A Measure of a Firm's Average Share Price

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A COMMON practical problem which faces research workers in the general area of the theory of the firm as well as financial analysts is that of measuring the average market price of a firm's shares, or a firm's average market valuation over a suitable period of time. The problem is of particular importance, for example, to economists concerned with formulating and testing theories relating to growth, dividend retentions, new-issue and investment policy of business units.‡ The significance of the market price of a firm's shares in such theories need hardly be stressed; the price of the shares reflects the market's evaluation of a firm's performance and of its future potentialities; it is also likely to represent a major decision variable, not only for investors, but also for entrepreneurs or managers. But since most share prices are subject to wide short-period fluctuations, it is obvious that some relatively long-run average, taken over a suitable period, would represent a better variable for hypothesis testing than the price taken on any particular day. Unfortunately, however, the task of obtaining such averages for large populations of firms, from the available primary sources, has proved so burdensome as to have deterred many investigators altogether, while forcing others to employ rather small samples drawn from one or two industries only. The present writers believe that they have found an alternative measure of average price which it is possible to use in econometric applications, is sufficiently accurate for many other purposes as well, and yet is very much easier to compute. It is described here in the hope that it may prove generally useful to other workers and, in particular, to those concerned with the behaviour of the firm. At the time of writing, although many extant theories are based on the hypothesis that boards of directors aim to maximize the welfare of the shareholders, by maximizing the market value of their equity, only a relatively small number of empirical applications have been able to include direct observations of this central objective variable.

It so happens that the "cards" relating to individual companies which are sent to U.K. subscribers to *Moody's Services* show the highest and lowest share prices recorded during each calendar year for all companies quoted on any British stock

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‡ There is a host of relevant literature both British and American. In a number of studies, failing an easily computable measure of the average share price, less satisfactory indicators (e.g. the price on the day after declaration of dividend) have been employed. For British examples, see G. R. Fisher (1961), Maurice Scott (1962) and E. V. Morgan and Cynthia Taylor (1957). For American examples, see Modigliani and Miller (1960), M. Gordon (1959), Friend and Puckett (1964) and the references contained therein.

exchange. Essentially, in this paper, we demonstrate the possibility of obtaining an appropriate measure of a firm's average share price during a year by using some combination of these extreme values only. We started by conducting an experiment with random samples of firms taken from two different industries over a period of four years. In Section I we describe the nature of these samples and the data on which the analysis of the next three sections is based. In Section II we explore whether either an arithmetic average or a geometric average of the two values may serve as a suitable measure, and we attempt probability statements about the absolute errors involved if such measures are, in fact, used. Section III investigates the possibility of reducing the errors by using measures which give different weights to the highest and lowest prices, the weights being estimated from the sample data themselves or taken from a suitable outside source. In Section IV we study some additional properties of the distribution of errors involved in the use of a particular function of the extreme values as a measure of average share price, and we suggest the modifications necessary before this measure can be used in further econometric applications. To verify the results obtained in the previous sections, in Section V we test some of them on a much larger random sample of firms taken from a different year and not restricted to any particular industry. Finally, in Section VI we sum up the main conclusions of our analysis.

I. SAMPLES AND DATA

We first drew two random samples of firms, one from the food processing industry and the other from non-electrical engineering, the sample frame being the Board of Trade Register of Quoted Companies in Britain in 1959–60.† The two industries were chosen for no other reason than that we happened to be working on them in connection with the study of takeovers. Ordinary share prices for all firms in the two samples which were quoted on the London Stock Exchange were recorded once every month (on the second Friday of each month—for the procedure followed, see Appendix B), for a period of four years for sample firms in the food industry (1957, 1958, 1959 and 1960) and three years (1957, 1959 and 1960) for sample firms in the non-electrical engineering industry.‡ (Thus we shall be considering seven samples of firms, four in the food industry and three in the N.E.E. industry.) Further, for each of the firms we recorded the highest and lowest share prices for the relevant calendar year from Moody's cards. Since a particular company's share price may not be regularly available for some years because its shares were too infrequently traded (see Section (ii) of Appendix B) and since in the first instance we excluded firms for which Moody's had adjusted the highest and lowest share prices for the year for scrip or bonus issues, the analysis in the next three sections is based on the following samples: (a) in the food industry, samples of 11 firms, 12 firms, 13 firms and 15 firms respectively for the years 1957, 1958, 1959 and 1960; (b) in the N.E.E. industry, samples of 14 firms each for the years 1957, 1959 and 1960. A list of names of the firms in the original samples as well as the firms excluded in the different years is given in Appendix B. The statistical implications of the exclusions are also discussed in this Appendix and it is shown that they do not weaken the conclusions arrived at in the text.

† The Register contained 321 companies in non-electrical engineering and 116 companies in food.

‡ Hereafter, non-electrical engineering industry is referred to as the N.E.E. industry.

We use notation as follows:

x_1 = Arithmetic average of a firm's once-a-month ordinary share prices during the year. Because of the serial correlation involved in share prices of firms during a year, an average of 12 share prices recorded once a month by the method given above is considered for the purposes of this paper a reasonable approximation to the "true" average.†

x_2 = Moody's highest share price for the relevant calendar year.

x_3 = Moody's lowest share price for the relevant calendar year.

In terms of this notation we shall consider various combinations of x_2 and x_3 and determine to what extent they approximate the value of x_1. The sample data, on which the analysis is based, are given in Table I in Appendix A.

II. BEHAVIOUR OF SIMPLE AVERAGES

We consider first two very simple measures; the mid-range, or arithmetic mean, of x_2 and x_3, which we denote by x_4, and the corresponding geometric mean, which is denoted by x_5. What errors are involved in using these measures as estimators of x_1?

We are concerned, of course, not only with the average error for all firms, but also with the individual errors. We therefore computed

$$\left[\left|\frac{(x_1-x_4)}{x_1}\right|(100)\right] \quad \text{and} \quad \left[\left|\frac{(x_1-x_5)}{x_1}\right|(100)\right]$$

for all firms in each sample, and the resulting frequency distributions, means and standard deviations are given in Appendix A (Tables IIA and IIB). The following points emerge from the consideration of Tables I, IIA and IIB.

First, in spite of the large range of variation which individual share prices of most firms display during a typical year, the use of mid-range as a measure of average share price involves only a relatively small average error. Except for one sample (N.E.E., 1959), absolute average error lies between 2 and 5 per cent. The respective standard deviations of these errors are also relatively small, N.E.E., 1959 again being an exception. Secondly, such error as does occur varies both between industries and years, reflecting the fact that the distributions of share prices are different in different years as well as in different industries.‡ However, apart from 1959, the inter-industry differences in average error are not as important as the inter-year differences. For 1959, the average error is more than 5 per cent in the food industry and 9 per cent in the N.E.E. industry—in the latter industry it has been brought to that level primarily because of one extremely high observation of

$$\left[\left|\frac{x_1-x_4}{x_1}\right|(100)\right].$$

The effect of this observation is damped when we use x_5 as a measure of x_1, but even then the average error for the N.E.E. industry in 1959 is 6 per cent. The relatively high average error in 1959 can be explained in terms of the general movement of share prices in that year; there was a stock market boom in the last two quarters of

† In one sense, of course, a true average can never be found because we do not know the number of shares traded at any particular price. This consideration reinforces the use of annual average of monthly share prices as a reasonable approximation to the average share price during the year.

‡ See further the regression analysis in the next section.

1959 which distorted the distribution of share prices for most firms during this year as compared with the other years.† The significance of this point will become clearer in the next section, when we apply regression analysis to the problem.

Finally, we may notice that though there may be some *a priori* grounds for expecting a geometric mean to serve better than an arithmetic mean, the tables do not in fact suggest that x_5 is any better a measure of x_1 than x_4; x_5 is therefore abandoned.

Having examined the average error, we now evidently need to look for an estimate of the possible range of errors for individual firms and a basis for inference from sample errors to errors in the population. For this purpose, in the absence of any justification for making any particular assumptions about the distribution of the variable

$$\left|\frac{x_1 - x_4}{x_1}\right|,$$

we use a non-parametric test statistic. We regard the sample proportion of firms with an error of more than 10 per cent

$$\left(\text{i.e.} \quad \left[\left|\frac{x_1 - x_4}{x_1}\right|(100)\right] \geqslant 10\right)$$

or more than 15 per cent

$$\left(\text{i.e.} \quad \left[\left|\frac{x_1 - x_4}{x_1}\right|(100)\right] \geqslant 15\right)$$

as a binomial variable and give estimates at the 95 per cent confidence level of the true proportion of firms with the same attributes.‡ However, since this test, like most other non-parametric tests, is rather insensitive when the sample size is small, we get a better notion of the range of error involved for individual firms if we combine all the samples.§ The sample cumulative distribution function for

$$\left[\left|\frac{x_1 - x_4}{x_1}\right|(100)\right]$$

for all samples combined is given in Table 1.

We observe from the table that 98 per cent of the firms have individual errors of 14 per cent or less. Thus only 2 per cent of the sample firms have individual errors of more than 14 per cent. From Snedecor's table of confidence limits for the true proportion (Snedecor, 1946, p. 45), we find that at 95 per cent confidence level, if 2 per cent of the sample firms have individual errors of 14 per cent or more, the fraction of firms in the population with the same attribute (i.e. with individual errors

† There was a sharp upward movement of nearly 60 points on the Moody's Ordinary Share Price Index during the last two quarters of 1959.

‡ An alternative non-parametric test statistic could have been the Kolomogrov–Smirnov test statistic, which on the basis of sample cumulative distribution function gives confidence limits for the true cumulative distribution function. However, that test was found to be less sensitive than the present one. (See Massey, 1951; Miller, 1956.)

§ It should be noted that we are combining samples solely for the purpose of getting an estimate of the possible range of errors for individual firms; there are, as was mentioned above, inter-year and inter-industry differences in the average error committed.

of 14 per cent or more) would be between 0 and 7 per cent.† Thus we can say with 95 per cent confidence that in no case would more than about 7 per cent of the firms have individual errors exceeding 14 per cent.‡ Similarly, we see that not more than 5 per cent of the sample firms have individual errors exceeding 12 per cent or more.

TABLE 1

Cumulative distribution of x_4 errors for all samples combined

Class interval	$F\left[\left\lvert\frac{x_1-x_4}{x_1}\right\rvert(100)\right]$
0·00 to 1·99	0·43
2·00 to 3·99	0·62
4·00 to 5·99	0·75
6·00 to 7·99	0·83
8·00 to 9·99	0·91
10·00 to 11·99	0·95
12·00 to 13·99	0·98
14·00 to 15·99	0·98
16·00 and above	1·00

This leads us to say with 95 per cent confidence that the fraction of firms in the population with the same attribute would lie between 2 and 11 per cent.

On the basis of the above analysis and the small sample standard deviations of the errors, we conclude that mid-range of the share prices during the year serves as a reasonably good measure of the (arithmetic) average annual share price for most firms. Not only is the average error for all firms small, but also only a very small proportion of firms would have individual errors exceeding 12 or 14 per cent.§ In comparison with the usual errors involved in the use of some other variables which are employed in the theory of the firm (e.g. net worth, etc.), the magnitude of error incurred in the use of mid-range as a measure of average annual share price is rather small.

III. WEIGHTED COMBINATIONS BASED ON REGRESSION ANALYSIS

Though the error in using x_4 as a measure of x_1 is small for most purposes, it seemed worth while to explore ways of reducing it further. The mid-range assigns weights of $\frac{1}{2}$ each to the highest and the lowest share prices during the year. We therefore investigated the possibility of using a different set of weights, where these would be estimated in the first instance from the sample data by regression analysis;

† Since Snedecor's tables for confidence limits is for $N = 100$ and our $N = 93$, the accurate confidence limits would be somewhat wider than those given in the text.

‡ At 99 per cent confidence level, the corresponding limits are 0 and 9 per cent.

§ This conclusion applies to all years and all industries, though one feels reluctant to apply it to N.E.E., 1959. But even here it must be observed that the high average error for this year is due to one extremely high value of

$$\left[\left\lvert\frac{x_1-x_4}{x_1}\right\rvert(100)\right]$$

The use of a weighted combination of x_2 and x_3 instead of x_4 results in a substantial reduction in this error as well, as is shown in the next section.

in effect, this method represents an empirical estimate of the effects of skewness. We used cross-section regression analysis for each of the samples to estimate these weights, using the following model:

$$\frac{x_1}{x_3} = \gamma_1 \frac{x_2}{x_3} + \gamma_2 + v,$$

where $v = u/x_3$ is the stochastic error term.

By using the above form we were able to skirt both the problem of multicollinearity and to normalize the share prices. The values of γ_1 and γ_2 as well as the related standard errors and R^2, obtained by applying the above regression model to the sample data are given in the following table.

TABLE 2

Results of regression analysis

	γ_2	γ_1	R^2
Food, 1957	0·56485±0·035	0·45036±0·02	0·98
Food, 1958	0·468±0·14	0·490±0·09	0·75
Food, 1959	0·40±0·08	0·53±0·05	0·92
Food, 1960	0·56±0·06	0·44±0·04	0·91
N.E.E., 1957	0·36±0·04	0·62±0·02	0·98
N.E.E., 1959	0·97±0·05	0·15±0·03	0·62
N.E.E., 1960	0·47±0·06	0·51±0·04	0·94

It will be seen that the estimated weights show significant inter-year and inter-industry variations, indicating a systematic source of variation in skewness and, hence, in the error involved in the use of sample averages. Are weighted combinations, therefore, to be considered superior? To answer this question, we computed the weighted combination of the highest and the lowest share prices, with weights γ_1 and γ_2 as given in Table 2 above, for all firms in all samples. Thus x_6, which is the notation for the weighted combination, are the predicted values of x_1 for each of the samples from the regression model. The distribution, mean value and the variance of

$$\left[\left|\frac{x_1 - x_6}{x_1}\right|(100)\right]$$

for all samples are given in Table IIIA in Appendix A.

$$\left|\frac{x_1 - x_6}{x_1}\right|,$$

of course, gives the absolute magnitude of the error committed if x_6 is used as a measure of x_1 for a particular firm, and the first two rows of the following table compare the mean values and standard deviations of

$$\left[\left|\frac{x_1 - x_6}{x_1}\right|(100)\right]$$

for the various samples with those of

$$\left[\left|\frac{x_1-x_4}{x_1}\right|(100)\right]$$

for the same samples.

TABLE 3

Errors using x_4, x_6, y_1

	Food				*N.E.E.*			*All samples combined*
	1957	1958	1959	1960	1957	1959	1960	
$M\left[\left\|\frac{x_1-x_4}{x_1}\right\|(100)\right]$	2·188	4·332	5·165	3·033	2·494	9·939	3·324	4·401
$S.D.\left[\left\|\frac{x_1-x_4}{x_1}\right\|(100)\right]$	1·304	3·688	3·661	3·050	2·846	11·630	2·828	5·864
$M\left[\left\|\frac{x_1-x_6}{x_1}\right\|(100)\right]$	1·624	3·516	4·302	2·180	2·124	3·761	3·203	2·967
$S.D.\left[\left\|\frac{x_1-x_6}{x_1}\right\|(100)\right]$	1·421	1·673	2·121	2·470	1·517	1·905	2·881	2·271
$M\left[\left\|\frac{x_1-y_1}{x_1}\right\|(100)\right]$	2·332	3·554	4·369	2·903	2·204	7·167	3·288	3·719
$S.D.\left[\left\|\frac{x_1-y_1}{x_1}\right\|(100)\right]$	1·617	1·780	2·331	3·022	2·608	9·688	2·825	4·675

It is seen from the above table that even though

$$M\left[\left|\frac{x_1-x_6}{x_1}\right|(100)\right]$$

is not significantly lower than

$$M\left[\left|\frac{x_1-x_4}{x_1}\right|(100)\right]$$

for most of the individual samples, mainly owing to their small size, it is so for all samples combined. Furthermore, the standard deviation of

$$\left[\left|\frac{x_1-x_6}{x_1}\right|(100)\right]$$

is not only significantly lower than the standard deviation of

$$\left[\left|\frac{x_1-x_4}{x_1}\right|(100)\right],$$

for all samples combined; it is interesting to see that it is so for four individual samples as well at the 5 per cent F level. In no case is

$$M\left[\left|\frac{x_1-x_4}{x_1}\right|(100)\right] \quad \text{or} \quad \text{S.D.}\left[\left|\frac{x_1-x_6}{x_1}\right|(100)\right]$$

significantly better than

$$M\left[\left|\frac{x_1-x_6}{x_1}\right|(100)\right] \quad \text{or} \quad \text{S.D.}\left[\left|\frac{x_1-x_4}{x_1}\right|(100)\right]$$

Not unexpectedly, the greatest improvement is in the N.E.E., 1959 sample where the average error is reduced from more than 9 per cent to about 4 per cent when x_6 is used as a measure of x_1. However, to the extent that the t test and the F test assume a normal distribution of the variables, the above evidence which indicates that x_6 is a better measure of x_1 than x_4 cannot be considered conclusive. But this evidence is greatly reinforced when, analogously to Table 1 for

$$\left[\left|\frac{x_1-x_4}{x_1}\right|(100)\right],$$

we consider the cumulative distribution for

$$\left[\left|\frac{x_1-x_6}{x_1}\right|(100)\right]$$

for all samples combined. Column 2 of Table 4 below, gives this cumulative distribution.

TABLE 4

Cumulative distribution of x_6 and y_1 errors

Class interval	$F\left[\left\|\frac{x_1-x_6}{x_1}\right\|(100)\right]$	$F\left[\frac{x_1-y_1}{x_1}\right\|(100)\right]$
0·00 to 1·99	0·40	0·38
2·00 to 3·99	0·73	0·70
4·00 to 5·99	0·88	0·83
6·00 to 7·99	0·96	0·92
8·00 to 9·99	0·99	0·96
10·00 to 11·99	1·00	0·97
12·00 to 13·99	1·00	0·99
14·00 to 15·99	1·00	0·99
16·00 to –	1·00	1·00

We find from the above table that only 1 per cent of the firms have individual errors

$$\left[\left|\frac{x_1-x_6}{x_1}\right|(100)\right]$$

of 10 per cent or more and only 4 per cent have errors of 8 per cent or more. Making the same kind of inferences as we did for

$$\left[\left|\frac{x_1-x_4}{x_1}\right|(100)\right],$$

we are led to say with 95 per cent confidence that the fraction of firms in the population with individual errors exceeding 10 per cent would be between 0 and 5 per cent and the fraction of firms in the population with individual errors exceeding 8 per cent would lie between 1 per cent and 10 per cent. It is therefore clear that x_6, the weighted combination of the firm's highest and lowest share prices during the year, serves as a better measure of x_1 than does x_4, and the errors involved in the use of this measure are indeed very small.

Next we investigated the possibility of using kx_4 as a measure of x_1, where the k's, unlike above, would not be estimated from the sample data but obtained from an outside source. Since all that x_6, a weighted combination of x_2 and x_3, does is to remove the systematic sources of inter-year and inter-industry variations in the use of x_4, as a measure of x_1, we thought that it might be possible to remove part of the inter-year variation by using a correction factor obtained from Moody's Ordinary Share Price Index, which is readily available. This index, based on a stratified random sample of 60 firms on the London Stock Exchange, gives the (unweighted geometric) average share price for these firms for each month during the calendar year as well as the highest and lowest price recorded during the year. We obtained the values of

$$k = \frac{\text{Average annual share price}}{\text{Mid-range of the highest and lowest share prices}}$$

for each year from this index and used the k's as a correction factor on x_4's in our samples.† Denoting kx_4 by y_1, we then investigated the errors implied in the use of y_1 as a measure of x_1.

Analogously to our previous procedure, the mean values, standard deviations and frequency functions of

$$\left[\left|\frac{x_1-y_1}{x_1}\right|(100)\right]$$

are given in Table IIIB, Appendix A; the mean values and standard deviations are reproduced for purposes of comparison in Row 3 of Table 3. Similarly, the cumulative distribution function of

$$\left[\left|\frac{x_1-y_1}{x_1}\right|(100)\right]$$

† The values of x_1, x_4 and $k = x_1/x_4$ for Moody's Ordinary Share Price Index (base, 1947 = 100) for the years 1957–61 are as follows:

	1957	*1958*	*1959*	*1960*	*1961*
x_1	135·9	139·1	199·0	251·6	260·35
x_4	134·8	143·95	209·85	252·1	265·95
$k = x_1/x_4$	1·0079	0·9664	0·98409	0·99795	0·97894

It may be noted that for this index itself

$$\left[\left|\frac{x_1-x_4}{x_1}\right|(100)\right]$$

is never more than 5 per cent.

is to be found in Column 3 of Table 4. It emerges from the considerations of these tables that y_1 does not do quite as well as x_6 as a measure of x_1, but seems to do better than x_4. Though

$$M\left[\left|\frac{x_1-y_1}{x_1}\right|(100)\right]$$

is not significantly smaller than

$$M\left[\left|\frac{x_1-x_4}{x_1}\right|(100)\right]$$

for all samples combined, the S.D. of

$$\left[\left|\frac{x_1-y_1}{x_1}\right|(100)\right]$$

is significantly less than that of

$$\left[\left|\frac{x_1-x_4}{x_1}\right|(100)\right]$$

at the 5 per cent F level. The estimates of the ranges of error involved for individual firms, if y_1 is used as a measure of x_1, can easily be obtained from the cumulative distribution function in Table 4, in a manner similar to that employed for x_6.

IV. SOME ADDITIONAL PROPERTIES OF THE DISTRIBUTION OF ERRORS

So far we have been concerned with the absolute magnitude of errors involved in the use of x_4, or some function of x_4, as a measure of x_1. However, if x_4 or $\gamma_1 x_2 + \gamma_2 x_3$ is to be used as a proxy for x_1 in regression analysis or in other econometric applications, it is necessary to know some additional properties of the distribution of errors. In particular, if for some year, $x_{4i} = x_{1i} + u_i$ for the ith firm, where u_i is the error term, one needs to know, among other statistics, the following:†

$$\text{(a) } \operatorname{cov}(u_i, u_j); \quad \text{(b) } \operatorname{cov}(u_i, x_{1i}); \quad \text{(c) } E(u_i); \quad \text{(d) } V(u_i). \tag{1}$$

We investigated the possibility of obtaining efficient estimates of the statistics at (1) from the sample data. The data, however, indicate that it is difficult to derive an estimate of $V(u_i)$ if x_{4i} or $\gamma_1 x_{2i} + \gamma_2 x_{3i}$ is used as a proxy for x_{1i} in some kind of an error-in-variable model, owing to an element of heteroscedasticity in the u_i's. But if $\beta \log x_{4i} + \log u_i$ is used as a proxy for $\log x_{1i}$, it can be shown that it is easily possible to obtain unbiased and efficient estimates of β's and $V(\log u_i)$, and good estimates of the other statistics at (1) from the sample data. These estimates may be obtained by fitting the following regression model to each of the samples:

$$\log x_{1i} = \beta \log x_{4i} + \log u_i,$$

where the usual assumptions are made that $E(\log u_i) = 0$,

$$V(\log u_i) = \sigma^2; \quad \operatorname{cov}(\log u_i, \log u_j) = 0; \quad \log u_i \text{ and } \log x_{4i} \tag{2}$$

are independently distributed.

† Cf. Kendall and Stuart (1961).

Unbiased and efficient estimates of β's and σ^2, obtained by least squares, for different years and different industries are given in Table 5.

Analysis of various tests performed on the residuals from the fitted regression for each sample shows that the assumption of constant variance made at (2) above is

TABLE 5

Estimates of β's and $V(\log u_i)$

	Food				N.E.E.		
	1957	1958	1959	1960	1957	1959	1960
$\hat{\beta}$	0·9988 ±0·00279	0·98663 ±0·0046	0·988 ±0·005	0·99425 ±0·003	1·0096 ±0·0036	0·97437 ±0·0113	0·998 ±0·0045
$V(\log u_i)$	0·000756	0·002051	0·003112	0·00159	0·001168	0·014394	0·002177

valid. Other tests made on the sample residuals show that the assumptions, $\text{cov}(\log u_i, \log u_j) = 0$, $E(\log u_i) = 0$ and the independence of $\log u_i$ and $\log x_{4i}$, underlying the regression model, also hold. Furthermore, the estimating procedure adopted for this model ensures that $\text{cov}(\log u_i, \log x_{1i}) = 0$ for the given industries and years.

Thus it seems that whenever annual average market valuation or annual average share price is to be used as an explanatory variable in an econometric model and it is possible to apply log transformations, $\hat{\beta} \log x_{4i}$ can readily be used as a proxy for $\log x_{1i}$. $\hat{\beta}$'s and the various statistics at (1) can be estimated by the above procedure from a small random sample of a cross section of firms for the year in question.†

V. VERIFICATION OF THE RESULTS OF THE PREVIOUS SECTIONS

To verify the results obtained in the previous sections, we tested some of them in this section on another random sample of firms, taken from a different year and not restricted to any particular industry. This sample consisted of 100 firms,‡ the sample frame being the 2,213 firms listed in the commercial and industrial section of the London Stock Exchange *Daily List* and the *Monthly Supplementary List* for January 1961. Analogously to our previous procedure we recorded x_1, x_2 and x_3 for the sample firms for 1961§ and computed for each of them x_4, $y_1 = kx_4$,

$$\left[\left|\frac{x_1 - x_4}{x_1}\right|(100)\right] \quad \text{and} \quad \left[\left|\frac{x_1 - y_1}{x_1}\right|(100)\right]$$

† Even though we have random samples from different industries and different years in the above analysis, we have found that the required estimates may be obtained by taking a random sample of firms from a particular year, not restricted to any particular industries. The estimates of β and other statistics for 1961 in the next section have been obtained in this manner.

‡ There were in fact 95 firms, 5 firms being included twice in the random sample selected. See Appendix B for fuller discussion of the procedure adopted and the list of names of firms included in this sample.

§ See Table IV, Appendix A.

The cumulative distribution function, mean value and standard deviation for the last two variables are given in Table 6.

TABLE 6

Errors using x_4 and y_1 for 1961 *sample*

Class interval	$F\left[\left\lvert\frac{x_1-x_4}{x_1}\right\rvert(100)\right]$	$F\left[\left\lvert\frac{x_1-y_1}{x_1}\right\rvert(100)\right]$
0·00 to 1·99	36	30
2·00 to 3·99	26	32
4·00 to 5·99	12	16
6·00 to 7·99	8	11
8·00 to 9·99	10	5
10·00 to 11·99	4	3
12·00 to 13·99	1	3
14·00 to 15·99	3	—
16·00 and above	—	—
N	100	100
Mean	4·237184	3·990152
S.D.	3·689	3·046

Though in the above table, $M\left[\left\lvert\frac{x_1-y_1}{x_1}\right\rvert(100)\right]$

is not significantly smaller than $M\left[\left\lvert\frac{x_1-x_4}{x_1}\right\rvert(100)\right]$

the *F*-test shows that the S.D. of $\left[\left\lvert\frac{x_1-y_1}{x_1}\right\rvert(100)\right]$

is significantly smaller than that of $\left[\left\lvert\frac{x_1-x_4}{x_1}\right\rvert(100)\right]$

at the 5 per cent level. The table also confirms the other main results obtained in relation to x_4 and y_1 in Sections II and III.

The conclusions arrived at in Section IV are also found to be valid for this sample. Unbiased estimates of β and $V(\log u)$ for 1961 are as follows:

$$\hat{\beta} = 0{\cdot}99176; \quad \text{S.E.} = \pm 0{\cdot}0018; \quad V(\log u) = 0{\cdot}002504.$$

VI. CONCLUSION AND LIMITATIONS OF THE ANALYSIS

The main conclusions which emerge from our analysis may now be summarized as follows:

(i) The mid-range (or the geometric average) of the firm's highest and lowest share prices during the year is a reasonably good estimate of its average share price for the year. The average absolute error incurred by the use of this measure is seldom more than 5 per cent though it varies from year to year and from industry to industry. The range of the error is also small and only a relatively insignificant proportion of firms are likely to have individual errors of more than 15 per cent.

(ii) It is possible to reduce the margin of error by using a weighted combination of the highest and lowest share prices, rather than the mid-range as a measure of average share prices, the weights being computed by cross-section regression analysis from a randomly selected sample of firms or taken from an outside source. We saw above that there are systematic sources of variation in the degree of approximation of the mid-range to average share prices of the firms due to inter-year and inter-industry differences. This suggests that one can reduce the margin of error in two stages: first by estimating weights by taking into account only the inter-year differences and disregarding the inter-industry differences and second by estimating weights which take into account both inter-year and inter-industry differences. For instance, suppose a more accurate measure of the firm's average share price than the mid-range is required. In the first instance $kx_4 = y_1$, instead of x_4, could be used as a measure of x_1, k being obtained from Moody's Ordinary Share Price Index. We saw in Section III that the use of y_1 accounts for a part of the inter-year source of variation and it will certainly reduce the range of errors for individual firms. If a more accurate measure is required, a small random sample of firms in a particular year could be taken, their average share prices and the highest and lowest share prices recorded and the optimal weights estimated by regression in the manner given in Section III. The effort required for this process would still be much smaller than that required for taking average share prices of hundreds of firms. A rough rule of thumb is that if there has been an abnormal general movement in a particular year of share prices† in either direction in either the first or second half of the year, it would be better to use a weighted combination of x_2 and x_3 rather than the mid-range as a measure of average share prices. If there has been no such movement, the errors involved with the use of mid-range or y_1 as a measure of average share prices are likely to be very small. As we saw above, the average error for both industries in 1957, for instance, was only about 2 per cent.

It should be clear from the foregoing that if a still greater degree of accuracy is desired, random samples would have to be drawn from particular industries in particular years. This, however, may not always prove economical.

(iii) For purposes of further econometric applications, we have studied some additional properties of the distribution of errors involved, if some function of mid-range is to be used as a measure of average share price. We have shown that with the indicated modifications, mid-range can in fact be readily employed as a proxy variable for average share price in econometric analysis of various problems involving inter-firm variations in the general area of the theory of the firm. From the point of view of efficient estimation of econometric models in this area from large populations of firms, we thus appear to be better placed in relation to average market valuation of the firm or its average share price than with regard to some other variable like profit rate or book value of capital. Though there are no errors of "measurement" in the latter variables, of a kind similar to that encountered in the use of a function of mid-range as a measure of average share price, there are conceptual errors which arise from the fact that the accounting concepts of these variables are different from

† Roughly speaking such abnormal movement may be defined in terms of a departure of more than 40 points in a half-year period on Moody's Ordinary Share Price Index.

the corresponding economic concepts (cf. Harcourt (1961)). The effect of these "conceptual" errors, as far as estimation of econometric models is concerned, is essentially the same as that of errors of "measurement", but unfortunately little work has been done on studying the relevant statistical properties of these errors.

Finally, the following limitations of our analysis and conclusions should be noted:

(i) Our sample firms are not random with regard to all quoted firms in the British economy; they are random only with regard to the firms quoted on the London Stock Exchange. To the extent that small firms tend to be quoted on the provincial stock exchanges, this limits the conclusions of our analysis accordingly.

(ii) We have dealt only with ordinary share prices. The behaviour of preference shares was not studied.

(iii) Lastly, we have considered in this paper a convenient measure of a firm's share price only during the period of a year. Although for a large range of problems in the theory of the firm, this is the most appropriate time period,† there may be some problems where a firm's average share price over a period of three months or of six months will be of greater interest. We have not studied this aspect of the matter and since the conclusions we have arrived at are sensitive to the time period considered, we shall not hazard a guess as to the applicability of our results to shorter periods. We believe, however, that we have shown the way in which an interested investigator may handle a problem of this kind.

† See, for instance, G. Stigler, *Capital and Rates of Return in Manufacturing Industries*, Princeton University Press, 1963, especially pp. 83–88; and Y. Grunfeld, "The determinants of corporate investment", in *The Demand for Durable Goods* (A. C. Harberger, Ed.), University of Chicago Press, 1960. It may be of interest to note that both Grunfeld and Stigler use the mid-range of a firm's highest and lowest share price during the year as a measure of its annual average share price. They do not, however, discuss the problems of accuracy of such a measure.

APPENDIX A

TABLE I

Annual average share price (x_1), highest share price during the year (x_2), lowest share during the year (x_3) and range of variation of share prices during the year $(x_2-x_3)/x_3$ for firms in the seven samples. All prices are in shillings

Food, 1957				*Food*, 1958				*Food*, 1959				*Food*, 1960			
x_1	x_2	x_3	$\frac{x_2-x_3}{x_3}$	x_1	x_2	x_3	$\frac{x_2-x_3}{x_3}$	x_1	x_2	x_3	$\frac{x_2-x_3}{x_3}$	x_1	x_2	x_3	$\frac{x_2-x_3}{x_3}$
62·84	74·00	48·75	0·518	56·33	63·50	50·00	0·270	66·00	73·75	56·50	0·305	55·770	67·00	48·00	0·396
1·20	1·33	1·00	0·330	1·16	1·58	1·00	0·580	2·75	4·00	1·42	1·817	2·753	3·58	2·08	0·721
18·73	19·50	17·50	0·114	24·16	29·50	19·50	0·513	36·92	48·00	29·83	0·609	48·208	50·50	44·50	0·135
19·30	22·50	17·75	0·268	2·25	2·75	1·83	0·503	23·66	28·75	22·50	0·278	23·400	26·25	20·75	0·265
42·97	47·00	38·75	0·213	46·92	61·00	38·75	0·574	3·00	4·25	2·33	0·824	5·825	7·83	4·00	0·958
4·09	5·25	3·08	0·705	2·98	3·58	2·33	0·536	66·00	78·00	55·50	0·405	74·000	101·83	65·00	0·567
0·56	0·83	0·33	1·515	35·92	47·75	27·00	0·769	4·17	4·92	3·58	0·374	4·333	5·00	3·83	0·305
45·38	51·50	39·00	0·321	0·75	1·00	0·50	1·000	51·92	61·50	43·50	0·414	59·417	65·50	50·75	0·291
3·98	4·50	3·33	0·351	15·33	19·83	13·08	0·516	0·92	1·08	0·66	0·636	1·127	1·58	0·83	0·904
64·51	76·50	55·75	0·372	50·33	68·25	41·25	0·655	21·42	28·08	18·58	0·511	25·594	28·75	23·00	0·250
4·12	5·25	3·25	0·615	4·22	5·50	3·58	0·536	2·50	3·25	2·25	0·444	72·604	90·00	58·00	0·552
				4·05	4·83	3·16	0·528	5·25	7·08	4·50	0·573	4·682	6·08	3·66	0·661
								78·98	91·50	70·00	0·307	3·404	4·75	2·16	1·199
												85·188	96·50	75·00	0·287
												6·406	7·41	5·08	0·459

N.E.E., 1957				*N.E.E.*, 1959				*N.E.E.*, 1960			
x_1	x_2	x_3	$\frac{x_1-x_3}{x_3}$	x_1	x_2	x_3	$\frac{x_1-x_3}{x_3}$	x_1	x_2	x_3	$\frac{x_1-x_3}{x_3}$
6·53	7·25	5·83	0·244	32·25	40·50	26·75	0·514	12·80	16·50	9·25	0·784
70·75	83·25	51·75	0·609	9·166	10·83	8·00	0·354	10·80	12·42	8·75	0·419
17·08	18·58	15·50	0·199	10·66	12·75	9·75	0·308	38·31	49·00	31·50	0·556
4·07	5·33	2·75	0·938	3·166	4·25	2·50	0·700	22·93	29·83	20·00	0·492
11·35	12·33	9·25	0·333	48·50	55·00	42·75	0·287	3·15	3·83	2·25	0·702
5·08	6·58	2·50	1·632	29·00	32·00	25·33	0·263	9·16	11·25	6·75	0·667
18·64	21·75	15·25	0·426	5·166	11·50	3·75	2·067	27·48	32·50	17·50	0·857
3·45	4·25	2·58	0·647	15·416	22·33	11·50	0·942	8·10	12·08	3·75	2·221
21·67	23·50	18·00	0·306	17·416	26·25	13·33	0·969	4·96	6·25	4·00	0·563
2·35	2·75	1·92	0·432	3·583	5·08	3·08	0·649	22·88	28·75	17·25	0·667
10·18	12·50	8·25	0·515	26·66	30·50	21·25	0·435	17·54	20·33	15·25	0·333
7·39	8·08	6·50	0·243	10·083	15·25	8·75	0·743	14·78	16·25	13·00	0·250
3·08	3·25	2·92	0·113	5·166	6·42	4·25	0·511	4·02	6·08	2·50	1·432
39·33	46·75	28·75	0·626	21·166	29·33	16·33	0·796	32·90	39·50	26·75	0·477

TABLE II(A)

The distribution of absolute percentage error $[|(x_1-x_4)/x_1|(100)]$ *in the use of* x_4 *as a measure of* x_1

	$\left[\left\lvert\frac{x_1-x_4}{x_1}\right\rvert(100)\right]$ *Food*				$\left[\left\lvert\frac{x_1-x_4}{x_1}\right\rvert(100)\right]$ *N.E.E.*		
	1957	1958	1959	1960	1957	1959	1960
0·00 to 1·99	5	6	5	7	9	2	6
2·00 to 3·99	5	0	1	5	0	3	4
4·00 to 5·99	1	1	2	1	4	2	1
6·00 to 7·99	0	3	0	1	0	2	1
8·00 to 9·99	0	1	3	0	0	1	2
10·00 to 11·99	0	1	2	0	1	0	0
12·00 to 13·99	0	0	0	1	0	2	0
14·00 to 15·99	0	0	0	0	0	0	0
16·00 and above	0	0	0	0	0	2†	0
N	11	12	13	15	14	14	14
Mean	02·188	04·332	05·165	03·033	02·494	09·939	03·324
S.D.	1·304	3·688	3·661	3·050	2·846	11·63	2·828

† Values are 19 per cent and 48 per cent respectively.

TABLE II(B)

The distribution of absolute percentage error $[|(x_1-x_5)/x_1|(100)]$ *in the use of* x_5 *as a measure of* x_1

	$\left[\left\lvert\frac{x_1-x_5}{x_1}\right\rvert(100)\right]$ *Food*				$\left[\left\lvert\frac{x_1-x_5}{x_1}\right\rvert(100)\right]$ *N.E.E.*		
	1957	1958	1959	1960	1957	1959	1960
0·00 to 1·99	6	4	4	10	5	4	4
2·00 to 3·99	3	3	2	2	2	4	5
4·00 to 5·99	1	4	1	2	4	2	1
6·00 to 7·99	1	0	3	0	2	1	2
8·00 to 9·99	0	1	2	1	0	0	0
10·00 to 11·99	0	0	0	0	0	1	0
12·00 to 13·99	0	0	1	0	0	0	1
14·00 to 15·99	0	0	0	0	0	1	0
16·00 and above	0	0	0	0	1	1†	1
N	11	12	13	15	14	14	14
Mean	02·502	03·421	04·894	02·391	04·482	06·119	04·819
S.D.	1·817	2·588	3·873	2·588	4·99	6·957	4·615

† Values are 20 per cent, 27 per cent and 17 per cent respectively.

TABLE III(A)

The distribution of absolute percentage error $[|(x_1-x_6)/x_1|(100)]$ *in the use of* x_6 *as a measure of* x_1

	$\left[\left\|\frac{x_1-x_6}{x_1}\right\|(100)\right]$ *Food*				$\left[\left\|\frac{x_1-x_6}{x_1}\right\|(100)\right]$ *N.E.E.*		
	1957	1958	1959	1960	1957	1959	1960
0·00 to 1·99	7	2	2	9	8	3	6
2·00 to 3·99	3	6	5	4	4	5	4
4·00 to 5·99	1	3	3	1	2	3	1
6·00 to 7·99	0	1	2	0	0	3	1
8·00 to 9·99	0	0	1	0	0	0	2
10·00 to 11·99	0	0	0	1	0	0	0
12·00 to 13·99	0	0	0	0	0	0	0
14·00 to 15·99	0	0	0	0	0	0	0
16·00 and above	0	0	0	0	0	0	0
N	11	12	13	15	14	14	14
Mean	1·6235	3·516	4·302	2·180	2·124	3·7607	3·203
S.D.	1·421	1·673	2·121	2·470	1·517	1·905	2·881

TABLE III(B)

The distribution of absolute percentage error $[|(x_1-y_1)/x_1|(100)]$ *in the use of* y_1 *as a measure of* x_1

	$\left[\left\|\frac{x_1-y_1}{x_1}\right\|(100)\right]$ *Food*				$\left[\left\|\frac{x_1-y_1}{x_1}\right\|(100)\right]$ *N.E.E.*		
	1957	1958	1959	1960	1957	1959	1960
0·00 to 1·99	5	3	1	8	8	4	6
2·00 to 3·99	4	5	5	5	4	3	4
4·00 to 5·99	2	3	4	0	1	1	1
6·00 to 7·99	0	1	2	1	0	4	1
8·00 to 9·99	0	0	0	0	1	0	2
10·00 to 11·99	0	0	1	0	0	0	0
12·00 to 13·99	0	0	0	1	0	1	0
14·00 to 15·99	0	0	0	0	0	0	0
16·00 and above	0	0	0	0	0	1	0
N	11	12	13	15	14	14	14
Mean	2·332331	3·55372	4·36906	2·90347	2·20407	7·1668	3·28774
S.D.	1·617	1·780	2·331	3·02205	2·6078	9·688	2·825

TABLE IV

Annual average share price (x_1), highest share price during the year (x_2), lowest share during the year (x_3) for firms in the 1961 sample. All prices are in shillings

x_1	x_2	x_3	x_1	x_2	x_3	x_1	x_2	x_3	x_1	x_2	x_3
49·66667	60·0	43·0	28·89583	34·5	22·0	58·53125	70·75	44·5	4·1875	4·58333	3·75
65·375	76·25	58·0	99·45833	123·75	69·0	19·98958	21·25	18·33333	6·23958	9·83333	3·08333
5·02083	7·0	3·58333	2·85417	4·08333	1·75	15·75	19·33333	12·58333	10·26563	14·0	7·25
46·80556	50·5	41·25	2·85417	4·08333	1·75	3·80556	4·58333	3·08333	16·22396	19·0	13·5
7·96591	9·0	6·5	10·61932	12·58333	8·25	11·97917	16·25	7·75	10·48958	16·5	7·58333
9·05208	11·25	6·75	17·5	17·5	17·0	18·64583	24·83333	14·58333	7·54688	10·5	6·0
2·98958	3·91666	1·5	8·04688	10·75	5·33333	6·95139	11·25	4·33333	35·92708	44·5	29·08333
2·98958	3·91666	1·5	0·96212	1·25	0·5	3·84722	5·25	2·75	4·19444	4·25	3·75
26·39583	29·75	23·58333	13·0625	19·0	10·0	3·97708	4·75	3·25	33·05208	43·25	26·5833
18·78125	25·25	15·75	13·0625	19·0	10·0	57·27083	67·25	45·0	12·10417	16·25	10·25
2·52778	3·5	1·75	11·26563	12·83333	9·75	13·275	16·0	9·5	12·83125	13·75	11·0
24·48229	27·5	22·75	21·84375	25·33333	18·08333	12·73958	16·25	10·83333	5·95	6·91666	4·58333
6·26736	8·25	4·25	6·06771	8·08333	4·75	61·22917	79·25	36·5	14·07813	18·0	11·5
19·75	22·0	16·75	30·875	40·25	22·83333	14·19792	15·5	12·33333	10·32292	12·0	8·25
78·04861	106·83333	58·75	17·4375	22·5	13·75	4·24208	6·75	2·5	34·13021	42·75	28·58333
3·39962	4·0	2·75	13·75	17·08333	10·75	7·21875	8·83333	5·58333	43·39583	54·0	36·0
13·88021	18·25	11·25	19·0	26·5	17·0	53·52083	73·25	46·58333	9·47222	11·58333	7·25
8·6875	10·5	7·91666	81·625	95·66666	65·0	4·23438	5·16666	3·58333	10·10417	11·75	8·5
12·11458	15·0	9·0	23·9375	30·0	18·5	5·02431	6·83333	4·08333	39·15104	48·75	26·5
12·11458	15·0	9·0	14·17708	19·5	9·83333	49·98958	61·25	38·0	7·54271	9·75	5·16666
2·25	2·58333	2·0	5·47917	7·08333	4·16666	4·24132	5·25	3·83333	64·9375	70·5	59·0
4·07292	5·83333	3·25	7·11979	8·83333	6·5	6·89583	9·0	5·83333	8·11458	11·58333	6·08333
3·67708	4·75	2·25	3·02431	3·83333	2·16666	6·89583	9·0	5·83333	8·97396	10·0	7·83333
111·34375	135·0	101·25	2·10606	2·5	1·5	3·36458	4·33333	2·5	6·45313	8·5	4·66666
1·42014	2·16666	0·91666	57·66667	90·0	43·0	95·725	106·25	90·0	4·26042	5·33333	3·58333

APPENDIX B

There are four sections to this Appendix. Section (i) gives the procedure followed in recording the once-a-month share prices used for computing x_1's for each firm; Section (ii) gives the names of the firms in the original random samples chosen from the two industries and the firms which were excluded for different reasons in the various years; Section (iii) examines the implication of these exclusions for the conclusions arrived at in the text; finally, Section (iv) describes the random sample of firms used in Section V of the text.

(i)

Ordinary share prices of the sample firms were taken from the London Stock Exchange "Dealings of the week" as reported in *The Financial Times* on the second Saturday of each month. The list of share prices given in Saturday's *Financial Times* records Friday's markings (i.e. the prices at which marked bargains actually took place) and also the latest markings during the week of any shares not dealt in on Friday. The procedure followed in recording share prices was to take for every firm the first unannotated marking (i.e. one without a footnote indicating special prices) given in the second Saturday's *Financial Times*. Thus most of the time, the marking or recorded share price referred to the second Friday of every month. If, however, there was no marking for a particular firm in the second Saturday's *Financial Times*, it was recorded from the third Saturday's *Financial Times*, failing which it was recorded from the first Saturday's *Financial Times*. After that, the fourth or the fifth Saturday in the month was considered.

(ii)

(a) *Food industry*

Out of 116 firms in the food industry, which were listed in the Board of Trade Register of Quoted Companies for 1959–60, a randomly selected sample of 25 firms was taken. Eight firms from this sample were quoted on the provincial stock exchanges only and one had been taken over. This left us with the following sample of 16 firms in the food industry which were quoted on the London Stock Exchange.

1. Bassett, Geo. & Co.
2. British Feeding-Meals Co.
3. Clarks Bread Co.
4. Fryer & Co., Nelson
5. Harveys Belgravia Foods
6. Hugon & Co.
7. Malga-Vita Suppliers
8. Manbre & Garton
9. Quorn Specialities Holdings
10. Ranks
11. Reckitt & Colman Holdings
12. Smithfield Animal Products Co.
13. Stratford-on-Avon Produce Canners
14. United Biscuits
15. Whiteside HS & Co.
16. Radcliffe Edible Products

For some years, some of these companies were not included in the samples used for our analysis because their shares were too infrequently traded—four or fewer observations were available for these firms during a calendar year. These firms were as follows:

Year	*No.*	*Names of the firms*
1957	3	Radcliffe Edible Products
		Harveys Belgravia Foods
		Stratford-on-Avon Produce Canners
1958	2	Radcliffe Edible Products
		Stratford-on-Avon Produce Canners
1959	1	Radcliffe Edible Products
1960	1	Radcliffe Edible Products

The firms which were not included in the analysis in the various years because of bonus or scrip issues are as follows (see Section I in the text):

Year	*No.*	*Names of the firms*
1957	2	Manbre & Garton
		Ranks
1958	1	Fryer & Co., Nelson
1959	2	Reckitt & Colman Holdings
		Smithfield Animal Products Co.
1960	0	

In 1958 United Biscuits was also excluded because there was a mistake in the recording of its highest and lowest share prices in Moody's cards. As a result of the above exclusions, the analysis in the text is based on a sample of 11 firms in the food industry in 1957, 12 firms in the food industry in 1958, 13 firms in the food industry in 1959 and 15 firms in the food industry in 1960.

(b) *Non-electrical engineering industry*

Out of 321 firms in this industry which were listed in the Board of Trade Register of Quoted Companies for 1959–60, a random sample of 22 firms was taken. Four of these 22 firms were quoted only on provincial stock exchanges. Thus we were left with the following sample of 18 firms in the N.E.E. industry which had a quotation on the London Stock Exchange.

1. Armstrong Shock Absorbers
2. Armstrong Whitworth Metal Industries
3. Babcock & Wilcox
4. Coventry Gauge & Tool Co.
5. Elliott, B.
6. Hall J. P. & Sons Holdings
7. Hills, West Bromwich
8. Keith Blackman
9. Kenwood Manufacturing Co.
10. Liner Concrete Machinery Co.
11. Monotype Corporation
12. Otis Elevator Co.
13. Padley & Venobles
14. Shaw Francis & Co.
15. Smith S. & Sons, England
16. Tecalemit
17. Tobenoil
18. Vickers

The firms which were excluded in the various years because their shares were too infrequently traded are as follows (see Section I in the text):

Year	*No.*	*Names of the firms*
1957	2	Kenwood Manufacturing Co.
		Otis Elevator Co.
1959	2	Kenwood Manufacturing Co.
		Otis Elevator Co.
1960	1	Otis Elevator Co.

The firms which were dropped in the various years because of scrip or bonus issue are as follows (see Section I in the text):

Year	*No.*	*Names of the firms*
1957	2	Armstrong Shock Absorbers
		Smith S. & Sons, England
1959	1	Armstrong Shock Absorbers
1960	2	Liner Concrete Machinery Co.
		Monotype Corporation

One firm, Hills, West Bromwich, was excluded from N.E.E., 1959 and N.E.E., 1960 samples by mistake. As a result of these exclusions, the analysis in the text is based on samples of 14 firms each in the N.E.E. industry for 1957, 1959 and 1960.

(iii)

We shall now briefly investigate whether the exclusion of the above-mentioned firms from the various samples, because their shares were too infrequently traded or because there was a scrip or bonus issue during the year, affects the conclusions arrived at in the text. We consider the former case first. *A priori*, it would appear that the average of the highest or the lowest share prices during the year would approximate a firm's average annual share price more closely if there were very infrequent trading in its shares. In the limiting case, if there were only one marking during the year, the highest, the lowest and the average share price would coincide. However, to verify, we performed the same operations on some of the firms which had been excluded from the two industry samples for the years 1958 and 1959 for this particular reason, as we did on the sample firms in the text. The results are summarized in Table V.

Similar operations were performed on the firms which had been excluded due to bonus issue from the N.E.E. sample for 1960 and from the food industry sample for 1958. The results are summarized in Table VI.

It is clear from the two tables that the exclusion of certain firms from the various samples because their shares were too infrequently traded or because of bonus issues does not in any way weaken the main conclusions arrived at. In fact, since in calculating x_8, the weights computed from the regression analysis in the text are applied here to observations not in the respective samples and the errors

$$\text{i.e.} \quad \left[\left|\frac{x_1 - x_8}{x_1}\right|(100)\right]$$

still lie within admissible limits, the findings of these tables confirm the conclusions of the text.

TABLE V

Name of firm	*Year*	*Industry*	x_1	x_2	x_3	x_4	$\left[\left\|\frac{x_1-x_4}{x_1}\right\|(100)\right]$	x_6†	$\left[\left\|\frac{x_1-x_6}{x_1}\right\|(100)\right]$
Radcliffe Edible Products	1958	Food	1·875	1·875	1·667	1·771	5·547	$0{\cdot}468x_3+0{\cdot}49x_2=$ 1·699	9·387
Stratford-on-Avon Produce Canners	1958	Food	2·271	2·625	1·833	2·229	1·849	$0{\cdot}468x_3+0{\cdot}49x_2=$ 2·144	5·592
Radcliffe Edible Products	1959	Food	2·000	2·625	1·333	1·979	1·050	$0{\cdot}400x_3+0{\cdot}53x_2=$ 1·924	3·800
Otis Elevator Co.	1959	N.E.E.	126·875	130·000	123·750	126·875	0·000	$0{\cdot}970x_3+0{\cdot}15x_2=$ 139·538	9·981

TABLE VI

Name of firm	*Year*	*Industry*	x_1	x_2	x_3	x_4	$\left[\left\|\frac{x_1-x_4}{x_1}\right\|(100)\right]$	x_8†	$\left[\left\|\frac{x_1-x_6}{x_1}\right\|(100)\right]$
Liner Concrete Machinery Co.	1960	N.E.E.	7·9479	10·75	6·580	8·66	9·040	$0{\cdot}470x_3+0{\cdot}51x_2=$ 8·575	7·890
Monotype Corporation	1960	N.E.E.	27·2400	34·00	20·166	27·08	5·730	$0{\cdot}470x_3+0{\cdot}51x_2=$ 26·818	1·549
Fryer & Co., Nelson	1958	Food	17·0560	22·25	14·833	18·5416	8·687	$0{\cdot}468x_3+0{\cdot}49x_2=$ 17·844	4·620

† The respective x_6's have been computed from the weights estimated by regression in Table 2 in the text. (See Section III in the text)

(iv)

The random sample used in Section V of the text was drawn from 2,213 firms listed in the industrial and commercial section of the Stock Exchange *Daily List* of January 3rd, 1961, and the *Monthly Supplementary List* for January 1961. Firms whose shares were too infrequently traded in and firms which had a bonus or a rights issue or were subsequently taken over during the year were not included in the sample. The sample consisted of 100 observations; there were 95 firms, 5 being included twice.

From the Stock Exchange official *Daily Lists* of the first few days of each month, which are kept in the Department of Applied Economics Library, we recorded the once-a-month share prices in a manner similar to that described in Section (i) above. The highest and lowest share prices were obtained from Moody's cards. The following companies were in the sample:

Acrow (Engineers) Ltd.
Amalg. Rental Co. Ltd
Bigwood (Joshua) & Son Ltd
Bluemel Bros. Ltd
Boosey & Hawkes Ltd
Boulton & Paul Ltd
Brayhead Ltd
Breedon & Cloud Hill Lime Works Ltd
British Home Stores Ltd
British Steel Constr. (Birmingham)
British Sugar (Corporation) Ltd
British Tar Products
Brown, Muff & Co. Ltd
Burton (Montague) Ltd
Capital & County Laundries Ltd
Carrington & Dewhurst Ltd
Chamberlain (W. W.) (Assoc.) Ltd
Cohen (George) 600 Group Ltd
Collier (S.) & Co. Ltd
Compressed Paper Packing Ltd
Dallas (John E.) & Sons Ltd
Dickinson (John) & Co. Ltd
Duckworth (John) & Son (Blackburn) Ltd
Duncan Fox & Co. Ltd
Dunhill (Alfred) Ltd
Easterns Ltd
Edworks (1936) Ltd
Elson & Robbins Ltd
Everest Shoe Co. Ltd
Excelsior Motor Co. Ltd
Fairweather (H.) & Co. Ltd
Fitch Lovell Ltd
Francis (F.) & Sons Holdings Ltd
Fry's (London) Ltd
General Electric Co. Ltd
Hattersley (Ormskirk) Ltd
Hazell Sun Ltd
Heenan Group Ltd
Henekeys Ltd
Hick, Hargreaves & Co. Ltd
Hillhead Hughes Ltd
Hollychrome Bricks Ltd
Humphreys Ltd
Jessups Holdings Ltd
K. & L. Timbers Ltd
Kinloch (Provision Merchants) Ltd
Lancashire Cotton Ltd
Lawrie (Alex.) & Co. Ltd
Lyle & Scott Ltd
Madame Tussauds Ltd
Makin (J. & J.) Paper Mills Ltd
Manders (Holdings) Ltd
Mann & Overton Ltd
Marling & Evans Ltd
Mason & Burns Ltd
Metal Industries Ltd
Mills (A. S.) & Co. Ltd
Moss Gear Co. Ltd
National Canning Co. Ltd
National United Laundries Ltd
Norfolk Hotel (Hove) Ltd
Oldham Twist Co. Ltd
Parsons (C. A.) & Co. Ltd
Permali Ltd
Pretoria Portland Cement Co. Ltd
Ratcliffe (Thomas) & Co. Ltd
Reddihough (John) Ltd
Redfern Holdings Ltd
Ritz Hotel (London) Ltd
Rotherham (Jeremiah) & Co.
Royston Industries Ltd
Samnuggur Jute Factory Co. Ltd
Sankey (J. H.) & Son (Holdings) Ltd
Saunders (H. A.) Ltd
Sena Sugar Estates Ltd
Shipton (E.) & Co. Holdings Ltd
Simon Engineering Ltd
South African Clothing Industries Ltd
Steetley Co. Ltd
Stirling Knitting Co. (Southport) Ltd
Terry (Herbert) & Sons Ltd
"The Times" Veneer Co. Ltd

Times Furnishing (Holdings) Ltd
Tomkinsons (Holdings) Ltd
United Molasses Co. Ltd
Universal Grinding Wheel Co. Ltd
Victoria Jute Co. Ltd
Viners Ltd
Walker & Martin Ltd
Warners' Holiday Camps Ltd
Warren (James) & Co. Ltd
Watford, Electric & Manufacturing Co. Ltd
Wesley (Harold) Ltd
Williams & Williams
Wombwell Foundry & Engineering Co. Ltd

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