Conditional Autoregregressive Range (CARR) Based Volatility Spillover Index For the Eurozone Markets

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Abstract: We examine the volatility spillovers among major Eurozone countries employing the Diebold and Yilmaz (2012) model with time-varying conditional ranges generated from conditional autoregressive range (CARR) model of Chou (2005). The empirical findings, based on a data set covering a fifteen year period (1998-2013), suggest a total volatility spillover index in a very high degree. 74.9% of total volatility in the Eurozone markets is attributed to spillover effects from other markets. Moreover, rolling window analysis shows that volatility spillover index is relatively higher during the turmoil periods.

Keywords: CARR, financial crisis, volatility spillover index, Eurozone

JEL Classifications: G01, G10, C32

1. Introduction

The world has experienced numerous financial crises in the last two decades, namely the Mexican crisis (1994), the Asian crisis (1998), the Russian crisis (1998), the Brazilian crisis (1999), the Argentine crisis (2002), the US subprime crisis (2008) and the European sovereign debt crisis (ongoing). These crises have induced excessive volatility and turmoil in global financial markets. Rising volatility spills over across the markets as the interdependence of financial markets increases. In this regard, investors seek to know how volatility is transmitted across markets through time.

In measuring volatility spillovers, researchers predominantly use generalized autoregressive conditional heteroscedastic (GARCH) as an underlying model which is based on closing prices. In other words, it ignores intraday information of price fluctuations and leads to loss of efficiency (Alizadeh, Brandt and Diebold, 2002; Brandt and Diebold, 2006). The price range is defined as the difference between the highest and lowest log prices over a fixed sampling interval and an alternative tool for measuring volatility since the pioneering work of Mandelbrot (1963). Range based models which were firstly introduced by Garman and Klass (1980) and Parkinson (1980) are widely used methods in the literature of financial volatility modeling. They are superior to GARCH models in the sense that, they incorporate extreme price fluctuations. Indeed, the information included in the opening, highest, lowest and closing prices of an asset is largely used in Japanese candlestick plots (Chou, 2010). Li and Hong (2011) assert that range based volatility estimators (Garman and Klass, 1980; Parkinson, 1980; Rogers and Satchell, 1991; Yang and Zhang, 2000) are claimed to be 5-14 times more efficient than the historical volatility.

Although historical volatility models are popular, they do not handle the time-varying evolution of the volatility process. In order to address this issue, Chou (2005) proposed the conditional autoregressive range (CARR) model which is a dynamic process for the high/low range of logarithmic asset prices over a fixed time interval. The model is very similar to the autoregressive conditional duration (ACD) model of Engle and Russel (1998). Analyzing out of sample volatility forecast of the S&P 500 index, the study of Chou documents that CARR model procures sharper volatility estimates as compared with standard GARCH model. Furthermore, Chou and Wang (2007) conduct volatility forecasting on the U.K. stock market (FTSE100) with CARR model and ascertain that the model provides a simple, yet efficient volatility forecasts.

Addition to univariate volatility models, it is also important to analyze the interactions of financial markets within the multivariate framework. Diebold and Yilmaz (2009) proposed a quantitative measure of volatility spillover by centering upon forecast error variance decompositions which permit to compute total spillover effects across markets. Based on the aggregate volatility spillovers they also provide a single spillover measure. They employed Garman-Klass (1980) as the range based volatility estimator and assume that volatility is fixed within periods but variable across periods. Analyzing four major indices and twelve emerging stock markets, they found that unconditional return and volatility spillovers are 29% and 31%, respectively. They also examine time-varying rolling sample analysis since dynamics of conditional return and volatility spillovers may be different. The spillover plots of rolling sample analysis suggest considerably different results from static full sample analysis. The volatility spillover plots provide evidence of wide fluctuations and give reactions to economic and financial events. Applying the same methodology, Diebold and Yilmaz (2011) studied equity markets spillovers in the Americas: Argentina, Brazil, Chile, Mexico and the U.S. and draw a conclusion that return spillovers show gradually evolving cycles but no bursts, while volatility spillovers indicate clear bursts associated with economic and financial events.

The aforementioned methodology proposed by Diebold and Yilmaz suffers from the variant ordering of vector autoregressive (VAR) system. The authors developed the method to overcome this problem and present measures of both total and directional volatility spillovers in their recent paper in 2012. Measuring the spillovers across US stock, bond, foreign exchange and commodities markets, they indicate quite limited spillovers until the global financial crisis. However, after the collapse of Lehman Brothers in September 2008, they pointed out important spillovers from the stock market to the other markets. Louzis (2013) employed the same methodology to probe price and volatility spillovers among the money, stock, foreign exchange and bond markets of the Euro area. The findings of the study suggest that stock market is the main transmitter of return and volatility spillovers during the sovereign debt crisis. Additionally, it is reported that bonds of periphery countries transmit volatility to other markets diachronically with the exception of the period 2011-2012.

Inspired by aforementioned studies, we analyze volatility spillovers across major Eurozone countries based on the conditional autogressive range (CARR) model. Sovereign debt crisis in the Eurozone since late 2009 has emphasized the importance of evaluating and monitoring the spillover effects among major Eurozone countries. The weekly data from January 1, 1998 to September 30, 2013 is employed and this sample witnesses a highly volatile period in the world economy and extant financial crises. This study contributes to the literature of volatility spillover effects in several ways. Particularly, this is the first paper studying volatility spillover effects among stock markets of Eurozone countries with Diebold and Yilmaz (2012) methodology. Additionally, to the best of our knowledge, none of the studies in the previous literature consider CARR model to compute volatility spillover index.

The remainder of this paper is organized as follows: Part II describes the related methodology; Part III examines the data employed, Part IV represents empirical results and discusses the findings; Part V concludes.

2. Methodology

2.1. Conditional Autoregressive Range (CARR) Model

Let P_t be the logarithm of a speculative asset price at time t, where t=1,2,...,T. The observed range R_t at time t is defined by Chou (2005) as follows;

$$R_t = \max\{P_t\} - \min\{P_t\} \tag{1}$$

,where $\max\{P_t\}$ and $\min\{P_t\}$ are the highest and lowest prices of the asset at time t, respectively.

Chou (2005) proposed the Conditional Autoregressive Range (CARR) model for the range as follows;

$$R_t = \lambda_t \varepsilon_t$$

$$\lambda_t = \omega + \sum_{i=1}^{q} \alpha_i R_{t-i} + \sum_{j=1}^{p} \beta_j \lambda_{t-j} \tag{2}$$

$$\varepsilon_t = f(.)$$

,where λ_t is the conditional mean of the range based on all available information up to time t. The distribution of the error term ε_t or the normalized range is assumed to have a density function f(.) with a unit mean.

Exponential distribution may be used as a choice of distribution, since ε_t is positively valued given that the range R_t and its expected value λ_t are positively valued. However, Chou (2005) asserts that even if the exponential density specification may provide consistent estimation, it is not efficient. The efficiency result can only be achieved if the conditional density is correctly specified. Thus, we estimate the CARR model with the Weibull distribution which has a more general density function. The log likelihood function of the CARR model with Weibull distributed innovations is given by;

$$L(\alpha_i, \beta_j, \theta; R_1, R_2, ..., R_T) = \sum_{t=1}^{T} \ln\left(\frac{\theta}{R_t}\right) + \theta \ln\left(\frac{\Gamma(1+1/\theta) R_t}{\lambda_t}\right) - \left(\frac{\Gamma(1+1/\theta) R_t}{\lambda_t}\right)^{\theta} \quad (3)$$

2.2. Volatility Spillover Effects

In this section, we briefly discuss the methodology proposed by Diebold and Yilmaz (2012) for calculating the volatility spillover index. In their pioneering work in 2009, they measure total spillover index based on the Cholesky decomposition which is variant to the ordering in a simple VAR system. In 2012, they developed the methodology to evaluate directional spillovers in a generalized VAR framework. In this way, they eliminate the dependence of the results on ordering of variables. Next, we summarize the methodology implemented for this study.

Assume a covariance stationary *N*-variable Vector Autoregressive (VAR) model at order of p;

$$y_t = \sum_{i=1}^{p} \phi_i y_{t-i} + \varepsilon_t \text{ , with } \varepsilon_t \sim i.i.d.\ (0,\Sigma) \quad (4)$$

where ϕ_i are N x N matrices of coefficients, ε_t is the vector of independently and identically distributed innovations and Σ is the variance-covariance matrix.

Moving average representation of the VAR(p) model is given as;

$$y_t = \sum_{i=0}^{\infty} A_i \varepsilon_{t-i}$$

where A_i are the NxN moving average coefficient matrices. The A_i coefficient matrices obey the following recursion;

$$A_i = \phi_1 A_{i-1} + \phi_2 A_{i-2} + ... + \phi_p A_{i-p}$$

where A_0 represents NxN identity matrix and A_0=0 for i<0.

Given the VAR framework, H-step ahead forecast error variance decompositions can be written as follows;

$$\theta_{ij}^{g}(H)=\frac{\sigma_{jj}^{-1}\sum_{h=0}^{H-1}(e_i^{'}A_h\sum e_j)^2}{\sum_{h=0}^{H-1}(e_i^{'}A_h\sum A_h^{'}e_i)} \tag{5}$$

where σ_{jj} represents the standard deviation of the error term for the *j*th equation, $\sum$ is the variance-covariance matrix and e_i is the selection vector which its ith element is one and the other elements are zeros.

In the generalized VAR model, the shocks to each variable are not orthogonalized as in the Cholesky factorization. Thus, the sum of the elements in each row of the variance decomposition matrix does not add to unity. We divide each elements of the decomposition matrix by the row sum hence we use the available information in the decomposition matrix to compute the spillover index as follows;

$$\tilde{\theta}_{ij}^{g}(H)=\frac{\theta_{ij}^{g}(H)}{\sum_{j=1}^{N}\theta_{ij}^{g}(H)}, \text{ with } \sum_{j=1}^{N}\tilde{\theta}_{ij}^{g}(H)=1 \text{ and } \sum_{i,j=1}^{N}\tilde{\theta}_{ij}^{g}(H)=N \tag{6}$$

Total spillover index is constructed via normalized entries of the variance decomposition matrix given in equation X. We calculate total spillover index based on H-step ahead forecasts with the following equation;

$$TS^{g}(H)=\frac{\sum_{i,j=1,i\neq j}^{N}\tilde{\theta}_{ij}^{g}(H)}{\sum_{i,j=1}^{N}\tilde{\theta}_{ij}^{g}(H)}x100=\frac{\sum_{i,j=1,i\neq j}^{N}\tilde{\theta}_{ij}^{g}(H)}{N}x100 \tag{7}$$

The equation Y is based on Cholesky factorization which is used by Diebold and Yilmaz (2009). The total spillover index computes contribution of volatility spillovers across the markets to the total forecast error variance.

Even though it is important to analyze total spillovers it is also essential to investigate the directions of spillover effects from and to a particular market. For this purpose, generalized VAR method enables us to compute directional volatility spillovers. Volatility spillovers directional from all other markets *j* to market *i* is given by;

$$DS_{i\leftarrow j}^{g}(H)=\frac{\sum_{j=1,i\neq j}^{N}\tilde{\theta}_{ij}^{g}(H)}{N}x100 \tag{8}$$

The following index evaluating the spillover effects transmitted by market *i* to all other markets *j* is as follows;

$$DS^{g}_{j\leftarrow i}(H) = \frac{\sum_{j=1, i\neq j}^{N} \tilde{\theta}^{g}_{ji}(H)}{N} x100 \qquad (9)$$

Using equations A and B, we are able to compute net directional spillover index for market *i* as;

$$NDS^{g}_{i}(H) = DS^{g}_{j\leftarrow i}(H) - DS^{g}_{i\leftarrow j}(H) \qquad (10)$$

The net directional spillover index is the difference between the total volatility shocks transmitted to and received from all other markets. Positive values of the index indicate that there exist spillover effects from market *i* to all other markets meanwhile the negative values imply that market *i* is a volatility spillover receiver.

3. Data Analysis

For this study, we use weekly ranges of major Eurozone countries: Italy, Germany, Greece, Netherlands, Spain, Austria and Belgium. We obtain the conditional ranges as volatility proxy for the spillover index computation. The data employed consists of 822 observations from January 1, 1998 to September 30, 2013. Table 1 presents the descriptive statistics of the range series under investigation. It is evident from the table that the series show leptokurtic behaviour as kurtosis coefficients for all the series are greater than 3. *Q*-statistics indicate serial correlation in the range series in a very high persistence degree.

Table 1: Descriptive Statistics for the Range Series

	Mean	Min	Max	St.Dev	Skewness	Kurtosis	Q(12)
Italy	2.979	0.104	15.761	2.139	1.986	5.904	1210***
Germany	4.743	0.865	26.337	3.023	2.276	7.844	2095***
Greece	3.751	0.227	21.306	2.683	2.020	6.404	555***
France	4.475	0.971	24.948	2.690	2.098	7.359	2001***
Netherlands	3.106	0.143	19.164	2.252	2.201	7.487	1469***
Spain	4.707	0.870	23.848	2.850	2.124	7.716	1597***
Austria	4.194	0.886	39.124	3.120	3.905	26.884	1891***
Belgium	3.811	0.621	24.877	2.594	2.835	13.432	1475***

4. Empirical Results

This section of the paper discusses the empirical findings of our study. For this purpose, we firstly fit a CARR(1, 1) model driven by Weibull distributed innovations (WCARR henceforth) to weekly ranges of the stock markets. Table 2 presents the results of WCARR(1, 1) model. All of the model coefficients are statistically significant at the 1% level, indicating that the WCARR(1, 1) is an appropriate model for analyzing the range series. Also, sum of the coeffients $\alpha+\beta$ are less than one, indicating that conditional range processes are covariance stationary. The Ljung-Box *Q*-statistics imply no remaining serial correlation in the residuals up to 12 lags. The parameters θ are significantly different than 1, suggesting that the data support Weibull distribution.

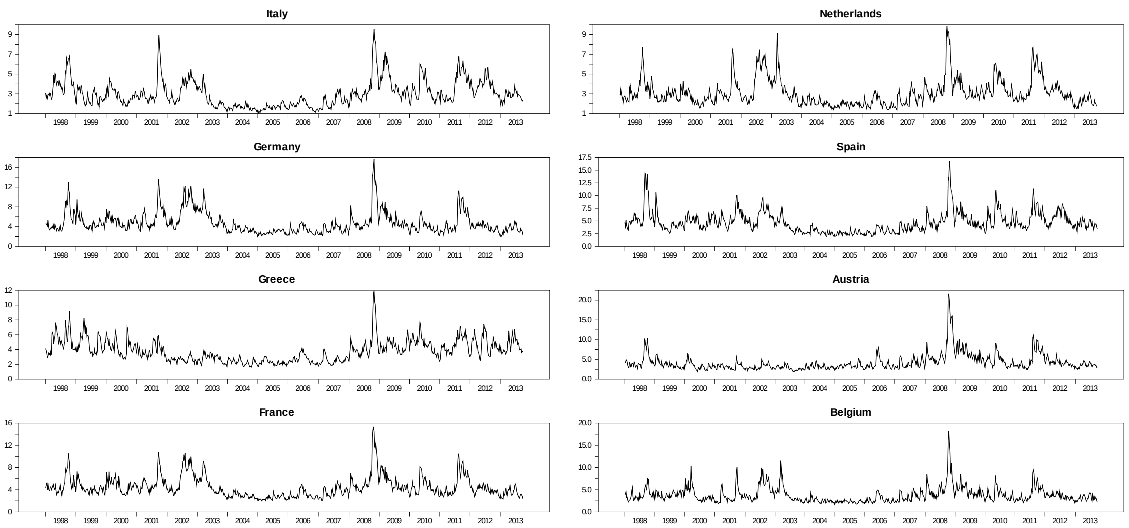
Table 2: WCARR(1, 1) Model estimation

	ω	**α**	**β**	**θ**	**Q(12)**
Italy	0.141***	0.255***	0.698***	1.938***	15.38
	(0.046)	(0.030)	(0.040)	(0.048)	[0.221]
Germany	0.406***	0.418***	0.492***	2.411***	6.63
	(0.092)	(0.038)	(0.047)	(0.055)	[0.881]
Greece	0.182***	0.241***	0.714***	1.805***	2.71
	(0.069)	(0.035)	(0.046)	(0.045)	[0.997]
France	0.369***	0.380***	0.533***	2.480***	13.94
	(0.083)	(0.036)	(0.046)	(0.058)	[0.304]
Netherlands	0.295***	0.293***	0.608***	1.973***	8.71
	(0.065)	(0.032)	(0.047)	(0.048)	[0.727]
Spain	0.419***	0.421***	0.489***	2.447***	4.49
	(0.101)	(0.042)	(0.055)	(0.058)	[0.973]
Austria	0.441***	0.395***	0.497***	2230***	5.12
	(0.103)	(0.040)	(0.054)	(0.052)	[0.954]
Belgium	0.508***	0.460***	0.403***	2298***	9.15
	(0.100)	(0.041)	(0.055)	(0.054)	[0.690]

Note: Standard errors are in parenthesis and p-values are in brackets. (***). (**) and (*) denotes 1%. 5% and 10% statistical significance levels. respectively.

After estimating the conditional ranges from WCARR(1, 1) model, we move on to next step in our analysis. We use estimated conditional ranges from WCARR(1, 1) model (see Figure 1) as input variables in volatility spillover index analysis.

Figure 1: WCARR(1, 1) Estimated Volatilities



Looking at the Figure 1, we observe that all stock markets show more or less similar volatility clustering behaviour. During stable periods between circa 2002 and 2007, all stock markets exhibit lesser volatility compared to turmoil periods. The effects of subprime mortgage crisis which escalated with the collapse of Lehman Brothers in 2008 are evident in all stock markets. Also, Eurozone crisis cause upward jumps in volatility processes of the European markets.

In order to examine volatility transmissions among stock markets, we employ volatility spillover index methodology proposed by Diebold and Yilmaz (2012). In this context, the results of the total spillovers for eight Eurozone stock markets are presented in Table 3. Its *ij*th elements of the spillover table indicate the forecast error variance of market *i* coming from shocks to market *j*. Forecast error variance analysis displays the contribution of each source of shock to the variance of the future forecast error for each endogenous variable. Hence, it splits the forecast error variance of a variable to its own shock and other variables' shocks in the system.

The own variance shares which represent the forecast error variance of market *i* resulting from its own shock are given in the diagonal elements of Table 3. Off-diagonal column elements of the table display the contribution of market *i* to the other market *j*. Whereas, the spillover effects received by market *i* from other market *j* are give in off-diagonal row elements of the table. Moreover, off-diagonal row and column sums show "directional spillovers from others", and "directional spillovers to others" respectively. In addition, net volatility spillovers are calculated by substracting "directional from others" from "directional to others".

The findings suggest that 74.9% of the forecast error variance in the markets originates from other markets, indicating that the Eurozone countries are highly interconnected. Inspecting net volatility spillovers, we can notice that France is the main transmitter of volatility and Greece is the main receiver.

Table 3: Volatility Spillover Table for Major Eurozone Countries

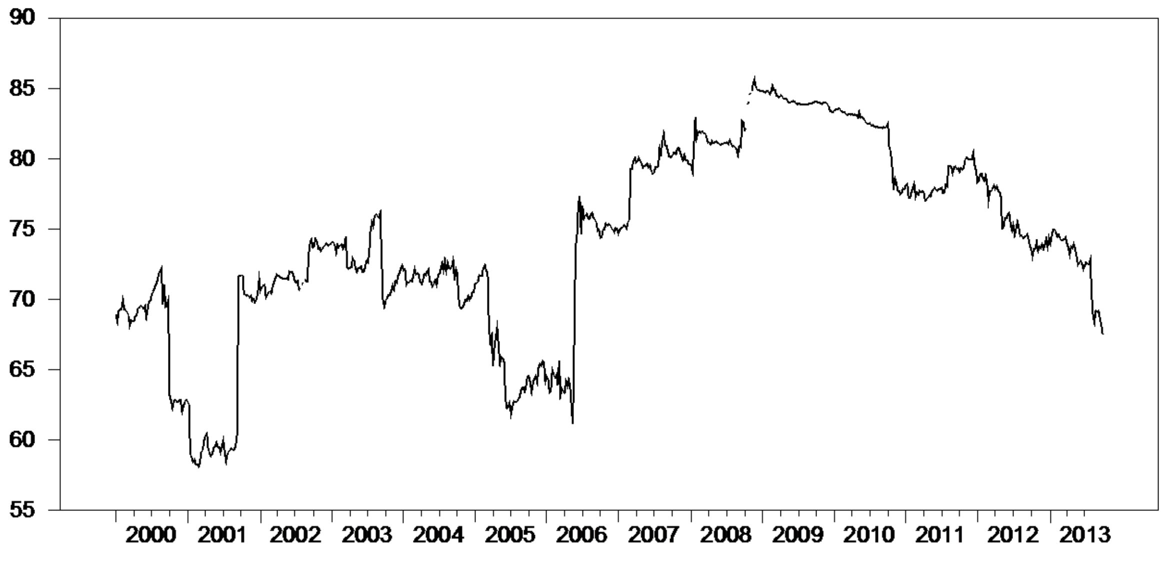
	It	Ger	Gr	Fr	Nd	Sp	At	Bel	Directional FROM Others
It	19.9	10.7	4.1	14.3	13.3	15.5	8.9	13.3	80
Ger	9	19.5	2.1	18.4	14.8	12.5	8	15.6	80
Gr	7.9	4.6	42.6	7.5	6.9	8.9	10.8	10.8	57
Fr	10.3	14.5	2.5	19.3	15.4	13	8.9	16	81
Nd	11.3	12.4	2.3	16.5	21.7	12.2	7.9	15.6	78
Sp	11.9	11.6	4.2	15.7	12	22.4	9.2	12.9	78
At	9.5	7.7	3.3	12.5	11.7	9.8	30.5	15	70
Bel	10	11.2	2.5	15.9	15.7	9.9	10.2	24.7	75
Directional TO Others	70	73	21	101	90	82	64	99	599
Net Volatility Spillovers	-10	-7	-36	20	12	4	-6	14	Total Spillover Index: 74.9%

The sample we use witnesses a highly volatile period in the world economy, including Dot.com bubble, 9/11 terrorist attacks, Afghan and Iraq Wars, FED intervention, US subprime mortgage crisis and European sovereign debt crisis. As mentioned by Diebold and Yilmaz (2012), we are unable to evaluate the impacts of aforementioned political, economic and financial crisis by unconditional volatility spillover index given in Table 3. To investigate time-varying spillover indices, we provide a visual presentation of rolling window framework in Figure 2. In this regard, we used 104 week rolling window with 10 step ahead forecasts. The plot reveals that volatility spillover effect among countries tend to increase during periods of political tensions and/or economic turmoils.

At a first glance, we notice that first sharp increase in volatility spillover index happened in mid 2000 resulting from Dot.com bubble. The next sudden rise in the index came right after the 9/11 terrorist attacks. After a tranquil period, between end-2001 and mid-2003, multinational invasion of Iraq caused a turmoil affecting global economic environment. Following a relatively calm era, FED decided to increase federal funds rate to tighten monitory policy in mid-2006. This action of FED led to capital outflow from world markets to the US. The worst financial crisis since the Great Depression hit the world economy as a whole and the collapse of Lehman Brothers induced a big chaos in the global markets. As a result volatility spillover index reached its peak around 85% at the end of 2008. Afterwards, ongoing European sovereign debt crisis has been impacting Eurozone countries. Therefore, for the period between late 2009 and late 2011 volatility spillover index remained relatively high.

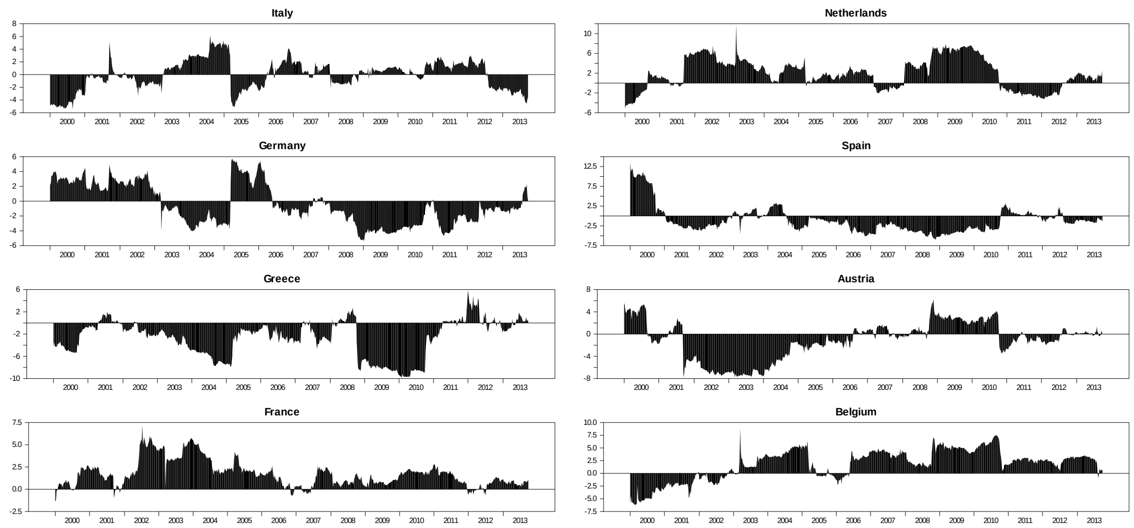
Overall, our empirical findings have some surprising pinpoints. The European sovereign debt crisis has lesser influence on Eurozone volatility spillover index than the US subprime mortgage crisis. Additionally, unsystematic events e.g. 9/11 terrorist attacks and FED intervention, triggered a sharp upward movements in volatility spillover index. Our results are indicative for policy makers and regulators to monitor the financial and economic climate in the Eurozone, and to keep a close eye on sudden changes in the world political and economic environment.

Figure 2: Total Volatility Spillover Plot



As pointed out by Louzis (2013), it is also beneficial to identify the directional spillover effects across markets for policy implications. In order to spot possible sources of volatility among Eurozone countries, net volatility spillovers are presented in Figure 3. The figure affirms the results of Table 3, implying that France, the Netherlands, Spain and Belgium are net transmitters and Italia, Germany, Greece and Austria are net receivers of volatility spillovers. When we look at the Figure 3, we can locate some indicative points. Firstly, for most of the sample period Greece is mainly volatility receiver but a transmitter after receiving financial support from IMF and ECB. Another interesting point is revealed by inspecting the net volatility spillovers of two biggest economies in the Eurozone. Until 2006 Germany is generally volatility transmitter, after that period it is subject to volatility contagion from other Eurozone countries. Whereas, France is the main volatility transmitter during whole period.

Figure 3: Net Directional Volatility Spillover Plots



5. Concluding Remarks

For this study, we construct a WCARR(1, 1) based volatility spillover index to investigate the directional dynamics of volatility among the European markets. We have implemented a modified version of the forecast-error variance decomposition model of Diebold and Yilmaz (2012) by incorporating conditional ranges generated from WCARR(1, 1) model as input variables for the spillover index calculation. Our empirical findings suggest a high level of total volatility spillovers for our whole sample period with sudden but temporary jumps being spotted during exogenous (9/11 terrorist attacks, FED intervention, US subprime crisis) and unsystematic (European sovereign debt crisis) shocks to the European markets. We can empirically say that during the periods of economic turmoil and political stress volatility spillover tends to be higher compared to stable periods.

A possible future development for this study can be the investigation of cross-country spillover effects in order to see the pairwise relationships between the markets. Also, examining volatility transmission between European markets and other economies (US, Russia, China etc.) would be beneficial in terms of understanding the volatility dynamics in the Eurozone.

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