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Niall Farrell and Mel Devine and William Lee and James Gleeson and Seán Lyons

Economic and Social Research Institute, Whitaker Square, Sir John Rogerson's Quay, Dublin, Ireland, Department of Economics, Trinity College, Dublin, Ireland, MACSI, Department of Mathematics and Statistics, University of Limerick, Limerick, Ireland

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Specifying An Efficient Renewable Energy Feed-in Tariff

Niall Farrell^{a,b,*}, Mel T. Devine^c, William T. Lee^c, James P. Gleeson^c, Seán Lyons^{a,b}

^aEconomic and Social Research Institute, Whitaker Square, Sir John Rogerson's Quay, Dublin, Ireland ^bDepartment of Economics, Trinity College, Dublin, Ireland ^cMACSI, Department of Mathematics and Statistics, University of Limerick, Limerick, Ireland

Abstract

This paper derives efficient pricing formulae for renewable energy Feed-in Tariff (FiT) designs that incorporate exposure to uncertain market prices by using option pricing theory. Such FiT designs are presented as a means to delineate market price risk amongst investors and policymakers when designing renewable energy support schemes. Sequential game theory provides the theoretical framework through which we model the strategic interaction of policymakers and investors during policy formulation. This model is solved using option pricing theory when a FiT is comprised of market prices combined with a guaranteed element. This solution also allows for an analytical formulation of the policy cost of subsidisation. Partial derivatives characterise sensitivity of policy cost and investor remuneration to deviations in market conditions beyond those expected. Analytical derivations provide a set of tools which may guide more efficient FiT policy and investment decisions. Numerical simulations demonstrate application for a stylised Irish case study, with a scenario analysis providing further insight into the relative sensitivity of policy cost and investor remuneration under different market conditions.

Keywords: Renewable Energy, Feed-in Tariff, Option Pricing, Renewable Energy Support Schemes

^{*}Corresponding author: niall.farrell@esri.ie; +353 (0)1 863 2097; Economic and Social Research Institute, Whitaker Square, Sir John Rogerson's Quay, Dublin, Ireland

Email addresses: niall.farrell@esri.ie (Niall Farrell), mel.devine@ul.ie (Mel T. Devine), william.lee@ul.ie (William T. Lee), james.gleeson@ul.ie (James P. Gleeson), sean.lvons@esri.ie (Seán Lyons)

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1. Introduction

The intermittent nature of many renewable energy sources combine with uncertain market prices to make renewable energy investment an inherently risky venture. Alongside this, renewable energy technologies currently generate electricity at a cost greater than fossil fuel-based alternatives. These factors can impede the achievement of renewable energy deployment goals with publicly-funded support mechanisms thus required in many jurisdictions (Fell and Linn, 2013; DCENR, 2006; del Rio and Bleda, 2012; E.C., 2005; International Energy Agency and Organisation for Economic Co-operation and Development, 2008). A Feed-in Tariff (FiT) guarantees a set payment per unit of electricity generated and has been found to limit investors' exposure to uncertain market prices to a greater extent than alternate mechanisms (Burer and Wustenhagen, 2009; Fagiani et al., 2013; International Energy Agency and Organisation for Economic Co-operation and Development, 2008; Ragwitz et al., 2007). Although theoretically less efficient than quantity-based mechanisms, this greater level of effectiveness has resulted in FiTs becoming the preferred support mechanism in many policy evaluations (Burer and Wustenhagen, 2009; Fagiani et al., 2013; International Energy Agency and Organisation for Economic Co-operation and Development, 2008; Ragwitz et al., 2007) whilst successful employment has resulted in a growing preference for FiT policies in many jurisdictions (International Energy Agency and Organisation for Economic Co-operation and Development, 2008, 2011).

Couture and Gagnon (2010) give a review of remuneration structures considered to date, whereby FiTs generally offer a variant of a fixed price or a constant premium. A fixed-price FiT removes risk associated with market price fluctuations from the investment decision and transfers these to the policymaker. This transfer of risk means that policymakers must incur the full risk of excessive cost should low wholesale electricity prices prevail. This topic of excessive consumer burden has become the subject of recent debate (Doherty and O'Malley, 2011; Rauch, 2013) as the potential outcome of a low wholesale electricity price may result in the cost of enforcing a FiT to increase. This risk has become a greater concern with the increasing penetration of low-price natural gas in the international energy mix (Rauch, 2013). Policymakers may wish to mitigate exposure to such excessive cost. On the other hand, wind provides policymakers with a hedge against any rise in fossil fuel prices and foregoing this benefit by allocating the entire degree of market price risk to investors through a premium policy, whilst also forcing investors to bear the

Abbreviations: DCENR: Department of Communications, Energy and Natural Resources; FiT: Feed-in Tariff; IEA: International Energy Agency; MW: Megawatt; OECD: Organisation for Economic Cooperation and Development; REFIT: Renewable Energy Feed-in Tariff; SEM: Single Electricity Market; SNSP: System Non-Synchronous Penetration limit; WWAP: Wind-Weighted Average Price

entire degree of market risk, may not be desirable. As such, policymakers may wish to share market price risk amongst both policymakers and investors through an alternate FiT design.

Couture and Gagnon (2010) and Kim and Lee (2012) outline the extent to which a number of different FiT designs expose investors to uncertain market prices. These structures, and potential augmentations, may allow for apportionment of market uncertainty. A guaranteed price floor may be enforced to reduce investors' exposure to low market prices, with potential investors receiving the benefit should the market price exceed the guaranteed floor. The Irish Renewable Energy Feed-in Tariff (REFIT) is designed in this way (DCENR, 2006). Such a design provides certainty for investors, who also receive the potential of any market 'upside' in excess of this floor. An efficient specification of such a design would imply that the expected value of this market 'upside' is taken into account, necessitating a lower floor. Thus, the market risk borne by the policymaker is reduced. If the investor did not want to bear market price risk to this extent, whilst the policymaker was willing to incur a greater degree of market price risk, then both parties could share the market upside according to a fixed % delineation and the guaranteed price floor altered to ensure expected remuneration remained constant. Alternatively, potential market upside may be split according to a predefined cap & floor, allowing for a similar expected value but a different delineation of market price risk.

It is the purpose of this paper to provide an analytical model to calculate such FiT structures with which policymakers may delineate market price risk. One can see that risk-sharing designs are comprised of a fixed and uncertain portion. An efficient price for the fixed portion should be set at a rate that incorporates the expected value of the uncertain portion. This has not been addressed in the literature. Doherty and O'Malley (2011) suggest that the Irish FiT is calculated based on the guaranteed portion alone, with the additional market upside potentially leading to overcompensation (DCENR, 2006; Doherty and O'Malley, 2011). This claim is validated when one examines the policy documents specifying the FiT rate in Ireland, where calculations of investment viability consider the FiT rate alone, omitting the expected value of market upside (DCENR, 2006). Kim and Lee (2012) do incorporate the value of market upside when valuing FiT policy. They do not specify a means to define an efficient FiT rate, but rather used numerical simulation to consider a range of possible values and choose that which yielded the optimal result. Although effective, numerical simulation is computationally intensive. Furthermore, the design of a simulation model is both labour intensive and project-specific. This places barriers for other policymakers to apply a given simulation-based modelling framework. As such, the development of an analytical framework of general applicability improves accessibility, replicability and transparency of FiT modelling procedures and results.

An appropriate theoretical framework must be decided upon to adequately characterise the strategic interaction between policymakers and investors when setting a FiT price. The approach

adopted is of a similar fashion to the electricity pricing analysis of Woo (1988), whilst also drawing on sequential game theory (Kalkuhl et al., 2012; Chang et al., 2013). To incorporate uncertain market prices into this framework, this paper uses option pricing theory (Black and Scholes, 1973) to calculate the expected value of a given combination of certain price guarantee and uncertain market remuneration. Financial options may take a number of forms with FiT policies sharing traits similar to European 'put' options. A European put option gives the buyer the right, but not the obligation, to sell an underlying asset (e.g. electricity) at a guaranteed 'strike' price (e.g. the guaranteed FiT price floor) at a particular time in the future. Implicit in this option is the possibility to sell at the prevailing market price should it be greater than the guaranteed option 'strike' price. A number of option pricing theories have been developed to estimate the value of such an arrangement, with the most widely used being the Black-Scholes model (Black and Scholes, 1973). A model similar to the Black-Scholes model (Black and Scholes, 1973) is used in this paper, following a number of authors who have used this and other option pricing theories to value a number of energy market commodities and derivatives; Pickles and Smith (1993) have used a binomial lattice option pricing approach to value undeveloped petroleum reserves; Smith (2005) have derived formulae to calculate the value of an option for repeated drilling in petroleum exploration; Overdahl and Matthews (1988) have drawn on option pricing theory to forecast crude oil prices, whilst Keppo and Rasanen (1999) have used option pricing to price retail electricity tariffs when consumption and wholesale price is uncertain. In an environmental context, Chambers et al. (1994) have used option pricing theory to compare the value of an ecological option to conserve against the value of national debt reduction, whilst Conrad (2000) have used option pricing to identify optimal timing of wilderness preservation, extraction or development.

The modelling framework of this paper, outlined in Section 2, draws together these separate fields of FiT policy design, game theory in renewable energy investment and option pricing theory. Section 3 presents optimal pricing rules for 3 classes of FiT; constant premium, shared 'upside' and cap & floor policy structures. Each of these tariff designs incorporate varying degrees of market uncertainty and these analytical derivations are the primary contribution to the understanding of FiT policy design offered by this paper.

Section 4 quantifies the sensitivity of each policy specification to unexpected market price outcomes, providing a further tool for FiT evaluation. The use of the tools presented in this paper are illustrated using a numerical example in Section 5. This provides quantitative insight into the preceding analytical solutions. A scenario analysis is used to compare sensitivities when underlying rates of volatility or growth are of different magnitudes. Finally, Section 6 offers some concluding comments.

2. Investment and market price models

2.1. Investment model

The interaction between policymakers and renewable energy investors in relation to FiT design is similar to a strategic leader game. In a leadership game, a leader (policymaker) chooses their strategy (FiT price) first with followers (investors) implementing their strategy (investment) conditional on that chosen by the leader (Chang et al., 2013; Fudenberg and Tirole, 1991). The leader is aware of the strategic response of the follower and chooses the FiT price that results in deployment of the desired quantity of renewable generation. This problem can be solved by using backward induction; the best response function of the follower (investor), conditional on the observed market price, is first calculated and substituted into the decision function of the leader (policymaker).

It is assumed that a policymaker wishes to incentivise the deployment of Q_I units of renewable capacity, which operate during t time periods in a time horizon [1, T]. The market is comprised of n investors who will deploy an aggregate Q units of renewable electricity generation capacity. For simplicity, it is assumed that all investment occurs during time period 0. Also, it is assumed that technological change is negligible during the considered time period, following policy implementations to date (DCENR, 2006). During time period 0, investors evaluate the investment decision comprised of the total discounted output from each time period t of operation, less the total capital and discounted operating costs. It is assumed that capital costs are incurred in time period 0 and all operation and maintenance costs incurred annually for T time periods thereafter. The total cost of installing Q units of electricity is thus assumed to be equal to the sum of capital (A) and operating (O) costs (including any required return to personnel, capital, etc.), discounted according to a discount rate r:

$$CQ = AQ + \sum_{t=1}^{T} e^{-rt} OQ \tag{1}$$

During each time period of operation, G_t units of electricity are generated. This is calculated according to equation (2).

$$G_t(Q) = b(Q)uvh \tag{2}$$

where h is the number of hours per time period t; v is operational availability net of maintenance and other such outages and u is the capacity factor for initial units. The parameter b reflects the nameplate capacity, augmented to incorporate any change in effective capacity/availability as Q changes. This captures the impact of increased curtailment, poorer site availability, etc. due to increasing levels of installation. As such, b(Q) is the nameplate capacity of all installed units, Q, scaled according to changes in the capacity factor from initial

units to the Q portfolio installed. b(Q) and hence G(Q), is assumed to follow a concave functional form. Although the most appropriate functional form is subject to the jurisdiction considered, the following general functional form is assumed for this problem:

$$b(Q) = Q_{max}(1 - e^{-\gamma Q}) \tag{3}$$

where Q_{max} is the maximum potential Q, whilst γ is a parameter controlling the rate of change which must be calibrated to observed data.

The market is comprised of potential investors in wind generation who are rational and wish to maximise profits. Aggregating the decision of n investors, where Π represents industry profit when $t \in [1, T]$, the industry-level investment decision may be formulated as;

$$\max_{Q} \Pi = \sum_{t=1}^{T} \left(P_t G_t(Q) \right) - CQ \tag{4}$$

The profit is calculated as the price received during period t, P_t , times the quantity of electricity generated during that period less the cost of generation during that period (CQ). As the objective function of (4) is concave, an optimal Q is achieved when;

$$\frac{\partial \Pi}{\partial Q} = \sum_{t=1}^{T} \left(P_t \frac{\partial G_t}{\partial Q} \right) - C = 0 \tag{5}$$

The policymaker's target requires the renewable energy industry to install Q_I units of capacity. For each trading period, the price received by the investor, P_t , is dependent on the particular FiT structure chosen. This may be the prevailing market price, S_t ; or a policy-oriented guaranteed FiT price floor, K, a FiT cap, \overline{S} , or a predefined portion of the market price S_t , denoted by θ . To meet the Q_I target, a policy maker must offer a combination of these prices such that the investment decision of (4) will yield Q_I units of generation.

It should also be noted that the duration of the FiT may not last for the full life of the wind plant installed. For example, the Irish FiT offers remuneration for 15 years (DCENR, 2006), whilst a wind turbine is expected to be in operation for 20 years (Doherty and O'Malley, 2011). As such, T_1 is taken to represent the final period of FiT remuneration, with $P_t = e^{-rt}E[S_t]$ when $t > T_1$.

Assuming that the policy maker wishes to minimise the net cost of subsidisation, represented as the net difference of all market prices and FiT outlays, the policymaker's price support decision is as follows;

$$\min_{K,\theta,\bar{S}} \quad F = \sum_{t=1}^{T_1} f_t G(Q) \tag{6}$$

subject to

$$Q = Q_I \tag{7}$$

$$\sum_{t=1}^{T} \left(P_t \frac{\partial G_t}{\partial Q} \right) - C = 0, \tag{8}$$

where f_t is the discounted expected cost to the policymaker of supporting the FiT policy during time t and P_t is the price received by the investor during time t. Assuming that the objective function in (6) is increasing for each of the different REFIT policy parameters $(K, \theta, \overline{S})$, the policy maker's problem is optimal when

$$\sum_{t=1}^{T} \left(P_t \frac{\partial G_t}{\partial Q} |_{Q_I} \right) - C = 0, \tag{9}$$

where $\frac{\partial G_t}{\partial Q}|_{Q_I}$ represents the derivative evaluated at Q_I .

2.2. Market price model

Policymakers and investors are faced with uncertainty as to the market price S_t . An appropriate methodology to incorporate this uncertainty into the expected price values is required in order to solve Equations (4) and (6). This uncertainty may be modelled by assuming an appropriate stochastic process (Skantze et al., 2000; Barlow, 2002).

Selection of an appropriate price process is determined by how well the evolution of the assumed process reflects that of the expected progression of the price process being modelled (Skantze et al., 2000). In certain jurisdictions, FiT payments that incorporate market prices are often calculated based on the average price over a given time period (DCENR, 2006; Folketinget Danish Parliament, 2008), with each period's price weighted according to the amount of wind generated during that period (DCENR, 2006; Devitt and Malaguzzi Valeri, 2011). This is known as the Wind Weighted Average Price (WWAP) and is offered for each unit of output generated during that period. The resolution of the time period chosen may vary, with Ireland offering a price floor tariff based on annual timesteps (DCENR, 2006), whilst Denmark offers a FiT for offshore wind based on annual timesteps (Folketinget Danish Parliament, 2008). Monthly timesteps have also been observed, with Denmark offering a FiT for onshore wind using this methodology (Folketinget Danish Parliament, 2008). For this paper, annual timesteps are considered where it is assumed that intra-annual variability is irrelevant and thus jump processes are ruled out. A Geometric Brownian Motion (GBM) stochastic process is thus used to simulate future annual WWAPs. GBM has been the price process employed in many studies to date when simulating annual electricity prices (e.g. Yang and Blyth, 2007; Heydari et al., 2012; Zhu, 2012) and is thus the approach taken in this paper. Extending this model to consider alternate timesteps and thus alternate price processes will be the subject of further study.

GBM may be used to model commodity prices or financial derivatives which contain a degree of random fluctuation. The parameters of this process are μ , which is the drift or average trend of growth; σ which is the volatility around the average trend; and dw which is an increment of a Wiener process. The Stochastic Differential Equation (SDE) for a GBM price process is given by equation (10) (Shreve et al., 2004);

$$dS = \mu S dt + \sigma S dw, \tag{10}$$

It should be noted that the 'merit order effect' of wind (Sensfuß et al., 2008) may result in lower rates of growth and volatility as the quantity Q of installed capacity increases. For a given Q such as Q_I , there is a single expected rate of growth and volatility. As such, the WWAP used for this application is modelled based on the growth and volatility of a Q_I level of installation. Assuming the policymaker wishes to install Q_I , then the WWAP parameters associated with Q_I are the only parameters that are of concern to the policymaker. If this Q_I level of installation is signalled to investors, and each individual investor is a price-taker and believes that this is a credible target that will be achieved, then each investor will anticipate that future WWAP values will follow those expected as a result of installing Q_I . This will then be incorporated into their investment decision. For simplicity of presentation we thus assume that WWAP follows that associated with a Q_I level of installation for this paper and thus explicitly modelling the relationship between Q and WWAP is not required. Augmenting this analysis to relax the assumption that market participants are price takers, and thus consider potential change in expected WWAP as a result of their choice of Q, is a possible avenue of further study.

The Black-Scholes model (Black and Scholes, 1973) assumes that markets are complete. It may be the case that this pricing structure is applied to an incomplete market, where the market price of risk may need to be taken into account. Should this be an issue, Appendix A outlines how the stochastic price process may be augmented to consider such market incompleteness by incorporating a market price of risk.

3. Pricing tariffs

As Section 1 has indicated, exposure to uncertain market prices may be delineated amongst investors and policymakers in a number of ways. Tariffs considered are outlined in Figure 1 and incorporate those commonly employed to date whilst also including mechanisms by which market price uncertainty may be delineated. Following policies employed to date, a tariff of constant premium in excess of the prevailing market price is first considered (Couture and Gagnon, 2010). Second, following the Irish price floor regime (DCENR, 2006), tariffs that comprise a floor and

a share of market 'upside' in excess of this floor are considered. A special case of this category is when no market 'upside' is received by the investor, and they thus receive a fixed price that is completely independent of market prices. Market upside in a shared upside regime may be delineated according to many different structures, with an alternative of a cap & floor regime being the third category considered by this paper. The cap & floor policy design draws on cap mechanisms discussed in the literature (Couture and Gagnon, 2010; Kim and Lee, 2012) along with those considered in certain jurisdictions (Queensland Competition Authority, 2013). The optimal pricing rules for each of these categories will now be derived, and the relative sensitivities discussed in more detail in this section and section 4.

Figure 1: FiT Payment structures



Payment structures: (left) constant premium, (middle) shared upside, (right) cap & floor.

3.1. Tariff of constant premium

The first tariff structure to be analysed in this paper is a constant premium in excess of the prevailing wholesale market price. This tariff structure incorporates the entire degree of market uncertainty in the price offered to investors whereby the price received comprises the prevailing S_t in addition to a constant price supplement X.

Equation (5) may be altered to specify payments under this FiT structure when $t \leq T_1$. Assuming that the undiscounted price for each time period t, is V_t , then V_t may be denoted;

$$V_t = X + S_t \tag{11}$$

Denoting P_t as the discounted expected value³ of V_t , then $P_t = e^{-rt} E[V_t]$. As such, the expected

³The expected value may be interpreted as the sum of each potential value, multiplied by the probability of occurrence.

value of remuneration under this tariff is equivalent to the expected value of X plus the expected value of market price S_t at time t. It is assumed that S_0 represents S at time period 0, the initial time period of analysis and the time period at which the investment at time t is evaluated. The expected value of GBM, and hence S_t is $S_0 e^{\mu t}$ (Hull, 2003). Thus, P_t when $t \leq T_1$ becomes:

$$P_t = Xe^{-rt} + S_0 e^{(\mu - r)t}$$
(12)

Using this definition for the price received by the investor, the investment decision of (5) becomes;

$$\left(\left\{\sum_{t=1}^{T_1} \left[e^{-rt}X + e^{(\mu-r)t}S_0\right] \frac{\partial G_t}{\partial Q}|_{Q_I}\right\} + \left\{\sum_{t=T_1}^T e^{(\mu-r)t}S_0 \frac{\partial G_t}{\partial Q}|_{Q_I}\right\}\right) - C = 0$$
(13)

Rearranging Equation (13) leads to a solution for X as follows:

$$X = \frac{C - \sum_{t=1}^{T} e^{(\mu-r)t} S_0 \frac{\partial G_t}{\partial Q} |_{Q_I}}{\sum_{t=1}^{T_1} e^{-rt} \frac{\partial G_t}{\partial Q} |_{Q_I}}.$$
(14)

The cost to the policymaker in offering this FiT(F) may be calculated as the sum of all X payments and may be formally represented as:

$$F = \sum_{t=1}^{T_1} X e^{-rt} G_t.$$
 (15)

The solution for an efficient constant (X) premium found in equation (14) may be interpreted as the total generating cost, less market remuneration, averaged over each time period of generation for the marginal generation unit. Output of the marginal unit, evaluated at Q_I , is represented by $\frac{\partial G_t}{\partial Q}|_{Q_I}$. Equation (14) shows that a greater level of curtailment reduces output and results in a greater X premium. One can see that increased cost has a positive impact on the X premium. A greater initial market price or a greater expected rate of market price growth has a negative impact on the X premium. Equation (14) shows that the investor is entirely exposed to any changes in the rate of market price growth, whilst equation (15) shows that the policy maker remains unaffected. The expected volatility of market prices has no impact on the expected value of the X premium as the investors/policymakers are assumed risk neutral.

3.2. Price floor plus shared market 'upside'

The second set of FiT structures analysed in this paper is a variable tariff that offers a guaranteed minimum price, but allows the investor and policymaker to share the gains due to higher prices should the market price exceed the guaranteed minimum. This portion of remuneration shall be referred to as the market 'upside'. This tariff structure provides certainty as to the return for

the investor, but also allows them to receive the benefit of any market 'upside' according to a predefined proportional split which may range anywhere between 0%-100%. When no market upside is offered to the investor the policy becomes a fixed tariff policy.

The payoff that a renewable energy investor receives at time t when $t \leq T_1$ may be characterised as follows;

$$P_t = \max(K, K + \theta(S_t - K))$$
(16)

where θ represents the percentage share of market upside received by the investor. The investor will thus generate Q_I units of electricity such that;

$$\frac{\partial \Pi}{\partial Q} : \left(\left\{ \sum_{t=1}^{T_1} \max(K, K + \theta(S_t - K)) \frac{\partial G_t}{\partial Q} |_{Q_I} \right\} + \left\{ \sum_{t=T_1}^{T} e^{(\mu - r)t} S_0 \frac{\partial G_t}{\partial Q} |_{Q_I} \right\} \right) - C = 0 \quad (17)$$

As Equations (16) and (17) illustrate, modelling of the expected payoff under this FiT structure requires the incorporation of uncertain price processes as outlined in Section 2.2. To identify the expected value of the payoff denoted by equation (16) when $t < T_1$ in the context of the stochastic market prices of Section 2.2, one must derive the expected value of achieving either K or the market price conditional on being greater than K. This is similar to a European 'put' option with the partial differential equation to describe the value of remuneration at time period t represented by equation (18). This is derived in Appendix B.

$$\frac{\partial P}{\partial \tau} + \mu S \frac{\partial P}{\partial S} + \frac{\sigma^2 S^2}{2} \frac{\partial^2 P}{\partial S^2} - rP = 0, \tag{18}$$

which has the terminal condition⁴:

$$P(S_t, t, t) = V_t. (19)$$

Equation 18 may be solved subject to a given terminal condition in order to derive the value of P_t under a different FiT structure. For the shared upside tariff the terminal condition $P(S_t, t, t) = \max(K, K + \theta(S_t - K))$ is considered, giving the following solution to Equation (18):

$$P(S_{\tau},\tau,t) = Ke^{-r(t-\tau)} + \theta \Big(S_{\tau} e^{(\mu-r)(t-\tau)} N(d_1) - Ke^{-r(t-\tau)} N(d_2) \Big),$$
(20)

where N(.) represents the cumulative distribution function of the standard normal distribution

⁴When equation 18 is being solved to find the expected cost of FiT, V_t should be replaced by f_t in equation 19.

while

$$d_1 = \frac{\ln(\frac{S_{\tau}}{K}) + (\mu + \frac{\sigma^2}{2})(t - \tau)}{\sigma\sqrt{t - \tau}},$$
(21)

$$d_2 = \frac{\ln(\frac{S_{\tau}}{K}) + (\mu - \frac{\sigma^2}{2})(t - \tau)}{\sigma\sqrt{t - \tau}}.$$
 (22)

All other variables are as defined as previously. The expression $(1 - N(d_2))$ is equivalent to the probability that the option will be exercised in a risk-neutral world. The expression $S_{\tau}e^{(\mu-r)(t-\tau)}N(d_1)$ represents the expected value, in a risk-neutral world, of a variable that is equal to S_t if the option is exercised and zero otherwise (Hull, 2003). For simplicity, the rest of this paper shall refer to the discounted expected value of FiT from time zero, i.e., $\tau = 0$ and $P(S_0, 0, t)$ will be labelled P_t . When $\tau = 0$ Equation (20) becomes

$$P_t = K e^{-rt} (1 - \theta N(d_2)) + \theta S_0 e^{(\mu - r)t} N(d_1)$$
(23)

with terminal condition $P(S_t, t, t) = V(t)$. Substituting this value for P_t into Equation (5) gives the optimal investment decision in terms of a shared upside policy, represented by equation (24).

$$\left(\sum_{t=1}^{T_1} \left[Ke^{-rt} (1 - \theta N(d_2)) + S_0 e^{(\mu - r)t} \theta N(d_1) \right] \frac{\partial G_t}{\partial Q} |_{Q_I} \right) + \left(\sum_{t=T_1}^T S_0 e^{(\mu - r)t} \frac{\partial G_t}{\partial Q} |_{Q_I} \right) = C$$
(24)

Rearranging (24) allows for θ and K pricing rules:

$$K(\theta) = \frac{C - \left(\left[\sum_{t=1}^{T_1} S_0 e^{(\mu-r)t} \theta N(d_1) \right] \frac{\partial G_t}{\partial Q} |_{Q_I} \right) + \left(\sum_{t=T_1}^{T} S_0 e^{(\mu-r)t} \frac{\partial G_t}{\partial Q} |_{Q_I} \right)}{\left[\sum_{t=1}^{T_1} e^{(\mu-r)t} \left[1 - \theta N(d_2) \right] \frac{\partial G_t}{\partial Q} |_{Q_I} \right]}$$
(25)

$$\theta(K) = \frac{C - \left(\sum_{t=T_1}^{T} S_0 e^{(\mu-r)t} \frac{\partial G_t}{\partial Q} |_{Q_I}\right) + \left(\sum_{t=1}^{T_1} K e^{-rt} \frac{\partial G_t}{\partial Q} |_{Q_I}\right)}{\sum_{t=1}^{T_1} \left(S_0 e^{(\mu-r)t} N(d_1) - K e^{-rt} N(d_2)\right) \frac{\partial G_t}{\partial Q} |_{Q_I}}$$
(26)

Equation (25) may be used to calculate an efficient price floor K that must be used with a given θ , whilst equation (26) gives the efficient θ that must be used with a given price floor K. This framework may also be used to calculate the cost of the FiT programme to the policymaker. For time period t, the cost of the FiT may be defined as

$$f_t = \max(0, K - S_t) - (1 - \theta) \max(0, S_t - K).$$
(27)

Using equation (27) as the terminal condition in equation (19), equation (18) may be used to obtain

the following discounted expected cost of the FiT:

$$EF_t = e^{-rt} E[f_t] = K e^{-rt} - S_0 e^{(\mu - r)t} + \theta \Big(S_0 e^{(\mu - r)t} N(d_1) - K e^{-rt} N(d_2) \Big).$$
(28)

Full derivations for (25) and (26) are provided in Appendix C.1, whilst equations (27) and (28) are derived in Appendix C.1.1.

Equations (25) and (26) provide a means to calculate an efficiently-priced K and θ combination. Equation (25) shows that the optimal price K is based around the average cost of generation but is partially determined by the proportion of total remuneration obtained from $\sum_t S_t$ and $\sum_t K$. These values are influenced by the risk-neutral probability of achieving either the market price S_t or the guaranteed price K, and weighted according to the proportion of total market upside received by an investor, represented by θ . The probability values change when K changes, as an increase in K results in an increase in the probability of $K > S_t$, holding everything else constant. This results in the inability to identify an explicit solution for K in terms of all other parameters. In practice, an iterative procedure may be used to calculate efficient K and θ combinations for a number of assumed K values using Equations (25) and (26).

When $\theta = 0$ an efficient K is completely independent of the stochastic market process derived in Section 2.2, with remuneration thus fixed for all time periods. In such circumstances, equation (25) may be interpreted as the total cost of generation divided by the total units generated. When θ is not equal to zero, the efficient price floor K is determined by the θ value chosen. When θ increases from zero to any number greater than that, the additional negative term in the numerator of (25), along with the θ term in the denominator, results in a smaller value for the optimal K, holding everything else constant. This results in an inverse relationship between efficient θ and K values, where a trade-off exists with respect to the degree of market upside a policymaker is willing to offer in exchange for a reduced guaranteed price premium K.

From this finding it may also be concluded that there is a unique locus of θ and K combinations, with one efficient K for each θ , holding all other variables constant. Equation (29) provides a means to quantify the shape of this locus and thus the sensitivity of θ to a change in the guaranteed price floor, K;

$$\frac{\partial K}{\partial \theta}\Big|_{\Pi} = -\frac{\sum_{t=1}^{T_1} [S_0 e^{(\mu-r)t} N(d_1) - K e^{-rt} N(d_2)] G_t}{\sum_{t=1}^{T_1} e^{-rt} [1 - \theta N(d_2)] G_t} \le 0$$
(29)

One can see the impact a shared upside policy may have from the policymaker's perspective by analysing equation (28), where policymaker exposure to market price fluctuation is weighted according to $(1 - \theta)$.

In summary, it has been shown that each θ structure offers the same level of FiT remuneration/cost in expectation, should observed market conditions follow those assumed ex-ante

when setting the tariff rate. However, the risk presented by a deviation in market price evolution is allocated differently. This trade-off is discussed further in Sections 4 and 5.

3.3. Price-floor FiT with cap

The third FiT structure analysed in this paper is a variable tariff that offers a guaranteed minimum price, which also shares a portion of market upside. However, instead of sharing a fixed portion of all market upside, the investor receives the entire amount of remuneration up to an upper limit or 'cap'. Should the market price exceed this cap, the investor receives the cap price and the policymaker receives market remuneration exceeding the cap. Denoting the cap as \bar{S} , the price received by the investor during time period t when $t < T_1$ may be characterised as follows:

$$V_t = \max(K, \min(S_t, \bar{S})) \tag{30}$$

Given the assumed stochastic market price process of Section 2.2, the partial differential equation to describe the value of remuneration at time period t represented by equation 18 (derived in Appendix B). When $t < T_1$, the expected value for P_t with a terminal condition of equation 30 is derived in Appendix C.2 and may be characterised as follows:

$$P_t = e^{-rt} E[V_t] = K e^{-rt} \left(1 - N(d_2) \right) + S_0 e^{(\mu - r)t} \left(N(d_1) - N(d_3) \right) + \bar{S} e^{-rt} N(d_4)$$
(31)

where d_1 and d_2 are as previously defined and

$$d_3 = \frac{\ln(\frac{S_{\tau}}{S}) + (\mu + \frac{\sigma^2}{2})(t - \tau)}{\sigma\sqrt{t - \tau}},$$
(32)

$$d_4 = \frac{\ln(\frac{S_\tau}{S}) + (\mu - \frac{\sigma^2}{2})(t - \tau)}{\sigma\sqrt{t - \tau}}.$$
(33)

Given this expected price, Equation (5) may be augmented in the following way to get the optimal investment rule under a cap & floor policy;

$$\sum_{t=1}^{T_1} \left(\left[Ke^{-rt} (1 - N(d_2)) + S_0 e^{(\mu - r)t} (N(d_1) - N(d_3)) + \bar{S}e^{-rt} N(d_4) \right] \frac{\partial G_t}{\partial Q} |_{Q_I} \right) + \sum_{t=T_1}^{T} \left(S_0 e^{(\mu - r)t} \frac{\partial G_t}{\partial Q} |_{Q_I} \right) - C = 0,$$
(34)

Rearranging Equation (34) allows for guaranteed floor, K, and cap, \overline{S} , pricing rules:

$$K = \frac{C - \{\sum_{t=1}^{T_1} [(S_0 e^{(\mu-r)t} (N(d_1) - N(d_3)) + \bar{S}e^{-rt} N(d_4))] \frac{\partial G_t}{\partial Q} |_{Q_I} \} - [\sum_{t=T_1}^{T} S_0 e^{(\mu-r)t} \frac{\partial G_t}{\partial Q} |_{Q_I}]}{\sum_{t=1}^{T_1} e^{-rt} (1 - N(d_2)) \frac{\partial G_t}{\partial Q} |_{Q_I}} |_{Q_I}}$$
(35)

and

$$\bar{S} = \frac{C - \{\sum_{t=1}^{T_1} [Ke^{-rt}(1 - N(d_2)) + S_0 e^{(\mu - r)t}(N(d_1) - N(d_3))] \frac{\partial G_t}{\partial Q}|_{Q_I}\} - [\sum_{t=T_1}^{T} S_0 e^{(\mu - r)t} \frac{\partial G_t}{\partial Q}|_{Q_I}\}}{\sum_{t=1}^{T_1} e^{-rt} N(d_4) \frac{\partial G_t}{\partial Q}|_{Q_I}}$$
(36)

Analogous to the previously discussed interpretations of $(1 - N(d_2))$ and $S_0 e^{(\mu-r)(t)} N(d_1)$ (Hull, 2003, for example), the expression $S_0 e^{(\mu-r)(t)} N(d_3)$ represents the expected value, in a risk-neutral world, of a variable that is equal to S_t if the market price is greater than \bar{S} and zero otherwise. As such, $S_0 e^{(\mu-r)t} (N(d_1) - N(d_3))$ represents the expected value, in a risk-neutral world, of a variable that is equal to S_t if the market price is within the cap & floor 'collar' and zero otherwise. $N(d_4)$ represents the expected probability, in a risk-neutral world, of exceeding \bar{S}

Alongside defining an efficient price, equations defining the cost of the FiT regime for the policymaker may be calculated. For a cap & floor policy, the cost of the FiT at time T is

$$f_t = \max(0, K - S_t) - \max(0, S_t - \bar{S}).$$
(37)

Using equation (37) as the terminal condition in equation (19), equation (18) may be used to obtain the following discounted expected cost of FiT:

$$EF_t = e^{-rt}E[f_t] = Ke^{-rt}(1 - N(d_2)) - S_0e^{(\mu - r)t}(1 - N(d_1)) - S_0e^{(\mu - r)t}N(d_3) + \bar{S}e^{-rt}N(d_4).$$
(38)

Details of how this solution is derived are provided in Appendix C.2.1.

Similar to the shared upside regime, a unique locus of efficient K and \overline{S} pairings exists. Equations (35) and (36) show that there is an inverse relationship between efficient K and \overline{S} values, with a higher K resulting in a lower \overline{S} . In a similar fashion to the shared upside regime, the shape of this locus and thus sensitivity of this relationship may be quantified by the partial derivative of equation (39):

$$\frac{\partial K}{\partial \bar{S}}|_{\Pi} = -\frac{\sum_{t} e^{-rt} N(d_4) G_t}{\sum_{t} e^{-rt} [1 - N(d_2)] G_t} \le 0$$
(39)

One can see that an increase in K results in smaller value for $S_0 e^{(\mu-r)(t)} N(d_1)$ and a greater value for $S_0 e^{(\mu-r)(t)} N(d_3)$. As a result, $S_0 e^{(\mu-r)t} (N(d_1) - N(d_3))$ will fall and thus the degree to which investors are exposed to market price uncertainty falls with an increase in K. Equation (38) shows that such an increase in K results in a reduced exposure to market prices for the policymaker. This trade-off is similar to that found with respect to the shared upside regime. Such exposure is compared in greater detail in Section 4.

4. Impact of error in assumed market price parameters

Should market prices evolve as expected, the remuneration/cost of the FiT policy will be as predicted by the formulae derived in Section 3. Should predictions contain a degree of error, then the eventual levels of policy cost and remuneration may be over or under estimated. The eventual outcome is an important consideration as the policymaker may be bound to meet the Q_I level of output, whilst minimising the cost of policy may also be of concern. The magnitude of these impacts will differ for each tariff structure when $t \leq T_1^5$, whilst the potential impact on investor behaviour is dependent on whether it becomes apparent before or after investors have committed to a given level of deployment.

4.1. Impact of parameter specification error on investor remuneration and behaviour

Equations (40) to (42) outline the impact an error in the specified rate of market price growth may have on the eventual rate of investor remuneration, whilst the sensitivities with respect to changes in market price volatility are observed in Equations (43) to (45).

$$XPremium: \frac{\partial \Pi}{\partial \mu} = \sum_{t=1}^{T_1} t S_0 e^{(\mu-r)t} G_t \ge 0$$
(40)

Shared Upside :
$$\frac{\partial \Pi}{\partial \mu} = \sum_{t=1}^{T_1} t S_0 e^{(\mu - r)t} G_t \theta N(d_1) \ge 0$$
 (41)

$$Cap \& floor: \frac{\partial \Pi}{\partial \mu} = \sum_{t=1}^{T_1} t S_0 e^{(\mu - r)t} G_t (N(d_1) - N(d_3)) \ge 0$$
(42)

$$X Premium : \frac{\partial \Pi}{\partial \sigma} = 0 \tag{43}$$

Shared Upside :
$$\frac{\partial \Pi}{\partial \sigma} = \sum_{t=1}^{T_1} \sqrt{t} S_0 e^{(\mu-r)t} G_t \theta N'(d_1) \ge 0$$
 (44)

$$Cap \& floor: \frac{\partial \Pi}{\partial \sigma} = \sum_{t=1}^{T_1} \sqrt{t} S_0 e^{(\mu-r)t} G_t (N'(d_1) - N'(d_3)) \ge 0$$
(45)

Exposure to market prices is represented by the terms $tS_0e^{(\mu-r)t}G_t$ and $\sqrt{t}S_0e^{(\mu-r)t}G_t$ for growth and volatility sensitivity, respectively. For each tariff structure, these terms are weighted by

⁵the expected value of remuneration when $t > T_1$ is the same for all policy options. As the purpose of this section is to compare sensitivity of each policy option, the analysis is limited to the relevant portion of remuneration that differs under different policy options. As such, remuneration when $t > T_1$ is thus omitted from this section of analysis

a term that corresponds to the probability of market exposure under each regime. For the constant premium tariff, this probability is equal to one with respect to error in market price growth, and zero with respect to changes in market price volatility⁶.

Analysing sensitivities to market price growth, the shared upside policy and cap & floor policies are weighted by $\theta(N(d_1))$ and $(N(d_1) - N(d_3))$ respectively. Investor remuneration under the shared upside policy is completely unaffected by changes in market price should $\theta = 0$. As such, a policy with a lower K, regardless of whether it is a cap & floor or shared upside policy, will always be more sensitive to error. It is due to this relationship that, when $\theta = 1$, investor remuneration under a shared upside policy will be more sensitive to changes in market price growth than a cap & floor policy, should $\overline{S} < \infty$.

The relative sensitivity of a shared upside policy to a cap & floor policy is less clear if $0 < \theta < 1$ and $\bar{S} < \infty$. The relative sensitivity of either tariff structure to a change in WWAP growth is determined by $\theta N(d_1)$ and $N(d_1) - N(d_3)$. Specifying the same K for both shared upside and cap & floor policies implies that $N(d_1)$ is the same for both and relative sensitivity may thus be compared. If $\theta N(d_1)$ is equal to $N(d_1) - N(d_3)$, then the impact on remuneration and the deployment of Q_I will be unaffected. If $\theta N(d_1) > N(d_1) - N(d_3)$, then the shared upside policy is more sensitive. As $N(d_1)$ is associated with the probability that S < K and K is assumed the same for both policies, then the change in $N(d_1)$ will be the same for both. θ will remain constant and thus any relative impact is determined by $N(d_3)$. $N(d_3)$ is associated with the probability that $S < \overline{S}$ and $\theta N(d_1) > N(d_1) - N(d_3)$ may occur if growth increases to such an extent that the probability of $S < \overline{S}$ decreases. As such, investor remuneration under a shared upside policy is more sensitive to an extreme ex-post increase in growth/volatility. However, should $N(d_3)$ fall relative to ex-ante assumptions, then $\theta N(d_1) < N(d_1) - N(d_3)$ and the investor receives greater remuneration under the cap & floor policy. As such, the cap & floor policy protects an investor from modest under remuneration to a greater extent than the shared upside policy, whilst removing the possibility of supernormal profit to a greater extent.

The impact of a change in the rate of volatility is represented by $\theta(N'(d_1))$ and $(N'(d_1) - N'(d_3))$ for a shared upside and cap/floor policy respectively. One can see that the relative impacts are the same as for changes in market price growth. However, the quantitative magnitude may differ. These magnitudes are explored in the numerical analysis of Section 5.

Should these impacts become apparent after a policy has been put in place but before an investor commits to a quantity of investment, the change in potential remuneration has a direct impact on the profit maximising output of Equation (5). As such an increase (decrease) in potential remuneration

⁶This finding is predicated on the assumption of risk neutrality. A full analysis of optimal pricing under different conditions of risk aversion is considered in a future analysis.

leads to an increase (decrease) in the optimal quantity deployed. Alternatively, these changes may occur after investors have committed to a quantity. As such, remuneration will be affected according to equations (40) to (45). This may reduce the profitability of the investment if negative or lead to supernormal profits if positive.

4.2. Impact of parameter specification error on policy cost

Defining the cost of policy as 'F', the impact an ex-post change in market prices may have on implementation cost is outlined in Equations (46) to (51). One can see that policy cost is completely unaffected by changes in both the rate of growth and volatility in market prices under a constant premium regime. A change in volatility has the opposite effect to a change in growth for shared upside and cap & floor policies. An increase in market price growth has a negative impact on policy cost, as the proportion of time that the market price is received by the investor grows, with a reduced policymaker subsidy required to supplement low market prices. Conversely, a change in market price volatility has a negative impact on policy cost as there is a greater probability that the cost will be less than floor price K. Interpreted in the context of the impact of volatility found by Section 4.1, an increase in volatility results in both an increase in policy cost is unaffected by the timescale at which an error in expected market price parameters becomes apparent.

$$X Premium : \frac{\partial F}{\partial \mu} = 0 \tag{46}$$

Shared Upside:
$$\frac{\partial F}{\partial \mu} = -\sum_{t=1}^{T_1} t S_0 e^{(\mu-r)t} G_t [1 - \theta N(d_1)] \leq 0$$
 (47)

$$Cap \& floor: \frac{\partial F}{\partial \mu} = -\sum_{t=1}^{T_1} t S_0 e^{(\mu - r)t} G_t (1 - N(d_1) + N(d_3)) \le 0$$
(48)

$$X Premium : \frac{\partial F}{\partial \sigma} = 0 \tag{49}$$

Shared Upside:
$$\frac{\partial F}{\partial \sigma} = \sum_{t=1}^{T_1} \theta S_0 \sqrt{t} e^{(\mu-r)t} G_t N'(d_1) \ge 0$$
 (50)

$$Cap \& floor: \frac{\partial F}{\partial \sigma} = \sum_{t=1}^{T_1} S_0 \sqrt{t} e^{(\mu - r)t} G_t (N'(d_1) - N'(d_3)) \ge 0$$
(51)

4.3. Comparing Ex-Post Sensitivities

Comparing equations (40) - (45) to (46) - (51) shows that an increase in volatility for both shared upside and cap/floor regimes imposes an additional cost in direct proportion to the profitability of investment.

If the rate of market price growth deviates from the rate assumed when setting the FiT price, the impact on policy cost is disproportionate to the impact on investor remuneration. For a shared upside regime, the relative impact is determined by the parameters $\theta N(d_1)$ and $1 - [\theta N(d_1)]$ for profit and cost respectively. For θ values greater than 0.5, $\theta N(d_1)$ will exceed $1 - [\theta N(d_1)]$ if one holds $N(d_1)$ constant. As such, changes in growth will have a greater impact on changes in policy cost than investor remuneration. A similar comparison may be made for a cap & floor policy, where the impact on profit and cost are represented by $(N(d_1)+N(d_3))$ and $(1-(N(d_1)+N(d_3)))$ respectively. Should $(N(d_1) + N(d_3))$ be greater than 0.5, then the impact on profit will be greater than the impact on policy cost. This may occur if $N(d_3)$ grows to a considerably high value due to extreme changes in growth. As such, policymakers may wish to take into account these factors when determining the distribution of risk presented by parameter specification.

5. An Illustration of Practical Model Implementation

This section demonstrates the application of the FiT pricing models of Section 3 and the tools for ex-post analysis derived in Section 4. A scenario analysis is used to give quantitative insight into relative sensitivities when the underlying market conditions vary.

5.1. Simulation Parameters

Using parameters outlined in Table 1, a stylised Irish case study following Doherty and O'Malley (2011) and Mc Garrigle et al. (2013) is considered. The annual Wind Weighted Average Price (WWAP) is modelled using GBM. It is assumed that a wind turbine is operational for 20 years, with FiT remuneration received during the initial 15 years and the expected WWAP considered for periods 16-20. Wind farm cost and availability data are taken from Doherty and O'Malley (2011).

The following parameters are used to calibrate the generation function of equation (2). Following Mc Garrigle et al. (2013), it is assumed that Ireland's 2020 target (Q_I) corresponds to a wind capacity of 4630MW being installed in Irish Single Electricity Market (SEM). The maximum possible capacity (Q_{max}) is assumed to be 16GW (SEAI, 2011) and the capacity factor is assumed constant at 0.35, whilst there is 95% availability. As the capacity factor is assumed constant, the b(Q) parameter equation (2) reflects the Q installed, adjusted to consider changes in the level of curtailment alone. The γ parameter is chosen such that the level of curtailment measured at Q_I approximates that of Mc Garrigle et al. (2013), who find that under assumptions of low offshore wind capacity and a 75% System Non-Synchronous Penetration limit (SNSP)⁷,

⁷SNSP is the systems capacity to safely produce a certain percentage of its generation from non-synchronous sources such as wind turbines. It has been suggested SNSP limits will be between 60% and 75% by 2020, with recommendations that a limit of 75% may be feasible (Mc Garrigle et al., 2013).

curtailment is 7.3% of total output. As such, the γ parameter is equal to 6.75×10^{-5} and using equation (2), these factors result in $G_t(Q)$ equalling 12,501,319.

The initial annual WWAP S_0 is assumed to be 52.41, following Doherty and O'Malley (2011). An analysis of SEM market data (SEMO, 2011) indicates that this approximates the Irish WWAP for 2010, with this simulation calibrated accordingly. Three future market price scenarios are considered. Scenario A is the central scenario as outlined in Table 1. The Irish electricity market has been in operation since 2007, with price data thus of insufficient duration to accurately calibrate WWAP parameters. To overcome this problem, Doherty and O'Malley (2011) model expected WWAP values according to expert opinion. Given the lack of existing data, this paper follows the precedent set by Doherty and O'Malley (2011) and calibrates growth and volatility parameters such that each distribution of annual prices is similar to those outlined in Doherty and O'Malley (2011). Rates of growth and volatility are chosen such that median, 10th and 80th percentile WWAP values correspond to those outlined in Doherty and O'Malley (2011).

Sensitivity to different underlying rates of growth and volatility is assessed by either doubling the rate of growth (Scenario B) or volatility (Scenario C) whilst holding all other variables constant at Table 1 values.

Parameter	Value	Source
Capital Cost (Wind, per MW)	€1.76m	(a)
Annual Operations and Maintenance Cost	2% of capital cost	(a)
SEM Installation target (Q_I)	4,630 MW	(b)
Capacity Factor (u)	0.35	(a)
Availability (v)	0.95	(a)
Maximum Q (Q_{max})	16 GW	(c)
γ	6.75×10^{-5}	
Generation during $t(G_t)$	12,501,319	own calculation
Electricity Price Growth (μ)	0.0155	Calibrated to (a)
Electricity Price Volatility (σ)	0.13	Calibrated to (a)
Initial WWAP (S_0)	€52.41	
Discount Rate (r)	0.06	

Table 1: B	Baseline S	Simulation	parameters
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(a) Doherty and O'Malley (2011); (b) Mc Garrigle et al. (2013) (c) SEAI (2011)

5.2. Simulation results

Table 2 illustrates efficient FiT prices using the formulae derived in Section 3. θ values equal to 0 and 1 are taken alongside representative values for low ($\theta = 0.2$), medium ($\theta = 0.6$) and

high ($\theta = 0.8$) intermediate θ values. To comprehensively compare shared upside and cap & floor policies, cap & floor scenarios with both low and high floors are displayed. For comparability, the floor chosen for each corresponds to the efficient floors for the shared upside policies of $\theta = 0.2$ and $\theta = 0.8$ respectively. A constant (X) premium is also analysed.

Firstly, a slight difference in FiT prices is observed when $\theta = 0$, representing the influence of market prices during the final 5 years of operation. When $\theta = 0$, a higher rate of growth results in a lower FiT in Scenario B, whilst there is a marginal change in the efficient FiT under a higher rate of volatility (Scenario C), precluding any conclusive finding. This is in contrast to the constant X premium policies, which fall to approximately one tenth of their baseline values due to a doubling of the rate of growth in Scenario B, quantifying the importance of correct growth specification under such a policy structure for certainty of investor remuneration. Under the risk-neutral assumptions of this application, the constant premium policy is insensitive to changes in volatility.

Table 2 shows the magnitude of the negative relationship between the θ parameter and the efficient price floor K for the shared upside policies when $\theta \neq 0$. For shared upside policies, an increase in the rate of growth (Scenario B) causes all floor prices to fall relative to Scenario A, whilst an increase in the rate of volatility (Scenario C) also causes floor prices to fall but to a lesser extent. Analysing cap & floor policies, Table 2 shows that an increase in the rate of price growth causes efficient cap prices to fall for both low and high floor price scenarios. Conversely, an increase in the rate of volatility leads to an increase in the price cap as market upside remuneration is subject to a wider dispersion which must be accounted for.

Relative to policies with high price floors, the efficient price cap for cap & floor policies with low price floors is marginally more sensitive to changes in the rate of growth. However, Table 2 shows that cap & floor policies with a low price floor are considerably more sensitive to changes in the rate of volatility. Section 4 has described how cap specification affects policy cost and investor remuneration, with these findings indicative of sensitivities to market price misspecification. The implications of such sensitivities across the considered scenarios are fully explored in tables 3 and 4.

	Baseline (A)	High Growth (B)	High Volatility (C)
Shared Upside (θ)			
θ : 0	71.05	67.13	71.02
θ : 0.2	70.19	65.24	68.65
θ : 0.6	68.20	60.16	62.94
θ : 0.8	67.02	56.17	59.30
θ : 1	65.67	47.69	54.63
Cap & Floor (high floor)			
Floor	70.19	65.24	68.65
Cap	74.83	70.18	80.15
Cap & Floor (low floor)			
Floor	67.02	56.17	59.30
Cap	103.14	98.72	163.27
Constant Premium			
Х	12.87	2.72	12.87

Table 2: FiT Prices for hypothetical example (\in /MWh)

Tables 3 and 4 quantify the sensitivity of policy cost and investor revenue to parameter specification error. Overall, changes in the rate of growth are more influential on policy cost and investor remuneration than changes in the rate of volatility. Table 3 shows a lower θ value leads to the greatest fall in policy cost under Scenario B, when the underlying rate of growth is greatest. Policy cost is less sensitive to a change in the rate of growth under Scenario C than Scenario A. It is also less sensitive under Scenario C than Scenario B, except when θ is high and $\cong 1$.

One can see that a change in volatility has the greatest impact on both policy cost and investor profit in Scenario A than both scenarios B and C. This indicates that scenarios of high volatility or growth result in volatility deviations having a lesser impact. For all scenarios the impact is greatest with a higher theta.

Changes in the rate of growth have the greatest impact in Scenario B, where the underlying rate of growth is greatest. Comparing the impact of changes in the rate of growth on cost and remuneration between scenarios A and C, Table 3 shows that the impact on cost is greater under Scenario A (when $\theta \neq 0$), whilst the impact on remuneration is greater under Scenario C. This would suggest that a higher underlying rate of volatility suppresses the sensitivity of cost with respect to a change in the rate of growth.

		Shared Upside $(\theta) \in (m)$				
		0	0.2	0.6	0.8	1
Baseline (A)	$\frac{\partial F}{\partial \mu}$	- 50,315	- 44,434	- 32,283	- 25,959	- 19,414
	$\frac{\partial F}{\partial \sigma}$	0	1,348	4,036	5,372	6,695
	$\frac{\partial \Pi}{\partial \mu}$	0	5,881	18,032	24,356	30,901
	$\frac{\partial \Pi}{\partial \sigma}$	0	1,348	4,036	5,372	6,695
High Growth (B)	$\frac{\partial F}{\partial \mu}$	- 58,596	- 49,866	- 31,371	- 21,190	- 8,879
	$\frac{\partial F}{\partial \sigma}$	0	1,303	3,714	4,692	4,946
	$\frac{\partial \Pi}{\partial u}$	0	8,731	27,225	37,406	49,717
	$\frac{\partial \Pi}{\partial \sigma}$	0	1,303	3,714	4,692	4,946
	00					
High Volatility (C)	$\frac{\partial F}{\partial \mu}$	- 50,315	- 43,675	- 29,832	- 22,499	- 14,689
	$\frac{\partial F}{\partial \sigma}$	0	1,302	3,840	5,048	6,171
	$\frac{\partial \Pi}{\partial \mu}$	0	6,640	20,483	27,816	35,626
	$\frac{\partial \Pi}{\partial \sigma}$	0	1,302	3,840	5,048	6,171

Table 3: Partial Derivatives: Shared Upside Policy

When comparing the results of Tables 3 and 4, common price floors suggest that it is best to compare the 'high floor' policy with a shared upside policy of $\theta = 0.2$, and a 'low floor' policy with a shared upside policy of $\theta = 0.8$. One can see that for both such comparisons, a change in growth leads to a greater policy cost reduction under a cap & floor policy than the shared upside policy. Predicted by Section 4, this is due to a greater proportion of market upside being in excess of the cap value, with the policymaker retaining a greater proportion of this under the cap & floor policy than the shared upside policy. One can see the magnitude of this effect in these simulations. A similar observation may be made with respect to change in profit, with the magnitude of change considerably less under a cap & floor policy.

Indeed, policy cost falls to a greater extent due to an increase in the rate of price growth under a high floor policy than a low floor policy. In both scenarios the policymaker is required to enforce the floor to a lesser extent, but a higher floor means that there is a lower cap, and thus the policymaker has ability to recoup a greater share of upside under this policy. One should note that the effect is reversed should growth be less than expected, with this result indicating that a higher floor requires greater subsidy.

A change in growth leads to the greatest cost reduction under Scenario B, when the underlying level of market price growth is high. Similarly, the wide margin offered by a low floor policy results in a change in growth having a greater impact on investor profit across all scenarios, the

magnitude of which is greater in the presence of a high degree of underlying price growth in Scenario B. Scenario C (high volatility) is the second most sensitive to this, then Scenario A (baseline). Sensitivities to volatility for both cost and profit are greatest when the underlying rate of growth is high (Scenario B).

A change in the rate of volatility has a positive impact on both policy cost and investor remuneration under Scenario A. One can see that this effect is greater for a low floor than a high floor policy, with a high floor policy considerably less than the equivalent shared upside policy. This suppression is due to the presence of the cap, demonstrating the hedging qualities against extreme changes offered by the cap & floor policy, as discussed in Section 4.

Indeed, these hedging qualities are demonstrated to a greater extent in Scenarios B and C, where both cost and remuneration fall with changes in the rate of volatility. This is due to the presence of the price cap and the influence of increasing average prices. An increase in volatility results in greater cost and remuneration during early years of deployment. However, as the average price increases with time, the price cap limits the benefit of positive volatility spikes received by the investor, transferring these to the policymaker. As such, deviations in volatility during the latter years of deployment benefit the policymaker more than the investor. For extreme deviations in either growth or volatility, these latter impacts outweigh the impacts during the early years, and thus policy cost and investor remuneration falls. This effect is more pronounced when there is a high underlying rate of growth than when there is a high underlying rate of volatility. This finding has important implications for the potential application of this policy structure. First, it demonstrates how a cap & floor allows policymakers to benefit from the hedging opportunities provided by wind to a greater extent than a shared upside policy. This example demonstrates that it is important that investors are bound to share market upside with policymakers under an adequate contract. This is especially important in the latter years of a cap & floor policy if the policymaker is likely to benefit to a greater extent than the investor. This is a contractual issue that must be considered under all FiT regimes. If high underlying rates of volatility or growth were anticipated, investors may be reluctant to enter such a power purchase agreement with a supplier. This would suggest that a cap & floor regime may not be suited to long term contracts in extreme market conditions demonstrated in Scenarios B and C. If such conditions were evident, a cap & floor FiT contract of short duration may be more appropriate.

Overall, Tables 3 and 4 show how the total impact of a change in market price processes may be delineated amongst investors and policymakers. Market price expectations and preference for robust policy cost or investor remuneration will influence the particular specification chosen by a policymaker. This analysis has given a quantitative insight into the relative magnitudes of each share under each policy option and provided further insight as to the behaviour of these effects under different scenarios of market conditions. Future analysis is proposed to incorporate quantitative measures of preference in deciding the circumstances under which each option may be preferred.

		Cap &	v	
		High Floor	Low Floor	Λ
Baseline (A)	$\frac{\partial F}{\partial \mu}$	- 48,871	- 40,833	0
	$\frac{\partial F}{\partial \sigma}$	3.21	509	0
	$\frac{\partial \Pi}{\partial \mu}$	1,444	9,483	50,315
	$\frac{\partial \Pi}{\partial \sigma}$	3.21	509	0
High Growth (B)	∂F	- 56 751	- 45 840	0
	$\frac{\partial \mu}{\partial F}$	- 262	- 1,311	0
	$\frac{\partial \Pi}{\partial \mu}$	1,845	12,755	58,596
	$\frac{\partial \Pi}{\partial \sigma}$	- 262	- 1,311	0
High Volatility (C)	$\frac{\partial F}{\partial \mu}$	- 48,610	- 39,129	0
	$\frac{\partial F}{\partial \sigma}$	- 140	- 160	0
	$\frac{\partial \Pi}{\partial \mu}$	1,705	11,185	50,315
	$\frac{\partial \Pi}{\partial \sigma}$	- 140	- 160	0

Table 4: Partial Derivatives: Cap & Floor and Constant (X) Premium (€m)

6. Conclusion

Many policy and academic studies have stated a preference for the use of Feed-in Tariff (FiT) regimes to support the deployment of renewable energy technologies. Tariff structures commonly employed tend to result in either investors or policymakers incurring the full degree of market price risk. It has been the purpose of this paper to analyse these tariff structures alongside different tariff structures that allow market price risk to be shared. In particular, this paper has developed a tractable and transparent means to define and compare efficient FiT rates for three classes of FiT structure. Each tariff structure varies in the way through which market price risk is shared amongst investors and policymakers. The analytical model presented incorporates elements of sequential decision making in game theory to characterise the strategic interaction of policymakers and investors. Option pricing theory has been used to characterise the expected value of a FiT regime when combinations of a certain guarantee and uncertain market prices are offered to investors. From this, efficient pricing rules have been derived for tariffs of constant premium, shared upside and cap & floor regimes. Partial derivatives are used to quantify sensitivity to ex-post changes in

the underlying market parameters. Numerical simulations have provided quantitative insight into efficient prices under each regime and the relative magnitude of changes in policy cost and investor profit due to ex-post market changes.

This work has provided the means to compare tariff structures and has applied these tools to compare the sensitivity of FiT designs to market price misspecification using an Irish case study. A scenario analysis has provided insight into how ex-post impacts change when market price parameters are proportionally different than the assumed Irish baseline scenario. Alongside providing tools for policy, the findings of this paper provide a modelling platform that may also aid future academic analyses of FiT policy. Although some insight has been offered as to the contextual suitability of certain tariff structures, this analysis has focussed on tariff definitions and has thus not addressed the issue of risk preference, which will be the subject of a follow-on study.

In a global energy market characterised by increasing proliferation of low-cost gas, wholesale energy prices are becoming increasingly uncertain. It is in this context that the potential cost of renewables deployment is becoming an ever-increasing concern in policy and academic debate. This paper provides a timely contribution by creating an analytical framework for FiT analyses through which the sharing of this market price risk in renewable energy deployment may be analysed.

Appendices

A. Augmentation of GBM when applying FiT pricing model in an incomplete market

Application of the Black-Scholes model assumes that the underlying market is complete, in that, there exist contracts to insure against all possible eventualities. However, electricity is a unique commodity as it cannot be stored and thus demand must equal supply at all moments in time (Lyle and Elliott, 2009; Burger et al., 2004; Tsitakis and Yannacopoulos, 2006). This characteristic affects the ability to hedge and thus the market for electricity is incomplete (Burger et al., 2004; Tsitakis and Yannacopoulos, 2006). The market price of risk may thus need to be incorporated into the Black-Scholes formula (Lemoine, 2009; Lyle and Elliott, 2009) and the SDE must be adjusted to account for this in the following way:

$$dS = (\mu^* - \lambda\sigma)Sdt + \sigma Sdw, \tag{52}$$

where λ represents the market price of risk (Hull, 2003; Wilmott et al., 1993) and μ^* represents rate of drift before risk is taken into account. In the conventional Black-Scholes pricing formula,

 $\lambda = \frac{\mu^* - r}{\sigma}$ Wilmott et al. (1993) and Equation (52) may be rewritten as

$$dS = rSdt + \sigma Sdw,\tag{53}$$

Consideration of this market risk when estimating a REFIT price may be carried out by defining $\mu = \mu^* - \lambda \sigma$ as the risk-adjusted drift. The SDE of Equation 52 then becomes

$$dS = \mu S dt + \sigma S dw. \tag{54}$$

As such, instead of estimating μ^* , μ is estimated. This approach follows that employed by (Burger et al., 2004; Benth et al., 2003; Schwartz and Smith, 2000; Albanese et al., 2006).

B. Deriving the partial differential equation governing the discounted expected FiT payoff

The Kolmogrov backward equation governing the probability $\rho(S_{\tau}|S_t)$ of getting a WWAP S at time τ , given a WWAP S_t at time t ($\tau \leq t$) is:

$$-\frac{\partial\rho}{\partial\tau} = \mu S \frac{\partial\rho}{\partial S} + \frac{\sigma^2 S^2}{2} \frac{\partial^2 \rho}{\partial S^2}.$$
(55)

with final condition $\rho(S_t|S_t) = \delta(S - S_t)$, where $\delta(.)$ represents the Dirac delta function (Wilmott et al., 1993; Wilmott, 2000). The discounted expected value of the FiT at time τ , given S_{τ} and expiry at time t may be defined as⁸

$$P(S_{\tau}, \tau, t) = e^{-r(t-\tau)} \int_0^\infty \rho(S_{\tau}|S_t) V_t dS_t.$$
 (56)

Now consider the following derivatives of Equation 56:

$$\frac{\partial P}{\partial \tau} = rP + e^{-r(t-\tau)} \int_0^\infty \frac{\partial \rho}{\partial \tau} V_t dS_t$$
(57)

$$\frac{\partial P}{\partial S} = e^{-r(t-\tau)} \int_0^\infty \frac{\partial \rho}{\partial S} V_t dS_t$$
(58)

$$\frac{\partial^2 P}{\partial S^2} = e^{-r(t-\tau)} \int_0^\infty \frac{\partial \rho}{\partial S^2} V_t dS_t$$
(59)

⁸When Equation 18 is being solved to find the discounted expected cost of FiT, V_t should be replaced by f_t in Equation 56.

Multiplying equation (55) by $e^{-r(t-\tau)}V_t$ and integrating over all possible values of V_t leads to

$$-e^{-r(t-\tau)}\int_0^\infty \frac{\partial\rho}{\partial\tau} V_t dS_t = \mu S e^{-r(t-\tau)} \int_0^\infty \frac{\partial\rho}{\partial S} V_t dS_t + \frac{\sigma^2 S^2}{2} e^{-r(t-\tau)} \int_0^\infty \frac{\partial^2\rho}{\partial S^2} V_t dS_t$$
(60)

When Equations (57) - (59) are substituted into Equation (60) the following PDE is obtained:

$$\frac{\partial P}{\partial \tau} + \mu S \frac{\partial P}{\partial S} + \frac{\sigma^2 S^2}{2} \frac{\partial^2 P}{\partial S^2} - rP = 0, \tag{61}$$

which has the terminal condition:

$$P(S_t, t, t) = \int_0^\infty \rho(S_t | S_t) V_t dS_t,$$
(62)

$$= \int_0^\infty \delta(S - S_t) V_t dS_t, \tag{63}$$

$$= V_t.$$
(64)

C. Solving the GBM partial differential equation for shared upside policy

Obtaining the discounted expected value and discounted expected Cost of FiT for the both Shared Upside and Cap & Floor policies (using Equation 61) is not trivial. In this section details of how these solutions are obtained are provided. The way in which they are derived is similar to that used to find solutions to the Black-Scholes PDE for European call and put options Wilmott et al. (1993).

Firstly consider the following transformations:

$$S = Ke^x, (65)$$

$$\tau = t - \frac{l}{\frac{\sigma^2}{2}},\tag{66}$$

$$P = Kw(x,l). \tag{67}$$

Using these transformations Equation 61 becomes

$$\frac{\partial w}{\partial l} = \frac{\partial^2 w}{\partial x^2} + \frac{\partial w}{\partial x}(E_1 - 1) - E_2 w \tag{68}$$

where $E_1 = \frac{\mu}{\frac{\sigma^2}{2}}$ and $E_2 = \frac{r}{\frac{\sigma^2}{2}}$. The terminal condition (Equation (19)), now an initial condition, becomes

$$w(x,0) = V'_t,$$
 (69)

where V_t^{\prime} is the REFIT payoff adjusted by the transformations in Equations (65) - (67). Now

consider the following change of variable

$$w(x,l) = e^{-\frac{1}{2}(E_1 - 1)x - (\frac{1}{4}(E_1 - 1)^2 + E_2)l}u(x,l).$$
(70)

This change means Equation (68) becomes

$$\frac{\partial u}{\partial l} = \frac{\partial^2 u}{\partial x^2},\tag{71}$$

with $-\infty < x < \infty$, l > 0 and initial condition

$$u(x,0) = u_0(x) = V_t'',$$
(72)

where V_t'' is the REFIT payoff adjusted by the change of variable in Equation (70). Equation (71) is the well known diffusion problem which has solution

$$u(x,l) = \frac{1}{2\sqrt{\pi l}} \int_{-\infty}^{\infty} u_0(s) e^{\frac{-(x-s)^2}{4l}} ds,$$
(73)

Wilmott et al. (1993).

C.1. Shared upside policy

We will firstly solve Equation (73) for the Shared Upside policy. The payoff of this policy is $V_t = \max(K, K + \theta(S - K))$ which means that, taking into account the transformations in Equations (65) - (67) and the change of variable in Equation (70), the initial condition of the diffusion equation (Equation (72)) becomes

$$u_0(x) = e^{\frac{1}{2}(E_1 - 1)} + \theta \max(e^{\frac{1}{2}(E_1 + 1)x} - e^{\frac{1}{2}(E_1 - 1)x}, 0),$$
(74)

which means that Equation (73) becomes

$$u(x,l) = \frac{1}{2\sqrt{\pi l}} \int_{-\infty}^{\infty} (e^{\frac{1}{2}(E_1-1)} + \theta \max(e^{\frac{1}{2}(E_1+1)s} - e^{\frac{1}{2}(E_1-1)s}, 0)) e^{\frac{-(x-s)^2}{4l}} ds.$$
(75)

When the change of variable $x' = (s - x)/\sqrt{2l}$ is considered, Equation (75) becomes

$$u(x,l) = \frac{1}{2\sqrt{\pi l}} \int_{-\infty}^{\infty} (e^{\frac{1}{2}(E_1-1)} + \theta \max(e^{\frac{1}{2}(E_1+1)(x+x'\sqrt{2l})} - e^{\frac{1}{2}(E_1-1)(x+x'\sqrt{2l})}, 0)) e^{\frac{1}{2}x'^2} dx', \quad (76)$$

which can be broken up into three integrals as follows

$$u(x,l) = \frac{1}{2\sqrt{\pi l}} \int_{-\infty}^{\infty} e^{\frac{1}{2}(E_1 - 1)} x'^2 dx' + \theta(\frac{1}{2\sqrt{\pi l}} \int_{-x/\sqrt{2l}}^{\infty} e^{\frac{1}{2}(E_1 + 1)(x + x'\sqrt{2l})} e^{\frac{1}{2}x'^2} dx' - \frac{1}{2\sqrt{\pi l}} \int_{-x/\sqrt{2l}}^{\infty} e^{\frac{1}{2}(E_1 - 1)(x + x'\sqrt{2l})} e^{\frac{1}{2}x'^2} dx'),$$
(77)

which we label I_0 , I_1 and I_2 respectively, i.e.,

$$u(x,l) = I_0 + \theta(I_1 - I_2).$$
(78)

We now consider the first integral I_0

$$I_0 = \frac{1}{2\sqrt{\pi l}} \int_{-\infty}^{\infty} e^{\frac{1}{2}(E_1 - 1) - \frac{1}{2}x'^2} dx',$$
(79)

$$= \frac{e^{\frac{1}{2}(E_1-1)x}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\frac{1}{2}(E_1-1)x'\sqrt{2l}-\frac{1}{2}x'^2} dx',$$
(80)

$$= \frac{e^{\frac{1}{2}(E_1-1)x}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(x' - \frac{((E_1-1)^2\sqrt{2l})}{2})^2} e^{\frac{(E_1-1)^2l}{4}} dx', \tag{81}$$

$$= \frac{e^{\frac{1}{2}(E_1-1)x} + \frac{1}{4}(E_1-1)^2 l}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(x' - \frac{((E_1-1)^2\sqrt{2l})}{2})^2} dx',$$
(82)

$$= \frac{e^{\frac{1}{2}(E_{1}-1)x} + \frac{1}{4}(E_{1}-1)^{2}l}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{\Lambda^{2}}{2}} dx', \qquad (83)$$

$$= e^{\frac{1}{2}(E_1 - 1)x + \frac{1}{4}(E_1 - 1)^2 l}.$$
(84)

Now consider the second integral I_1

$$I_{1} = \frac{1}{2\sqrt{\pi l}} \int_{-x/\sqrt{2l}}^{\infty} e^{\frac{1}{2}(E_{1}+1) - \frac{1}{2}{x'}^{2}} dx', \qquad (85)$$

$$= \frac{e^{\frac{1}{2}(E_1+1)x}}{\sqrt{2\pi}} \int_{-x/\sqrt{2l}}^{\infty} e^{\frac{1}{2}(E_1+1)x'\sqrt{2l}-\frac{1}{2}{x'}^2} dx',$$
(86)

$$= \frac{e^{\frac{1}{2}(E_1+1)x}}{\sqrt{2\pi}} \int_{-x/\sqrt{2l}}^{\infty} e^{-\frac{1}{2}(x' - \frac{((E_1+1)^2\sqrt{2l})}{2})^2} e^{\frac{(E_1+1)^2l}{4}} dx',$$
(87)

$$= \frac{e^{\frac{1}{2}(E_1+1)x} + \frac{1}{4}(E_1+1)^2 l}{\sqrt{2\pi}} \int_{-x/\sqrt{2l}}^{\infty} e^{-\frac{1}{2}(x' - \frac{((E_1+1)^2\sqrt{2l})}{2})^2} dx',$$
(88)

$$= \frac{e^{\frac{1}{2}(E_1+1)x} + \frac{1}{4}(E_1+1)^2 l}{\sqrt{2\pi}} \int_{-x/\sqrt{2l}}^{\infty} e^{-\frac{\Lambda^2}{2}} dx',$$
(89)

$$= e^{\frac{1}{2}(E_1+1)x + \frac{1}{4}(E_1+1)^{2l}} N(d_1),$$
(90)

where

$$d_1 = \frac{x}{\sqrt{2l}} + \frac{1}{2}(E_1 + 1)\sqrt{2l},\tag{91}$$

and

$$N(\Lambda) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\Lambda} e^{\frac{1}{2}\lambda^2} d\lambda,$$
(92)

represents the cumulative distribution function of the standard normal distribution. The solution for the integral I_2 is obtained in a similar manner to that of I_1 with $E_1 + 1$ replaced by $E_1 - 1$ throughout giving

$$I_2 = e^{\frac{1}{2}(E_1 - 1)x + \frac{1}{4}(E_1 - 1)^2 l} N(d_2),$$
(93)

where

$$d_2 = \frac{x}{\sqrt{2l}} + \frac{1}{2}(E_1 - 1)\sqrt{2l}.$$
(94)

Using Equations (78), (84), (90) and (93) gives the following solution to Equation (75):

$$u(x,l) = e^{\frac{1}{2}(E_1-1)x + \frac{1}{4}(E_1-1)^2l} + \theta(e^{\frac{1}{2}(E_1+1)x + \frac{1}{4}(E_1+1)^2l}N(d_1) - e^{\frac{1}{2}(E_1-1)x + \frac{1}{4}(E_1-1)^2l}N(d_2)).$$
(95)

When the change of variable in Equation (70) and the transformations $x = \ln(\frac{S}{K}), l = \frac{1}{2}\sigma^2(t-\tau)$, and P = Kw(x, l) are recovered, Equation (95) becomes

$$P(S_{\tau},\tau,t) = Ke^{-r(t-\tau)} + \theta(S_{\tau}e^{(\mu-r)(t-\tau)}N(d_1) - Ke^{-r(t-\tau)}N(d_2)),$$
(96)

with

$$d_1 = \frac{\ln(\frac{S_{\tau}}{K}) + (\mu + \frac{\sigma^2}{2})(t - \tau)}{\sigma\sqrt{t - \tau}},$$
(97)

$$d_2 = \frac{\ln(\frac{S_{\tau}}{K}) + (\mu - \frac{\sigma^2}{2})(t - \tau)}{\sigma\sqrt{t - \tau}}.$$
(98)

Equation (96) represents the discounted expected value of REFIT under the Shared-upside policy.

C.1.1. Cost of FiT for shared upside policy

We now consider solving Equation (73) for the Cost of FiT to the policymakers, under the shared upside policy, i.e., when $f_t = \max(0, K - S_t) - (1 - \theta) \max(0, S_t - K)$. When the transformations in Equations (65) - (67) and the change of variable in Equation (70) are taken into account for this cost, the initial condition of the diffusion equation (Equation (72)) becomes

$$u_0(x) = \max(e^{\frac{1}{2}(E_1-1)x} - e^{\frac{1}{2}(E_1+1)x}, 0) - (1-\theta)\max(e^{\frac{1}{2}(E_1+1)x} - e^{\frac{1}{2}(E_1-1)x}, 0).$$
(99)

When this initial condition and the change of variable $x' = (s - x)/\sqrt{2l}$ is considered, the solution to the diffusion Equation (73) becomes

$$u(x,l) = \frac{1}{2\sqrt{\pi l}} \int_{-\infty}^{\infty} (\max(e^{\frac{1}{2}(E_1-1)(x+x'\sqrt{2l})} - e^{\frac{1}{2}(E_1+1)(x+x'\sqrt{2l})}, 0) - (\max(e^{\frac{1}{2}(E_1+1)(x+x'\sqrt{2l})} - e^{\frac{1}{2}(E_1-1)(x+x'\sqrt{2l})}, 0)e^{\frac{1}{2}x'^2} dx'.$$
(100)

In similar manner to Equation (77), Equation (100) can be broken into four integrals as follows

$$\begin{split} u(x,l) &= \frac{1}{2\sqrt{\pi l}} \int_{x/\sqrt{2l}}^{\infty} e^{\frac{1}{2}(E_1 - 1)(x + x'\sqrt{2l})} e^{\frac{1}{2}x'^2} dx' \\ &- \frac{1}{2\sqrt{\pi l}} \int_{x/\sqrt{2l}}^{\infty} e^{\frac{1}{2}(E_1 + 1)(x + x'\sqrt{2l})} e^{\frac{1}{2}x'^2} dx', \\ -(1 - \theta)(\frac{1}{2\sqrt{\pi l}} \int_{-x/\sqrt{2l}}^{\infty} e^{\frac{1}{2}(E_1 + 1)(x + x'\sqrt{2l})} e^{\frac{1}{2}x'^2} dx' \\ &- \frac{1}{2\sqrt{\pi l}} \int_{-x/\sqrt{2l}}^{\infty} e^{\frac{1}{2}(E_1 - 1)(x + x'\sqrt{2l})} e^{\frac{1}{2}x'^2} dx'), \end{split}$$

which we label I_3 , I_4 , I_1 and I_2 respectively, i.e.,

$$u(x,l) = I_3 - I_4 - (1-\theta)(I_1 - I_2).$$
(101)

The integrals I_1 and I_2 are as previously defined while the integrals I_3 , I_4 are obtained in a similar manner to I_2 and I_1 respectively but the lower limit of integrals changed from $-x/\sqrt{2l}$ to $x/\sqrt{2l}$. Hence,

$$I_3 = e^{\frac{1}{2}(E_1 - 1)x + \frac{1}{4}(E_1 - 1)^{2l}} N(-d_2),$$
(102)

$$I_4 = e^{\frac{1}{2}(E_1+1)x + \frac{1}{4}(E_1+1)^2 l} N(-d_1).$$
(103)

Hence, the solution to Equation 100 is

$$u(x,l) = e^{\frac{1}{2}(E_1-1)x + \frac{1}{4}(E_1-1)^{2l}}N(-d_2) - e^{\frac{1}{2}(E_1+1)x + \frac{1}{4}(E_1+1)^{2l}}N(-d_1) - (1-\theta)e^{\frac{1}{2}(E_1+1)x + \frac{1}{4}(E_1+1)^{2l}}N(d_1) - e^{\frac{1}{2}(E_1-1)x + \frac{1}{4}(E_1-1)^{2l}}N(d_2).$$
(104)

When the change of variable in Equation (70) and the transformations $x = \ln(\frac{S}{K})$, $l = \frac{1}{2}\sigma^2(t-\tau)$, and P = Kw(x, l) are recovered Equation (104) becomes

$$EF(S_{\tau},\tau,t) = Ke^{-r(t-\tau)}N(-d_2) - S_{\tau}e^{(\mu-r)(t-\tau)}N(-d_1) - (1-\theta)(S_{\tau}e^{(\mu-r)(t-\tau)}N(d_1) - Ke^{-r(t-\tau)}N(d_2)).$$
(105)

Using the fact that $N(-d_1) = 1 - N(d_1)$ and $N(-d_2) = 1 - N(d_2)$ Equation 105 can be rewritten as

$$EF(S_{\tau},\tau,t) = Ke^{-r(t-\tau)} - S_{\tau}e^{(\mu-r)(t-\tau)} + \theta(S_{\tau}e^{(\mu-r)(t-\tau)}N(d_1) - Ke^{-r(t-\tau)}N(d_2)).$$
(106)

Equation (106) represents the discounted expected cost of REFIT under the Shared-upside policy.

C.2. Cap & floor policy

We now consider solving Equation (73) for the Cap & floor policy which has the payoff $V_t = \max(K, \min(S_t, \overline{S}))$. When the transformations in Equations (65) - (67) and the change of variable in Equation (70) are taken into account for this payoff, the initial condition of the diffusion equation (Equation (72)) becomes

$$u_0(x) = e^{\frac{1}{2}(E_1 - 1)} + \max(e^{\frac{1}{2}(E_1 + 1)x} - e^{\frac{1}{2}(E_1 - 1)x}, 0) - \max(e^{\frac{1}{2}(E_1 + 1)x} - \frac{\bar{S}}{K}e^{\frac{1}{2}(E_1 - 1)x}, 0).$$
(107)

When this initial condition and the change of variable $x' = (s - x)/\sqrt{2l}$ is considered, the solution to the diffusion Equation (73) becomes

$$u(x,l) = \frac{1}{2\sqrt{\pi l}} \int_{-\infty}^{\infty} (e^{\frac{1}{2}(E_1-1)} + \max(e^{\frac{1}{2}(E_1+1)(x+x'\sqrt{2l})} - e^{\frac{1}{2}(E_1-1)(x+x'\sqrt{2l})}, 0) - e^{\frac{1}{2}(E_1-1)(x+x'\sqrt{2l})}) dx = 0$$

$$\max(e^{\frac{1}{2}(E_1+1)(x+x'\sqrt{2l})} - \frac{\bar{S}}{K}e^{\frac{1}{2}(E_1-1)(x+x'\sqrt{2l})}, 0))e^{\frac{1}{2}x'^2}dx'.$$
(108)

Again, in similar manner to Equation (77), Equation (108) can be broken into five integrals as follows

$$u(x,l) = \frac{1}{2\sqrt{\pi l}} \int_{-\infty}^{\infty} e^{\frac{1}{2}(E_{1}-1)x'^{2}} dx' + \frac{1}{2\sqrt{\pi l}} \int_{-x/\sqrt{2l}}^{\infty} e^{\frac{1}{2}(E_{1}+1)(x+x'\sqrt{2l})} e^{\frac{1}{2}x'^{2}} dx' - \frac{1}{2\sqrt{\pi l}} \int_{-x/\sqrt{2l}}^{\infty} e^{\frac{1}{2}(E_{1}-1)(x+x'\sqrt{2l})} e^{\frac{1}{2}x'^{2}} dx' , - \frac{1}{2\sqrt{\pi l}} \int_{\ln(\frac{\bar{S}}{\bar{K}})-x/\sqrt{2l}}^{\infty} e^{\frac{1}{2}(E_{1}+1)(x+x'\sqrt{2l})} e^{\frac{1}{2}x'^{2}} dx' + \frac{\bar{S}}{K} \frac{1}{2\sqrt{\pi l}} \int_{\ln(\frac{\bar{S}}{\bar{K}})-x/\sqrt{2l}}^{\infty} e^{\frac{1}{2}(E_{1}-1)(x+x'\sqrt{2l})} e^{\frac{1}{2}x'^{2}} dx' ,$$
(109)

which we label I_0 , I_1 , I_2 , I_5 and I_6 respectively, i.e.,

$$u(x,l) = I_0 + I_1 - I_2 - I_5 + I_6.$$
(110)

The integrals I_0 , I_1 and I_2 are as previously defined while the integrals I_5 and I_6 are derived in a similar manner to I_1 and I_2 respectively (See Equations (85) - (98)), except with the lower limits on the integrals being equal to $\ln(\frac{\bar{S}}{K}) - x/\sqrt{2l}$. Thus,

$$I_5 = e^{\frac{1}{2}(E_1+1)x + \frac{1}{4}(E_1+1)^2 l} N(d_3), \qquad (111)$$

$$I_6 = \frac{S}{K} e^{\frac{1}{2}(E_1 - 1)x + \frac{1}{4}(E_1 - 1)^2 l} N(d_4), \qquad (112)$$

where

$$d_3 = \frac{x - \ln(\frac{S}{K})}{\sqrt{2l}} + \frac{1}{2}(E_1 + 1)\sqrt{2l}, \qquad (113)$$

$$d_4 = \frac{x - \ln(\frac{S}{K})}{\sqrt{2l}} + \frac{1}{2}(E_1 - 1)\sqrt{2l}.$$
 (114)

Using Equations (84), (110) and (102) - (112) the following is a solution to Equation (108)

$$u(x,l) = e^{\frac{1}{2}(E_1-1)x + \frac{1}{4}(E_1-1)^2 l} + e^{\frac{1}{2}(E_1+1)x + \frac{1}{4}(E_1+1)^2 l} N(d_1) - e^{\frac{1}{2}(E_1-1)x + \frac{1}{4}(E_1-1)^2 l} N(d_2)$$

$$-e^{\frac{1}{2}(E_1+1)x + \frac{1}{4}(E_1+1)^2 l} N(d_3) + \frac{\bar{S}}{K} e^{\frac{1}{2}(E_1-1)x + \frac{1}{4}(E_1-1)^2 l} N(d_4).$$
(115)

When the change of variable in Equation (70) and the transformations $x = \ln(\frac{S}{K})$, $l = \frac{1}{2}\sigma^2(t-\tau)$, and P = Kw(x, l) are recovered Equation (115) becomes

$$P(S_{\tau},\tau,t) = Ke^{-r(t-\tau)} + S_{\tau}e^{(\mu-r)(t-\tau)}N(d_1) - Ke^{-r(t-\tau)}N(d_2)$$

-S_{\tau}e^{(\mu-r)(t-\tau)}N(d_3) + \bar{S}e^{-r(t-\tau)}N(d_4), (116)

with

$$d_3 = \frac{\ln(\frac{S_{\tau}}{S}) + (\mu + \frac{\sigma^2}{2})(t - \tau)}{\sigma\sqrt{t - \tau}},$$
(117)

$$d_4 = \frac{\ln(\frac{S_{\tau}}{S}) + (\mu - \frac{\sigma^2}{2})(t - \tau)}{\sigma\sqrt{t - \tau}}.$$
(118)

Equation (116) represents the discounted expected value of REFIT under the Cap & floor policy.

C.2.1. Cost of Fit cap & floor policy

We now consider solving Equation (73) for the Cost of FiT to the policymakers, under the cap & floor policy, i.e., when $f_t = \max(0, K - S_t) - \max(0, S_t - \overline{S})$. When the transformations in Equations (65) - (67) and the change of variable in Equation (70) are taken into account for this payoff, the initial condition of the diffusion equation (Equation (72)) becomes

$$u_0(x) = \max(e^{\frac{1}{2}(E_1-1)x} - e^{\frac{1}{2}(E_1+1)x}, 0) - \max(e^{\frac{1}{2}(E_1+1)x} - \frac{\bar{S}}{K}e^{\frac{1}{2}(E_1-1)x}, 0).$$
(119)

When this initial condition and the change of variable $x' = (s-x)/\sqrt{2l}$ is considered, the solution to the diffusion Equation (73) becomes

$$u(x,l) = \frac{1}{2\sqrt{\pi l}} \int_{-\infty}^{\infty} (\max(e^{\frac{1}{2}(E_1-1)(x+x'\sqrt{2l})} - e^{\frac{1}{2}(E_1+1)(x+x'\sqrt{2l})}, 0) - (\max(e^{\frac{1}{2}(E_1+1)(x+x'\sqrt{2l})} - \frac{\bar{S}}{K}e^{\frac{1}{2}(E_1-1)(x+x'\sqrt{2l})}, 0)e^{\frac{1}{2}x'^2}dx'.$$
(120)

Again, in similar manner to Equation (77), Equation (120) can be broken into four integrals as follows

$$\begin{split} u(x,l) &= \frac{1}{2\sqrt{\pi l}} \int_{x/\sqrt{2l}}^{\infty} e^{\frac{1}{2}(E_1 - 1)(x + x'\sqrt{2l})} e^{\frac{1}{2}{x'}^2} dx' \\ &- \frac{1}{2\sqrt{\pi l}} \int_{x/\sqrt{2l}}^{\infty} e^{\frac{1}{2}(E_1 + 1)(x + x'\sqrt{2l})} e^{\frac{1}{2}{x'}^2} dx', \\ &- \frac{1}{2\sqrt{\pi l}} \int_{\ln(\frac{\bar{S}}{\bar{K}}) - x/\sqrt{2l}}^{\infty} e^{\frac{1}{2}(E_1 + 1)(x + x'\sqrt{2l})} e^{\frac{1}{2}{x'}^2} dx' \end{split}$$

$$+\frac{\bar{S}}{K}\frac{1}{2\sqrt{\pi l}}\int_{\ln(\frac{\bar{S}}{K})-x/\sqrt{2l}}^{\infty}e^{\frac{1}{2}(E_{1}-1)(x+x'\sqrt{2l})}e^{\frac{1}{2}x'^{2}}dx',$$
(121)

which we label I_3 , I_4 , I_5 and I_6 respectively, i.e.,

$$u(x,l) = I_3 - I_4 - I_5 + I_6.$$
(122)

Each of these integrals are as previously defined. Hence the solution to Equation (120) is

$$u(x,l) = e^{\frac{1}{2}(E_1-1)x + \frac{1}{4}(E_1-1)^{2l}}N(-d_2) - e^{\frac{1}{2}(E_1+1)x + \frac{1}{4}(E_1+1)^{2l}}N(-d_1) - e^{\frac{1}{2}(E_1+1)x + \frac{1}{4}(E_1+1)^{2l}}N(d_3) + \frac{\bar{S}}{K}e^{\frac{1}{2}(E_1-1)x + \frac{1}{4}(E_1-1)^{2l}}N(d_4).$$
(123)

When the change of variable in Equation (70) and the transformations $x = \ln(\frac{S}{K})$, $l = \frac{1}{2}\sigma^2(t-\tau)$, and P = Kw(x, l) are recovered Equation (123) becomes

$$EF(S_{\tau},\tau,t) = Ke^{-r(t-\tau)}(1-N(d_2)) - S_{\tau}e^{(\mu-r)(t-\tau)}(1-N(d_1)) - S_{\tau}e^{(\mu-r)(t-\tau)}N(d_3) + \bar{S}e^{-r(t-\tau)}N(d_4).$$
(124)

Equation (124) represents the discounted expected cost of REFIT under the Cap & Floor policy.

Bibliography

- Albanese, C., Lo, H., Tompaidis, S., 2006. A numerical method for pricing electricity derivtives for jump-diffusion processes based on continuous time latticesMPRA Paper, Imperial College London and University of Texas Austin.
- Barlow, M. T., 2002. A diffusion model for electricity prices. Mathematical Finance 12 (4), 287-298. URL http://doi.wiley.com/10.1111/j.1467-9965.2002.tb00125.x
- Benth, F. E., Ekeland, L., Hauge, R., Nielsen, B. F., 2003. A note on arbitrage-free pricing of forward contract in energy markets. Applied Mathematical Finance **10**, 325–336.
- Black, F., Scholes, M., 1973. The pricing of options and corporate liabilities. Journal of Political Economy 81 (3), 637–654.
- Burer, M. J., Wustenhagen, R., 2009. Which renewable energy policy is a venture capitalist's best friend? empirical evidence from a survey of international cleantech investors. Energy Policy 37 (12), 4997–5006.

URL http://linkinghub.elsevier.com/retrieve/pii/S0301421509004807

- Burger, M., Klar, B., Mller, A., Schindlmayr, G., 2004. A spot market model for pricing derivatives in electricity markets. Qunatitative Finance **4**, 109–112.
- Chambers, C. M., Chambers, P. E., Whitehead, J. C., 1994. Conservation organizations and the option value to preserve: an application to debt-for-nature swaps. Ecological Economics 9 (2), 135–143.
 URL http://ideas.repec.org/a/eee/ecolec/v9y1994i2p135–143.html

- Chang, M.-C., Hu, J.-L., Han, T.-F., 2013. An analysis of a feed-in tariff in taiwans electricity market. International Journal of Electrical Power & Energy Systems 44 (1), 916–920. URL http://linkinghub.elsevier.com/retrieve/pii/S0142061512004863
- Conrad, J. M., 2000. Wilderness: options to preserve, extract, or develop. Resource and Energy Economics 22 (3), 205–219.
- Couture, T., Gagnon, Y., 2010. An analysis of feed-in tariff remuneration models: Implications for renewable energy investment. Energy Policy 38 (2), 955–965. URL http://linkinghub.elsevier.com/retrieve/pii/S0301421509007940
- DCENR, 2006. Correspondence between the European Commission and Ireland on REFIT 1 state aid application. URL http://www.dcenr.gov.ie/Energy/Sustainable+and+Renewable+Energy+Divisi
- del Rio, P., Bleda, M., 2012. Comparing the innovation effects of support schemes for renewable electricity technologies: A function of innovation approach. Energy Policy 50, 272–282. URL http://linkinghub.elsevier.com/retrieve/pii/S0301421512005952
- Devitt, C., Malaguzzi Valeri, L., 2011. The effect of refit on irish wholesale electricity prices. The Economic and Social Review 42 (3), 343369. URL http://ideas.repec.org/a/eso/journl/v42y2011i3p343-369.html
- Doherty, R., O'Malley, M., 2011. The efficiency of Ireland's Renewable Energy Feed-In Tariff (REFIT) for wind generation. Energy Policy 39 (9), 4911–4919. URL http://linkinghub.elsevier.com/retrieve/pii/S0301421511004800
- E.C., 2005. The Support of Renewable Energy Sources. Tech. Rep. COM (2005) 627, European Commission, Brussels.
- Fagiani, R., Barqun, J., Hakvoort, R., 2013. Risk-based assessment of the cost-efficiency and the effectivity of renewable energy support schemes: Certificate markets versus feed-in tariffs.

Energy Policy 55, 648-661. URL http://linkinghub.elsevier.com/retrieve/pii/S0301421512011330

- Fell, H., Linn, J., 2013. Renewable electricity policies, heterogeneity, and cost effectiveness. Journal of Environmental Economics and Management (0), -. URL http://www.sciencedirect.com/science/article/pii/S0095069613000442
- Folketinget Danish Parliament, 2008. Promotion of renewable energy act. Act no. 1392 of 27 December 2008.
- Fudenberg, D., Tirole, J., 1991. Game theory. MIT Press, Cambridge, Massachusetts.
- Heydari, S., Ovenden, N., Siddiqui, A., 2012. Real options analysis of investment in carbon capture and sequestration technology. Computational Management Science 9 (1), 109–138. URL http://dx.doi.org/10.1007/s10287-010-0124-5
- Hull, J. C., 2003. Options, Futures, and Other Derivatives. Pearson Prentice-Hall, London.
- International Energy Agency, Organisation for Economic Co-operation and Development, 2008. Deploying renewables principles for effective policies: in support of the G8 plan of action. OECD/IEA, Paris. URL http://www.myilibrary.com?id=192197
- International Energy Agency, Organisation for Economic Co-operation and Development, 2011. Deploying renewables 2011 best and future policy practice. OECD/IEA, Paris. URL http://dx.doi.org/10.1787/9789264124912-en
- Kalkuhl, M., Edenhofer, O., Lessmann, K., 2012. Learning or lock-in: Optimal technology policies to support mitigation. Resource and Energy Economics 34 (1), 1 – 23. URL http://www.sciencedirect.com/science/article/pii/S0928765511000479
- Keppo, J., Rasanen, M., 1999. Pricing of electricity tariffs in competitive markets. Energy Economics 21 (3), 213 – 223. URL http://www.sciencedirect.com/science/article/pii/S0140988399000055
- Kim, K.-K., Lee, C.-G., 2012. Evaluation and optimization of feed-in tariffs. Energy Policy 49, 192–203. URL http://linkinghub.elsevier.com/retrieve/pii/S0301421512004831
- Lemoine, D. M., 2009. Valuing plug-in hybrid electric vehicles' battery capacity using a real options frameworkWorking Paper, University of California, Berkeley.

- Lyle, M. R., Elliott, R. J., 2009. A 'simple' hyprid model for power derivatives. Energy Economics **31**, 757–767.
- Mc Garrigle, E., Deane, J., Leahy, P., 2013. How much wind energy will be curtailed on the 2020 irish power system? Renewable Energy 55, 544–553. URL http://linkinghub.elsevier.com/retrieve/pii/S0960148113000384
- Overdahl, J. A., Matthews, H. L., 1988. The use of nymex options to forecast crude oil prices. The Energy Journal 0 (Number 4). URL http://ideas.repec.org/a/aen/journl/1988v09-04-a07.html
- Pickles, E., Smith, J. L., 1993. Petroleum property valuation: A binomial lattice implementation of option pricing theory. The Energy Journal 14 (2), pp. 1–26. URL http://www.jstor.org/stable/41322495
- Queensland Competition Authority, 2013. Estimating fair а and reasonable solar feed-in tariff for queensland [online]. http://www.qca.org.au/files/ER-QCA-FinalReport-ReviewofSolarFeedinTariffQLD-0313.pdf [Accessed: 24 August 2013].
- Ragwitz, M., Held, A., Resch, G., Faber, T., Haas, R., Huber, C., Coenraads, R., Voogt, M., Reece, G., Morthorst, P., Jensen, S., Konstantinaviciute, I., Heyder, B., 2007. Assessment and Optimization of Renewable Energy Support Schemes in the European Electricity Market: Final Report. Optimization of Renewable Energy Support (OPTRES) project for the European Commission, DG TREN, and Intelligent Energy for Europe (IEE), Karlsruhe, Germany.
- Rauch, J. N., 2013. The effect of different market conditions on the hedge value of long-term contracts for zero-fuel-cost renewable resources. The Electricity Journal 26 (5), 44 – 51. URL http://www.sciencedirect.com/science/article/pii/S1040619013001152
- Schwartz, E., Smith, J. E., 2000. Short-term variations and long-term dynamics in commodity prices. Management Science **46**, 893–911.
- SEAI, 2011. Seai wind energy roadmap 2011-2050. Report of the SEAI Energy Modelling Group.
- SEMO, 2011. SEM data publication reports [online]. http://www.sem-o.com/marketdata/Pages/default.aspx [Accessed: 14 July 2013].
- Sensfuß, F., Ragwitz, M., Genoese, M., 2008. The merit-order effect: A detailed analysis of the price effect of renewable electricity generation on spot market prices in germany. Energy Policy

36 (8), 3086-3094.

URL http://linkinghub.elsevier.com/retrieve/pii/S0301421508001717

- Shreve, S. E., Chalasani, P., Jha, S., 2004. Stochastic calculus for finance. Vol. 1. Springer New York.
- Skantze, P., Ilic, M., Chapman, J., 2000. Stochastic modeling of electric power prices in a multi-market environment. Vol. 2. IEEE, pp. 1109–1114. URL http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=850096
- Smith, J. L., 2005. Petroleum prospect valuation: The option to drill again. The Energy Journal 26 (4), pp. 53–68. URL http://www.jstor.org/stable/41323073
- Tsitakis, D. Xanthopoulos, S., Yannacopoulos, A., 2006. A closed-from solution for the price of cross-commodity electricity derivatives. Physica A **371**, 543–511.
- Wilmott, P., 2000. Paul Wilmott on Quantatiive Finance. Wiley, West Sussex, England.
- Wilmott, P., Dewynne, J., Howison, S., 1993. Option Pricing: Mathematical Models and Computation. Oxford Financial Press, Oxford, England.
- Woo, C.-K., 1988. Optimal electricity rates and consumption externality. Resources and Energy 10 (4), 277–292.
- Yang, M., Blyth, W., 2007. Modeling investment risks and uncertainties with real options approach. International Energy Agency.
- Zhu, L., 2012. A simulation based real options approach for the investment evaluation of nuclear power. Computers & Industrial Engineering 63 (3), 585–593.