On the Stratonovich – Kalman - Bucy filtering algorithm application for accurate characterization of financial time series with use of state-space model by central banks

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Abstract – The central banks introduce and implement the monetary and financial stabilities policies, going from the accurate estimations of national macro-financial indicators such as the Gross Domestic Product (GDP). Analyzing the dependence of the GDP on the time, the central banks accurately estimate the missing observations in the financial time series with the application of different interpolation models, based on the various filtering algorithms. The Stratonovich – Kalman – Bucy filtering algorithm in the state space interpolation model is used with the purpose to interpolate the real GDP by the US Federal Reserve and other central banks. We overviewed the Stratonovich – Kalman – Bucy filtering algorithm theory and its numerous applications. We describe the technique of the accurate characterization of the economic and financial time series with application of state space methods with the Stratonovich – Kalman - Bucy filtering algorithm, focusing on the estimation of Gross Domestic Product by the Swiss National Bank. Applying the integrative thinking principles, we developed the software program and performed the computer modeling, using the Stratonovich – Kalman – Bucy filtering algorithm for the accurate characterization of the Australian GDP, German GDP and the USA GDP in the frames of the state-space model in Matlab. We also used the Hodrick-Prescott filter to estimate the corresponding output gaps in Australia, Germany and the USA. We found that the Australia, Germany on one side and the USA on other side have the different business cycles. We believe that the central banks can use our special software program with the aim to greatly improve the national macroeconomic indicators forecast by making the accurate characterization of the financial time-series with the application of the state-space models, based on the *Stratonovich – Kalman – Bucy filtering algorithm.*

JEL: C15, C32, C51, C52, E5, E31, E32.

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Keywords: Wiener filtering theory, Stratonovich optimal non-linear filtering theory, Stratonovich – Kalman – Bucy filtering algorithm, state space interpolation technique**,** financial time-series, nonlinearities, stochastic volatility; Markov switching, Bayesian estimation. Gaussian distribution, econophysics, econometrics, central bank, integrative thinking.

Introduction

The economic and financial principles by the *Austrian school of economic thinking* in *Menger (1871), von Böhm-Bawerk (1884, 1889, 1921), von Mises (1912, 1949), Hayek (1931, 1935, 1948, 1980, 2008), Hazlitt (1946), Rothbard (1962, 2004)* had a considerable scientific influence on the modern *Monetarism theories* by the *American* scientists of the *Austrian* origin at the *Chicago school of economic thinking* in the *XX – XXI* centuries. In our time, the *Chicago school of economic thinking* has a reputation of a world renowned expert in the finances, influencing the *US* policymakers, governmental officials, congressmen, senators, who work on the *US Federal Reserve System* governance policies introduction and execution. The *central bank* of the *United States*, the *US Federal Reserve System*, was founded in the *Federal Reserve Act*, passed by the *US Congress* in *1913* in *Willis (1923), Meltzer (2003, 2009a, b), Bernanke (2013).* The main purpose of the *US Federal Reserve System* was: "provide a means by which periodic panics which shake the *American Republic* and do it enormous injury shall be stopped" in *Owen (1919), Bernanke (2013).* Analyzing the historical developments, *Dr. Ben Shalom Bernanke, Chairman of the US Federal Reserve System* distinguishes the following historical periods in the *US Federal Reserve System* operation in *Bernanke (2013)*: 1) The *Great Experiment* of the *US Federal Reserve System* founding in *1913*; 2) The *Great Depression* in *1922–1933*; 3) The *Stable Inflation* in 1950s – 1960s, *Great Inflation* in mid *1960s – end 1970s*, and *Disinflation* in *1979–1984*; 4) The *Great Moderation in 1984–2007*; 5) The *Great Recession* in *2008–until now. In Fig. 1, the first Federal Reserve System Board in 1914 is shown in Fox, Alvarez, Braunstein, Emerson, Johnson, Johnson, Malphrus, Reinhart, Roseman, Spillenkothen, Stockton (2005).*

Fig. 1. *The first Federal Reserve System Board in 1914 (after Fox, Alvarez, Braunstein, Emerson, Johnson, Johnson, Malphrus, Reinhart, Roseman, Spillenkothen, Stockton (2005)).*

As the principal monetary authority of a nation, the *US Federal Reserve System* (*central bank)* performs the key functions towards the introduction and implementation of:

1. *Monetary stability policy,* aiming to stabilize the prices and increase the confidence in the currency by setting and reaching the inflation target through the realization of transparent effective programs on the interest rates and asset purchases in the money markets, and
2. *Financial stability policy,* aiming to detect and reduce the systemic risks to the national financial system by identifying and monitoring the possible systemic threats to the financial stability and by taking an action to reduce those threats by improving the financial infrastructure, by setting the banking capital requirements, by acting as the lender of last resort.

Presently, the *US Federal Reserve System's* main purpose is to provide the nation with a safer, more flexible, and more stable monetary and financial system in *Fox, Alvarez, Braunstein, Emerson, Johnson, Johnson, Malphrus, Reinhart, Roseman, Spillenkothen, Stockton (2005).*

The *Federal Reserve System's* main duties are in *Fox, Alvarez, Braunstein, Emerson, Johnson, Johnson, Malphrus, Reinhart, Roseman, Spillenkothen, Stockton (2005)*:

1. Conducting the nation's monetary policy by influencing the monetary and credit conditions in the economy in pursuit of maximum employment, stable prices, and moderate long-term interest rates.

2. Supervising and regulating the banking institutions to ensure the safety and soundness of the nation's banking and financial system and to protect the credit rights of consumers.

3. Maintaining the stability of the financial system and containing systemic risk that may arise in financial markets.

4. Providing the financial services to depository institutions, the *US Government*, and foreign official institutions, including playing a major role in operating the nation's payments system.

The implementation of the monetary policy by the *US Federal Reserve System* is a challenging task in *Fox, Alvarez, Braunstein, Emerson, Johnson, Johnson, Malphrus, Reinhart, Roseman, Spillenkothen, Stockton (2005)*: "The *Federal Reserve* implements *U.S. monetary policy* by affecting conditions in the market for balances that depository institutions hold at the *Federal Reserve Banks*. The operating objectives or targets that it has used to effect desired conditions in this market have varied over the years. At one time, the *FOMC* sought to achieve a specific quantity of balances, but now it sets a target for the interest rate at which those balances are traded between depository institutions—the federal funds rate. By conducting open market operations, imposing reserve requirements, permitting depository institutions to hold contractual clearing balances, and extending credit through its discount window facility, the *Federal Reserve* exercises considerable control over the demand for and supply of Federal Reserve balances and the federal funds rate. Through its control of the federal funds rate, the *Federal Reserve* is able to foster financial and monetary conditions consistent with its monetary policy objectives.”

In Fig. 2, the *US Federal Reserve System* is depicted in *Fox, Alvarez, Braunstein, Emerson, Johnson, Johnson, Malphrus, Reinhart, Roseman, Spillenkothen, Stockton (2005).*

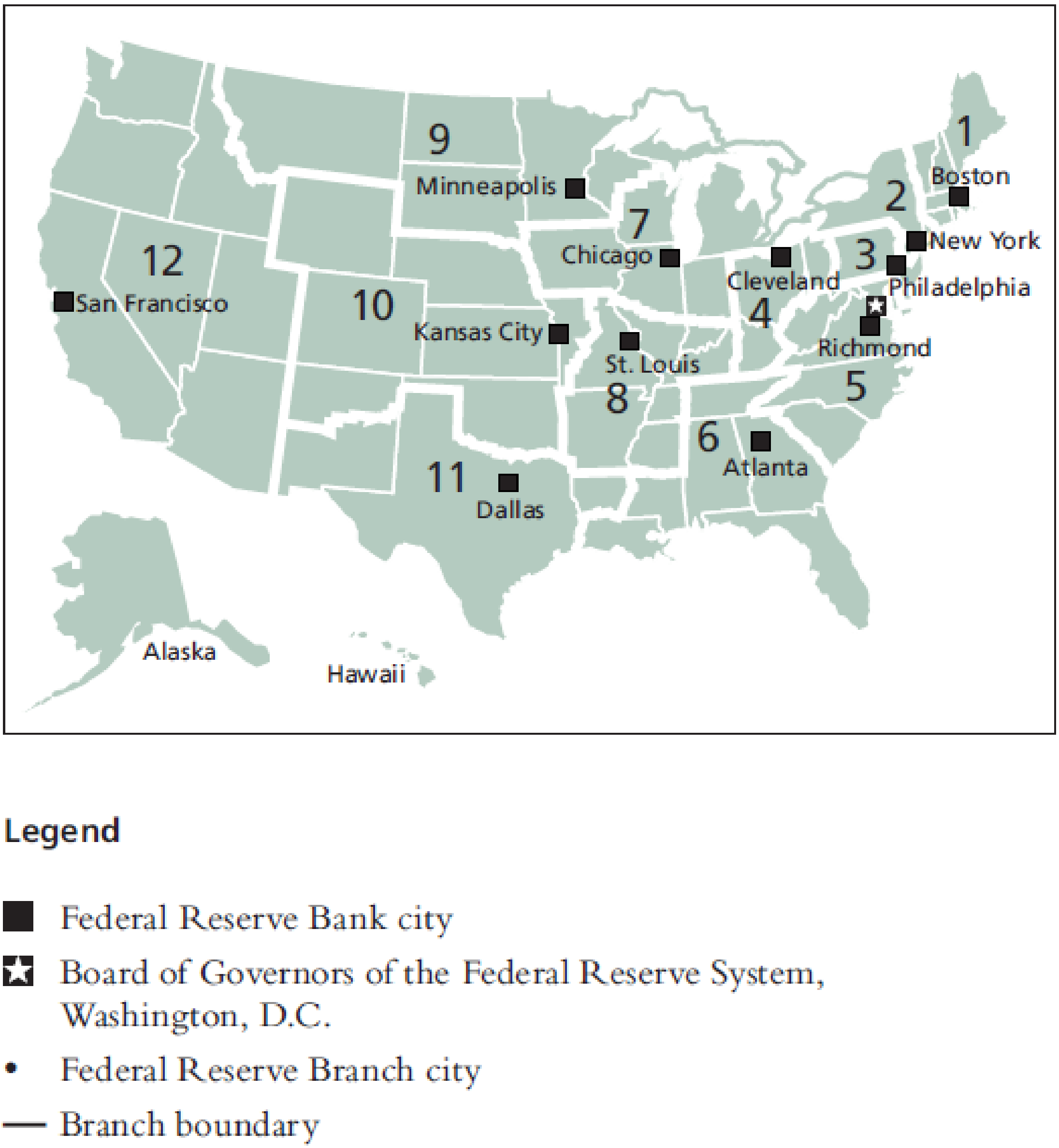


Fig. 2. *The US Federal Reserve System (after Fox, Alvarez, Braunstein, Emerson, Johnson, Johnson, Malphrus, Reinhart, Roseman, Spillenkothen, Stockton (2005)).*

In Tab. 1, the *US Federal Reserve district banks and branches* are shown in *Fox, Alvarez, Braunstein, Emerson, Johnson, Johnson, Malphrus, Reinhart, Roseman, Spillenkothen, Stockton (2005).*

Number	Letter	Bank	Branch
1	A	Boston	
2	B	New York	Buffalo, New York
3	C	Philadelphia	
4	D	Cleveland	Cincinnati, Ohio Pittsburgh, Pennsylvania
5	E	Richmond	Baltimore, Maryland Charlotte, North Carolina
6	F	Atlanta	Birmingham, Alabama Jacksonville, Florida Miami, Florida Nashville, Tennessee New Orleans, Louisiana
7	G	Chicago	Detroit, Michigan
8	H	St. Louis	Little Rock, Arkansas Louisville, Kentucky Memphis, Tennessee
9	I	Minneapolis	Helena, Montana
10	J	Kansas City	Denver, Colorado Oklahoma City, Oklahoma Omaha, Nebraska
11	K	Dallas	El Paso, Texas Houston, Texas San Antonio, Texas
12	L	San Francisco	Los Angeles, California Portland, Oregon Salt Lake City, Utah Seattle, Washington

Tab. 1. *The US Federal Reserve district banks and branches (after Fox, Alvarez, Braunstein, Emerson, Johnson, Johnson, Malphrus, Reinhart, Roseman, Spillenkothen, Stockton (2005)).*

In Fig. 3, the market for balances at the US Federal Reserve is shown in Fox, Alvarez, Braunstein, Emerson, Johnson, Johnson, Malphrus, Reinhart, Roseman, Spillenkothen, Stockton (2005).

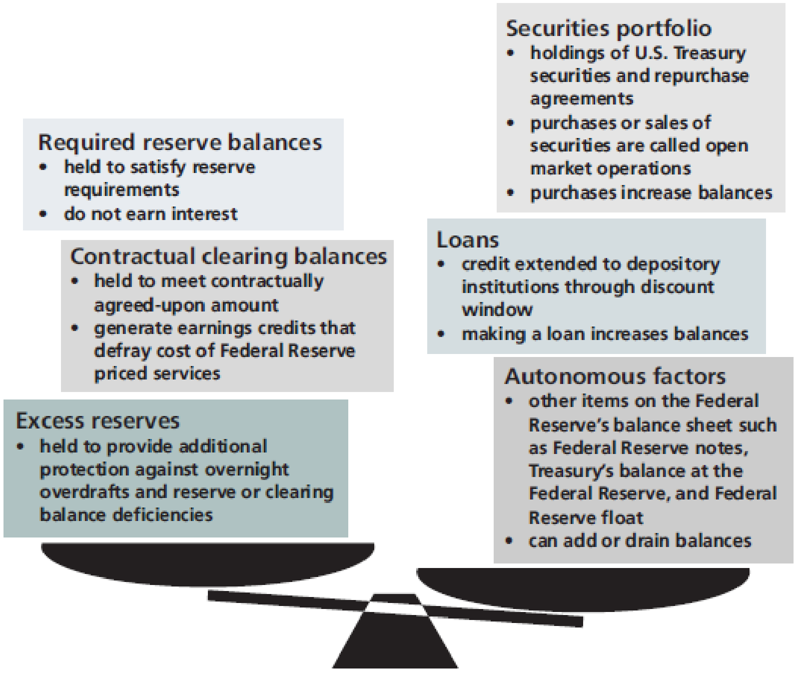


Fig. 3. *The market for balances at the US Federal Reserve (after Fox, Alvarez, Braunstein, Emerson, Johnson, Johnson, Malphrus, Reinhart, Roseman, Spillenkothen, Stockton (2005)).*

The central bank's *monetary policy* is usually guided by the basic principles and needs to be flexible enough to respond to the financial fluctuations in the capital markets in *Ferguson (2003)*. In the researched case of the *Swiss National Bank (SNB)*, the *SNB's "best-practice" monetary policy framework* is based on the following important principles in *Baltensperger, Hildebrand, Jordan (2007)*:

1. Priority given to long-term price stability as a firm nominal anchor, with an explicit quantitative definition of what is meant by price stability;

2. A medium-term orientation in the pursuit of this objective, giving scope for short-run flexibility to dampen real economic fluctuations;

3. A forward-looking approach in the pursuit of its objectives, through the use of an inflation forecast as its main indicator;

4. Flexible implementation of monetary policy, through the announcement of a target range for the three-month *CHF Libor* as an operational target;

5. Transparency and accountability as central principles of a successful policy concept.

Let us explain that the *central banks* introduce the changes into the *monetary stability policy* and the *financial stability policy,* going from the estimated economic and financial indicators such as the *Gross Domestic Product (GDP)* in *Taylor (1999).* Aiming to complete the accurate characterizations of the different economic and financial indicators, the *central banks* estimate the missing observations in the various economic time series with the application of the different *interpolation models,* including *the state-space model,* in *Bernanke, Gertler, Watson (1997), Cuche, Hess (2000).* ***The Stratonovich – Kalman – Bucy filtering algorithm in Stratonovich (1959a, b, 1960a, b), Kalman, Koepcke (1958, 1959), Kalman, Bertram (1958, 1959), Kalman (1960a, b, 1963), Kalman, Bucy (1961) represents one of the possible interpolation models, which has been effectively used to interpolate the real Gross Domestic Product (GDP) by the Federal Reserve in the USA, Swiss National Bank in Switzerland and by some other central banks in various countries in Bernanke, Gertler, Watson (1997), Cuche, Hess (2000), Proietti, Luati (2012a).***

The other macroeconomic applications of the state-space interpolation models may also include in *Proietti, Luati (2012a)*:

• ***The extraction of signals such as trends and cycles in macroeconomic time series***: see *Watson (1986), Clark (1987), Harvey and Jaeger (1993), Hodrick and Prescott (1997), Morley, Nelson and Zivot (2003), Proietti (2006), Luati and Proietti (2011).*

• ***The dynamic factor models, for the extraction of a single index of coincident indicators***, see *Stock and Watson (1989), Frale et al. (2011), and for large dimensional systems Jungbacker, Koopman and van der Wel (2011).*

• ***The stochastic volatility models***: see *Shephard (2005)* and *Stock and Watson (2007)* for applications to *US* inflation.

• ***The time varying auto-regressions with stochastic volatility***: see *Primiceri (2005), Cogley, Primiceri and Sargent (2010).*

• ***The structural change in macroeconomics***: see *Kim and Nelson (1999).*

• ***The class of dynamic stochastic general equilibrium (DSGE) models***: see *Sargent (1989), Fernandez-Villaverde and Rubio-Ramirez (2005), Smets and Wouters (2003), Fernandez-Villaverde (2010).*

In this research article, we intend to continue our scientific investigations on *the nonlinearities in the finances,* which have been conducted over the recent years in *Ledenyov V O, Ledenyov D O (2012a, b), Ledenyov D O, Ledenyov V O (2012c, d), Ledenyov D O, Ledenyov V O (2013a, b, c, d, e, f).*

Stratonovich – Kalman – Bucy filtering algorithm and its applications

The ***Stratonovich – Kalman – Bucy filtering algorithm*** was invented in *the science of radio-physics,* hence let us make a brief overview of ***the analogue and digital signals processing techniques*** with the purpose to understand an essence of the *Stratonovich – Kalman – Bucy* filtering algorithm in *Stratonovich (1959a, b, 1960a, b), Kalman, Koepcke (1958, 1959), Kalman, Bertram (1958, 1959), Kalman (1960a, b, 1963), Kalman, Bucy (1961).*

Discussing ***the analogue signal processing***, it is worth to say that, in the *theory of electrodynamics* and the *theory of radio-physics*, it is a well known fact that the *analogue signal* with the encoded *information* can be transmitted by the *signal carrier* over the wireless, wireline or optical channels in *Wanhammar (1999), Ledenyov D O, Ledenyov V O (2012e)*. This *analogue signal* can be accurately characterized by its changing *amplitude*, *frequency*, *phase* and *power* over the certain *time period* in *Ledenyov D O, Ledenyov V O (2012e)*. The encoding of the *information* into the *analogue signal* can be done with the help of various *modulation processes* by changing the analogue signal's parameters such as the amplitude (*amplitude modulation*), frequency (*frequency modulation*), phase (*phase modulation*) and power (*pulse code modulation*) over the time in *Ledenyov D O, Ledenyov V O (2012e)*. The *analogue signal* can be continuously transmitted over the transmission channel for some time period (*the continuous wave (CW) signal*) or it can be discretely transmitted over the transmission channel for some time(*the discrete signal*). In the last case, the *analogue signal* can be represented as a sequence of the discrete magnitudes of physical parameters of the analogue signal in *Ledenyov D O, Ledenyov V O (2012e)*. The *analogue signals filtering* with the *frequency selective signal filters* is needed in the cases, when it is necessary to transmit or receive the selected analogue signal over the certain frequency in the frequency domain only in *Ledenyov D O, Ledenyov V O (2012e)*. The *analogue signals filtering* is well described in the book: "Nonlinearities in microwave superconductivity" in *Ledenyov D O, Ledenyov V O (2012e)*: "The *High Temperature Superconducting (HTS)* microwave electromagnetic signal filter is one of the essential microwave components in modern wireless communication systems in which the complete and independent measurement of the entire signal space to identify and decode the information in the spectral transmission sequences over the wireless channel is made. The main functions of microwave filter are to select the information signal carrier in the frequency domain and amplify its amplitude by the resonance."

Discussing ***the digital signal processing techniques***, it makes sense to explain that the *analogue signal* can also be uniformly sampled over the time, using the *Nyquist theorem*, with the help of the *Analogue to Digital (A/D)* converter to obtain the *digital signal*; or the *digital signa*l can be de-sampled over the time with the help of the *Digital to Analogue (D/A)* converter to obtain the *analogue signal* in *Wanhammar (1999)*. The *analogue signal processing* can be performed, using the *analogue signal processing algorithms* such as the *Fourier transform*, *Laplace transform, etc*. in *Wanhammar (1999).* The *digital signal processing* can be performed, using the digital signal processing algorithms such as the *Discrete Fourier transform (DFT), Fast Fourier transform (FFT), Cooley-Turkey Fast Fourier transform (CT FFT), Sande-Tukey Fast Fourier transform (ST FFT), Inverse Fast Fourier transform (Inverse FFT), Discrete Cosine transform (DCT), Wavelet transform, z-transform, etc.* in *Wanhammar (1999).* As explained in *Wanhammar (1999)*: "The main purpose of a signal processing system is generally to reduce or retain the information in a signal." The digital signal processing is usually done for the *Linear Shift Invariant (LSI)* systems, which are linear and time-invariant in *Wanhammar (1999)*. The frequency response of the *Linear Shift Invariant (LSI)* system can be characterized by the *frequency function, magnitude function, attenuation function, phase function, group delay function, and transfer function* in *Wanhammar (1999)*. The digital filters can also be classified in the *Finite-length Impulse Response (FIR)* filters and *Infinite-length Impulse Response (IIR)* filters, depending on their response functions characteristics in *Wanhammar (1999).*

Going to the discussion on **the Stratonovich – Kalman – Bucy filtering algorithm**, it is interesting to highlight the fact that, since the beginning of the *XX* century, the ***nonlinearities*** and ***nonlinear physical systems*** represented the subjects of strong research interest in the natural sciences, including the *radio-physics (the analogue signal processing)* in *Mandel'shtam (1948-1955), Andronov (1956), Rytov (1957)*; the nuclear physics in *Fermi, Pasta, Ulam (1955), Femi (1971-1972). The nonlinearities in the microwave superconductivity* were comprehensively researched in *Ledenyov D O, Ledenyov V O (2012e).*

Analyzing the time series, *Ruslan L. Stratonovich* created ***the optimal non-linear filtering theory*** in *1959* in *Stratonovich (1959a, b, 1960a, b)*. During next few years, the ***optimal non-linear filtering theory*** has been extensively complemented by the various research findings; and its foundational principles have been used to develop the *Stratonovich – Kalman – Bucy filtering algorithm* in *1959-1963* in *Stratonovich (1959a, b, 1960a, b), Kalman, Koepcke (1958, 1959), Kalman, Bertram (1958, 1959), Kalman (1960a, b, 1963), Kalman, Bucy (1961).*

The ***Stratonovich – Kalman – Bucy filter*** is clearly defined as in *Wikipedia (2013)*: "The *Kalman filter*, also known as *Linear Quadratic Estimation (LQE)*, is an algorithm that uses a series of measurements observed over time, containing noise (random variations) and other inaccuracies, and produces estimates of unknown variables that tend to be more precise than those based on a single measurement alone. More formally, the *Kalman filter* operates recursively on streams of noisy input data to produce a statistically optimal estimate of the underlying system state. The filter is named for *Rudolf (Rudy) E. Kálmán*, one of the primary developers of its theory."

The ***Stratonovich – Kalman – Bucy filtering algorithm*** is also described in *Wikipedia (2013)*: "The algorithm works in a two-step process. In the prediction step, the *Kalman filter* produces estimates of the current state variables, along with their uncertainties. Once the outcome of the next measurement (necessarily corrupted with some amount of error, including random noise) is observed, these estimates are updated using a weighted average, with more weight being given to estimates with higher certainty. Because of the algorithm's recursive nature, it can run in real time using only the present input measurements and the previously calculated state; no additional past information is required."

Athans (1974) write: "The *Kalman filter* represents one of the major contributions in *modern control theory*. Since its original development (references [I] and [2]), it has been rederived from several points of view bringing into focus its properties from both a probabilistic and optimization viewpoint (references [3] to [9]). Its importance in stochastic control can be appreciated in view of the numerous applications (references [5], [6], [9] to [16])."

Kleeman (1995) explains: "A *Kalman filter* is an *optimal estimator* – i.e. it infers parameters of interest from indirect, inaccurate and uncertain observations. The process of finding the "best estimate" from noisy data amounts to "filtering out" the noise. If all noise is *Gaussian*, the *Kalman filter* minimizes the mean square error of the estimated parameters. However a *Kalman filter* also doesn't just clean up the data measurements, but also *projects* these measurements onto the state estimate."

Welch, Bishop (2001) notes: "The *Kalman filter* is essentially a set of mathematical equations that implement a predictor-corrector type estimator that is *optimal* in the sense that it minimizes the estimated *error* covariance—when some presumed conditions are met."

Pasricha (2006) states: "The *Kalman filter* is a recursive linear filter, first developed as a discrete filter for use in engineering applications and subsequently adopted by statisticians and econometricians. The basic idea behind the filter is simple - to arrive at a conditional density function of the unobservables using *Bayes' Theorem*, the functional form of relationship with observables, an equation of motion and assumptions regarding the distribution of error terms. The filter uses the current observation to predict the next period's value of unobservable and then uses the realization next period to update that forecast. The linear *Kalman filter* is optimal, i.e.

Minimum Mean Squared Error estimator if the observed variable and the noise are jointly *Gaussian*."

Proietti, Luati (2012a) come up with the explanation: "The *Kalman filter* (*Kalman, 1960; Kalman and Bucy, 1961*) is a fundamental algorithm for the statistical treatment of a state space model. Under the *Gaussian assumption*, it produces the minimum mean square estimator of the state vector along with its mean square error matrix, conditional on past information; this is used to build the one-step-ahead predictor of y_t and its mean square error matrix. Due to the independence of the one-step-ahead prediction errors, the likelihood can be evaluated via the prediction error decomposition."

It is possible to think about the ***Stratonovich – Kalman – Bucy filter*** as a device, which can estimate the state of a dynamic system from a series of incomplete and noisy measurements. It can be used to predict a current state by using the previous one, or estimate an updated state by using the previous state and current measurement. The predicted and estimated measurements are calculated from their corresponding states as discussed in *Matlab (R2012)*.

The *Stratonovich – Kalman – Bucy filtering algorithm* is extensively used in the following types of signal processing filters in *Wikipedia (2013)*: the *alpha beta filter*, *ensemble Kalman filter*, *extended Kalman filter*, *iterated extended Kalman filter*, *fast Kalman filter*, *invariant extended Kalman filter*, *kernel adaptive Kalman filter*, *non-linear Kalman filter*, *Schmidt–Kalman filter*, *hybrid Kalman filter*, and *Wiener filter* as explained in *Jazwinski (1970), Bozic (1979), Bar-Shalom, Maybeck (1990), Xiao-Rong Li (1993)*.

There is a big number of practical technical applications of the *Stratonovich – Kalman – Bucy filtering algorithm*, for example: the *Fast Stratonovich – Kalman – Bucy adaptive filter* is implemented in the *equalizers* with the short training times in *Wanhammar (1999)*. In the general case, the equalization of wireless channel is achieved by the application of the digital filters such as the *Recursive Least Square (RLS) lattice filters* with the aim to compensate for the various distortions in the wireless channel and to eliminate the inter-symbol interference in the chips sequence during the spread spectrum communication over the wireless channel, because of the *Ritz RF signal fading model* or the *Raleigh RF signal fading model*, for instance, in the cases of both the *Wideband Code Division Multiple Access (WCDMA)* communication or the *Direct Sequence Spread Spectrum (DSSS)* communication over the constantly fading wireless channel in *Wanhammar (1999), Ledenyov D O, Ledenyov V O (2012e)*.

Some other technical applications of the *Stratonovich – Kalman – Bucy* filtering algorithm include in *Wikipedia (2013)*: the *attitude and heading reference systems, autopilots, battery state of charge estimators, brain-computer interface systems, equalizers in receivers for* *spread spectrum signals communications, tracking and vertex fitting of charged particles in particle detectors systems, tracking of objects in computer vision systems, GDP interpolation in macroeconomics, Volterra series interpolation in economics, inertial guidance systems in on board navigation systems, radar tracking systems, GPS satellite navigation systems, systems for sensor-less control of AC motor variable-frequency drives.*

Stratonovich – Kalman – Bucy filtering algorithm theory

Let us begin with the comprehensive discussion on the *Stratonovich – Kalman – Bucy filtering algorithm theory* by bringing some interesting facts on the history of science to our close attention. *Wiener (1949)* completed a research on the extrapolation, interpolation and smoothing of stationary time series. *Stratonovich (1959a, b)* made a research on the selection of the useful signals from the noise in the nonlinear systems, attempting to create *the theory of optimal non-linear filtering* of random functions. *Stratonovich (1960a, b)* applied the *Markov processes theory* to the *theory of optimal non-linear filtering*. *Kalman, Koepcke (1958a, 1959b), Kalman, Bertram (1958b, 1959a), Kalman (1960a)* conducted the innovative researches on *the theory of linear sampling control systems*. At later date, *Kalman (1960b)* focused on *the linear filtering and prediction research problems*, formulating and solving the *Wiener problem* from the “state” point of view. *Kalman (1963)* accented his research attention on the development of new methods in *the Wiener filtering theory*. Let us emphasis that *Kalman (1960b)* considered some theoretical aspects of both the *general linear continuous-dynamic system* and the *general linear discrete-dynamic system.*

The block diagram of the *general linear continuous-dynamic system* is shown in Fig. 4 and the block diagram of the *general linear discrete-dynamic system* is depicted in Fig. 5 in *Kalman (1960b).*

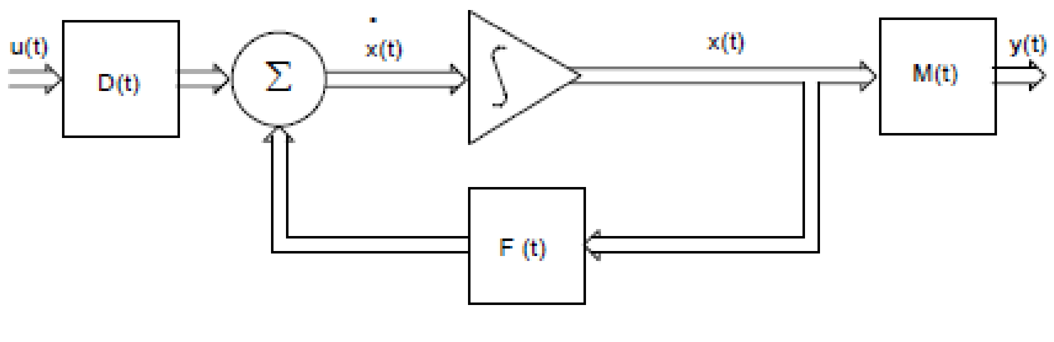


Fig. 4. *Matrix block diagram of the general linear continuous-dynamic system (after Kalman (1960b)).*

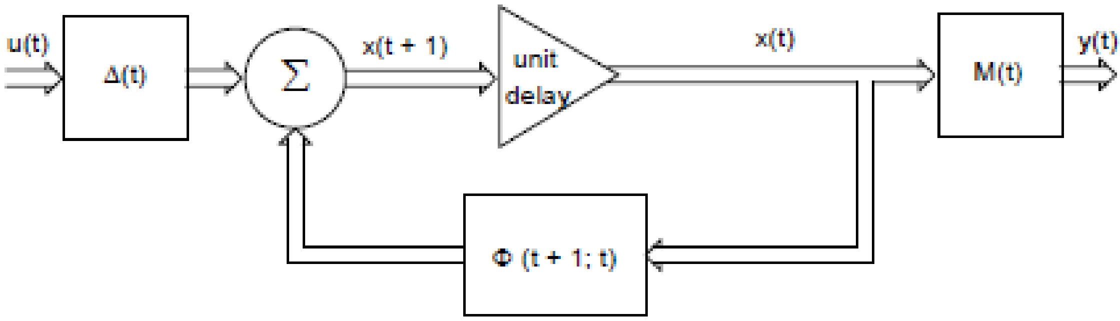


Fig. 5. *Matrix block diagram of the general linear discrete-dynamic system (after Kalman (1960b)).*

The block diagram of *optimal filter* is shown in Fig. 6 and the block diagram of *optimal controller* is represented in Fig. 7 in *Kalman (1960b).*

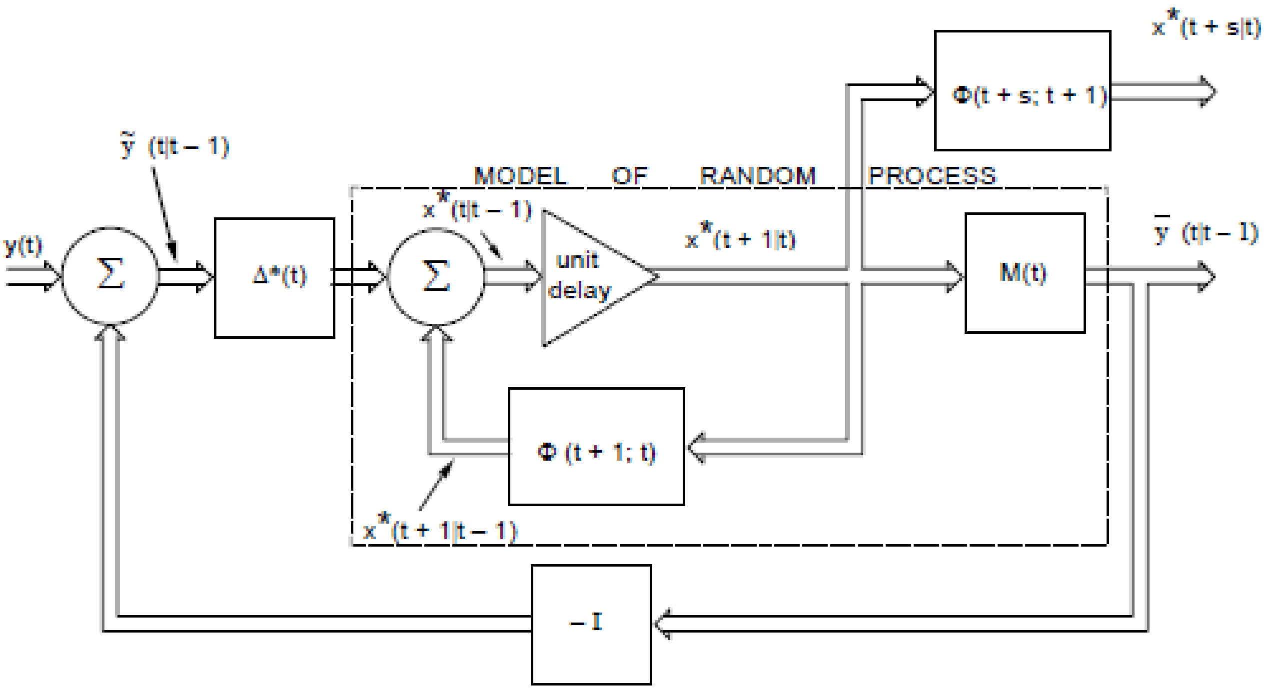


Fig. 6. *Matrix block diagram of optimal filter (after Kalman (1960b)).*

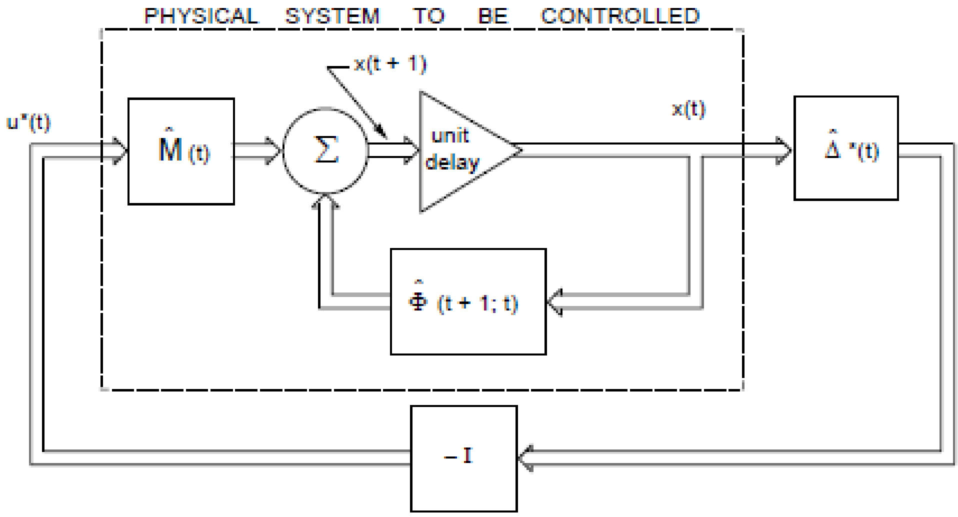


Fig. 7. *Matrix block diagram of optimal controller (after Kalman (1960b)).*

Let us learn more on the theory of the *Stratonovich – Kalman – Bucy filtering* in *Wikipedia (2013)*: “The *Kalman filters* are based on linear dynamic systems discretized in the time domain. They are modeled on a *Markov chain* built on linear operators perturbed by *Gaussian noise*. The state of the system is represented as a vector of real numbers. At each discrete time increment, a linear operator is applied to the state to generate the new state, with some noise mixed in, and optionally some information from the controls on the system if they are known. Then, another linear operator mixed with more noise generates the observed outputs from the true ("hidden") state. The *Kalman filter* may be regarded as analogous to the *hidden Markov model*, with the key difference that the hidden state variables take values in a continuous space (as opposed to a discrete state space as in the *hidden Markov model*). Additionally, the *hidden Markov model* can represent an arbitrary distribution for the next value of the state variables, in contrast to the *Gaussian noise* model that is used for the *Kalman filter*. There is a strong duality between the equations of the *Kalman Filter* and those of the *hidden Markov model*”.

The *Kalman filter* model assumes the true state at time *k* is evolved from the state at *(k−1)* according to in *Wikipedia (2013)*

$$\mathbf{x_k} = \mathbf{F_k x_{k-1}} + \mathbf{B_k u_k} + \mathbf{w_k}$$

where $\mathbf{F}_k$ is the state transition model, which is applied to the previous state $\mathbf{x}_{k-1}$;
$\mathbf{B}_k$ is the control-input model which is applied to the control vector $\mathbf{u}_k$;

$\mathbf{w}_k$ is the process noise which is assumed to be drawn from a zero mean multivariate normal distribution with covariance $\mathbf{Q}_k$.

$$\mathbf{w_k} \sim \mathbf{N}(0, \mathbf{Q_k})$$

At time k an observation (or measurement) $\mathbf{z}_k$ of the true state $\mathbf{x}_k$ is made according to in *Wikipedia (2013)*

$$\mathbf{z_k} = \mathbf{H_k}\mathbf{x_k} + \mathbf{v_k}$$

where $\mathbf{H}_k$ is the observation model, which maps the true state space into the observed space and $\mathbf{v}_k$ is the observation noise, which is assumed to be zero mean *Gaussian white noise* with covariance $\mathbf{R}_k$.

$$\mathbf{v_k} \sim \mathbf{N}(0, \mathbf{R_k})$$

The initial state, and the noise vectors at each step $\{\mathbf{x}_0, \mathbf{w}_1, ..., \mathbf{w}_k, \mathbf{v}_1 ... \mathbf{v}_k\}$ are all assumed to be mutually independent.

In Fig. 8, the model, underlying the *Kalman filter* is shown in *Wikipedia (2013)*.

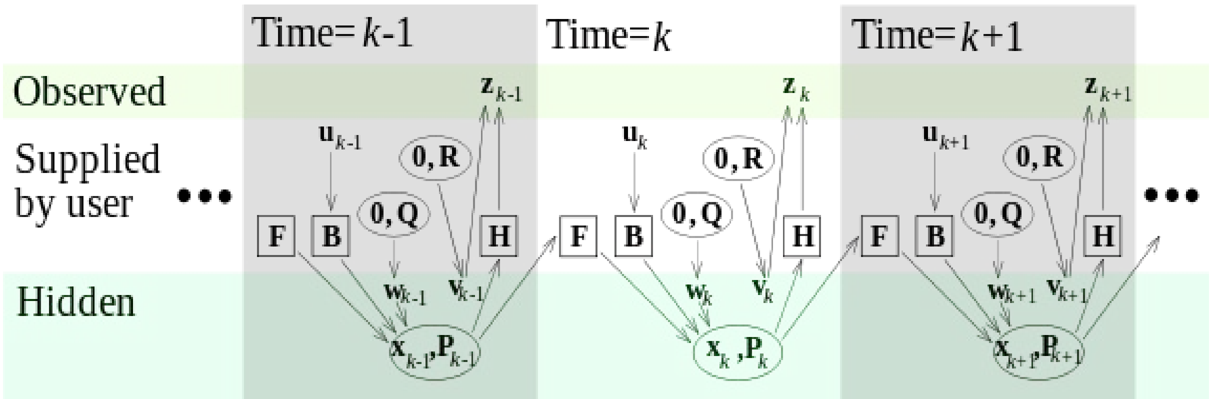


Fig. 8. *Model underlying the Kalman filter. Squares represent matrices. Ellipses represent multivariate normal distributions (with the mean and covariance matrix enclosed). Unenclosed values are vectors. In the simple case, the various matrices are constant with time, and thus the subscripts are dropped, but the Kalman filter allows any of them to change each time step (after Wikipedia (2013)).*

“The *Kalman filter* is a recursive estimator. This means that only the estimated state from the previous time step and the current measurement are needed to compute the estimate for the current state. In contrast to batch estimation techniques, no history of observations and/or estimates is required. In what follows, the notation $\hat{\mathbf{x}}_{\mathbf{n}|\mathbf{m}}$ represents the estimate of **x** at time *n* given observations up to, and including at time *m*,” as explained in *Wikipedia (2013).*

The state of the *Kalman filter* is represented by the two variables in *Wikipedia (2013)*:

1. $\hat{\mathbf{x}}_{\mathbf{k}|\mathbf{k}}$, the a *posteriori* state estimate at time k given observations up to and including at time k;
2. $\mathbf{P}_{\mathbf{k}|\mathbf{k}}$, the a *posteriori* error covariance matrix (a measure of the estimated accuracy of the state estimate).

“The *Kalman filter* can be written as a single equation, however it is most often conceptualized as two distinct phases: "*Predict*" and "*Update*". The *predict phase* uses the state estimate from the previous timestep to produce an estimate of the state at the current timestep. This predicted state estimate is also known as the *a priori* state estimate because, although it is an estimate of the state at the current timestep, it does not include observation information from the current timestep. In the *update phase*, the current *a priori* prediction is combined with current observation information to refine the state estimate. This improved estimate is termed the *a posteriori* state estimate.

Typically, the two phases alternate, with the prediction advancing the state until the next scheduled observation, and the update incorporating the observation. However, this is not necessary; if an observation is unavailable for some reason, the update may be skipped and multiple prediction steps performed. Likewise, if multiple independent observations are available at the same time, multiple update steps may be performed (typically with different observation matrices $\mathbf{H}_k$),” as explained in *Wikipedia (2013).*

Predict phase:

Predicted (*a priori*) state estimate: $$\hat{\mathbf{x}}_{\mathbf{k}|\mathbf{k}-1} = \mathbf{F}_{\mathbf{k}}\hat{\mathbf{x}}_{\mathbf{k}-1|\mathbf{k}-1} + \mathbf{B}_{\mathbf{k}-1}\mathbf{u}_{\mathbf{k}-1}$$

Predicted (*a priori*) estimate covariance: $$\mathbf{P}_{\mathbf{k}|\mathbf{k}-1} = \mathbf{F}_{\mathbf{k}}\mathbf{P}_{\mathbf{k}-1|\mathbf{k}-1}\mathbf{F}_{\mathbf{k}}^{\mathbf{T}} + \mathbf{Q}_{\mathbf{k}}$$

Update phase:

Innovation or measurement residual: $$\tilde{\mathbf{y}}_{\mathbf{k}} = \mathbf{z}_{\mathbf{k}} - \mathbf{H}_{\mathbf{k}}\hat{\mathbf{x}}_{\mathbf{k}|\mathbf{k}-1}$$

Innovation (or residual) covariance: $$\mathbf{S}_{\mathbf{k}} = \mathbf{H}_{\mathbf{k}}\mathbf{P}_{\mathbf{k}|\mathbf{k}-1}\mathbf{H}_{\mathbf{k}}^{\mathbf{T}} + \mathbf{R}_{\mathbf{k}}$$

Optimal *Kalman* gain: $$\mathbf{K_k} = \mathbf{P_{k|k-1}} \mathbf{H_k^T} \mathbf{S_k^{-1}}$$

Updated (*a posteriori*) state estimate: $$\hat{\mathbf{x}}_{\mathbf{k|k}-1} = \hat{\mathbf{x}}_{\mathbf{k|k}-1} + \mathbf{K_k}\tilde{\mathbf{y}}_\mathbf{k}$$

Updated (*a posteriori*) estimate covariance: $$\mathbf{P_{k|k}} = \left(\mathbf{I} - \mathbf{K_k H_k}\right)\mathbf{P_{k|k}}_{-1}$$

Invariants:

"If the model is accurate, and the values for $\hat{\mathbf{x}}_{0|0}$ and $\mathbf{P}_{0|0}$ accurately reflect the distribution of the initial state values, then the following invariants are preserved: (all the estimates have a mean error of zero)," see in *Wikipedia (2013)*

$$\mathbf{E}\left[\mathbf{x_k} - \hat{\mathbf{x}}_\mathbf{k|k}\right] = \mathbf{E}\left[\mathbf{x_k} - \hat{\mathbf{x}}_{\mathbf{k|k}-1}\right] = 0$$

$$\mathbf{E}\left[\tilde{\mathbf{y}}_\mathbf{k}\right] = 0$$

where $\mathbf{E}\left[\xi\right]$ is the expected value of ξ, and the covariance matrices accurately reflect the covariance of estimates in *Wikipedia (2013)*

$$\mathbf{P_{k|k}} = \operatorname{cov}\left(\mathbf{x_k} - \hat{\mathbf{x}}_\mathbf{k|k}\right)$$

$$\mathbf{P}_{\mathbf{k|k}-1} = \operatorname{cov}\left(\mathbf{x_k} - \hat{\mathbf{x}}_{\mathbf{k|k}-1}\right)$$

$$\mathbf{S_k} = \operatorname{cov}\left(\tilde{\mathbf{y}}_\mathbf{k}\right)$$

Let us express a general opinion in *Wikipedia (2013)*: "Practical implementation of the *Kalman filter* is often difficult due to the inability in getting a good estimate of the noise covariance matrices $\mathbf{Q}_k$ and $\mathbf{R}_k$. Extensive research has been done in this field to estimate these covariances from data. One of the more promising approaches to do this is the *Autocovariance Least-Squares (ALS)* technique that uses autocovariances of routine operating data to estimate the covariances in *Rajamani (2007), Rajamani, Rawlings (2009)*."

Let us provide an additional comment in *Wikipedia (2013)*: "It is known from the theory that the *Kalman filter* is optimal in case that

a) the model perfectly matches the real system,

b) the entering noise is white, and

c) the covariances of the noise are exactly known.

Several methods for the *noise covariance estimation* have been proposed during past decades. One, *ALS*, was mentioned in the previous paragraph. After the covariances are identified, it is useful to evaluate the performance of the filter, i.e. whether it is possible to improve the state estimation quality. It is well known that, if the *Kalman filter* works optimally, the innovation sequence (the output prediction error) is a white noise. The *whiteness property* reflects the state estimation quality. For evaluation of the filter performance it is necessary to inspect the whiteness property of the innovations. Several different methods can be used for this purpose. Three optimality tests with numerical examples are described in *Matisko, Havlena (2012)*."

We would like to explain that there are many types of algorithms for the *recursive estimation*, including the *Stratonovich – Kalman – Bucy filtering algorithm*, the *forgetting factor algorithm*, *the unnormalized and normalized gradient algorithm,* which have been used to solve the different mathematical tasks in the system identification problem in *Ljung (1999)*. The following set of equations summarizes the *Stratonovich – Kalman – Bucy filtering algorithm* in *Matlab (R2012)*:

$$\hat{\theta}(\boldsymbol{t}) = \hat{\theta}(\boldsymbol{t}-1) + \boldsymbol{K}(\boldsymbol{t})\big(\boldsymbol{y}(\boldsymbol{t}) - \hat{\boldsymbol{y}}(\boldsymbol{t})\big)$$

$$\hat{\boldsymbol{y}}(\boldsymbol{t}) = \Psi^{\boldsymbol{T}}(\boldsymbol{t})\hat{\theta}(\boldsymbol{t}-1)$$

$$\boldsymbol{K}(\boldsymbol{t}) = \boldsymbol{Q}(\boldsymbol{t})\Psi(\boldsymbol{t})$$

$$\boldsymbol{Q}(\boldsymbol{t}) = \frac{\boldsymbol{P}(\boldsymbol{t}-1)}{\boldsymbol{R}_2 + \Psi(\boldsymbol{t})^{\boldsymbol{T}}\boldsymbol{P}(\boldsymbol{t}-1)\Psi(\boldsymbol{t})}$$

$$\boldsymbol{P}(\boldsymbol{t}) = \boldsymbol{P}(\boldsymbol{t}-1) + \boldsymbol{R}_1 - \frac{\boldsymbol{P}(\boldsymbol{t}-1)\Psi(\boldsymbol{t})\Psi(\boldsymbol{t})^{\boldsymbol{T}}\boldsymbol{P}(\boldsymbol{t}-1)}{\boldsymbol{R}_2 + \Psi(\boldsymbol{t})^{\boldsymbol{T}}\boldsymbol{P}(\boldsymbol{t}-1)\Psi(\boldsymbol{t})}$$

This formulation assumes the linear-regression form of the model in *Matlab (R2012)*:

$$\boldsymbol{y}(\boldsymbol{t}) = \Psi^{\boldsymbol{T}}(\boldsymbol{t})\theta_0(\boldsymbol{t}) + \boldsymbol{e}(\boldsymbol{t})$$

The *Stratonovich – Kalman – Bucy filter* is used to obtain *Q(t)*.

This formulation also assumes that the true parameters $\theta_0(\boldsymbol{t})$ are described by a random walk in *Matlab (R2012)*:

$$\theta_0(\boldsymbol{t}) = \theta_0(\boldsymbol{t}-1) + \boldsymbol{w}(\boldsymbol{t})$$

where *w(t)* is the *Gaussian white noise* with the following covariance matrix, or drift matrix R_1 in *Matlab (R2012):*

$$\boldsymbol{Ew}(\boldsymbol{t})\boldsymbol{w}(\boldsymbol{t})^{\boldsymbol{T}} = \boldsymbol{R}_1$$

R_2 is the variance of the innovations *e(t)* in the following equation in *Matlab (R2012)*:

$$\boldsymbol{y}(\boldsymbol{t}) = \Psi^{\boldsymbol{T}}(\boldsymbol{t})\theta_0(\boldsymbol{t}) + \boldsymbol{e}(\boldsymbol{t})$$

The *Stratonovich – Kalman – Bucy filtering algorithm* is entirely specified by the sequence of data *y(t)*, the gradient $\Psi(\boldsymbol{t})$ R_1, R_2, and the initial conditions $\theta(\boldsymbol{t}=0)$(initial guess of the parameters) and $\boldsymbol{P}(\boldsymbol{t}=0)$ (covariance matrix that indicates parameters errors in *Matlab (R2012)*.

Summarizing the above research statements, we would like to say that the *Stratonovich – Kalman – Bucy filtering* is used to predict or estimate the state in the dynamic system. The *Wiener filtering*, *Stratonovich – Kalman – Bucy filtering* and related scientific problems have been extensively researched in the numerous scientific articles in *Wiener (1949), Bartlett (1954), Tukey (1957), Stratonovich (1959a, b, 1960a, b), Kalman, Koepcke (1958, 1959), Kalman, Bertram (1958, 1959), Kalman (1960a, b, 1963), Kalman, Bucy (1961), Friedman (1962), Bryson, Ho (1969), Bucy, Joseph (1970), Jazwinski (1970), Sorenson (1970), Chow, Lin (1971, 1976), Maybeck (1972, 1974, 1990), Willner (1973), Leondes, Pearson (1973), Akaike (1974), Dempster, Laird, Rubin (1977), Griffiths (1977), Schwarz (1978), Falconer, Ljung (1978), Anderson, Moore (1979), Bozic (1979), Priestley (1981), Lewis (1986), Proakis, Manolakis (1988), Caines (1988), de Jong (1988, 1989, 1991), de Jong, Chu-Chun-Lin (1994), Bar-Shalom, Maybeck (1990), Franklin, Powell, Workman (1990), Brockwell, Davis (1991), Jang (1991), Brown, Hwang (1992, 1997), Xiao-Rong Li (1993), Gordon, Salmond, Smith (1993), Farhmeir, Tutz (1994), Grimble (1994), Lee, Ricker (1994), Ricker, Lee (1995), Fuller (1996), Hayes (1996), Haykin (1996), Golub, van Loan (1996), Julier, Uhlmann (1997), Ljung (1999), Wanhammar (1999), Welch, Bishop (2001), Litvin, Konrad, Karl (2003), de Jong, Penzer (2004), van Willigenburg, De Koning (2004), Voss, Timmer, Kurths (2004), Capp´e, Moulines, Ryd´en (2005), Misra, Enge (2006), Rajamani (2007), Andreasen (2008), Rajamani, Rawlings (2009), Xia Y, Tong H (2011), Matisko, Havlena (2012), Proietti, Luati (2012a, b), Durbin, Koopman (2012).*

Accurate characterization of economic and financial time series models with application of state space models, using the Stratonovich – Kalman - Bucy filtering algorithm

Let us explain that, going from the consideration of the *modern financial systems properties*, it is possible to conclude that the *financial systems* can normally be classified as the *diffusion systems*, which can be accurately described by the *drift and diffusion coefficients* in *Bernanke (1979), Shiryaev (1998a), Ledenyov D O, Ledenyov V O (2013f)*. The financial variables, including the drift and diffusion coefficients, can have the nonlinear time dependences. *Xiaohong Chen, Hansen, Carrasco (2009)* state: "Nonlinearities in the drift and diffusion coefficients influence temporal dependence in scalar diffusion models." Moreover, the accurate characterization of the modern financial system may result in a need to interpolate the values of financial data with the missing or unobservable parameters, in the case of the incomplete data sets over the certain observation period. The application of the *Stratonovich – Kalman – Bucy* filtering algorithm can solve these financial engineering problems.

Athans (1974) write: “"In spite of the recent interest in *modern control theory* by mathematical economists the potential advantages of *Kalman filtering* methods have not been fully appreciated by economists and management scientists. One of the reasons is that the straightforward application of *Kalman filtering* methods involves estimation of state variables, whenever the actual measurements are corrupted by white noise. In most economic applications, the measurements of the endogenous and exogenous variables are assumed exact. In this paper, we shall indicate that the *Kalman filtering* algorithm does have potential use for an important class of economic problems, namely those involving the refinement of the parameter estimates (arid of their variances) in an econometric model. Right at the start we should like to emphasis that the use of the *Kalman filtering* techniques is viewed not as a replacement, but rather as a supplement, to traditional econometric methods. We visualize that the *Kalman filtering* methods should become useful only after an econometrician has constructed the mathematical model of a microeconomic or macroeconomic systems. Thus it may represent a final "tune-up" of the econometric model."

Pasricha (2006) explains: ““The *Kalman Filter* is a powerful tool and has been adapted for a wide variety of economic applications. It is essentially a least squares (*Gauss Markov*) procedure and therefore gives *Minimum Mean Square Estimators*, with the normality assumption. Even where the normality assumption is dropped, the *Kalman filter* minimizes any symmetric loss function, including one with kinks. Not only is it used directly in economic problems that can be represented in state-space forms, it is used in the background as part of several other estimation techniques, like the *Quasi-Maximum Likelihood* estimation procedure and estimation of *Markov Switching models*."

There are many applications of the *Stratonovich – Kalman – Bucy filtering* in finances and economics in *Pasricha (2006)*:

1. The evaluation of the international reserves demand by estimating the time varying parameters in a linear regression, using the Kalman Filter: "Another application of the filter is to obtain *GLS* estimates for the model $y_t = \beta' x_t + u_t$, where the error term u_t is *Gaussian ARMA(p,q)* with known parameters." "The classical regression model, $y_t = \beta' x_t + u_t$, where u_t is white noise, assumes that the relationship between the explanatory and explained variables remains constant through the estimation period. When this assumption is an unreasonable one (for example, while studying macroeconomic relationships for countries that have undergone structural reforms during the sample period), and the model is specified as one with β_t'*s*, the *Kalman filter* can be used to estimate the parameters."

2. Modeling of economic regime changes by the Markov switching models in the state-space form, using the Kalman Filter: "A number of macroeconomic and financial variables can plausibly be modeled to have different statistical and dynamic properties depending on the state of the nature and for the probabilities of moving from one state of nature to another to be well defined and constant. For example, the persistence of shocks to stock returns may be different during boom times than during recessions. These can be modeled using *Markov Switching* model if we assume that the switch between the boom and recession is governed by a *Markov chain* (and could alternatively be modeled using the Stochastic Volatility models discussed in Section 3.5 below)." "In the unobserved components models (see Section 3.4 below), for example where *GDP* is decomposed into trend and cyclical components, the trend component of the *GDP* may be modeled as a random walk with drift, where the latter evolves according to a *Markov chain*. Models of *Markov Switching* that can be put in state-space form can be estimated using the *Kalman Filter*."

3. Estimation of the exchange rate risk premia by applying the Kalman filter with the correlated error terms: "…in the market for exchange rates, where new information that causes the spot rate to jump may also cause the risk premium to change. Examples of such new information include shocks to money supply and interest rates, a switch in currency regime, a repudiation of debt by the country or announced change in currency's convertibility." *Cheung (1993)* uses the Kalman filter algorithm to solve this problem."

4. Analysis of the unobserved components model by using the extended Kalman filter: "*Extended Kalman filter* is simply the standard *Kalman filter* applied to a first order *Taylor's approximation* of a non-linear state-space model around its last estimate. This technique can be used, for example, to decompose the trend and cyclical components of the *GDP* when the parameters are also allowed to be time-varying."

5. Analysis of the stochastic volatility models, using the Kalman filter: "Financial data have been observed to have certain regularities in statistical properties, including leptokurtic distributions, volatility clustering (clustering of high and low volatility episodes), leverage effects (higher volatility during falling prices and lower volatility during stock market booms) and persistence of volatility. The financial econometrics literature spawns econometric models that seek to capture many of these stylized facts of the data. The most popular approach uses *GARCH* models, where the variance is postulated to be a linear function of squared past observations and variances. Another approach is *Stochastic Volatility (SV)* models, first proposed by *Taylor (1986)*, where log of the volatility is modeled as a linear, unobserved stochastic *AR* process." The *Stochastic Volatility (SV)* models incorporate the *Stratonovich – Kalman – Bucy filtering algorithm.*

As mentioned above, the *Stratonovich – Kalman – Bucy filtering algorithm* can be used in the process of precise estimation of the *GDP*. The *Stratonovich – Kalman – Bucy filtering algorithm* can be described as in *Cuche, Hess (2000)*: "A useful method for extracting signals is to write down a model linking the unobserved and observed variables in a state-space representation according to *Kalman (1960, 1963)*. The multivariate *Kalman filter* is an algorithm for sequentially updating a linear projection on the vector of interest."

Cuche, Hess (2000) write a general state-space representation in the form of a system of the two vector equations (1) and (2), explaining that the first state equation describes the dynamics of the state vector ($\boldsymbol{\xi}_t$) with the unobserved variables to estimate, and the second measurement equation links the state vector to the vector with the observed variables ($\mathbf{y}^+_t$)

$$\xi_{\mathbf{t}+1} = \mathbf{F_t}\xi_{\mathbf{t}} + \mathbf{C}'_{\mathbf{t}}\mathbf{x}_{\mathbf{t}+1} + \mathbf{R_t}\mathbf{u}_{\mathbf{t}+1}, \quad (1)$$

$$\mathbf{y}^+_{\mathbf{t}} = \mathbf{A}'_{\mathbf{t}}\mathbf{x}^*_{\mathbf{t}} + \mathbf{H}'_{\mathbf{t}}\xi_{\mathbf{t}} + \mathbf{N_t}\mathbf{v_t}. \quad (2)$$

where $t = 1, \dots ,T$; T is the number of monthly observations.

Cuche, Hess (2000) clarify: “In addition to the unobserved and the observed variables of interest, vector equations (1) and (2) contain the so-called related series ($\mathbf{x}_t$) and ($\mathbf{x}^*_t$) as exogenous variables in each equation. Both equations have multinormally distributed error terms

$$\begin{pmatrix} \mathbf{u_t} \\ \mathbf{v_t} \end{pmatrix} \sim \mathbf{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix} \bullet \begin{pmatrix} \mathbf{Q} & 0 \\ 0 & \mathbf{G} \end{pmatrix} \right)$$

Premultiplied by matrices $\mathbf{R}_t$ and $\mathbf{N}_t$, these orthogonal disturbances transform into nonorthogonal residuals within each vector equation. The coefficients matrices $\mathbf{F}_t$, $\mathbf{C}'_t$, $\mathbf{R}_t$, $\mathbf{A}'_t$, $\mathbf{H}'_t$, $\mathbf{N}_t$, and the two variance-covariance matrices $\mathbf{Q}$ and $\mathbf{G}$ are estimated by maximizing the log-likelihood function of this system.”

Cuche, Hess (2000) evaluated the alternative interpolation models for *Swiss GDP* and produced a monthly deseasonalized real *GDP* available for researchers and practitioners. *Cuche, Hess (2000)* write that a method, based on the *Stratonovich – Kalman – Bucy* filtering algorithm, allows to create a setup with a wide range of interpolation models.

Let us point out that *Cuche, Hess (2000)* adapted the general state-space representation in equations (1) and (2) to the considered problem by making an inclusion of related series and by assuming a presence of stochastic processes for the monthly *GDP*. *Cuche, Hess (2000)* explain that the state vector equation (3) describes the vector dynamics of the unobserved variable, monthly *GDP* $\boldsymbol{y}_t$, stacked in the state vector $\xi = \begin{pmatrix} \mathbf{y_t} & \mathbf{y_{t-1}} & \mathbf{y_{t-2}} \end{pmatrix}$; and the equation (4) relates the state vector to the observed quarterly *GDP* y^+_t .

$$\xi_{\mathbf{t}+1} = \mathbf{F_t}\xi_{\mathbf{t}} + \mathbf{C}'_{\mathbf{t}}\mathbf{x}_{\mathbf{t}+1} + \mathbf{R_t}\mathbf{u}_{\mathbf{t}+1}, \quad (3)$$

$$\mathbf{y}^+_{\mathbf{t}} = \mathbf{a}'_{\mathbf{t}}\mathbf{x}^*_{\mathbf{t}} + \mathbf{h}'_{\mathbf{t}}\xi_{\mathbf{t}}. \quad (4)$$

In Fig. 9, the overview of various interpolation models is presented in *Cuche, Hess (2000).*

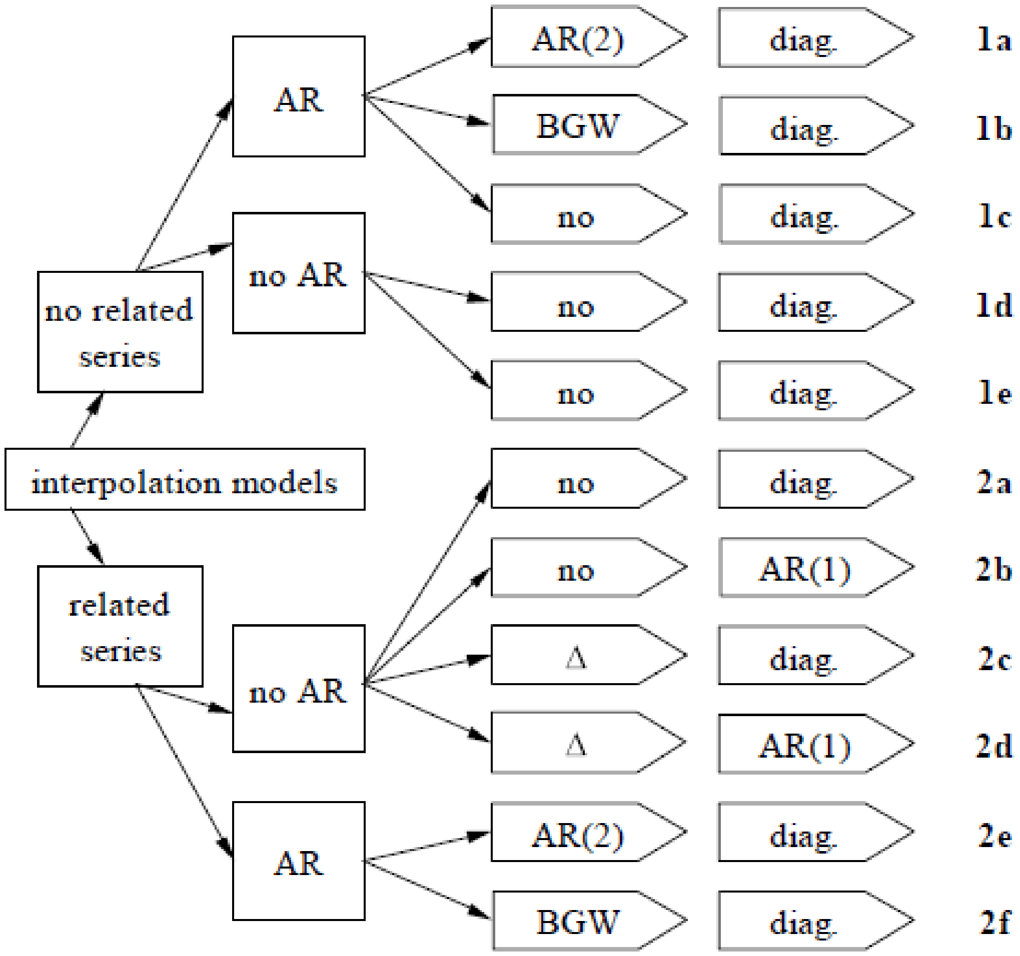


Fig. 9. *Overview of interpolation models (after Cuche, Hess (2000)).*

It is necessary to note that the *Models 1a, 1b, 1c, 1e* are designed without the related series. *Cuche, Hess (2000)* write: "We assume that there is enough information in the autocovariance function of the quarterly series and in the assumed low-order autoregressive (*AR*) process of monthly *GDP*." The *Models 2a-b, 2c-d* are designed with the related series in order to extract information for the interpolation of monthly *GDP*. *Cuche, Hess (2000)* comment: "Within this group, we distinguish between the assumptions that monthly *GDP* does not follow an autoregressive process (*Models 2a-d*) and that it does (*Models 2e-f*)."

Tab. 2 provides the basic summary statistics of the quarterly and monthly series, which are used for the interpolation in *Cuche, Hess (2000).* Fig. 10 shows the *GDP* and related series in *Cuche, Hess (2000).*Tab. 3 presents the interpolation results in *Cuche, Hess (2000).*

As it can be seen, *Cuche, Hess (2000)* evaluated the alternative interpolation models for the *Swiss GDP*, suggesting that the approach with the four related series, including the exports, imports, retail sales, and non-utilized construction loans, is a best approach for the interpolation of *Swiss GDP*.

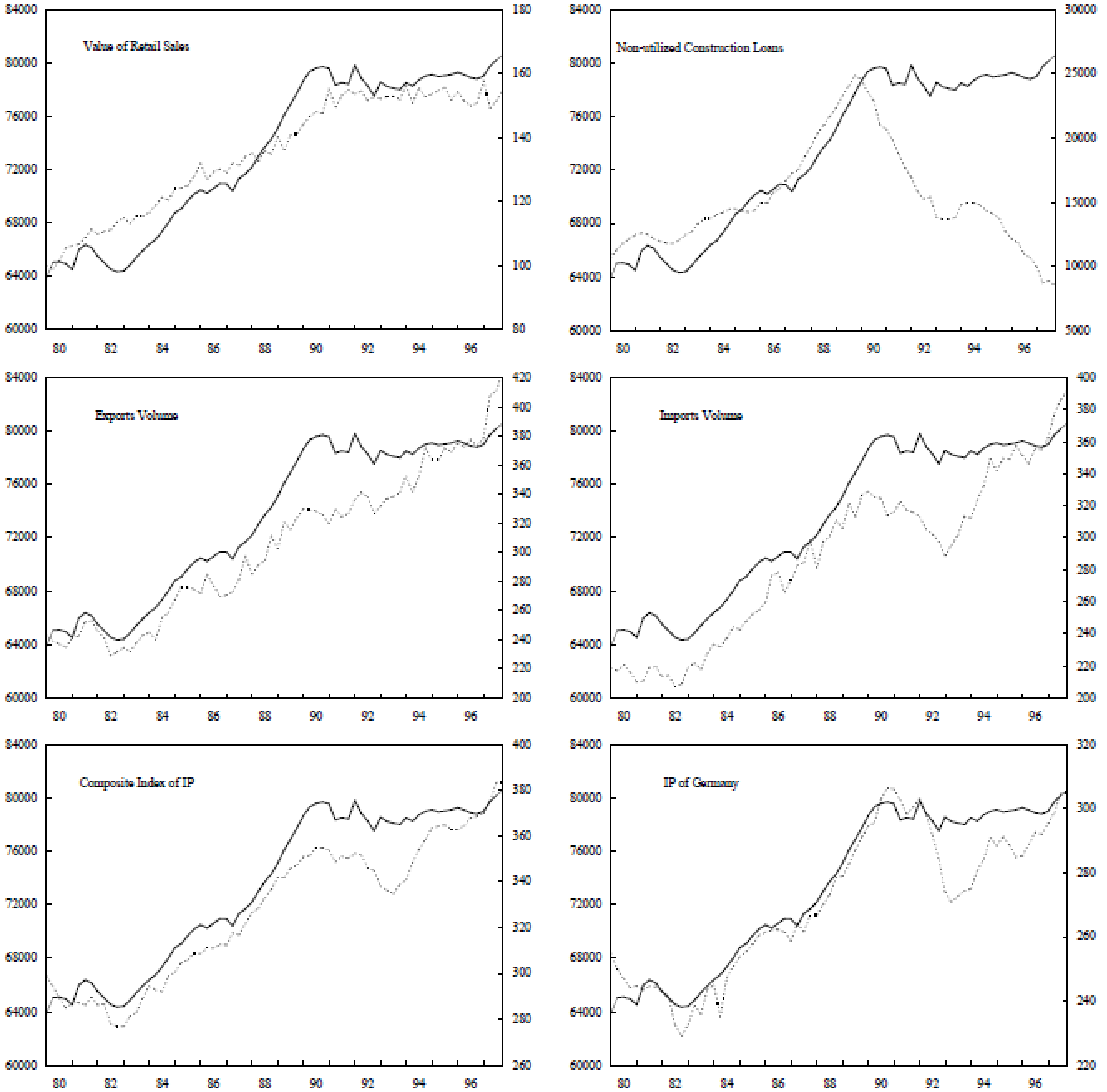
Descriptive Statistics

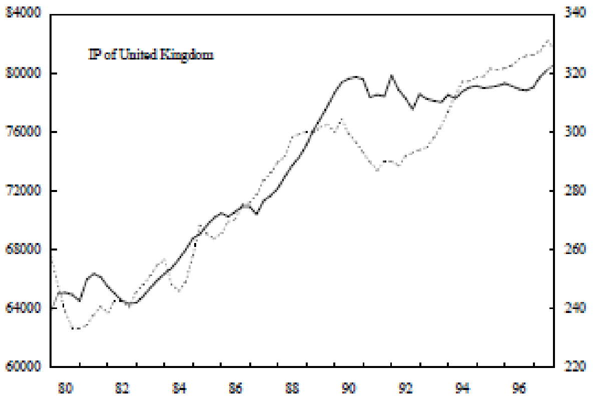
	μ	σ	AR(1)	JB	ADF
gdp	1.33	2.95	0.25	0.05	-4.38*
x^{rs}	3.20	46.29	-0.65*	77.88*	-11.04*
x^{nl}	-0.97	21.39	0.25*	199.18*	-2.98*
x^{X}	4.09	49.03	-0.57*	53.17*	-7.91*
x^{M}	4.29	51.11	-0.61*	85.52*	-8.06*
x^{gip}	1.40	21.73	-0.43*	1370.58*	-6.26*
x^{ukip}	1.31	13.17	-0.22*	6.71*	-5.30*
x^{comip}	1.52	12.16	-0.32*	68.53*	-5.63*

Cross-correlations with gdp

	-3	-2	-1	0	1	2	3
x^{rs}	0.01	0.12	-0.01	0.09	0.13	0.12	0.02
x^{nl}	0.30	0.37	0.30	0.23	0.23	0.14	0.16
x^{X}	0.10	0.15	0.18	0.26	0.25	-0.02	-0.07
x^{M}	0.16	0.17	0.20	0.09	0.26	-0.05	0.03
x^{gip}	0.03	0.12	0.34	0.25	0.35	0.21	0.14
x^{ukip}	0.03	0.09	0.17	0.05	-0.01	-0.09	-0.17
x^{comip}	0.05	0.18	0.41	0.27	0.30	0.19	0.09

Tab. 2. *Data description, where GDP is the Gross Domestic Product, x^{rs} is the value of retail sales to proxy for private consumption, x^{nl} is the value of non-utilized construction loans to proxy for investment, x^{X} is the value of exports, x^{M} is the value of imports, x^{gip} is the composite IP index of the Germany, x^{ukip} is the composite IP index of the UK, x^{comip} is the composite IP index of the Switzerland, μ is the mean, σ is the standard deviation, AR(1) is the first order autoregressive coefficient, JB is the Jarque-Bera test, ADF is the augmented Dickey-Fuller test (after Cuche, Hess (2000)).*





GDP uses solid line in mio CHF; related series use dashed line. Non-utilized construction loans in mio CHF, other related series as index points. Source: Datastream and State Secretariat of Economic Affairs.

Fig. 10.*GDP and Related Series (after Cuche, Hess (2000)).*

Model	1, 1a	2, 1c	3, 2a	4, 2a
Series	-	-	x^{comip}	x^{nl}
AIC	8.05	10.08	14.66	17.31
log L	-562.56	-573.41	-636.59	-733.33
μ	1.32	1.33	1.20	1.25
σ	4.30	5.11	9.93	5.22
AR(1)	0.21*	0.06	-0.37*	-0.02
JB	308.06*	15.59*	12.09**	192.13*
ADF	-5.47*	-5.96*	-5.79*	-5.30*
MSE 1e	3307.38	4146.21	18659.04	7123.27
MSE AQ	196862.53	144095.19	206864.82	208184.95

Model	5, 2a	6, 2b	7, 2b	8, 2b
Series	$x^{rs}, x^{nl}, x^{X}, x^{M}$	x^{comip}	x^{nl}	$x^{rs}, x^{nl}, x^{X}, x^{M}$
AIC	13.06	15.89	17.32	13.34
log L	-575.38	-681.65	-733.97	-585.70
μ	1.27	1.29	1.30	1.41
σ	14.13	6.42	3.49	12.21
AR(1)	-0.53*	-0.14**	0.73*	-0.53*
JB	2.77	17.93*	16.83*	0.52
ADF	-6.40*	-5.53*	-4.20*	-6.85*
MSE 1e	32328.09	8884.46	4245.59	23601.90
MSE AQ	248244.08	129891.30	97166.50	221468.03

Model	9, 2c	10, 2c	11, 2c	12, 2d
Series	x^{comip}	x^{nl}	$x^{rs}, x^{nl}, x^{X}, x^{M}$	x^{comip}
AIC	15.63	14.42	14.38	15.27
log L	-677.97	-627.11	-623.52	-664.89
μ	1.30	1.30	1.34	1.27
σ	3.65	3.51	4.66	6.80
AR(1)	0.63*	0.72*	0.14**	-0.23*
JB	6.58**	15.93*	9.62*	13.45*
ADF	-4.30*	-4.17*	-5.57*	-4.91*
MSE 1e	4394.56	4287.16	5674.41	10947.86
MSE AQ	139871.77	102001.75	79448.95	120193.62

Model	13, 2d	14, 2d	15, 2e	16, 2e
Series	x^{n}	$x^{rs}, x^{nl}, x^{X}, x^{M}$	x^{nl}	$x^{rs}, x^{nl}, x^{X}, x^{M}$
AIC	14.30	14.09	8.05	8.04
log L	-622.86	-614.83	-562.55	-562.55
μ	1.28	1.35	1.28	1.29
σ	4.69	7.96	4.30	4.58
AR(1)	0.10	-0.36*	0.22*	0.11
JB	332.63*	15.08*	306.50*	156.83*
ADF	-4.92*	-5.63*	-5.45*	-5.56*
MSE 1e	6696.91	11778.87	3282.52	3701.94
MSE AQ	151188.26	141821.70	147968.33	163693.19

Tab. 3. *Interpolation results (after Cuche, Hess (2000)).*

The application of the *Stratonovich – Kalman – Bucy filtering algorithm* and related scientific problems in the economics and finances have been researched in *Athans (1974), Fernandez (1981), Geweke, Singleton (1981), Litterman (1983), Meinhold, Singpurwalla (1983), Engle, Watson (1983), Harvey, Pierse (1984), Engle, Lilien, Watson (1985), de Jong (1991), Doran (1992), Tanizaki (1993), Venegas, de Alba, Ordorica (1995), Hodrick, Prescott (1997), Krelle (1997), Cuche, Hess (2000), Durbin, Koopman (2000), Morley, Nelson, Zivot (2002), Bahmani, Brown (2004), Broto, Ruiz (2004), Fernàndez-Villaverde, Primiceri (2005), Fernàndez-Villaverde, Rubio-Ramirez (2005, 2007), Ozbek, Ozale (2005), Proietti (2006), Ochoa (2006), Horváth (2006), Cardamone (2006), Pasricha (2006), Bignasca, Rossi (2007),* Dramani, Laye (2007), Paschke, Prokopczuk (2007), *Roncalli, Weisang (2008), Proietti (2008), Osman, Louis, Balli (2008), Gonzalez-Astudillo (2009), Bationo, Hounkpodote (2009), Mapa, Sandoval, Yap (2009), Chang, Miller, Park (2009), Fernàndez-Villaverde (2010), Theoret, and Racicot (2010), Lai, Te (2011), Jungbacker, Koopman, van der Wel (2011), Proietti, Luati (2012a, b), Darvas, Varga (2012), Hang Qian (2012).*

Stratonovich – Kalman – Bucy filtering algorithm for accurate characterization of financial and economic time-series with use of state-space model in Matlab: Gross Domestic Product

Let us begin with the consideration of the ***steady state filter*** and the ***time varying filter***, which are designed and simulated in *Matlab (R2012).*

Problem description:

Let us use the following discrete plant:

x(n+1) = Ax(n) + Bu(n),

y(n) = Cx(n) + Du(n),

where

```
A = [1.1269   -0.4940    0.1129,
       1.0000         0          0,
            0    1.0000          0];
B = [-0.3832
       0.5919
       0.5191];
C = [1 0 0];
```

D = 0.

Let us design the *Stratonovich – Kalman – Bucy filter* to estimate the output *y* based on the noisy measurements *yv[n] = C x[n] + v[n]*

The steady-state Stratonovich-Kalman-Bucy filter design:

Let us use the function ***KALMAN*** to design a steady-state *Stratonovich – Kalman – Bucy filter*. This function determines the optimal steady-state filter gain *M* based on the process noise covariance *Q* and the sensor noise covariance *R*. First, let us specify the plant + noise model. Let us set the sample time to *-1* to mark the plant as discrete:

Plant = ss(A,[B B],C,0,-1,'inputname',{'u' 'w'},'outputname','y').

Let us specify the process noise covariance (*Q*):

Q = 2.3; % A number greater than zero.

Let us specify the sensor noise covariance (*R*):

R = 1; % A number greater than zero.

Let us now design the steady-state *Stratonovich – Kalman – Bucy filter* with the equations:

Time update:

x[n+1|n] = Ax[n|n-1] + Bu[n],

Measurement update:

x[n|n] = x[n|n-1] + M (yv[n] - Cx[n|n-1]),

where *M = optimal innovation gain,*

using the *KALMAN* command:

[kalmf,L,~,M,Z] = kalman(Plant,Q,R);

The first output of the *Stratonovich – Kalman – Bucy filter KALMF* is the plant output estimate *y_e = Cx[n|n]*, and the remaining outputs are the state estimates. Let us keep only the first output *y_e*:

kalmf = kalmf(1,:);

M,   % innovation gain

```
M =
    0.5345
    0.0101
   -0.4776
```

To see how this filter works, let us generate some data and compare the filtered response with the true plant response in Fig. 11:

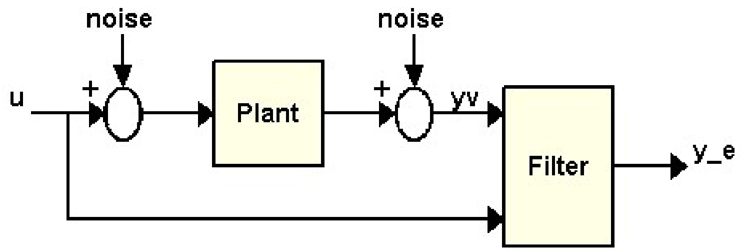


Fig. 11. *Stratonovich – Kalman – Bucy filter scheme (after Matlab (R2012).*

To simulate the system above, let us generate the response of each part separately or generate both together. To simulate each separately, first let us use *LSIM* with the plant and then with the filter. The following example simulates both together.

% First, build a complete plant model with *u,w,v* as inputs and

% *y* and *yv* as outputs:

a = A;

*b = [B B 0*B];*

c = [C;C];

d = [0 0 0;0 0 1];

P = ss(a,b,c,d,-1,'inputname',{'u' 'w' 'v'},'outputname',{'y' 'yv'});

Next, let us connect the plant model and the *Stratonovich – Kalman – Bucy filter* in parallel by specifying *u* as a shared input:

sys = parallel(P,kalmf,1,1,[],[]);

Finally, let us connect the plant output *yv* to the filter input *yv*. Note: *yv* is the 4^{th} input of *SYS* and also its 2^{nd} output:

SimModel = feedback(sys,1,4,2,1);

SimModel = SimModel([1 3],[1 2 3]);   % Delete yv form I/O

The resulting simulation model has *w,v,u* as inputs and *y,y_e* as outputs:

SimModel.inputname

ans =

'w'

'v'

'u'

SimModel.outputname

ans =

'y'

'y_e'

Let us simulate the filter behavior. Let us generate the sinusoidal input vector (known):

```
t = (0:100)';
u = sin(t/5)
```

Let us generate the process noise and sensor noise vectors:

```
rng(10,'twister');
w = sqrt(Q)*randn(length(t),1);
v = sqrt(R)*randn(length(t),1);
```

Let us now simulate the response using *LSIM*:

```
clf;
out = lsim(SimModel,[w,v,u]);

y = out(:,1);    % true response
ye = out(:,2);  % filtered response
yv = y + v;     % measured response
```

Let us compare the true response with the filtered response:

```
clf
subplot(211), plot(t,y,'b',t,ye,'r--'),
xlabel('No. of samples'), ylabel('Output')
title('Kalman filter response')
subplot(212), plot(t,y-yv,'g',t,y-ye,'r--'),
xlabel('No. of samples'), ylabel('Error')
```

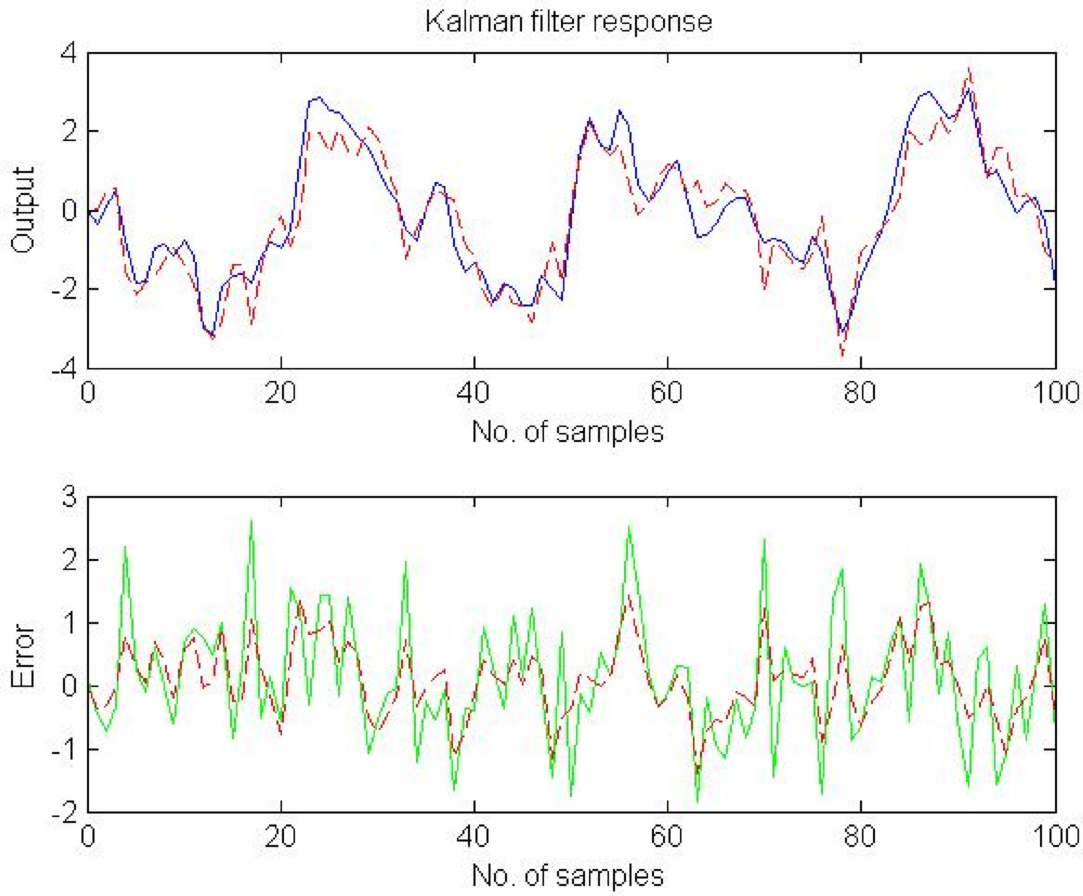


Fig. 12. *Stratonovich – Kalman – Bucy filter response (after Matlab (R2012).*

As shown in the *2nd* plot in Fig. 12, the *Stratonovich – Kalman – Bucy filter* reduces the error *y-yv* due to measurement noise. To confirm this, let us compare the error covariances:

MeasErr = y-yv;

*MeasErrCov = sum(MeasErr.*MeasErr)/length(MeasErr);*

EstErr = y-ye;

*EstErrCov = sum(EstErr.*EstErr)/length(EstErr);*

The covariance of error before filtering (*measurement error*):

MeasErrCov

MeasErrCov =

0.9871

The covariance of error after filtering (*estimation error*):

EstErrCov

EstErrCov =

0.3479

The time-varying Stratonovich-Kalman-Bucy filter design:

Let us design a time-varying *Stratonovich – Kalman – Bucy filter* to perform the same task. A time-varying Kalman filter can perform well even when the noise covariance is not stationary. However, in this demonstration, let us use the stationary covariance.

The time varying *Stratonovich – Kalman – Bucy filter* has the following update equations.

Time update:

$x[n+1|n] = Ax[n|n] + Bu[n];$

$P[n+1|n] = AP[n|n]A' + B*Q*B';$

Measurement update:

$x[n|n] = x[n|n-1] + M[n](yv[n] - Cx[n|n-1])$

$M[n] = P[n|n-1]\, C'\, (CP[n|n-1]C'+R)^{-1}$

$P[n|n] = (I-M[n]C)\, P[n|n-1]$

First, let us generate the noisy plant response:

```
sys = ss(A,B,C,D,-1);
y = lsim(sys,u+w);   % w = process noise
yv = y + v;          % v = measurement noise
```

Next, let us implement the filter recursions in a *FOR* loop:

```
P=B*Q*B';        % Initial error covariance
x=zeros(3,1);    % Initial condition on the state
ye = zeros(length(t),1);
ycov = zeros(length(t),1);
errcov = zeros(length(t),1);

for i=1:length(t)
  % Measurement update
  Mn = P*C'/(C*P*C'+R);
  x = x + Mn*(yv(i)-C*x);  % x[n|n]
  P = (eye(3)-Mn*C)*P;     % P[n|n]

  ye(i) = C*x;
  errcov(i) = C*P*C';
```

% Time update

*x = A*x + B*u(i);   % x[n+1|n]*

*P = A*P*A' + B*Q*B';   % P[n+1|n]*

end

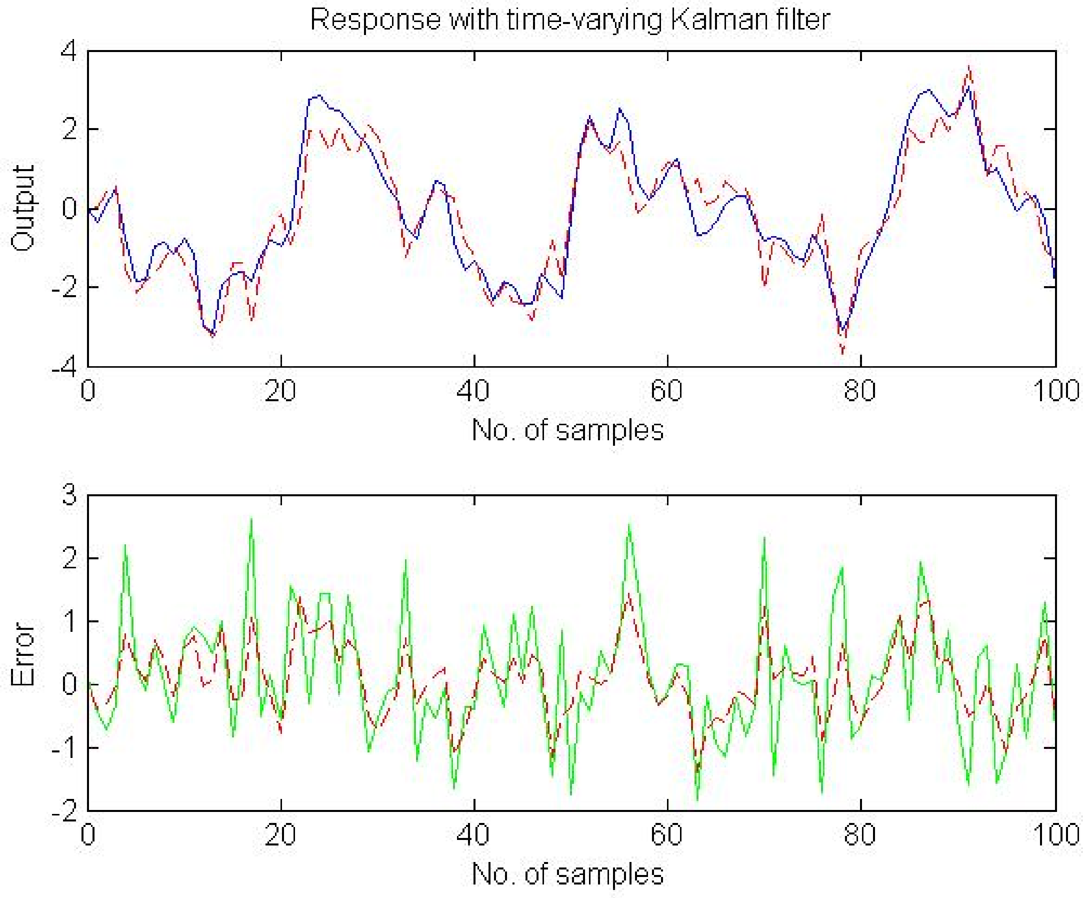


Fig. 13. *Time-varying Stratonovich – Kalman – Bucy filter response (after Matlab (R2012).*

Now, let us compare the true response with the filtered response as shown in Fig. 13:

subplot(211), plot(t,y,'b',t,ye,'r--'),

xlabel('No. of samples'), ylabel('Output')

title('Response with time-varying Kalman filter')

subplot(212), plot(t,y-yv,'g',t,y-ye,'r--'),

xlabel('No. of samples'), ylabel('Error')

The time varying filter also estimates the output covariance during the estimation. Let us plot the output covariance to see, if the filter has reached the steady state (as we would expect with the stationary input noise):

```
subplot(211)
plot(t,errcov), ylabel('Error Covar'),
title('Output covariance estimated by time-varying Kalman filter')
subplot(212), plot(t,y-yv,'g',t,y-ye,'r--'),
xlabel('No. of samples'), ylabel('Error')
```

From the covariance plot in Fig. 13, it can be seen that the output covariance did reach a steady state in about *5* samples. From then on, the time varying filter has the same performance as the steady state version.

Let us compare the covariance errors:

```
MeasErr = y-yv;
MeasErrCov = sum(MeasErr.*MeasErr)/length(MeasErr);
EstErr = y-ye;
EstErrCov = sum(EstErr.*EstErr)/length(EstErr);
```

The covariance of error before the filtering (measurement error):

```
MeasErrCov
MeasErrCov =
   0.9871
```

The covariance of error after the filtering (estimation error):

```
EstErrCov
EstErrCov =
   0.3479
```

Let us verify that the steady-state and final values of the Kalman gain matrices coincide:

```
M,Mn
M =
   0.5345
   0.0101
  -0.4776
Mn =
   0.5345
   0.0101
  -0.4776
```

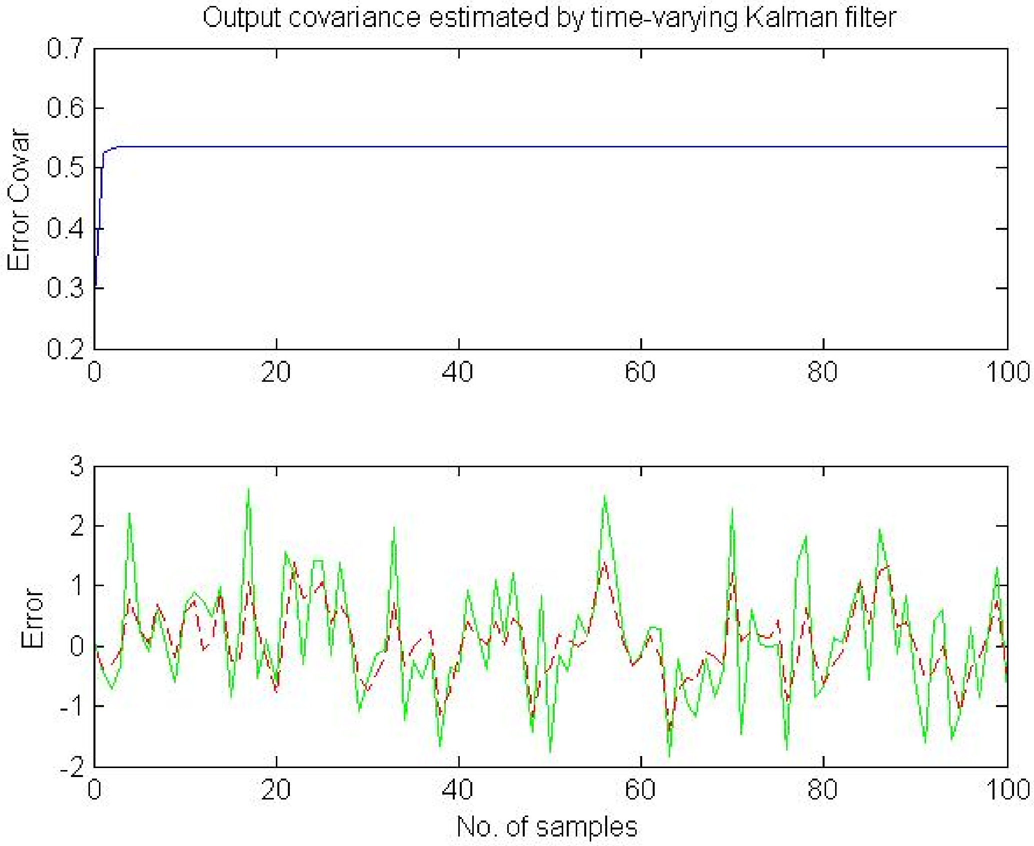


Fig. 14. *Output covariance estimated by time-varying Kalman filter (after Matlab (R2012).*

We developed the original software program and performed the computer modeling, using the *Stratonovich – Kalman – Bucy filtering algorithm* for the accurate characterization of the *Australian GDP*, *German GDP* and the *USA GDP* in the frames of the *state-space model* in *Matlab* in *Ledenyov D O, Ledenyov V O (2013g)*. We also used the *Hodrick-Prescott filter* to estimate the corresponding output gaps in *Australia, Germany* and the *USA* in *Ledenyov D O, Ledenyov V O (2013g)*, because as it is discussed in *Osman, Louis, Balli (2008)*: "The output gap is considered to be an important indicator of the cyclical position of the economy and the identification of changes in the pattern of business cycle evolution. Hence, knowledge of this variable together with other macroeconomic variables play a key role in explaining future economic forecasts particularly in the level of *real GDP, price and wage inflation*." It is necessary to explain that: "The *output gap* is measured by decomposing the actual output (real *GDP*) into structural and conjunctural components using different methodological techniques. The structural component is usually described as the trend component or the "potential output", while the latter is termed as the "output gap" which is the irregular components of the actual output and it includes temporary elements that are shaped by business cycle and other very short-run fluctuations" as explained in *Osman, Louis, Balli (2008).* We found that the *Australia, Germany* and the *USA* have the different business cycles, because the *Australian* and *Germany economies* are in the period of economic growth and the *USA economy* is in the contraction phase, caused by the economic recession.

Conclusion

We explained the fact that the central banks introduce and implement the monetary and financial stabilities policies, going from the accurate estimations of national macro-financial indicators such as the *Gross Domestic Product (GDP).* Analyzing the dependence of the *GDP* on the time, the central banks accurately estimate the missing observations in the financial time series with the application of different interpolation models, based on the various filtering algorithms. We highlighted the fact that that the *Stratonovich – Kalman – Bucy filtering algorithm* was applied in the state space interpolation model with the purpose to interpolate the real *GDP* by the *US Federal Reserve* for the first time. Therefore, we decided to overview the *Stratonovich – Kalman – Bucy filtering algorithm* theory and its numerous applications, focusing on the precise characterization of various time series. We describe the technique of the accurate characterization of the economic and financial time series with the application of state space methods with the use of the *Stratonovich – Kalman - Bucy filtering algorithm,* focusing on the recent data on the estimation of the *Gross Domestic Product* by the *Swiss National Bank.* Applying the integrative thinking principles, we developed the software program and performed the computer modeling, using the *Stratonovich – Kalman – Bucy filtering algorithm* for the accurate characterization of the *Australian GDP, German GDP* and the *USA GDP* in the frames of the *state-space model* in *Matlab* in *Ledenyov D O, Ledenyov V O (2013g).* We also used the *Hodrick-Prescott filter* to estimate the corresponding output gaps in *Australia, Germany* and the *USA* in *Ledenyov D O, Ledenyov V O (2013g).* We found that the *Australia, Germany* on one side and the *USA* on other side have the different business cycles, because the *Australian, Germany economies* are in the period of economic growth and the *USA economy* is in the contraction phase, caused by the economic recession. We believe that the central banks can use our special software program with the aim to greatly improve the national macroeconomic indicators forecast by making the accurate characterization of the financial time-series with the application of the state-space models, based on the *Stratonovich – Kalman – Bucy filtering algorithm.*

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