Optimal Carbon Taxes with Non-Constant Time Preference

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Abstract

The paper derives an explicit formula for the near-term carbon price in a dynamic stochastic general equilibrium climate model in which agents employ arbitrary non-constant time preference rates. The paper uses a simplified version of the model in Golosov et al. (2011), though we argue that the added assumptions are unlikely to matter for our conclusions. The formula is derived first under the assumption that the initial decision-maker has a commitment device, then solving for the unique subgame perfect equilibrium. Somewhat remarkably, the near-term carbon price is the same in both cases. We further show that the near-term carbon price remains unchanged for all potential beliefs about the time preference structure of future generations. It follows that concerns about time inconsistency can be safely ignored when applying the derived formula. The carbon price is the same as the Pigouvian tax in the equilibrium with commitment, and it is bigger than the Pigouvian tax in the equilibrium without commitment provided damages are sufficiently persistent. The formula reduces to the carbon price formula in Golosov et al. (2011) when discounting is constant, and it reduces to the carbon price formula in Gerlagh and Liski (2012) when discounting is quasi-hyperbolic.

KEYWORDS: hyperbolic discounting, time inconsistency, optimal carbon price

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[Declining discount rates] "solve one problem by creating another. Unless the discount rate is constant, the policy path is subject to 'time inconsistency'. Suppose an intelligent decision-maker plans a strategy for the long future, beginning today. Five years from now, she reconsiders the strategy, having followed it so far. She will want to change to a different strategy for no other reason than the passage of time. ...
This sounds like a poor way to run a railroad."

Robert Solow in forward to Portney and Weyant (1999)

1 Introduction

The discounted utility model, which assumes a constant discount rate, has strong distributional consequences when applied to long-term environmental problems in which severe consequences occur with long delay. Provided the discount rate is calibrated to match market interest rates, the model ensures that large costs will be imposed on future generations that could have been avoided at relatively little cost today[1]. This feature of the model is unsettling in part because the justification for a constant discount rate in social cost benefit analysis is weak. When introducing the discounted utility model, Samuelson (1937) stated "any connection between utility as discussed here and any welfare concept is disavowed" (Frederick et al. 2002). Moreover, in axiom systems for the model (Koopmans 1960; Fishburn 1970; Fishburn and Rubinstein 1982) the primary justification for the stationarity property that lurks behind the constancy of the discount rate is the assurance of time consistency. But as discussed below, time consistency is possible under non-constant discounting also. In addition to normative concerns, a variety of studies from psychology and economics suggest that, for many individuals, subjective discount rates are better approximated by a hyperbolic discounting function than by a constant exponential function (Thaler 1981; Loewenstein 1987; Ainslee 1991; Cropper et al. 1994; Kirby and Herrenstein 1995).

A discounting function in which the rate of time preference declines over time is intriguing in part because it is possible to match market interest rates over the horizon for which such instruments exist (30 to 40 years at most) then have rates decline in a way that increases the sensitivity of current actions to distant consequences (Chichilnisky 1996; Heal 2005). A declining rate of time preference (DRTP) can be justified in three ways. First, by invoking behavioral evidence, as mentioned above. Second, if there exists a "correct" discount rate, but it is viewed as an uncertain random variable with a probability distribution that is permanent over time, the problem solved by an expected utility maximizing decision-maker is formally equivalent to one with DRTP (Azfar 1999; Weitzman 2001). Finally, if citizens possess different discount rates and a social planner aggregates preferences by taking a weighted average of individual utilities, the resulting maximization problem is again equivalent to one with DRTP (Gollier and Zeckhauser 2005).

Whatever the rationale for non-constant time preference rates[2] (NCTP), their use introduces a potential problem that does not arise when the discount rate is constant—time inconsistency. Under NCTP, the marginal rate of substitution between future periods shifts as the contemporaneous decision-maker progresses in time. As a result, absent a commitment device, the long-run policy plan that a decision maker in the initial period would optimally choose is not a plausible equilibrium since future generations would prefer to deviate. In

[1]For example, with a discount rate of 5%, a decision maker today would pay at most sixty thousand dollars to avoid a billion dollars of damages in 200 years.

[2]In the paper, we allow for the general possibility of non-constant time preference rates, though we are primarily interested in the case of declining time preference rates.

principle, time inconsistency can be resolved in a straightforward way by considering a model that correctly anticipates how future agents are expected to respond; mainly, by requiring that the equilibrium be subgame perfect (Strotz 1955). But in practice, solving for subgame perfect equilibria in dynamic economic models can be difficult. To make progress, most applications of NCTP employ one or more of the following simplifying assumptions.

The first "assumption" has to do with the equilibrium concept. In the original growth literature that explored time inconsistency issues under NCTP (Strotz 1955, Pollak 1968, Phelps and Pollak 1968, Goldman 1980) the equilibrium was derived using backward induction in a finite horizon model[3]. Peleg and Yaari (1973) call this a Strotz-Pollak equilibrium. The more recent literature, initiated by Laibson (1997), emphasizes symmetric Markov perfect equilibra. A variety of solution strategies have been devised to solve for this type of equilibrium in stationary, infinite horizon dynamic models (Harris and Laibson 2001; Karp 2007; Fujii and Karp 2008). Nevertheless, the equilibrium concept has two potential flaws. First, it does not respect the order of decision-makers in the timeline. In long-term environmental applications, it is not clear that policy-makers would accept recommendations that view the current generation as being artificially constrained by potential strategic threats from future generations. Second, there are typically a continuum of strategies that satisfy the conditions for a symmetric Markov perfect equilibrium (Harris and Laibson 2001; Krussel and Smith 2003; Karp 2005; Karp 2007). A Strotz-Pollak equilibrium has the advantage that it clearly respects the order of decision-makers in the timeline, and it is more likely to be unique.

The second assumption that is often employed is to restrict attention to quasi-hyperbolic discounting functions. Quasi-hyperbolic discounting applies a higher rate of time preference between the first and second periods, after which the same constant lower rate is applied across all subsequent periods (Phelps and Pollak 1968, Laibson 1997). Applied to stationary, infinite-horizon dynamic models, quasi-hyperbolic discounting makes it possible to preserve a highly-tractable recursive structure (Harris and Laibson 2001). The trouble is inaccuracy. Under all three justifications for DRTP, time preference rates decline continuously in time. Translating these continuous paths into a qualitatively similar quasi-hyperbolic path introduces approximation error that becomes large in settings like climate policy where the long-run path of discount rates matters a lot. This point is demonstrated in our quantitative section.

In this paper, we derive an explicit formula for the near-term carbon price under arbitrary NCTP, doing so in a setting in which time inconsistency concerns are resolved in a clean and compelling way. The formula is derived first under the assumption that a commitment device is available, then using backward induction to solve for the unique Strotz-Pollak equilibrium. The near-term carbon price is the same in both cases. In particular, while the full trajectory of optimal policies differ in subsequent periods under the two equilibria, this difference has no effect on the first-period carbon price. This implies that time inconsistency concerns are in a significant sense *irrelevant* in the considered setting[4].

Without commitment, it can be argued that the first-period carbon price summarizes everything decision-makers need to know (on the benefit side) to evaluate current climate related investment projects. It clearly provides enough information to evaluate projects that

[3]This was extended to infinite horizon settings by taking the infinite-horizon limit of a sequence of equilibria to truncated models (as we do in this paper).

[4]Note that time inconsistency arises when the problem is solved in a way that falsely presumes that a commitment device is available. Our result shows that whether a commitment device is assumed or not, the optimal policy today is the same.

are to be implemented within the period. Meanwhile, plans that require coordination across multiple periods are arguably infeasible without commitment. In our calibrated model, the period length is a decade, so our no commitment assumption means that the commitment length of a given policy regime is at most ten years. Recent experience with global climate treaties supports the conclusion that climate policy commitment beyond ten years is unlikely.

Indeed, our irrelevance finding is even stronger. In addressing time inconsistency concerns, the most straightforward solution is to look for a subgame perfect equilibrium in which decision makers in different periods are identical. But as hard as it is for the current generation to "know" its own time preference structure, they presumably know significantly less about the preference structure of subsequent generations (Beltratti, Chichilnisky and Heal 1998). Accepting this critique, one could be led to entertain a wide range of possible assumptions about the path of time preference rates likely to be applied by future generations. In section 4.3, we show that, for any path of time preference rates that one might assume for future generations, the first-period carbon price derived using backward induction remains unchanged. We refer to this result as "strong irrelevance"[5].

Naturally, our strong results are a product of the considered model. We employ a somewhat simplified version of the dynamic, stochastic integrated assessment model developed in Golosov et al. (2011). The crucial assumptions for our results are (1) logarithmic preferences and (2) a damage function that multiplies output while taking a "linear-exponential" functional form. As discussed in Golosov et al. (2011), log utility implies that the marginal utility at which future damages are valued is inversely proportional to output. Because damages are directly proportional to output, the dependence on future output cancels as these terms are multiplied when evaluating current policy. Meanwhile, the "linear-exponential" feature of the damage function, combined with log preferences, gives rise to a value function that is linear in prior emissions. This feature of the analytical solution is behind a certainty equivalence result: though the damage elasticity parameter follows a Markov process, the carbon price formula depends only on the expected value of the parameter in each future period. These features together explain why the formula depends only on current output, along with the time preference rates, the carbon cycle parameters, and the expected value of future damage elasticities. In particular, all dependence on the future paths of consumption, capital, and emission stocks are notably absent. The same result is found in Golosov et al. (2011), and indeed our formula reduces to theirs in the case of constant discounting.

A potential criticism is that our strong results depend on the specific functional form assumptions in Golosov et al. (2011). As emphasized in that paper, these assumptions are close to those in much of the prior literature on climate policy; nevertheless, the situation in which income and substitution effects exactly cancel so that current policy is insensitive to future response behavior is clearly a special case. But there are two considerable advantages of using this setting to consider the implications of DRTP, and we argue that these advantages more than make up for the potential arbitrariness that accompanies our reliance on specific functional forms. First, in dynamic analyses of climate policy with NCTP and no commitment, the optimal choice by the current generation will typically depend on the likely response behavior (i.e., the state-contingent policy rules) assumed for future generations. But any attempt to model the preferences and response behavior of future generations is itself

[5] Golosov et al. (2011) note that their formula for the near-term carbon price would be unaffected if the model was changed to incorporate endogenous technology choice. The reason is that these alternative assumptions would affect the future paths of the endogenous variables, but the future paths of those variables do not affect the near-term carbon price. Our strong irrelevance finding is closely related to this alternative irrelevance finding in Golosov (2011).

unavoidably arbitrary. A model in which current policy is insensitive to these assumptions may therefore be less arbitrary then a criterion for selecting among the myriad of potential specifications for describing the time preferences and choice procedure of future generations.

Second, the argument for DRTP becomes much cleaner within the Golosov et al. (2011) framework. In particular, under what are arguably the two most compelling justifications for DRTP—(1) that the correct rate of time preference is uncertain and (2) that the planner seeks to aggregate diverse views across society—an important concern in most applications is that they impose the accompanying assumption that current information is frozen in time. Thus, there is no room for learning about the correct discount rate (under uncertainty), or for the distribution of views in society to evolve (under aggregation). Given the long time horizon in our application, this assumption is implausible. But with strong irrelevance, the concern falls away. When computing the path of discount rates that is relevant for determining the near term carbon price, we only need to consider current uncertainty or the current distribution of views. The expectation that these may change over time would imply that the path of time preference rates to be used by subsequent decision-makers would change, but this has no effect on the optimal carbon price today.

The model in the paper simplifies the Golosov et al. (2011) framework in two ways—though we argue in section 4.4 that these simplifications are unlikely to matter for our results. The simplifications follow recent work by Gerlagh and Liski (2012). Unlike those authors, we retain the stochastic portion of the original Golosov et al. (2011) model. In addition, we allow time preference rates to follow an arbitrary non-constant path, and we emphasize the Strotz-Pollak equilibrium, which we show to be unique. Gerlagh and Liski (2012) consider the case of quasi-hyperbolic discounting, and they solve for a particular symmetric Markov perfect equilibrium[6]. The first simplification is to assume that production in the energy extraction sector depends on labor and fossil fuel inputs only. In contrast, energy production in Golosov et al. (2011) depends also on energy and capital inputs, both of which are assumed to be mobile across sectors. The second simplification is to assume that fossil fuel reserves are effectively infinite[7].

The paper also solves for the Pigouvian taxes under each equilibrium. Following Gerlagh and Liski (2012), we take this to be the present value marginal externality cost discounted at the equilibrium real return. Since the future real return is different under the two equilibria, the corresponding Pigouvian taxes are also different. We find that the optimal first-period carbon tax (from our formula) equals the Pigouvian tax under commitment, but it is greater than the Pigouvian tax without commitment provided the delay between emissions and damages is sufficiently long. In the latter case, the wedge between the optimal carbon price and the Pigouvian tax without commitment reflects the "commitment value" of climate policy. As discussed in Gerlagh and Liski (2012), the idea of commitment value in this setting is analogous to the value that Laibson (2007) attributes to commitment devices in self-control problems. In the current setting, climate policy provides a mechanism for transferring wealth to distant generations that partially bypasses the control of intermediate generations who,

[6]Our paper builds on the work of Gerlagh and Liski (2012) in multiple ways, including our computation of the implied equilibrium Pigouvian taxes in section 5. A further important contribution of Gerlagh and Liski (2012) is to derive an alternative motivation for the linear coefficients in the Golosov et al. (2011) linear-exponential damage function that is consistent with a carbon cycle in which the delay between emissions and damages is more consistent with some prior integrated assessment models, including DICE (Nordhaus 2008).

[7]This assumption is plausible along an optimal policy path since most models would stop extracting fossil fuels before existing coal deposits are fully exploited. In addition, as we argue later, the assumption of finite reserves does not affect the near term carbon price in Golosov et al. (2011) and we see no reason to expect that it would affect the carbon price under non-constant discounting either.

under hyperbolic discounting, are more impatient.

A further contribution of the paper is to demonstrate a solution strategy for deriving the Strotz-Pollak equilibrium in a discrete time dynamic model with non-constant discounting. The solution strategy is conceptually related to the heuristic device in Harris and Laibson (2001). This is used to solve for symmetric Markov perfect equilibria in infinite horizon models with quasi-hyperbolic discounting. Fujii and Karp (2008) generalize the approach to allow time preference rates to be non-constant over a finite period. Our algorithm differs in that we solve for the backward induction solution in a truncated (finite-horizon) version of the model, then take the infinite horizon limit. Our approach is therefore more closely related to finite horizon dynamic programming. The standard finite horizon dynamic programming algorithm is modified to ensure that the continuation value functions appropriately account for the distinct time preferences of hyperbolic agents at different dates.

Our irrelevance finding under log utility has precedence in the literature. Phelps and Pollak (1968) consider a finite horizon growth model with linear production and quasi-hyperbolic time preference. They refer to savings in the full commitment equilibrium as first-best national savings and to savings in the no commitment (backward induction) equilibrium as second-best national savings. The effect of commitment on national savings in the initial period hinges on the elasticity of marginal utility. When the elasticity is one (corresponding to log utility) first- and second-best savings are the same. Indeed, they show that no matter what savings rate is adopted by subsequent generations when solving the problem using backward induction, the optimal savings choice for the initial generation remains unchanged. This is analogous to our strong irrelevance result. Phelps and Pollak (1968) conclude, "This logarithmic case must be added to the curious list of examples in which first-best and second-best decisions do not differ."

The results in Phelps and Pollak (1968) do not carry over to the more general Ramsey model. Barro (1999) solves for the unique Cournot-Nash equilibrium in a version of the Ramsey model with logarithmic preferences that allows for a continuously declining path of time preference rates. He also solves the model by assuming a commitment device. In both cases, the marginal propensity to consume out of wealth is a constant. Moreover, in the initial period, the marginal propensity to consume out of wealth is the same constant in both equilibria. Nevertheless, the amount of savings in the initial period differs across the two equilibria. The reason is that wealth is forward-looking, and it differs under the two scenarios[8].

The paper contributes to the literature in which optimal climate policy is derived in the presence of declining time preference rates. Nordhaus (1999), Mastrandrea and Schneider (2001), and Guo et al. (2006) all solve for optimal carbon prices under hyperbolic discounting. All of these authors solve their respective problems numerically, while assuming a commitment device. Karp (2005) solves for the (set of) symmetric Markov perfect equilibria in a one dimensional stock pollution abatement problem when discounting is quasi-hyperbolic. Gerlagh and Liski (2011) also solve for a symmetric Markov perfect equilibrium under quasi-hyperbolic discounting. Karp (2007) develops methods to derive a Markov perfect equilibrium in a continuous time dynamic setting with arbitrary NCTP, and he applies these methods to a carbon abatement problem. In addition, Fujii and Karp (2008) develop numerical methods to solve for the symmetric Markov perfect equilibria in a one-state-variable setting with

[8]In the model employed here, where physical capital depreciates 100% in each time period, the marginal propensity to consume out of current income is a constant. Moreover, it is the same constant in both equilibria. Because income depends only on the inherited state variables—and therefore is not forward-looking—current savings is the same in both equilibria.

arbitrary NCTP. Finally, Tsur and Karp (2011) derive optimal climate policy while allowing for arbitrary NCTP in a binary choice setting.

The paper is organized as follows. Section 2 presents the model. Section 3 solves for optimal carbon policy under commitment and presents the main formula of the paper. Section 4 develops a solution strategy to solve the model using backward induction, then shows that time inconsistency concerns are irrelevant in the considered model. Section 5 solves for the corresponding Pigouvian taxes for the equilibria with and without commitment, then compares these to the optimal carbon price. Section 6 calibrates the near-term SCC, and section 7 concludes.

2 Model

The model is a simplified version of the integrated assessment model in Golosov et al. (2011). The simplifications build on closely-related recent work by Gerlagh and Liski (2012). Unlike Gerlagh and Liski (2012) we follow Golosov et al. (2011) in assuming that the elasticity parameter in the damage function is stochastic. In addition, we allow decision-makers to employ an arbitrary path of time preference rates.

2.1 Preferences, technology, and climate

Time is discrete and runs from $t = 1, \ldots, T$. The time horizon is initially finite, though we later consider the infinite-horizon limit. There is assumed to be a representative household in each period who derives utility from the time path of consumption. Future utility is discounted using a known sequence of potentially non-constant discount factors $\{\beta_j\}$. For each j, $0 < \beta_j < 1$. We are typically interested in the case in which the pure rate of time preference (PRTP) declines weakly in time, thus where $\{\beta_j\}$ increases, though it is not necessary to assume this for our results. For each time horizon s periods ahead, we define a cumulative discount factor $R(s) = \prod_{j=1}^{s} \beta_j$. This gives the utility value today of an extra unit of utility received s periods from now. By assumption, $R(0) = 1$. Household preferences in different periods are initially assumed to be identical; nevertheless, in section 4.3 we consider the case in which each generation has its own distinct path of time preference rates.

The utility function of the representative household in generation t is

$$\mathbb{E}_t \sum_{s=0}^{T-t} R(s) \ln(C_{t+s}).$$

$\mathbb{E}_t$ is the mathematical expectation conditional on information available in period t. Preferences are logarithmic.

The economy has two sectors: a final-goods sector and an energy extraction sector. Energy is an input in the final-goods sector. Carbon emissions arise as an externality associated with energy production. Units are selected so Z_t denotes both fossil fuel consumption in the energy sector and carbon emissions. Atmospheric carbon accumulates according to

$$S_t = \sum_{k=1}^{t-1} (1 - d_k) Z_{t-k}, \tag{2.1}$$

where $1 - d_k$ is the k-period-ahead carbon depreciation rate. It is the fraction of a unit of carbon emissions that remains in the atmosphere k periods after it is emitted.

Cumulative emissions lead to climate damages that impact economic output through a multiplicative damage function that takes the following "exponential–linear" form:

$$\omega(S_t) = \exp(-\gamma_t S_t). \tag{2.2}$$

γ_t is an elasticity that denotes the percent output loss associated with an extra unit of atmospheric carbon. It follows a known Markov process that is assumed to be bounded[9].

Output in the final-goods sector, net of environmental damages and abatement costs, is determined by an aggregate production function that takes the following form:

$$Y_t = K_t^{\alpha} A_t(E_t, N_{1,t})\omega(S_t). \tag{2.3}$$

Capital, K_t, enters in a Cobb-Douglass manner. In addition, output depends on an unspecified energy-labor composite function, $A_t(\cdot,\cdot)$, and on climate damages, $\omega(\cdot)$.

In addition, output in the energy extraction sector is determined by the following production function:

$$E_t = G_t(Z_t, N_{2,t}). \tag{2.4}$$

Both $A_t(\cdot,\cdot)$ and $G_t(\cdot,\cdot)$ are left unspecified. This is done because the main formula derived in the paper does not depend on the specific functional forms assumed for them. Both $A_t(\cdot,\cdot)$ and $G_t(\cdot,\cdot)$ are also allowed to depend on time to reflect potential technological change. Production in the energy sector depends on labor in addition to fossil fuels. Labor is mobile between sectors and satisfies the constaint

$$N_t = N_{1,t} + N_{2,t}. \tag{2.5}$$

Golsov et al. (2012) further assume that capital and energy are mobile across sectors, but we follow Gerlagh and Liski (2012) in abstracting from this additional complexity.

To close the model, we need to specify an aggregate resource constraint for the final-goods sector. We assume that physical capital depreciates 100% in each period[10]. This implies

$$Y_t = C_t + K_{t+1}. \tag{2.6}$$

Output each period is allocated between consumption and next period's capital stock.

The model resembles the Brock-Mirman optimal growth example with log utility, Cobb Douglass production, and 100% depreciation (Ljungqvist and Sargent 2004, Brock and Mirman 1973). This model is often used to motivate the "guess and verify" method for dynamic programming because the value function takes an extremely simple analytical form—linear in the log of capital . As we will see, the model here gives rise to value functions that are also analytically tractable.

2.2 The planner's problem with commitment

We initially consider the problem in which a decision-maker (or planner) in the initial period employs a commitment device. This enables them to choose the entire time path of state-contingent policies to be implemented by subsequent generations. Since the first-period

[9]Boundedness ensures that all relevant expectations are finite.

[10]Golosov et al. (2011) ostensibly allow for incomplete depreciation, but only in the section of their paper in which household savings behavior is described by a constant savings rate. In the portion of their paper in which households maximize intertemporal utility, the same 100% depreciation rate is assumed. This is an extreme assumption that is only partly offset by the fact that the period length in the model is taken to be a decade.

decision-maker seeks to maximize its own utility, it fixes the path of time preference rates in the way that it prefers given its own time perspective. In sequence form, the problem is

$$\max_{\{k_{t+1},z_t\}} \mathbb{E}_1 \sum_{t=1}^{T} R(t-1)\ln(c_t),$$

subject to (2.1), (2.2), (2.3), (2.4), (2.5) and (2.6),

as well as non-negativity constraints on consumption and capital.

3 The optimal carbon price with commitment

In this section, we derive a formula for the first period carbon price when a decision maker in that period has a commitment device. The formula is the central result of the paper. As we show in the next section, it gives the near-term carbon price not only when there is a commitment device, but also when concerns about time inconsistency are fully accounted for.

To solve for the optimal carbon price, we exploit some tractable analytical features of the model. The same features are emphasized in Gerlagh and Liski (2012). Those authors show, with quasi-hyperbolic discounting, that a symmetric Markov perfect equilibrium exists for which the equilibrium value function is linear in the log of capital and linear in prior emissions[11]. The same general analytical structure is preserved in our setting when the full commitment problem is solved under NCTP. It is also preserved when the model is solved using backward induction.

Before presenting the result, we define some notation. Let $R(l,m) = \prod_{j=l}^{m} \beta_j$ be the cumulative discount factor between period l and period $m+1$. It is the price at date l of a unit of utility to be received at date $m+1$. For convenience, we define $R(j,k) \equiv 1$ anytime $k \leq j$. For $n > m$, the cumulative discount factor can be decomposed:

$$R(n) \equiv R(1,n) = R(1,m)R(m+1,n).$$

Proposition 1 *Consider the planner's problem under commitment described in section 2.2. In the limit as the time horizon goes to infinity, the optimal carbon tax in period* 1 *is*[12]

$$\frac{F_z(1)}{Y_1} = \mathbb{E}_1 \left[\sum_{k=1}^{\infty} R(k)\gamma_{1+k}(1-d_k)\cdot\Gamma(k) \right], \tag{3.1}$$

where

$$\Gamma(k) = \frac{\sum_{m=0}^{\infty} \alpha^m R(k+1,k+m)}{\sum_{n=0}^{\infty} \alpha^n R(n)}.$$

Proof. We fix the path of discount factors at those preferred by the initial generation, then apply finite horizon dynamic programming. Because the time horizon is finite, the problem is nonstationary, so the value function is indexed at each time step by the number of periods remaining. The number of periods remaining is indicated by a superscript on the value function.

[11] Gerlagh and Liski (2012) assume that future damages are deterministic, while we assume that the elasticity of future damages follows a known stochastic process.

[12] $F_z(1)$ is shorthand for the partial derivative of the final-goods production function with respect to fossil fuel input Z; the "1" indicates that all variables are evaluated at their period 1 values.

Before starting, we simplify notation. The energy-labor composite function can be viewed as a function of N_{1t}, N_{2t} and Z_t. For convenience, we denote this by $\tilde{E}(Z_t, N_{1t}) \equiv A_t(G_t(Z_t, N_{2t}), N_{1t})$. Given N_{1t}, N_{2t} is redundant by (2.5), so we suppress it in the notation. We also suppress the time subscript on $\tilde{E}$.

In the last period, the optimal policy is $K_{T+1} = 0$ and $Z_T = \bar{Z}$, where $\bar{Z}$ is the maximum feasible rate of fossil fuel consumption (which is assumed to bind in the last period only). This implies

$$\begin{aligned} V^{(0)}(K_T, S_T, \gamma_T) &= ln[K_T^{\alpha}\tilde{E}(N_{1t}, \bar{Z})\omega(S_T)] \\ &= \alpha ln(K_T) + ln(\tilde{E}(N_{1t}, \bar{Z})) - \sum_{j=1}^{T-1} \gamma_T(1-d_j)Z_{T-j}. \end{aligned}$$

In $T-1$,

$$\begin{aligned} V^{(1)}(K_{T-1}, S_{T-1}, \gamma_{T-1}) &= max_{K_T, Z_{T-1}} ln[K_{T-1}^{\alpha}\tilde{E}(N_{1t}, Z_{T-1})\omega(S_{T-1}) - K_T] \\ &+ \beta_{T-1}\mathbb{E}_{T-1}\left[\ldots + \alpha ln(K_T) - \sum_{j=1}^{T-1}\gamma_T(1-d_j)Z_{T-j}\right]. \end{aligned}$$

Throughout, "…" indicates inessential constants. Taking first-order conditions and substituting gives

$$\begin{aligned} V^{(1)}(K_{T-1}, S_{T-1}, \gamma_{T-1}) &= \ldots + \alpha(1+\alpha\beta_{T-1})ln(K_{T-1}) \\ - \sum_{m=1}^{T-2}\{(1+\alpha\beta_{T-1})\gamma_{T-1}(1-d_m) &+ \beta_{T-1}(1-d_{m+1})\mathbb{E}_{T-1}[\gamma_T]\}Z_{T-m-1}. \end{aligned}$$

This implies the following value function coefficients when $j = 1$:

$$\theta_1 = \alpha(1+\alpha\beta_{T-1}) \tag{3.2}$$

and

$$\zeta_{m,1} = (1+\alpha\beta_{T-1})\gamma_{T-1}(1-d_m) + \beta_{T-1}(1-d_{m+1})\mathbb{E}_{T-1}[\gamma_T]. \tag{3.3}$$

Next, proceeding by induction, suppose the value function takes the following form when there are $j \geq 1$ periods remaining:

$$V^{(j)}(K_{T-j}, S_{T-j}, \gamma_{T-j}) = \Gamma_j + \theta_j ln(K_{T-j}) - \sum_{m=1}^{T-j-1}\zeta_{m,j}Z_{T-j-m}. \tag{3.4}$$

The problem in period $T-(j+1)$ is then

$$\begin{aligned} V^{(j+1)}(K_{T-j-1}, S_{T-j-1}, \gamma_{T-j-1}) = \max_{K_{T-j}, Z_{T-j-1}} & ln(K_{T-j-1}^{\alpha}\tilde{E}(N_{1t}, Z_{T-j-1})\omega(S_{T-j-1}) - K_{T-j}) \\ &+\beta_{T-j-1}\mathbb{E}_{T-j-1}[\ldots + \theta_j ln(K_{T-j}) - \sum_{m=1}^{T-j-1}\zeta_{m,j}Z_{T-j-m}]. \end{aligned}$$

Taking first-order conditions and simplifying gives

$$\begin{aligned} V^{(j+1)}(K_{T-j-1}, S_{T-j-1}, \gamma_{T-j-1}) &= \ldots + \alpha(1+\beta_{T-j-1}\theta_j)\ln(K_{T-j-1}) \\ - \sum_{m=1}^{T-j-2}\{(1+\beta_{T-j-1}\theta_j)\gamma_{T-j-1}(1-d_m) &+ \beta_{T-j-1}\mathbb{E}_{T-j-1}[\zeta_{m+1,j}]\}Z_{t-j-1-m}. \end{aligned}$$

This confirms the inductive hypothesis. It also implies the following recursive equations that can be used to construct the value function coefficients:

$$\theta_{j+1} = \alpha(1 + \beta_{T-j-1}\theta_j) \tag{3.5}$$

and

$$\zeta_{m,j+1} = (1 + \beta_{T-j-1}\theta_j)\gamma_{T-j-1}(1 - d_m) + \beta_{T-j-1}\mathbb{E}_{T-j-1}[\zeta_{m+1,j}]. \tag{3.6}$$

Iterating on (3.5) starting from (3.2) gives

$$\theta_j = \alpha\left[1 + \sum_{k=1}^{j}\left(\prod_{l=0}^{k-1}\alpha\beta_{T-j+l}\right)\right]. \tag{3.7}$$

Iterating on (3.6) starting from (3.3), applying the Law of Iterated Expectations, gives

$$\zeta_{m,j} = \sum_{k=0}^{j-1}\left[\left(\prod_{l=0}^{k-1}\beta_{T-j+l}\right)\left(1 + \sum_{n=1}^{j-k}\prod_{p=0}^{n-1}\alpha\beta_{T-j+p+k}\right)\right](1 - d_{m+k})\mathbb{E}_{T-j}[\gamma_{T-j+k}] \tag{3.8}$$

$$+\prod_{q=0}^{j-1}\beta_{T-j+q}(1 - d_{m+j})\mathbb{E}_{T-j}[\gamma_T]. \tag{3.9}$$

Next, taking the first-order condition with respect to Z_1 for the first period problem gives

$$\frac{F_z(1)}{Y_1} = \frac{\beta_1\mathbb{E}_1[\zeta_{1,T-2}]}{\sum_{n=0}^{T-1}\alpha^n R(n)}.$$

Taking the limit as the time horizon goes to infinity gives (3.1). ∎

Equation (3.1) shows that the optimal carbon price as a fraction of output is a constant that depends only on the expected value of the future damage elasticities, on the carbon cycle parameters, and on the path of time preference rates. Notably, the formula does not depend on the endogenous paths for carbon stocks, output, or consumption. As discussed in the introduction, this peculiar finding is a consequence of the combined assumptions that utility is logarithmic and damages the assumed multiplicative form.

The formula demonstrates a form of certainty equivalence that is noted also in Golosov et al. (2011). In particular, the only feature of uncertainty about future damages that affects the current decision is the expected value of the future elasticity parameter conditional on the information set today. It follows that fat-tailed damages affect the optimal carbon tax in this model only so far as they affect the expected value of future realizations of the damage parameter[13].

To develop intuition for the formula, it is useful to consider the case in which time preference rates are constant: $\beta_j = \beta$ for all j. In this case, the numerator and denominator in the ratio term on the right-hand side of (3.1) are equal, so the term equals one. The overall expression then reduces to

$$\frac{F_z(1)}{Y_1} = \mathbb{E}_1\left[\sum_{k=1}^{\infty}\beta^k\gamma_{1+k}(1 - d_k)\right]. \tag{3.10}$$

[13]See Weitzman (2009) and Nordhaus (2011) for further discussion of the potential implications of "fat-tailed" uncertainty for climate policy.

This is identical to the carbon price formula presented in Proposition 1 of Golosov et al. (2011)[14]. The finding that the two formulas coincide is unexpected since the models differ somewhat. These differences do not matter for determining the near-term carbon price.

The intuition for the carbon price in (3.10) is straightforward. γ_{1+k} is the elasticity of damages (as a fraction of output) with respect to an extra unit of atmospheric carbon k periods ahead. Meanwhile, $1 - d_k$ is the fraction of an extra unit of period 1 emissions that remain in the atmostphere in k periods. The product of the two terms gives the k-period-ahead damages (as a fraction of output) that can be attributed to an extra unit of emissions in period 1. It follows that the right-hand side of (3.10) is the marginal externality cost of emissions, and also, as Golosov et al. (2011) show, the Pigouvian tax.

In moving from constant discounting to hyperbolic discounting, one might expect the equilibrium carbon price to take the same form, obtained by replacing the cumulative discount factor in (3.10) with the cumulative discount factor under hyperbolic discounting: i.e., replace β^k with $R(k)$. But formula (3.1) shows that in fact the so-far-ignored $\Gamma(k)$ terms create a wedge between this naive formulation and the optimal carbon price. It is easy to show that the $\Gamma(k)$ terms make the carbon price bigger.

To see this, compare the sums in the numerator and the denominator of $\Gamma(k)$. $R(k+1, k+m)$ gives the cumulative discount factor between period $k+1$ and period $k+m+1$, a span of m periods. $R(1,n)$ gives the cumulative discount factor between period 1 and period $n+1$, a span of n periods. Comparing term by term (equating the indices m and n) the covered spans are the same. The difference is that the cumulative discount factors in the numerator select sections of the discount factor sequence that are further in the future. Under constant discounting, this doesn't matter (i.e., the ratio equals one). But when time preference rates decline, the cumulative discount factor terms in the numerator are larger. Therefore, a sufficient condition for

$$\frac{\sum_{m=0}^{\infty} \alpha^m R(k+1, k+m)}{\sum_{n=0}^{\infty} \alpha^n R(1,n)} > 1$$

is for discount rates to decline weakly in all periods and to decline strictly in at least one.

4 Time inconsistency irrelevance

In deriving the formula for the carbon price in equation (3.1), we assumed that decision-makers in the initial period could use a commitment device. But if future decisions are instead modeled in a way that is consistent with the preferences (and time perspective) of the agents living at that time, then the expected future trajectory of the climate-economy system will change in ways that would typically affect the optimal choice of carbon policy today. Despite this expectation, we show in this section that the optimal first-period carbon price is unchanged when we solve instead using backward induction. In particular, the optimal carbon price under backward induction is also given by equation (3.1). We further show that this result continues to hold no matter what one assumes about the time preference structure of future generations.

[14] One small difference arises because Golosov et al. (2011) have emissions affect damages immediately, while we follow Gerlagh and Liski (2012) in having the initiation of damages be delayed one period. In effect, $\frac{\partial S_t}{\partial Z_t} = 0$ in our formulation, while it is allowed to be positive in Golosov et al. (2011). When Golosov et al. (2011) calibrate their model to match the Nordhaus (2008) carbon cycle as closely as possible, they assume $\frac{\partial S_t}{\partial Z_t} = 0$ (in which case their formula matches (3.10) exactly).

Our finding implies that time consistency concerns can be safely ignored when applying the carbon price formula provided in the paper. When the model is calibrated in section 6, the period length is taken to be a decade. Since the current policy regime cannot plausibly commit action beyond that span of time[15], the information contained in the the first-period carbon price is all the current policy regime needs to know to justify current decisions.

4.1 Time Consistent Backward Induction

To solve the problem without commitment, we present an algorithm that can be used to solve for the backward induction equilibrium in a finite horizon model in which decision-makers employ an arbitrary path of PRTPs.

4.1.1 Algorithm

We illustrate the algorithm using a fairly generic dynamic model. We assume that enough assumptions have been placed on the model to ensure that the Strotz-Pollak equilibrium exists and is unique. This is a strong assumption, but it suffices for our purposes because we can demonstrate that this holds for the model considered in the paper.

Time runs from $t = 1, \ldots, T$. A control vector z is chosen in each period, and a state vector S evolves according to the following transition equation: $S_{t+1} = G(S_t, z_t)$. There is no commitment device, so the decision maker each period controls only the current decision. Agents are identical, and they each employ the same sequence of (nonconstant) discount factors $\{\beta_t\}$. The payoff each period is given by the return function $h(S, z)$.

Throughout, V denotes the continuation value from the perspective of the contemporaneous generation. Superscripts indicate the number of periods remaining. In the last period, the Bellman equation for the last period decision maker is

$$V^{(0)}(S) = \max_z h(S, z).$$

In period T-1, the contemporaneous decision-maker solves

$$V^{(1)}(S) = \max_z \left[h(S, z) + \beta_1 V^{(0)}(G(S, z)) \right]. \tag{4.1}$$

This can be used to generate the corresponding policy function $z = \phi_1(S)$. Note that $V^{(1)}(S)$ captures the continuation value from the perspective of the period T-1 decision maker, but not the relevant continuation value from the perspective of earlier generations. For example, the decision-maker in $T-2$ would apply β_2 in place of β_1 in (4.1) to get the relevant continuation value. Similarly, the decision-maker in $T-3$ would apply β_3 in place of β_1.

To account for the distinct time perspective of agents in each prior period, we construct an "auxiliary value function" for each[16]. As a matter of notation, $W_t^{(j)}(S)$ will denote the continuation value with j periods remaining as viewed from the perspective of a decision maker in period $t < T - j$. For the $T-1$ period problem, there are $T-2$ such auxiliary value

[15] For example, there was widespread support for the Kyoto protocol when it was initially adopted in 1997, but twelve years later, the international community was unable to negotiate a compelling continuation treaty at the Copenhagen Climate Summit.

[16] The term "auxiliary value function" is adopted in Harris and Laibson (2001). Our usage of the term is related but distinct.

functions. They are constructed as follows[17]:

$$
\begin{aligned}
W_{T-2}^{(1)}(S) &= h(S, \phi_1(S)) + \beta_2 V^{(0)}[G(S, \phi_1(S))] \\
W_{T-3}^{(1)}(S) &= h(S, \phi_1(S)) + \beta_3 V^{(0)}[G(S, \phi_1(S))] \\
&\vdots \\
W_1^{(1)}(S) &= h(S, \phi_1(S)) + \beta_{T-1} V^{(0)}[G(S, \phi_1(S))].
\end{aligned}
$$

Next, consider the problem in $T-2$. Again, start with the contemporaneous decision maker.

$$V^{(2)}(S) = \max_z \left[h(S, z) + \beta_1 W_{T-2}^{(1)}[G(S, z)] \right].$$

Note that the relevant continuation value for a decision-maker in $T-2$ is the auxiliary value function $W_{T-2}^{(1)}$. Solving this problem gives the period $T-2$ policy function $z = \phi_2(S)$. This in turn is used to construct the auxiliary value functions associated with the continuation value from $T-2$ forward:

$$
\begin{aligned}
W_{T-3}^{(2)}(S) &= h(S, \phi_2(S)) + \beta_2 W_{T-3}^{(1)}[G(S, z)] \\
&\vdots \\
W_1^{(2)}(S) &= h(S, \phi_2(S)) + \beta_{T-2} W_1^{(1)}[G(S, z)].
\end{aligned}
$$

The procedure is repeated until arriving in period 1. At that point,

$$V^{(T-1)}(S) = \max_z \left[h(S, z) + \beta_1 W_1^{(T-2)}[G(S, z)] \right].$$

4.2 The optimal carbon price without commitment

The algorithm just described can be used to solve for the sequence of policy functions along the Strotz-Pollak equilibrium path, including the equilibrium carbon policy in period one. Doing so leads to the following result.

Proposition 2 *Consider the abatement problem described in section 2 and assume a commitment device is unavailable. The first period carbon price calculated along the Strotz-Pollak equilibrium path is the same as found in the problem with full commitment. In both cases, the optimal first period carbon price in the infinite horizon limit of the model is given by expression* (3.1)*.*

Proof. The proof resembles that of proposition 1. As there, we work backwards from the initial period. The key difference is that we employ the backward induction algorithm from section 4.1, rather than simply imposing the path of discount factors that would be applied by a first period decision maker.

In the last period, $K_{T+1} = 0$ and $Z = \bar{Z}$, so

$$V^{(0)}(K_T, S_T, \gamma_T) = \alpha ln(K_T) + ln(\tilde{E}(N_{1t}, \bar{Z})) - \sum_{j=1}^{T-1} \gamma_T (1 - d_j) Z_{T-j}.$$

[17] When using this approach as a numerical algorithm, it is worth noting that the auxiliary value functions can be constructed without any additional optimization steps.

Because no discount factor is used to construct this value function, the corresponding auxiliary value functions are identical:

$$W_t^{(0)}(K_T, S_T, \gamma_T) = V^{(0)}(K_T, S_T, \gamma_T), \quad t = 1, \ldots, T-1.$$

In $T-1$, the value function (for a contemporaneous decision maker) solves

$$\begin{aligned} V^{(1)}(K_{T-1}, S_{T-1}, \gamma_{T-1}) &= max_{K_T, Z_{T-1}} ln[K_{T-1}^{\alpha} \tilde{E}(N_{1t}, Z_{T-1})\omega(S_{T-1}) - K_T] + \beta_1 \mathbb{E}_{T-1} W_{T-1}^{(0)}(K_T, S_T, \gamma_T) \\ &= max_{K_T, Z_{T-1}} ln[K_{T-1}^{\alpha} \tilde{E}(N_{1t}, Z_{T-1})\omega(S_{T-1}) - K_T] \\ &+ \beta_1 \mathbb{E}_{T-1} \left[\alpha ln(K_T) + ln(\tilde{E}(N_{1t}, 0)) - \sum_{j=1}^{T-1} \gamma_T (1-d_j) Z_{T-j} \right]. \end{aligned}$$

The first-order conditions for K_T and Z_{T-1} imply

$$K_T = \frac{\alpha\beta_1}{1+\alpha\beta_1} Y_{T-1} \tag{4.2}$$

and

$$\frac{\tilde{E}_z(N_{1t}, Z_{T-1}^*)}{\tilde{E}(N_{1t}, Z_{T-1}^*)} = \frac{\beta_1(1-d_j)\mathbb{E}_{T-1}[\gamma_T]}{1+\alpha\beta_1}. \tag{4.3}$$

Conditional on information available in $T-1$, Z_{T-1}^* can be viewed as a constant that is independent of K_{T-1} and S_{T-1}. Substituting and simplifying gives

$$\begin{aligned} &V^{(1)}(K_{T-1}, S_{T-1}, \gamma_{T-1}) \\ &= ln[\frac{1}{1+\alpha\beta_1} Y_{T-1}] + \alpha\beta_1 ln(\frac{\alpha\beta_1}{1+\alpha\beta_1} Y_{T-1}) + \beta_1 ln(\tilde{E}(N_{1t}, 0)) - \beta_1 \sum_{m=1}^{T-1} (1-d_m)\mathbb{E}_{T-1}[\gamma_T] Z_{T-m} \\ &= \ldots + \alpha(1+\alpha\beta_1) ln(K_{T-1}) - \sum_{m=1}^{T-2} \{(1+\alpha\beta_1)\gamma_{T-1}(1-d_m) + \beta_1(1-d_{m+1})\mathbb{E}_{T-1}[\gamma_T]\} Z_{T-m-1}. \end{aligned}$$

To construct the corresponding auxiliary value function for $t < T-1$, we again use (4.2) and (4.3), but we change the discount factor applied between $T-1$ and T from β_1 to β_{T-t}. We would also change the next period auxiliary value function, but in this case, it is unnecessary because they are the same. Substituting and simplifying gives

$$\begin{aligned} &W_t^{(1)}(K_{T-1}, S_{T-1}, \gamma_{T-1}) \\ &= ln[\frac{1}{1+\alpha\beta_1} Y_{T-1}] + \alpha\beta_{T-t} ln(\frac{\alpha\beta_1}{1+\alpha\beta_1} Y_{T-1}) + \beta_{T-t} ln(\tilde{E}(N_{1t}, 0)) - \beta_{T-t} \sum_{m=1}^{T-1} (1-d_m)\mathbb{E}_{T-1}[\gamma_T] Z_{T-m} \\ &= \ldots + \alpha(1+\alpha\beta_{T-t}) ln(K_{T-1}) - \sum_{m=1}^{T-2} \{(1+\alpha\beta_{T-t})\gamma_{T-1}(1-d_m) + \beta_{T-t}(1-d_{m+1})\mathbb{E}_{T-1}[\gamma_T]\} Z_{T-m-1}. \end{aligned}$$

Note that β_1, carried over from the policy rule chosen by the $T-1$ decision-maker, ends up in the constant term (indicated here by "...").

The rest of the proof proceeds by induction. The hypothesis is that the date $T-j$ continuation value, viewed from the perspective of $t < T-j$, takes the form

$$W_t^{(j)}(K_{T-j}, S_{T-j}) = \Gamma_j^t + \theta_j^t ln(K_{T-j}) - \sum_{m=1}^{T-j-1} \zeta_{m,j}^t Z_{T-j-m}.$$

The coefficients depend on the number of periods remaining (including information that becomes available in that period), and they depend on the time perspective, "t".

The hypothesis is confirmed for $j = 1$ by the initial analysis above. In that case, the value function coefficients are

$$\theta_1^t = \alpha(1 + \alpha\beta_{T-t}) \tag{4.4}$$

and

$$\zeta_{m,1}^t = (1 + \alpha\beta_{T-t})\gamma_{T-1}(1 - d_m) + \beta_{T-t}(1 - d_{m+1})\mathbb{E}_{T-1}[\gamma_T]. \tag{4.5}$$

Next, consider the problem for a decision-maker at date $T - (j + 1)$.

$$V^{(j+1)}(K_{T-j-1}, S_{T-j-1}, \gamma_{T-j-1}) = \max_{K_{T-j}, Z_{T-j-1}} ln(K_{T-j-1}^{\alpha}\tilde{E}(N_{1t}, Z_{T-j-1})\omega(S_{T-j-1}) - K_{T-j})$$
$$+ \beta_1\mathbb{E}_{T-j-1}[\Gamma_j^{T-j-1} + \theta_j^{T-j-1} ln(K_{T-j}) - \sum_{m=1}^{T-j-1} \zeta_{m,j}^{T-j-1} z_{T-j-m}].$$

Taking first-order conditions and simplifying gives

$$\begin{aligned}
&V^{(j+1)}(K_{T-j-1}, S_{T-j-1}, \gamma_{T-j-1}) \\
&= \ldots + ln(\frac{1}{1 + \beta_1\theta_j^{T-j-1}} Y_{T-j-1}) \\
&+ \beta_1\mathbb{E}_{T-j-1}[\Gamma_j^{T-j-1} + \theta_j^{T-j-1} ln(\frac{\beta_1\theta_j^{T-j-1}}{1 + \beta_1\theta_j^{T-j-1}} y_{T-j-1}) - \sum_{m=1}^{T-j-1} \zeta_{m,j}^{T-j-1} z_{T-j-m}] \\
&= \ldots + \alpha(1 + \beta_1\theta_j^{T-j-1}) \ln(K_{T-j-1}) \\
&- \sum_{m=1}^{T-j-2} \{(1 + \beta_1\theta_j^{T-j-1})\gamma_{T-j-1}(1 - d_m) + \beta_1\mathbb{E}_{T-j-1}[\zeta_{m+1,j}^{T-j-1}]\} Z_{t-j-1-m}.
\end{aligned}$$

To construct the corresponding auxiliary value functions for $t < T - (j + 1)$, continue to use the first-order conditions from the problem above, but replace β_1 with β_{T-j-t} and replace $W_{T-j-1}^{(j)}(K_{T-j}, S_{T-j}, \gamma_{T-j})$ with $W_t^{(j)}(K_{T-j}, S_{T-j}, \gamma_{T-j})$.

$$\begin{aligned}
&W_t^{(j+1)}(K_{T-j-1}, S_{T-j-1}, \gamma_{T-j-1}) \\
&= \ldots + ln(\frac{1}{1 + \beta_1\theta_j^{T-j}} Y_{T-j-1}) + \beta_{T-j-t}\mathbb{E}_{T-j-1}[\Gamma_j^t + \theta_j^t ln(\frac{\beta_1\theta_j^{T-j}}{1 + \beta_1\theta_j^{T-j}} y_{T-j-1}) - \sum_{m=1}^{T-j-1} \zeta_{m,j}^t Z_{T-j-m}] \\
&= \ldots + \alpha(1 + \beta_{T-j-t}\theta_j^t) \ln(K_{T-j-1}) \\
&- \sum_{m=1}^{T-j-2} \{(1 + \beta_{T-j-t}\theta_j^t)\gamma_{T-j-1}(1 - d_m) + \beta_{T-j-t}\mathbb{E}_{T-j-1}[\zeta_{m+1,j}^t]\} Z_{t-j-1-m}.
\end{aligned}$$

The coefficients are

$$\theta_{j+1}^t = \alpha(1 + \beta_{T-j-t}\theta_j^t) \tag{4.6}$$

and

$$\zeta_{m,j+1}^t = (1 + \beta_{T-j-t}\theta_j^t)\gamma_{T-j-1}(1 - d_m) + \beta_{T-j-t}\mathbb{E}_{T-j-1}[\zeta_{m+1,j}^t]. \tag{4.7}$$

This shows that the relevant auxiliary value function coefficients for a decision maker in a given period t is determined by an independent recursive system. Fixing the time perspective at $t = 1$, it is easy to see that θ_1^1 from (4.4) equals θ_1 from (3.2) and $\zeta_{m,1}^1$ from (4.5) equals $\zeta_{m,1}$ from (3.3). Moreover, the recursive relationship for θ_j^1 and $\zeta_{m,j}^1$ in (4.6) and (4.7) is the same as the recursive relationship for θ_j and $\zeta_{m,j}$ in (3.5) and (3.6). It follows that the continuation value relevant to the first-period decision differs across the equilibrium scenarios by at most a constant[18]. Since the constant term in the value function does not affect the optimal policy decision, it follows that the optimal carbon price chosen by an agent in the first period is the same in the two equilibria. ■

The proposition shows that the general formula for the first period carbon price in (3) is robust to concerns about time inconsistency. This finding is strengthened in the next subsection. But before presenting that, we first discuss an interpretation of the equilibrium derived in Gerlagh and Liski (2012) that is implied by Proposition 2.

Gerlagh and Liski (2012) solve for a symmetric Markov perfect equilibrium under which the policy functions are characterized by a particularly simple simple form: the optimal choice of capital in $t+1$ is linear in output at date t and the marginal product of final goods production with respect to fossil fuel inputs in t is linear in period t consumption. (Indeed, our equilibrium takes the same form.) In general, one would expect there to be multiple symmetric Markov perfect equilibria, and Gerlagh and Liski (2012) do not show that the symmetric Markov perfect equilibrium for which they solve is unique; rather, they motivate their particular equilibrium on the grounds that the simplicity might appeal to policy-makers. Proposition 2 can be viewed as strengthening the Gerlagh and Liski (2012) result because it can be shown to imply that the symmetric Markov perfect equilibrium in Gerlagh and Liksi (2012) is also the unique Strotz-Pollak equilibrium. Thus, their equilibrium is not just one arbitrary time consistent equilibrium among many; rather, it is the unique one that clearly respects the order of decision makers in the timeline.

To compare the carbon price formula in Lemma 4 of Gerlagh and Liski (2012) with the formula here, it is necessary to assume in their model (as we do) that $\Delta_y = 1$ and $\Delta_u = 0$. In addition, because they assume that the damage function elasticity parameter is a known constant, we rewrite the damage function in the following "generic" form to facilitate comparison:

$$\omega(S_t) = \exp(-\sum_{k=1}^{t-1} \phi_k Z_{t-k}).$$

Then setting the discount factors to $(\rho, \theta, \theta, \ldots)$, where $\theta > \rho$, the first period carbon price reduces to the following in both papers:

$$\frac{F_z(1)}{Y_1} = \frac{\rho}{1 + \alpha(\rho - \theta)} \sum_{k=1}^{\infty} \theta^{k-1} \phi_k. \tag{4.8}$$

4.3 Strong irrelevance

The "irrelevance" result from Proposition 2 can be strengthened. In resolving time inconsistency, Proposition 2 stepped away from the assumption of a commitment device by solving instead for the backward induction solution when decision makers in every period are identical. But anticipating the preferences of future generations is a notoriously difficult problem (Beltratti, Chichilnisky and Heal 1998) and one could plausibly maintain a wide variety of

[18]In fact, though it is not tracked explicitly in the writing of the proof shown here, the constant term is higher under commitment. This means that the wealth level of the first period agent is higher with commitment then without.

possible assumptions about the structure of time preferences to be applied by future generations. The following result shows that the initial period carbon price is unchanged no matter what one assumes about the path of time preference rates to be employed by future generations. It ensures that the carbon price in Proposition 1 is robust to time inconsistency concerns in the strongest possible way.

Proposition 3 *Consider the abatement problem described in section 2 and assume a commitment device is unavailable. Assume further that agents in different time periods have their own path of time preference rates, which could differ in arbitrary ways across generations. Then first period carbon price calculated along the Strotz-Pollak equilibrium path is the same as found with full commitment. In the infinite horizon limit, it is given by expression* (3.1).

Proof. The proof is identical to that of Proposition 2 with some notational changes to allow for an arbitrary of path of discount factors for each later generation. The result follows from the fact that the discount factors used by later generations, which get embedded into each period's policy function, end up in the constant term of the auxiliary value function for earlier generations. As a result, while these subsequent decisions (and the path of discount factors that motivate them) affect the wealth level of earlier decision-makers, they do not affect the optimal choice of carbon policy in the initial period. ■

4.4 The role of simplifying assumptions

It is worth considering the importance, for our results, of the simplifying assumptions that we maintain relative to Golosov et al. (2011). As discussed, we simplify in two ways. First, by treating the stock of fossil fuels as infinite, and second by simplifying production in the energy sector.

First note that none of these features of the model appear in the carbon price formula derived in Golosov et al. (2011). So they do not affect the near-term carbon price under constant discounting. Of course, the way in which energy production and resource scarcity are modeled does affect the long-term dynamic path of the climate-economy system, but these changes do not bare on our results. The question is if something in the nature of backward induction with hyperbolic agents would create a mechanism under which these features of the model would start to matter for the near-term carbon price. We do not see a case for such a mechanism.

For the assumption regarding production in the energy extraction sector, we think this point is particularly clear. In particular, as in the full Golosov et al. (2011) model with constant discounting, our formula depends only loosely on the assumptions regarding production in the energy sector and the way in which energy gets incorporated into final-goods production. In particular, the derived formula allows for arbitrary functional forms for both the energy sector production function and the labor-energy composite function in final-goods production. Since the details of these functional forms do not in any way matter for determining the near-term carbon price, it is hard to see why it would suddenly matter—and only for the hyperbolic case—when energy production is assumed to also depend on capital.

Of course, these arguments comprise only a conjecture. One could defend the conjecture numerically. In particular, we could use finite-horizon dynamic programming to solve numerically for the near-term carbon price with commitment, then use the algorithm from section 4.1.1 to solve numerically for the carbon price without commitment. To do so, we would need to specify functional forms for the energy sector production function and the energy-labor composite in final-goods production. The exercise would therefore only prove the conjecture for the considered scenarios. To accomplish this, the suggested numerical problems would involve a four-dimensional state space. The numerical problems are likely feasible, but it would be challenging, and we view it as outside the scope of the current paper.

5 Pigouvian taxes

A Pigouvian tax would set the tax on carbon equal to the marginal externality cost of emissions, with future costs discounted at a rate that is consistent with the equilibrium real return on capital. The relevant utility discount factor in period t, which we denote ϕ_t, depends on the equilibrium path of consumption as follows[19]

$$u'(C_t) = \phi_t \mathbb{E}_t u'(C_{t+1}) F_k(K_{t+1}, N_{t+1}, S_{t+1}, \gamma_{t+1}).$$

Log utility and the fact that final-goods production is Cobb Douglass in capital implies

$$\phi_t = \frac{1}{\alpha} \cdot \mathbb{E}_t \left[\frac{C_{t+1}}{Y_{t+1}} \cdot \frac{K_{t+1}}{C_t} \right]. \tag{5.1}$$

Since the relevant discount rate depends on the equilibrium path of consumption, the calculated Pigouvian tax depends on which equilibrium is considered. Since our analysis compares two equilibria, we compute two Pigouvian taxes.

5.0.1 Pigouvian tax with commitment

To derive the period one Pigouvian tax corresponding to the equilibrium with commitment, we combine equation (5.1) with the full commitment equilibrium savings rule[20]:

$$K_{t+1} = \frac{\beta_t \theta_{T-t-1}}{1 + \beta_t \theta_{T-t-1}} Y_t$$

and

$$C_t = \frac{1}{1 + \beta_t \theta_{T-t-1}} Y_t.$$

It follows that

$$\frac{K_{t+1}}{C_t} = \beta_t \theta_{T-t-1}.$$

Moreover[21],

$$\frac{C_{t+1}}{Y_{t+1}} = \frac{1}{1 + \beta_{t+1} \theta_{T-t-2}} = \frac{\alpha}{\theta_{T-t-1}}.$$

Substituting and simplifying yields

$$\phi_t = \beta_t,$$

[19] Our derivation builds on work in Gerlagh and Liski (2012). The setting differs somewhat since our model is non-stationary and stochastic.

[20] The savings rule falls out from the proof of Proposition 1 with $t = T - j - 1$.

[21] The second equality follows from equation (3.5).

which holds for all t. With commitment, the initial-period decision maker sets savings and consumption decisions to ensure the marginal rate of substitution between adjacent periods, viewed from the initial period, equals the marginal rate of transformation in equilibrium.

Since this is the same subjective discount factor that is applied when solving the full commitment equilibrium, it follows that the period one Pigouvian tax and the period one optimal carbon tax in the equilibrium with commitment are the same.

5.1 Pigouvian tax without commitment

For $t < T$, the savings rule in the no commitment equilibrium is[22]

$$K_{t+1} = \frac{\Omega_{T-t}}{1+\Omega_{T-t}} Y_t, \tag{5.2}$$

where

$$\Omega_{T-t} = \sum_{m=1}^{T-t} \prod_{n=1}^{m} (\alpha\beta_n). \tag{5.3}$$

Substituting into equation (5.1) as before gives

$$\phi_t = \frac{\Omega_{T-t}}{\alpha(1+\Omega_{T-t-1})},$$

which can be simplified to give

$$\phi_t = \beta_1 \cdot \frac{1 + \alpha\beta_2 + \alpha\beta_2\alpha\beta_3 + \ldots + \alpha\beta_2\cdots\alpha\beta_{T-t}}{1 + \alpha\beta_1 + \alpha\beta_1\alpha\beta_2 + \ldots + \alpha\beta_1\cdots\alpha\beta_{T-t-1}} > \beta_1. \tag{5.4}$$

In addition, it is easy to verify in the last period that $\phi_{T-1} = \beta_1$.

Without commitment, the agent in control in period t discounts utility between t and $t+1$ using the subjective discount factor β_1. Provided the preferences of subsequent agents aligned with their own, they would ensure that the expected marginal rate of transformation equaled $\frac{u'(C_t)}{\beta_1 u'(C_{t+1})}$. This is the case in $T-1$ since the preferences of the generation in the very last period align with those of agents in earlier periods (they consume everything, as prior generations would have wanted). But for $t < T-1$, it is not the case. The decision maker in t saves more to account for the fact that subsequent decision-makers are going to save less then they would have if the hyperbolic agent in t could force their hand with a commitment device.

The more interesting question is to compare the Pigouvian discount rates, $\{\phi_t\}$, with the discount rates applied in the full commitment equilibrium, $\{\beta_t\}$. Since the optimal carbon price under commitment equals the optimal carbon price without commitment (from Proposition 2) this comparison will make it possible to compare the optimal carbon price without commitment to the Pigouvian tax without commitment.

Proposition 4 *In the equilibrium without commitment, the relevant subjective discount factor for use in constructing the Pigouvian tax starts out above the corresponding discount factor that would be applied under commitment, then declines monotonically. For large t, it is below the subjective discount factor under commitment. In particular,* $\frac{\phi_1}{\beta 1} > 1$, $\frac{\phi_{T-1}}{\beta T-1} < 1$, *and* $\frac{\phi_{t+1}}{\beta t+1} \leq \frac{\phi_t}{\beta t}$ *for* $1 \leq t \leq T-1$.

[22] The savings rule follows from the inductive step in the proof of proposition 2.

Proof. That $\frac{\phi_1}{\beta 1} > 1$ and $\frac{\phi_{T-1}}{\beta T-1} < 1$ follows from inspection. To prove that $\frac{\phi_{t+1}}{\beta t+1} \leq \frac{\phi_t}{\beta t}$, suppose instead that $\frac{\phi_{t+1}}{\beta t+1} > \frac{\phi_t}{\beta t}$. In particular, suppose

$$\frac{\Omega_{T-t-1}}{\alpha\beta_{t+1}(1+\Omega_{T-t-2})} > \frac{\Omega_{T-t}}{\alpha\beta_t(1+\Omega_{T-t-1})}. \tag{5.5}$$

Defining $\Psi_n = \prod_{m=1}^{n} \alpha\beta_m$, (5.5) can be rewritten as

$$\frac{\Omega_{T-t-1}}{\alpha\beta_{t+1}(1+\Omega_{T-t-2})} > \frac{\Omega_{T-t-1}+\Psi_{T-t}}{\alpha\beta_t(1+\Omega_{T-t-2}+\Psi_{T-t-1})}.$$

Cross-multiplying, this is equivalent to

$$\alpha\beta_t\Omega_{T-t-1} + \alpha\beta_t\Omega_{T-t-1}\Omega_{T-t-2} + \alpha\beta_t\Psi_{T-t-1}\Omega_{T-t-1} >$$
$$\alpha\beta_{t+1}\Omega_{T-t-1} + \alpha\beta_{t+1}\Omega_{T-t-1}\Omega_{T-t-2} + \alpha\beta_{t+1}\Psi_{T-t}(1+\Omega_{T-t-2}).$$

But $\alpha\beta_{t+1}\Omega_{T-t-1} \geq \alpha\beta_t\Omega_{T-t-1}$, $\alpha\beta_{t+1}\Omega_{T-t-1}\Omega_{T-t-2} \geq \alpha\beta_t\Omega_{T-t-1}\Omega_{T-t-2}$ and $\alpha\beta_{t+1}\Psi_{T-t}(1+\Omega_{T-t-2}) > \alpha\beta_t\Psi_{T-t-1}\Omega_{T-t-1}$, which gives a contradiction. ■

It follows that the relationship between the optimal first-period carbon tax and the first-period Pigouvian tax in the equilibrium without commitment is ambiguous[23]. If climate damages all happen immediately, the Pigouvian tax would be bigger. But if climate damages occur with sufficient delay—more likely—the optimal carbon tax is bigger.

Gerlagh and Liski (2011) argue that the delay between emissions and damages in most integrated assessment models is significantly greater than is implied under the Golosov et al. (2011) carbon cycle. They derive an alternative motivation for the linear coefficients in the Golosov et al. (2011) linear-exponential damage function. Their calibration of this alternative model implies a much greater delay between emissions and damages than in Golosov et al. (2011). Adopting the Gerlagh and Liski (2012) carbon cycle would imply a greater separation between the optimal carbon tax and the Pigouvian tax then is found in our quantitative results. As discussed in Gerlagh and Liski (2012), the wedge between the optimal carbon tax and the Pigouvian tax reflects the commitment value[24] of climate policy when generations discount the future hyperbolically. In particular, the long delay between climate-related investments and the corresponding payoffs provides a mechanism to transfer wealth across generations in a way that partially leapfrogs intermediate generations whose intertemporal preferences don't align with those of earlier generations.

6 Quantitative analysis

6.1 Calibration of time preference rates

An advantage of having an explicit formula is that it is easy to compare the effect of alternative specifications. The introduction mentioned three justifications for DRTP. The first was behavioral evidence showing that the observed choice behavior of many individuals is consistent with a hyperbolic discounting function (Frederick et al. 2002). A number of papers in macroeconomics use this evidence to calibrate a declining path of time preference rates (For example, Laibson 1997 and Barro 1999). It is not clear, however, how this evidence should be

[23]Gerlagh and Liski (2012) make the same point.

[24]As Gerlagh and Liski (2012) note, the commitment value in this case is analogous to Laibson's finding that commitment devices have value in self-control problems (Laibson, 2007).

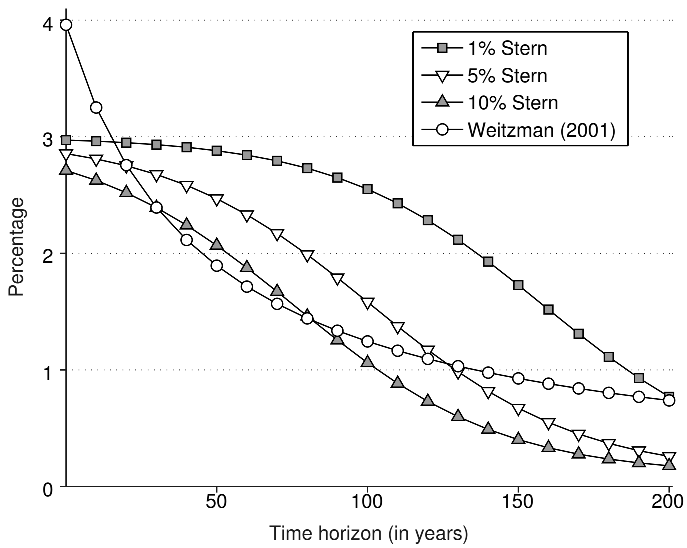

Figure 1: Comparison of alternative hyperbolic discounting paths.

used when evaluating public sector projects (Guo et al. 2006). The trouble is that empirical studies that estimate the parameters of a hyperbolic discounting function imply very high short-run discount rates. For example, in Barro (1999), the continuous path of PRTPs starts at 50%. Thus, even though the appropriate rate may decline to near zero in the very long run (Cropper et al. 1994) the analysis would be overwhelmed by the very high initial rates.

We focus instead on the other justifications for DRTP, in part because the Golosov et al. (2011) model is a convenient setting in which to consider them. We compare four calibrations, presented in figure 1. Each can be interpreted either through the uncertainty lens, under which the "correct" time preference rate is a random variable with a known probability distribution, or through the aggregation lens, where the assumed distribution reflects the distribution of views in society. For simplicity, we adopt the uncertainty lens when describing assumptions.

The first three scenarios reflect the high-profile debate in the economics of climate change between Nordhaus (2008) and Stern. Nordhaus (2008) argues that the PRTP and the elasticity of marginal utility should be jointly calibrated to ensure that the savings decisions of households in the model are consistent with the real return on capital—about 5.5% in Nordhaus (2008). Under log utility, this implies a PRTP of 3.0% in DICE. Stern (2007), in contrast, argues that the intergenerational distributional consequences of this assumption are too extreme when applied to climate change. On ethical grounds, he argues that the PRTP under log utility should be 0.1%. Our goal is not to resolve this debate. Rather, we consider a variety of scenarios for the probability distribution that a global decision-maker today might assign to these competing perspectives. The scenarios attribute respectively 1%, 5%, and 10% weight to the Stern model, with the remaining weight assigned to Nordhaus. We do not offer a deep reason for viewing the Stern rate as dramatically less probable then the Nordhaus rate, though we suspect that a review of general-interest economics journals would reveal a strong preference for the Nordhaus view. More importantly, our scenarios show that the effect of entertaining even a small chance that the Stern rate is correct (or putting a small weight on those members of society who think it is) can be large.

The final calibration builds on the expert elicitation survey in Weitzman (2001). To motivate his calculation of DRTP, Weitzman (2001) surveyed over 2000 professional economists, asking for their "professionally considered gut opinion" about the best discount rate to use in cost benefit analysis for climate policy. Weitzman's survey is the closest thing we have to an expert elicitation study for climate change discounting that is representative for the economics profession. A problem with applying his results to the current situation is that he looks at consumption discount rates, while we are interested in the PRTP[25]. The celebrated *Ramsey formula*, derived from the first-order conditions for the optimal consumption path in the Ramsey growth model, shows that the (instantaneous) consumption discount rate is comprised by the sum of two terms: the PRTP and the product of the consumption growth rate and the elasticity of marginal utility. Provided future growth is positive, the consumption discount rate is bigger than the PRTP. It is therefore reasonable to view Weitzman's distribution as a conservative estimate of the distribution of PRTPs[26].

As noted in the introduction, the strong irrelevance result in proposition 3 strengthens the case for this calibration procedure. The main critique leveled against the calculation of certainty-equivalent discount rates in Weitzman (2001)—see, for example, Newell and Pizer (2003)—is that it hinges on the assumption that current uncertainty (or the current distribution of views) is static. But this criticism does not apply when Weitzman's argument is used in the context of the integrated assessment model here. In this case, even if one expects uncertainty (or the distribution of views) to change over time, which presumably it must, these changes do not matter for determining current policy.

6.2 Calibration of the carbon cycle and damages

To calibrate our damage function, we follow Golosov et al. (2011). Those authors assume that the expected value of the future damage elasticity parameter, conditional on information today, is the same for all future periods. Moreover, they calibrate their damage function to coincide with two data points from a meta analysis of damages in Nordhaus (2000). The first calibration point estimates that a 2.5 degree Celsius increase in mean global temperature would lead to a 0.48% loss of GWP. In addition, they allow for a 6.8% chance that damages from a 6 degree rise in temperatures would be catastrophic, leading to a 30% loss of GWP. These considerations imply an expected damage elasticity of 2.379×10^{-5}.

Our calibration of the carbon cycle follows Golosov et al. (2011) with two adjustments. They calibrate the decay structure of atmospheric carbon dioxide to be consistent with recent evidence that the geometric decay structure in most climate policy models is incorrect. The revised scientific understanding of atmospheric carbon decay is provided in the following quote from the IPCC (IPCC 2007), which is included in Golosov et al. (2011): "About half of a CO2 pulse to the atmosphere is removed over a timescale of 30 years; a further 30% is removed within a few centuries; and the remaining 20% will typically stay in the atmosphere for many thousands of years". To replicate this, they assume that fraction ϕ_0 of emissions fall out of the atmosphere immediately. A further fraction ϕ_L remain forever. And the remaining

[25]The consumption discount rate is the rate at which consumption units are discounted, while the PRTP is the rate at which utility units are discounted.

[26]If Weitzman had instead surveyed economists about the best PRTP for climate policy, one would expect the resulting distribution to put uniformly more weight on lower rates relative to what we get from the study of consumption discount rates; this hypothetical survey would lead to a higher carbon price than obtained with the existing study.

carbon decays at a constant geometric rate. This implies the following formula for decay:

$$1 - d_s = \phi_L + (1 - \phi_L)\phi_0(1 - \phi)^s.$$

They calibrate this model by assuming $\phi_L = 0.2$, $\phi_0 = 0.393$, and $\phi = 0.0228$. We adopt the same parameter values, but we modify the decay sequence in two ways.

First, we alter the assumption that some fraction of the carbon stock remains in the atmosphere "forever". This is a reasonable approximation for the purpose of evaluating the optimal carbon tax provided the discount rate is moderately high. In that case, nothing beyond a few hundred years matters in determining the carbon price, so there is no difference between assuming that a portion of emissions remains for a couple thousand years (as suggested in the IPCC report) or that it remains literally forever. But this assumption is no longer harmless if the subjective discount rate is very low (as, for example, in Stern 2007) or if it declines over time to near zero. In that case, the effect of a portion of emissions remaining in the atmosphere can be enormous. For example, if the subjective discount rate declines to zero in finite time, our formula implies that when a portion of emissions remains forever the near term carbon price is infinite. To avoid this possibility, we instead take the IPCC description literally and assume that the fraction ϕ_0 remains in the atmosphere for 2000 years.

Our second modification is to assume that $1 - d_0 = 0$, meaning that current emissions do not effect damages until after a one period delay. Golosov et al. (2011) assume the same thing in the their robustness section when they modify their calibration to more closely replicate the carbon price in Nordhaus (2008). However, in their main calibration, they assume $1-d_0 = 1$. Gerlagh and Liski (2012) argue that the implied time lag between emissions and damages implied by this assumption is inconsistent with most integrated assessment models[27], including DICE (Nordhaus 2008).

6.3 Results

The results are shown in table 1. We report the optimal tax under each discounting scenario alongside the implied Pigouvian tax in the corresponding no commitment equilibrium. The results are roughly in line with prior estimates in the literature. For example, Nordhaus and Boyer (2000) apply the declining discount rate scheme suggested by the UK HM Treasury (2003) Green Book[28]. Applied in DICE, this scheme roughly doubles the carbon price compared to their baseline calibration. In our analysis, the optimal tax doubles under the 5% Stern scenario, and it increases by 65% under Weitzman's (2001) gamma discounting[29].

[27]Part of the problem is that the Golosov et al. (2011) model does not include temperature inertia, so stocks induce damages instantaneously, while more complicated IAMs account for the time lag induced by temperature inertia.

[28]This is the most significant use of declining social discount rates in actual policy making. The implied rates are motivated by the uncertainty-based logic in Newell and Pizer (2003). The recommended discount rates follow a step function that begins at 3.5% for the first 30 years. It then declines in increments until 300 years, after which it remains constant at 1%.

[29]As noted, our use of Weitzman's (2001) results are conservative since we treat his estimated consumption discount factors as pure rates of time preference.

	Optimal tax	Pigouvian tax
Nordhaus (2008)	23	23
1% Stern	28	24
5% Stern	46	25
10% Stern	69	27
Stern (2007)	442	442
Weitzman (2001)	38	20

Table 1: The optimal carbon tax is computed using fomula (3.1). The Pigouvian tax is the present value marginal externality cost at the equilibrium interest rate. Units are dollars per ton carbon. Results reported for the hyperbolic discounting paths in figure 1 along with two constant paths. "Nordhaus (2008)" assumes a constant PRTP of 3.0%; "Stern (2007)" assumes a constant PRTP of 0.1%.

When the PRTP is constant, the optimal tax and the Pigouvian tax are equal. This is consistent with the finding in Golosov et al. (2011) that the optimal tax equals the marginal externality cost discounted at the equilibrium interest rate. When the PRTP declines, the optimal tax is bigger than the Pigouvian tax, reflecting the commitment value of climate policy in the model. The magnitude of the wedge increases with the rate of decline in the path of discount rates; thus, the wedge is small for the 1% Stern scenario, while under "Weitzman (2001)" the optimal tax is almost double the Pigouvian tax.

Another feature of the results is that the Pigouvian taxes are similar in magnitude for the four DRTP scenarios, even as the optimal taxes differ by a lot. The explanation for this can be seen by reexamining equation (5.4). As indicated, the equation shows that ϕ_t is bigger than β_1 for all t. But the magnitude of this effect is small. As a result, to fairly close approximation, $\phi_t \approx \beta_1$, for all t. The finding is not too surprising since β_1 reflects the subjective discount rate that the decision maker who controls policy in period t would like to have prevail between t and $t+1$. Because of this, the Pigouvian taxes under the Stern-Nordhaus scenarios are close to the optimal tax using the PRTP from Nordhaus (2008), while the Pigouvian tax under Weitzman (2001), where β_1 is calculated using a declining PRTP path that starts at 4.0%, is somewhat smaller.

6.3.1 Evaluation of the quasi-hyperbolic approximation

One contribution of the paper is to derive a formula for the near-term carbon price that allows for an arbitrary path of time preference rates. Gerlagh and Liski (2011), using a similar model, derive a formula for the carbon price under quasi-hyperbolic discounting. In this section, we consider how important it might be to move beyond the quasi-hyperbolic approximation when evaluating a problem like climate change in which time lags span centuries.

Figure 2 shows two plausible quasi-hyperbolic approximations to the continuous "Weitzman (2001)" path. *Approximation A* applies a PRTP of 3.3% over the first decade, then 1.6% thereafter. *Approximation B* applies a PRTP of 2.4% over the first three decades, followed by 0.9% thereafter. Looking at the figure, both approximations appear plausible, and any attempt to choose among them would presumably be to a large degree arbitrary. Using formula (3.1) to calculate the carbon price under each approximation, we get a carbon price of $41 under *approximation A* and $56 under *approximation B*. So the carbon price under *approximation A* turns out to be quite close to the carbon price of $38 that we computed us-

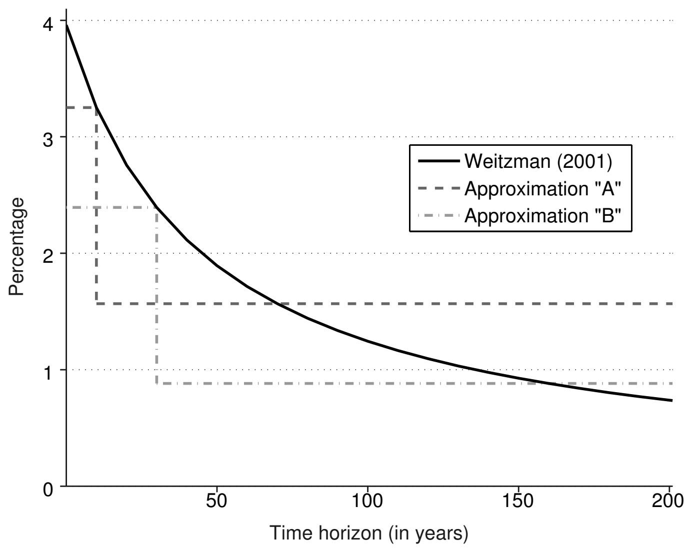

Figure 2: Comparison of alternative hyperbolic discounting paths.

ing the original "Weitzman (2001)" path. In contrast, the carbon price using *approximation B* is about 50% too high. The implication isn't that *approximation A* is necessarily better—whether it is or not will depend on the calibration—rather, it is that the approximation error that arises when the analysis is forced into the quasi-hyperbolic specification is quantitatively important.

7 Conclusion

The paper considers a somewhat simplified version of the dynamic stochastic general equilibrium climate policy model in Golsov et al. (2011). We derive a formula for the near-term carbon price when the pure rate of time preference is non-constant. We calibrate the climate model roughly in line with the calibration in Golosov et al. (2011). Starting with Nordhaus's (2008) baseline PRTP under log utility, we find that the effect of a 5% chance that the Stern (2007) discount rate is correct—or equivalently, the effect of a planner who aggregates preferences across a population in which 5% of citizens view the Stern rate as correct—is to double the carbon price relative to what it is when the PRTP is consistent with Nordhaus (2008).

The paper demonstrates two reasons why the considered model provides an extremely convenient setting in which to consider the implications of non-constant time preference rates for climate policy. First, time inconsistency concerns, which have bedeviled efforts to incorporate declining time preference rates into climate policy decision making, fall away completely. The optimal near term carbon price with commitment is the same as the optimal near term carbon price in the equilibrium without commitment, and it remains unchanged for any beliefs that the current generation might hold about the PRTP path (constant, declining or otherwise) to be applied by subsequent generations.

Second, the motivation for using a declining rate of time preference can be significantly strengthened in the considered model. An important criticism of most applications of both the uncertainty-based logic for a declining rate of time preference (Azfar 1999; Weitzman 2001) and the closely-related preference aggregation argument (Gollier and Zeckhauser 2005) is that they implausibly assume that current information is static. Weitzman (2001) anticipated this concern when he noted: "I think it suffices to think of [the gamma discounting formula] as defining a table of technocratic time-dependent weights, which give future-dollar equivalence values for making one-time irreversible decisions. The possibility of being able later to revisit and revise time-inconsistent investment choices introduces a set of complicated issues that are better treated separately." Using the model in this paper, the thorny issues that arise when Weitzman's (2001) formulation is embedded in a fully dynamic setting are resolved in a very simple way: because future changes in the distribution of beliefs (or the distribution of preferences) do not affect optimal policy today, they can be ignored when computing the near term carbon price.

References

Ainslie, G. (1991): "Derivation of "rational" economic behavior from hyperbolic discount curves," *The American Economic Review*, 334–340.

Azfar, O. (1999): "Rationalizing hyperbolic discounting," *Journal of Economic Behavior & Organization*, 38, 245–252.

Barro, R. (1999): "Ramsey meets Laibson in the neoclassical growth model," *The Quarterly Journal of Economics*, 114, 1125–1152.

Beltratti, A., G. Chichilnisky, and G. Heal (1998): "Preservation, uncertain future preferences and irreversibility. In: Sustainability: Dynamics and uncertainty, editors Chichilnisky, G. and Heal, G. and Vercelli, A." *Kluwer Academic Publishers*.

Brock, W. and L. Mirman (1972): "Optimal economic growth and uncertainty: The discounted case," *Journal of Economic Theory*, 4, 479–513.

Chichilnisky, G. (1996): "An axiomatic approach to sustainable development," *Social choice and welfare*, 13, 231–257.

Cropper, M., S. Aydede, and P. Portney (1994): "Preferences for life saving programs: How the public discounts time and age," *Journal of Risk and Uncertainty*, 8, 243–265.

Fishburn, P. (1970): *Utility theory for decision making*, Wiley.

Fishburn, P. and A. Rubinstein (1982): "Time preference," *International Economic Review*, 23, 677–694.

Frederick, S., G. Loewenstein, and T. O'Donoghue (2002): "Time discounting and time preference: A critical review," *Journal of Economic Literature*, 40, 351–401.

Fujii, T. and L. Karp (2008): "Numerical analysis of non-constant pure rate of time preference: A model of climate policy," *Journal of Environmental Economics and Management*, 56, 83–101.

Gerlagh, R. and M. Liski (2012): "Carbon prices for the next thousand years," *CESifo Working Paper Series*.

Goldman, S. (1980): "Consistent plans," *The Review of Economic Studies*, 533–537.

Gollier, C. and R. Zeckhauser (2005): "Aggregation of heterogeneous time preferences," *Journal of Political Economy*, 113, 878–896.

Golosov, M., J. Hassler, P. Krusell, and A. Tsyvinski (2011): "Optimal taxes on fossil fuel in general equilibrium," *NBER Working Paper 17348*.

Guo, J., C. Hepburn, R. Tol, and D. Anthoff (2006): "Discounting and the social cost of carbon: a closer look at uncertainty," *Environmental Science & Policy*, 9, 205–216.

Harris, C. and D. Laibson (2004): "Dynamic choices of hyperbolic consumers," *Econometrica*, 69, 935–957.

Heal, G. (2005): "Intertemporal welfare economics and the environment," *Handbook of Environmental Economics*, 3, 1105–1145.

Karp, L. (2005): "Global warming and hyperbolic discounting," *Journal of Public Economics*, 89, 261–282.

——— (2007): "Non-constant discounting in continuous time," *Journal of Economic Theory*, 132, 557–568.

Karp, L. and Y. Tsur (2011): "Time perspective and climate change policy," *Journal of Environmental Economics and Management*, 62, 1–14.

Kirby, K. and R. Herrnstein (1995): "Preference reversals due to myopic discounting of delayed reward," *Psychological Science*, 6, 83–89.

Koopmans, T. (1960): "Stationary ordinal utility and impatience," *Econometrica*, 287–309.

Laibson, D. (1997): "Golden eggs and hyperbolic discounting," *The Quarterly Journal of Economics*, 112, 443–478.

Ljungqvist, L. and T. Sargent (2004): *Recursive macroeconomic theory*, MIT Press.

Loewenstein, G. (1987): "Anticipation and the valuation of delayed consumption," *The Economic Journal*, 666–684.

Mastrandrea, M. and S. Schneider (2001): "Integrated assessment of abrupt climatic changes," *Climate Policy*, 1, 433–449.

Nordhaus, W. (2011): "The economics of tail events with an application to climate change," *Review of Environmental Economics and Policy*, 5, 240–257.

Peleg, B. and M. Yaari (1973): "On the existence of a consistent course of action when tastes are changing," *The Review of Economic Studies*, 391–401.

Phelps, E. and R. Pollak (1968): "On second-best national saving and game-equilibrium growth," *The Review of Economic Studies*, 35, 185–199.

Pollak, R. (1968): "Consistent planning," *The Review of Economic Studies*, 35, 201–208.

Samuelson, P. (1937): "A note on measurement of utility," *The Review of Economic Studies*, 4, 155–161.

Stern, N. (2007): *The economics of climate change: The Stern review*, Cambridge University Press.

Strotz, R. (1955): "Myopia and inconsistency in dynamic utility maximization," *The Review of Economic Studies*, 23, 165–180.

Thaler, R. (1981): "Some empirical evidence on dynamic inconsistency," *Economics Letters*, 8, 201–207.

Weitzman, M. (2001): "Gamma discounting," *The American Economic Review*, 91, 260–271.

——— (2009): "On modeling and interpreting the economics of catastrophic climate change," *The Review of Economics and Statistics*, 91, 1–19.