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Abstract This article shows how the endogenous human capital affects the labor market equilibrium when jobs provided by firms can be either unskilled or skilled and workers differ in their education level which can be either low-educated or high-educated. We develop an equilibrium search model in which the high-educated workers are assumed to be able to accept either the unskilled jobs or the skilled jobs, while the low-educated workers can only accept the unskilled jobs. The market equilibrium is characterized by deriving the unemployment rate and the human capital distributions when the growth rate of the human capital is an endogenous variable. The results demonstrate that the structure proportion of the offered jobs affects the equilibrium which shows there is a threshold that can distinguish whether the equilibrium is separating or cross-skill. In addition, the cross-skill equilibrium solution implies the high-educated workers are more likely to own higher pay rates than the low-educated workers with same tenure. It also yields a new insight on the effect of the structure proportion of workers on the profits, which implies the profits of the firms decrease with the increasing number of the low-educated workers. Moreover, the profit of the firms offering the skilled jobs is greater than those offering the unskilled jobs until there is only very few high-educated workers.

Keywords Job search · Labor market equilibrium · Contracts · Human capital accumulation · Education level

1 Introduction

The talents, which have high level of education, are the most precious fortune and resources of an enterprise. Moreover, they are the direct motivities of economic development and social advancement. However, anti-intellectualism,

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which is defined as "opposing or hostile to intellectuals or to an intellectual view or approach", may be generally found in many countries in recent years. It has some negative influences on the development of higher education and socioeconomic. In addition, it will make people lose the passion in education development and affect the promotion of social civilization. The effect of endogenous changes of human capital on the market equilibrium is one of the urgent problems in the studies of labor market. It is therefore crucial to develop effective job search strategies and market equilibrium analysis for the problem.

The unsymmetrical distribution between job opportunities and labor resources makes the real labor market different from other markets. In the labor market, workers of different education levels hope to find suitable jobs within the shortest time, while firms of different production levels expect to maximize their benefits through offering workers the wage as low as possible. Search theory has been an important theory in the labor market which is full of uncertainty and information completeness. Since the pioneering research of Stigler (1961, 1962), many researchers have done tremendously significant works in the search theory. McCall (1965, 1970) developed a sequential search model to analyze the job search behaviors of the new entrants. Given that the searcher is assumed to know both the distribution of wages for particular skills and the cost of generating a job offer, the optimal stopping rule for the job searcher is to reject all the offers below a single reservation wage. Phelps et al (1971) who is the first one to propose the concept of job search theory assumed that firms offer diversity wages. Each worker has incomplete information about the distribution of wages and learns about it from her search outcomes. In the early work on job search, most simple models did not consider on-the-job search, analyzed equilibrium in the steady-state condition, and assumed the distribution of wage is given exogenously, and leisure have no value, etc. But more recently, job search models relax the assumptions of the canonical model by allowing workers can be heterogeneous, an endogenous wage distribution, on-the-job search and unsteady-state conditions, to have better explanations for the search behaviors of workers, the process of the firms offering wage, and the phenomenon of job creation and job destruction in the labor market.

Over the past a few decades, there has been a number of significant developments in the study of labor market equilibrium. Maybe the most commonly used equilibrium search models are on the basis of the work of Burdett and Mortensen (1998) (for simplicity B/M). B/M assumed that each firm posts a single wage, both employed and unemployed workers can search for better job opportunities. They found that the unique equilibrium distribution of wage offers is non-degenerate, even if firms and workers are homogeneous. However, an obvious weakness of the simplest B/M model is that wage of worker will not increase unless changing employer. Having recognized that, Burdett and Coles (2003) (for simplicity B/C) and Stevens (2004) extended B/M by assuming that firms post wage-tenure contract rather than a single wage. Burdett and Coles (2010b) generalized previous work by assuming firms have different productivities. More productive firms always offer more desirable wage-tenure contracts and a worker who quits to a more productive firm may accept a wage cut. Carrillo-Tudela (2009b) studied the labor market equilibrium in which firms offer wage-experience contracts on the basis of B/M. Carrillo-Tudela (2009a) subsequently made an important assumption that firms cannot decide their wage offers on unemployment and employment duration. Shi (2009) analyzed the equilibrium in a labor market in which firms offer wage-tenure contracts to direct the search by risk-averse workers. From the aspects of contract form, this paper differs from the aforementioned papers in the important aspect that the firms offer wage-human capital contracts in order to analyze the effects of human capital accumulation on the labor market equilibrium.

Recent years, a number of theses have combined human capital accumulation to the B/M as studied here. For instance, Rubinstein and Weiss (2007) analyzed the accumulation of human capital and on-the-job search, however, not considering market equilibrium. The model structured by Robin et al (2011) is a useful tool to study labor markets as they structured a model on a panel of Danish matched employer-employee data to analyze the determinants of individual wage dynamics. Dolado et al (2008) who examined the effects of transitory skill mismatches in a matching model with heterogeneous jobs and workers. Gonzalez and Shi (2010) integrated learning from search into an equilibrium framework and applied lattice-theoretic techniques to analyze learning from experience. In an insightful study, Burdett and Coles (2010a) assumed that workers accumulate general human capital through learning-by-dong, and they found the equilibrium approach to identify the effects between experience and tenure on workers' wages. Bring in training, Fu (2011) yielded new insights on wage dispersion and wage dynamics which shows that endogenous training breaks the perfect correlation between work experience and human capital.

The model addressed in this thesis is related to the work of Burdett et al (2011). Burdett et al (2011) structured an equilibrium labor market by allowing workers learn by doing. Their paper implied that learning-by-doing increases equilibrium wage dispersion and considered that all firms are equally productive, and workers are heterogeneous defined by initial productivity. In contrast to the previous discussion, we discuss an equilibrium search model in which firms offer unskilled jobs or skilled jobs with different distribution and workers with low-education or high-education search for better job opportunities. The introduction of heterogeneous firms refines the structure of the Labor market which has important consequences for the optimal search profits. High-educated workers are assumed to accept unskilled jobs for which they are over-qualified. The structure proportion of jobs effects the equilibrium which shows there is a threshold that can distinguish whether exists a separating equilibrium or a cross-skill equilibrium. In addition, the cross-skill equilibrium solution implies that the high-educated workers are more likely to own higher pay rates than that the low-educated workers with the same tenure are likely to. It also implies both the profits of offering the unskilled jobs and offering skilled jobs decrease with the increasing number of the low-educated workers. Moreover, the profit of the firms offering the skilled jobs is greater than the profit of the firms offering the unskilled jobs until there is only very few high-educated workers. Another important difference is that we allow for the growth rate of an employed worker's human capital to be an endogenous variable. Unlike other papers such as Burdett and Coles (2010a), Carrillo-Tudela (2010), Burdett et al (2011) and Fu (2011) in which the growth rate of an employed worker's productivity is given exogenously, this allows us to clarify the endogenous growth rate of human capital which can be determined by death shock, job destruction shock, the fraction of the unskilled jobs and the arrival rate of jobs.

The rest of this paper is organized as follows. Section 2 outlines a framework describing the workers' job search strategies with different education level and the firms' optimal contract. Section 3 gives a definition of market equilibrium and illustrates the distributions of pay rate in different types of jobs. Section 4 presents the effect of human capital accumulation on equilibrium pay rate. Using numerical example, Section 5 describes how the proportion of the low-educated workers effect profit and the support of the pay rate. The paper is concluded in Section 6 with further work pointed out.

2 The model

Consider a labor market in steady state in which time is discrete. There is a continuum of risk neutral workers and firms, each of measure one.

The workers differ in their education level j which can be either low-educated ($j = 0$) or high-educated ($j = 1$). An exogenous fraction $\alpha \in [0, 1]$ of the workers is low-educated, while the remaining fraction $1 - \alpha$ is high-educated. After a low-educated worker has worked for τ periods up to period t, and her human capital is denoted by $h_t = (1 + g)^\tau$, where $0 < g < 1$. While for a high-educated worker, her human capital $h_t = e^{g\tau}$. For each new entrant, $\tau = 0$ and $h_t = 1$. The lives of workers are of uncertain duration. Any worker dies with probability δ per period and δ also describes the inflow rate of new entrants per period. All the dead workers are replaced with newly joined workers, so the population is balanced. There are job destruction shocks in which each employed worker is displaced into unemployment with probability σ per period. Assuming that there is no recall if a worker quit or reject a job offer. The object of a worker is to maximize her total expected lifetime utility.

Jobs are either unskilled or skilled. A fraction β of all the jobs in the labor market which can be performed by both types of workers are called unskilled jobs; while $1 - \beta$ of all the jobs which only requires high-educated workers are called skilled jobs. Any firm generates revenue p from each unit of human capital he employs. Each firm posts a job offer contract θ^i at zero cost on a "take it or leave it" basis in the period t, where θ^i is the pay rate per unit of human capital and i specifies the type of jobs, which can be either unskilled jobs ($i = 0$) or skilled jobs ($i = 1$). Let $F_0(\theta^0)$ describe the probability of receiving an unskilled job offer which is no greater than θ^0. Likewise, $F_1(\theta^1)$ describes the probability of receiving a skilled job offer which is no greater than

θ^1. Further, let $\underline{\theta}^i$ and $\overline{\theta}^i$ denote the infimum and supremum of the support of F_i. Let λ denote the Poisson arrival rate of these offers. Low-educated workers meet these offers at the same rate as high-educated do, but they do not qualify for the skilled jobs. The effective arrival rate of unskilled job offers faced by any low-educated worker is $\beta\lambda$, independent of their employment status; while for high-educated workers, the effective arrival rate depends on the type of jobs. If a job is from unskilled jobs, the arrival rate is $\beta\lambda$; if it is from skilled jobs, the offer arrival rate is $(1-\beta)\lambda$. For simplicity, we assume that firms only offer one type of job and the choice is irreversible. The object of each firm is thus to maximize steady state flow profit.

Assume that workers and firms meet randomly. For a low-educated worker with human capital h_t, she will meet an outside firm which offers unskilled job contract θ^0 with probability $\beta\lambda$. If the match succeeds, the worker is paid $\theta^0 h_t$ while the firm's profit is $\left(p-\theta^0\right) h_t$ at the production stage. Meanwhile for a high-educated worker with human capital h_t, it depends on the type of the outside jobs. With probability $\beta\lambda$ the worker will meet an unskilled jobs offering θ^0. Then the worker is paid $\theta^0 h_t$ and the firm's profit is $\left(p-\theta^0\right) h_t$ at the production stage. With probability $(1-\beta)\lambda$ the worker will meet a skilled job offering θ^1. If the match succeeds, the worker is paid $\theta^1 h_t$ and the firm's profit is $\left(p-\theta^1\right) h_t$. If the match does not succeed, the worker returns to her previous status. In this situation, the outside firm doesn't have any profit. For an unemployed worker, she has income bh_t, where b can be interpreted as home production or leisure, and $0 < b < p$.

2.1 Worker behavior

When a job arrivals, a worker must choose between keeping the current status or accepting an outside offer. Let x_t be the worker's decision variable which can be either 0 or 1, in which $x_t = 0$ means that the worker will keep the current status, while $x_t = 1$ means that the worker will accept the new offer.

2.1.1 Low-educated workers' payoffs and job search strategies

In this section, the search behavior of only low-educated workers is considered. The low-educated workers' objective function is

$$\max_{x_t} \mathrm{E} \sum_{t=0}^{\infty} u\left(\theta_t, h_t, x_t\right),$$

where θ_t, h_t are the state variables in t period.

First consider the unemployed workers with low-education. The pay rate θ_t of the workers satisfies

$$\theta_{t+1} = \xi\left\{(1-\zeta)b + \zeta\left[(1-x_t)b + x_t\theta^0\right]\right\},$$

where ξ is a random variable denoting whether a low-educated worker dies at that period with $\Pr\{\xi = 0\} = \delta$ which means the worker dies and $\Pr\{\xi =$

$1\} = 1 - \delta$ which means the worker is still in the labor market. While ζ is also a random variable, denoting whether a low-educated worker receives an unskilled job with $\Pr\{\zeta_t = 1\} = \beta\lambda$ and $\Pr\{\zeta = 0\} = 1 - \beta\lambda$. In addition, the human capital of the unemployed worker does not grow. The expected lifetime utility of an unemployed worker with the human capital h_t is indicated by $V_{u0}(h_t)$. When an unemployed worker with low-education meets an outside job offering θ^0, she would compare the expected lifetime utility of accepting the offer, denoted by $V_0(\theta^0; h_t)$, with that of unemployment $V_{u0}(h_t)$. If the worker chooses $V_0(\theta^0; h_t)$, it means that she accepts a new offer θ^0. Otherwise, she still keeps the current status. Therefore, the Bellman equation

$$V_{u0}(h_t) = \max\{bh_t + \mathrm{E}_{(\theta^0)} V_0(\theta_{t+1}; h_{t+1})\},$$

which can be rewritten as follows

$$V_{u0}(h_t) = bh_t + (1 - \delta - \beta\lambda)V_{u0}(h_t) + \beta\lambda \mathrm{E}_{(\theta^0)} \max\{V_{u0}(h_t), V_0(\theta^0; h_t)\}. \tag{1}$$

Next consider the employed workers with low-education. Different from the unemployed workers, there is a job destruction shock for the employed workers with low-education. The pay rate θ_t of the employed worker with low-education satisfies

$$\theta_{t+1} = \xi \left\{(1 - \eta)b + \eta \left[(1 - \zeta)\theta_t + \zeta \left((1 - x_t)\theta_t + x_t\theta^0\right)\right]\right\},$$

where η is a random variable denoting whether there exists a job destruction shock with $\Pr\{\eta = 0\} = \sigma$ which means the worker is displaced into unemployment and $\Pr\{\eta = 1\} = 1 - \sigma$ which means there is no job destruction shock, and independent with ξ and ζ. Let $V_0(\theta_t; h_t)$ denote the expected lifetime utility of a worker whose human capital is h_t and employed by a firm offering θ_t in period t. Given an employed worker with human capital h_t, she needs to choose the larger one between $V_0(\theta_t; h_{t+1})$ and $V_0(\theta^0; h_{t+1})$. In addition, her human capital h_t will be $(1 + g)h_t$ next period. Therefore, the Bellman equation

$$V_0(\theta_t; h_t) = \max\{\theta_t h_t + \mathrm{E}_{(\theta^0)} V_0(\theta_{t+1}; h_{t+1})\},$$

which can be written as

$$\begin{aligned} V_0(\theta_t; h_t) = &\theta_t h_t + \sigma V_{u0}((1 + g)h_t) + (1 - \delta - \sigma - \beta\lambda)V_0(\theta_t; (1 + g)h_t) \\ &+ \beta\lambda \mathrm{E}_{(\theta^0)} \max\{V_0(\theta_t; (1 + g)h_t), V_0(\theta^0; (1 + g)h_t)\}. \end{aligned} \tag{2}$$

The employed worker has a job from which she gets θ_t for each unit of her human capital in period t, which corresponds to the first term on the right-hand side in Eq.(2). With probability σ, the worker is displaced into unemployment by a job destruction shock, which is the second term on the right-hand side in Eq.(2). If the worker does not receives any offer, she keeps with the current firm, which is the third term on the right-hand side in Eq.(2). Next period the worker gets a new offer with probability $\beta\lambda$, upon which she chooses whether to stay with the current job or to accept the new job, which is the last term on the right-hand side in Eq.(2).

The employed worker with low-education leaves the firm when she dies or is displaced into unemployment or receives an outside offer which is higher than her current offer. Hence, the separate probability

$$\psi_0(\theta_t) = \delta + \sigma + \beta\lambda(1 - F_0(\theta_t)),$$

which describes the separation rate of a low-education worker employed by a firm offering θ_t.

Employed workers would accept any outsider offer which is higher than her current offer; while for unemployed worker, they would accept any outsider offer which is no less than the reservation pay rate described in Proposition 1.

Proposition 1 *For the unemployed workers with low-education, the reservation pay rate per unit of human capital, denoted by θ_r^0, can be characterized by the following equation*

$$\frac{\delta(b(1+g) - \theta_r^0)}{g} = b + \beta\lambda \int_{\theta_r^0}^{\overline{\theta}^0} \frac{1 - F_0(\theta)}{1 - (1+g)\,(1 - \psi_0(\theta))} \mathrm{d}\theta. \tag{3}$$

Moreover, the optimal job search implies that any unemployed worker with low-education accepts job offer θ^0 if and only if $\theta^0 \geq \theta_r^0$.

Proof It is obvious that a worker's income, whether the worker is unemployed or employed, is always proportional to h_t, that is, there exists a number v_{u0} and a function $v_0(\theta_t)$ such that $V_{u0}(h_t) = v_{u0}h_t$ and $V_0(\theta_t; h_t) = v_0(\theta_t)h_t$ respectively. In this way, Eq.(1) can be written as

$$\delta v_{u0} = b + \beta\lambda \int_{\theta_r^0}^{\overline{\theta}^0} (v_0(\theta) - v_{u0}) \mathrm{d}F_0(\theta), \tag{4}$$

while Eq.(2) can be written as

$$v_0(\theta_t) = \theta_t + (1+g) \left[\sigma v_{u0} + (1 - \delta - \sigma)v_0(\theta_t) + \beta\lambda \int_{\theta_t}^{\overline{\theta}^0} (v_0(\theta) - v_0(\theta_t)) \mathrm{d}F_0(\theta) \right] \tag{5}$$

Since θ_r^0 is the reservation pay rate per unit of human capital of the unemployed workers with low-education, there is no difference between accepting the offer θ_r^0 and keeping unemployment, i.e., $V_0(\theta_r^0; h_t) = V_{u0}(h_t)$. Therefore, we get $v_0(\theta_r^0) = v_{u0}$. Let $\theta_t = \theta_r^0$ in Eq.(5). Thus,

$$v_{u0} = \theta_r^0 + (1+g) \left[(1 - \delta)v_{u0} + \beta\lambda \int_{\theta_r^0}^{\overline{\theta}^0} (v_0(\theta) - v_{u0}) \mathrm{d}F_0(\theta) \right]. \tag{6}$$

Note that $\int_{\theta_r^0}^{\overline{\theta}^0} (v_0(\theta)) - v_{u0}) \mathrm{d}F_0(\theta) = \frac{\delta v_{u0} - b}{\beta\lambda}$ by (4), and then (6) can be written as

$$g v_{u0} = b(1+g) - \theta_r^0. \tag{7}$$

On the other hand, differentiating (5) with respect to θ_t yields

$$\frac{\mathrm{d}v_0(\theta_t)}{\mathrm{d}\theta_t} = \frac{1}{1-(1+g)\,(1-\psi_0(\theta_t))}. \tag{8}$$

Therefore, (4) can be written as

$$\begin{aligned} \delta v_{u0} &= b + \beta\lambda \int_{\theta_r^0}^{\overline{\theta}^0} (v_0(\theta) - v_{u0})\mathrm{d}F_0(\theta) \\ &= b + \beta\lambda \left[\int_{\theta_r^0}^{\overline{\theta}^0} (1-F_0(\theta))\frac{\mathrm{d}v_0(\theta)}{\mathrm{d}\theta}\mathrm{d}\theta\right] \qquad \text{(integrate by parts)} \\ &= b + \beta\lambda \int_{\theta_r^0}^{\overline{\theta}^0} \frac{1-F_0(\theta)}{1-(1+g)(1-\psi_0(\theta))}\mathrm{d}\theta \qquad \text{(by(8))} \end{aligned} \tag{9}$$

Substituting (7) into (9) yields Eq.(3). Moreover, the optimal job search implies that any unemployed worker with low-education accepts job offer θ^0 if and only if $\theta^0 \geq \theta_r^0$. This completes the proof.

Proposition 2 *The growth rate of human capital g is an endogenous variable which is increasing with the death shock δ and the job destruction shock σ, while decreasing with the fraction of the unskilled jobs β and the arrival rate of jobs λ.*

Proof Calculating the derivative of Eq.(3) with θ_r^0, so as to get

$$F_0(\theta_r^0) = 1 - \frac{\delta[1-(1+g)(1-\delta-\sigma)]}{\beta\lambda(g-\delta-g\delta)}. \tag{10}$$

All the unskilled jobs offer $\theta^0 \geq \theta_r^0$, $\theta^0 \in [\underline{\theta}^0, \overline{\theta}^0]$, otherwise there is no worker accepts the offer. Thus in the market equilibrium, $F_0(\theta_r^0) = 0$, and from Eq.(10) we can get

$$g = \frac{\delta(\delta+\sigma+\beta\lambda)}{(\beta\lambda+\delta)(1-\delta)-\delta\sigma}. \tag{11}$$

Therefore, g is determined by δ, σ, β and λ. It is easily to prove that the partial derivatives of g have the following properties:

$$\begin{aligned} \frac{\partial g}{\partial \delta} &= \frac{(\beta\lambda+\delta)^2+\sigma\beta\lambda}{[(\beta\lambda+\delta)(1-\delta)-\delta\sigma]^2} > 0, \\ \frac{\partial g}{\partial \sigma} &= \frac{(\beta\lambda+\delta)\delta}{[(\beta\lambda+\delta)(1-\delta)-\delta\sigma]^2} > 0, \\ \frac{\partial g}{\partial \beta} &= \frac{-\delta\lambda\sigma}{[(\beta\lambda+\delta)(1-\delta)-\delta\sigma]^2} < 0, \\ \frac{\partial g}{\partial \lambda} &= \frac{-\delta\beta\sigma}{[(\beta\lambda+\delta)(1-\delta)-\delta\sigma]^2} < 0. \end{aligned}$$

As a result, g is increasing with δ and σ, decreasing with β and λ. This completes the proof.

Facing the high death rate or the staff reduction in large, workers normally raise the growth rate of the human capital actively which increases production indirectly in order to continue to be employed. That is why the rise of the death shock or the job destruction shock result in the growth rate of the human capital increases. If there are more unskilled jobs in the labor market, resulting in demotivation, the growth rate of the human capital reduces. When the situation of employment is better, such as the arrival rate of the jobs is higher, the workers obviously have less competition which leads to the growth rate of the human capital becomes slower.

Proposition 3 *The reservation pay rate of unemployed workers with low-education θ_r^0 which satisfies Eq.(3) is unique.*

Proof From Eq.(3), we can get

$$\theta_r^0 = -\frac{g\beta\lambda}{\delta}\int_{\theta_r^0}^{\overline{\theta}^0} \frac{1-F_0(\theta)}{1-(1+g)\left(1-\psi_0(\theta)\right)}\mathrm{d}\theta - \frac{gb}{\delta} + (1+g)b.$$

Let

$$T(x) = -\frac{g\beta\lambda}{\delta}\int_{x}^{\overline{\theta}^0} \frac{1-F_0(\theta)}{1-(1+g)\left(1-\psi_0(\theta)\right)}\mathrm{d}\theta - \frac{gb}{\delta} + (1+g)b.$$

$\forall x_1, x_2 \in [\underline{\theta}^0, \overline{\theta}^0]$,

$$|T(x_1)-T(x_2)| = \left|\frac{g\beta\lambda}{\delta}\int_{x_1}^{x_2} \frac{1-F_0(\theta)}{1-(1+g)\left(1-\delta-\sigma-\beta\lambda(1-F_0(\theta))\right)}\mathrm{d}\theta\right|.$$

$\exists \epsilon \in [x_1, x_2] \in (\underline{\theta}^0, \overline{\theta}^0)$,

$$|T(x_1)-T(x_2)| = \frac{g\beta\lambda}{\delta}\left|\frac{(x_2-x_1)(1-F_0(\epsilon))}{1-(1+g)(1-\delta-\sigma-\beta\lambda(1-F_0(\epsilon)))}\right|$$

.

Since

$$g = \frac{\delta(\delta+\sigma+\beta\lambda)}{(\beta\lambda+\delta)(1-\delta)-\delta\sigma},$$

it is easily to find that

$$0 < \frac{g\beta\lambda(1-F_0(\epsilon))}{\delta\left[1-(1+g)(1-\delta-\sigma-\beta\lambda(1-F_0(\epsilon)))\right]} < 1.$$

So that

$$|T(x_1)-T(x_2)| < |x_2-x_1|.$$

From Banach fixed point theorem, we can prove that there exists a unique point $\theta_r^0 \in [\underline{\theta}^0, \overline{\theta}^0]$ satisfying Eq.(3).

Obviously, the term on the left-hand side of Eq.(3) is linear in θ_r^0 with slope $-\frac{\delta}{g}$, and the straight line intercepts the x-axis at $\theta_0^r = b(1+g)$. The terms on the right-hand side of Eq.(3) describe a curve which is continuous and strictly decreasing with first derivative

$$-\frac{\beta\lambda[1-F_0(\theta_r^0)]}{[1-(1+g)\,(1-\psi_0(\theta_r^0))]} = -\frac{\delta}{g} < 0,$$

that means the straight line is tangent to the curve at θ_r^0. It is clear that the terms on the right-hand side are positive strictly. Therefore, the straight line and the curve must have a unique intersection at some $\theta_r^0 < b(1+g)$ described in Figure 1 which implies θ_r^0 is unique. This completes the proof.

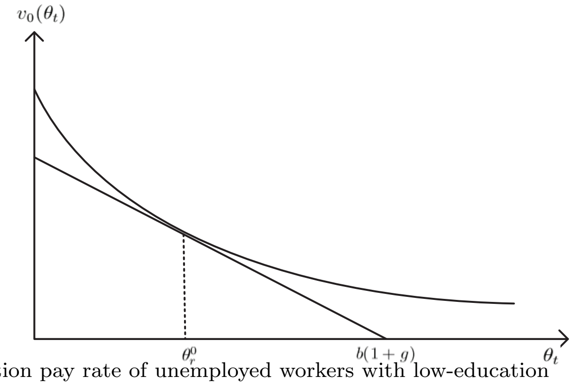

Fig. 1 Reservation pay rate of unemployed workers with low-education

2.1.2 High-educated workers' payoffs and job search strategies

For the high-educated workers, they can receive both the unskilled jobs and the skilled jobs, and their objective function is

$$\max_{x_t} \mathrm{E} \sum_{t=0}^{\infty} u\left(\theta_t, h_t, x_t\right),$$

where θ_t and h_t are the state variables in period t.

First consider the unemployed workers with high-education. The pay rate θ_t of the workers satisfies

$$\theta_{t+1} = \xi\left\{(1-y)b + y\left[(1-x_t)b + x_t\left((1-z)(\theta^0+c) + z\theta^1\right)\right]\right\},$$

where ξ, η, ζ, y and z are all independent radon variables. ξ, η and ζ are described as the same as the above section, y denotes whether the unemployed worker with high-education receives an outsider offer with $\Pr\{y=0\} = 1-\lambda$ means that the worker does not receive any outside job and $\Pr\{y=1\} = \lambda$ which means that the worker receives an outside offer, and z represents the kind of the outsider offer received with $\Pr\{z=0\} = \beta\lambda$ which means the offer is from the unskilled jobs and $\Pr\{z=1\} = (1-\beta)\lambda$ which means the offer is from the skilled jobs. y and z are independent with ξ, η and ζ. In addition, the human capital of the unemployed worker with high-education does not grow. The Bellman equation

$$V_{u1}(h_t) = \max\{bh_t + \mathrm{E}_{(\theta^0)} V_1(\theta_{t+1}; h_{t+1})\},$$

which can be rewritten as follows

$$\begin{aligned} V_{u1}(h_t) = & bh_t + (1-\delta-\lambda)V_{u1}(h_t) + \beta\lambda \mathrm{E}_{(\theta^0)} \max\{V_{u1}(h_t), V_1(\theta^0; h_t)\} \\ & +(1-\beta)\lambda \mathrm{E}_{(\theta^1)} \max\{V_{u1}(h_t), V_1(\theta^1; h_t)\}. \end{aligned} \tag{12}$$

Next consider the employed workers with high-education. The worker's pay rate θ_t satisfies

$$\theta_{t+1} = \xi \left\{(1-\eta)b + \eta \left[(1-y)\theta_t + y\left((1-x_t)\theta_t + x_t\left((1-z)\theta^0 + z\theta^1\right)\right)\right]\right\}.$$

Given an employed worker with the human capital h_t and the pay rate θ_t in t period, the Bellman equation

$$V_1(\theta_t; h_t) = \max\{\theta_t h_t + \mathrm{E}_{(\theta^1)} V_1(\theta_{t+1}; h_{t+1})\},$$

which can be written as

$$\begin{aligned} V_1(\theta_t; h_t) = & \theta_t h_t + \sigma V_{u1}(e^g h_t) + (1-\delta-\sigma-\lambda)V_1(\theta_t; e^g h_t) \\ & +\beta\lambda \mathrm{E}_{(\theta^0)} \max\{V_1(\theta_t; e^g h_t), V_1(\theta^0 + c; e^g h_t)\} \\ & +(1-\beta)\lambda \mathrm{E}_{(\theta^1)} \max\{V_1(\theta_t; e^g h_t), V_1(\theta^1; e^g h_t)\}. \end{aligned} \tag{13}$$

The employed worker leaves the firm when she dies or is displaced into unemployment or receives an outside offer which is higher than her current offer. Given the pay rate θ_t, the employed worker leaves a firm at the rate

$$\psi_1(\theta_t) = \delta + \sigma + \beta\lambda(1 - F_0(\theta_t)) + (1-\beta)\lambda(1 - F_1(\theta_t)),$$

which is the separation rate of an employed worker with high-education. Similarly, the employed worker with high-education accepts any outsider offer which is higher than her current offer, while the unemployed worker with high-education accepts any outsider offer which is no less than the reservation pay rate described in Proposition 4.

Proposition 4 *For the unemployed workers with high-education, the reservation pay rate per unit of human capital, denoted by θ_r^1, can be described by the following equation*

$$\frac{\delta\left(e^g b - \theta_r^1\right)}{e^g - 1} = b + \lambda \left[\int_{\theta_r^1}^{\bar{\theta}^0} \frac{\beta\left(1 - F_0(\theta)\right)}{1 - e^g\left(1 - \psi_1(\theta)\right)} \mathrm{d}\theta + \int_{\theta_r^1}^{\bar{\theta}^1} \frac{(1-\beta)\left(1 - F_1(\theta)\right)}{1 - e^g\left(1 - \psi_1(\theta)\right)} \mathrm{d}\theta\right]. \tag{14}$$

Furthermore, the optimal job search implies that an unemployed worker with high-education accepts job offer θ^1 if and only if $\theta^1 \geq \theta_r^1$.

Proof It is obvious that a worker's income is always proportional to h_t, that is, there exists a number v_{u1} and a function $v_1(\theta_t)$ such that $V_{u1}(h_t) = v_{u1}h_t$ and $V_1(\theta_t; h_t) = v_1(\theta_t)h_t$ respectively. In this way, Eq.(12) implies that v_{u1} satisfies

$$\delta v_{u1} = b + \lambda \left[\beta \int_{\theta_r^1}^{\overline{\theta}^0} (v_1(\theta) - v_{u1})\mathrm{d}F_0(\theta) + (1-\beta) \int_{\theta_r^1}^{\overline{\theta}^1} (v_1(\theta) - v_{u1})\mathrm{d}F_1(\theta)\right], \tag{15}$$

while Eq.(13) implies that $v_1(\theta_t)$ satisfies

$$\begin{aligned} v_1(\theta_t) =& \theta_t + \sigma v_{u1} + e^g[(1-\delta-\sigma)v_1(\theta_t) + \beta\lambda \int_{\theta_t}^{\overline{\theta}^0} (v_1(\theta) - v_1(\theta_t))\mathrm{d}F_0(\theta) \\ &+(1-\beta)\lambda \int_{\theta_t}^{\overline{\theta}^1} (v_1(\theta) - v_1(\theta_t))\mathrm{d}F_1(\theta)]. \end{aligned} \tag{16}$$

Since θ_r^1 is the reservation pay rate per unit of human capital of the unemployed workers with high-education, there is no difference between accepting the offer θ_r^1 and keeping unemployment, i.e., $V_1(\theta_r^1; h_t) = V_{u0}(h_t)$. Therefore, we can get $v_1(\theta_r^1) = v_{u1}$. Thus,

$$\begin{aligned} v_{u1} =& \theta_t + e^g \left[(1-\delta)v_{u1} + \beta\lambda \int_{\theta_t}^{\overline{\theta}^0} (v_1(\theta) - v_{u1})\mathrm{d}F_0(\theta) \right. \\ &\left. + (1-\beta)\lambda \int_{\theta_t}^{\overline{\theta}^1} (v_1(\theta) - v_{u1})\mathrm{d}F_1(\theta)\right]. \end{aligned} \tag{17}$$

Let $\theta_t = \theta_r^1$ in (17). That is

$$\begin{aligned} v_{u1} =& \theta_r^1 + e^g[(1-\delta)v_{u1} + \beta\lambda \int_{\theta_r^1}^{\overline{\theta}^0} (v_1(\theta) - v_{u1})\mathrm{d}F_0(\theta) \\ &+(1-\beta)\lambda \int_{\theta_r^1}^{\overline{\theta}^1} (v_1(\theta) - v_{u1})\mathrm{d}F_1(\theta)]. \end{aligned} \tag{18}$$

Note that $\beta \int_{\theta_r^1}^{\overline{\theta}^0} (v_1(\theta)-v_{u1})\mathrm{d}F_0(\theta)+(1-\beta) \int_{\theta_r^1}^{\overline{\theta}^1} (v_1(\theta)-v_{u1})\mathrm{d}F_1(\theta) = \frac{\delta v_{u1} - b}{\lambda}$ by (15), thus (18) can be written as

$$(e^g - 1)v_{u1} = e^g b - \theta_r^1. \tag{19}$$

On the other hand, differentiating (16) with respect to θ_t implies $v_1(\theta_t)$ is determined by the following differential equation

$$\frac{\mathrm{d}v_1(\theta_t)}{\mathrm{d}\theta_t} = \frac{1}{1 - e^g\left(1 - \psi_1(\theta_t)\right)}. \tag{20}$$

Therefore, (15) can be written as

$$\begin{aligned}\delta v_{u1} &= b+\lambda\left[\beta\int_{\theta_r^1}^{\overline{\theta}^0}(v_1(\theta)-v_{u1})\mathrm{d}F_0(\theta)+(1-\beta)\int_{\theta_r^1}^{\overline{\theta}^1}(v_1(\theta)-v_{u1})\mathrm{d}F_1(\theta)\right]\\ &= b+\lambda\left[\beta\int_{\theta_r^1}^{\overline{\theta}^0}(1-F_0(\theta))\frac{\mathrm{d}v_0(\theta)}{\mathrm{d}\theta}\mathrm{d}\theta+(1-\beta)\int_{\theta_r^1}^{\overline{\theta}^1}(1-F_1(\theta))\frac{\mathrm{d}v_0(\theta)}{\mathrm{d}\theta}\mathrm{d}\theta\right]\\ &\quad\text{(integrate by parts)}\\ &= b+\lambda\left[\beta\int_{\theta_r^1}^{\overline{\theta}^0}\frac{1-F_0(\theta)}{1-e^g\,(1-\psi_1(\theta))}\mathrm{d}\theta+(1-\beta)\int_{\theta_r^1}^{\overline{\theta}^1}\frac{1-F_1(\theta)}{1-e^g\,(1-\psi_1(\theta))}\mathrm{d}\theta\right]\\ &\quad\text{(by(20))}\end{aligned} \tag{21}$$

Substituting (19) into (21) yields Eq.(14). Moreover, the optimal job search implies that any unemployed worker with high-education accepts job offer θ^1 if and only if $\theta^1 \geq \theta_r^1$. This completes the proof.

Proposition 5 *When the fraction of the unskilled jobs is greater than β, there exists a cross-skill equilibrium that is the high-educated workers choose either the unskilled job or the skilled job; when the fraction of the unskilled jobs is no greater than β, there exists a separating equilibrium that is the high-educated workers only choose the skilled jobs, where β satisfies*

$$\exp\left(\frac{\delta(\delta+\sigma+\beta\lambda)}{(\beta\lambda+\delta)(1-\delta)-\delta\sigma}\right)=\frac{(1-\beta)\lambda+\delta}{(1-\delta)[(1-\beta)\lambda+\delta]-\delta\sigma}.$$

Proof Calculating the derivative of Eq.(14) with θ_r^1, so as to get

$$\beta F_0(\theta_r^1)+(1-\beta)F_1(\theta_r^1)=1-\frac{\delta[1-e^g(1-\delta-\sigma)]}{\lambda(e^g-1-\delta e^g)}. \tag{22}$$

All the skilled jobs offer $\theta^1 \geq \theta_r^1$, $\theta^1 \in [\underline{\theta}^1, \overline{\theta}^1]$, otherwise there is no worker accepts the offer. Thus in the market equilibrium, $F_1(\theta_r^1) = 0$, and Eq.(22) can be written as

$$F_0(\theta_r^1)=\frac{1}{\beta}-\frac{\delta[1-e^g(1-\delta-\sigma)]}{\beta\lambda(e^g-1-\delta e^g)}. \tag{23}$$

For the unskilled jobs, if $\overline{\theta}^0 \geq \theta_r^1$ which means that the high-educated workers choose either the unskilled job or the skilled job, we can get $F_0(\theta_r^1) < 1$, and by Eq.(23), there exists

$$e^g<\frac{(1-\beta)\lambda+\delta}{[(1-\beta)\lambda+\delta](1-\delta)-\delta\sigma}.$$

Since

$$g=\frac{\delta(\delta+\sigma+\beta\lambda)}{(\beta\lambda+\delta)(1-\delta)-\delta\sigma},$$

it is easily to get

$$exp\left(\frac{\delta(\delta+\sigma+\beta\lambda)}{(\beta\lambda+\delta)(1-\delta)-\delta\sigma}\right)<\frac{(1-\beta)\lambda+\delta}{[(1-\beta)\lambda+\delta](1-\delta)-\delta\sigma}.$$

Therefore, when the fraction of the unskilled jobs is greater than β, the high-educated workers choose either the unskilled job or the skilled job.

If $\overline{\theta}^0<\theta_r^1$ which implies that the high-educated workers only choose the skilled jobs, we can get $F_0(\theta_r^1)=1$, and by Eq.(23), there exists

$$e^g=\frac{(1-\beta)\lambda+\delta}{[(1-\beta)\lambda+\delta](1-\delta)-\delta\sigma}.$$

Since

$$g=\frac{\delta(\delta+\sigma+\beta\lambda)}{(\beta\lambda+\delta)(1-\delta)-\delta\sigma},$$

it is easily to get

$$\exp\left(\frac{\delta(\delta+\sigma+\beta\lambda)}{(\beta\lambda+\delta)(1-\delta)-\delta\sigma}\right)=\frac{(1-\beta)\lambda+\delta}{[(1-\beta)\lambda+\delta](1-\delta)-\delta\sigma}. \tag{24}$$

When the fraction of the unskilled jobs is no greater than β, the high-educated workers only choose the skilled jobs. This completes the proof.

Proposition 6 *The reservation pay rate of the unemployed workers with high-education θ_r^1 which satisfies Eq.(14) is unique.*

Proof From Eq.(14), we can get

$$\begin{aligned}\theta_r^1=&e^gb-\frac{(e^g-1)b}{\delta}-\frac{(e^g-1)\lambda}{\delta}\left[\int_{\theta_r^1}^{\overline{\theta}^0}\frac{\beta\left(1-F_0(\theta)\right)}{1-e^g\left(1-\psi_1(\theta)\right)}\mathrm{d}\theta\right.\\&\left.+\int_{\theta_r^1}^{\overline{\theta}^1}\frac{(1-\beta)\left(1-F_1(\theta)\right)}{1-e^g\left(1-\psi_1(\theta)\right)}\mathrm{d}\theta\right].\end{aligned} \tag{25}$$

Let

$$\begin{aligned}T(x)=e^gb-\frac{(e^g-1)b}{\delta}-\frac{(e^g-1)\lambda}{\delta}&\left[\int_{x}^{\overline{\theta}^0}\frac{\beta\left(1-F_0(\theta)\right)}{1-e^g\left(1-\psi_1(\theta)\right)}\mathrm{d}\theta\right.\\&\left.+\int_{x}^{\overline{\theta}^1}\frac{(1-\beta)\left(1-F_1(\theta)\right)}{1-e^g\left(1-\psi_1(\theta)\right)}\mathrm{d}\theta\right].\end{aligned} \tag{26}$$

$\forall x_1,x_2\in[\underline{\theta}^0,\overline{\theta}^0]$,

$$|T(x_1)-T(x_2)|=\left|\frac{(e^g-1)\lambda}{\delta}\left[\int_{x_1}^{x_2}\frac{1-\beta F_0(\theta)-(1-\beta)F_1(\theta)}{1-e^g(1-\psi_1(\theta))}\mathrm{d}\theta\right]\right|$$

$\exists \epsilon \in [x_1, x_2] \in (\underline{\theta}^0, \overline{\theta}^0)$,

$$|T(x_1) - T(x_2)| = \frac{(e^g-1)\lambda}{\delta} \int_{x_1}^{x_2} \frac{1-\beta F_0(\theta)-(1-\beta)F_1(\theta)}{1-e^g\left[1-\delta-\sigma-\lambda+\beta F_0(\theta)+(1-\beta)F_1(\theta)\right]} \mathrm{d}\theta$$
$$= \left| \frac{(e^g-1)\lambda}{\delta} \left[(x_2-x_1) \frac{1-\beta F_0(\epsilon)-(1-\beta)F_1(\epsilon)}{1-e^g(1-\psi_1(\epsilon))} \right] \right|$$
$$< \frac{(e^g-1)\lambda}{\delta[1-e^g(1-\delta-\sigma-\lambda)]} |x_2-x_1| .$$

It is easily to find that $0 < \frac{(1-\sigma)(e^g-1)\lambda}{\delta[1-e^g(1-\delta-\sigma-\lambda)]} < 1$. From Banach fixed point theorem, we can prove that there exists a unique point $\theta_r^1 \in [\underline{\theta}^0, \overline{\theta}^0]$ satisfied Eq.(14).

Obviously, the term on the left-hand side of Eq.(14) is linear in θ_r^1 with slope $-\frac{\delta}{e^g-1}$ and the straight line intercepts the x-axis at $\theta_1^r = e^g b$. The terms on the right-hand side of Eq.(14) describe a curve which is continuous and strictly decreasing with the first derivative

$$-\frac{\lambda[1-\beta F_0(\theta_r^1)-(1-\beta)F_1(\theta_r^1)]}{1-e^g\,(1-\psi_1(\theta_r^1))} = -\frac{\delta}{e^g-1} < 0,$$

which means the straight line is tangent to the curve at θ_r^1. It is clear that the terms on the right-hand side are positive strictly. Therefore, the straight and the curve must have a unique intersection at some $\theta_r^1 < e^g b$ described in Figure 5 which shows that θ_r^1 is unique. This completes the proof.

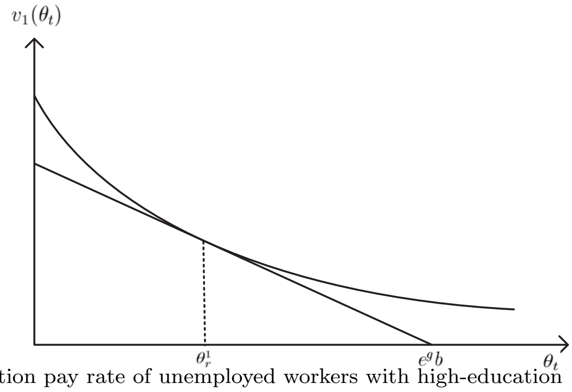

Fig. 2 Reservation pay rate of unemployed workers with high-education

2.2 Firm's payoffs

The firm's optimization problem can reduce to choosing an optimal pay rate to maximize the steady state profit. Let $N_j(h)$ denote the probability of the unemployed workers of type j whose human capital is h, $G_j(\theta, h)$ denote the probability of employed workers with human capital h and pay rate θ, and γ_j denote the unemployment rate of the workers with type j, $j = 0, 1$.

For a firm offering unskilled job contract θ^0 to the low-educated workers with human capital $(1+g)^\tau$, the steady state profit

$$
\begin{aligned}
\pi_{00} =& \alpha\lambda\alpha\gamma_0 \sum_{\tau=0}^{\infty} \left[N_0\left((1+g)^\tau\right) \sum_{s=0}^{\infty} (1-\psi_0(\theta^0))^s (p-\theta^0)(1+g)^{s+\tau} \right] \\
&+ \alpha\lambda\alpha(1-\gamma_0) \sum_{\tau=0}^{\infty} \left[\int_{\underline{\theta}^0}^{\theta^0} G_0(\theta, (1+g)^\tau) \mathrm{d}\theta \sum_{s=0}^{\infty} (1-\psi_0(\theta^0))^s (p-\theta^0)(1+g)^{s+\tau} \right] \\
=& \frac{\lambda\alpha^2(p-\theta^0)}{1-(1-\psi_0(\theta^0))(1+g)} \left[\gamma_0 \sum_{\tau=0}^{\infty} N_0\left((1+g)^\tau\right)(1+g)^\tau \right. \\
&\left. + (1-\gamma_0) \sum_{\tau=0}^{\infty} \int_{\underline{\theta}^0}^{\theta^0} G_0(\theta, (1+g)^\tau)(1+g)^\tau \mathrm{d}\theta \right].
\end{aligned}
\tag{27}
$$

The steady state flow profit equals the hiring rate of the firm, multiplied by the expected profit of each hire. The first term in the above equation is the flow profit due to attracting unemployed workers with low-education whose human capital is $(1+g)^\tau$, where τ is the experience. The second term is the flow profit due to attracting employed workers with low-education whose human capital is $(1+g)^\tau$ and her current offer is no greater than θ^0.

If the firm offering unskilled job contract θ^0 to the high-educated workers with human capital $e^{g\tau}$, the steady state profit

$$
\begin{aligned}
\pi_{01} =& (1-\alpha)\lambda(1-\alpha)\gamma_1 \sum_{\tau=0}^{\infty} \left[N_1\left(e^{g\tau}\right) \sum_{s=0}^{\infty} (1-\psi_1(\theta^0))^s (p-\theta^0) e^{g(s+\tau)} \right] \\
&+ (1-\alpha)\lambda(1-\alpha)(1-\gamma_1) \sum_{\tau=0}^{\infty} \left[\int_{\underline{\theta}^1}^{\theta^0} G_1(\theta, e^{g\tau}) \mathrm{d}\theta \sum_{s=0}^{\infty} (1-\psi_1(\theta^0))^s (p-\theta^0-c) e^{g(s+\tau)} \right] \\
=& \frac{\lambda(1-\alpha)^2(p-\theta^0-c)}{1-(1-\psi_1(\theta^0))e^g} \left[\gamma_1 \sum_{\tau=0}^{\infty} N_1\left(e^{g\tau}\right) e^{g\tau} + (1-\gamma_1) \sum_{\tau=0}^{\infty} \int_{\underline{\theta}^1}^{\theta^0} G_1(\theta, e^{g\tau}) e^{g\tau} \mathrm{d}\theta \right].
\end{aligned}
\tag{28}
$$

The firm offering unskilled jobs can attract both types of workers, thus the optimization problem can be described as follows:

$$
\begin{aligned}
&\max_{\theta^0} \, (\pi_{00}+\pi_{01}) \\
&\text{s.t.} \begin{cases}
V_0(\theta_t;h_t) = \theta_t h_t + (1+g)\left[\sigma V_{u0}(h_t) + (1-\delta-\sigma)V_0(\theta_t;h_t) \right. \\
\qquad \left. +\beta\lambda \int_{\theta_t}^{\overline{\theta}^0} (V_0(\theta^0;h_t) - V_0(\theta_t;h_t))\mathrm{d}F_0(\theta^0)\right] \\
\delta V_{u0}(h_t) = bh_t + \beta\lambda \int_{\theta_r^0}^{\overline{\theta}^0} (V_0(\theta^0;h_t) - V_{u0}(h_t))\mathrm{d}F_0(\theta^0) \\
V_1(\theta_t;h_t) = \theta_t h_t + e^g \left[\sigma V_{u1}(h_t) + (1-\delta-\sigma)V_1(\theta_t;h_t) \right. \\
\qquad + \beta\lambda \int_{\theta_t}^{\overline{\theta}^0} (V_1(\theta^0;h_t) - V_1(\theta_t;h_t))\mathrm{d}F_0(\theta^0) \\
\qquad \left. +(1-\beta)\lambda \int_{\theta_t}^{\overline{\theta}^1} (V_1(\theta^1;h_t) - V_1(\theta_t;h_t))\mathrm{d}F_1(\theta^1)\right] \\
\delta V_{u1}(h_t) = bh_t + \lambda\left[\beta \int_{\theta_r^1}^{\overline{\theta}^0} (V_1(\theta^0;h_t) - V_{u1}(h_t))\mathrm{d}F_0(\theta^0) \right. \\
\qquad \left. +(1-\beta) \int_{\theta_r^1}^{\overline{\theta}^1} (V_1(\theta^1;h_t) - V_{u1}(h_t))\mathrm{d}F_1(\theta^1)\right].
\end{cases}
\end{aligned}
\tag{29}
$$

If a firm offering a skilled job contract θ^1 to the high-educated workers with human capital $e^{g\tau}$, the steady state profit

$$
\begin{aligned}
\pi_{11} &= \lambda(1-\alpha)\gamma_1 \sum_{\tau=0}^{\infty}\left[N_1\left(e^{g\tau}\right)\sum_{s=0}^{\infty}(1-\psi_1(\theta^1))^s(p-\theta^1)e^{g(s+\tau)}\right] \\
&\quad +\lambda(1-\alpha)(1-\gamma_1)\sum_{\tau=0}^{\infty}\left[\int_{\underline{\theta}^1}^{\theta^1} G_1(\theta,e^{g\tau})\mathrm{d}\theta \sum_{s=0}^{\infty}(1-\psi_1(\theta^1))^s(p-\theta^1)e^{g(s+\tau)}\right] \\
&= \frac{\lambda(1-\alpha)(p-\theta^1)}{1-(1-\psi_1(\theta^1))e^g}\left[\gamma_1\sum_{\tau=0}^{\infty}N_1\left(e^{g\tau}\right)e^{g\tau} + (1-\gamma_1)\sum_{\tau=0}^{\infty}\int_{\underline{\theta}^1}^{\theta^1} G_1(\theta,e^{g\tau})e^{g\tau}\mathrm{d}\theta\right].
\end{aligned}
\tag{30}
$$

The firm offering skilled jobs only attracts high-educated workers, then the optimization problem can be described as follows:

$$\max_{\theta^1} \ \pi_{11}$$
$$\text{s.t.} \begin{cases} V_1(\theta_t;h_t) = \theta_t h_t + e^g \Big[\sigma V_{u1}(h_t) + (1-\delta-\sigma)V_1(\theta_t;h_t) \\ \qquad + \beta\lambda \int_{\theta_t}^{\overline{\theta}^0} (V_1(\theta^0;h_t) - V_1(\theta_t;h_t))\mathrm{d}F_0(\theta^0) \\ \qquad + (1-\beta)\lambda \int_{\theta_t}^{\overline{\theta}^1} (V_1(\theta^1;h_t) - V_1(\theta_t;h_t))\mathrm{d}F_1(\theta^1)\Big] \\ \delta V_{u1}(h_t) = bh_t + \lambda \Big[\beta \int_{\theta_r^1}^{\overline{\theta}^0} (V_1(\theta^0;h_t) - V_{u1}(h_t))\mathrm{d}F_0(\theta^0) \\ \qquad + (1-\beta) \int_{\theta_r^1}^{\overline{\theta}^1} (V_1(\theta^1;h_t) - V_{u1}(h_t))\mathrm{d}F_1(\theta^1)\Big] \end{cases} \tag{31}$$

Note that the object of each firm is to maximize steady state flow profit, that is, the unskilled firms choose θ^0 to maximize $\pi_0 = \pi_{00} + \pi_{01}$, while the skilled firms choose θ^1 to maximize $\pi_1 = \pi_{11}$. Moreover, let π_i^* denote the optimal value satisfying the above restrain condition, $i = 0, 1$.

3 Market equilibrium

We now formally define a market equilibrium.

Definition 1 A market equilibrium is a set $\{\theta_r^0, \theta_r^1, \gamma_0, \gamma_1, N_0(\cdot), N_1(\cdot), G_0(\cdot), G_1(\cdot), F_0(\cdot), F_1(\cdot)\}$ that satisfies the following requirements:

1) θ_r^0 is the optimal reservation pay rate per unit of human capital of any unemployed low-educated worker, and θ_r^1 is the optimal reservation pay rate per unit of human capital of any unemployed high-educated worker, given in Propositions 1 and 4, respectively;
2) $\gamma_0, N_0(\cdot), G_0(\cdot)$ are consistent with the steady state turnover and pay rate distribution $F_0(\cdot)$; likewise, $\gamma_1, N_1(\cdot), G_1(\cdot)$ are consistent with steady state turnover and pay rate distributions $F_0(\cdot)$ and $F_1(\cdot)$;
3) the constant profit conditions are satisfied, i.e.,

$$\begin{aligned} &\pi_{00} + \pi_{01} = \pi_0^* > 0, && \text{for all } \ \theta^0 \text{ where } \mathrm{d}F_0(\theta^0) > 0, \\ &\pi_{00} + \pi_{01} \le \pi_0^*, && \text{for all } \ \theta^0 \text{ where } \mathrm{d}F_0(\theta^0) = 0, \\ &\pi_{11} = \pi_1^* > 0, && \text{for all } \ \theta^1 \text{ where } \mathrm{d}F_0(\theta^1) > 0 \text{ or } \mathrm{d}F_1(\theta^1) > 0, \\ &\pi_{11} \le \pi_1^*, && \text{for all } \ \theta^1 \text{ where } \mathrm{d}F_0(\theta^1) = 0 \text{ and } \mathrm{d}F_1(\theta^1) = 0, \end{aligned}$$

where π_{00} is the profit of the firms offering the unskilled jobs to the low-educated workers, π_{01} is the profit of the firms offering the unskilled jobs to the high-educated workers, π_{11} is the profit of the firms offering the skilled jobs to the high-educated workers, π_0^* is the total optimal profit of offering the unskilled jobs and π_1^* is the total optimal profit of the firms offering the skilled jobs in a equilibrium market. The constant profit conditions imply that all equilibrium offers of the same type i enjoy the same profit π_i^* with the workers' optimal strategy.

In Section 3.1, we firstly use the steady state turnover arguments to solve for γ_j, $j = 0, 1$. And then in Section 3.2, we determine the distribution functions $N_j(\cdot)$ and $G_j(\cdot)$. Lastly, we find F_i in Section 3.3 so that the above constant profit conditions are satisfied, $i = 0, 1$.

3.1 Solve γ_0, $N_0(\cdot)$ and $G_0(\cdot)$ for the low-educated workers

To solve γ_0, we first consider the steady state turnover in the pool of unemployed workers whose number is $\gamma_0\alpha$. The total outflow from this pool is $(\beta\lambda+\delta)\gamma_0\alpha$, which either finds a job or leaves the market. While the inflow in the pool is composed of the new entrants and the employed workers who are displaced into the unemployment, which is $\delta\alpha+\sigma(1-\gamma_0)\alpha$. Equating the outflow with the inflow, the equilibrium unemployment rate of the low-educated workers is

$$\gamma_0 = \frac{\sigma + \delta}{\delta + \sigma + \beta\lambda}. \tag{32}$$

To solve $N_0(h)$, we next consider the steady state turnover in the pool of the unemployed workers with human capital h. When $h = 1$, the outflow $(\beta\lambda + \delta)\gamma_0\alpha N_0(1)$ consists of the workers who either find an unskilled job or leave the market. And the inflow is composed only of the new entrants which is $\delta\alpha$, as the human capital of the workers who are laid off from the employment is at least $(1 + g)$. Setting the outflow equal to the inflow yields

$$N_0(1)= \frac{\delta}{\gamma_0(\beta\lambda + \delta)}. \tag{33}$$

When $h \in \{(1 + g)^m\}_{m=1}^{\infty}$, the steady-state turnover requires

$$(\beta\lambda + \delta)\gamma_0\alpha N_0(h) = \sigma(1 - \gamma_0)\alpha \int_{\theta_r^0}^{\overline{\theta}^0} G_0(\theta, h)\mathrm{d}\theta, \tag{34}$$

where the left hand side describes the outflow of the workers with human capital h who find a new unskilled job or leave the labor market, and the right hand side describes the inflow of the employed workers with human capital h and current offer θ who are displaced into the unemployment. Solving for (34) yields

$$N_0(h)= \frac{\sigma(1 - \gamma_0)}{(\beta\lambda + \delta)\gamma_0} \int_{\theta_r^0}^{\overline{\theta}^0} G_0(\theta, h)\mathrm{d}\theta. \tag{35}$$

Therefore,

$$N_0(h) = \begin{cases} \dfrac{\delta}{\gamma_0(\beta\lambda+\delta)}, & \text{if } h = 1 \\ \dfrac{\sigma(1-\gamma_0)}{(\beta\lambda+\delta)\gamma_0} \displaystyle\int_{\theta_r^0}^{\overline{\theta}^0} G_0(\theta,h)\mathrm{d}\theta, & \text{if } h \in \{(1+g)^m\}_{m=1}^{\infty}. \end{cases}$$

To solve $G_0(\theta, h)$, we finally consider the steady state turnover in the pool of the employed workers with human capital h, where $h \in \{(1+g)^m\}_{m=1}^{\infty}$ and her current pay rate θ is no greater than θ^0. The workers with human capital h will leave this pool for sure regardless of whether they stay (if they stay, their human capital becomes $(1+g)h$) or are displaced into unemployment or leave the market, so the total outflow is

$$(1-\gamma_0)\alpha \int_{\theta_r^0}^{\theta^0} G_0(\theta,h)\mathrm{d}\theta.$$

The workers with human capital $(1+g)^{-1}h$ who were employed or unemployed will join this pool group if they find a job offering pay rate which is no greater than θ^0. The inflow of these workers is

$$(1-\gamma_0)\alpha\left[1-\psi_0(\theta^0)\right] \int_{\theta_r^0}^{\theta^0} G_0\left(\theta, \frac{h}{1+g}\right)\mathrm{d}\theta + \gamma_0\alpha N_0\left(h\right)\beta\lambda F_0(\theta^0).$$

Equating the outflow with the inflow implies

$$\begin{aligned}(1-\gamma_0)\int_{\theta_r^0}^{\theta^0} G_0(\theta,h)\mathrm{d}\theta =& (1-\gamma_0)\left[1-\psi_0(\theta^0)\right]\int_{\theta_r^0}^{\theta^0} G_0(\theta,\frac{h}{1+g})\mathrm{d}\theta \\ &+\gamma_0 N_0\left(h\right)\beta\lambda F_0(\theta^0).\end{aligned} \tag{36}$$

Proposition 7 *For the low-educated workers, the steady state turnover in a market equilibrium implies*

$$N_0(h) = \frac{\delta\sigma\beta\lambda}{\gamma_0(\beta\lambda+\delta)^2} q_0^m,$$

and $G_0(\theta, h)$ satisfies

$$\begin{aligned}\int_{\theta_r^0}^{\theta^0} G_0(\theta,h)\mathrm{d}\theta =& \frac{\delta\beta\lambda F_0(\theta^0)(\beta\lambda+\delta-\beta\lambda\sigma)}{(1-\gamma_0)(\beta\lambda+\delta)\left[\sigma(1-\psi_0(\theta^0))+(1-F_0(\theta^0))(\beta\lambda+\delta)\right]} \\ &\times\left\{(1-F_0(\theta^0))\left[1-\psi_0(\theta^0)\right]^m + \frac{\sigma}{\beta\lambda+\delta}q_0^{m+1}\right\},\end{aligned}$$

for all $\theta^0 \in [\theta_r^0, \overline{\theta}^0]$ and $h \in \{(1+g)^m\}_{m=0}^{\infty}$, where $q_0 = \frac{(1-\delta-\sigma)(\beta\lambda+\delta)}{\beta\lambda(1-\sigma)+\delta} < 1$.

Proof Let $h = 1$ and $\theta^0 = \overline{\theta}^0$ in (36). Thus,

$$\int_{\theta_r^0}^{\overline{\theta}^0} G_0(\theta, 1)\mathrm{d}\theta = \frac{\gamma_0\beta\lambda}{1-\gamma_0} N_0(1),$$

and using (33) to substitute out $N_0(1)$ obtains

$$\int_{\theta_r^0}^{\overline{\theta}^0} G_0(\theta, 1)\mathrm{d}\theta = \frac{\delta\beta\lambda}{(1-\gamma_0)(\beta\lambda+\delta)}.$$

Let $h \in \{(1+g)^m\}_{m=1}^{\infty}$ and $\theta^0 = \overline{\theta}^0$ in (36) once again. Therefore,

$$\begin{aligned}(1-\gamma_0)\int_{\theta_r^0}^{\overline{\theta}^0} G_0(\theta, (1+g)^m)\mathrm{d}\theta =& (1-\gamma_0)\left[1-\psi_0(\overline{\theta}^0)\right]\int_{\theta_r^0}^{\overline{\theta}^0} G_0(\theta, (1+g)^{m-1})\mathrm{d}\theta \\ &+\gamma_0 N_0\left((1+g)^m\right)\beta\lambda,\end{aligned}$$

and using (35) to substitute out $N_0(\cdot)$ yields

$$\int_{\theta_r^0}^{\overline{\theta}^0} G_0(\theta, (1+g)^m)\mathrm{d}\theta = q_0 \int_{\theta_r^0}^{\overline{\theta}^0} G_0(\theta, (1+g)^{m-1})\mathrm{d}\theta,$$

where

$$q_0 = \frac{(1-\delta-\sigma)(\beta\lambda+\delta)}{\beta\lambda+\delta-\sigma\beta\lambda} < 1.$$

Therefore, when $h \in \{(1+g)^m\}_{m=0}^{\infty}$,

$$\int_{\theta_r^0}^{\overline{\theta}^0} G_0(\theta, (1+g)^m)\mathrm{d}\theta = \frac{\delta\beta\lambda}{(1-\gamma_0)(\beta\lambda+\delta)} q_0^m. \tag{37}$$

Using (37) in (35) obtains

$$N_0((1+g)^m) = \frac{\delta\sigma\beta\lambda}{\gamma_0(\beta\lambda+\delta)^2} q_0^m. \tag{38}$$

Given θ_0, if $h = 1$, (36) can be written as

$$\int_{\theta_r^0}^{\theta^0} G_0(\theta, 1)\mathrm{d}\theta = \frac{\delta\beta\lambda F_0(\theta^0)}{(1-\gamma_0)(\beta\lambda+\delta)},$$

and if $h \in \{(1+g)^m\}_{m=1}^{\infty}$, (36) can be written as

$$\int_{\theta_r^0}^{\theta^0} G_0(\theta, (1+g)^m)\mathrm{d}\theta = \left[1-\psi_0(\theta^0)\right]\int_{\theta_r^0}^{\theta^0} G_0(\theta, (1+g)^{m-1})\mathrm{d}\theta + \frac{\sigma\delta\beta^2\lambda^2 F_0(\theta^0)}{(1-\gamma_0)(\beta\lambda+\delta)^2} q_0^m.$$

Then we can get that $G_0(\theta, h)$ satisfies

$$\begin{aligned}\int_{\theta_r^0}^{\theta^0} G_0(\theta, h)\mathrm{d}\theta =& \frac{\delta\beta\lambda F_0(\theta^0)(\beta\lambda+\delta-\lambda\sigma)}{(1-\gamma_0)(\beta\lambda+\delta)\left[\sigma(1-\psi_0(\theta^0)) + (1-F_0(\theta^0))(\beta\lambda+\delta)\right]} \\ &\times\left\{(1-F_0(\theta^0))\left[1-\psi_0(\theta^0)\right]^m + \frac{\sigma}{\beta\lambda+\delta} q_0^{m+1}\right\}.\end{aligned} \tag{39}$$

This completes the proof.

3.2 Solve γ_1, $N_1(\cdot)$ and $G_1(\cdot)$ for the high-educated workers

The inflow and the outflow of the high-educated workers is similar to the low-educated workers.

To solve γ_1, we first consider the steady state turnover in the pool of the unemployed workers with high-education whose number is $\gamma_1(1-\alpha)$. Equating the outflow $(\delta+\lambda)\gamma_1(1-\alpha)$ with the inflow $\delta(1-\alpha)+\sigma(1-\gamma_1)(1-\alpha)$, the equilibrium unemployment rate of the high-educated workers is

$$\gamma_1 = \frac{\delta+\sigma}{\delta+\sigma+\lambda}. \tag{40}$$

To solve $N_1(h)$, we next consider the steady state turnover in the pool of the unemployed workers with human capital h. When $h=1$, the outflow $(\delta+\lambda)\gamma_1(1-\alpha)N_1(1)$ consists of the workers who leave the market or either find an unskilled job or a skilled job. And the inflow is composed only of the new entrants which is $\delta(1-\alpha)$. Setting the outflow equal to the inflow implies that $N_1(1)$ satisfies

$$N_1(1) = \frac{\delta}{\gamma_1(\delta+\lambda)}. \tag{41}$$

When $h=\{e^{ng}\}_{n=1}^{\infty}$, the steady state turnover requires

$$(\delta+\lambda)\gamma_1(1-\alpha)N_1(e^{ng}) = \sigma(1-\gamma_1)(1-\alpha)\int_{\theta_r^1}^{\overline{\theta}^1} G_1(\theta,e^{ng})\mathrm{d}\theta, \tag{42}$$

where the left hand side describes the outflow of unemployed workers with human capital h who find a new job or leave the labor market, and the right hand side describes the inflow of employed workers with human capital e^{ng} who are displaced into unemployment. Solving for $N_1(e^{ng})$ implies

$$N_1(e^{ng}) = \frac{\sigma(1-\gamma_1)}{(\delta+\lambda)\gamma_1}\int_{\theta_r^1}^{\overline{\theta}^1} G_1(\theta,e^{ng})\mathrm{d}\theta. \tag{43}$$

Therefore,

$$N_1(h) = \begin{cases} \dfrac{\delta}{\gamma_1(\delta+\lambda)}, & \text{if } h=1 \\ \dfrac{\sigma(1-\gamma_1)}{(\delta+\lambda)\gamma_1}\displaystyle\int_{\theta_r^1}^{\overline{\theta}^1} G_1(\theta,e^{ng})\mathrm{d}\theta, & \text{if } h\in\{e^{ng}\}_{n=1}^{\infty}. \end{cases}$$

To solve $G_1(h)$, we finally consider the steady state turnover in the pool of the employed workers with human capital h where $h\in\{e^{ng}\}_{n=1}^{\infty}$ and her current pay rate θ is no greater than θ^1. The workers with human capital h will leave this pool for sure regardless of whether they stay (if they stay, their human capital become $e^g h$) or are displaced into unemployment or leave the market, so the total outflow is

$$(1-\gamma_1)(1-\alpha)\int_{\theta_r^1}^{\theta^1} G_1(\theta,h)\mathrm{d}\theta.$$

The workers with human capital $e^{-g}h$ who were employed or unemployed will join this pool group if they find a job offering pay rate which is no greater than θ^1. The inflow of these workers is

$$(1-\gamma_1)(1-\alpha)\left(1-\psi_1(\theta^1)\right)\int_{\theta_r^1}^{\theta^1} G_1\left(\theta,\frac{h}{e^g}\right)\mathrm{d}\theta$$
$$+\gamma_1(1-\alpha)N_1(h)\lambda\left(\beta F_0(\theta^1)+(1-\beta)F_1(\theta^1)\right).$$

Equating the outflow with the inflow yields

$$(1-\gamma_1)\int_{\theta_r^1}^{\theta^1} G_1(\theta,h)\mathrm{d}\theta = (1-\gamma_1)\left(1-\psi_1(\theta^1)\right)\int_{\theta_r^1}^{\theta^1} G_1(\theta,\frac{h}{e^g})\mathrm{d}\theta + \gamma_1 N_1(h)\lambda\left(\beta F_0(\theta^1)+(1-\beta)F_1(\theta^1)\right). \quad (44)$$

Proposition 8 *For the high-educated workers, the steady state turnover in a market equilibrium implies:*

$$N_1(e^{ng}) = \frac{\delta\sigma\lambda}{\gamma_1(\lambda+\delta)^2}q_1^n,$$

and $G_1(h)$ *satisfies*

$$\int_{\theta_r^1}^{\theta^1} G_1(\theta,e^{ng})\mathrm{d}\theta = \frac{\delta\lambda(\beta F_0(\theta^1)+(1-\beta)F_1(\theta^1))(\lambda+\delta-\sigma\lambda)}{(1-\gamma_1)(\delta+\lambda)\left[\sigma(1-\psi_1(\theta^1))+(1-\beta F_0(\theta^1)-(1-\beta)F_1(\theta^1))(\lambda+\delta)\right]} \times\left[\left(1-\beta F_0(\theta^1)-(1-\beta)F_1(\theta^1)\right)(1-\psi_1(\theta^1))^n+\frac{\sigma}{\gamma+\delta}q_1^{n+1}\right],$$

for all $\theta\in[\theta_r^1,\overline{\theta}^1]$ *and* $h\in\{e^{ng}\}_{n=1}^{\infty}$*, where* $q_1=\dfrac{(1-\delta-\sigma)(\lambda+\delta)}{\lambda+\delta-\sigma\lambda}<1.$

Proof Use the same method like the above subsection to consider the high-educated workers. Let $h=1$ and $\theta^1=\overline{\theta}^1$ in (44). Thus,

$$\int_{\theta_r^1}^{\overline{\theta}^1} G_1(\theta,1)\mathrm{d}\theta = \frac{\gamma_1\lambda}{(1-\gamma_1)}N_1(1),$$

and using (41) to substitute out $N_1(1)$ yields

$$\int_{\theta_r^1}^{\overline{\theta}^1} G_1(\theta,1)\mathrm{d}\theta = \frac{\delta\lambda}{(1-\gamma_1)(\lambda+\delta)}.$$

Let $h \in \{e^{ng}\}_{n=1}^{\infty}$ and $\theta^1 = \overline{\theta}^1$ in (44). Hence,

$$(1-\gamma_1)\int_{\theta_r^1}^{\overline{\theta}^1} G_1(\theta, e^{ng})\mathrm{d}\theta = (1-\gamma_1)\int_{\theta_r^1}^{\overline{\theta}^1} G_1(\theta, e^{(n-1)g})\left(1-\psi_1(\overline{\theta}^1)\right)\mathrm{d}\theta + \gamma_1\lambda N_1(e^{ng}).$$

and using (43) to substitute out $N_1(.)$ yields

$$\int_{\theta_r^1}^{\overline{\theta}^1} G_1(\theta, e^{ng})\mathrm{d}\theta = q_1\int_{\theta_r^1}^{\overline{\theta}^1} G_1(\theta, e^{(n-1)g})\mathrm{d}\theta,$$

where

$$q_1 = \frac{(1-\delta-\sigma)(\lambda+\delta)}{\lambda+\delta-\sigma\lambda} < 1.$$

Therefore, when $h \in \{e^{ng}\}_{n=1}^{\infty}$,

$$\int_{\theta_r^1}^{\overline{\theta}^1} G_1(\theta, e^{ng})\mathrm{d}\theta = \frac{\delta\lambda}{(1-\gamma_1)(\lambda+\delta)}q_1^n. \tag{45}$$

Using (45) in (43) implies

$$N_1(e^{ng}) = \frac{\delta\sigma\lambda}{\gamma_1\left(\lambda+\delta\right)^2}q_1^n. \tag{46}$$

Given θ_1, if $h=1$, (44) can be written as,

$$\int_{\theta_r^1}^{\theta^1} G_1(\theta, 1)\mathrm{d}\theta = \frac{\delta\lambda\left(\beta F_0(\theta^1)+(1-\beta)F_1(\theta^1)\right)}{(1-\gamma_1)(\lambda+\delta)},$$

and if $h \in \{e^{ng}\}_{n=1}^{\infty}$, (44) can be written as,

$$\int_{\theta_r^1}^{\theta^1} G_1(\theta, e^{ng})\mathrm{d}\theta = \left(1-\psi_1(\theta^1)\right)\int_{\theta_r^1}^{\theta^1} G_1(\theta, e^{(n-1)g})\mathrm{d}\theta + \frac{\sigma\delta\lambda^2[\beta F_0(\theta^1)+(1-\beta)F_1(\theta^1)]}{(1-\gamma_1)(\lambda+\delta)^2}q_1^n.$$

Then we can get that $G_1(\theta, h)$ satisfies

$$\begin{aligned}\int_{\theta_r^1}^{\theta^1} G_1(\theta, e^{ng})\mathrm{d}\theta =& \frac{\delta\lambda(\beta F_0(\theta^1)+(1-\beta)F_1(\theta^1))(\lambda+\delta-\sigma\lambda)}{(1-\gamma_1)(\delta+\lambda)\left[\sigma(1-\psi_1(\theta^1))+(1-\beta F_0(\theta^1)-(1-\beta)F_1(\theta^1))(\lambda+\delta)\right]} \\ &\times\left[\left(1-\beta F_0(\theta^1)-(1-\beta)F_1(\theta^1)\right)(1-\psi_1(\theta^1))^n + \frac{\sigma}{\gamma+\delta}q_1^{n+1}\right].\end{aligned} \tag{47}$$

This completes the proof.

3.3 Solve $F_0(\theta^0)$ and $F_1(\theta^1)$

Using (38) and (39) to substitute out $N_0((1+g)^\tau)$ and $\int_{\underline{\theta}^0}^{\theta^0} G_0(\theta,(1+g)^\tau)\mathrm{d}\theta$ in (27), π_{00} can be written as

$$\pi_{00}(\theta^0) = \frac{\beta\lambda^2\alpha^2(p-\theta^0)\delta}{[1-(1-\psi_0(\theta^0))(1+g)]\,(\beta\lambda+\delta)\,[\sigma(1-\psi_0(\theta^0))+(1-F_0(\theta^0))(\beta\lambda+\delta)]} \times\left[\frac{\sigma(\beta\lambda+\delta+\sigma)(1-\sigma-\delta F_0(\theta^0))}{[1-q_0(1+g)](\beta\lambda+\delta)}+\frac{(\beta\lambda+\delta-\beta\lambda\sigma)F_0(\theta^0)(1-F_0(\theta^0))}{1-(1-\psi_0(\theta^0))(1+g)}\right],$$

where

$$\psi_0(\theta^0)=\delta+\sigma+\beta\lambda(1-F_0(\theta^0)).$$

Moreover, substitute $N_1(e^{g\tau})$ and $\int_{\underline{\theta}^1}^{\theta^0} G_1(\theta,e^{g\tau})\mathrm{d}\theta$ into (28) to get the following formula

$$\pi_{01}(\theta^0) = \frac{\lambda^2(1-\alpha)^2(p-\theta^0)\delta}{[1-(1-\psi_1(\theta^0))e^g]\,(\lambda+\delta)\,[\sigma(1-\psi_1(\theta^0))+(1-\beta F_0(\theta^0)-(1-\beta)F_1(\theta^0)))(\lambda+\delta)]} \times\left[\frac{\sigma(\lambda+\delta+\sigma)\left[1-\sigma-\delta(\beta F_0(\theta^0)+(1-\beta)F_1(\theta^0))\right]}{(1-q_1e^g)(\lambda+\delta)} + \frac{(\lambda+\delta-\sigma\lambda)(\beta F_0(\theta^0)+(1-\beta)F_1(\theta^0))(1-\beta F_0(\theta^0)-(1-\beta)F_1(\theta^0))}{1-(1-\psi_1(\theta^0))e^g}\right],$$

where

$$\psi_1(\theta^0)=\delta+\sigma+\beta\lambda(1-F_0(\theta^0))+(1-\beta)\lambda(1-F_1(\theta^0)).$$

Note that $\pi_0(\theta^0)=\pi_{00}(\theta^0)+\pi_{01}(\theta^0)$ for all $\theta^0\in[\underline{\theta}^0,\overline{\theta}^0]$. By the constant profit conditions, we have $\pi_0(\theta^0)=\pi_0^*$ for all $\theta^0\in[\underline{\theta}^0,\overline{\theta}^0]$. To obtain the equilibrium profit for the firms offering unskilled jobs π_0^*, set $\theta^0=\underline{\theta}^0$, we can obtain

$$\pi_0^* = \frac{\beta\lambda^2\alpha^2\delta\sigma(p-\underline{\theta}^0)}{[1-(1-\delta-\sigma-\beta\lambda)(1+g)](\beta\lambda+\delta)^2[1-q_0(1+g)]} + \frac{\lambda^2(1-\alpha)^2\delta\sigma(p-\underline{\theta}^0)}{[1-(1-\delta-\sigma-\lambda)e^g](\lambda+\delta)^2(1-q_1e^g)}. \tag{48}$$

To solve for $F_0(\cdot)$ and $F_1(\cdot)$, we divide $[\underline{\theta}^0,\overline{\theta}^0]$ into two intervals $[\underline{\theta}^0,\underline{\theta}^1]$ and $(\underline{\theta}^1,\overline{\theta}^0]$. In interval $[\underline{\theta}^0,\underline{\theta}^1]$, there are only the unskilled jobs, while in interval

$(\underline{\theta}^1, \overline{\theta}^0]$, there are both the unskilled jobs and the skilled jobs. And then we should determine $\underline{\theta}^1$. Assume that $\theta^0 = \underline{\theta}^1$,

$$\begin{aligned}\pi_0(\underline{\theta}^1) =& \frac{\beta\lambda^2\alpha^2(p-\underline{\theta}^1)\delta}{\left[1-(1-\psi_0(\underline{\theta}^1))(1+g)\right](\beta\lambda+\delta)\left[\sigma(1-\psi_0(\underline{\theta}^1))+(1-F_0(\underline{\theta}^1))(\beta\lambda+\delta)\right]}\\ &\times\left[\frac{\sigma(\beta\lambda+\delta+\sigma)(1-\sigma-\delta F_0(\underline{\theta}^1))}{[1-q_0(1+g)](\beta\lambda+\delta)}+\frac{(\beta\lambda+\delta-\beta\lambda\sigma)F_0(\underline{\theta}^1)(1-F_0(\underline{\theta}^1))}{1-(1-\psi_0(\underline{\theta}^1))(1+g)}\right]\\ &+\frac{\lambda^2(1-\alpha)^2(p-\underline{\theta}^1)\delta}{\left[1-(1-\psi_1(\underline{\theta}^1))e^g\right](\lambda+\delta)\left[\sigma(1-\psi_1(\underline{\theta}^1))+(1-\beta F_0(\underline{\theta}^1)))(\lambda+\delta)\right]}\\ &\times\left[\frac{\sigma(\lambda+\delta+\sigma)(1-\sigma-\delta\beta F_0(\underline{\theta}^1))}{(1-q_1e^g)(\lambda+\delta)}+\frac{(\lambda+\delta-\sigma\lambda)\beta F_0(\underline{\theta}^1)(1-\beta F_0(\underline{\theta}^1))}{1-(1-\psi_1(\underline{\theta}^1))e^g}\right],\end{aligned} \tag{49}$$

where

$$\psi_0(\underline{\theta}^1) = \delta+\sigma+\beta\lambda(1-F_0(\underline{\theta}^1)),$$

$$\psi_1(\underline{\theta}^1) = \delta+\sigma+\lambda-\beta\lambda F_0(\underline{\theta}^1),$$

and

$$F_0(\underline{\theta}^1) = \frac{1}{\beta}-\frac{\delta[1-e^g(1-\delta-\sigma)]}{\beta\lambda(e^g-1-\delta e^g)}.$$

As $\pi_0(\underline{\theta}^1) = \pi_0^*$, we can determine $\underline{\theta}^1$. When $\theta^0 \in [\underline{\theta}^0, \underline{\theta}^1]$,

$$\begin{aligned}\pi_0(\theta^0) =& \frac{\beta\lambda^2\alpha^2(p-\theta^0)\delta}{[1-(1-\psi_0(\theta^0))(1+g)](\beta\lambda+\delta)[\sigma(1-\psi_0(\theta^0))+(1-F_0(\theta^0))(\beta\lambda+\delta)]}\\ &\times\left[\frac{\sigma(\beta\lambda+\delta+\sigma)(1-\sigma-\delta F_0(\theta^0))}{[1-q_0(1+g)](\beta\lambda+\delta)}+\frac{(\beta\lambda+\delta-\beta\lambda\sigma)F_0(\theta^0)(1-F_0(\theta^0))}{1-(1-\psi_0(\theta^0))(1+g)}\right]\\ &+\frac{\lambda^2(1-\alpha)^2(p-\theta^0)\delta}{[1-(1-\psi_1(\theta^0))e^g](\lambda+\delta)[\sigma(1-\psi_1(\theta^0))+(1-\beta F_0(\theta^0)))(\lambda+\delta)]}\\ &\times\left[\frac{\sigma(\lambda+\delta+\sigma)(1-\sigma-\delta\beta F_0(\theta^0))}{(1-q_1e^g)(\lambda+\delta)}+\frac{(\lambda+\delta-\sigma\lambda)\beta F_0(\theta^0)(1-\beta F_0(\theta^0))}{1-(1-\psi_1(\theta^0))e^g}\right],\end{aligned} \tag{50}$$

where

$$\psi_0(\theta^0) = \delta+\sigma+\beta\lambda(1-F_0(\theta^0)),$$

and

$$\psi_1(\theta^0) = \delta+\sigma+\lambda-\beta\lambda F_0(\theta^0).$$

Thus we can get the expression for $F_0(\theta^0)$ by Eqs. (48) and (50), where $\theta^0 \in [\underline{\theta}^0, \underline{\theta}^1]$. When $\theta^0 \in (\underline{\theta}^1, \overline{\theta}^0]$,

$$\begin{aligned}\pi_0(\theta^0) =& \frac{\beta\lambda^2\alpha^2(p-\theta^0)\delta}{[1-(1-\psi_0(\theta^0))(1+g)]\,(\beta\lambda+\delta)\,[\sigma(1-\psi_0(\theta^0))+(1-F_0(\theta^0))(\beta\lambda+\delta)]} \\ &\times\left[\frac{\sigma(\beta\lambda+\delta+\sigma)(1-\sigma-\delta F_0(\theta^0))}{[1-q_0(1+g)](\beta\lambda+\delta)}+\frac{(\beta\lambda+\delta-\beta\lambda\sigma)F_0(\theta^0)(1-F_0(\theta^0))}{1-(1-\psi_0(\theta^0))(1+g)}\right] \\ &+\frac{\lambda^2(1-\alpha)^2(p-\theta^0)\delta}{[1-(1-\psi_1(\theta^0))e^g]\,(\lambda+\delta)\,[\sigma(1-\psi_1(\theta^0))+(1-\beta F_0(\theta^0)-(1-\beta)F_1(\theta^0)))(\lambda+\delta)]} \\ &\times\left[\frac{\sigma(\lambda+\delta+\sigma)\left[1-\sigma-\delta(\beta F_0(\theta^0)+(1-\beta)F_1(\theta^0))\right]}{(1-q_1e^g)(\lambda+\delta)}\right. \\ &+\left.\frac{(\lambda+\delta-\sigma\lambda)(\beta F_0(\theta^0)+(1-\beta)F_1(\theta^0))(1-\beta F_0(\theta^0)-(1-\beta)F_1(\theta^0))}{1-(1-\psi_1(\theta^0))e^g}\right],\end{aligned} \tag{51}$$

where

$$\psi_0(\theta^0) = \delta+\sigma+\beta\lambda(1-F_0(\theta^0)),$$

and

$$\psi_1(\theta^0) = \delta+\sigma+\beta\lambda(1-F_0(\theta^0))+(1-\beta)\lambda(1-F_1(\theta^0)).$$

Setting $\theta^0 = \overline{\theta}^0$ yields

$$\begin{aligned}\pi_0(\overline{\theta}^0) =& \frac{\beta\lambda^2\alpha^2(p-\overline{\theta}^0)\delta(\beta\lambda+\delta+\sigma)}{[1-(1-\delta+\sigma)(1+g)]\,(\beta\lambda+\delta)^2[1-q_0(1+g)]} \\ &+\frac{\lambda^2(1-\alpha)^2(p-\overline{\theta}^0)\delta}{\left[1-(1-\psi_1(\overline{\theta}^0))e^g\right](\lambda+\delta)\left[\sigma(1-\psi_1(\overline{\theta}^0))+(1-\beta)(1-F_1(\overline{\theta}^0))(\lambda+\delta)\right]} \\ &\times\left[\frac{\sigma(\lambda+\delta+\sigma)\left[1-\sigma-\delta(\beta+(1-\beta)F_1(\overline{\theta}^0))\right]}{(1-q_1e^g)(\lambda+\delta)}\right. \\ &+\left.\frac{(\lambda+\delta-\sigma\lambda)(\beta+(1-\beta)F_1(\overline{\theta}^0))(1-\beta)(1-F_1(\overline{\theta}^0))}{1-(1-\psi_1(\overline{\theta}^0))e^g}\right],\end{aligned} \tag{52}$$

where

$$\psi_1(\overline{\theta}^0) = \delta+\sigma+(1-\beta)\lambda(1-F_1(\overline{\theta}^0)).$$

Using (46) and (47) to substitute out $N_1(e^{g\tau})$ and $\int_{\underline{\theta}^1}^{\theta^1} G_1(\theta, e^{g\tau})\mathrm{d}\theta$ in (30), we can get the following formula

$$\begin{aligned}\pi_{11}(\theta^1) =& \frac{\lambda^2(1-\alpha)(p-\theta^1)\delta}{[1-(1-\psi_1(\theta^1))e^g](\lambda+\delta)[\sigma(1-\psi_1(\theta^1))+(1-\beta F_0(\theta^1)-(1-\beta)F_1(\theta^1)))(\lambda+\delta)]}\\ &\times\left[\frac{\sigma(\lambda+\delta+\sigma)\left[1-\sigma-\delta(\beta F_0(\theta^1)+(1-\beta)F_1(\theta^1))\right]}{(1-q_1e^g)(\lambda+\delta)}\right.\\ &+\left.\frac{(\lambda+\delta-\sigma\lambda)(\beta F_0(\theta^1)+(1-\beta)F_1(\theta^1))(1-\beta F_0(\theta^1)-(1-\beta)F_1(\theta^1))}{1-(1-\psi_1(\theta^1))e^g}\right],\end{aligned}$$

where

$$\psi_1(\theta^1) = \delta+\sigma+\beta\lambda(1-F_0(\theta^1))+(1-\beta)\lambda(1-F_1(\theta^1)).$$

Note that $\pi_1(\theta^1) = \pi_{11}(\theta^1)$ for all $\theta^1 \in [\underline{\theta}^1, \overline{\theta}^1]$. By the constant profit conditions, we have $\pi_1(\theta^1) = \pi_1^*$ for all $\theta^1 \in [\underline{\theta}^1, \overline{\theta}^1]$. To obtain the equilibrium profit for the firms offering skilled jobs, set $\theta^1 = \underline{\theta}^1$, we obtain

$$\begin{aligned}\pi_1^* =& \frac{\lambda^2(1-\alpha)(p-\underline{\theta}^1)\delta}{\left[1-(1-\psi_1(\underline{\theta}^1))e^g\right](\lambda+\delta)\left[\sigma(1-\psi_1(\underline{\theta}^1))+(1-\beta F_0(\underline{\theta}^1)))(\lambda+\delta)\right]}\\ &\times\left[\frac{\sigma(\lambda+\delta+\sigma)(1-\sigma-\delta\beta F_0(\underline{\theta}^1))}{(1-q_1e^g)(\lambda+\delta)}+\frac{(\lambda+\delta-\sigma\lambda)\beta F_0(\underline{\theta}^1)(1-\beta F_0(\underline{\theta}^1))}{1-(1-\psi_1(\underline{\theta}^1))e^g}\right],\end{aligned} \tag{53}$$

where

$$\psi_0(\underline{\theta}^1) = \delta+\sigma+\beta\lambda(1-F_0(\underline{\theta}^1)),$$

and

$$F_0(\underline{\theta}^1) = \frac{1}{\beta} - \frac{\delta[1-e^g(1-\delta-\sigma)]}{\beta\lambda(e^g-1-\delta e^g)}.$$

Given $\theta^1 = \overline{\theta}^1$,

$$\pi_1(\overline{\theta}^1) = \frac{\lambda^2(1-\alpha)(p-\overline{\theta}^1)\delta(\lambda+\delta+\sigma)}{[1-(1-\delta-\sigma)e^g](\delta+\lambda)^2(1-q_1e^g)}.$$

As $\pi_1(\overline{\theta}^1) = \pi_1^*$, we can obtain $\overline{\theta}^1$. Divide $\theta^1 \in [\underline{\theta}^1, \overline{\theta}^1]$ into two intervals $[\underline{\theta}^1, \overline{\theta}^0]$ and $(\overline{\theta}^0, \overline{\theta}^1]$, and then we should determine $\overline{\theta}^0$. Setting $\theta^1 = \overline{\theta}^0$ yields

$$\begin{aligned}\pi_1(\overline{\theta}^0) =& \frac{\lambda^2(1-\alpha)(p-\overline{\theta}^0)\delta}{\left[1-(1-\psi_1(\overline{\theta}^0))e^g\right](\lambda+\delta)\left[\sigma(1-\psi_1(\overline{\theta}^0))+(1-\beta)(1-F_1(\overline{\theta}^0))(\lambda+\delta)\right]}\\ &\times\left[\frac{\sigma(\lambda+\delta+\sigma)\left[1-\sigma-\delta(\beta+(1-\beta)F_1(\overline{\theta}^0))\right]}{(1-q_1e^g)(\lambda+\delta)}\right.\\ &\left.+\frac{(\lambda+\delta-\sigma\lambda)(\beta+(1-\beta)F_1(\overline{\theta}^0))(1-\beta)(1-F_1(\overline{\theta}^0))}{1-(1-\psi_1(\overline{\theta}^0))e^g}\right],\end{aligned} \tag{54}$$

where

$$\psi_1(\overline{\theta}^0) = \delta+\sigma+(1-\beta)\lambda(1-F_1(\overline{\theta}^0)).$$

We can get the expressions for $\overline{\theta}^0$ and $F_1(\overline{\theta}^0)$, as Eqs. (52) and (54) are two equations about $\overline{\theta}^0$ and $F_1(\overline{\theta}^0)$. When $\theta^1 \in [\underline{\theta}^1, \overline{\theta}^0]$, there exists

$$\begin{aligned}\pi_1(\theta^1) =& \frac{\lambda^2(1-\alpha)(p-\theta^1)\delta}{[1-(1-\psi_1(\theta^1))e^g](\lambda+\delta)[\sigma(1-\psi_1(\theta^1))+(1-\beta F_0(\theta^1)-(1-\beta)F_1(\theta^1)))(\lambda+\delta)]}\\ &\times\left[\frac{\sigma(\lambda+\delta+\sigma)\left[1-\sigma-\delta(\beta F_0(\theta^1)+(1-\beta)F_1(\theta^1))\right]}{(1-q_1e^g)(\lambda+\delta)}\right.\\ &\left.+\frac{(\lambda+\delta-\sigma\lambda)(\beta F_0(\theta^1)+(1-\beta)F_1(\theta^1))(1-\beta F_0(\theta^1)-(1-\beta)F_1(\theta^1))}{1-(1-\psi_1(\theta^1))e^g}\right],\end{aligned} \tag{55}$$

where

$$\psi_1(\theta^0) = \delta+\sigma+\beta\lambda(1-F_0(\theta^0))+(1-\beta)\lambda(1-F_1(\theta^0)).$$

We obtain the expression for $F_0(\theta^1)$ and $F_1(\theta^1)$ by Eqs. (48), (51), (53) and (55), when $\theta^1 \in [\underline{\theta}^1, \overline{\theta}^0]$. Moreover, when $\theta^1 \in (\overline{\theta}^0, \overline{\theta}^1]$,

$$\begin{aligned}\pi_1(\theta^1) =& \frac{\lambda^2(1-\alpha)(p-\theta^1)\delta}{[1-(1-\psi_1(\theta^1))e^g](\lambda+\delta)[\sigma(1-\psi_1(\theta^1))+(1-\beta)(1-F_1(\theta^1))(\lambda+\delta)]}\\ &\times\left[\frac{\sigma(\lambda+\delta+\sigma)\left[1-\sigma-\delta(\beta+(1-\beta)F_1(\theta^1))\right]}{(1-q_1e^g)(\lambda+\delta)}\right.\\ &\left.+\frac{(\lambda+\delta-\sigma\lambda)(\beta+(1-\beta)F_1(\theta^1))(1-\beta)(1-F_1(\theta^1))}{1-(1-\psi_1(\theta^1))e^g}\right],\end{aligned} \tag{56}$$

where

$$\psi_1(\theta^1) = \delta+\sigma+(1-\beta)\lambda(1-F_1(\theta^1)).$$

We can get the expression for $F_1(\theta^1)$ by Eqs. (53) and (56), when $\theta^1 \in (\overline{\theta}^0, \overline{\theta}^1]$.

4 Human capital and equilibrium pay rate

In this section, we first research the effect of the low-educated workers' human capital on equilibrium pay rate. Next, we consider the effect of the high-educated workers' human capital on the equilibrium pay rate. Similar to the findings of Burdett et al. Burdett et al (2011), there is positive correlation between the human capital of workers and the equilibrium pay rates. Last, we discuss differences between these two kinds of effects.

4.1 The effect of low-educated workers' human capital on equilibrium pay rate

Denote $\int_{\theta_r^0}^{\theta^0} G_0(\theta|h)\mathrm{d}\theta = \frac{\int_{\theta_r^0}^{\theta^0} G_0(\theta,h)\mathrm{d}\theta}{\int_{\theta_r^0}^{\bar{\theta}^0} G_0(\theta,h)\mathrm{d}\theta}$, which implies the probability of the low-educated workers' pay rate is less than θ^0 conditional on human capital $h=(1+g)^m$. Using (39) can obtain that

$$\int_{\theta_r^0}^{\theta^0} G_0(\theta|h)\mathrm{d}\theta = \frac{(\beta\lambda+\delta-\beta\lambda\sigma)F_0(\theta^0)}{\sigma(1-\psi_0(\theta^0))+(\beta\lambda+\delta)(1-F_0(\theta^0))}\left\{(1-F_0(\theta^0))\left(\frac{1-\psi_0(\theta^0)}{q_0}\right)^m + \frac{\sigma(1-\delta-\sigma)}{\beta\lambda+\delta-\beta\lambda\sigma}\right\}. \tag{57}$$

Notice that $\frac{(\beta\lambda+\delta-\beta\lambda\sigma)F_0(\theta^0)}{\sigma(1-\psi_0(\theta^0))+(\beta\lambda+\delta)(1-F_0(\theta^0))} > 0$, $1-F_0(\theta^0)>0$ and $\frac{\sigma(1-\delta-\sigma)}{\beta\lambda+\delta-\beta\lambda\sigma} > 0$. Since $0 < 1-\psi_0(\theta^0) < 1$ and $0 < q_0 < 1$, we can get $\frac{1-\psi_0(\theta^0)}{q_0} > 0$. On the other hand, $\frac{1-\psi_0(\theta^0)}{q_0} = \frac{\left[1-\delta-\sigma-\beta\lambda(1-F_0(\theta^0))\right](\beta\lambda+\delta-\beta\lambda\sigma)}{(1-\delta-\sigma)(\beta\lambda+\delta)} < 1$, therefore, $0 < \frac{1-\psi_0(\theta^0)}{q_0} < 1$. Thus, it can prove that the conditional probability is decreasing in m. That is to say, a low-educated worker with higher human capital is more likely to have a higher pay rate.

Specially, when $m=0$, Eq.(57) can be written as

$$\int_{\theta_r^0}^{\theta^0} G_0(\theta|1)\mathrm{d}\theta = F_0(\theta^0).$$

It shows that for the new entrants with low-educated, their pay rates are randomly drawn from $F_0(\theta^0)$. Further, when $m\to\infty$,

$$\int_{\theta_r^0}^{\theta^0} G_0(\theta|\infty)\mathrm{d}\theta = \frac{\sigma(1-\delta-\sigma)F_0(\theta^0)}{\sigma(1-\psi_0(\theta^0))+(\beta\lambda+\delta)(1-F_0(\theta^0))},$$

which implies that $\int_{\theta_r^0}^{\theta^0} G_0(\theta|\infty)\mathrm{d}\theta$ is non-degenerate.

Proposition 9 *The probability of the low-educated workers' equilibrium pay rate which is less than θ^0 conditional on human capital h, $\int_{\theta_r^0}^{\theta^0} G_0(\theta|h)\mathrm{d}\theta$, is decreasing in h.*

4.2 The effect of high-educated workers' human capital on equilibrium pay rate

Note that $\int_{\theta_r^1}^{\theta^1} G_1(\theta|h)\mathrm{d}\theta = \int_{\theta_r^1}^{\theta^1} G_1(\theta,h)\mathrm{d}\theta / \int_{\theta_r^1}^{\overline{\theta}^1} G_1(\theta,h)\mathrm{d}\theta$, which implies the probability of the high-educated workers' pay rate is less than θ^1 condition on human capital h. Using (47) obtains that

$$\begin{aligned}\int_{\theta_r^1}^{\theta^1} G_1(\theta|h)\mathrm{d}\theta = &\frac{(\lambda+\delta-\sigma\lambda)(\beta F_0(\theta^1)+(1-\beta)F_1(\theta^1))}{\sigma(1-\psi_1(\theta^1))+(\lambda+\delta)\left[1-\beta F_0(\theta^1)-(1-\beta)F_1(\theta^1)\right]} \\ &\times\left\{\left[1-\beta F_0(\theta^1)-(1-\beta)F_1(\theta^1)\right]\left(\frac{1-\psi_1(\theta^1)}{q_1}\right)^n+\frac{\sigma(1-\delta-\sigma)}{\lambda+\delta-\sigma\lambda}\right\}.\end{aligned} \tag{58}$$

Note that $\frac{(\lambda+\delta-\sigma\lambda)(\beta F_0(\theta^1)+(1-\beta)F_1(\theta^1))}{\sigma(1-\psi_1(\theta^1))+(\lambda+\delta)[1-\beta F_0(\theta^1)-(1-\beta)F_1(\theta^1)]} > 0$, $1-\beta F_0(\theta^1)-(1-\beta)F_1(\theta^1) > 0$, $\frac{\sigma(1-\delta-\sigma)}{\lambda+\delta-\sigma\lambda} > 0$ and $0 < \frac{1-\psi_1(\theta^1)}{q_1} < 1$, which implies that the conditional probability is decreasing in n. That is to say, a high-educated worker with higher human capital is more likely to have a higher pay rate.

Specially, when $n=0$, Eq.(58) can be written as

$$\int_{\theta_r^1}^{\theta^1} G_1(\theta|1)\mathrm{d}\theta = \beta F_0(\theta^1)+(1-\beta)F_1(\theta^1).$$

For the new entrants with high-educated, their pay rates are randomly drawn from $\beta F_0(\theta^1)+(1-\beta)F_1(\theta^1)$. When $n\to\infty$,

$$\int_{\theta_r^1}^{\theta^1} G_1(\theta|\infty)\mathrm{d}\theta = \frac{\sigma(1-\delta-\sigma)\left[\beta F_0(\theta^1)+(1-\beta)F_1(\theta^1)\right]}{\sigma(1-\psi_1(\theta^1))+(\lambda+\delta)\left[1-\beta F_0(\theta^1))-(1-\beta)F_1(\theta^1))\right]},$$

which implies $\int_{\theta_r^1}^{\theta^1} G_1(\theta|\infty)\mathrm{d}\theta$ is non-degenerate.

Proposition 10 *The probability of the high-educated workers' equilibrium pay rate which is less than θ^1 conditional on human capital h, $\int_{\theta_r^1}^{\theta^1} G_1(\theta|h)\mathrm{d}\theta$, is decreasing in h.*

4.3 Comparative analysis

We compare the effect of the low-educated workers' tenure m on conditional probability $\int_{\theta_r^0}^{\theta^0} G_0(\theta|h)\mathrm{d}\theta$ with the effect of the high-educated workers' tenure n on conditional probability $\int_{\theta_r^1}^{\theta^1} G_1(\theta|h)\mathrm{d}\theta$. Assume that $m = n = \tau$ and $\theta^0 = \theta^1 = \theta'$. When $\tau = 0$,

$$\int_{\theta_r^0}^{\theta'} G_0(\theta|1)\mathrm{d}\theta = F_0(\theta'),$$

and

$$\int_{\theta_r^1}^{\theta'} G_1(\theta|1)\mathrm{d}\theta = \beta F_0(\theta') + (1-\beta)F_1(\theta').$$

Since

$$F_0(\theta') \geq \beta F_0(\theta') + (1-\beta)F_1(\theta'),$$

we can get

$$\int_{\theta_r^0}^{\theta'} G_0(\theta|1)\mathrm{d}\theta \geq \int_{\theta_r^1}^{\theta'} G_1(\theta|1)\mathrm{d}\theta,$$

which indicates that for the new entrants, the high-educated workers have relatively higher probability to enjoy a higher pay rate. As

$$\frac{1-\psi_0(\theta')}{q_0} = \frac{[1-\delta-\sigma-\beta\lambda(1-F_0(\theta'))]\,(\beta\lambda+\delta-\beta\lambda\sigma)}{(1-\delta-\sigma)(\beta\lambda+\delta)},$$

and

$$\frac{1-\psi_1(\theta')}{q_1} = \frac{[1-\delta-\sigma-\beta\lambda(1-F_0(\theta'))-(1-\beta)\lambda(1-F_1(\theta'))]\,(\lambda+\delta-\sigma\lambda)}{(1-\delta-\sigma)(\lambda+\delta)},$$

it is easily to get

$$0 < \frac{1-\psi_1(\theta')}{q_1} < \frac{1-\psi_0(\theta')}{q_0} < 1.$$

From (57) and (58), we can get that with the increase of tenure, the conditional probability of the high-educated workers falls faster than that of the low-educated workers which presents that with same tenure, the high-educated workers are more likely to own higher pay rates.

5 Numerical example

In this section, we perform a numerical example in a more general situation of the model, so as to show how the profit of the firms offering the unskilled jobs to the low-educated workers π_{00}, the profit of the firms offering the unskilled jobs to the high-educated workers π_{01}, the total profit of the firms offering the unskilled jobs π_0, the profit of the firms offering the skilled jobs π_1 and the supports of the pay rate change with respect to the proportion of the low-educated workers. Following Burdett et al (2011) who consider a year as the reference time unit and assume workers have a 40 year expected working lifetime, we set $\delta = 0.025$. Following Jolivet et al. (2006) who estimates American turnover parameters, we set $\sigma = 0.055$ and $\lambda = 0.15$. In addition, we set $\beta = 0.9$, so that there exists a cross-skill equilibrium.

Table 1 The effects of different proportion of the low-educated workers

α	π_{00}	π_{01}	π_0	π_1	$[\underline{\theta}^0, \overline{\theta}^0]$	$[\underline{\theta}^1, \overline{\theta}^1]$
0.1	0.0250	9.1397	9.1647	10.1545	[0.3, 0.9531]	[0.4690, 0.9608]
0.2	0.0999	7.2214	7.3213	9.0245	[0.3, 0.9534]	[0.4691, 0.9609]
0.3	0.2247	5.5290	5.7537	7.8905	[0.3, 0.9531]	[0.4695, 0.9715]
0.4	0.3994	4.0621	4.4615	6.7543	[0.3, 0.9531]	[0.4702, 0.9609]
0.5	0.6241	2.8209	3.445	5.6159	[0.3, 0.9533]	[0.4714, 0.9610]
0.6	0.8987	1.8054	2.7041	4.4766	[0.3, 0.9535]	[0.4733, 0.9535]
0.7	1.2232	1.0155	2.2387	3.3396	[0.3, 0.9537]	[0.4761, 0.9614]
0.8	1.5977	0.4513	2.049	2.2136	[0.3, 0.9540]	[0.4791, 0.9616]
0.9	2.0221	0.1128	2.1349	1.1021	[0.3, 0.9542]	[0.4813, 0.9618]

From Table 1, it is apparent that π_{00} is increasing along with the proportion of low-educated workers α, while π_{01} is decreasing with the proportion. In the real life, the firms offering unskilled jobs are more easily to employ the low-educated workers and more hardly to find the high-educated workers that is why π_{00} is increasing with α, while π_{01} is decreasing. In addition, π_{01} is greater than π_{00} if $\alpha < 0.7$ and π_{00} is greater than π_{01} if $\alpha \geq 0.7$ which indicate that the profit of the firms offering the unskilled jobs to the high-educated workers is greater than that of offering the unskilled jobs to the low-educated workers when there are more high-educated workers as it is more easy to employ the high-educated workers to gain more profit. Further, π_0 decreases with α if $\alpha \leq 80\%$, while π_0 increases with α if $\alpha > 80\%$ as in this situation there are many low-educated workers and the profit of the firms offering the unskilled job to the low-educated workers is very big so that the total profit of the firms offering the unskilled job increases with the proportion of the low-educated workers.

For the skilled jobs, the profit decreases with the increasing number of the low-educated workers. Moreover, the profit of the firms offering the skilled jobs is greater than that of the firms offering the unskilled jobs until there is only very few high-educated workers. Given $\underline{\theta}^0$, with the decreasing number of the high-educated workers, $\underline{\theta}^1$ decreases, while there is no distinct tendency in $\overline{\theta}^0$ and $\overline{\theta}^1$. That is, along with the number of the high-educated workers decreasing, the reservation pay rate of the high-educated workers increases.

6 Conclusion

In this paper, we construct and analyze an equilibrium search model in a labor market where firms post wage-human capital contracts and risk neutral workers search for better job opportunities whether employed or unemployed. There are heterogeneous firms (unskilled or skilled) and workers (low-educated or high-educated), and high-educated workers may accept unskilled jobs for which they are over-qualified. In addition, the structure proportion of the offered jobs affects the equilibrium, which shows there exists a threshold that can distinguish whether the equilibrium is separating or cross-skill. The cross-skill equilibrium solution implies the workers with higher human capital are more likely to earn higher pay rates and the high-educated workers are more likely to own higher pay rates than that the low-educated workers with the same tenure are likely to.

Numerical simulations show the profit of the firms offering unskilled jobs to low-educated workers is increasing with the proportion of low-educated workers, while the profit of offering unskilled jobs to high-educated workers is decreasing with the proportion. Moreover, the profit of offering the unskilled jobs to the high-educated workers is greater than the profit of the firms offering the unskilled jobs to the low-educated workers when there are more high-educated workers. The total profit of the firms offering the unskilled jobs decrease with the increasing number of the low-educated workers until the great majority of workers are low-educated worker. The profit of the firms offering skilled jobs decreases with the increasing number of the low-educated workers. Moreover, the profit of the firms offering the skilled jobs is greater than the profit of offering the unskilled jobs until there is only very few high-educated workers. Along with the number of the high-educated workers decreasing, the reservation pay rate of the high-educated workers increases. One of the most interesting conclusions is the growth rate of human capital is an endogenous variable which is determined by death shock, job destruction shock, the fraction of the unskilled jobs and the arrival rate of jobs.

This work has produced encouraging results, but further developments will help to enhance its potential and applicability. For instance, an important issue is to take into account workers' pensions. This paper only consider death shock and job destruction shock; however, the workers are not only interesting in current wage but also care about the pensions. Another interesting piece of further research is to classify human capital which can be general (related to experience) or specific (related to job tenure). These are left for future research.

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