u L}$$

Similarly, the unit capital compensation is:

$$\frac{\partial (qKIT)}{\partial KIT} = (r - D \ln p + \beta_8) \frac{K}{KIT} + (r - D \ln p + \beta_{10}) \frac{C}{\gamma KIT}$$

where β_8 and β_{10} are the risk premia relative to traditional and innovative capital stocks and q is unit capital compensation.

Substitute into the production function (4) the capital share evaluated in the base period:

$$Y_t = \left[\pi_0 \left(\frac{Y_0}{KIT_0} \right)^{-\beta_1} (KIT_t)^{-\beta_1} + \left(\beta_3 \beta_2 e^{\mu_K \ (t-t_0)} L_t \right)^{-\beta_1} \right]^{-\frac{1}{\beta_1}}$$

Following an analogous procedure for the labor share (6), we have:

$$Y_t = Y_0 \left[\pi_0 \left(\frac{KIT_t}{KIT_0} \right)^{-\beta_1} + (1 - \pi_0) \left(e^{\mu_K \ (t-t_0)} \frac{L_t}{L_0} \right)^{-\beta_1} \right]^{-\frac{1}{\beta_1}} \tag{7}$$

In the index number form, the production function is:

$$\frac{Y_t}{Y_0} = \left[\pi_0 \left(\frac{KIT_t}{KIT_0} \right)^{-\beta_1} + (1 - \pi_0) \left(e^{\mu_K \ (t-t_0)} \frac{L_t}{L_0} \right)^{-\beta_1} \right]^{-\frac{1}{\beta_1}} \tag{8}$$

For simplicity, this last equation will be rewritten as:

$$y_t = \left[\pi_0 \left(kit \right)^{-\beta_1} + (1 - \pi_0) \left(e^{\mu_K \ (t-t_0)} l_t \right)^{-\beta_1} \right]^{-\frac{1}{\beta_1}} \tag{9}$$

In the capital intensive form with inputs expressed in efficiency units the equation becomes

$$\frac{y_t}{e^{\mu_K \ (t-t_0)} l_t} = \left[\pi_0 \left(\frac{kit}{e^{\mu_K \ (t-t_0)} l_t} \right)^{-\beta_1} + (1 - \pi_0) \right]^{-\frac{1}{\beta_1}}$$

There are two points worth making about equation 8. First, under imperfect competition with a non-zero mark-up, the distribution parameter π_0 equals the share of capital income over total factor income, the sum of labour and capital income. Second, in the normalized production function the only key parameter is β_1 which is related to the elasticity of substitution, σ_1.

Before performing any estimation exercise using normalization, we need to fix income shares in the benchmark period. Employing observed data for capital, labour and output and our parameters estimates of table 1, the capital share for the Italian economy, see equation (5), is:

$$\pi_0 = (\beta_3)^{-\beta_1} \left(\frac{Y_0}{KIT_0} \right)^{\beta_1} = 0.24$$

so that labour income share is

$$1 - \pi_0 = 0.76$$

Since these estimates are quite close to those present in different databanks (such as OECD, EU KLEMS, AMECO), we adopt these shares for the reference period.

6 Estimation results

6.1 Single-equation approach

The single-equation approach has been used for parameters estimation following two alternative routes: the production function and the optimizing behavior present in the equations of the income shares. We discuss these two estimation directions in the following subsections.

6.1.1 Technology

Let us begin with the estimation of the production function (9). We first estimate only β_1, setting the other parameters $(\gamma_1, \lambda_K, \lambda_C)$ at their values in table (1), with nonlinear least squares. Specifically, we used the production function in log form:

$$\ln(y_t) = -\frac{1}{\hat{\beta}_1} \ln\left[\pi_0 kit^{-\hat{\beta}_1} + (1 - \pi_0) e^{\mu_K \ (t-t_0)} l_t^{-\hat{\beta}_1}\right]$$

where $\hat{\beta}_1$ is the estimated value on observed NDP.

This produces an estimate equal to $\hat{\beta}_1 = 13.6$, which is significantly different from zero at the 5 percent level and has an R-squared equal to $R^2 = 0.97$. The implied value of the elasticity of substitution is $\hat{\sigma}_1 = \frac{1}{1+\hat{\beta}_1} = 0.068$. Notwithstanding the high significance level and the good fit, this estimates presents at least two problems. First, the implied level of σ_1 is quite low and "unrealistic". Second, and more importantly, the Durbin-Watson statistics is very low ($DW = 0.12$) indicating the existence of serial correlation in the residuals.[11] The strong residual autocorrelation invalidates the $\hat{\beta}_1$ estimate and suggests the presence of a misspecification problem.

As we saw above, ICT played a key role in the Italian economic dynamics. Hence, we try to fix, at least partly, the specification problem explicitly extending the estimation to the weight of ICT. Consequently, we jointly estimate the elasticity of substitution and the role of ICT in increasing the efficiency of traditional capital. This gives rise to estimates, $\hat{\beta}_1 = 11.5$ and $\hat{\gamma}_1 = 0.055$, both significant at the 5 percent level, with an R-squared equal to $R^2 = 0.97$. This slightly increases the estimate of $\hat{\beta}_1$ – the elasticity of substitution becomes $\hat{\sigma}_1 = 0.08$ – but the serial correlation remains high ($DW = 0.13$).

Up to now we estimated the parameters using the observed variables, which do not take into account adjustment costs. One of the methods frequently used in the literature to recover "desired" or long run values is to filter the time series. The adopted procedure transform variables in the frequency domain excluding medium and high frequencies, keeping only low frequencies. In the time domain this allows to get long run variables. So, what do we get from the filtering? We tried several filters – the Baxter-King and Christiano-Fitzgerald versions, with different hypothesis about the trend – but results seem insensitive to these transformations. These are $\hat{\beta}_1 = 5.32$ ($\sigma_1 = 0.16$) and $\hat{\gamma}_1 =$

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[11]Here, and in what follows, we tested for residual correlation computing ADF and Breusch-Godfrey statistics. The tests always confirms the DW results. For brevity, these tests are not reported.

0.076, with a high fit; however, the $\hat{\beta}_1$ estimate is significant only at 10 percent level and, above all, the residual are still serially correlated.[12]

Table 2 Single Equation Estimations – Technology
(standard errors in parenthesis

	Non-filtered	Non-filtered	Filtered
$\hat{\sigma}_1$	0.068 (0.0134)	0.08 (0.0131)	0.16 (0.1364)
$\hat{\gamma}_1$	calibrated	0.055 (0.0086)	0.076 (0.0032)
R^2	0.97	0.97	0.98
D.W.	0.12	0.13	0.275

6.1.2 Income shares

Let us now turn to the estimation of the first-order conditions related to firm's optimizing behavior. We use the income share equations which embody the first-order conditions. In writing the production function in its index form, we emplyed the mid-sample period as a reference. Income shares in the base period were determined as follows:

$$1 - \pi_0 = (\beta_3)^{-\beta_1} \left(\frac{Y_0}{\beta_2 L_0} \right)^{\beta_1}$$

More generally, the labour share in period t can be written as:

$$1 - \pi_t = (\beta_3)^{-\beta_1} \left(\frac{Y_t}{\beta_2 e^{\mu_K (t-t_0)} L_t} \right)^{\beta_1}$$

where $\mu_K = \lambda_K + \gamma_1 \lambda_C$. Dividing side by side the last two equations, we obtain

$$1 - \pi_t = (1 - \pi_0) \left(\frac{y_t}{e^{\mu_K (t-t_0)} l_t} \right)^{\beta_1}.$$

This equation has a straightforward economic interpretation: the labor income share is directly related, via β_1 and thus the elasticity of substitution, to the productivity of labor expressed in efficiency units.

Taking logs of the last expression, we get:

$$\ln(1 - \pi_t) = \ln(1 - \pi_0) + \beta_1 \ln \left(\frac{y_t}{e^{\mu_K (t-t_0)} l_t} \right) \tag{10}$$

As in the case of production function estimation, we set the lambdas at the values specified in table 1 and estimate the two deep parameters γ_1 and σ_1. The results, using both

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[12] We also replicated the specification of Mallick (2012), who assumes Hicks-neutral technical progress, obtaining very similar results for Italy ($\sigma_1 = 0.15$). He does not, however, address the serial correlation problem.

observed and filtered data for the variables involved in the previous equation, we get for β_1 values of 1.05 ($\hat{\sigma}_1 = 0.49$) and 1.23 ($\hat{\sigma}_1 = 0.45$), and for γ_1, 0.17 and 0.15. Differently from the production function estimation, however, the R-squared more than halved. What remains unchanged is the high serial correlation in the residuals.

An analogous estimation can be done for the capital income share. The equation estimated is:

$$\pi_t = \pi_0 \left(\frac{y_t}{k_t c_t^{\gamma_1}} \right)$$

and in log form:

$$\ln(\pi_t) = \ln(\pi_0) + \beta_1 \ln \left(\frac{y_t}{k_t c_t^{\gamma_1}} \right).$$

As in the case of labour, the capital income share is directly linked to the productivity of capital expressed in efficiency units through the ICT capital contribution. Estimating this equation as above with observed and "long run" data, we obtain a value for β_1 approximately equal to 4 ($\hat{\sigma}_1 = 0.2$). What at first sight appears counterintuitive is the sign of the estimated γ_1, which is negative. However, this is a finding not uncommon in the literature (see for instance Antras 2004).

Finally, we estimate the ratio of income shares:

$$\frac{\pi_t}{1-\pi_t} = \frac{\pi_0}{1-\pi_0} \left(\frac{e^{\mu_K \ (t-t_0)} l_t}{k_t c_t^{\gamma_1}} \right)^{\beta_1}$$

and in log form:

$$\ln \left(\frac{\pi_t}{1-\pi_t} \right) = \ln \left(\frac{\pi_0}{1-\pi_0} \right) + \beta_1 \ln \left(\frac{e^{\mu_K \ (t-t_0)} l_t}{k_t c_t^{\gamma_1}} \right).$$

In words, the ratio of capital to labour income share in inversely related to labour-capital ratio, both expressed in efficiency units. Figure 2 does not support one of more accepted stylized facts in economic literature, the stationarity of income shares. In the sample period considered, there has been a continuous increase in the capital share – not only in Italy but, as the figure shows, in most industrialized countries. Even if the profit share seems to be stabilized at a new level in recent years, it creates an estimation problem which is not easy to solve. In fact, we made attempts, both with raw and filtered data, to deal with this problem without obtaining satisfactory economic results: for instance, the estimated weight of ICT is implausibly high (about 40%, while the weight of the ICT capital stock in the total capital stock is in the range of 3-6%). Moreover, the residuals remain serially correlated.

Table 3 Single Equation Estimations – Income shares
(standard errors in parenthesis

	Labour income share		Capital income share		Income Share ratio	
	Non-filtered	Filtered	Non-filtered	Filtered	Non-filtered	Filtered
$\hat{\sigma}_1$	0.49 (0.076)	0.45 (0.054)	0.22 (0.020)	0.2 (0.015)	0.5 (0.09)	0.33 (0.026)
$\hat{\gamma}_1$	0.17 (0.024)	0.15 (0.015)	-0.52 (0.037)	-0.57 (0.044)	0.42 (0.135)	0.38 (0.024)
R^2	0.38	0.50	0.58	0.59	0.82	0.89
D.W.	0.045	0.047	0.054	0.071	0.025	0.04

To sum up the results discussed so far, the single equation approaches suffer of a misspecification problem. The problem with using just one equation – such as the production function $Y = f(K, L)$ alone – is that it assumes the observations are taken from a static economy in equilibrium. We can see no way that can hold. These three variables are perhaps the most heavily inter-related in theory: both K and L will be functions of demand, and demand (for given prices and wages) must be met from domestic output, imports or variations in stocks. K will depend on some investment function which alone will lead to lags. L will depend almost certainly on demand and the current (installed) production frontier, so even leaving aside simultaneous equation bias, there will be some form of serial correlation (probably moving average disturbances) within the model. Similar problems arise in the context of income share estimation.

6.2 System approach

Since the single equation approach seems unsuitable for jointly estimating the two technical parameters of interest, we turn our attention to the system approach, which is also the most frequently used in the literature.

The system estimated is:

$$\begin{cases} \ln\left(y_t\right) = -\frac{1}{\hat{\beta}_1}\ln\left[\pi_0 kit^{-\hat{\beta}_1} + \left(1 - \pi_0\right)e^{\mu_K\ (t-t_0)}l_t^{-\hat{\beta}_1}\right] \\ \ln\left(\frac{\pi_t}{1-\pi_t}\right) = \ln\left(\frac{\pi_0}{1-\pi_0}\right) + \beta_1\ln\left(\frac{e^{\mu_K\ (t-t_0)}l_t}{k_t c_t^{\gamma_1}}\right) \end{cases} \tag{11}$$

We estimated (11) using non-linear SUR, obtaining $\sigma_1 = 0.364$ and $\gamma_1 = 0.24$. However, the estimation of this system strongly indicates the presence of serial correlation (the D.W. is close to 0). The residual autocorrelation is confirmed by the multivariate Box-Pierce/Ljung-Box Q-statistics. To correct for this problem, we decide to add autoregressive components. We run unit root tests both for the NDP and the income ratio share, indicating that the former is $I\left(1\right)$ while the latter is $I\left(2\right)$.[13] This leads us to include one autoregressive term in the first equation of the system and two in the second equation. The estimation procedure reduces, but does not solve, the serial correlation

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[13]We also tried to correct for serial correlation in the single equation estimation. However, the strategy of including an autoregressive component did not affect the results.

in the residuals; furthermore, it gives a wrong sign in the $\hat{\beta}_1$ estimate.[14] A look at the residual correlogram drives to increase the number of autoregressive terms in the equation of the income share ratio. Increasing the order of the autoregressive process almost solves the serial correlation problem but the economic content of this econometric manipulation has very limited value.[15] Furthermore, the estimation of the elasticity of substitution is close to 1, a result at odds with the findings of the recent literature.

7 The theoretical roots of the estimation problem

As seen above, the traditional approach has some weaknesses. Indeed, a key issue arises in estimating the technological parameters. To see the problem at hand in the simplest way, suppose that the production function underlying the economy may be represented as $y = f(K, L, \theta)$, where θ is a vector of parameters. These are technological parameters indicating the way in which factors of production are brought together to produce output.

If the economy has frictions, rigidities etc. which reduce the efficiency of production, these rigidities must be taken into account in the estimation of the parameters of the production function for, otherwise, the standard errors of the estimates will be biased. Some of those rigidities will be unavoidable; it takes time to install capital, build a new plant, etc., but it can be assumed firms will take whatever steps they can to minimize costs associated with those rigidities. The same will apply to regulations; although firms are assumed to minimize costs by choosing the optimal point on the production frontier, depending on factor costs, rigidities will encourage or force the firm to operate at some other, sub-optimal point. This sub-optimal point may be at a different point on the same "iso-technology" frontier or the whole frontier may be sub-optimal.

If data were available on costs, it might be possible to build these into the production function but generally that is not the case at the aggregate level anyway. Also, to the extent that firms take steps to reach the optimal position from their current sub-optimal position, estimation of the production function is likely to result in auto-correlated errors.

The divergence between sub-optimal and optimal variables is often cast in terms of the difference between observable short run data and their long run values. Observable data do not include adjustment costs while the long run values – on which the estimates should be based – are already cost adjusted but are unobservable.

The divergence between optimal and sub-optimal positions is a problem with time-series analysis; auto-correlated errors are often eliminated statistically but if they have an economic cause originating from a misspecification problem, the model should be re-specified accordingly. The estimated model should be based on the idea that firms optimize their intertemporal profits subject to the production function but taking account of rigidities, adjustment costs and other frictions. This produces a structural dynamic model which, at least to an approximation, enables the true parameters of the production

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[14]The strategy of filtering data does not improve the results.

[15]Specifically, we included autoregressive components at lags 1 up to 5, and at lags 9 and 10 for the equation of income share ratio; for the production function, we add 1 autoregressive element. That way, the R squared is 0.99 and the D.W. increased to 1.9.

function to be separated from the costs of adjustment. The parameters θ then are not biased by those costs.

Our model is based on the idea that firms optimize their intertemporal profits subject to the production function but taking account of rigidities, adjustment costs and other frictions. This produces the dynamic model which, at least to an approximation, enables the true parameters of the production function to be separated from the costs of adjustment. The parameters θ then are not biased by those costs.

Once the model specified to include these costs has been estimated, so if the specification is correct, the parameters will be unbiased, the partial equilibrium of the economy may be calculated under the assumption that costs of rigidities are zero. Observations of economic variables include these costs (so output would be higher if these costs did not exist), while the calculated values from the unbiased estimates exclude them so (estimated) output in the latter case should be higher than observed.

A more formal way of looking at this from a general point of view, is to think of a theoretical function

$$\tilde{y}(t) = f(x(t), \theta) + u(t), \tag{12}$$

where the $u(t)$ are a set of errors that would arise if this relationship, and in particular $\tilde{y}(t)$, could be observed. This relationship could then be estimated directly.

If this relationship is subject to adjustment costs, rigidities, frictions etc., the function above could be considered as embedded in a more general relationship, for instance

$$y(t) = g[f(x(t), \theta), z(t), \pi] + \xi(t), \tag{13}$$

which may depend on other variables $z(t)$ and parameters π, which gives a better representation of the economy. Thus $y(t)$ is the variable, corresponding to $\tilde{y}(t)$, that is observed. In that case, it is this second equation that should be estimated for all of the parameters θ, π. If this second equation is the correct specification of the model that produces the observed $y(t)$, estimating the first equation on the assumption that $\tilde{y}(t) = y(t)$ would produce biased estimates of the parameters vector θ. Thus $y(t)$ is the variable corresponding to that observed. In that case, it is equation (13) that should be estimated for all of the parameters θ, π.

If (13) were the true model but (12) is estimated using the observed values of $y(t)$ it is likely that residuals in (12) will then be serially correlated. Take for instance the behavior of factor markets. These are very often characterized by frictions and rigidities arising from many sources that affect adjustment process. In many countries, the employment protection legislation is evidence of the existence of institutional factors that delay or hinder the achievement of equilibrium in the labor market; at the same time, they make the wage unresponsive to the excess of demand or supply. Similarly, the optimal or "desired" capital stock cannot be instantaneously obtained given a variety of adjustment costs. Some scholars (see, for example, Antras 2004, Leon-Ledesma et al. 2010) do not consider the presence of those frictions and rigidities assuming that the economic system is in equilibrium at any point in time. Although this assumption may be convenient for theoretical work, it causes an error in the specification of the structure underlying the model thus giving rise to serially correlated residuals.

As this has an economic cause, that is it is due to a misspecification of using $y(t)$ with (12) rather than (13), it should be eliminated by using the correct specification rather than by some statistical means. Because of the dynamics in the true model, if we wished to use values calculated from (13) to re-estimate (12) directly, we would need to calculate "observations" of these variables from (13) first, but such estimates would almost certainly be inconsistent.[16] In the following we will see that the standard approach suffers from such a misspecification problem.

8   The misspecification problem

Our model is formulated as a dynamic disequilibrium system in continuos time. The approach followed derives partial equilibrium functions by firms' profit maximization subject to technological constraints, in turn subject also to short run constraint on their behavior. This implies that these partial equilibrium functions are embedded in adjustment framework to give the dynamic model. The specification includes but is not limited to partial adjustment of some variables, e.g. the traditional or innovative stock of capital, to the difference between its actual and "desired" levels.

In our model the adjustment takes two forms. First-order process assume that the variable under consideration adjust to its partial equilibrium level in the following way, $Dy(t) = \alpha[\hat{y}(t) - y(t)]$, where $\hat{y}(t)$ is the equilibrium or desired level, α is the speed of adjustment and D is the operator d/dt. Second-order adjustment assume instead that it is the rate of change of the variable to adjust to its equilibrium level, $D^2 y(t) = \alpha_1 \{\alpha_2[\hat{y}(t) - y(t)] - Dy(t)\}$, where the first term in parenthesis describes the adjustment of the variable to its desired level.

For instance, let us have a look at the second order (time) derivative of the log of traditional capital which implicitly defines the investment equation in our model, repeated here for the reader's convenience:

$$\dot{k} = \alpha_1 \left[\alpha_2 \left(\frac{\partial f}{\partial K} - (r - \beta_7 D \ln p + \beta_8) \right) - (k - \mu_K) \right]$$

where $k = D \ln(K)$, $\dot{k} = D^2 \ln(K)$ and μ_K is the growth rate of labor efficiency. Inside the parentheses, we model the adjustment of the marginal product of capital to its user cost, defined by the real interest rate plus a risk premium (β_8). The speed at which firms make this adjustment is given by α_2, or, in other words, how long it takes to adjust the existing capital stock to its desired level. As time goes by, however, this desired level changes at the velocity μ_K. Inside the square brackets we find this second long run adjustment process, which runs at α_1, the speed of the accumulation process. Of course,

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[16]As all the FIML or similar estimators are asymptotic, a vector of parameters is consistent if and only if the probability limit of the estimate of the vector of parameters theta equals the true value of θ as the sample size goes to infinity for the estimate to be consistent. This condition is roughly asymptotic equivalent – but not same as – to the condition in OLS that a parameter is unbiased if and only if the expected value of its estimate is equal to the true value.

as the estimates in Saltari et al. (2012) confirm, the first adjustment takes a much shorter time than the second one.

All the other equations in our model are specified in a similar manner, i.e. as dynamic equations. This implies that the model is recursive in the sense that it is expressed as a system of differential equations in which the derivative of each endogenous variable depends on the levels of all the other variables.[17]

These particular features may help, at least in principle, in solving the residual correlation and misspecification problems seen above. As the reader may recall, the misspecification derives from the "fundamental tension", as Chirinko (2008) dubs it, between the short run observable data and the long run nature of the elasticity of substitution. As the estimation results showed, the ways out of this problem proposed in the literature have not been useful. Our modelling strategy takes into account, though implicitly, a variety of adjustment costs without leaving out the optimization hypothesis. Turning again to the accumulation equation, the alphas embody the adjustment lags with which the firm reach their optimal capital stock.

As we said above, the standard procedure assume instead an instantaneous adjustment between the factor rental price and its marginal product in estimating the technical parameters.

To remain in the previous example, this implies that the technical parameter estimation comes out from the following equation:

$$r_t = \left(\frac{\partial f}{\partial K}\right)_t \tag{14}$$

where r_t is the real interest rate and, using equation (2), the marginal product of capital is:

$$\left(\frac{\partial f}{\partial K}\right)_t = \beta_3 C_t^{\gamma_1}\left(\frac{Y_t}{\beta_3 K_t C_t^{\gamma_1}}\right)^{1+\beta_1}$$

Written in log, the previous equality (14) is:

$$\log(r_t) = -\beta_1 \log(\beta_3) - \beta_1 \gamma_1 \log(C_t) + (1 + \beta_1)\log\left(\frac{Y_t}{K_t}\right)$$

The standard form in which this equation is estimated equation is:

$$\log\left(\frac{Y_t}{K_t}\right) = \theta_0 + \theta_1 \log(r_t) + \theta_2 \log(C_t) \tag{15}$$

where θ_0 is a constant.

To estimate this equation, we simulated our model that by construction embodies a generic form of adjustment costs. More precisely, we employ for the endogenous variables (K_t, C_t) the results of the simulation, while the exogenous variables (Y_t, r_t) were set

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[17]More details on these dynamic disequilibrium models can be found in Gandolfo (1981) and Wymer (1996).

at their observed values. Notice that, in estimating equation (15), we are deliberately neglecting these adjustment costs. That way, we are introducing a specification bias.

Even leaving aside the unrealistic estimated values for the elasticity of substitution (near zero and thus quasi-Leontief) and the parameter γ_1 technical progress (implausibly high), the estimation results of equation (15) show the typical features we emphasized above. Indeed, it presents the usual combination of an high R squared (0.93) and strong serial correlation in the residuals (the D.W. is 0.55), that are symptoms of a misspecification in the model. Similar results hold estimating the first order condition for labour input.

Taking into account frictions and rigidities, as in our model, we reckon to provide a better representation of the economic system. Although our framework cannot always guarantee the solution to the specification problem, in this case it turns out that residuals are not serially correlated. Multivariate Portmanteau or Q (Ljung-Box) statistic is equal to 101.4 for the first two autocorrelations.[18] As a consequence, the null hypothesis that the residuals are not serially correlated cannot be rejected because the Q statistic is below the critical value in the region of the upper tail.[19] This is not a surprising result since a second order differential equation model gives rise to a second order moving-average error process that is taken into account explicitly in the estimation procedure. To the extent that observations generated by a second order system inherently incorporate a first or second order moving average process depending on whether the variables are stocks or flows, at least in a linear model and to an approximation in a non-linear model, that too can be taken into account and the variables transformed to remove the serial correlation (see Wymer, 1972).

9   Conclusions

A growing number of papers has shown that the elasticity of substitution is a key technological parameters for boosting economic efficiency. Perhaps the most innovative and interesting result of this literature is that the elasticity of substitution well below 1, i.e. the Cobb-Douglas assumption is biased upward.

However, in our opinion these new estimates are in turn affected by a theoretical weakness. The elasticity of substitution is a long run technological parameter whose estimation is constrained by the availability of short run data. This problem has been solved employing two different econometric strategies: on the one hand, making use of a theoretical framework to account for the delayed adjustment to the long run optimizing relationship; on the other, filtering the data in such a way to retain only long run components.

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[18]Augmented Dickey-Fueller statistic may also be run for the single equations of the model. Although these are not appropriate for a FIML estimator, the single equation results, for what they are worth, show no relevance to non-linear differential equation systems.

[19]Approximate critical values of Chi-Square distribution with 98 degrees of freedom are at 5 per cent level equal to 122.1, while at 1 per cent level is 133.5.

This paper has emphasized that these estimates have a serial correlation problem deriving from unsolved theoretical issues: it is difficult to explicitly specify the appropriate adjustment costs and data filtering are subject to the usual ad-hock criticism. The mode we proposed is a tentative solution strategy to these problems in that it incorporates frictions and, as a disequilibrium model, it is intrinsically dynamic. The test results seem to confirm that our strategy is effective.

A distinguished feature of our model is the capital-augmenting technical progress which gives a key role to the ICT capital stock, differently from the existing literature where it is generally assumed constant. The next step of our research project is to refine the ICT modelling by giving it an autonomous role.

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Appendix

The core of the model is composed by the following seven differential equation (for more details, see Saltari et al. 2012):

1. Investment functions:

   (a) Traditional capital

   $$\dot{k} = \alpha_1 \left[\alpha_2 \left(\frac{\partial f}{\partial K} - (r - \beta_7 D \ln p + \beta_8) \right) - (k - \mu_K) \right] \tag{A.1}$$

   (b) ICT capital

   $$\dot{c} = \alpha_3 \left[\alpha_4 \left(\frac{\partial f_I}{\partial C} - (r - \beta_9 D \ln p + \beta_{10}) \right) - (c - \mu_C) \right] \tag{A.2}$$

   where in Equation (A.1) $\mu_K = \lambda_K + (\gamma_S - \gamma_1) \lambda_S + \gamma_u \lambda_u$ and in Equation (A.2) $\mu_C = \lambda_s + \lambda_c$.[20]

2. Skilled labour:

   (a) Demand for skilled labour

   $$\dot{\ell}_s = \alpha_5 \left[\alpha_6 \ln \left(\frac{\partial f}{\partial L_s} \bigg/ \frac{w_s}{p} \right) + \alpha'_6 \ln \left(\frac{\partial f_I}{\partial L_{I s}} \bigg/ \frac{w_s}{p} \right) - (\ell_s - \lambda_s) \right] \tag{A.3}$$

   (b) Skilled wages

   $$D^2 \ln w_s = \alpha_7 [\alpha_8 \ln \left(\frac{\partial f}{\partial L_s} \frac{w_s}{p} \right) + \alpha'_8 \ln \left(\frac{\partial f_I}{\partial L_{I s}} \frac{w_s}{p} \right) -$$
   $$(\alpha_7 + \alpha_8 + \alpha'_8) \, (D \ln w_s - \beta_{11} D \ln p - \lambda_K - \gamma_1 \lambda_C)] \tag{A.4}$$

   where β_{11} measures money illusion.[21]

3. Unskilled labour:

   (a) Employment

   $$\dot{\ell}_u = \alpha_9 \alpha_{10} \ln \left(L_u^d / L_u \right) - (\alpha_9 + \alpha_{10}) (\ell_u - \lambda_u) \tag{A.5}$$

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[20] While the investment equations allow for money illusion in specifing the real interest rate, estimates showed that β_7 and β_9 were not significantly different from 1 and in the final estimates they were set to 1.

[21] Estimates of β_{11} were not significantly different from 1 showing there is no money illusion in the determination of real wages. In the final model β_{11} was set to 1.

(b) Unskilled wages

$$D^2 \ln w_u = \alpha_{11} \left[\alpha_{12} \ln \left(\frac{L_u^d}{L_u^s} \right) - \left(D \ln \frac{w_u}{p} - \lambda_K - \gamma_1 \lambda_C \right) \right] \quad \text{(A.6)}$$

where $L_u^s = L_{u0} \left(\frac{w_u}{p} \right)^{\beta_{12}} e^{\lambda_u t}$. In the model, changes in the unskilled labor supply depend on the real wage, with elasticity β_{12}. Thus, the effect on labor supply will be largely symmetrical at the margin for increases and decreases of real wages. However this is only one side of the labor market. We should also take into account the demand side. Unless the elasticity of real wages in the supply function is one, changes in nominal wages have a differing effect on prices and hence on real wages. The price effect then feeds back into invest-ment, capital, and thus on the demand for labour via its marginal product.

4. Price determination:

The marginal cost of labour is obtained in the usual way as a ratio between the mean wage and the marginal product of labour, where labour is defined as a Cobb-Douglas function of the two labor components, $L = L_s^{\gamma_u} L_u^{\gamma_u}$. The short term marginal cost is a weighted average of skilled and unskilled wage rates

$$mc \left(\frac{\partial L}{\partial Y} \right) = \left(\frac{w_s L_s}{\gamma_s} + \frac{w_u L_u}{\gamma_u} \right) L_s^{-\gamma_u} L_u^{-\gamma_u} (\beta_2 \beta_3)^{-1} e^{-(\lambda_K + \gamma_1 \lambda_C) t} \left[1 + \left(\beta_2 e^{(\lambda_K + \gamma_1 \lambda_C) t} \psi \right)^{\beta_1} \right]^{\frac{1+\beta_1}{\beta_1}}$$

where $\psi = \frac{L}{KC^{\gamma_1}}$.

The dynamics of price determination are described by a second-order process:

$$D^2 \ln(p) = \alpha_{15} \ln \left(\frac{\beta_{13} \tau mc \left(\frac{\partial L}{\partial Y} \right)}{p} \right) + \alpha_{13} \left(D \ln \left(\frac{w_s}{p} \right) - \lambda_c \right) +$$

$$+ \alpha_{14} \left\{ D \ln \left(\frac{w_u}{p} \right) - (\lambda_k + \gamma_1 \lambda_c) \right\} + \alpha_{16} \ln \left\{ \frac{\upsilon M}{pY} \left(1 + \lambda_m - \lambda_m e^{-\lambda_m t} \right) \right\} \quad \text{(A.7)}$$

where β_{13} is the mark-up and $mc \left(\frac{\partial L}{\partial Y} \right)$ is the marginal cost determined as follows. Further, $\upsilon = \overline{\ln p} + \overline{\ln Y} - \overline{\ln M}$ is the mean velocity over the sample and is assumed to vary at a rate λ_ν.

$$I = \beta_7 \left[C^{-\beta_6} + (\beta_8 \exp(\lambda_C) L_I)^{-\beta_6} \right]^{-\frac{1}{\beta_6}}$$