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Manuel Gonzalez-Astudillo [*]
Federal Reserve Board
Washington, D.C., USA
manuel.p.gonzalez-astudillo@frb.gov

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Abstract

In this paper, we formulate and solve a New Keynesian model with monetary and fiscal policy rules whose coefficients are time-varying and interdependent. We implement time variation in the policy rules by specifying coefficients that are logistic functions of correlated latent factors and propose a solution method that allows for these characteristics. The paper uses Bayesian methods to estimate the policy rules with time-varying coefficients, endogeneity, and stochastic volatility in a limited-information framework. Results show that monetary policy switches regime more frequently than fiscal policy, and that there is a non-negligible degree of interdependence between policies. Policy experiments reveal that contractionary monetary policy lowers inflation in the short run and increases it in the long run. Also, lump-sum taxes affect output and inflation, as the literature on the fiscal theory of the price level suggests, but the effects are attenuated with respect to a pure fiscal regime.

Keywords: Time-varying policy rule coefficients, Monetary and Fiscal policy interactions, Nonlinear state-space models.

JEL Classification Numbers: C11, C32, E63.

1 Introduction

The great recession has raised awareness of the role of monetary-fiscal policy interactions in determining the behavior of economic aggregates. In the United States, for example, reductions in discount and funds rates and even unconventional quantitative easing measures have accompanied fiscal stimulus policies. Moreover, given the high projected fiscal liabilities in developed as well as in some developing countries, interactions are likely to figure more prominently in determining economic outcomes over the coming decades.

The considerations behind monetary-fiscal policy interactions have been part of the monetary and fiscal policymaking literature for a long time. Friedman (1948) proposed a scheme of monetary-fiscal policy interactions to deliver stability in cyclical fluctuations. The seminal work of Sargent and Wallace (1981) provides a formal investigation that illustrates how monetary and fiscal policies interact to determine inflation. The authors show that under fiscal dominance —where a partial monetization of government debt is necessary to avoid default— the ability of the monetary authority to control inflation disappears. Leeper (1991) examined monetary-fiscal policy interactions within the dynamic stochastic general equilibrium (DSGE) framework. The work of Canzoneri, Cumby and Diba (2010) surveys the positive and normative aspects of monetary-fiscal policy interactions in the existing literature.

Conventionally, there have been two approaches to how each of the policy authorities responds to deviations of its variables of interest from target. One approach specifies a central bank that follows a Taylor (1993) rule under which the nominal interest rate increases more than proportionally when inflation increases. Thus, monetary policy provides the nominal anchor to deliver price-level determinacy. In this approach, the fiscal authority follows a rule under which (lump-sum) taxes stabilize debt. This first approach has been referred to as monetary dominance (see Carlstrom and Fuerst, 2000). We will call it a monetary (M) regime.

The other approach postulates that the fiscal authority does not adjust taxes to stabilize debt. In this case, the central bank cannot stabilize the price level. Thus, fiscal policy provides the nominal anchor for price-level determinacy through expectations about future surpluses, given a level of outstanding nominal liabilities (see Leeper, 1991; Sims, 1994; Woodford, 1994, 1995, 1996; Cochrane, 1998, 2005, 2001). This approach has been referred to as the fiscal theory of the price level. We will call it a fiscal (F) regime.

When the two approaches are considered together, there are four possible combinations of monetary and fiscal policy stances that have been referred to as monetary-fiscal policy interactions. The four combinations are: (i) an M regime; (ii) an F regime; (iii) a regime where no authority provides the nominal anchor and the price level is indeterminate; (iv) a regime where both authorities try to provide the nominal anchor and debt is unbounded.

This paper formulates and solves a New Keynesian model that incorporates monetary and fiscal policy rules whose coefficients are time-varying and interdependent. Time variation and interdependence allow for co-movements in monetary and fiscal policymaking, thereby introducing a direct channel of interactions. This channel influences expectations about future monetary and fiscal policymaking, affecting the dynamics of the variables in equilibrium. In particular, we show that when there are co-movements in monetary and fiscal policymaking in the direction of stable and determinate equilibria —the M and F regimes— the volatilities of output and inflation are reduced, compared to the case where

co-movements in that direction are absent.

As Davig and Leeper (2007) emphasize, policymaking is a complicated process of analyzing and interpreting data, receiving advice, and applying judgment. During some periods policymakers may give more attention to inflation or debt stabilization, while in other periods they may give more attention to output stabilization. Furthermore, a substantial empirical literature argues that policy rules have not remained invariant over the past six decades (see Clarida, Gali and Gertler, 2000; Favero and Monacelli, 2003; Lubik and Schorfheide, 2004; Davig and Leeper, 2006; Fernandez-Villaverde, Guerron and Rubio-Ramirez, 2010; Bianchi, 2012). In this paper, policy rule coefficients move across regimes as functions of exogenous latent factors. In particular, the policy rule coefficients are logistic functions of these latent factors. The logistic function allows the policy rule coefficients to move between two regimes —as with a two-state Markov-switching specification— with a smooth transition between regimes. We assume that one latent factor influences the evolution of the coefficients of the monetary policy rule and that a different latent factor influences the evolution of the coefficients of the fiscal policy rule.

We introduce interdependence between monetary and fiscal policy by allowing correlation between the two latent factors driving the evolution of the monetary and the fiscal policy rule coefficients. This correlation can be seen as coordination —explicit or implicit— in policymaking. The correlation between the latent factors allows policy interactions through co-movements in the evolution of the policy rule coefficients.

Given the logistic specification of the policy rule coefficients and the existence of latent factors as additional states, the model is intrinsically nonlinear. We devise a solution method that takes into consideration these nonlinearities. The solution method incorporates agents' expectations on the joint evolution of the policy rule coefficients. This makes the model appropriate to analyze the impacts of monetary and fiscal policies in a framework of interactions.

We employ a limited-information analysis to estimate the policy rule coefficients using Bayesian methods appropriate for nonlinear state-state models. The results indicate that the policy rule coefficients are far from constant. In particular, the monetary policy rule coefficient on inflation switches more frequently than the fiscal policy rule coefficient on debt. There is also a degree of policy interdependence: fiscal policy that focuses on keeping debt under control tends to accompany monetary policy that focuses on keeping inflation under control, and monetary policy that does not increase interest rates actively tends to accompany fiscal policy that does not pay attention to debt increases.

Finally, we calibrate the model to carry out policy analysis. The policy experiments show that contractionary monetary policy can lower inflation, at least in the short-run. The experiments also show that (lump-sum) taxes have effects on output and inflation, as the literature on the fiscal theory of the price level suggests, but the effects are attenuated with respect to a pure fiscal regime.

2   Interactions Between Monetary and Fiscal Policy

In this section, we introduce the concept of interdependence in policymaking that will be central to the analysis of the subsequent sections. We describe the theoretical framework of

monetary-fiscal policy interactions and the empirical findings in the literature with respect to policy interactions. Then, we present how we implement interdependence in policymaking through interdependence of the policy rule coefficients.

The theoretical framework of monetary-fiscal policy interactions in a DSGE model was first introduced by Leeper (1991), who shows how different combinations of the magnitudes of monetary and fiscal policy rule coefficients lead to different equilibrium outcomes and local dynamics. Leeper coins the terms "active" and "passive" monetary and fiscal policies to describe how the central bank adjusts interest rates with respect to inflation deviations from target, and how fiscal policy adjusts taxes to changes in public debt. A change more than proportional in nominal interest rates with respect to inflation deviations from target is called "active" monetary policy, while a Ricardian view of fiscal policy, where taxes adjust enough to cover interest payments and to retire debt, is called "passive" fiscal policy. The alternative scenarios with respect to monetary and fiscal policies are called "passive" monetary policy and "active" fiscal policy.[1] Leeper finds that the model delivers a bounded unique rational expectations equilibrium as long as monetary policy is active and fiscal policy is passive —regime M— or if monetary policy is passive and fiscal policy is active —regime F. Also, the model delivers indeterminacy if both monetary policy and fiscal policy are passive, and no bounded solution if both are active. Other papers along this line are Sims (1994) and Leith and Wren-Lewis (2000).

Davig and Leeper (2006) estimate Markov-switching models of monetary and fiscal policy rules with U.S. data. Their results show that there have been numerous switches in monetary and fiscal policy rule coefficients. In particular, whenever the interest rate rule pays more (less) attention to inflation deviations, less (more) weight is given to output deviations. Also, when the tax rule pays more (less) attention to debt deviations, more (less) weight is given to output deviations —in line with an automatic stabilizers argument. These switches deliver the four regimes of policy interactions described above.

Bianchi (2012) conducts a full-information estimation of a Markov-switching DSGE model with policy rule coefficients that switch among three states. The results show that an M regime was in place starting in the 1990s, that an F regime was in place during the 1970s and that a no-bounded-solution regime was in place during the 1980s.

We explicitly introduce co-movements between the coefficients of the monetary and the fiscal policy rules. This setup generalizes the Markov-switching setup in Davig and Leeper (2006) and Bianchi (2012) by introducing interdependence in the evolution of the policy rule coefficients. With this addition, the model allows for a direct channel of interactions in policymaking, so that agents form expectations accordingly, not only in terms of the individual future evolution of policy rule coefficients, but as a framework of joint future policymaking. Below we specify the policy rules, the time variability characteristic of their coefficients, and the way in which we introduce policymaking interdependence.

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[1]Any positive response of taxes to debt constitutes a passive fiscal policy. If one only wants to consider equilibria with bounded real debt, then real taxes have to respond to real debt deviations with increases higher than the real interest rate.
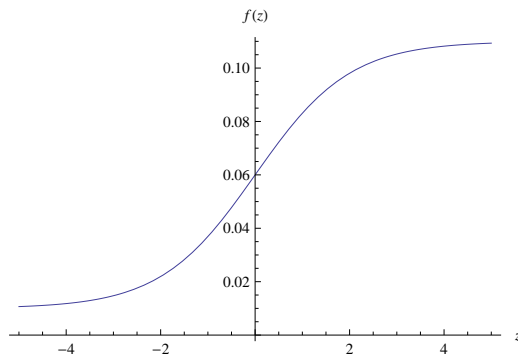
2.1 Policy Rules

We specify policy rules with coefficients that are time varying. The time-varying coefficients of a particular policy rule are logistic functions of a latent state. More specifically, if ϱ_t is a time-varying coefficient of a policy rule, it has the following functional form:

$$
\begin{aligned}
\varrho_t &\equiv \varrho(z_t) \\
&= \varrho_0 + \frac{\varrho_1}{1 + \exp\left(-\varrho_2(z_t - \varrho_3)\right)},
\end{aligned}
$$

where $z_t = \rho_z z_{t-1} + \xi_t$ is a latent factor, $0 \leq \rho_z \leq 1$ and $\xi_t \sim \text{iid}\mathbb{N}(0,1)$.

Under this specification, ϱ_0 denotes the lower (upper) bound of ϱ_t, while $\varrho_0 + \varrho_1$ denotes its upper (lower) bound (if $\varrho_1 < 0$). $\varrho_2 > 0$ is a transition coefficient affecting the speed of the transition between the lower and the upper bounds, and ϱ_3 is a location parameter determining the value of z_t at which ϱ_t crosses the y-axis. A graph for $\varrho(z_t)$ with $\varrho_0 = 0.01$, $\varrho_1 = 0.1$, $\varrho_2 = 1$, and $\varrho_3 = 0$ is reproduced in Figure 1.

Figure 1: Logistic Function

Two approaches have been proposed in the empirical literature to model time-varying policy rule coefficients. One is a Markov-switching specification with finite number of states (see Davig and Leeper, 2006; Eo, 2009; Davig and Doh, 2009; Bianchi, 2012). The second is a random-coefficient specification (see Kim and Nelson, 2006a; Boivin, 2006; Fernandez-Villaverde et al., 2010). Yksel et al. (2013) offers a survey of the literature on estimation of Taylor rules with time-varying coefficients. An advantage of the Markov-switching specification is that it implies bounded coefficients, which could be important in terms of determinacy and relevance of the equilibrium. With respect to the random-coefficient specification, an advantage is that it implies smooth transitions between states. The logistic specification proposed in this paper generalizes the two approaches: On one hand it allows a policy rule coefficient to switch smoothly from one regime to another, while on the other it allows for a bounded evolution of the coefficient.[2]

The latent factors can be seen as representing a combination of political and institutional determinants of policymaking. We assume that the latent factors are exogenous with respect to the variables in the model, and that the information that they provide is part of the

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[2]The Markov-switching specification is a particular case of the logistic specification, when $\varrho_2 \to \infty$.

information set of the agents. That is, agents and policymakers share the same information set. This assumption is needed to solve the model, as will be seen.

2.1.1 Monetary Policy Rule

Monetary policy takes place by means of an interest rate feedback rule of the form

$$R_t = R_{t-1}^{\rho_R} \bar{R}_t^{(1-\rho_R)} \exp\left(\varepsilon_t^R\right),$$

where $\rho_R \in (0,1)$ indicates the degree of interest rate smoothing, and $\varepsilon_t^R \sim \text{iidN}(0, \sigma_R^2)$ is a monetary policy shock. $\bar{R}_t$ is the target short-term nominal interest rate. The central bank sets the interest rate to react to deviations of inflation from target and to the output gap according to

$$\bar{R}_t = R \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\alpha^\pi(z_t^m)} \left(\frac{Y_t}{Y_t^*}\right)^{\alpha^y(z_t^m)},$$

where $\Pi_t = P_t/P_{t-1}$ is the gross inflation rate, Y_t is output, Y_t^* is output in the absence of price rigidities, and R is the steady state nominal interest rate, which is guaranteed to be state independent if the target inflation rate, $\bar{\Pi}$, is set equal to Π, the steady state inflation.[3]

The time varying monetary policy rule coefficients are denoted by $\alpha^\pi(z_t^m)$ for inflation deviations from steady state, and $\alpha^y(z_t^m)$ for the output gap. Both are logistic functions of the monetary policy latent factor, z_t^m.

2.1.2 Fiscal Policy Rule

The fiscal rule is a feedback rule for the ratio of lump-sum taxes net of transfers to output, $\tau_t = T_t/Y_t$, of the form

$$\tau_t = \tau_{t-1}^{\rho_\tau} \bar{\tau}_t^{(1-\rho_\tau)} \exp\left(\varepsilon_t^\tau\right),$$

where $\rho_\tau \in (0,1)$ indicates the degree of tax rate smoothing, and $\varepsilon_t^\tau \sim \text{iidN}(0, \sigma_\tau^2)$ is a fiscal policy shock. $\bar{\tau}_t$ is the target level of the ratio of taxes net of transfers to output. The fiscal authority sets lump-sum taxes to respond to debt deviations and the output gap according to

$$\bar{\tau}_t = \tau \left(\frac{b_{t-1}}{\bar{b}}\right)^{\gamma^b(z_t^f)} \left(\frac{Y_t}{Y_t^*}\right)^{\gamma^y(z_t^f)},$$

where $b_t = B_t/(P_t Y_t)$ denotes the debt-to-output ratio in period t, and $\bar{b}$ is its target level. τ denotes the steady state level of τ_t, which is guaranteed to be state independent in the steady state equilibrium if $\bar{b}$ is set equal to its steady state value, denoted by b.

The time-varying fiscal policy rule coefficients are $\gamma^b(z_t^f)$ for debt deviations from steady state and $\gamma^y(z_t^f)$ for the output gap. All are logistic functions of the fiscal policy latent factor, z_t^f.

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[3]The target inflation rate is constant to allow the linearization of the model around the steady state conditional on a realization of the latent factor at each period.

2.2 Interdependence in Policymaking

To incorporate direct interactions between policies, we specify the latent factors driving the evolution of the monetary and fiscal policy rule coefficients as follows:

$$z_t^m = \rho_{z^m} z_{t-1}^m + \xi_t^m, \tag{1}$$

$$z_t^f = \rho_{z^f} z_{t-1}^\tau + \xi_t^f, \tag{2}$$

where ξ_t^m and ξ_t^f are normally distributed with zero mean, unit variance and $\mathrm{corr}(\xi_t^m, \xi_t^f) = \kappa$. Notice that under this specification, if κ is different from zero, there exist explicit interactions or interdependent changes between monetary and fiscal policymaking.

In this context, policy authorities react more or less strongly to deviations of their feedback variables from target —inflation and the output gap for the monetary authority, and debt and the output gap for the fiscal authority— depending on the values of the policy rule coefficients as driven by the evolution of the latent factors, z_t^m and z_t^f. These latent factors, ultimately, are going to determine the nature of the possible combinations between monetary and fiscal policymaking. The full specification of the monetary and fiscal policy rule coefficients is as follows:

$$\alpha^\pi(z_t^m) = \alpha_0^\pi + \frac{\alpha_1^\pi}{1 + \exp(-\alpha_2^\pi(z_t^m - \alpha_3^\pi))}, \tag{3}$$

$$\alpha^y(z_t^m) = \alpha_0^y + \frac{\alpha_1^y}{1 + \exp(-\alpha_2^y(z_t^m - \alpha_3^y))}, \tag{4}$$

$$\gamma^b(z_t^f) = \gamma_0^b + \frac{\gamma_1^b}{1 + \exp(-\gamma_2^b(z_t^f - \gamma_3^b))}, \tag{5}$$

$$\gamma^y(z_t^f) = \gamma_0^y + \frac{\gamma_1^y}{1 + \exp(-\gamma_2^y(z_t^f - \gamma_3^y))}. \tag{6}$$

A constant-coefficient version of this model delivers an M Regime when α^π and γ^b are sufficiently high —that is, when the monetary authority reacts strongly to inflation deviations and the fiscal authority reacts strongly to debt deviations. The model delivers an F regime when α^π and γ^b are sufficiently low —that is, when the fiscal authority reacts weakly to debt deviations and the monetary authority reacts weakly to inflation deviations. With time-varying coefficients, we can choose values of the parameters defining each of the policy rule coefficients (3)-(6) in a way that the model delivers determinate equilibria in the long-run with short-run deviations from these equilibria.

The interactions with long-run determinacy of the equilibria are only well defined if $\kappa \geq 0$. In that case, at a given point in time, a high value of $\alpha^\pi(z_t^m)$ —given by a high value of z_t^m— is likely to be associated with a high value of $\gamma^b(z_t^f)$ —given by a high value of z_t^f. Also, at a given point in time, a low value of $\alpha^\pi(z_t^m)$ —given by a low value of z_t^m— is likely to be associated with a low value of $\gamma^b(z_t^f)$ —given by a low value of z_t^f. Then, $\kappa > 0$ tends to send the economy towards the M and F regimes. $\kappa = 0$ is the case present in the existing literature, where interactions depend on the values of the policy rule coefficients.

3   The Model

The economy is populated by a representative household, a final-goods-producing firm, a continuum of intermediate-goods-producing firms, a monetary authority and a fiscal authority. The model extends the setup in An and Schorfheide (2007) to incorporate a fiscal policy rule and time-varying policy rule coefficients. Appendix A details the derivations.

3.1   Households

The representative household derives utility from consumption, C_t,[4] relative to a habit stock, A_t, that is given by the level of technology of the economy, and real money balances, M_t/P_t; and derives disutility from working hours, H_t. Hence, a representative household chooses consumption, real balances, bond holdings and working hours to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t/A_t)^{1-\sigma}}{1-\sigma} + \chi_M \log{(M_t/P_t)} - \chi_H \frac{H_t^{1+\varphi}}{1+\varphi} \right),$$

where $0 < \beta < 1$ is the discount factor, $\sigma > 0$ is the inverse of the elasticity of intertemporal substitution, $\varphi > 0$ is the inverse of the Frisch elasticity of labor supply, and $\chi_M > 0$ and $\chi_H > 0$ are constants that determine the steady state level of real money balances and hours worked. The household saves in the form of nominal government bonds, B_t, that pay a gross interest rate R_t each period, and by accumulating money balances that do not pay interests. It supplies labor services to the firms taking the nominal wage, W_t, as given; it also receives its aggregate share on the firms' nominal profits, D_t, and pays lump-sum taxes, T_t. Thus, the household's budget constraint is expressed as

$$P_t C_t + M_t + B_t + P_t T_t \le H_t W_t + D_t + M_{t-1} + R_{t-1} B_{t-1} \text{ for } t \ge 0,$$

given the initial value of nominal assets $M_{-1} + R_{-1} B_{-1}$, and where the transversality condition that rules out Ponzi schemes holds.

3.2   Firms

There are two types of producers: perfectly competitive final goods producers and a continuum of monopolistic intermediate goods producers.

3.2.1   Final Goods Producers

Given the composite good price, P_t, and intermediate goods prices, $P_t(j)$, for $j \in [0,1]$, producers assemble the intermediate goods, $Y_t(j)$, to obtain a composite final good, Y_t,

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[4]C_t is a composite consumption good given by $C_t = \left(\int_0^1 C_t(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}$ and $\theta \ge 1$. The household chooses $C_t(j)$ to minimize expenditure on the continuum of goods indexed by $j \in [0,1]$ which yields j's good demand as $C_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\theta} C_t$, where P_t is the final good price at t and $P_t(j)$ is the price of the consumption good indexed by j.

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according to a CES technology, so that

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\theta_t - 1}{\theta_t}} dj \right)^{\frac{\theta_t}{\theta_t - 1}}, \tag{7}$$

where $\theta_t \in [0, 1]$ is the (time varying) price elasticity of demand for each intermediate good. Here θ_t represents a markup, or cost-push, shock in the Phillips curve relationship. This cost-push shock follows the autoregresive process

$$\log \theta_t = (1 - \rho_\theta) \log \theta + \rho_\theta \log \theta_{t-1} + \varepsilon_t^\theta,$$

with $\rho_\theta \in (0, 1)$, $\theta > 1$ and $\varepsilon_t^\theta \sim$ iid $\mathbb{N}(0, \sigma_\theta^2)$.

Final good producers choose the demand of intermediate goods, $Y_t(j)$, to maximize profits given by

$$P_t Y_t - \int_0^1 P_t(j) Y_t(j) dj.$$

Optimization yields the demand function of intermediate good j,

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\theta_t} Y_t. \tag{8}$$

Combining (8) and (7) yields the expression of the final good price

$$P_t = \left(\int_0^1 P_t(j)^{(1-\theta_t)} dj \right)^{\frac{1}{1-\theta_t}}.$$

3.2.2 Intermediate Goods Producers

Intermediate goods firms produce type j good according to the linear technology

$$Y_t(j) = A_t L_t(j), \tag{9}$$

where $L_t(j)$ are hours of work employed by the producer of intermediate good j, and A_t is an exogenous technology shock identical across producers following the stochastic process

$$A_t = \delta A_{t-1} \exp(\nu_t),$$

where δ is a trend, and ν_t is a stochastic component following the process

$$\nu_t = \rho_\nu \nu_{t-1} + \varepsilon_t^\nu,$$

with $\rho_\nu \in (0, 1)$ and $\varepsilon_t^\nu \sim$ iid $\mathbb{N}(0, \sigma_\nu^2)$.

Intermediate good producers face an explicit cost of adjusting their price, measured in

units of the finished good, and given by

$$\frac{\phi}{2} \left(\frac{P_t(j)}{\Pi P_{t-1}(j)} - 1 \right)^2 Y_t,$$

where $\phi \geq 0$ measures the magnitude of the price adjustment cost, and Π is the steady state gross inflation rate associated with the final good.

Producers in the intermediate goods sector take wages as given and behave as monopolistic competitors in their goods market, choosing the price for their product taking the demand in (8) as given. Hence, firm j chooses its labor input, $L_t(j)$, and its price, $P_t(j)$, to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \mathrm{MRS}_{0,t} \left[\frac{P_t(j)}{P_t} Y_t(j) - \frac{W_t}{P_t} L_t(j) - \frac{\phi}{2} \left(\frac{P_t(j)}{\Pi P_{t-1}(j)} - 1 \right)^2 Y_t \right], \tag{10}$$

where $\mathrm{MRS}_{0,t}$ is the household's marginal rate of substitution between periods 0 and t, which is given exogenously to the firm. Notice that firm j's nominal labor cost is given by $W_t Y_t(j)/A_t$, and its real marginal labor cost is given by $\psi_t = W_t/P_t A_t$, which is the same across firms in the intermediate goods sector.

3.3 Government

The government finances purchases of goods, G_t, with a combination of lump-sum taxes, T_t, and money creation, $M_t - M_{t-1}$, so that the implied process for nominal debt, B_t, satisfies the budget constraint:

$$B_t + M_t + P_t T_t = P_t G_t + M_{t-1} + R_{t-1} B_{t-1} \text{ for } t \geq 0, \tag{11}$$

given $M_{-1} + R_{-1}B_{-1}$. Each period, the government demand of the final good is given by

$$G_t = \zeta_t Y_t,$$

where $\zeta_t \in (0,1)$ is an exogenous process defined by the transformation $g_t = 1/(1 - \zeta_t)$ with

$$\ln g_t = (1 - \rho_g) \ln g + \rho_g \ln g_{t-1} + \varepsilon_t^g,$$

where $\rho_g \in (0,1)$, $g = 1/(1 - \zeta)$ with ζ being the steady state ratio of government spending to output, and $\varepsilon_t^g \sim \mathrm{iid}\ \mathbb{N}(0, \sigma_g^2)$.

3.4 Symmetric Equilibrium

In a symmetric equilibrium, all the intermediate-goods-producing firms make identical decisions, the money supply equals money demand, labor supply equals labor demand, and the net supply of government bonds is zero. Hence, the equilibrium conditions for $t \geq 0$ are

given by

$$Y_t = C_t + G_t + \frac{\phi}{2}\left(\frac{\Pi_t}{\Pi} - 1\right)^2 Y_t, \tag{12}$$

$$0 = 1 - \theta_t + \theta_t\psi_t - \phi\left(\frac{\Pi_t}{\Pi} - 1\right)\frac{\Pi_t}{\Pi} +$$
$$+ \beta\phi\mathbb{E}_t\left(\frac{C_t/A_t}{C_{t+1}/A_{t+1}}\right)^\sigma \frac{Y_{t+1}/A_{t+1}}{Y_t/A_t}\left(\frac{\Pi_{t+1}}{\Pi} - 1\right)\frac{\Pi_{t+1}}{\Pi} \tag{13}$$

$$1 = \beta R_t\mathbb{E}_t\left(\frac{C_t/A_t}{C_{t+1}/A_{t+1}}\right)^\sigma \frac{A_t}{A_{t+1}}\frac{1}{\Pi_{t+1}}, \tag{14}$$

$$\frac{W_t}{P_t} = \chi_H L_t^\varphi A_t\left(\frac{C_t}{A_t}\right)^\sigma, \tag{15}$$

$$\frac{M_t}{P_t} = \chi_M A_t\left(\frac{C_t}{A_t}\right)^\sigma\left(\frac{R_t}{R_t - 1}\right), \tag{16}$$

with $B_{-1}, R_{-1} > 0, A_{-1} > 0$ and $M_{-1} > 0$ given. The symmetric equilibrium is complemented with the monetary and fiscal policy rules, and the exogenous processes for G_t, θ_t, and A_t.

3.5 Frictionless Equilibrium

The frictionless equilibrium is given by the above equilibrium with no frictions ($\phi = 0$). Aggregate output in the frictionless equilibrium is given by

$$Y_t^* = A_t\left(\frac{\frac{\theta_t - 1}{\theta_t}}{\chi_H}\right)^{1/(\sigma+\varphi)} g_t^{\sigma/(\sigma+\varphi)}. \tag{17}$$

The above is the potential output over which the output gap in the monetary and fiscal policy rules is defined.

3.6 Steady-State Equilibrium

Since technology, A_t, is a non-stationary process, it introduces a stochastic trend in output, consumption, real money balances, and the real wage. We define the stationary variables as: $y_t = Y_t/A_t, c_t = C_t/A_t, w_t = W_t/(A_tP_t)$ and $v_t = Y_t/(M_t/P_t)$. The steady-state equilibrium is the stationary equilibrium in the absence of shocks, and is defined by the following equations:

$$\Pi = \frac{R\beta}{\delta}, \tag{18}$$

$$y = \left(\frac{\frac{\theta - 1}{\theta}}{\chi_H}\right)^{1/(\sigma+\varphi)} g^{\sigma/(\sigma+\varphi)} = y^*, \tag{19}$$

$$c = \left(\frac{\frac{\theta-1}{\theta}}{\chi_H} \right)^{1/(\sigma+\varphi)} g^{\varphi/(\sigma+\varphi)}, \tag{20}$$

$$\frac{1}{v} = \chi_M y^{(\sigma-1)} g^{-\sigma} \left(\frac{R}{R-1} \right), \tag{21}$$

$$b = \left(\frac{\beta}{\beta-1} \right) \left(1 - \frac{1}{g} - \tau - \frac{1}{v} \left(1 - \frac{1}{\Pi} \frac{1}{\delta} \right) \right), \tag{22}$$

where b is the steady state level of the debt-to-output ratio, $B_t/(P_t Y_t)$.

3.7 Log-linearized Model and Solution Method

Given that the coefficients of the policy rules are time varying, and the time variation depends on the latent factors, the log-linearization is performed conditioning on the latent factors being at their current values each period. That is the essence of the quasi-linearity of the model. We present the model in log-deviations from the non-stochastic steady state, and show a way to solve it using a method in line with the minimum state variable (MSV) solution approach (McCallum, 1983).

Conditioning on a value of the latent factors at period t, z_t^m and z_t^f, the log-linearized equations characterizing the economy in equilibrium are ($\hat{x}_t = \ln(x_t/x)$ denotes the log-deviation of variable x_t relative to its non-stochastic steady state, x):

$$\hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - \frac{1}{\sigma}(\hat{R}_t - \mathbb{E}_t \hat{\Pi}_{t+1}) + (1 - \rho_g)\hat{g}_t + \frac{\rho_\nu}{\sigma}\hat{\nu}_t \tag{23}$$

$$\hat{\Pi}_t = \beta \mathbb{E}_t \hat{\Pi}_{t+1} + \frac{(\theta-1)(\varphi+\sigma)}{\phi}(\hat{y}_t - \hat{y}_t^*) \tag{24}$$

$$\hat{v}_t = (1 - \sigma)\hat{y}_t + \sigma\hat{g}_t + \left(\frac{1}{R-1} \right)\hat{R}_t \tag{25}$$

$$\hat{b}_t = \frac{1}{bg}\hat{g}_t - \frac{\tau}{b}\hat{\tau}_t + \frac{1}{bv}\hat{v}_t - \frac{1}{bv\Pi\delta}\hat{v}_{t-1} - \left(\frac{1}{bv\Pi\delta} + \frac{1}{\beta} \right)(\hat{\Pi}_t + \Delta\hat{Y}_t) + \frac{1}{\beta}(\hat{R}_{t-1} + \hat{b}_{t-1}) \tag{26}$$

$$\hat{y}_t^* = \frac{1}{(\varphi+\sigma)(\theta-1)}\hat{\theta}_t + \frac{\sigma}{\varphi+\sigma}\hat{g}_t \tag{27}$$

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R)\left(\alpha^\pi(z_t^m)\hat{\Pi}_t + \alpha^y(z_t^m)(\hat{y}_t - \hat{y}_t^*) \right) + \varepsilon_t^R \tag{28}$$

$$\hat{\tau}_t = \rho_\tau \hat{\tau}_{t-1} + (1 - \rho_\tau)\left(\gamma^b(z_t^f)\hat{b}_{t-1} + \gamma^y(z_t^f)(\hat{y}_t - \hat{y}_t^*) \right) + \varepsilon_t^\tau, \tag{29}$$

where $\Delta\hat{Y}_t = \hat{y}_t - \hat{y}_{t-1} + \hat{\nu}_t$. The exogenous shocks that complete the equilibrium are the government spending shock, the cost-push shock, and the technology shock, given by

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \varepsilon_t^g \tag{30}$$

$$\hat{\theta}_t = \rho_\theta \hat{\theta}_{t-1} + \varepsilon_t^\theta \tag{31}$$

$$\hat{\nu}_t = \rho_\nu \hat{\nu}_{t-1} + \varepsilon_t^\nu. \tag{32}$$

To solve the model, let $\boldsymbol{\omega}_t = [\hat{y}_t, \hat{\pi}_t]'$, $\mathbf{k}_t = [\hat{v}_t, \hat{b}_t, \hat{R}_t, \hat{\tau}_t, \Delta\hat{Y}_t, y_{t-1}, \hat{y}_t^*]'$, $\mathbf{u}_t = [\hat{g}_t, \hat{\theta}_t, \hat{\nu}_t, \varepsilon_t^R, \varepsilon_t^\tau]'$,

$\boldsymbol{\varepsilon}_t = [\varepsilon_t^g, \varepsilon_t^\theta, \varepsilon_t^\nu, \varepsilon_t^R, \varepsilon_t^\tau]'$, $\mathbf{z}_t = [z_t^m, z_t^f]'$ and rewrite (23)-(32) as

$$0 = \mathbf{A}(\mathbf{z}_t)\mathbf{k}_t + \mathbf{B}(z_t^f)\mathbf{k}_{t-1} + \mathbf{C}(\mathbf{z}_t)\boldsymbol{\omega}_t + \mathbf{D}\mathbf{u}_t \tag{33}$$

$$0 = \mathbf{G}\mathbf{k}_t + \mathbf{J}\mathbb{E}_t\boldsymbol{\omega}_{t+1} + \mathbf{K}\boldsymbol{\omega}_t + \mathbf{M}\mathbf{u}_t \tag{34}$$

$$\mathbf{u}_{t+1} = \mathbf{N}\mathbf{u}_t + \boldsymbol{\varepsilon}_{t+1}, \tag{35}$$

where $\mathbf{A}, \mathbf{B}(z_t^f), \mathbf{C}(\mathbf{z}_t), \mathbf{D}, \mathbf{G}, \mathbf{J}, \mathbf{K}, \mathbf{M}$ and $\mathbf{N}$ are appropriate coefficient matrices shown in Appendix B.

The proposed solution is given by

$$\mathbf{k}_t = \mathbf{P}(\mathbf{z}_t)\mathbf{k}_{t-1} + \mathbf{Q}(\mathbf{z}_t)\mathbf{u}_t, \tag{36}$$

$$\boldsymbol{\omega}_t = \mathbf{R}(\mathbf{z}_t)\mathbf{k}_{t-1} + \mathbf{S}(\mathbf{z}_t)\mathbf{u}_t, \tag{37}$$

where, for $\mathbf{F}(\mathbf{z}_t) = \{\mathbf{P}(\mathbf{z}_t), \mathbf{Q}(\mathbf{z}_t), \mathbf{R}(\mathbf{z}_t), \mathbf{S}(\mathbf{z}_t)\}$, the $i, j - th$ entry is given by

$$F(\mathbf{z}_t) = F_0 + F_1 \frac{\frac{1}{1+\exp\left(-F_{2m}(z_t^m - F_{3m})\right)} \frac{1}{1+\exp\left(-F_{2f}\left(z_t^f - F_{3f}\right)\right)}}{1 - F_4 \frac{\exp\left(-F_{2m}(z_t^m - F_{3m})\right)}{1+\exp\left(-F_{2m}(z_t^m - F_{3m})\right)} \frac{\exp\left(-F_{2f}\left(z_t^f - F_{3f}\right)\right)}{1+\exp\left(-F_{2f}\left(z_t^f - F_{3f}\right)\right)}}, \tag{38}$$

with $F_4 \in [0, 1]$. This functional form is known as a bivariate logistic function and was introduced by Ali et al. (1978).[5] The parameter F_4 allows the model to capture the effect that the dependence between latent factors has on the expectations formations of the agents about the future evolution of the coefficients of the solution. Appendix C illustrates the procedure to obtain the parameters of the bivariate logistic functions based on the coefficients of the structural model.

We solve the model using an undetermined coefficients method approach where not only the solution has to be guessed and verified, but also the functional form of the coefficients of the solution. Within the logistic specification of policy rules, the bivariate logistic function (38) satisfies this last requirement. Appendix E shows that the coefficients of the solution indeed follow a bivariate logistic function.

3.8   On Existence, Stability and Uniqueness of the Solution

Existence of the solution is guaranteed by using the undetermined coefficients method. Time-varying coefficients pose a difficulty at guaranteeing stability and/or uniqueness of the solution, in particular if one thinks of stability and/or uniqueness holding in each period of time. The method presented finds a solution that is based on the values of the time-varying policy rule coefficients at their limits, or long-run bounds. These limiting coefficient values are chosen to deliver stability and uniqueness of the solution in a constant-coefficient version of the model, offering well-defined bounds between which the economy evolves and between which agents form expectations. Davig and Leeper (2007) and Farmer et al. (2008) emphasize that stability and uniqueness of Markov-switching rational expectations models

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[5]For identification of the latent factors and the coefficients of the policy rules in a estimation setting it is necessary to impose that $F_{2m} > 0$ and $F_{2f} > 0$.

have to be discussed within a framework of how agents form expectations about the future evolution of policy rule coefficients, and that is the approach taken at deriving the solution of the model in this paper. Here, we take expectations to the realization of future policy rule coefficients as given by realization of logistic functions of the latent factors.[6]

4 Estimation Strategy

This section presents the estimation of the policy rules with time-varying coefficients driven by latent factors as specified for the New Keynesian model presented in Section 3. The estimation employs a limited-information estimation approach using Bayesian methods, which allows obtaining the set of parameters characterizing the policy rules, denoted by Θ_y, the set of parameters of the latent factors, denoted by Θ_z, and the latent factors themselves.

4.1 Time-varying Coefficients, Stochastic Volatility and Endogeneity

Let INT_t denote the demeaned nominal federal funds rate in period t, TAX_t the demeaned ratio of federal receipts net of transfers to output in period t, INF_t the demeaned annual inflation rate in period t, GAP_t the output gap in period t and DBT_t the demeaned average debt to output ratio over the last four quarters. The state-space model is composed of the observation equations

$$\text{INT}_t = \rho_R \text{INT}_{t-1} + (1 - \rho_R)\left(\alpha^\pi(z_t^m)\text{INF}_t + \alpha^y(z_t^m)\text{GAP}_t\right) + \upsilon_t^R \tag{39}$$

$$\text{TAX}_t = \rho_\tau \text{TAX}_{t-1} + (1 - \rho_\tau)\left(\gamma^b(z_t^f)\text{DBT}_t + \gamma^y(z_t^f)\text{GAP}_t\right) + \upsilon_t^\tau, \tag{40}$$

and the transition equations

$$z_t^m = \rho_{z^m} z_{t-1}^m + \xi_t^m \tag{41}$$

$$z_t^f = \rho_{z^f} z_{t-1}^\tau + \xi_t^f. \tag{42}$$

Assumptions about the distributions of υ_t^R and υ_t^τ are made explicit in the following section.

4.1.1 Stochastic Volatility

The existence of stochastic volatility in the shocks of policy rules with time-varying coefficients has been documented by Davig and Leeper (2006), Fernandez-Villaverde et al. (2010), Bianchi (2012) and Fernandez-Villaverde et al. (2011b), who find that not only switches in policy rule coefficients are detectable in estimation, but also a fair amount of

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[6]The debate on the determinacy of the solution of DSGE models with time-varying coefficients has attracted attention of the literatures in recent years, and it is still an open field to future research (see Davig and Leeper, 2006, 2007; Fernandez-Villaverde et al., 2010; Farmer et al., 2011; Cho, 2013; Foerster et al., 2013).

stochastic volatility.[7] Hence, the distribution of the error terms in the policy rules is specified as $\upsilon_t^R \sim \mathbb{N}(0, \sigma_{R,t}^2)$ and $\upsilon_t^\tau \sim \mathbb{N}(0, \sigma_{\tau,t}^2)$, where

$$\ln \sigma_{R,t} = (1 - \rho_{\sigma_R}) \ln \sigma_R + \rho_{\sigma_R} \ln \sigma_{R,t-1} + \eta_R \xi_t^R \tag{43}$$

$$\ln \sigma_{\tau,t} = (1 - \rho_{\sigma_\tau}) \ln \sigma_\tau + \rho_{\sigma_\tau} \ln \sigma_{\tau,t-1} + \eta_\tau \xi_t^\tau \tag{44}$$

with $\rho_{\sigma_R} \in [0, 1)$, $\rho_{\sigma_\tau} \in [0, 1)$, $\xi_t^R \sim \text{iid}\mathbb{N}(0, 1)$ and $\xi_t^\tau \sim \text{iid}\mathbb{N}(0, 1)$.

In what follows, let $\mathbf{h}_t = [\ln \sigma_{R,t}, \ln \sigma_{\tau,t}]$ and let the set of parameters of the stochastic volatility processes be denoted Θ_h.

Equations (43) and (44) are added to the state-space model (39)-(42) to introduce stochastic volatility to the specification of the policy rules with time-varying coefficients.

4.1.2 Endogeneity

Since the work of Clarida et al. (2000), the estimation of monetary policy rules with constant coefficients, in particular the Taylor rule, has taken into account the endogeneity that exists between the shocks of the policy rule and inflation and output. The instrument set used in their GMM estimation contains four lags of: inflation, the output gap, the Federal funds rate, the short-long spread, and commodity price inflation.

With respect to fiscal policy rules with constant coefficients, Li (2009) illustrates the endogeneity/simultaneity problem that arises when estimating a fiscal policy rule like the one presented in this work. In estimating a fiscal policy rule that reacts to contemporary debt and the output gap, Claeys (2008) uses a set of instrumental variables in his GMM estimation that contains lags of: the output gap, debt, unit labor costs, growth in labor productivity, NAIRU, a broad money aggregate, a synthetic interest rate of the EURO area, oil price index, and the SEK/DEM exchange rate.

In terms of estimating linear equations with time-varying coefficients, either in the conventional random-coefficient or Markov-switching setups, Kim (2006) and Kim (2009) establish a Heckman-type two-stage maximum likelihood estimation technique to deal with the endogeneity problem to yield consistent estimates of the hyper-parameters, as well as to provide correct inferences on the time-varying coefficients. Kim and Nelson (2006b) estimate a random coefficients monetary policy rule for the United States using as the set of instruments four lags of: the Federal funds rate, output gap, inflation, commodity price inflation, and M2 growth. In related work, Bae et al. (2011) estimate a Markov-switching coefficients monetary policy rule for the United States using as the set of instruments three lags of: the Federal funds rate, GDP gap, inflation, commodity price changes, and the spread between the long-term bond rate and the three-month Treasury Bill rate.

The set of instruments that we use for both the monetary and the fiscal policy rules is given by four lags of: inflation, the output gap, government spending as proportion of GDP, M2 growth, and commodity price inflation. In a constant-coefficient version of the policy rules, the GMM estimation obtains the following results with respect to the instrument set: (i) the J test statistics of overidentification restrictions for both of the rules do not reject the null hypothesis that the instrument set is appropriate at the 5% level of significance; (ii) the

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[7]Sims and Zha (2006), on the other hand, argue that only changes in volatility can be detected in estimation, and not changes in coefficients.

exogeneity C test statistics imply that different subsets of instruments are exogenous at the 5% level of significance; (iii) the Cragg-Donald test statistics reject the null hypothesis of weak instruments at the 5% level of significance for both policy rules. Appendix F describes the setup of the model to correct for endogeneity. Details on the implementation of the Bayesian estimation of Θ_y, Θ_z, Θ_h, $\{\mathbf{z}_s\}_{s=0}^T$ and $\{\mathbf{h}_s\}_{s=0}^T$ appear in Appendix H.

4.2 Data

We use quarterly data from 1960:1 to 2008:3. The sample is not extended beyond 2008:3 to avoid having to deal with the zero lower bound (ZLB) of interest rates. It is still possible to estimate the interdependence between monetary and fiscal policy under the ZLB, but that constitutes a subject of future research. Inflation is the percentage change over the last four quarters of the price level given by the GDP price deflator. The nominal interest rate is the quarterly average of the federal funds rate. The output gap is the log difference between real GDP and the Congressional Budget Office's measure of potential real GDP. M2 growth is the percentage change over the last four quarters of seasonally adjusted M2. Commodity price inflation is the percentage change over the last four quarters of the commodity price index. Government spending is the federal consumption expenditures and gross investment. These variables are obtained from FRED. Lagged debt is the average debt-output ratio over the previous four quarters, where debt is the TreasuryDirect par value of gross marketable federal debt held by the public. Tax net of transfers corresponds to the seasonally adjusted quarterly current receipts of the federal government from which the current transfer payments have been deducted. This variable is obtained from the NIPA Table 3.2.

5 Estimation Results

The choice of prior distributions, hyper parameters, means of $5,000$ draws from the posterior distribution after trimming the first $1,000,000$ out of $2,000,000$ draws and thinning every 200th draw, along with 90% confidence sets appear in Table 1.[8] In order to keep the estimation relatively simple, we impose two restrictions that do not change the results qualitatively: First, we assume that the output gap coefficients of both policy rules are not time varying, which allows us to focus on capturing the interdependence between monetary and fiscal policy making in terms of the co-movement of the Taylor rule coefficient of inflation and the tax rule coefficient of lagged debt. Second, we assume that the location parameters of the logistic policy rule coefficients, α_3^π and γ_3^b, are zero.

Figure 2 shows that the estimated model does acceptably well at explaining the observed time series of interest and tax rates.

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[8]The Raftery and Lewis (1992) diagnostic test determines that 606,826 draws from the posterior distribution should be taken to estimate the 50th percentile within 0.01 with a 95% confidence level. It also determines that thinning to achieve an independent chain should occur every 115th draw.

Table 1: Results from the Bayesian Estimation

Parameters	Prior			Posterior	
	Density	Mean	SD	Mean	90% Conf. Set
α_0^π	Gamma	0.8	0.2	0.57	[0.35, 0.84]
α_1^π	Gamma	1.2	0.3	1.06	[0.61, 1.68]
α_2^π	Gamma	10	8	5.05	[0.10, 15.72]
α^y	Gamma	0.5	0.4	0.25	[0.08, 0.52]
ρ_R	Beta	0.9	0.05	0.96	[0.91, 0.99]
γ_0^b	Normal	-0.02	0.05	0.00	[-0.06, 0.06]
γ_1^b	Gamma	0.1	0.05	0.09	[0.03, 0.20]
γ_2^b	Gamma	10	8	4.03	[0.04, 14.70]
γ^y	Gamma	0.5	0.4	0.76	[0.44, 1.16]
ρ_τ	Beta	0.9	0.05	0.94	[0.93, 0.96]
ρ_{z^m}	Beta	0.9	0.05	0.86	[0.74, 0.95]
ρ_{z^f}	Beta	0.9	0.05	0.88	[0.78, 0.95]
z_0^m	Normal	0	2	-0.41	[-2.66, 1.93]
z_0^f	Normal	0	2	0.05	[-2.26, 2.35]
κ	—			0.20	[0.02, 0.47]
$\ln \sigma_R$	—			-2.05	[-2.41, -1.70]
ρ_{σ_R}	—			0.77	[0.63, 0.90]
η_R	—			0.57	[0.39, 0.78]
$\ln \sigma_{R,0}$	—			-2.93	[-5.18, -0.68]
$\ln \sigma_\tau$	—			-1.15	[-1.45, -0.89]
ρ_{σ_τ}	—			0.48	[0.27, 0.64]
η_τ	—			0.88	[0.65, 1.11]
$\ln \sigma_{\tau,0}$	—			-5.85	[-12.48, 0.59]

— denotes a flat prior.

Figure 2: Observed and Predicted Series
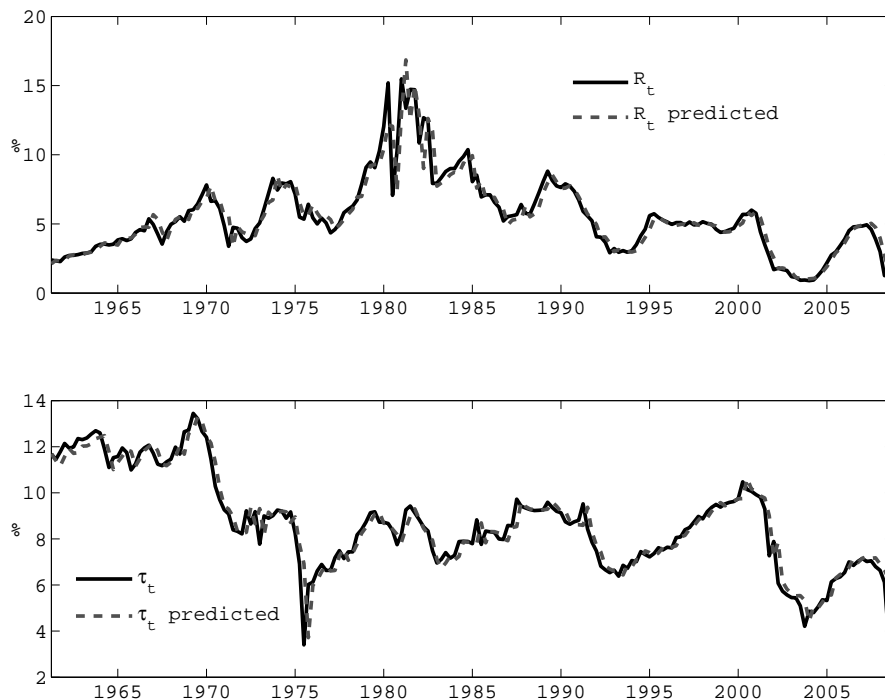
5.1 Choice of Prior Distributions

With respect to the monetary policy rule coefficients, there exist results in the literature about the values that they take in different regimes. Clarida et al. (2000) estimate that the inflation coefficient is 0.83 in the pre-Volcker era and 2.15 in the Volcker-Greenspan era. Lubik and Schorfheide (2004) find that the coefficient on inflation is estimated at 0.77 or 0.89, depending on the prior used, in the pre-1982 era, and 2.19 in the post-1982 era. Bianchi (2012) finds that the coefficient on inflation is estimated to be 0.94, 1.25, or 1.6 in a three-regime Markov-switching specification. Davig and Leeper (2011) find that the inflation coefficient switches between 0.53 and 1.29. We specify the prior distribution that characterizes the lower bound of the inflation coefficient with a mean of 0.8, and the upper bound with a prior distribution whose mean is 2 (the sum of 0.8 and 1.2 in Table 1). In regard to the fiscal policy rule coefficients, Bianchi (2012) finds that the coefficient on debt is estimated to be 0.0006, -0.0007, or -0.0036. Davig and Leeper (2011) estimate that the coefficient on debt switches between -0.025 and 0.071. We set the prior mean of the lower bound of the debt coefficient to -0.02, and the mean of the prior distribution that sets the upper bound of the coefficient to 0.08 (the sum of -0.02 and 0.1 in Table 1). The policy rule coefficients on the output gap for both policy rules have 0.5 as the prior mean.

The prior distribution specification of the transition coefficients of both logistic policy rule coefficients is set to have a mean of 10 with a standard deviation of 8. We recall that the larger the coefficient is, the more rapid the transition between states is. Values of the coefficient greater than 10 imply a specification that mimics closely a Markov-switching-like transition. The choice of this prior distribution allows the coefficient to take low or high

values.

The smoothing coefficients of the policy rules have prior distributions that are standard in terms of the persistence that they represent for interest and tax rates in the data.

A coefficient that deserves attention is the correlation coefficient between the shocks of the latent factors, which determines the degree of interdependence between monetary and fiscal policymaking. The prior distribution chosen is uniform on [0,1]. We recall that a positive value of this parameter implies that monetary and fiscal policy co-move in such a way that they tend to deliver outcomes in the M or the F regime more frequently than outcomes in the indeterminacy or no solution regimes. Hence, by choosing a prior distribution on the nonnegative domain, we are imposing a model with either co-movements towards the determinate regimes or absence of co-movements in that direction, but we are not allowing co-movements in the direction of the indeterminate or unbounded debt regimes.

Finally, following Fernandez-Villaverde et al. (2011a), we do not specify prior distributions for the parameters of the stochastic volatility processes.

5.2 Parameter Estimates

The estimated parameters of the monetary and fiscal policy rule coefficients in Table 1 have the expected magnitudes and signs. In particular, for the monetary policy rule, the inflation coefficient takes values in (0.57, 1.63), while the output gap coefficient has a posterior mean of 0.25. With respect to the fiscal policy rule, the debt coefficient takes on values in (0.001, 0.095), while the output gap coefficient has a posterior mean of 0.76. The speed of transition of the policy rule coefficients is estimated to be higher for the inflation coefficient of the monetary policy rule than for the debt coefficient of the tax rule. This implies that the fiscal authority has a slower transition across states (in terms of how taxes react with respect to deviations of debt) than the monetary authority does (in terms of how the interest rate reacts with respect to inflation). This result is consistent with the legislative and implementation lags of fiscal policymaking.

The latent factors show relatively high persistence, with the fiscal policy latent factor being somewhat more persistent than the monetary policy one. This result implies that fiscal policy regimes are likely to last longer than monetary policy regimes.

Finally, the correlation between the latent factors has an estimated posterior mean of 0.20 and a 90% confidence set given by (0.01, 0.47). This result implies that there is a degree of direct interactions between policies. In particular, monetary tightenings to stabilize inflation tend to be accompanied by fiscal policy that stabilizes debt (M regime), while fiscal policy that departs from debt stabilization tends to be accompanied by interest rates that react weakly to inflation (F regime).

5.3 Evolution of Policy Rule Coefficients

Figure 3 shows the smoothed estimates of the monetary policy rule coefficient for inflation on the left axis, and of the fiscal policy rule coefficient for debt on the right axis. Two facts are apparent from this figure: First, the fiscal policy rule coefficient on debt was low during most of the 1970s, the second half of the 1980s, and the first half of the 1990s. Second, the monetary policy rule coefficient on inflation was high during the second half of the 1960s

Figure 3: Evolution of Policy Rule Coefficients and NBER Recession Periods
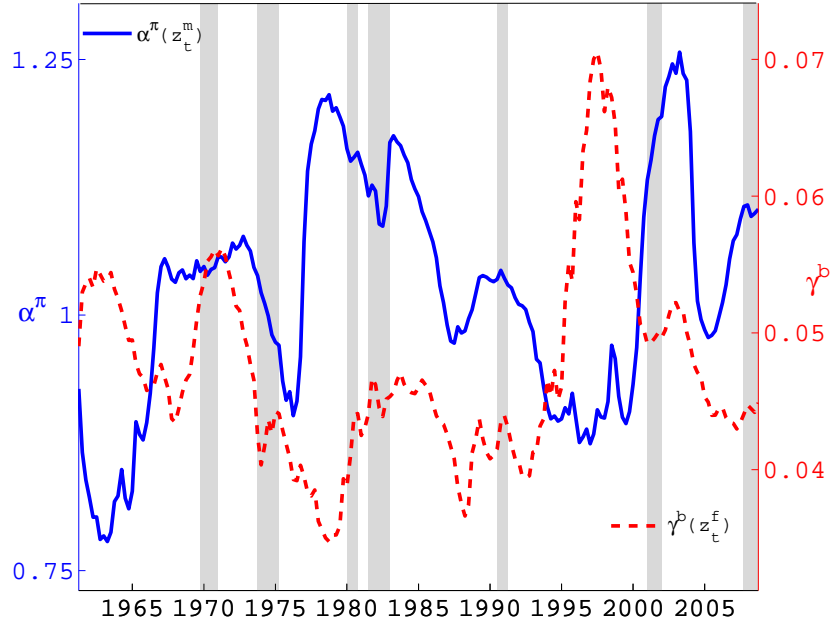
and the first half of the 1970s, during the first half of the 1980s, and during the first half of the 2000s. In the next paragraphs, we discuss the evolution of each of the policy rule coefficients to compare the econometric results with the narrative about monetary and fiscal policymaking and with similar results in the literature.

The evolution of the monetary policy rule coefficient on inflation reveals that the Federal Reserve conducted a hawkish monetary policy during the second half of 1960s, as Davig and Leeper (2011), Bianchi (2010), Eo (2009) and Fernandez-Villaverde et al. (2010) find, with a tendency to ease it towards the end of the decade. During the mid 1970s the interest rate did not react strongly to inflation. Most studies in the literature find a passive monetary policy during this decade, except Boivin (2006), who finds that monetary policy was tight during the first half of the 1970s. At the end of the 1970s monetary policy switched rapidly to a hawkish regime. This switch in policymaking is also found by Davig and Doh (2009), Eo (2009) and Bianchi (2010) who find, based on estimations of Markov-switching policy rule coefficients, that the active monetary policy periods started around, or a little earlier than, the mid 1980s. The graph shows that monetary policy continued to be relatively hawkish during most of the 1980s. Starting in the late 1980s, the estimates of the model suggest a relatively weak response to inflation, which lasted through much of the 1990s. A study that finds a similar result is Fernandez-Villaverde et al. (2010).[9] In 2000s, monetary policy increased its reaction to inflation until 2005 and then the strength of the reaction declined. The empirical evidence is divided with respect to this result: On one hand Eo (2009), Davig and Doh (2009) and Bianchi (2010) find that monetary policy was actively fighting inflation, while on another Fernandez-Villaverde et al. (2010) and Davig and Leeper (2011) find the

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[9]Kim and Nelson (2006a) find that their confidence interval starts including the passive monetary policy region at the beginning of the 1990s.

opposite.

For a narrative perspective on the results, we rely on the history of the monetary policy of the Federal Reserve as told by Hetzel (2008). He compares Fed Chairman William Martin to Fed Chairman Paul Volcker and Fed Chairman Alan Greenspan in that Martin believed that raising short-term interest rates in an expansion was a way to preempt inflation. That allows to understand the behavior of monetary policy, according to the graph, during the second half of the 1960s. Hetzel also mentions the weak reaction of interest rates to inflation during the 1970s due to the focus of the central bank to promote employment and the belief that inflation was a nonmonetary phenomenon. According to Hetzel, the 1980s saw the commitment of the Federal Reserve to money targets allowing the FOMC to raise interest rates by whatever extent necessary to lower inflation. The results show that the reaction coefficient of the interest rate to inflation was indeed relatively high.

After tightening monetary policy at the end of the 1980s to counteract concerns about inflation, the results show a decline in the reaction of the federal funds rate to inflation at the beginning of the 1990s to help with the recovery after the recession. With inflation declining during this decade and interest rates remaining nearly flat, except for the rise between 1993 and 1995 due to the "1994 inflation scare" (see Hetzel, 2008), the reaction coefficient on inflation declines until the events of the Asian and Russian crises. During 1998 the Federal Reserve moved the interest rate in the same direction as declining inflation, causing a spike in the evolution of the inflation coefficient. In the 2000s, with declining inflation and interest rates between 2001 and the beginning of 2003, the reaction coefficient to inflation rises rapidly to reach a peak in the second quarter of 2003 when concerns about interest rates being close to the zero-bound arose for the first time. With inflation stable and interest rates rising during 2004, the reaction coefficient declines. Then the coefficient rises at the end of the sample, between 2005 and 2008, reflecting interest rates that risen more than what inflation risen during the period.

For the debt policy rule coefficient, the results show a coefficient responding weakly to lagged debt during the 1970s, a result mostly in the same lines as those in Davig and Leeper (2006), Bianchi (2012), and Favero and Monacelli (2003). The results also show a low debt coefficient during the second half of the 1980s and the first half of the 1990s, a result that is compatible with the findings in Bianchi (2012).

In terms of the narrative, the tax reductions in the revenue act of 1964 can help understand the decline of the coefficient during the 1960s, while the revenue and expenditure control act of 1968 and the tax reform act of 1969 can help explain the rise of the coefficient at the end of the decade. The subsequent tax reforms of 1971, 1975 and 1976 help explain the decline in the debt coefficient during the 1970s. The rise of the coefficient observed at the end of the decade has to do with the fact that the debt-GDP ratio rises between 1975 and 1980, inclusive, and the net revenue-GDP ratio rose in that period. That relationship breaks up during the 1980s due to the tax reforms of the decade, in particular the recovery tax act of 1981. During the 1990s, the coefficient rises rapidly, most likely due to the changes implemented in the budget reconciliation act of 1993. The taxpayer relief act of 1997 can help understand the decline in the coefficient at the end of the decade. The decline of the debt coefficient in the 2000s can be attributed to the tax relief acts of 2001 and 2003, and the subsequent stimulus acts of 2004 and 2008.

In a historical perspective, the evolution of the estimated policy rule coefficients suggest

Figure 4: Evolution of Estimated Stochastic Volatilities
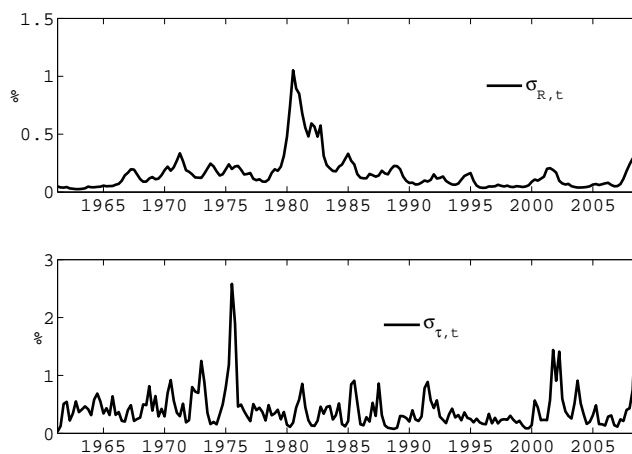
that policy leaned towards a fiscal regime during some part of the 1970s, a result emphasized by Bianchi (2012). On the other hand, the estimates favored a monetary regime during the second half of the 1960s, the first half of the 1980s, and the first half of the 2000s. The macroeconomic model allows short-run dynamics that visit temporarily all the regimes, including the indeterminate and the no-bounded-solution regimes, but the long-run expectations of a mixed regime imply dynamics that tend to move the economy between the two determinate regimes.

5.4 Evolution of Estimated Stochastic Volatilities

Figure 4 shows the evolution of the estimated stochastic volatilities for the interest and tax rate rules. The estimation shows that the volatility of interest rates was significantly higher during the beginning of the 1980s due to the important change in conducting monetary policy. Also, volatility is somewhat lower starting in the 1990s, a phenomenon referred to as the great moderation (see Stock and Watson, 2002), and stays low with a small peak at the end of the sample period when the great recession hit. On the other hand, fiscal policy volatility shows spikes in the mid 1970s, the first half of the 2000s, and the beginning of the great recession. All these events are associated with some kind of fiscal stimulus. It is worth noting that the persistence of the volatility of monetary policymaking, given by the estimated coefficient ρ_{σ_R}, is higher than the persistence of the volatility of fiscal policymaking, given by the estimated coefficient ρ_{σ_τ}, as can be seen in Table 1.

6 Monetary and Fiscal Policy Analysis

This section presents the results of monetary and fiscal policy experiments through impulse-response functions. The section also presents the effect of policy interdependence in

reducing the volatility of output and inflation.

6.1 Nonlinear Impulse Response Analysis

We perform policy experiments with the model solved using the proposed solution method and calibrated according to the estimated parameters of the policy rule coefficients. Since the estimation is not constrained to deliver a continuous solution to the macroeconomic model as presented in Appendix E, we calibrate the model with coefficients that are consistent with continuity of the solution. In particular, we calibrate the lower bound of the monetary policy rule coefficient on inflation, α_0^π, to be 0.85 instead of 0.57. The fiscal policy rule coefficients, γ_0^b and γ_0^b, are calibrated to 0.03 and 0.07 instead of 0 and 0.09, respectively.

For the markup in steady state and the price adjustment cost parameter, Keen and Wang (2007) show that, given a steady state markup and a fraction of firms that re-optimize each period, there is a corresponding value for the price adjustment cost parameter. We set the markup to 20% and the fraction of re-optimizing firms to 25% each period (a firm re-optimizes every 12 months). These values correspond, approximately, to $\theta = 6$ and $\phi = 60$. Other parameters that need to be specified are: δ, the steady state gross quarterly rate of output growth, which is set to 1.0081, the average over the sample period that implies a steady state annual growth rate of approximately 3.25%; Π, the steady state gross quarterly rate of price inflation, which is set to 1.0084, the average over the sample period that implies a steady state annual inflation of approximately 3.4%; b, the steady state level of debt to output, which is set to 0.3354, the average over the sample period; ζ, the ratio of government spending to output is set to 0.081, also the average over the sample period; $1/vb$, the ratio of outstanding money balances to debt, is set to 0.2 following Kim (2003); β is set to 0.99; σ is set to 1; and φ is set to 1, so that the Frisch elasticity of labor supply is unity.

We perform two policy experiments. First, a one-time i.i.d shock on the interest rate. Second, a one-time i.i.d shock on the tax-to-output ratio. The responses to these shocks are calculated under three scenarios: starting at regime M and staying there forever, starting at regime F and staying there forever, and a Mixed regime.

In the Mixed regime, the latent factors start at their means, which we set to zero according to the confidence sets in Table 1. These zero values imply corresponding values for the initial monetary and fiscal policy rule coefficients ($\alpha^\pi(0) = 1.38$ and $\gamma^b(0) = 0.065$). Over the response horizon, the policy rule coefficients evolve according to their logistic specifications and the joint evolution of the latent factors. To compute the responses in the Mixed regime, we take the average of the responses for given realizations of the policy rule coefficients. Note that in the Mixed regime the initial values of the policy rule coefficients imply that the economy is in regime M. However, because agents expect regime F to take place for some time in the future, the dynamics of the Mixed regime are not those of a pure M regime, but of a regime that incorporates the features of both regimes M and F.

6.1.1 A Monetary Contraction

Figure 5 shows the responses of some of the variables of the model to a one-time 25 basis point i.i.d. shock on the interest rate.

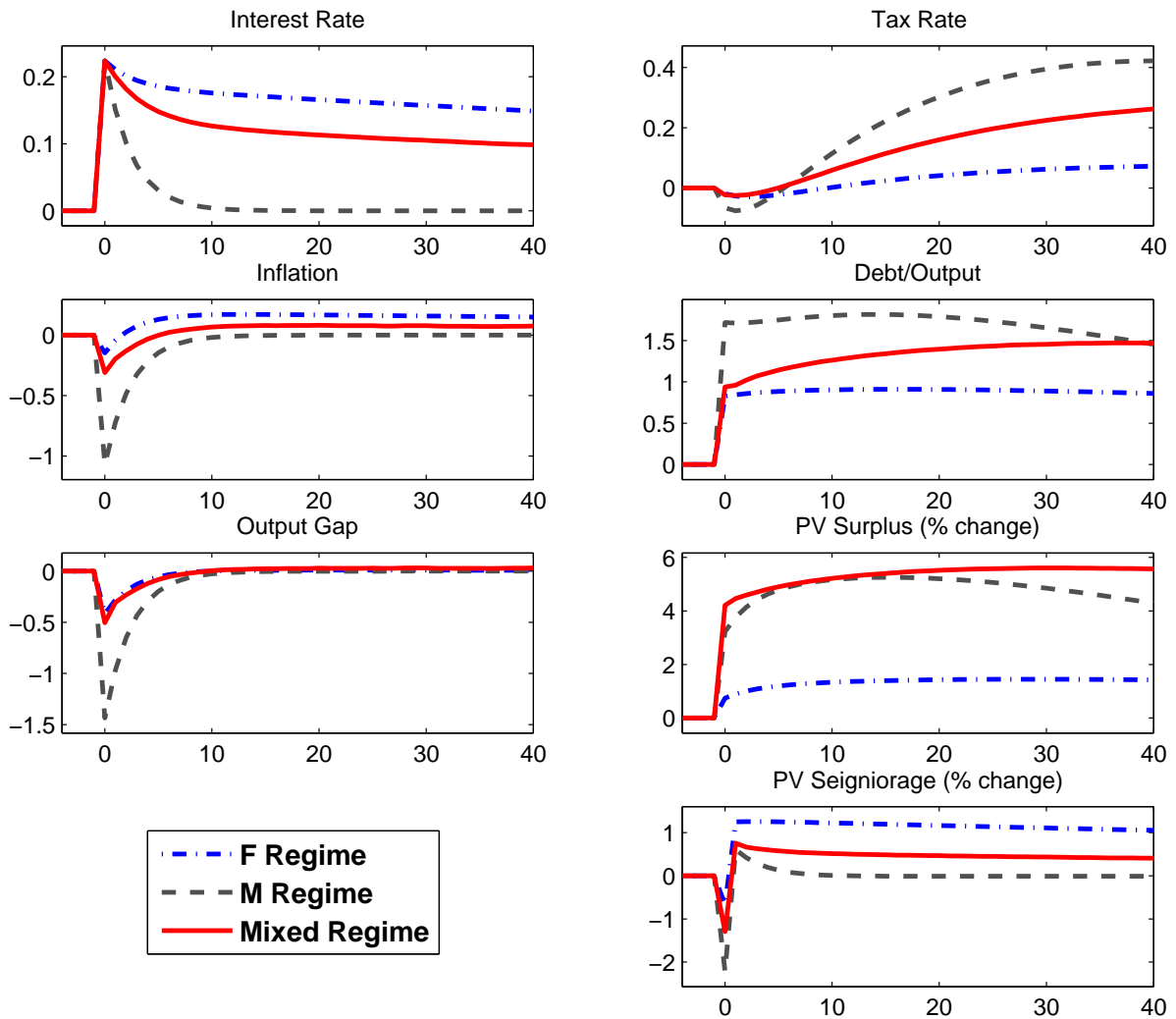Figure 5: Response to a 25 bp Increase in the Interest Rate
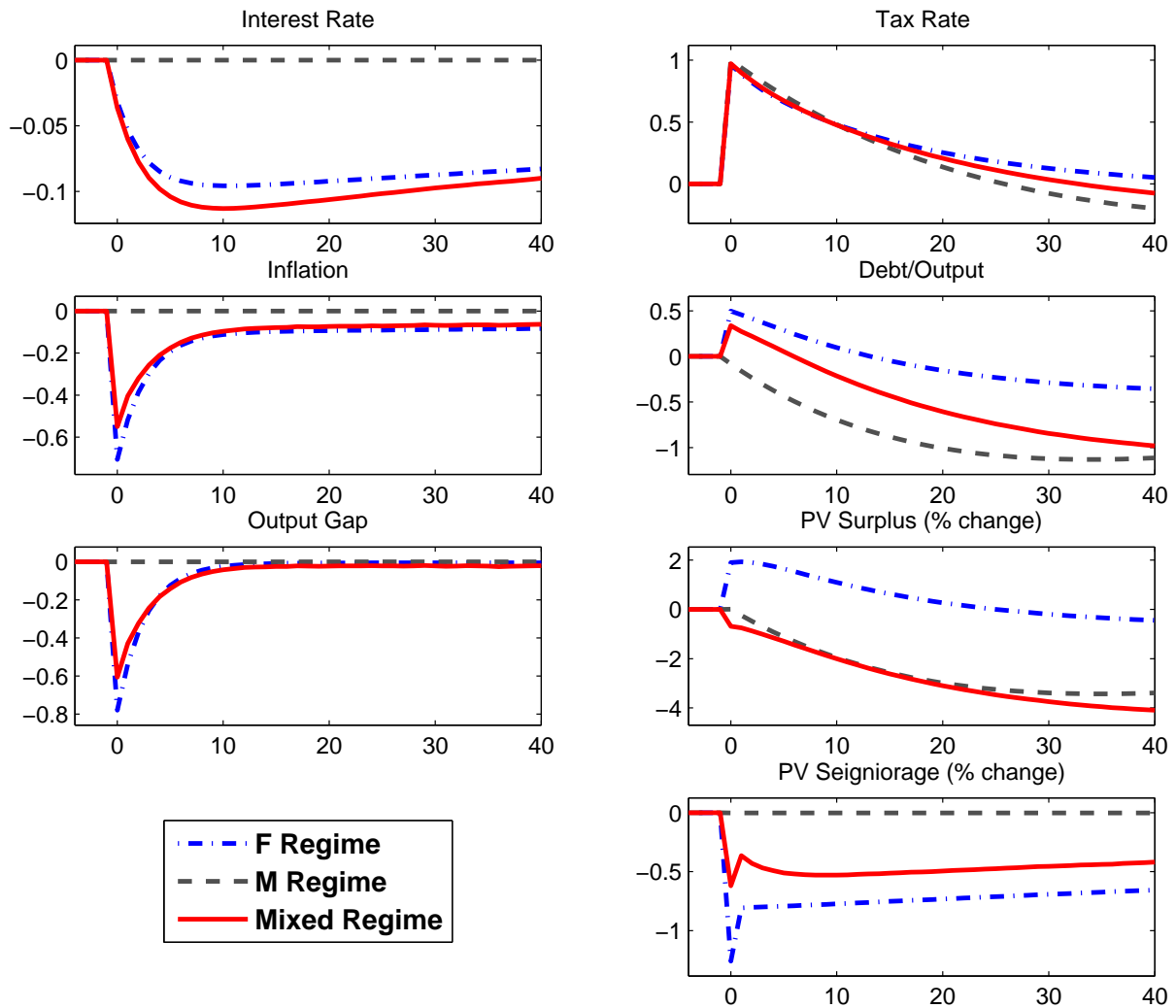
Figure 6: Response to a 100 bp Increase in the Tax Rate

Interest Rate

Tax Rate

Inflation

Debt/Output

Output Gap

PV Surplus (% change)

- · - · F Regime
- - - M Regime
—— Mixed Regime

PV Seigniorage (% change)

The effects of a monetary contraction in a New Keynesian model in regime M are well known: an open market operation that sells debt to households, and that is expected to be corrected in the future via higher taxes, does not change household wealth and only increases the nominal interest rate. With sticky prices the real interest rate increases, reducing consumption, output and inflation. As the nominal interest rate decreases, as well as the real interest rate, the output gap returns to zero and inflation to its steady state. Taxes first decrease due to feedback from the output gap and then increase due to their Ricardian response to lagged debt, which is now higher. Given the persistence of taxes, the present value of expected future surpluses increases and slowly returns to steady state, while the present value of expected future seignorage decreases on impact.

In regime F, an open market operation that sells debt to households, which is not expected to be corrected in the future via higher taxes, increases household's wealth. The increase in wealth attenuates the liquidity-induced contraction in demand. Inflation drops on impact given that the liquidity effect dominates the wealth effect.[10] The real interest rate increases on impact, causing output to fall. Note that a "price puzzle", in the sense of Sims (1992), is present. Over the response horizon, inflation increases due to some unpleasant monetarist arithmetic: Taxes are not expected to increase to pay back the debt and some monetization of the deficit is expected. Taxes decrease on impact, reacting mainly to the output gap, and then increase above steady state due to the initial increase in debt, since the debt coefficient of the fiscal policy rule is small but positive. Given the persistence of taxes, the tax increase above steady state makes the present value of expected future surpluses increase on impact.

In the Mixed regime, agents compute their responses based on their expectation of the evolution of future regimes. As a result, the effects of a monetary contraction on inflation and output are between the two previously described regimes. Agents' expectations incorporate the behavior of the economy under both regimes M and F. Thus, there is lower inflation on impact and there is a wealth effect that translates into marginally higher inflation in the long run. Output also falls on impact due to higher real interest rates. These results mean that, as long as the economy does not start at one of the limiting regimes and stays there forever, some of the wealth effect derived from regime F will influence the economy's response to a monetary contraction. That is, a monetary contraction decreases inflation on impact, but the long-run effect is an increase of inflation above steady state. Sims (2011) describes this feature of monetary policy as "stepping on a rake".

The impact response of inflation and output to a 25 basis points increase in the interest rate is shown in Figure 7 for a combination of values of $\alpha^{\pi}(z^m)$ and $\gamma^b(z^f)$ for $z^m = z^f$ in [-10,10]. The responses of inflation and output range from -1.03% and -1.37%, respectively, in regime M ($z^m = z^f = 10$), to -0.15% and -0.42%, respectively, in regime F ($z^m = z^f = -10$). In between these limits there exists a continuum of impact responses given by the possible combinations of the monetary and fiscal policy rule coefficients. Hence, the size of the impact response of inflation and output to a monetary policy shock will depend on the state of the

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[10]To determine the direction of the initial response of inflation in regime F, we use the following expression:

$$R_{-1}b_{-1}\frac{1}{\Pi_0}\frac{1}{\Delta Y_0} = \mathbb{E}_0 \sum_{t=0}^{\infty} \text{MRS}_{0,t}\left(s_t + m_t\right),$$

where s_t and m_t denote the primary surplus and seignorage, respectively, in output terms.

Figure 7: Response on Impact to a 25 bp Increase in the Interest Rate
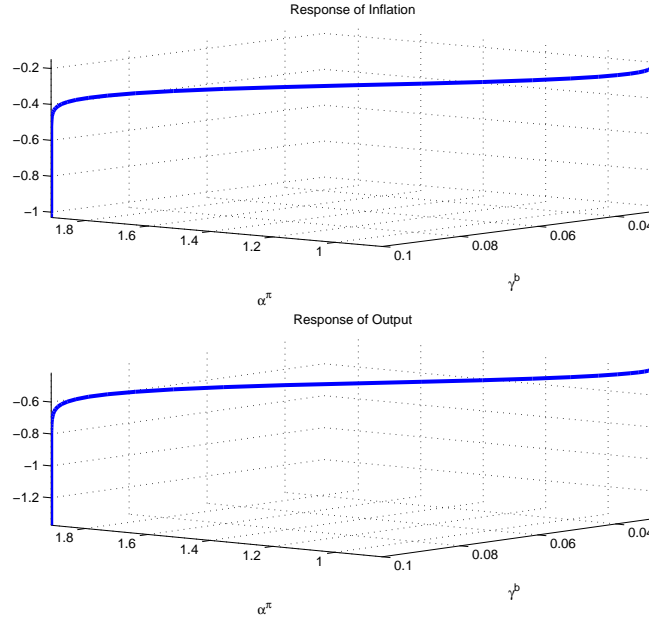
monetary and fiscal policy rule coefficients at the time of policy implementation.

### 6.1.2 A Fiscal Contraction - Tax increase

Figure 6 shows the responses of some of the variables of the model to a one-time 100 basis point i.i.d shock on the tax-to-output ratio.

In regime M, a tax increase that retires debt does not have any effect on output or inflation due to the Ricardian nature of the equilibrium: higher taxes today are expected to be fully compensated with lower taxes in the future, with no wealth effect on households. Wealth is unchanged because, on impact, the present value of expected future surpluses and seignorage are unchanged. Taxes decrease after impact in reaction to the reduction in debt and, given the persistence of taxes, remain below steady state at long horizons. Debt falls, anticipating the decrease in the present value of expected future surpluses. There are no changes in seignorage.

A tax increase in regime F reduces wealth since the increase is not expected to be reversed in the future. The decrease in wealth reduces demand for goods and, hence, inflation. Since the nominal interest rate responds passively to changes in inflation, the real interest rate increases, reducing consumption and output. The present value of expected future surpluses increases due to persistently higher taxes that will not be compensated for in the future. The present value of expected future seignorage declines on impact due to lower inflation and interest rates. Overall, the change in the expected present value of government net receipts implies that debt initially increases before returning to steady state.

In the Mixed regime, as with the monetary contraction, agents' behavior is based on their expectation of the evolution of future regimes. As a result, the effects of a fiscal contraction are between the two previously described regimes. That is, a change in taxes has effects on inflation and output due to the wealth effect that emerges because agents do not expect the

Figure 8: Response on Impact to a 100 bp Increase in the Tax Rate
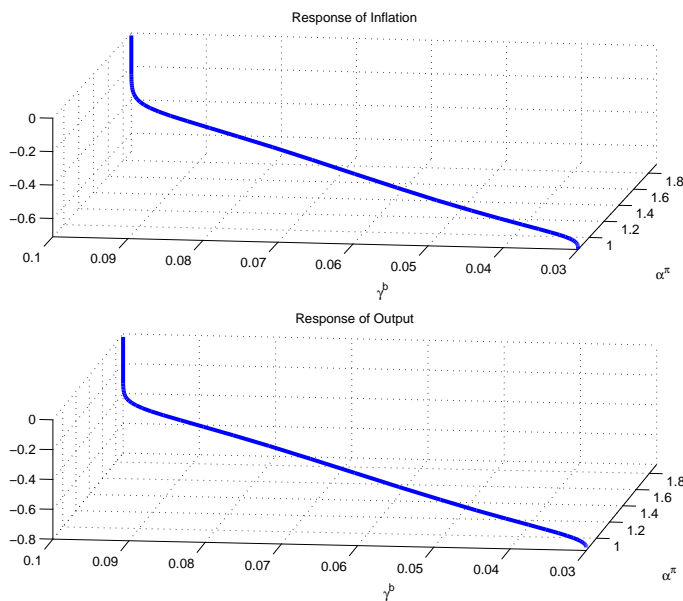
initial increase in taxes to be completely reversed in the future. However, the effects are attenuated with respect to a pure F regime.

The impact response of inflation and output to a 100 basis point increase in the tax-to-output ratio is shown in Figure 8 for a combination of values of $\alpha^\pi(z^m)$ and $\gamma^b(z^f)$ for $z^m = z^f$ in [-10,10]. The responses of inflation and output range from 0% and 0%, respectively, in regime M ($z^m = z^f = 10$), to -0.71% and -0.78%, respectively, in regime F ($z^m = z^f = -10$). In between these limits there exists a continuum of impact responses dependent on the combinations of monetary and fiscal policy rule coefficients. Hence, the magnitude of the impact responses of inflation and output to a fiscal policy shock will depend on the state of the monetary and fiscal policy rule coefficients at the time of policy implementation.

6.2   Policy Interdependence: Its Effects

In this section we analyze the effect of interdependence in monetary and fiscal policy making as measured by a positive correlation coefficient between the latent factors, κ. In Section 5.2, we showed that the estimated posterior mean of the correlation coefficient between the latent factors is 0.2. As mentioned previously, this value implies that monetary tightenings to stabilize inflation tend to be accompanied by fiscal policy that stabilizes debt (regime M) while fiscal policy that deviates from debt stabilization tends to be accompanied by a loose monetary policy that contributes to keep debt stable (regime F).

The existing literature on Markov-switching policy rule coefficients assumes that independent states drive the switches between regimes of the coefficients of each policy rule. In that sense, there are not explicit interactions between monetary and fiscal policymaking. In the context of the current paper, that would be analogous to the case where $\kappa = 0$.

To examine the effect of policy interdependence ($\kappa > 0$), we solve and simulate the model for different values of the correlation coefficient between the latent factors. We simulate the

Figure 9: Interdependence and Unconditional Volatilities of Output and Inflation
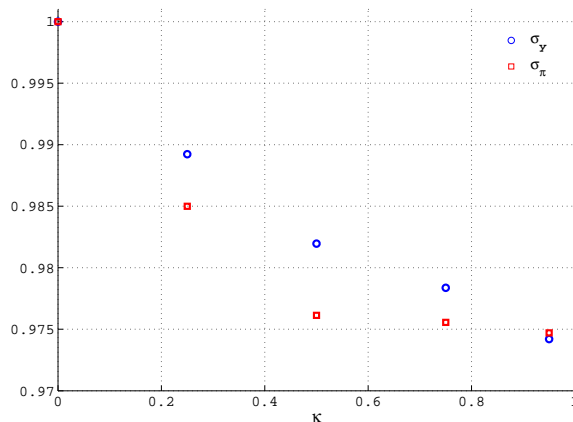
model for 1,000 periods 1,000 times, including the evolution of latent factors, and take the standard deviation of output and inflation.[11] Figure 9 illustrates the results of the exercise where the unconditional standard deviations have been normalized to one when $\kappa = 0$, which is the case of absence of coordination.

As can be seen from the figure, the stronger the association between monetary and fiscal policymaking, as measured by the degree of synchronization of changes in the coefficients of the policy rules, the smaller the unconditional standard deviation of the series. The results show that going from a scenario of independent switching in the coefficients of the policy rules to a scenario of relatively low interdependence, as measured by $\kappa = 0.25$, would result in a decrease of between 1% and 2% in the unconditional standard deviation of inflation or output.

To understand this result, recall that the only two determinate equilibria arise in regimes M and F, while the other two equilibria imply either an indeterminacy or a solution with unbounded debt. Under the indeterminate equilibrium, the presence of sunspot shocks may imply a higher volatility of inflation, whereas under the unbounded debt equilibrium there is not a stationary solution to the macroeconomic model. Hence, policy co-movements in the direction of the M and F regimes would help stabilize output and inflation with respect to the case of no policy co-movements or co-movements away from determinate regimes.

7 Concluding Remarks

In this paper, we formulated and solved a New Keynesian model with time-varying policy rule coefficients that allows for interdependence between monetary and fiscal policymaking. This specification permits analyzing the effects of monetary and fiscal policy in an environment where agents expect the economy to evolve between times where the nominal anchor is provided by monetary policymaking and times where the nominal anchor is provided by

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[11]The standard deviation of the shocks is kept fixed across simulations.

fiscal policymaking.

We estimated the policy rules, and obtained results showing that there have been fluctuations in the monetary and fiscal policy rule coefficients. Additionally, policymaking shows important persistence, with fiscal policy being somewhat more persistent than monetary policy. The results also show that there is a degree of policy interdependence, given by a positive correlation coefficient between the latent factors that drive the evolution of policy rule coefficients. More specifically, fiscal policy that focuses on keeping debt under control tends to accompany monetary policy that focuses on keeping inflation under control, and monetary policy that keeps interest rates with a low reaction to inflation tends to accompany fiscal policy that departs from debt stabilization.

Policy experiments show that when agents expect the economy to evolve between a regime of monetary price determination and a regime of fiscal price determination, contractionary monetary policy lowers inflation in the short run and increases it in the long run. The experiments reveal that (lump-sum) taxes have effects on output and inflation, as the literature on the fiscal theory of the price level suggests, but effects are attenuated relative to a pure fiscal regime.

Appendix

A   Model Setup

The representative household solves the following problem:

$$\max_{\{C_t, Mt/P_t, B_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t/A_t)^{1-\sigma}}{1-\sigma} + \chi_M \log\left(M_t^d/P_t\right) - \chi_H \frac{H_t^{1+\varphi}}{1+\varphi} \right)$$

subject to

$$C_t + \frac{M_t^d}{P_t} + \frac{B_t}{P_t} + \frac{T_t}{P_t} \leq H_t \frac{W_t}{P_t} + \frac{D_t}{P_t} + \frac{M_{t-1}^d}{P_t} + R_{t-1}\frac{B_{t-1}}{P_t} \text{ for } t \geq 0,$$

$$\frac{M_{-1} + R_{-1}B_{-1}}{P_{-1}} \text{ given,}$$

$$\lim_{t \to \infty} MRS_{0,t} \frac{M_t + B_t}{P_t} = 0,$$

where $MRS_{0,t}$ denotes the marginal rate of substitution between period 0 and period t. The necessary first order conditions are:

$$C_t \quad : \quad \frac{1}{A_t}\left(\frac{C_t}{A_t}\right)^{-\sigma} - \lambda_t = 0 \tag{45}$$

$$H_t \quad : \quad -\chi_H H_t^{\varphi} + \lambda_t \frac{W_t}{P_t} = 0 \tag{46}$$

$$\frac{M_t^d}{P_t} \quad : \quad \chi_M \left(\frac{M_t^d}{P_t}\right)^{-1} - \lambda_t + \beta \mathbb{E}_t \lambda_{t+1} \frac{P_t}{P_{t+1}} = 0 \tag{47}$$

$$B_t \quad : \quad -\frac{\lambda_t}{P_t} + \beta R_t \mathbb{E}_t \frac{\lambda_{t+1}}{P_{t+1}} = 0, \tag{48}$$

$$\lambda_t \quad : \quad C_t + \frac{M_t^d}{P_t} + \frac{B_t}{P_t} + \frac{T_t}{P_t} - H_t \frac{W_t}{P_t} - \frac{D_t}{P_t} - \frac{M_{t-1}^d}{P_t} - R_{t-1}\frac{B_{t-1}}{P_t} = 0 \tag{49}$$

where λ_t is the Lagrange multiplier associated to the budget constraint at time t.

From (45) and (48),

$$1 = \beta R_t \mathbb{E}_t \left(\frac{C_t/A_t}{C_{t+1}/A_{t+1}}\right)^{\sigma} \frac{A_t}{A_{t+1}} \frac{P_t}{P_{t+1}}. \tag{50}$$

From (45) and (46)

$$\chi_H H_t^{\varphi} A_t \left(\frac{C_t}{A_t}\right)^{\sigma} = \frac{W_t}{P_t}. \tag{51}$$

From (45), (47) and (48),

$$\frac{M_t}{P_t} = \chi_M A_t \left(\frac{C_t}{A_t}\right)^\sigma \left(\frac{R_t}{R_t - 1}\right). \tag{52}$$

Profits of intermediate firm j are given by

$$\frac{D_t(j)}{P_t} = \frac{P_t(j)}{P_t}Y_t(j) - \frac{W_t}{P_t}L_t(j) - \frac{\phi}{2}\left(\frac{P_t(j)}{\Pi P_{t-1}(j)} - 1\right)^2 Y_t. \tag{53}$$

Substituting (8) and (9) in (53) yields

$$\frac{D_t(j)}{P_t} = \left[\left(\frac{P_t(j)}{P_t}\right)^{1-\theta_t} - \psi_t\left(\frac{P_t(j)}{P_t}\right)^{-\theta_t} - \frac{\phi}{2}\left(\frac{P_t(j)}{\Pi P_{t-1}(j)} - 1\right)^2\right] Y_t.$$

Then, intermediate firm j chooses $P_t(j)$ to maximize

$$\mathbb{E}_t \sum_{k=0}^{\infty} \mathrm{MRS}_{t,t+k}\left[\left(\frac{P_t(j)}{P_t}\right)^{1-\theta_t} - \psi_t\left(\frac{P_t(j)}{P_t}\right)^{-\theta_t} - \frac{\phi}{2}\left(\frac{P_t(j)}{\Pi P_{t-1}(j)} - 1\right)^2\right] Y_t.$$

The first order condition to this maximization problem is

$$
\begin{aligned}
0 = {} & \lambda_t Y_t \left[(1-\theta_t)\left(\frac{P_t(j)}{P_t}\right)^{-\theta_t}\frac{1}{P_t} + \theta_t\frac{\psi_t}{P_t}\left(\frac{P_t(j)}{P_t}\right)^{-\theta_t-1} - \phi\left(\frac{P_t(j)}{\Pi P_{t-1}} - 1\right)\frac{1}{\Pi P_{t-1}(j)}\right] + \\
& + \beta\lambda_{t+1}Y_{t+1}\phi\left(\frac{P_{t+1}(j)}{\Pi P_t} - 1\right)\frac{P_{t+1}(j)}{\Pi P_t(j)^2}.
\end{aligned}
\tag{54}
$$

In a symmetric equilibrium, $P_t(j) = P_t$, $L_t(j) = L_t$ and $Y_t(j) = Y_t$, hence

$$\frac{D_t(j)}{P_t} = \frac{D_t}{P_t} = \left[1 - \psi_t - \frac{\phi}{2}\left(\frac{P_t}{\Pi P_{t-1}} - 1\right)^2\right] Y_t. \tag{55}$$

In equilibrium, $M_t^d = M_t^s = M_t$, $B_t = 0$ and $H_t = L_t$. Then, (11), (55) and (49) imply (12). In the symmetric equilibrium, substituting (45) into (54) yields (13). Finally, the symmetric equilibrium yields (15), (52) is (16), and (50) is (14).

Before proceeding to the log-linearization of the model, it is convenient to write (11) in terms of nominal output. The resulting expression is

$$b_t = 1 - \frac{1}{g_t} - \tau_t - \frac{1}{v_t} + \frac{1}{v_{t-1}}\frac{1}{\Pi_t}\frac{1}{\Delta Y_t} + R_{t-1}b_{t-1}\frac{1}{\Pi_t}\frac{1}{\Delta Y_t}, \tag{56}$$

where $\Delta Y_t = Y_t/Y_{t-1}$.

To obtain (17), start with (15) and recall the definition of $\psi_T = W_t/P_t A_t$ to get

$$\psi_t = \chi_H L_t^\varphi A_t \left(\frac{C_t}{A_t}\right)^\sigma.$$

In the symmetric equilibrium, from (9), $Y_t = A_t L_t$, and without price rigidities, $\psi_t = \frac{\theta_t - 1}{\theta_t}$ and $C_t = Y_t/g_t$, hence

$$\frac{\theta_t - 1}{\theta_t} = \chi_H \left(\frac{Y_t^*}{A_t}\right)^{\varphi+\sigma} g_t^{-\sigma}.$$

Once the variables have been de-trended by dividing them by A_t, the absence of shocks yields (18) from (14), (21) from (16), (19) from combining (12) and (17), (20) from (19) and $c_t = y_t/g_t$, and (22) from (56) with $\Delta y = \delta$.

B Matrices for Solving the Model

The matrices in system (33)-(35) are given by

$$\mathbf{G} = \begin{bmatrix} 0 & 0 & -1/\sigma & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -(\theta-1)(\varphi+\sigma)/\phi \end{bmatrix}, \qquad \mathbf{J} = \begin{bmatrix} 1 & 1/\sigma \\ 0 & \beta \end{bmatrix},$$

$$\mathbf{K} = \begin{bmatrix} -1 & 0 \\ (\theta-1)(\varphi+\sigma)/\phi & -1 \end{bmatrix}, \qquad \mathbf{M} = \begin{bmatrix} 1-\rho_g & 0 & \rho_\nu/\sigma & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathbf{A}(z_t) = \begin{bmatrix} -1 & 0 & 1/(R-1) & 0 & 0 & 0 & 0 \\ 1/vb & -1 & 0 & -\tau/b & -(1/bv\Pi\delta + 1/\beta) & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & -(1-\rho_R)\alpha^y(z_t^m) \\ 0 & 0 & 0 & -1 & 0 & 0 & -(1-\rho_\tau)\gamma^y(z_t^f) \\ 0 & 0 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix},$$

$$\mathbf{B}(z_t^f) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1/v\Pi\delta b & 1/\beta & 1/\beta & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_R & 0 & 0 & 0 & 0 \\ 0 & (1-\rho_\tau)\gamma^b(z_t^f) & 0 & \rho_\tau & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathbf{C}(z_t) = \begin{bmatrix} 1-\sigma & 0 \\ 0 & -(1/bv\Pi\delta + 1/\beta) \\ (1-\rho_R)\alpha^y(z_t^m) & (1-\rho_R)\alpha^\pi(z_t^m) \\ (1-\rho_\tau)\gamma^y(z_t^f) & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\mathbf{D} = \begin{bmatrix} \sigma & 0 & 0 & 0 & 0 \\ 1/bg & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \sigma/(\varphi+\sigma)+1/(\varphi+\sigma) & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathbf{N} = \begin{bmatrix} \rho_g & 0 & 0 & 0 & 0 \\ 0 & \rho_\theta & 0 & 0 & 0 \\ 0 & 0 & \rho_\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

C   Obtaining the Coefficients of the Logistic Functions

Each of the elements in the matrices $\mathbf{P}(z_t), \mathbf{Q}(z_t), \mathbf{R}(z_t), \mathbf{S}(z_t)$ have the bivariate logistic functional form (38), which is reproduced here for convenience and is illustrated in Figure 10:

$$F(\mathbf{z}_t) = F_0 + F_1 \frac{\frac{1}{1+\exp\left(-F_{2m}(z_t^m - F_{3m})\right)} \frac{1}{1+\exp\left(-F_{2f}\left(z_t^f - F_{3f}\right)\right)}}{1 - F_4 \frac{\exp\left(-F_{2m}(z_t^m - F_{3m})\right)}{1+\exp\left(-F_{2m}(z_t^m - F_{3m})\right)} \frac{\exp\left(-F_{2f}\left(z_t^f - F_{3f}\right)\right)}{1+\exp\left(-F_{2f}\left(z_t^f - F_{3f}\right)\right)}}.$$

Figure 10: Bivariate Logistic Function
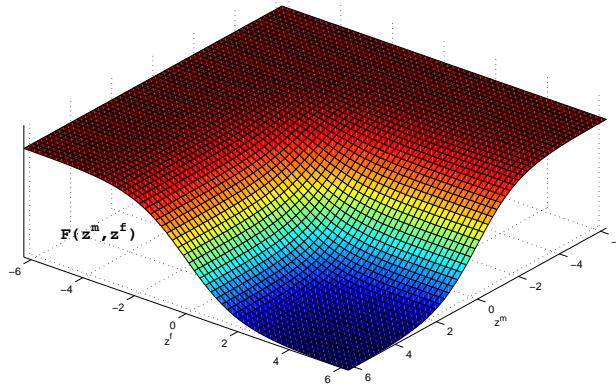
To avoid computational costs, we find the solutions to the coefficients in $\mathbf{R}(z_t)$ and $\mathbf{S}(z_t)$ only, solving for inflation and output, leaving the full solution of the model to be accounted for the structural equations of the state variables (25)-(32). Given the structure of the vector of shocks, $\mathbf{S}(z_t)$ is a matrix with 10 distinct elements. Given the structure of the vector of state variables, $\mathbf{R}(z_t)$ is a matrix with 10 distinct elements as well, although it has 14 entries.

Therefore, in total there are $20 \times 6 = 120$ coefficients of the logistic functions to be found to obtain a solution.

C.1 Finding F_0 and F_1

Notice that the time-varying policy rule coefficients have the following limiting combinations (bounds):

	$z_t^m \to +\infty$	$z_t^m \to -\infty$
$z_t^f \to +\infty$	$\alpha^\pi(z_t^m) = \alpha_0^\pi + \alpha_1^\pi \quad \alpha^y(z_t^m) = \alpha_0^y + \alpha_1^y$ $\gamma^b(z_t^f) = \gamma_0^b + \gamma_1^b \quad \gamma^y(z_t^f) = \gamma_0^y + \gamma_1^y$	
$z_t^f \to -\infty$		$\alpha^\pi(z_t^m) = \alpha_0^\pi \quad \alpha^y(z_t^m) = \alpha_0^y$ $\gamma^b(z_t^f) = \gamma_0^b \quad \gamma^y(z_t^f) = \gamma_0^y$

Also, notice that $F(\mathbf{z}_t)$ has the following limiting expressions:

	$z_t^m \to +\infty$	$z_t^m \to -\infty$
$z_t^f \to +\infty$	$F(\mathbf{z}_t) = F_0 + F_1$	
$z_t^f \to -\infty$		$F(\mathbf{z}_t) = F_0$

To obtain F_0 and F_1, we solve the constant-coefficient versions of the model two times using Uhlig (1998), at the two different limiting combinations of the latent factors z_t^m and z_t^f. We solve for the two bounds making sure that the solutions deliver determinacy of the equilibria at the bounds.

C.2 Finding $F_{2m}, F_{2f}, F_{3m}, F_{3f}$

Substitute (36)-(37) in (33)-(35) to obtain

$$\left[\mathbf{A}(z_t)\mathbf{P}(z_t) + \mathbf{C}(z_t)\mathbf{R}(z_t) + \mathbf{B}(z_t^f)\right] k_{t-1} + \left[\mathbf{A}(z_t)\mathbf{Q}(z_t) + \mathbf{C}(z_t)\mathbf{S}(z_t) + \mathbf{D}\right] u_t = 0,$$

$$\left\{\left[\mathbf{J}\bar{\mathbf{R}}(z_t) + \mathbf{G}\right]\mathbf{P}(z_t) + \mathbf{K}\mathbf{R}(z_t)\right\} k_{t-1} + \left\{\left[\mathbf{J}\bar{\mathbf{R}}(z_t) + \mathbf{G}\right]\mathbf{Q}(z_t) + \mathbf{J}\bar{\mathbf{S}}(z_t)\mathbf{N} + \mathbf{K}\mathbf{S}(z_t) + \mathbf{M}\right\} u_t = 0,$$

where $\bar{\mathbf{R}}(z_t) \equiv \mathbb{E}_t \mathbf{R}(z_{t+1})$ and $\bar{\mathbf{S}}(z_t) \equiv \mathbb{E}_t \mathbf{S}(z_{t+1})$. By the undetermined coefficients method, we have

$$\mathbf{A}(z_t)\mathbf{P}(z_t) + \mathbf{C}(z_t)\mathbf{R}(z_t) + \mathbf{B}(z_t^f) = 0$$
$$\mathbf{A}(z_t)\mathbf{Q}(z_t) + \mathbf{C}(z_t)\mathbf{S}(z_t) + \mathbf{D} = 0$$
$$\left[\mathbf{J}\bar{\mathbf{R}}(z_t) + \mathbf{G}\right]\mathbf{P}(z_t) + \mathbf{K}\mathbf{R}(z_t) = 0$$
$$\left[\mathbf{J}\bar{\mathbf{R}}(z_t) + \mathbf{G}\right]\mathbf{Q}(z_t) + \mathbf{J}\bar{\mathbf{S}}(z_t)\mathbf{N} + \mathbf{K}\mathbf{S}(z_t) + \mathbf{M} = 0.$$

Then, solving for $\mathbf{R}(z_t)$ and $\mathbf{S}(z_t)$, we have

$$[\mathbf{K} + [\mathbf{G} + \mathbf{J}\bar{\mathbf{R}}(\mathbf{z}_t)][-\mathbf{A}(\mathbf{z}_t)]^{-1}\mathbf{C}(\mathbf{z}_t)]\mathbf{R}(\mathbf{z}_t) = -[\mathbf{G} + \mathbf{J}\bar{\mathbf{R}}(\mathbf{z}_t)][-\mathbf{A}(\mathbf{z}_t)]^{-1}\mathbf{B}(z_t^f)$$
$$[\mathbf{K} + [\mathbf{G} + \mathbf{J}\bar{\mathbf{R}}(\mathbf{z}_t)][-\mathbf{A}(\mathbf{z}_t)]^{-1}\mathbf{C}(\mathbf{z}_t)]\mathbf{S}(\mathbf{z}_t) = -\mathbf{J}\bar{\mathbf{S}}(\mathbf{z}_t)\mathbf{N} - \mathbf{M} - [\mathbf{G} + \mathbf{J}\bar{\mathbf{R}}(\mathbf{z}_t)][-\mathbf{A}(\mathbf{z}_t)]^{-1}\mathbf{D}.$$

Let

$$\mathbf{T}(\mathbf{z}_t) \equiv \mathbf{K} + [\mathbf{G} + \mathbf{J}\bar{\mathbf{R}}(\mathbf{z}_t)][-\mathbf{A}(\mathbf{z}_t)]^{-1}\mathbf{C}(\mathbf{z}_t) \tag{57}$$

$$\mathbf{U}(\mathbf{z}_t) \equiv -[\mathbf{G} + \mathbf{J}\bar{\mathbf{R}}(\mathbf{z}_t)][-\mathbf{A}(\mathbf{z}_t)]^{-1}\mathbf{B}(z_t^f) \tag{58}$$

$$\mathbf{V}(\mathbf{z}_t) \equiv -\mathbf{J}\bar{\mathbf{S}}(\mathbf{z}_t)\mathbf{N} - \mathbf{M} - [\mathbf{G} + \mathbf{J}\bar{\mathbf{R}}(\mathbf{z}_t)][-\mathbf{A}(\mathbf{z}_t)]^{-1}\mathbf{D}. \tag{59}$$

Then, using the relevant elements of $\mathbf{R}(z_t)$ and $\mathbf{S}(z_t)$, we can write

$$\begin{bmatrix} \mathbf{T}(\mathbf{z}_t) & 0 \\ 0 & \mathbf{T}(\mathbf{z}_t) \end{bmatrix} \begin{bmatrix} \mathbf{R}(\mathbf{z}_t) \\ \mathbf{S}(\mathbf{z}_t) \end{bmatrix} = \begin{bmatrix} \mathbf{U}(\mathbf{z}_t) \\ \mathbf{V}(\mathbf{z}_t) \end{bmatrix}. \tag{60}$$

Given F_0, F_1 and F_4, $F(\mathbf{z}_t)$ takes the following expressions that include F_{2m}, F_{2f}, F_{3m} and F_{3f}:

- $$\lim_{z_t^f \to \infty} F(0, z_t^f) = F_0 + \frac{F_1}{1 + \exp(F_{2m}F_{3m})}$$

- $$\lim_{z_t^m \to \infty} F(z_t^m, 0) = F_0 + \frac{F_1}{1 + \exp(F_{2f}F_{3f})}$$

- $$\frac{\partial}{\partial z_t^m} F(z_t^m, 0)|_{z_t^m=0} = g_m\left(F_1, F_{2m}, F_{3m}, F_{2f}, F_{3f}, F_4\right)$$

- $$\frac{\partial}{\partial z_t^f} F(z_t^m, 0)|_{z_t^f=0} = g_f\left(F_1, F_{2m}, F_{3m}, F_{2f}, F_{3f}, F_4\right),$$

where $g_m(\cdot)$ and $g_f(\cdot)$ are nonlinear functions of their respective arguments.

Then, to find F_{2m}, F_{2f}, F_{3m} and F_{3f} we solve the following system of equations for the relevant elements of $\mathbf{R}(z_t)$ and $\mathbf{S}(z_t)$:

$$\begin{bmatrix} \lim_{z_t^f \to \infty} \mathbf{T}(0, z_t^f) & 0 \\ 0 & \lim_{z_t^f \to \infty} \mathbf{T}(0, z_t^f) \end{bmatrix} \begin{bmatrix} \lim_{z_t^f \to \infty} \mathbf{R}(0, z_t^f) \\ \lim_{z_t^f \to \infty} \mathbf{S}(0, z_t^f) \end{bmatrix} = \begin{bmatrix} \lim_{z_t^f \to \infty} \mathbf{U}(0, z_t^f) \\ \lim_{z_t^f \to \infty} \mathbf{V}(0, z_t^f) \end{bmatrix},$$

$$\begin{bmatrix} \lim_{z_t^m \to \infty} \mathbf{T}(z_t^m, 0) & 0 \\ 0 & \lim_{z_t^m \to \infty} \mathbf{T}(z_t^m, 0) \end{bmatrix} \begin{bmatrix} \lim_{z_t^m \to \infty} \mathbf{R}(z_t^m, 0) \\ \lim_{z_t^m \to \infty} \mathbf{S}(z_t^m, 0) \end{bmatrix} = \begin{bmatrix} \lim_{z_t^m \to \infty} \mathbf{U}(z_t^m, 0) \\ \lim_{z_t^m \to \infty} \mathbf{V}(z_t^m, 0) \end{bmatrix},$$

$$\begin{bmatrix} \frac{\partial}{\partial z_t^m} \mathbf{T}(z_t^m, 0)|_{z_t^m=0} & 0 \\ 0 & \frac{\partial}{\partial z_t^m} \mathbf{T}(z_t^m, 0)|_{z_t^m=0} \end{bmatrix} \begin{bmatrix} \mathbf{R}(0,0) \\ \mathbf{S}(0,0) \end{bmatrix} +$$

$$+ \begin{bmatrix} \mathbf{T}(0,0) & 0 \\ 0 & \mathbf{T}(0,0) \end{bmatrix} \begin{bmatrix} \frac{\partial}{\partial z_t^m} \mathbf{R}(z_t^m, 0)|_{z_t^m=0} \\ \frac{\partial}{\partial z_t^m} \mathbf{S}(z_t^m, 0)|_{z_t^m=0} \end{bmatrix} =$$

$$
= \begin{bmatrix} \dfrac{\partial}{\partial z_t^m} \, \mathbf{U}(z_t^m,0)|_{z_t^m=0} \\ \dfrac{\partial}{\partial z_t^m} \, \mathbf{V}(z_t^m,0)|_{z_t^m=0} \end{bmatrix},
$$

$$
\begin{bmatrix} \dfrac{\partial}{\partial z_t^f} \, \mathbf{T}(0,z_t^f)\Big|_{z_t^f=0} & 0 \\ 0 & \dfrac{\partial}{\partial z_t^f} \, \mathbf{T}(0,z_t^f)\Big|_{z_t^f=0} \end{bmatrix} \begin{bmatrix} \mathbf{R}(0,0) \\ \mathbf{S}(0,0) \end{bmatrix} +
$$

$$
+ \begin{bmatrix} \mathbf{T}(0,0) & 0 \\ 0 & \mathbf{T}(0,0) \end{bmatrix} \begin{bmatrix} \dfrac{\partial}{\partial z_t^f} \, \mathbf{R}(0,z_t^f)\Big|_{z_t^f=0} \\ \dfrac{\partial}{\partial z_t^f} \, \mathbf{S}(0,z_t^f)\Big|_{z_t^f=0} \end{bmatrix} =
$$

$$
= \begin{bmatrix} \dfrac{\partial}{\partial z_t^f} \, \mathbf{U}(0,z_t^f)\Big|_{z_t^f=0} \\ \dfrac{\partial}{\partial z_t^f} \, \mathbf{V}(0,z_t^f)\Big|_{z_t^f=0} \end{bmatrix},
$$

where

$$
\lim_{z_t^f \to +\infty} \mathbf{A}(0,z_t^f) = \begin{bmatrix}
-1 & 0 & 1/(R-1) & 0 & 0 & 0 & 0 \\
1/vb & -1 & 0 & -\tau/b & -(1/v\Pi\delta + 1/\beta) & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & -(1-\rho_R)\left(\alpha_0^y + \frac{\alpha_1^y}{1+\exp(\alpha_2^y\alpha_3^y)}\right) \\
0 & 0 & 0 & -1 & 0 & 0 & -(1-\rho_\tau)(\gamma_0^y + \gamma_1^y) \\
0 & 0 & 0 & 0 & -1 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1
\end{bmatrix}
$$

$$
\lim_{z_t^m \to +\infty} \mathbf{A}(z_t^m,0) = \begin{bmatrix}
-1 & 0 & 1/(R-1) & 0 & 0 & 0 & 0 \\
1/vb & -1 & 0 & -\tau/b & -(1/v\Pi\delta + 1/\beta) & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & -(1-\rho_R)(\alpha_0^y + \alpha_1^y) \\
0 & 0 & 0 & -1 & 0 & 0 & -(1-\rho_\tau)\left(\gamma_0^y + \frac{\gamma_1^y}{1+\exp(\gamma_2^y\gamma_3^y)}\right) \\
0 & 0 & 0 & 0 & -1 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1
\end{bmatrix}
$$

$$
\lim_{z_t^f \to +\infty} \mathbf{C}(0,z_t^f) = \begin{bmatrix}
1-\sigma & 0 \\
0 & -(1/v\Pi\delta + 1/\beta) \\
(1-\rho_R)\left(\alpha_0^y + \frac{\alpha_1^y}{1+\exp(\alpha_2^y\alpha_3^y)}\right) & (1-\rho_R)\left(\alpha_0^\pi + \frac{\alpha_1^\pi}{1+\exp(\alpha_2^\pi\alpha_3^\pi)}\right) \\
(1-\rho_\tau)(\gamma_0^y + \gamma_1^y) & 0 \\
1 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix},
$$

$$
\lim_{z_t^f \to +\infty} \mathbf{C}(z_t^m,0) = \begin{bmatrix}
1-\sigma & 0 \\
0 & -(1/v\Pi\delta + 1/\beta) \\
(1-\rho_R)(\alpha_0^y + \alpha_1^y) & (1-\rho_R)(\alpha_0^\pi + \alpha_1^\pi) \\
(1-\rho_\tau)\left(\gamma_0^y + \frac{\gamma_1^y}{1+\exp(\gamma_2^y\gamma_3^y)}\right) & 0 \\
1 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix},
$$

$$\mathbf{B}(0) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1/v\Pi\delta & 1/\beta & 1/\beta & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_R & 0 & 0 & 0 & 0 \\ 0 & (1-\rho_\tau)\left(\gamma_0^b + \frac{\gamma_1^b}{1+\exp(\gamma_2^b\gamma_3^b)}\right) & 0 & \rho_\tau & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\lim_{z_t^f \to +\infty} \mathbf{B}(z_t^f) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1/v\Pi\delta & 1/\beta & 1/\beta & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_R & 0 & 0 & 0 & 0 \\ 0 & (1-\rho_\tau)\left(\gamma_0^b + \gamma_1^b\right) & 0 & \rho_\tau & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

$$\frac{\partial}{\partial z_t^m} \mathbf{A}(z_t^m, 0)\big|_{z_t^m=0} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -(1-\rho_R)\frac{\alpha_1^y \alpha_2^y \exp(\alpha_2^y \alpha_3^y)}{(1+\exp(\alpha_2^y \alpha_3^y))^2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\frac{\partial}{\partial z_t^m} \mathbf{C}(z_t^m, 0)\big|_{z_t^m=0} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ (1-\rho_R)\frac{\alpha_1^y \alpha_2^y \exp(\alpha_2^y \alpha_3^y)}{(1+\exp(\alpha_2^y \alpha_3^y))^2} & (1-\rho_R)\frac{\alpha_1^\pi \alpha_2^\pi \exp(\alpha_2^\pi \alpha_3^\pi)}{(1+\exp(\alpha_2^\pi \alpha_3^\pi))^2} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\frac{\partial}{\partial z_t^f} \mathbf{B}(z_t^f)\big|_{z_t^f=0} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & (1-\rho_\tau)\frac{\gamma_1 \gamma_2 \exp(\gamma_2 \gamma_3)}{(1+\exp(\gamma_2 \gamma_3))^2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

C.3   Finding F_4

Given $F_0, F_1, F_{2m}, F_{2f}, F_{3m}$ and F_{3f}, we have the following expression that includes F_4:

$$F(0) = F_0 + F_1 \frac{\frac{1}{1+\exp(F_{2m}F_{3m})} \cdot \frac{1}{1+\exp\left(F_{2f}F_{3f}\right)}}{1 - F4\frac{\exp(F_{2m}F_{3m})}{1+\exp(F_{2m}F_{3m})} \cdot \frac{\exp\left(F_{2f}F_{3f}\right)}{1+\exp\left(F_{2f}F_{3f}\right)}}$$

Then, to find F_4 we solve the following system of equations:

$$\begin{bmatrix} \mathbf{T}(0) & 0 \\ 0 & \mathbf{T}(0) \end{bmatrix} \begin{bmatrix} \mathbf{R}(0) \\ \mathbf{S}(0) \end{bmatrix} = \begin{bmatrix} \mathbf{U}(0) \\ \mathbf{V}(0) \end{bmatrix}.$$

D Computation of $\bar{\mathbf{R}}(z_t)$ and $\bar{\mathbf{S}}(z_t)$

To obtain $\mathbb{E}_t \mathbf{R}(\mathbf{z}_{t+1})$ and $\mathbb{E}_t \mathbf{S}(\mathbf{z}_{t+1})$ we perform a second order Taylor expansion to

$$
\begin{aligned}
F(\mathbf{z}_{t+1}) &= F(z_{t+1}^m, z_{t+1}^f) \\
&= F(\rho_{z^m} z_t^m + \xi_{t+1}^m, \rho_{z^f} z_t^f + \xi_{t+1}^f)
\end{aligned}
$$

around $\xi_{t+1}^m = \xi_{t+1}^f = 0$ as follows:

$$
\begin{aligned}
F(z_{t+1}^m, z_{t+1}^f) &\approx F(\rho_{z^m} z_t^m, \rho_{z^f} z_t^f) + F_m(\rho_{z^m} z_t^m, \rho_{z^f} z_t^f)\xi_{t+1}^m + F_f(\rho_{z^m} z_t^m, \rho_{z^f} z_t^f)\xi_{t+1}^f + \\
&\quad + 0.5 F_{mm}(\rho_{z^m} z_t^m, \rho_{z^f} z_t^f)(\xi_{t+1}^m)^2 + 0.5 F_{ff}(\rho_{z^m} z_t^m, \rho_{z^f} z_t^f)(\xi_{t+1}^f)^2 + \\
&\quad + F_{mf}(\rho_{z^m} z_t^m, \rho_{z^f} z_t^f)\xi_{t+1}^m \xi_{t+1}^f,
\end{aligned}
$$

where $F_i(\cdot)$ and $F_{ij}(\cdot)$ denote the partial derivative of $F(\cdot)$ with respect to ξ^i and the partial derivative of $F(\cdot)$ with respect to ξ^i first and then with respect to ξ^j, for $j = m, f$, respectively.

Then, taking expectation conditional to z_t^m and z_t^f, we get

$$
\begin{aligned}
\mathbb{E}\left(F(z_{t+1}^m, z_{t+1}^f) | z_t^m, z_t^f\right) &\approx F(\rho_{z^m} z_t^m, \rho_{z^f} z_t^f) + \\
&\quad + 0.5 F_{mm}(\rho_{z^m} z_t^m, \rho_{z^f} z_t^f) + 0.5 F_{ff}(\rho_{z^m} z_t^m, \rho_{z^f} z_t^f) + \\
&\quad + F_{mf}(\rho_{z^m} z_t^m, \rho_{z^f} z_t^f)\kappa.
\end{aligned}
$$

Notice that the expectation of the coefficient matrices involves κ, the correlation coefficient between the latent factors that drive the evolution of the monetary and fiscal policy rule coefficients. When taking expectations, the agents of the model incorporate the degree of interdependence in policymaking.

E Verifying the Guessed Functional Form

Having obtained the coefficients of the logistic functions that characterize the solution, it is necessary to verify that the guessed functional forms for $\mathbf{R}(z_t)$ and $\mathbf{S}(z_t)$ are indeed logistic. Recall the system of equations in (60):

$$
\begin{bmatrix} \mathbf{T}(\mathbf{z}_t) & 0 \\ 0 & \mathbf{T}(\mathbf{z}_t) \end{bmatrix} \begin{bmatrix} \mathbf{R}(\mathbf{z}_t) \\ \mathbf{S}(\mathbf{z}_t) \end{bmatrix} = \begin{bmatrix} \mathbf{U}(\mathbf{z}_t) \\ \mathbf{V}(\mathbf{z}_t) \end{bmatrix}.
$$

Notice that if both z_t^m and z_t^f were white noise processes, then both $\bar{\mathbf{R}}(\mathbf{z}_t)$ and $\bar{\mathbf{S}}(\mathbf{z}_t)$ would be constant matrices. In that case, if the coefficients on the output gap of both policy rules were constant, $\mathbf{T}(\cdot)$ in (57) would be a function of z_t^m only through $\mathbf{C}(z_t^m)$, $\mathbf{U}(\cdot)$ in (58) would be a function of z_t^f only through $\mathbf{B}(z_t^f)$, and $\mathbf{V}(\cdot)$ would be constant. This implies that some of the elements of $\mathbf{R}(\cdot)$ are bivariate logistic functions, and that all the elements of $\mathbf{S}(\cdot)$ are univariate logistic functions that depend on z_t^m. In particular, only the entries in the second column of $\mathbf{R}(\cdot)$, those that relate the behavior of inflation and the output

gap with the evolution of lagged debt, depend on both z_t^m and z_t^f; all the other elements are univariate logistic functions dependent on z_t^m. Hence, in the absence of persistence in the latent factors, it is easy to show that the solution matrices follow indeed bivariate (or univariate) logistic functions. It can be shown that the same argument holds if the policy rule coefficients on the output gap are time varying.

In any case, however, and in particular when the latent factors are persistent, the solution has to be such that in (60) we do not 'divide by zero'. The issue has to do with the fact that since the transition between states is continuous, as opposed to the Markov-switching setup of policy rule coefficients, for some values of $\mathbf{z}_t$ there may exist a discontinuity in the solution. To see this, consider the simple Fisherian model:

$$R_t = \mathbb{E}_t \pi_{t+1} + u_t$$
$$R_t = \alpha(z_t)\pi_t$$
$$u_{t+1} = \rho_u u_t + \varepsilon_{t+1}, \qquad \varepsilon_t \sim \text{iid } \mathbb{N}(0, \sigma), \qquad |\rho_u| < 1$$
$$z_{t+1} = \rho_z z_t + \xi_{t+1}, \qquad \xi_t \sim \text{iid } \mathbb{N}(0, 1), \qquad 0 \leq \rho_z \leq 1$$
$$z_t \perp u_s \quad \forall s, t$$
$$\alpha(z_t) = \alpha_0 + \frac{\alpha_1}{1 + \exp(-\alpha_2 z_t)},$$

where the notation is the same as in the new Keynesian model, since this is a particular version of the model presented in the paper.

We guess that the fundamental solution is given by

$$\pi_t = a(z_t)u_t,$$

where

$$a(z_t) = a_0 + \frac{a_1}{1 + \exp(-a_2(z_t - a_3))}.$$

Solving for $a(z_t)$, we have

$$\frac{\rho_u \mathbb{E}_t a(z_{t+1}) + 1}{\alpha(z_t)} = a(z_t).$$

Performing a first-order Taylor expansion to $a(z_{t+1})$ around $\xi_{t+1} = 0$, we get

$$\bar{a}(z_t) = a(\rho_z z_t) = a_0 + \frac{a_1}{1 + \exp(-a_2(\rho_z z_t - a_3))}$$

If $\rho_z = 1$, it is easy to see that

$$a(z_t) = \frac{1}{\alpha(z_t) - \rho_u}.$$

Hence, for the solution to be logistic —and continuous, we need $\alpha(z_t) > \rho_u \ \forall z_t$, which is the same as $\alpha_0 > \rho_u$, given the monotonicity of $\alpha(z_t)$.

To derive the analogous condition in the more general case of the model presented in the

paper, rewrite (60) for $\mathbf{S}(\mathbf{z}_t)$ ignoring the last two terms of (59), as follows:

$$\mathbf{T}(\mathbf{z}_t)\mathbf{S}(\mathbf{z}_t) = -\mathbf{J}\bar{\mathbf{S}}(\mathbf{z}_t)\mathbf{N},$$

which can be rewritten as

$$(I_5 \otimes \mathbf{T}(\mathbf{z}_t)) \operatorname{vec}(\mathbf{S}(\mathbf{z}_t)) = -\left(\mathbf{N}^\top \otimes \mathbf{J}\right) \operatorname{vec}\left(\bar{\mathbf{S}}(\mathbf{z}_t)\right).$$

Hence, a sufficient condition for the continuity of the solution is that each of the elements of $-\left(I_5 \otimes \mathbf{T}(-\infty)\right)$ is greater than the corresponding element of $\left(\mathbf{N}^\top \otimes \mathbf{J}\right)$.

F Endogeneity Setup

Let GSP_t be the demeaned government spending to output ratio in period t, $\mathrm{M2G}_t$ the annual rate of growth of M2 in period t, and CMP_t commodity price inflation. In order to account for the existence of endogeneity, the observation equations (39)-(40) of the state-space model have to be modified by introducing a system of simultaneous equations. To that end, let

$$y_{1,t} = \mathrm{INT}_t,$$
$$y_{2,t} = \mathrm{TAX}_t,$$
$$\mathbf{x}_{1,t} = [\mathrm{INF}_t, \mathrm{GAP}_t],$$
$$\mathbf{x}_{2,t} = [\mathrm{DBT}_{t-1}, \mathrm{GAP}_t],$$
$$\boldsymbol{\alpha}(z_t^m) = [\alpha^\pi(z_t^m), \alpha^y(z_t^m)]',$$
$$\boldsymbol{\gamma}(z_t^f) = [\gamma^b(z_t^f), \gamma^y(z_t^f)]',$$
$$\mathbf{w}_{1,t} = [\{\mathrm{INF}_{t-s}\}_{s=1}^4, \{\mathrm{GAP}_{t-s}\}_{s=1}^4, \{\mathrm{GSP}_{t-s}\}_{s=1}^4, \{\mathrm{M2G}_{t-s}\}_{s=1}^4, \{\mathrm{CMP}_{t-s}\}_{s=1}^4],$$
$$\mathbf{w}_{2,t} = [\mathrm{DBT}_{t-1}, \{\mathrm{INF}_{t-s}\}_{s=1}^4, \{\mathrm{GAP}_{t-s}\}_{s=1}^4, \{\mathrm{GSP}_{t-s}\}_{s=1}^4, \{\mathrm{M2G}_{t-s}\}_{s=1}^4, \{\mathrm{CMP}_{t-s}\}_{s=1}^4].$$

Hence, conditional on $\mathbf{z}_t$, the state-space model (39)-(44) can be divided into two models: one for the interest rate equation, and another for the tax rate equation. The observation equations of each of the models are:

$$y_{1,t} = \rho_R y_{1,t-1} + (1 - \rho_R)\mathbf{x}_{1,t}\boldsymbol{\alpha}(z_t^m) + v_t^R \tag{61}$$
$$\mathbf{x}_{1,t} = \mathbf{w}_{1,t}\Pi_1 + \mathbf{v}_{1,t}, \tag{62}$$

and

$$y_{2,t} = \rho_\tau y_{2,t-1} + (1 - \rho_\tau)\mathbf{x}_{2,t}\boldsymbol{\gamma}(z_t^f) + v_t^\tau \tag{63}$$
$$\mathbf{x}_{2,t} = \mathbf{w}_{2,t}\Pi_2 + \mathbf{v}_{2,t}. \tag{64}$$

Here, Π_1 and Π_2 are conformable parameter matrices, and $\mathbf{v}_{i,t} \sim$ iid $\mathbb{N}(0, \Psi_i)$ for $i = 1, 2$. We introduce endogeneity in (61)-(62) and (63)-(64) by specifying

$$v_t^{R*} = \mathbf{v}_{1,t}\boldsymbol{\delta}_1 + e_t^R,$$

$$v_t^{\tau*} = \mathbf{v}_{2,t}\boldsymbol{\delta}_2 + e_t^\tau,$$

where $v_t^j = \sigma_{j,t}v_t^{j*}$ for $j = R, \tau$, and

$$e_t^R|y_{1,t-1}, \mathbf{v}_{1,t} \sim \text{iid } \mathbb{N}\left(0, 1 - \boldsymbol{\delta}_1'\Psi_1\boldsymbol{\delta}_1\right),$$
$$e_t^\tau|y_{2,t-1}, \mathbf{v}_{2,t} \sim \text{iid } \mathbb{N}\left(0, 1 - \boldsymbol{\delta}_2'\Psi_2\boldsymbol{\delta}_2\right).$$

Let $\mathbf{y}_t = [y_{1,t}, y_{2,t}]'$ and $\mathbf{Y}_t = \{\mathbf{y}_s\}_{s=1}^t$. Appendix G shows how to obtain the conditional likelihood function of $\mathbf{Y}_T$.

G The Likelihood Function of the Model with Endogeneity, Time-Varying Coefficients and Stochastic Volatility

Notice that the joint density function of $y_{i,t}$ and $\mathbf{v}_{i,t}$, conditional on $y_{i,t-1}$, for $i = 1, 2$, and the latent factors and stochastic volatility (these last two omitted from the density functions below) can be written as

$$p\left(y_{i,t}, \mathbf{v}_{i,t}|y_{i,t-1}, \Theta_{y_i}, \Theta_{\mathbf{v}_i}\right) = p_y\left(y_{i,t}|y_{i,t-1}, \mathbf{v}_{i,t}, \Theta_{y_i}, \Theta_{\mathbf{v}_i}\right)p_{\mathbf{v}}(\mathbf{v}_{i,t}|\Theta_{\mathbf{v}_i})$$
$$= p_y\left(y_{i,t}|y_{i,t-1}, \mathbf{x}_{i,t}, \mathbf{v}_{i,t}, \Theta_{y_i}, \Theta_{\mathbf{x}_i}\right)p_{\mathbf{x}}(\mathbf{x}_{i,t}|\mathbf{w}_{i,t}, \Theta_{\mathbf{x}_i}),$$

where $\Theta_{\mathbf{v}_i}$ for $i = 1, 2$ is the set of parameters $\{\boldsymbol{\delta}_i, \Pi_i, \Psi_i\}$. Adding the conditionality on latent factors and stochastic volatility,

$$y_{1,t}|y_{1,t-1}, \mathbf{x}_{1,t}, \mathbf{v}_{1,t}, z_t^m, \sigma_{R,t}, \Theta_{y_1}, \Theta_{\mathbf{x}_1} \sim \mathbb{N}\left(\rho_R y_{1,t-1} + (1-\rho_R)\mathbf{x}_{1,t}\boldsymbol{\alpha}(z_t^m) + \sigma_{R,t}\left(\mathbf{v}_{1,t}\boldsymbol{\delta}_1 + e_t^R\right), 1 - \boldsymbol{\delta}_1'\Psi_1\boldsymbol{\delta}_1\right)$$

$$y_{2,t}|y_{2,t-1}, \mathbf{x}_{2,t}, \mathbf{v}_{2,t}, z_t^f, \sigma_{\tau,t}, \Theta_{y_2}, \Theta_{\mathbf{x}_2} \sim \mathbb{N}\left(\rho_\tau y_{2,t-1} + (1-\rho_\tau)\mathbf{x}_{2,t}\boldsymbol{\gamma}(z_t^f) + \sigma_{\tau,t}\left(\mathbf{v}_{2,t}\boldsymbol{\delta}_2 + e_t^\tau\right), 1 - \boldsymbol{\delta}_2'\Psi_2\boldsymbol{\delta}_2\right)$$

$$\mathbf{x}_{i,t}|\mathbf{w}_{i,t}, \Theta_{\mathbf{x}_i} \sim \mathbb{N}\left(\mathbf{w}_{i,t}\Pi_i, \Psi_i\right), \text{ for } i = 1, 2.$$

Let $\mathbf{Y}_{i,t} = \{\mathbf{y}_{i,s}\}_{s=1}^t$, for $i = 1, 2$ and let $\mathbf{X}_{i,t}$ and $\mathbf{V}_{i,t}$ be defined in a similar fashion. Let $\mathbf{Z}_{k,t} = \{z_s^k\}_{s=1}^t$ for $k = m, f$ and let $\mathbf{H}_{j,t} = \{\sigma_{j,s}\}_{s=1}^t$ for $j = R, \tau$. Then, the conditional log-likelihood functions of $\mathbf{Y}_{1,T}$ given $\mathbf{Y}_{1,T-1}, \mathbf{X}_{1,t}, \mathbf{V}_{1,t}, \mathbf{Z}_{m,t}, \mathbf{H}_{R,t}$, and of $\mathbf{Y}_{2,T}$ given $\mathbf{Y}_{2,T-1}, \mathbf{X}_{2,t}, \mathbf{V}_{2,t}, \mathbf{Z}_{f,t}, \mathbf{H}_{\tau,t}$ are, respectively,

$$\mathfrak{L}_{1,T}\left(\Theta_{y_1}(\hat{\Theta}_{\mathbf{x}_1})\right) = \sum_{t=1}^T l_{1,t}\left(\Theta_{y_1}(\hat{\Theta}_{\mathbf{x}_1})\right),$$

$$\mathfrak{L}_{2,T}\left(\Theta_{y_2}(\hat{\Theta}_{\mathbf{x}_2})\right) = \sum_{t=1}^T l_{2,t}\left(\Theta_{y_2}(\hat{\Theta}_{\mathbf{x}_2})\right),$$

where

$$\hat{\Theta}_{\mathbf{x}_i} = \max_{\Theta_{\mathbf{x}_i}} \sum_{t=1}^T \log\left(p_{\mathbf{x}}(\mathbf{x}_{i,t}|\mathbf{w}_{i,t}, \Theta_{\mathbf{x}_i})\right)$$

is the maximum likelihood estimator of $\Theta_{\mathbf{x}_i}$ for $i = 1, 2$, and

$$l_{1,t}\left(\Theta_{y_1}(\hat{\Theta}_{\mathbf{x}_1})\right) = \log\left(p_y\left(y_{1,t}|y_{1,t-1}, \mathbf{x}_{1,t}, \mathbf{v}_{1,t}, z_t^m, \sigma_{R,t}, \Theta_{y_1}, \hat{\Theta}_{\mathbf{x}_1}\right)\right),$$

$$l_{2,t}\left(\Theta_{y_2}(\hat{\Theta}_{\mathbf{x}_2})\right) = \log\left(p_y\left(y_{2,t}|y_{2,t-1}, \mathbf{x}_{2,t}, \mathbf{v}_{2,t}, z_t^f, \sigma_{\tau,t}, \Theta_{y_2}, \hat{\Theta}_{\mathbf{x}_2}\right)\right).$$

This is the two-stage conditional maximum likelihood estimation procedure suggested by Vuong (1984). Let $\mathbf{X}_t = \cup_{i=1}^2 \mathbf{X}_{i,t}$, and let $\mathbf{V}_t$ be defined in a similar way. Let $\mathbf{Z}_t = \{\mathbf{z}_s\}_{s=1}^t$ and $\mathbf{H}_t = \{\mathbf{h}_s\}_{s=1}^t$. The log-likelihood function of $\mathbf{Y}_T$ given $\mathbf{Y}_{T-1}, \mathbf{X}_T, \mathbf{V}_T, \mathbf{Z}_T, \mathbf{H}_T$ is

$$\mathfrak{L}_T\left(\Theta_y(\hat{\Theta}_{\mathbf{x}})\right) = \sum_{t=1}^T l_t\left(\Theta_y(\hat{\Theta}_{\mathbf{x}})\right),$$

where

$$l_t\left(\Theta_y(\hat{\Theta}_{\mathbf{x}})\right) = l_{1,t}\left(\Theta_{y_1}(\hat{\Theta}_{\mathbf{x}_1})\right) + l_{2,t}\left(\Theta_{y_2}(\hat{\Theta}_{\mathbf{x}_2})\right),$$

and $\hat{\Theta}_{\mathbf{x}} = \hat{\Theta}_{\mathbf{x}_1} \cup \hat{\Theta}_{\mathbf{x}_2}$, $\Theta_y = \Theta_{y_1} \cup \Theta_{y_2}$.

H Bayesian Estimation

The method presented here extends the estimation method implemented in Geweke and Tanizaki (2001) by allowing endogeneity. Let $p_{\mathbf{y}}(\mathbf{y}_t|\mathbf{X}_t, \mathbf{V}_t, \mathbf{Z}_t, \mathbf{H}_t, \Theta_y)$ denote the conditional density of $\mathbf{y}_t$ given $\mathbf{X}_t, \mathbf{V}_t, \mathbf{Z}_t, \mathbf{H}_t$ and Θ_y. Let $p_{\mathbf{z}}(\mathbf{z}_t|\mathbf{z}_{t-1}, \Theta_z)$ denote the conditional density of $\mathbf{z}_t$ given $\mathbf{z}_{t-1}$ and Θ_z. Let $p_{\mathbf{h}}(\mathbf{h}_t|\mathbf{h}_{t-1}, \Theta_h)$ denote the conditional density of $\mathbf{h}_t$ given $\mathbf{h}_{t-1}$ and Θ_h. Define $\mathbf{Z}_{t+1}^* = \{\mathbf{z}_s\}_{s=t+1}^T$ and $\mathbf{H}_{t+1}^* = \{\mathbf{h}_s\}_{s=t+1}^T$. Under this setup, the joint density of $\mathbf{Z}_T, \mathbf{H}_T$ and $\mathbf{Y}_T$ given $\mathbf{X}_T, \mathbf{V}_t, \Theta_z, \Theta_h$ and Θ_y is given by

$$p(\mathbf{Z}_T, \mathbf{H}_T, \mathbf{Y}_T|\mathbf{X}_T, \mathbf{V}_T, \Theta_z, \Theta_h, \Theta_y) = p_{\mathbf{z}}(\mathbf{Z}_T|\mathbf{X}_T, \mathbf{V}_T, \Theta_z)p_{\mathbf{h}}(\mathbf{H}_T|\mathbf{X}_T, \mathbf{V}_T, \mathbf{Z}_T, \Theta_h)p_{\mathbf{y}}(\mathbf{Y}_T|\mathbf{X}_T, \mathbf{V}_T, \mathbf{Z}_T, \mathbf{H}_T, \Theta_y)$$

$$= p_{\mathbf{z}}(\mathbf{Z}_T|\Theta_z)p_{\mathbf{h}}(\mathbf{H}_T|\Theta_h)p_{\mathbf{y}}(\mathbf{Y}_T|\mathbf{X}_T, \mathbf{V}_T, \mathbf{Z}_T, \mathbf{H}_T, \Theta_y),$$

where the last equality follows from the Markov property of $\{\mathbf{z}_s\}_{s=0}^t$ and $\{\mathbf{h}_s\}_{s=0}^t$. Then, if $\mathbf{z}_0$ and $\mathbf{h}_0$ are assumed to be stochastic,

$$p_{\mathbf{z}}(\mathbf{Z}_T|\Theta_z) = p_{\mathbf{z}}(\mathbf{z}_0|\Theta_z)\prod_{t=1}^T p_{\mathbf{z}}(\mathbf{z}_t|\mathbf{z}_{t-1}, \Theta_z),$$

$$p_{\mathbf{h}}(\mathbf{H}_T|\Theta_h) = p_{\mathbf{h}}(\mathbf{h}_0|\Theta_h)\prod_{t=1}^T p_{\mathbf{h}}(\mathbf{h}_t|\mathbf{h}_{t-1}, \Theta_h),$$

$$p_{\mathbf{y}}(\mathbf{Y}_T|\mathbf{X}_T, \mathbf{V}_T, \mathbf{Z}_T, \mathbf{H}_T, \Theta_y) = \prod_{t=1}^T p_{\mathbf{y}}(\mathbf{y}_t|\mathbf{X}_t, \mathbf{V}_t, \mathbf{Z}_t, \mathbf{H}_t, \Theta_y),$$

$$p(\mathbf{z}_t|\mathbf{Z}_{t-1}, \mathbf{Z}_{t+1}^*, \mathbf{Y}_T, \mathbf{X}_T, \mathbf{V}_T, \mathbf{H}_T, \Theta_y, \Theta_z, \Theta_h) \propto \tag{65}$$

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$$
\begin{cases}
p_{\mathbf{y}}(\mathbf{y}_t|\mathbf{X}_t, \mathbf{V}_t, \mathbf{Z}_t, \mathbf{H}_t, \Theta_y)p_{\mathbf{z}}(\mathbf{z}_t|\mathbf{z}_{t-1}, \Theta_z)p_{\mathbf{z}}(\mathbf{z}_{t+1}|\mathbf{z}_t, \Theta_z), & \text{if } t \leq T-1 \\
p_{\mathbf{y}}(\mathbf{y}_t|\mathbf{X}_t, \mathbf{V}_t, \mathbf{Z}_t, \mathbf{H}_t, \Theta_y)p_{\mathbf{z}}(\mathbf{z}_t|\mathbf{z}_{t-1}, \Theta_z), & \text{if } t = T,
\end{cases}
$$

$$
p(\mathbf{h}_t|\mathbf{H}_{t-1}, \mathbf{H}_{t+1}^*, \mathbf{Y}_T, \mathbf{X}_T, \mathbf{V}_T, \mathbf{Z}_T, \Theta_y, \Theta_z, \Theta_h) \propto \tag{66}
$$

$$
\begin{cases}
p_{\mathbf{y}}(\mathbf{y}_t|\mathbf{X}_t, \mathbf{V}_t, \mathbf{Z}_t, \mathbf{H}_t, \Theta_y)p_{\mathbf{h}}(\mathbf{h}_t|\mathbf{h}_{t-1}, \Theta_h)p_{\mathbf{h}}(\mathbf{h}_{t+1}|\mathbf{h}_t, \Theta_h), & \text{if } t \leq T-1 \\
p_{\mathbf{y}}(\mathbf{y}_t|\mathbf{X}_t, \mathbf{V}_t, \mathbf{Z}_t, \mathbf{H}_t, \Theta_y)p_{\mathbf{h}}(\mathbf{h}_t|\mathbf{h}_{t-1}, \Theta_h), & \text{if } t = T,
\end{cases}
$$

$$
p(\Theta_y|\mathbf{Y}_T, \mathbf{X}_T, \mathbf{V}_T, \mathbf{Z}_T, \mathbf{H}_T, \Theta_z, \Theta_h) \propto p_{\mathbf{y}}(\mathbf{Y}_T|\mathbf{X}_T, \mathbf{Z}_T, \mathbf{H}_T, \Theta_y)p_{\Theta_y}(\Theta_y), \tag{67}
$$

$$
p(\Theta_z|\mathbf{Y}_T, \mathbf{X}_T, \mathbf{V}_T, \mathbf{Z}_T, \Theta_y, \Theta_h) \propto p_{\mathbf{z}}(\mathbf{Z}_T|\Theta_z)p_{\Theta_z}(\Theta_z), \tag{68}
$$

$$
p(\Theta_h|\mathbf{Y}_T, \mathbf{X}_T, \mathbf{V}_T, \mathbf{H}_T, \Theta_y, \Theta_z) \propto p_{\mathbf{h}}(\mathbf{H}_T|\Theta_h)p_{\Theta_h}(\Theta_h), \tag{69}
$$

where $p_{\Theta_y}(\Theta_y)$, $p_{\Theta_z}(\Theta_z)$ and $p_{\Theta_h}(\Theta_h)$ are the prior densities of Θ_y, Θ_z and Θ_h, respectively. From the posterior densities (67)-(69), the smoothing random draws are generated as follows:

Step 0. **Take appropriate initial values for Θ_y, Θ_z, $\{\mathbf{z}_t\}_{t=0}^T$, Θ_h and $\{\mathbf{h}_t\}_{t=0}^T$, and fix $\hat{\mathbf{\Pi}}_i$, $\hat{\mathbf{\Psi}}_i$ for i=1,2.**[12]

Step 1. **Generate a random draw of $\mathbf{z}_t$ from $p(\mathbf{z}_t|\mathbf{Z}_{t-1}, \mathbf{Z}_{t+1}^*, \mathbf{Y}_T, \mathbf{X}_T, \mathbf{V}_T, \mathbf{H}_T, \Theta_y, \Theta_z, \Theta_h)$ for $t = 1, 2, \ldots, T$. Draw $\mathbf{z}_t^{(i)}$ using the normal proposal density**

$$
\mathbf{z}_t^{(i)} \sim \mathbb{N}(\mathbf{z}_t^{(i-1)}, cK_t^z)
$$

where K_t^z is the filtered variance-covariance matrix of the random coefficients of a random-coefficient specification of the policy rules, and c is a proper scale coefficient. The algorithm accepts $\mathbf{z}_t^{(i)}$ with probability r:

$$
r = \min\left\{\frac{p\left(\mathbf{z}_t^{(i)}|\mathbf{Z}_{t-1}^{(i-1)}, \mathbf{Z}_{t+1}^{*(i-1)}, \mathbf{Y}_T, \mathbf{X}_T, \mathbf{V}_T, \mathbf{H}_T^{(i-1)}, \Theta_y^{(i-1)}, \Theta_z^{(i-1)}, \Theta_h^{(i-1)}\right)}{p\left(\mathbf{z}_t^{(i-1)}|\mathbf{Z}_{t-1}^{(i-1)}, \mathbf{Z}_{t+1}^{*(i-1)}, \mathbf{Y}_T, \mathbf{X}_T, \mathbf{V}_T, \mathbf{H}_T^{(i-1)}, \Theta_y^{(i-1)}, \Theta_z^{(i-1)}, \Theta_h^{(i-1)}\right)}, 1\right\}.
$$

Step 2. **Generate a random draw of Θ_y from $p(\Theta_y|\mathbf{Y}_T, \mathbf{X}_T, \mathbf{V}_T, \mathbf{Z}_T, \mathbf{H}_T, \Theta_z, \Theta_h)$.**

(a) Partition $\Theta_{y_1} = \Theta_{y_{1.1}} \cup \Theta_{y_{1.2}} \cup \Theta_{y_{1.3}}$, where $\Theta_{y_{1.1}} = \{\alpha_0^\pi, \alpha_1^\pi, \alpha_2^\pi, \alpha_3^\pi, \alpha_0^y, \alpha_1^y, \alpha_2^y, \alpha_3^y\}$, $\Theta_{y_{1.2}} = \rho_R$ and $\Theta_{y_{1.3}} = \boldsymbol{\delta}_1$. Partition $\Theta_{y_2} = \Theta_{y_{2.1}} \cup \Theta_{y_{2.2}} \cup \Theta_{y_{2.3}}$, where $\Theta_{y_{2.1}} = \{\gamma_0^b, \gamma_1^b, \gamma_2^b, \gamma_3^b, \gamma_0^y, \gamma_1^y, \gamma_2^y, \gamma_3^y\}$, $\Theta_{y_{2.2}} = \rho_\tau$ and $\Theta_{y_{2.3}} = \boldsymbol{\delta}_2$.

---

[12]The initial latent factors, $\{\mathbf{z}_t\}_{t=0}^T$, are smoothed estimates of a random coefficients model of the policy rules with constant volatility. The initial volatility processes, $\{\mathbf{h}_t\}_{t=0}^T$, are smoothed estimates of a stochastic volatility model of the policy rules with constant coefficients. The initial values of the parameters of the policy rules, Θ_y, are obtained from the maximization of the likelihood function given the initial processes for the latent factors and stochastic volatilities. The initial values for Θ_z come from a least-squares regression of current against lagged initial latent factors. The initial values for Θ_h come from a least-squares regression of current against lagged initial log stochastic volatilities.

$\hat{\mathbf{\Pi}}_i$ and $\hat{\mathbf{\Psi}}_i$ for $i = 1, 2$ are the least squares estimates of the parameters in equations (62) and (64), respectively.

(b) Generate a random draw of $\Theta_{y1}^{(i)}$ sequentially, as follows:

   i. Generate a random draw of a transformation of $\Theta_{y1.1}^{(i)}$, $\tilde{\Theta}_{y1.1}^{(i)}$, using the normal proposal density

$$\tilde{\Theta}_{y1.1}^{(i)} \sim \mathbb{N}(\tilde{\Theta}_{y1.1}^{(i-1)}, cS_{1.1}),$$

where $S_{1.1}$ is the variance-covariance matrix of the maximum likelihood estimator of $\tilde{\Theta}_{y1.1}$ given the initial latent factors and stochastic volatilities, and c is a scale coefficient. The algorithm accepts $\Theta_{y1.1}^{i}$ with probability r:

$$r = \min\left\{\frac{p\left(\Theta_{y1.1}^{(i)} \cup \Theta_{y1.2}^{(i-1)} \cup \Theta_{y1.3}^{(i-1)} | \mathbf{Y}_{1,T}, \mathbf{X}_{1,T}, \mathbf{V}_{1,T}, \mathbf{Z}_T^{(i)}, \mathbf{H}_T^{(i-1)}, \Theta_z^{(i-1)}, \Theta_h^{(i-1)}\right)}{p\left(\Theta_{y1.1}^{(i-1)} \cup \Theta_{y1.2}^{(i-1)} \cup \Theta_{y1.3}^{(i-1)} | \mathbf{Y}_{1,T}, \mathbf{X}_{1,T}, \mathbf{V}_{1,T}, \mathbf{Z}_T^{(i)}, \mathbf{H}_T^{(i-1)}, \Theta_z^{(i-1)}, \Theta_h^{(i-1)}\right)}, 1\right\}.$$

   ii. Generate a random draw of a transformation of $\Theta_{y1.2}^{(i)}$, $\tilde{\Theta}_{y1.2}^{(i)}$, using the normal proposal density

$$\tilde{\Theta}_{y1.2}^{(i)} \sim \mathbb{N}(\tilde{\Theta}_{y1.2}^{(i)}, cS_{1.2})$$

where $S_{1.2}$ is the variance-covariance matrix of the maximum likelihood estimator of $\tilde{\Theta}_{y1.2}$ given the initial latent factors and stochastic volatilities, and c is a scale coefficient. The algorithm accepts $\Theta_{y1.2}^{(i)}$ with probability r:

$$r = \min\left\{\frac{p\left(\Theta_{y1.1}^{(i)} \cup \Theta_{y1.2}^{(i)} \cup \Theta_{y1.3}^{(i-1)} | \mathbf{Y}_{1,T}, \mathbf{X}_{1,T}, \mathbf{V}_{1,T}, \mathbf{Z}_T^{(i)}, \mathbf{H}_T^{(i-1)}, \Theta_z^{(i-1)}, \Theta_h^{(i-1)}\right)}{p\left(\Theta_{y1.1}^{(i)} \cup \Theta_{y1.2}^{(i-1)} \cup \Theta_{y1.3}^{(i-1)} | \mathbf{Y}_{1,T}, \mathbf{X}_{1,T}, \mathbf{V}_{1,T}, \mathbf{Z}_T^{(i)}, \mathbf{H}_T^{(i-1)}, \Theta_z^{(i-1)}, \Theta_h^{(i-1)}\right)}, 1\right\}.$$

   iii. Generate a random draw of $\Theta_{y1.3}^{(i)}$ using the normal proposal density

$$\Theta_{y1.3}^{(i)} \sim \mathbb{N}(\hat{\Theta}_{y1.3}, S_{1.3})$$

where $\hat{\Theta}_{y1.3}$ is the least squares estimator of $\Theta_{y1.3}$ using the residuals of least squares estimations of (61) and (62), and $S_{1.3}$ is its least squares estimated variance-convariance matrix. The algorithm accepts $\Theta_{y1.3}^{(i)}$ with probability r:

$$r = \min\left\{\frac{p\left(\Theta_{y1.1}^{(i)} \cup \Theta_{y1.2}^{(i)} \cup \Theta_{y1.3}^{(i)} | \mathbf{Y}_{1,T}, \mathbf{X}_{1,T}, \mathbf{V}_{1,T}, \mathbf{Z}_T^{(i)}, \mathbf{H}_T^{(i-1)}, \Theta_z^{(i-1)}, \Theta_h^{(i-1)}\right)}{p\left(\Theta_{y1.1}^{(i)} \cup \Theta_{y1.2}^{(i)} \cup \Theta_{y1.3}^{(i-1)} | \mathbf{Y}_{1,T}, \mathbf{X}_{1,T}, \mathbf{V}_{1,T}, \mathbf{Z}_T^{(i)}, \mathbf{H}_T^{(i-1)}, \Theta_z^{(i-1)}, \Theta_h^{(i-1)}\right)} \frac{g(\Theta_{y1.3}^{(i-1)})}{g(\Theta_{y1.3}^{(i)})}, 1\right\},$$

where $g(\cdot)$ is the proposal density.

(c) Generate a random draw of $\Theta_{y2}^{(i)}$ sequentially, as follows:

   i. Generate a random draw of a transformation of $\Theta_{y2.1}^{(i)}$, $\tilde{\Theta}_{y2.1}^{(i)}$, using the normal proposal density

$$\tilde{\Theta}_{y2.1}^{(i)} \sim \mathbb{N}(\tilde{\Theta}_{y2.1}^{(i-1)}, cS_{2.1}),$$

where $S_{2.1}$ is the variance-covariance matrix of the maximum likelihood estimator of $\tilde{\Theta}_{y2.1}$ given the initial latent factors and stochastic volatilities, and c is a scale coefficient. The algorithm accepts $\Theta_{y2.1}^{(i)}$ with probability r:

$$r = \min\left\{\frac{p\left(\Theta_{y2.1}^{(i)} \cup \Theta_{y2.2}^{(i-1)} \cup \Theta_{y2.3}^{(i-1)} | \mathbf{Y}_{2,T}, \mathbf{X}_{2,T}, \mathbf{V}_{2,T}, \mathbf{Z}_T^{(i)}, \mathbf{H}_T^{(i-1)}, \Theta_z^{(i-1)}, \Theta_h^{(i-1)}\right)}{p\left(\Theta_{y2.1}^{(i-1)} \cup \Theta_{y2.2}^{(i-1)} \cup \Theta_{y2.3}^{(i-1)} | \mathbf{Y}_{2,T}, \mathbf{X}_{2,T}, \mathbf{V}_{2,T}, \mathbf{Z}_T^{(i)}, \mathbf{H}_T^{(i-1)}, \Theta_z^{(i-1)}, \Theta_h^{(i-1)}\right)}, 1\right\}.$$

ii. Generate a random draw of a transformation of $\Theta^{(i)}_{y2.2}$, $\tilde{\Theta}^{(i)}_{y2.2}$, using the normal proposal density

$$\tilde{\Theta}^{(i)}_{y2.2} \sim \mathbb{N}(\tilde{\Theta}^{(i)}_{y2.2}, cS_{2.2})$$

where $S_{2.2}$ is the variance-covariance matrix of the maximum likelihood estimator of $\tilde{\Theta}_{y2.2}$ given the initial latent factors and stochastic volatilities, and c is a scale coefficient. The algorithm accepts $\Theta^{(i)}_{y2.2}$ with probability r:

$$r = \min\left\{ \frac{p\left(\Theta^{(i)}_{y2.1} \cup \Theta^{(i)}_{y2.2} \cup \Theta^{(i-1)}_{y2.3} | \mathbf{Y}_{2,T}, \mathbf{X}_{2,T}, \mathbf{V}_{2,T}, \mathbf{Z}^{(i)}_T, \mathbf{H}^{(i-1)}_T, \Theta^{(i-1)}_z, \Theta^{(i-1)}_h \right)}{p\left(\Theta^{(i)}_{y2.1} \cup \Theta^{(i-1)}_{y2.2} \cup \Theta^{(i-1)}_{y2.3} | \mathbf{Y}_{2,T}, \mathbf{X}_{2,T}, \mathbf{V}_{2,T}, \mathbf{Z}^{(i)}_T, \mathbf{H}^{(i-1)}_T, \Theta^{(i-1)}_z, \Theta^{(i-1)}_h \right)}, 1 \right\}.$$

iii. Generate a random draw of $\Theta^{(i)}_{y2.3}$ using the normal proposal density

$$\Theta^{(i)}_{y2.3} \sim \mathbb{N}(\hat{\Theta}_{y2.3}, S_{2.3})$$

where $\hat{\Theta}_{y2.3}$ is the least squares estimator of $\Theta_{y2.3}$ using the residuals of least squares estimations of (63) and (64), and $S_{2.3}$ is its least squares estimated variance-convariance matrix. The algorithm accepts $\Theta^{(i)}_{y2.3}$ with probability r:

$$r = \min\left\{ \frac{p\left(\Theta^{(i)}_{y2.1} \cup \Theta^{(i)}_{y2.2} \cup \Theta^{(i)}_{y2.3} | \mathbf{Y}_{2,T}, \mathbf{X}_{2,T}, \mathbf{V}_{2,T}, \mathbf{Z}^{(i)}_T, \mathbf{H}^{(i-1)}_T, \Theta^{(i-1)}_z, \Theta^{(i-1)}_h \right)}{p\left(\Theta^{(i)}_{y2.1} \cup \Theta^{(i)}_{y2.2} \cup \Theta^{(i-1)}_{y2.3} | \mathbf{Y}_{2,T}, \mathbf{X}_{2,T}, \mathbf{V}_{2,T}, \mathbf{Z}^{(i)}_T, \mathbf{H}^{(i-1)}_T, \Theta^{(i-1)}_z, \Theta^{(i-1)}_h \right)} \frac{g(\Theta^{(i-1)}_{y2.3})}{g(\Theta^{(i)}_{y2.3})}, 1 \right\},$$

where $g(\cdot)$ is the proposal density.

Step 3. **Generate a random draw of $\mathbf{h}_t$ from $p(\mathbf{h}_t | \mathbf{H}_{t-1}, \mathbf{H}^*_{t+1}, \mathbf{Y}_T, \mathbf{X}_T, \mathbf{V}_T, \mathbf{Z}_T, \Theta_y, \Theta_z, \Theta_h)$ for $t = 1, 2, \ldots, T$.** Draw $\mathbf{h}^{(i)}_t$ using the normal proposal density

$$\mathbf{h}^{(i)}_t \sim \mathbb{N}(\mathbf{h}^{(i-1)}_t, cK^h_t)$$

where K^h_t is the filtered variance-covariance matrix of the volatility of a constant-coefficient specification of the policy rules, and c is a proper scale coefficient. The algorithm accepts $\mathbf{h}^{(i)}_t$ with probability r:

$$r = \min\left\{ \frac{p\left(\mathbf{h}^{(i)}_t | \mathbf{H}^{(i-1)}_{t-1}, \mathbf{H}^{*(i-1)}_{t+1}, \mathbf{Y}_T, \mathbf{X}_T, \mathbf{V}_T, \mathbf{Z}^{(i)}_T, \Theta^{(i)}_y, \Theta^{(i-1)}_z, \Theta^{(i-1)}_h \right)}{p\left(\mathbf{h}^{(i-1)}_t | \mathbf{H}^{(i-1)}_{t-1}, \mathbf{H}^{*(i-1)}_{t+1}, \mathbf{Y}_T, \mathbf{X}_T, \mathbf{V}_T, \mathbf{Z}^{(i)}_T, \Theta^{(i)}_y, \Theta^{(i-1)}_z, \Theta^{(i-1)}_h \right)}, 1 \right\}.$$

Step 4. **Generate a random draw of Θ_z from $p(\Theta_z | \mathbf{Y}_T, \mathbf{X}_T, \mathbf{V}_T, \mathbf{Z}_T, \Theta_y, \Theta_h)$ sequentially, as follows:**

(a) Partition $\Theta_z = \Theta_{z1} \cup \Theta_{z2}$ where $\Theta_{z1} = \{\rho_{zm}, \rho_{zf}\}$ and $\Theta_{z2} = \{\kappa\}$.

(b) Generate a random draw $\{\rho^{(i)}_{zm}, \rho^{(i)}_{zf}\}$ using two independent Beta proposal densities (expressed in terms of means and standard deviations)

$$\rho^{(i)}_{zm} \sim \text{Beta}(\hat{\rho}_{zm}, \hat{\sigma}_{\hat{\rho}_{zm}})$$
$$\rho^{(i)}_{zf} \sim \text{Beta}(\hat{\rho}_{zm}, \hat{\sigma}_{\hat{\rho}_{zf}}),$$

where $\hat{\rho}_{zm}$ is the ordinary least squares estimate of ρ_{zm} in (41) using $\{z_t^{m(i)}\}_{t=1}^T$, and $\hat{\sigma}_{\hat{\rho}_{zm}}$ is its standard error. The same applies for $\hat{\rho}_{zf}$ and $\hat{\sigma}_{\hat{\rho}_{zf}}$, which come from the estimation of (42) using $\{z_t^{f(i)}\}_{t=1}^T$. The algorithm accepts $\{\rho_{zm}^{(i)}, \rho_{zf}^{(i)}\}$ with probability r:

$$r = \min\left\{ \frac{p\left(\Theta_{z1}^{(i)} \cup \Theta_{z2}^{(i-1)} | \mathbf{Y}_T, \mathbf{X}_T, \mathbf{V}_T, \mathbf{Z}_T^{(i)}, \Theta_y, \Theta_h\right)}{p\left(\Theta_{z1}^{(i-1)} \cup \Theta_{z2}^{(i-1)} | \mathbf{Y}_T, \mathbf{X}_T, \mathbf{V}_T, \mathbf{Z}_T^{(i)}, \Theta_y, \Theta_h\right)} \frac{g(\rho_{zm}^{(i-1)}, \rho_{zf}^{(i-1)})}{g(\rho_{zm}^{(i)}, \rho_{zf}^{(i)})}, 1 \right\}.$$

(c) Generate a random draw $\kappa^{(i)}$ using a four-parameter Beta proposal density with range on [-1,1] (expressed in terms of mean and standard deviation)

$$\kappa^{(i)} \sim \text{TransformedBeta}(\hat{\kappa}, (1 - \hat{\kappa}^2)/\sqrt{n-1}),$$

where $\hat{\kappa}$ is the correlation coefficient between the residuals of equations (41) and (42) using $\rho_{zm}^{(i)}$ and $\rho_{zf}^{(i)}$ as estimated coefficients where corresponds. The algorithm accepts $\kappa^{(i)}$ with probability r:

$$r = \min\left\{ \frac{p\left(\Theta_{z1}^{(i)} \cup \Theta_{z2}^{(i)} | \mathbf{Y}_T, \mathbf{X}_T, \mathbf{V}_T, \mathbf{Z}_T^{(i)}, \Theta_y, \Theta_h\right)}{p\left(\Theta_{z1}^{(i)} \cup \Theta_{z2}^{(i-1)} | \mathbf{Y}_T, \mathbf{X}_T, \mathbf{V}_T, \mathbf{Z}_T^{(i)}, \Theta_y, \Theta_h\right)} \frac{g(\kappa^{(i-1)})}{g(\kappa^{(i)})}, 1 \right\}.$$

Step 5. Generate a random draw of Θ_h from $p(\Theta_h | \mathbf{Y}_T, \mathbf{X}_T, \mathbf{V}_T, \mathbf{H}_T, \Theta_y, \Theta_z)$.

(a) Partition $\Theta_h = \Theta_{h1} \cup \Theta_{h2}$ where $\Theta_{h1} = \{\ln \sigma_R, \rho_{\sigma_R}, \ln \sigma_\tau, \rho_{\sigma_\tau}\}$ and $\Theta_{h2} = \{\eta_R, \eta_\tau\}$.

(b) Generate a random draw $\{\ln \sigma_R^{(i)}, \rho_{\sigma_R}^{(i)}, \ln \sigma_\tau^{(i)}, \rho_{\sigma_\tau}^{(i)}\}$ using the independent proposal densities (expressed in terms of means and standard deviations)

$$\rho_{\sigma_R}^{(i)} \sim \text{Beta}(\hat{\rho}_{\sigma_R}, \hat{\sigma}_{\hat{\rho}_{\sigma_R}})$$
$$\rho_{\sigma_\tau}^{(i)} \sim \text{Beta}(\hat{\rho}_{\sigma_\tau}, \hat{\sigma}_{\hat{\rho}_{\sigma_\tau}})$$
$$c_R^{(i)} \sim \mathbb{N}(\hat{c}_R, \hat{\sigma}_{c_R})$$
$$c_\tau^{(i)} \sim \mathbb{N}(\hat{c}_\tau, \hat{\sigma}_{c_\tau}),$$

with $\ln \sigma_R^{(i)} = c_R^{(i)}/(1 - \rho_{\sigma_R}^{(i)})$ and $\ln \sigma_\tau^{(i)} = c_\tau^{(i)}/(1 - \rho_{\sigma_\tau}^{(i)})$, where $\hat{c}_R, \hat{\rho}_{\sigma_R}$ are the least squares estimates of $(1 - \rho_{\sigma_R}) \ln \sigma_R$ and ρ_{σ_R} in (43), respectively, using $\{\ln \sigma_{R,t}^{(i)}\}_{t=0}^T$, and where $\hat{c}_\tau, \hat{\rho}_{\sigma_\tau}$ are obtained similarly from (44) using $\{\ln \sigma_{\tau,t}^{(i)}\}_{t=0}^T$. The algorithm accepts $\{\ln \sigma_R^{(i)}, \rho_{\sigma_R}^{(i)}, \ln \sigma_\tau^{(i)}, \rho_{\sigma_\tau}^{(i)}\}$ with probability r:

$$r = \min\left\{ \frac{p\left(\Theta_{h1}^{(i)} \cup \Theta_{h2}^{(i-1)} | \mathbf{Y}_T, \mathbf{X}_T, \mathbf{V}_T, \mathbf{H}_T^{(i)}, \Theta_y, \Theta_z\right)}{p\left(\Theta_{h1}^{(i-1)} \cup \Theta_{h2}^{(i-1)} | \mathbf{Y}_T, \mathbf{X}_T, \mathbf{V}_T, \mathbf{H}_T^{(i)}, \Theta_y, \Theta_z\right)} \frac{g(\ln \sigma_R^{(i-1)}, \rho_{\sigma_R}^{(i-1)}, \ln \sigma_\tau^{(i-1)}, \rho_{\sigma_\tau}^{(i-1)})}{g(\ln \sigma_R^{(i)}, \rho_{\sigma_R}^{(i)}, \ln \sigma_\tau^{(i)}, \rho_{\sigma_\tau}^{(i)})}, 1 \right\}.$$

(c) Generate a random draw $\{\eta_R^{(i)}, \eta_\tau^{(i)}\}$ using the inverted gamma proposal densities

$$\eta_R^{(i)} \sim \text{IG}(\tilde{\xi}'^{R(i)} \tilde{\xi}^{R(i)}, \text{df})$$

$$\eta_\tau^{(i)} \sim \text{IG}(\tilde{\xi}'^{\tau(i)} \tilde{\xi}^{\tau(i)}, \text{df}),$$

where $\tilde{\xi}_t^{R(i)} = \ln \sigma_{R,t}^{(i)} - (1 - \rho_{\sigma_R}^{(i)}) \ln \sigma_R^{(i)} - \rho_{\sigma_R}^{(i)} \ln \sigma_{R,t-1}^{(i)}$ and $\tilde{\xi}_t^{\tau(i)} = \ln \sigma_{\tau,t}^{(i)} - (1 - \rho_{\sigma_\tau}^{(i)}) \ln \sigma_\tau^{(i)} - \rho_{\sigma_\tau}^{(i)} \ln \sigma_{\tau,t-1}^{(i)}$ are residuals, and $\tilde{\xi}^{R(i)}$ and $\tilde{\xi}^{\tau(i)}$ are vectors that stack the respective residuals. The algorithm accepts $\{\eta_R^{(i)}, \eta_\tau^{(i)}\}$ with probability r:

$$r = \min \left\{ \frac{p\left(\Theta_{h1}^{(i)} \cup \Theta_{h2}^{(i)} | \mathbf{Y}_T, \mathbf{X}_T, \mathbf{V}_T, \mathbf{H}_T^{(i)}, \Theta_y, \Theta_z\right)}{p\left(\Theta_{h1}^{(i)} \cup \Theta_{h2}^{(i-1)} | \mathbf{Y}_T, \mathbf{X}_T, \mathbf{V}_T, \mathbf{H}_T^{(i)}, \Theta_y, \Theta_z\right)} \frac{g(\eta_R^{(i-1)}, \eta_\tau^{(i-1)})}{g(\eta_R^{(i)}, \eta_\tau^{(i)})}, 1 \right\}.$$

Step 6. **Repeat steps 1-4 N times to obtain N random draws of $\mathbf{Z}_T$, $\mathbf{H}_T$, Θ_y, Θ_z and Θ_h.**

In steps (ii)-(vi) the random draws of $\mathbf{Z}$, $\mathbf{H}$, Θ_y, Θ_z and Θ_h are updated. This sampling method is referred to as the Gibbs sampler. To generate the random draws of $\mathbf{z}_t$, $\mathbf{h}_t$ for $t = 1, 2, \ldots, T$, Θ_y, Θ_z and Θ_h, we use the Metropolis-Hastings (M-H) algorithm. That is, the Gibbs sampler and the M-H algorithm are combined to obtain the smoothing random draws from the state-space model.

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