# Skill Diversity and Leadership in Team Production 

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# Skill Diversity and Leadership in Team Production* 

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#### Abstract

Using a team-production model with heterogeneous workers, we examine the short- and long-run efficiency effects of skill diversity and leadership in teams. Our analysis focuses on workers' strategic incentives to manipulate their skills. In the short run, heterogeneous pairing (pairing workers with different skills) yields a greater total production than homogeneous pairing. However, in the long run, homogeneous paring may yield a greater total production because of gradual improvements in workers' skill. We also show new potential benefits of leadership: assigning a leader to a team yields a smaller total production in the short run, but, a greater production in the long run by preventing workers from consistently reducing their skills.


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Keywords: Team production; Voluntary provision of public goods; Leadership; Efficient role assignment; Team-building

## 1 Introduction

How to best organize workers in teams and assign leadership roles are important issues not only for managers in charge of team production, but also for the firm overall. One important objective of managers is to provide workers with the right incentives to dedicate considerable effort to team production. As is well known, workers may by tempted to free ride on other workers' efforts when rewards are based on team performance rather than individual performance. The other important objective of managers is to encourage workers to improve their skills and productivity. The company can enjoy long-term benefits in team production if managers successfully provide workers the right incentives to voluntarily improve their skills by establishing an appropriate team structure.

[^0]However, different methods of forming teams and assigning leadership roles may have different impacts on workers' incentives to free ride and upgrade their skills. This raises some practical questions about the desirable organizational structure of teamwork. Is a team composed of workers with similar level of productivity more encouraged to improve their productivity than a team composed of workers with varying productivity levels? How does the assignment of leadership roles affect team production on the short term and skill development on the long term? How do managers' contract terms affect their decisions regarding organizational structures?

To address the issues, this paper examines the optimal design of teams by employing a teamproduction model within a firm. We develop the simplest possible model in which a manager divides workers into teams of two, and team production is iterated many times between the same pair of workers. Workers choose their contribution to team production, and their skills in contributing change gradually over time. Therefore, the manager has short- and long-term objectives: to increase the total team production taking the workers' skill levels as given, and to provide workers with the right incentives to improve their skills. In other words, in the short run, the manager establish teams to maximize total production taking the workers' skill levels as given, and in the long run, he/she should consider the effect of changes in workers' skills on team production.

Within the above framework, we investigate how managers should pair workers with different productivities into teams and how they should assign leadership roles in order to achieve shortand long-term objectives. In particular, we consider two ways to pair up workers and two role assignments. One is homogeneous pairing-where the manager pairs up workers with the same skills-and the other is heterogeneous pairing-where the manager pairs up workers with different skills. The two types of role assignment are as follows: horizontal teamwork, where workers in each team have no particular role, and vertical teamwork, where one worker is assigned a leadership role. Theoretically, the difference between horizontal and vertical teamwork is the different timing of the moves between workers in each pair: horizontal teamwork is modeled by the simultaneous-move (Nash) game of team production, whereas vertical teamwork is modeled by the sequential-move (Stackelberg) game.

We show that strategic incentives for free riding or manipulating skills depend on the initial skill gaps and workers' preference for rewards based on their team production. In a horizontal teamwork game, we find that both workers have no incentives to improve their skills (in fact, they even have wrong incentives to lower their skills) if the initial skill gaps between them and preferences for team rewards are small. Although a decrease in one worker's skills will diminish the overall efficiency in team production, it will also decrease the other worker's free-riding incentives. Therefore, if the latter effect dominates the former, both workers have strategic incentives to be less-skilled. On the other hand, both workers have the right incentives to improve their skills if the skill gaps are small and the preference for team rewards is large. We also find that there is a tendency that a relatively high-skilled worker has incentives to further increase his/her skills, but a relatively low-skilled worker has incentives to decrease his/her skills (or has no incentives to improve his/her skills) if the initial skill gaps between them are large. In other words, the initial skill gaps between workers in a pair may be widened, but cannot be
reduced over time.
Comparing homogeneous and heterogeneous pairings in horizontal teamwork, we find that, in the short-term perspective, heterogeneous pairing yields equal or greater total production than homogeneous pairing. However, when workers' preferences for team rewards are large, homogeneous pairing can provide all workers with the right incentives to improve their skills, while heterogeneous pairing may not. In that case, homogeneous pairing results in greater total production than heterogeneous pairing in the long term. On the other hand, when workers' preferences for team rewards are small, heterogeneous pairing yields greater total production in both short and long terms. This is because, in this case, all homogeneously paired workers persistently try to reduce their skills (or at least to not actively improve them) for strategic reasons.

We then find that in vertical teamwork game, it is impossible for all workers to have incentives to reduce their skills in equilibrium. This is in contrast to the case of horizontal teamwork. In particular, a leader never has incentives to reduce his/her skills, while a follower may have. In addition, in the short term, a greater or equal total production can be achieved with heterogeneous pairing compared to homogeneous pairing by by assigning a relatively low-skilled worker as a leader. As in the horizontal teamwork game, heterogeneous pairing is preferable for a manager in both short- and long-term perspectives when workers' preferences for team rewards are small.

Finally, comparing the equilibria between horizontal and vertical teamwork games, we find that horizontal teamwork yields greater production than vertical teamwork in both short and long term when workers' preferences for team rewards are large. However, when their preferences are small, managers may face tradeoffs between short- and long-term efficiency in team production; in the short run, horizontal teamwork leads to greater production. However, this is not necessarily the case in the long run because it may provide workers with the wrong incentives to continuously decrease their skills. Under vertical teamwork, there is no case where no one tries to improve their skills. Therefore, we can state that the benefit of having a leader is to prevent workers from persistently reducing their skills. In addition, the tradeoffs may distort the manager's decision in organizing teams, depending on the duration of his/her employment contract. In particular, if a manager's contract duration is relatively short term, he/she may employ horizontal teamwork to obtain greater short-term team performances, which may decrease long-term performances.

In the literature on the theory of the private provision of public goods, there is a well-known paradoxical result on skills in contributing public goods: players may have strategic incentives to reduce their own skills or to adopt inferior technologies in contributing public goods (Buchholz and Konrad 1994; Ihori 1996; Buchholz et al. 1998; Hattori 2005). The disadvantage of being more skilled is due to the fact that an increase in the contribution skills of one agent induces the other agents to contribute less. These strategic incentives also play a significant role in our study. However, our study differs from the previous studies in the following three important aspects. First, we consider a situation where agents (workers) have heterogeneous skills. This enables us to answer the question of what type of worker, high-skilled or low-skilled, has incentives
to improve (or reduce) their skills. Second, we consider the vertical structure (i.e., sequentialmove situation) as well as the horizontal structure (i.e., standard simultaneous-move situation) within a team. Through a comparison of these structures, we provide a new interpretation of the benefits of leadership. Finally, we consider the problem of how managers should organize heterogeneous workers into teams in order to increase the total production of the team (i.e., total provision of public goods).

Several theoretical studies have compared the outcome of simultaneous and sequential contribution games. In the standard model of private provision of a public good, Varian (1994) shows that a sequential-move contribution mechanism yields a smaller or equal amount of public goods than a simultaneous-move mechanism. However, when other factors are introduced, the resulting advantage of simultaneous-move over sequential-move mechanism with respect to total amount of public goods may not be robust. Romano and Yildirim (2001) show the advantage of a sequential-move mechanism in cases where players are motivated by "warm glow" or "snob appeal." Vesterlund (2003) and Andreoni (2006) also show the advantages of a sequentialmove mechanism when there is asymmetric information on the value of the public good. Our study considers the strategic incentives of players to improve (or reduce) skills and shows that a simultaneous-move mechanism-versus a sequential-move mechanism-has an advantage in the short run, but may not work in the long run.

This paper is also related to the literature on endogenous timing in a public good or a team production game. Kempf and Rota-Graziosi (2010) develop an endogenous timing model where two countries provide public goods, and show that the complementarity or substitutability of public goods is crucial for the endogenous emergence of leadership. In their study, if the public goods are substitutes (complements), then both players' contributions are strategic substitutes (complements), and thus simultaneous-move (sequential-move) equilibrium occurs. In our study, a leadership role is assigned to workers by a manager who does not directly engage in team production or providing public goods. Because a vertical teamwork (sequential-move) situation yields less team production (total provision of public goods) in the case where players' contributions are strategic substitutes, there seems to be no reason for the manager to assign a leadership role to some workers. However, as our results suggest, leadership assignment may result in a better outcome for the manager when we take into account workers' strategic incentives to manipulate their skills.

In order to focus on workers' strategic incentives to improve or reduce their skills and the resulting managerial implication in effective team building, we do not consider the issue of asymmetric information on the value of team output or uncertainty in the productivity of the team as in Hermalin (1998), Vesterlund (2003) and Huck and Rey-Biel (2006). ${ }^{1}$ Instead, we consider the benefits of leadership in a situation where workers with asymmetric skills can gradually change their skill levels.

[^1]This paper is organized as follows: Section 2 presents the basic structure of the model. Section 3 derives the equilibria of a horizontal teamwork game and analyzes workers' strategic incentives to improve/reduce their skills. Then, we compare equilibria in two types of teams: homogeneous or heterogeneous teams. Section 4 conducts the same analysis in the framework of a vertical teamwork game. Section 5 discusses the benefits of leadership by comparing horizontal and vertical teamwork. Section 6 discusses three possible extensions of the model and the relationship between the results and existing empirical and experimental studies. Section 7 concludes the paper.

## 2 Basic Structure of the Model

Our team-production model is an extended version of the standard model of private provision of public goods, such as the one proposed by Bergstrom et al. (1986). Consider a division in a firm that consists of a manager and $2 M$ number of workers. The workers are paired into $M$ teams of two and engage together in team production iteratively many rounds. ${ }^{2}$ In each round of team production, each worker $i(i \in\{1,2\})$ in team $k(k \in\{1, \cdots, M\})$ obtains utility from leisure time $\left(x_{k i}\right)$ and rewards $\left(G_{k}\right)$. The rewards, $G_{k}$, are assumed to be paid on the basis of team performance in each round (i.e., the rewards are public goods for members in a team). The team performance in team $k$ is given by

$$
G_{k}=\beta_{k i} g_{k i}+\beta_{k j} g_{k j} \quad j \neq i
$$

where $g_{k i}$ represents the effort contributed to team production expended by worker $i$ in team $k$ (which is measured in units of time), and $\beta_{k i}>0$ represents worker $k i$ 's productivity or contribution skill per unit of time. As described in detail later, workers have the opportunity to marginally improve or reduce their own contribution skills $\beta_{k i}$ in the intervals between each round of team production. The manager cannot observe an individual's effort, $g_{k i}$, but only the amount the team produces, $G_{k}$. Therefore, the rewards paid to each worker $i$ in team $k\left(G_{k}\right)$ are decided on the basis of team performance, and for simplicity, are assumed to be equal to $G_{k}$.

Each worker ki has a following Cobb-Douglas utility function:

$$
U_{k i}\left(x_{k i}, G_{k}\right)=(1-\alpha) \ln x_{k i}+\alpha \ln G_{k}
$$

where $\alpha \in(0,1)$ represents the preference for rewards compared to leisure time. In each round of team production, each worker has a unit of time endowment to be allocated between his/her effort to the team production, $g_{k i}$, and leisure, $x_{k i} \equiv 1-g_{k i}$.

The manager decides how to pair workers and whether to assign a leadership role to one of the two workers in each team, in order to increase the total production. In the short term, the effect of changes in workers' skills are negligible, so the manager's objective is to simply

[^2]

Figure 1: Structure of the game
maximize the total production of all teams in each time of production, $F \equiv \sum_{k=1}^{M} G_{k}$, given the workers' initial skills. However, in the long term, the manager has to care about the gradual changes in workers' skills because the changes affect the total production over time. Therefore, the manager's long-term objective is to increase total production over time by providing workers with the right incentives to improve their skills. We consider two ways for pairing the workers: workers with same skills can be paired-up or workers with different skills can be paired-up. We call the former homogeneous paring and the latter heterogeneous paring. In addition, we consider two types of role assignment: horizontal and vertical teamwork. In horizontal teamwork, workers have no particular role in a team, so they simultaneously decide how much effort to contribute to team production. ${ }^{3}$ In vertical teamwork, the manager assigns one of the two workers in each pair as a leader. Note that the leadership role here means solely moving first. Therefore, the situation of vertical teamwork is modeled by sequential contribution games.

Workers are assumed to have an opportunity to marginally increase or decrease their own skills between the production stages. We assume that workers' incentives to manipulate their skills are simply given by the sign of the marginal payoffs of the increase in skill. ${ }^{4}$ If a worker's marginal payoffs of improving skills is positive (negative), he/she has incentives to marginally improve (reduce) their skills and his/her skills are marginally increased (decreased) in the intervals between the production stages.

Figure 1 illustrates the structure of the game: First, anticipating the equilibrium behavior of workers in the subsequent stages, the manager decides how to pair up the workers (i.e., homogeneous or heterogeneous pairing) and whether to assign a leadership role to one of the two workers in each team (i.e., horizontal or vertical teamwork), in order to increase the sum of team production. Then, each worker non-cooperatively decides how much effort to contribute to team production. This contribution stage is iterated many rounds. ${ }^{5}$ In the intervals between

[^3]the contribution stages, workers' skill levels change gradually depending on their incentives.

## 3 Horizontal (Nash) Teamwork

### 3.1 Equilibrium in the contribution stage

In the horizontal teamwork game, each worker $k i$ simultaneously chooses his/her contribution, $g_{k i}$, to maximize his/her utility $U_{k i}$, taking the other worker's choice of $g_{k j}(j \neq i)$ as given (i.e., we are considering a Nash equilibrium). Solving the utility maximization, we obtain the reaction functions as follows:

$$
\begin{equation*}
g_{k i}=R\left(g_{k j}\right)=\max \left\{\alpha-\frac{(1-\alpha) \beta_{k j} g_{k j}}{\beta_{k i}}, 0\right\} \quad j \neq i . \tag{1}
\end{equation*}
$$

Because $R^{\prime}\left(g_{k j}\right) \leq 0$, the individual contributions to team production are strategic substitutes. Solving the above reaction functions, we derive the equilibrium contribution pair $\left(g_{k i}^{N}, g_{k j}^{N}\right)$ as

$$
\left(g_{k i}^{N}, g_{k j}^{N}\right)= \begin{cases}(0, \alpha) & \text { if } \frac{\beta_{k i}}{\beta_{k j}} \leq(1-\alpha) \\ \left(\frac{\beta_{k i}-(1-\alpha) \beta_{k j}}{(2-\alpha) \beta_{k i}}, \frac{\beta_{k j}-(1-\alpha) \beta_{k i}}{(2-\alpha) \beta_{k j}}\right) & \text { if }(1-\alpha)<\frac{\beta_{k i}}{\beta_{k j}}<\frac{1}{1-\alpha} \\ (\alpha, 0) & \text { if } \frac{\beta_{k i}}{\beta_{k j}} \geq \frac{1}{1-\alpha} .\end{cases}
$$

The above equations mean that if worker $i$ has much lower skills than worker $j$ (the case for $\beta_{k i} / \beta_{k j} \leq(1-\alpha)$ ), worker $i$ would be a free rider, and if worker $i$ has much higher skills than worker $j$ (the case for $\beta_{k i} / \beta_{k j} \geq 1 /(1-\alpha)$ ), worker $i$ would be the sole contributor. Hereafter we call these corner-solution equilibria full specialization. Otherwise (the case for $\left.(1-\alpha)<\beta_{k i} / \beta_{k j}<1 /(1-\alpha)\right)$, both workers contribute a positive amount of time in an equilibrium. We call these interior-solution equilibria collaboration.

The equilibrium team performance $G_{k}^{N}$ is given by

$$
G_{k}^{N}= \begin{cases}\alpha \beta_{k j} & \text { if worker } i \text { is a free rider } \\ \frac{2 \alpha \bar{\beta}_{k}}{2-\alpha} & \text { if both workers collaborate }\end{cases}
$$

where $\bar{\beta}_{k} \equiv\left(\beta_{k 1}+\beta_{k 2}\right) / 2$ is the average productivity in team $k$. Then, we obtain the equilibrium utilities as $U_{k i}^{N} \equiv U_{k i}\left(x_{k i}^{N}, G_{k}^{N}\right)$, where $x_{k i}^{N}=1-g_{k i}^{N}$.

[^4]
### 3.2 Strategic incentives to improve/reduce skills

We derive the following comparative static results:

$$
\frac{d U_{k i}^{N}}{d \beta_{k i}}= \begin{cases}0 & \text { if worker } i \text { is a free rider, } \\ \frac{2 \alpha \bar{\beta}_{k}-\beta_{k j}}{2 \beta_{k i} \bar{\beta}_{k}} & \text { if both workers collaborate } \\ \frac{\alpha}{\beta_{k i}} & \text { if worker } i \text { is a sole-contributor. }\end{cases}
$$

When $d U_{k i}^{N} / d \beta_{k i}>0$, i.e., worker $i$ 's marginal utility of increasing his/her skills is positive, worker $i$ has incentives to improve his/her skills. On the other hand, when $d U_{k i}^{N} / d \beta_{k i}<0$, worker $i$ has incentives to reduce his/her skills for strategic reasons. Obviously, a worker has no incentives to change his/her skill levels if he is a free rider, but has if he/she is a sole contributor. When both workers collaborate, it holds that

$$
\frac{d U_{k i}^{N}}{d \beta_{k i}} \gtreqless 0 \Leftrightarrow \frac{\beta_{k i}}{\beta_{k j}} \gtreqless \frac{1-\alpha}{\alpha} .
$$

Then, we have the following proposition.

## Proposition 1

In horizontal teamwork,
(i) in a full specialization equilibrium, a free rider has no incentives to change his/her skill levels, but a sole contributor always has incentives to improve his/her skills,
(ii) in a collaboration equilibrium, there are three possible cases: (a) both workers have incentives to reduce their skills if the skill gap between them and the preferences for rewards are small; (b) both workers have incentives to improve their skills if the skill gap is small and the preference for rewards is large; and (c) a relatively high-skilled worker has incentives to improve his/her skills, whereas a relatively low-skilled worker has incentives to reduce his/her skills if the skill gap is large.

This proposition is closely related to the result obtained by Ihori (1996). Within the standard framework of private provision of public goods, Ihori (1996) shows that each player may have strategic incentives to reduce his/her contribution skills. Our Proposition 1 clarifies that such results crucially depend on the preferences for public good (team rewards) and/or the skill gaps between players. Although a decrease in one worker's skills will diminish the overall efficiency in team production, the worker is better off by decreasing his/her skills because it induces the other worker to contribute more. Therefore, if the latter effect dominates the former, both workers have strategic incentives to be less skilled. ${ }^{6}$

[^5]

Figure 2: Marginal utility of changing productivity in horizontal teamwork case

Proposition 1 also suggests that there is a tendency that a relatively high-skilled worker has incentives to improve his/her skills, while a relatively low-skilled worker does not, but the reverse is not the case. The intuition of this is simple: a marginal increase in the skills of one worker increases his contributions, but-at the same time-it induces the other worker to decrease her contributions. When the skill level of the former worker is higher (lower) than that of the latter worker, this negative reaction has a small (large) impact on the former worker. Therefore, the relatively high-skilled worker has incentives to heighten his/her contribution skills, but the relatively low-skilled one does not.

Figure 2 depicts workers' incentives to change their skill levels. In the figure, $(\cdot, \cdot)$ represents the sign of both workers' marginal utility of changing skill levels (the first and second elements are the sign of $d U_{k 1}^{N} / d \beta_{k 1}$ and of $d U_{k 2}^{N} / d \beta_{k 2}$, respectively). Worker 1 is a sole contributor and worker 2 is a free rider in the region $(+, 0)$, and the opposite occurs in the region $(0,+)$. In these regions, a sole contributor will have incentives to improve his/her skills, but a free rider will not. In the region $(-,-)$, both workers have incentives to reduce their skills. In the region $(+,+)$, both workers have incentives to improve their skills. Note that when the skill gaps between workers are large, represented by regions $(+,-)$ and $(-,+)$, the relatively high-skilled worker has incentives to improve his/her skills, but the low-skilled worker has incentives to reduce his/her skills. The figure also indicates that if workers are symmetric in their skills (represented by the dotted line), then all of them have incentives to reduce their skills when $\alpha<0.5$ and to improve their skills when $\alpha>0.5$.

## Corollary 1

In horizontal teamwork, the equilibrium of the contribution stage eventually converges to full specialization if the initial skill gaps within teams are large. Both workers persistently reduce their contribution skills when $\alpha<0.5$ and improve them when $\alpha>0.5$ if the initial skills are
symmetric between workers.
Workers have an opportunity to marginally change their skill levels in the intervals between the contribution stages. In Figure 2, each arrow represents the direction of change in skills of two workers in a pair. The figure indicates that in the specialization case (that is, regions $(+, 0)$ and $(0,+))$, skill gaps between the two gradually widen over time. In the collaboration case-if the initial skill gaps are relatively large-the skill gaps widen over time, which eventually leads to full-specialization. However, if two workers are symmetric, then their skills persistently increase (decrease) over time when $\alpha$ is greater (smaller) than 0.5. These properties are crucial for the manager's problem of effective team building.

### 3.3 Manager's problem: homogeneous vs. heterogeneous paring

Here we consider the manager's problem of pairing workers. As previously mentioned, the manager's objectives are twofold: to increase the sum of team production and to encourage workers to improve their skills. Assume that there are two types of workers: one half $(M)$ are high-skilled workers, who have a skill parameter $\beta_{h}$, and the other half $(M)$ are low-skilled workers, who have $\beta_{l}\left(\beta_{h}>\beta_{l}\right)$. This assumption of half-and-half distribution of high- and low-skilled workers is made for simplicity only, and-as shown later-all of our results also hold for any distribution of high- and low-skilled workers. There are two ways of pairing workers: homogeneous pairing, where workers of the same type are paired into a team, and heterogeneous pairing, where workers of different types are paired.

When a manager employs homogeneous pairing, there are $M / 2$ high-skilled pairs (consisting of high-skilled workers) and $M / 2$ low-skilled pairs (consisting of low-skilled workers). In this case, there is no possibility that a full-specialization equilibrium occurs in every team. Then, the team production of each high-skilled and low-skilled team is, respectively, given by

$$
G_{h h}^{N}=\frac{2 \alpha \beta_{h}}{2-\alpha}, \quad G_{l l}^{N}=\frac{2 \alpha \beta_{l}}{2-\alpha},
$$

where the subscript $h h(l l)$ refers to the variables of high-skilled (low-skilled) pairs. Therefore, the sum of the teams' production, $F_{\text {homo }}^{N}$, is given by

$$
F_{\text {homo }}^{N}=\frac{M}{2} G_{h h}^{N}+\frac{M}{2} G_{l l}^{N}=\frac{\alpha\left(\beta_{h}+\beta_{l}\right) M}{2-\alpha} .
$$

When the manager employs heterogeneous pairing, there are $M$ symmetric teams (that consist of high- and low-skilled workers). In this case, either a collaboration or full-specialization equilibrium may occur depending on the skill gaps between $\beta_{h}$ and $\beta_{l}$. Because the relatively low-skilled worker will become a free rider in the specialization case, we obtain each team's performances as

$$
G_{h l}^{N}= \begin{cases}\frac{\alpha\left(\beta_{h}+\beta_{l}\right)}{2-\alpha} & \text { if both workers collaborate } \\ \alpha \beta_{h} & \text { if low-skilled worker is a free rider }\end{cases}
$$

and the sum of teams' production under heterogeneous pairing is $F_{\text {hetero }}^{N}=M \times G_{h l}^{N}$. Therefore, we find that $F_{\text {homo }}^{N}=F_{\text {hetero }}^{N}$ if both workers collaborate in heterogeneous pairing, and $F_{\text {homo }}^{N}<$
$F_{\text {hetero }}^{N}$ if full specialization occurs in heterogeneous pairing. In other words, from a short-term standpoint (i.e., given that the workers' skills are constant), the company's performance is equal or greater when heterogenous pairing rather than homogeneous pairing is employed. However, from a long-term standpoint (i.e., considering gradual changes in workers' skills), homogeneous pairing may provide the right incentives to improve their skills and thereby yield a greater total production. ${ }^{7}$

Combining the above with Proposition 1, we have the following proposition:

## Proposition 2

In horizontal teamwork, the sum of team production under heterogeneous pairing is equal or greater than homogeneous pairing in the short run. However, heterogeneous pairing may not provide workers the right incentives to improve their skills when the preferences for team rewards are large ( $\alpha>0.5$ ). In this case, homogeneous pairing may yield greater performance in the long run. On the other hand, when the preferences for team rewards are small ( $\alpha \leq 0.5$ ), homogeneous pairing provides workers with the wrong incentives to decrease their skills. In that case, heterogeneous pairing yields greater performance in both short and long run.

The proposition is easily understood from Figure 2, which indicates that homogeneous pairing results in persistent decreases (increases) in skills for both high- and low-skilled pairs if $\alpha<0.5$ $(\alpha>0.5)$. On the other hand, heterogeneous pairing results in full specialization when the skill gaps are large. In that case, heterogeneous pairing only encourages the sole contributor in a team to improve his/her skills. Therefore, homogeneous pairing can encourage workers to improve their skills more (less) than the heterogeneous team if the workers' preferences for team rewards are large (small). Therefore, from a long-term standpoint, the company's performance in the case of homogeneous pairing may be greater than that in the case of heterogeneous pairing when $\alpha$ is large.

## 4 Vertical (Stackelberg) teamwork

This section considers vertical teamwork, where the manager assigns one worker in each pair as a leader (i.e., one worker acts as a Stackelberg leader and the other acts as a follower in each pair). As shown in Figure 1, in a contribution stage, the leader of each pair chooses his/her effort to put into team production first, and then the follower observes the decision of the leader and chooses his/her effort.

[^6]
### 4.1 Equilibrium in the contribution stage

In case of vertical teamwork, each worker in team $k$ is indexed by $k i(k \in\{1, \cdots, M\}$ and $i \in\{L, F\}$ ), where $L$ represents a leader and $F$ a follower. By backward induction, we first solve worker $k F$ 's problem, given worker $k L$ 's choices. Worker F's reaction function is the same as (1), so worker L chooses $g_{k L}$ so as to maximize $U_{k L}$, anticipating the follower's reaction. Solving the problem, we derive the equilibrium contribution pair in team $k$ as

$$
\left(g_{k L}^{S}, g_{k F}^{S}\right)= \begin{cases}(0, \alpha) & \text { if } \frac{\beta_{k L}}{\beta_{k F}} \leq \frac{1-\alpha}{\alpha} \\ \left(\alpha-(1-\alpha) \frac{\beta_{k F}}{\beta_{k L}}, 1-\alpha+\alpha^{2}-\alpha(1-\alpha) \frac{\beta_{k L}}{\beta_{k F}}\right) & \text { if } \frac{1-\alpha}{\alpha}<\frac{\beta_{k L}}{\beta_{k F}}<\frac{1-\alpha+\alpha^{2}}{\alpha(1-\alpha)} \\ (\alpha, 0) & \text { if } \frac{\beta_{k L}}{\beta_{k F}} \geq \frac{1-\alpha+\alpha^{2}}{\alpha(1-\alpha)} .\end{cases}
$$

We find that, compared to the horizontal teamwork case, a leader is more likely to become a free rider, which is consistent with the results shown in Varian (1994): a leader can enjoy a strategic advantage by moving first. Then, the equilibrium team production of team $k$ can be obtained as

$$
G_{k}^{S}= \begin{cases}\alpha \beta_{k j} & \text { if worker } i \text { is a free rider } \\ \alpha^{2}\left(\beta_{k F}+\beta_{k L}\right) & \text { if both workers collaborate }\end{cases}
$$

which indicates that the team performances in case of full specialization are identical between horizontal and vertical teamworks, but differ in the collaboration cases. We then obtain the equilibrium utilities for worker $k i$ as $U_{k i}^{S} \equiv U_{k i}\left(x_{k i}^{S}, G^{S}\right)$, where $x_{k i}^{S}=1-g_{k i}^{S}$.

### 4.2 Strategic incentives to improve/reduce skills

Then, we derive the following comparative static results:

$$
\frac{d U_{k i}^{S}}{d \beta_{k i}}= \begin{cases}0 & \text { if worker } i \text { is a free rider, } \\ \frac{2 \alpha \overline{\beta_{k}}-\beta_{k j}}{2 \beta_{k i} \bar{\beta}} & \text { if both workers collaborate } \\ \frac{\alpha}{\beta_{k i}} & \text { if worker } i \text { is a sole-contributor. }\end{cases}
$$

We have the following proposition.

## Proposition 3

In vertical teamwork,
(i) in a full-specialization equilibrium, a free rider has no incentives to change his/her skill levels, while a sole-contributor always has incentives to improve his/her skills,
(ii) in a collaboration equilibrium, (a) a leader always has incentives to improve his/her skills and (b) a follower has incentives to improve his/her skills when $\beta_{F}>\beta_{L}$, and incentives to reduce his/her skills when $\beta_{F}<\beta_{L}$ and $\alpha$ is small.


Figure 3: Marginal utility of changing productivity in vertical teamwork case

The proposition states that a follower may have incentives to reduce his/her skills, but a leader never has such incentives in equilibrium. Especially in a collaboration equilibrium, worker $L$ always gains by an improvement in his/her skills. The intuition is simple: a worker is better off by lowering his/her skills in a horizontal teamwork game because it will induce the other worker to contribute more, but will diminish the overall efficiency of team production. In contrast, in a vertical teamwork game, a leader can determine a follower's response (contribution) by committing his/her own contribution first, without sacrificing the overall production efficiency. Therefore, a leader has no incentives to reduce his/her contribution skills.

Figure 3 depicts the direction of a workers' change in skills in vertical teamwork. In the figure, $(\cdot, \cdot)$ represents the sign of both workers' marginal utility of changing skill levels (the first and second element is the sign of $d U_{k L}^{N} / d \beta_{k L}$ and of $d U_{k F}^{N} / d \beta_{k F}$, respectively). From Proposition 3 and the figure, we obtain the following corollary.

## Corollary 2

In vertical teamwork, the equilibrium of the contribution stage eventually converges to fullspecialization situations when $\alpha<0.5$ from any initial skill configuration between workers. There is no case where both workers in a pair persistently reduce their skills.

In comparing Figure 3 with Figure 2, two points should be emphasized in the vertical teamwork game. First, the figure shows that there is no region in which both workers have incentives to reduce their skills. Second, it is possible that the skill gap between workers shrinks when the relatively low-skilled worker is appointed as a follower and becomes the sole contributor, which is represented by the area above $\beta_{L} / \beta_{F}=1$ of $(0,+)$ region in the figure. In this case, only the low-skilled follower has incentives to improve his/her skills, and thereby the skill gap shrinks in an early phase and is reversed afterward. These points are quite in contrast to the results
obtained in the horizontal teamwork case.

### 4.3 Manager's problem: homogeneous vs. heterogeneous paring

We consider a manager's problem of designing teams. As in the horizontal teamwork game, we assume, for simplicity, that there are half $(M)$ high-skilled workers, who have $\beta_{h}$, and half $(M)$ low-skilled workers, who have $\beta_{l}\left(\beta_{h}>\beta_{l}\right)$. The manager decides whether to employ homogeneous or heterogeneous pairing. In the heterogeneous pairing case, the manager also assign one worker in each pair as a leader.

When the manager employs homogeneous pairing, full specialization occurs when $\alpha \leq 0.5$ and a collaboration occurs when $\alpha>0.5$, as shown in Figure 3. The team production in the pair of high-skilled workers $G_{h h}^{S}$ is given by

$$
G_{h h}^{S}= \begin{cases}\alpha \beta_{h} & \text { for } \alpha \leq 0.5 \\ 2 \alpha^{2} \beta_{h} & \text { for } \alpha>0.5\end{cases}
$$

and the team production in the pair of low-skilled workers $G_{l l}^{S}$ is given by

$$
G_{l l}^{S}= \begin{cases}\alpha \beta_{l} & \text { for } \alpha \leq 0.5 \\ 2 \alpha^{2} \beta_{l} & \text { for } \alpha>0.5\end{cases}
$$

Therefore, the sum of team production in case of homogeneous pairing, $F_{h o m o}^{S} \equiv(M / 2)\left(G_{h h}^{S}+\right.$ $\left.G_{l l}^{S}\right)$, is given by

$$
F_{\text {homo }}^{S}= \begin{cases}\alpha \bar{\beta} M & \text { for } \alpha \leq 0.5 \\ 2 \alpha^{2} \bar{\beta} M & \text { for } \alpha>0.5\end{cases}
$$

where $\bar{\beta} \equiv\left(\beta_{h}+\beta_{l}\right) / 2$ is the average skills in all workers.
When the manager employs heterogeneous pairing, either a collaboration or full-specialization situation may occur depending on whether the high-skilled or low-skilled worker is assigned to be a leader. First, we consider the $\alpha \leq 0.5$ case. Figure 3 indicates that the manager may yield the full-specialization situation by assigning the low-skilled worker as a leader. In this case, the sum of team production is given by $\alpha \beta_{h} M$. On the other hand, if the manager assigns the highskilled worker as a leader, then either a collaboration or a full-specialization situation occurs, and the sum of team production is given by $2 \alpha^{2} \bar{\beta} M$ (collaboration) or $\alpha \beta_{l} M$ (full specialization). Because $\alpha \beta_{h} M>2 \alpha^{2} \bar{\beta} M$ and $\alpha \beta_{h} M>\alpha \beta_{l} M$ hold for $\alpha \leq 0.5$, we find that the manager prefers full specialization, where a low-skilled worker is assigned as a leader, rather than a collaboration or full-specialization situation, where a high-skilled worker is assigned as a leader.

Second, we consider the $\alpha>0.5$ case. Figure 3 indicates that when the skill gap between the high- and low-skilled workers in a team is small, no matter which worker the manager assigns as a leader, the collaboration situation occurs, yielding $G_{h l}^{S}=2 \alpha^{2} \bar{\beta}$. When the skill gaps are large enough, the manager can obtain either the full-specialization situation and $G_{h l}^{S}=\alpha \beta_{h}$ by assigning the low-skilled worker as a leader, or the collaboration situation and $G_{h l}^{S}=2 \alpha^{2} \bar{\beta}$ by assigning the high-skilled worker as a leader. Because the former is necessarily greater than the
latter in this case, the manager prefers the full-specialization situation when the skill gaps are large enough.

In summary, the sum of team production in the case of vertical and heterogeneous pairing is

$$
F_{\text {hetero }}^{S}= \begin{cases}\alpha \beta_{h} M & \text { for } \alpha \leq 0.5 \\ 2 \alpha^{2} \bar{\beta} M & \text { for } \alpha>0.5 \text { and skill gaps are small (collaboration) } \\ \alpha \beta_{h} M & \text { for } \alpha>0.5 \text { and skill gaps are large (full specialization) }\end{cases}
$$

Therefore, we find that $F_{\text {homo }}^{S}<F_{\text {hetero }}^{S}$ when $\alpha \leq 0.5, F_{\text {homo }}^{S}<F_{\text {hetero }}^{S}$ when $\alpha>0.5$ and skill gaps are large, and $F_{\text {homo }}^{S}=F_{\text {hetero }}^{S}$ when $\alpha>0.5$ and skill gaps are small. ${ }^{8}$

## Proposition 4

In vertical teamwork, the sum of team production under heterogeneous pairing in which a lowskilled worker is assigned as a leader is equal or greater than homogeneous pairing. However, heterogeneous pairing may not provide workers with the right incentives to improve their skills when the preferences for rewards are large ( $\alpha>0.5$ ). In this case, homogeneous pairing may yield a greater performance in the long run. When the preferences for team rewards are small ( $\alpha \leq 0.5$ ), both paring methods provide workers with the incentives to change their skill levels in the same direction, and therefore, heterogeneous pairing yields a greater performance in both short and long run.

Homogeneous pairing provides all workers with the right incentives to improve their skills when the preference for team rewards is large. On the other hand, heterogeneous pairing provides only one worker in each team the right incentives to improve his/her skills when there are large skill gaps in the pairs. Therefore, in this case, homogeneous pairing has an advantage over heterogeneous pairing from a long-term standpoint.

## 5 Why are Leaders Necessary?

This section compares horizontal teamwork with vertical teamwork and considers the benefits of leaders for both managers and the firm.

Comparing $G^{N}$ with $G^{S}$, we find that $G^{N}>G^{S}$ holds under the parameter values where workers collaborate in horizontal teamwork and $G^{N}=G^{S}$ holds under the parameter values where full specialization occurs in horizontal teamwork. This result is consistent with that shown in Varian (1994, Corollary, p.179). This is because each worker's contributions to team production are strategic substitutes, and the leader's contributions in the sequential game are never larger than those made in the simultaneous contribution game. Therefore, in the short run, the horizontal teamwork is preferable for the manager and generates a greater sum of team production.

However, as shown in the previous sections, horizontal teamwork in the long run does not necessarily yield a greater team performance if we consider workers' strategic incentives to

[^7]improve or decrease their skills. In particular, horizontal teamwork provides both workers the incentives to persistently reduce their skills when their preference parameters for team rewards are small (represented by $(-,-)$ region in Figure 2); in contrast, such a dismal situation never happens in vertical teamwork. Therefore, we have the following proposition.

## Proposition 5

When the preferences for rewards are large ( $\alpha>0.5$ ), both in the short and long run, horizontal teamwork yields a greater sum of team production than vertical teamwork. When the preferences for rewards are small ( $\alpha \leq 0.5$ ), horizontal teamwork initially yields a greater team production, but in the long run, vertical teamwork may provide a greater sum of team production through gradual changes in workers' skill levels.

The proposition implies a trade-offs between the short- and long-term performances in team production: In the short run, horizontal teamwork leads to greater production than vertical teamwork, but is not necessarily the case in the long run, because all workers may consistently try to reduce their skills in horizontal teamwork. The result suggests a potential benefit of leadership in team production: Leadership can prevent workers from persistently decreasing their skills. Furthermore, the tradeoffs may distort a manager's team assignment decisions, depending on the duration of his/her employment contract. When the manager's contract terms are relatively short term, he/she may employ horizontal pairing to have greater short-team gains, disregarding the long-term effect of workers' strategic incentives to reduce their skills on team performances.

## 6 Discussion

In this section, we briefly discuss three possible extensions of the basic model. Then, we discuss the contributions of our analysis, especially in the relationship between our theoretical results and existing empirical and experimental works.

First, the future possibility of team reorganization and workers' rational expectations over the possibility will affect the present incentives of workers to manipulate their skills. Consider a situation where a manager divides workers into homogeneous pairs without a leader. Proposition 1 indicates that all workers will have incentives to reduce their skills when the preferences for team rewards are small. However, if workers take the future regrouping of teams into account, they may in fact have incentives to improve their skills, because although it may decrease their present payoffs, it will raise future payoffs by increasing the chance of being paired with highskilled worker. In contrast, under a heterogeneous pairing system, rational workers may reduce their skills (or may not improve their skills) in order to increase the chance of being paired with high-skilled worker in the future. Therefore, in this sense, homogeneous pairing may provide workers with more incentives to improve skills than heterogeneous pairing. A similar scenario applies when comparing horizontal and vertical teamwork. As Proposition 4 indicates, in a case where a manager employs vertical teamwork, he can attain maximum total production by assigning a relatively low-skilled worker as a leader. If workers expect the present team to
be regrouped in the future, they may strategically reduce their contribution skills to obtain the position of a leader in the future and enjoy the first-mover advantages. In other words, leadership assignment for increasing short-term production may not be suitable for providing workers with the right incentives to upgrade skills.

Second, our results on the benefits of leadership also hold for the case where more than two workers are required to engage in team production. In the case where workers are divided into teams of more then two, each worker is more likely to have incentives to decrease his/her contribution skills, as Ihori (1996) shows. This is because reducing skills has little impact on overall production efficiency in team production, but has a greater positive spillover effect from increases in the rest of the teammates' contributions. In other words, in a horizontal teamwork case, the result that all workers have incentives to reduce their skills is more likely to occur if the number of workers in a team is large. In addition, in a vertical teamwork case in which one worker is a leader and the rest are followers, the leader never has incentives to reduce his/her skills because he/she can determine followers' responses (contributions) by committing his/her contributions in advance. Therefore, the existence of a leader is more likely to prevent workers from persistently lowering their skills if the number of team members is large.

Third, we focus on a fully noncooperative game, thereby excluding the possibility of some forms of cooperation within pairs. Do workers have strategic incentives to reduce their contribution skills when workers cooperate in team production? Consider a situation where cooperation (or a joint payoff-maximizing outcome) between workers in each pair is sustained by, for example, trigger strategies in a repeated game setting, as in Pecorino (1999). In this case, increasing skills, which is given before the supergame of contribution game, may hinder cooperation by increasing the other worker's incentives to deviate from the joint payoff-maximizing outcome. This is because an increase in one worker's skills necessarily increases the other worker's payoff in a one shot noncooperative game. Next, consider a cooperative situation of Nash bargaining between two workers. Clearly, an increase in one worker's contribution skills enlarges the joint-utility possibility frontier, which benefits both workers. However, as shown in this paper, workers are worse off by increasing their contribution skills under non-cooperative situations, which means that it may decrease the payoffs in the disagreement (or threat) point of the Nash bargaining problem. Therefore, we conjecture that workers may still have strategic incentives to reduce their skills even if they cooperate with each other.

Our study relates to two strands of literature: (i) the comparison between sequential and simultaneous-move in the voluntary provision of public goods, (ii) the efficiency effect of leadership and heterogeneous/homogeneous grouping in team production.

First, several papers on voluntary provision of public goods theoretically compare equilibria with sequential and simultaneous moves, as we did in this paper. Varian (1994) shows that if one agent can commit first (a sequential-move game), the leader can free ride on the follower's effort and that the total amount of public goods will be smaller than in the non-commitment case (a simultaneous-move game). Gächter et al. (2010) experimentally confirm that the total
contribution to public goods is lower under sequential than under simultaneous contributions. ${ }^{9}$ However, they find that leaders do not attain their first-mover advantage, which is not consistent with the theoretical predicted. Their laboratory experiment does not consider the long-term effect of changes in players' skills because the contributions by the subject (player) are monetary and the contribution stages are iterated at most 15 rounds in their experiment. Although the mechanism differs, our theoretical results also find the second mover's incentives to reduce his/her contribution skills in order to free ride on the first mover's effort: the followers can induce more effort from the leaders by committing to lower their productivity. ${ }^{10}$

Second, our comparison of homogeneous and heterogeneous pairing relates to the literature on the effect of diversity on team performance. Focusing on the complementarity in workers' skills, some studies investigate the effect of diversity in productivity on the performance of team production. ${ }^{11}$ For example, examining detailed company data in a garment factory in Napa, California, Hamilton et al. $(2003,2012)$ find evidence that more heterogeneous teams are more productive than the teams of the same ability, with the average ability held constant. Our Proposition 2 also shows that if $\alpha$ is small, heterogeneous teams yield a greater outputs than homogenous teams in both short- and long-run perspectives. Furthermore, Hamilton et al. (2003, p.493) show that "higher-ability workers appear to improve team productivity more than low-ability workers do" in the sense that "an increase in the productivity of the most able team member has a significantly larger impact on team productivity than an equivalent improvement in the ability of the least able member." Our study-showing that there is a tendency for the relatively high-skilled (low-skilled) worker to improve (reduce) his/her skillmay be consistent with their finding. Hamilton et al. (2003) conclude that the reason why teams with a greater spread in ability are more productive can be attributed to a complementarity in production among workforces. In particular, they indicate that the worker complementarity within teams comes from bargaining in team behavior, where high-ability workers can impose a higher norm of team output, and from mutual learning, where more able workers teach their

[^8]less-able colleagues to be more productive. Note that our results are consistent with their empirical findings-although we do not include worker complementarity in production, but focus on workers' free-riding incentives and strategic manipulation of their skills.

## 7 Conclusion

Facilitating productive teamwork between employees is one of the keys to success in any business environment. In this paper, using a simple model of team production with heterogeneous workers, we examine the short- and long-run efficiency effects of skill diversity and leadership on team production.

The results obtained in this paper are summarized as follows. First, in horizontal teamwork, where there is no leader assigned, the workers have the wrong (right) incentives to reduce (improve) their skills if the skill gaps are small and the preferences for team rewards are small (large). If the skill gaps are relatively large, there is a tendency for the relatively high-skilled (low-skilled) worker to improve (reduce) his/her skill. In contrast, in vertical teamwork, where one worker is assigned a leadership role, the leader never has incentives to reduce his/her skills.

Second, in both horizontal and vertical teamwork, heterogeneous pairing yields greater or equal total production than homogeneous pairing in the short term. On the other hand, in the long term, homogeneous pairing may yield a greater total production because of the gradual improvement in workers' skills, depending on the initial skill gaps and workers' preference for team rewards.

Finally, by comparing horizontal teamwork performance with vertical teamwork performance, we find that a manager may face tradeoffs between short- and long-term team productions with regard to the assignment of a leader. In the short run, horizontal teamwork leads to a greater production, but in the long run, this is not necessarily the case because it may provide workers with the wrong incentives to persistently reduce their skills. Therefore, we offer a new explanation for the benefit of leadership in team production: the existence of a leader can prevent workers from continuously reducing their skills. We suggest that if a manager's contract duration is relatively short term, he/she may employ the horizontal teamwork to obtain greater short-term team performances, thereby negatively affecting the company's profits on the long term.

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[^1]:    ${ }^{1}$ Hermalin (1998) shows that if some workers are better informed of the value of team production than other workers, by acting as an example by exerting high effort, a well-informed leader can induce the team members to also exert high effort. For a comprehensive survey of the economic analyses of leadership, see also Hermalin (2013)

[^2]:    ${ }^{2}$ We assume a situation where the production of outputs or services requires pairs of workers as a fixed input, for example, double-handed sailing (sailing with two persons on board), assembling components in a factory, security guards, and a driver and a navigator in fieldworks. Each worker's contributions are assumed to be variable inputs for team production.

[^3]:    ${ }^{3}$ Note that in our model setting, there are no interactions between teams.
    ${ }^{4}$ In other words, we do not explicitly set up the stage where workers choose their skills and do not derive the Nash equilibrium of this stage. For simplicity, this paper does not consider endogenous determination of skills such as, for example, ex-ante investment or learning activities. However, if we take the ex-ante determination of skills into our model, our results do not change qualitatively. In addition, our assumption of a gradual change in skills comes from the fact that workers can change their amount of effort much easily than their contribution skills
    ${ }^{5}$ In this paper, we exclude the possibility of cooperation in infinitely repeated games by some history-dependent

[^4]:    strategies (e.g., trigger or tit-for-tat strategies).

[^5]:    ${ }^{6}$ Note that the strategic incentives of workers to decrease their skills result from the assumption that the rewards are public goods for team members. Suppose instead that the rewards are based on relative performances or contributions of team members (i.e., rewards are private goods). In this case, workers may have more incentives to increase their skills when the skill gaps between them decrease because of competition within the team. Proposition 1 indicates that workers may have more incentives to reduce their skills as the skill gaps decrease when the rewards are based on team performance.

[^6]:    ${ }^{7}$ All of our comparison results also hold for any distribution of high- and low-skilled workers (not only a fiftyfifty distribution). Suppose we have $2 a$ number of high-skilled and $2 b$ number of low-skilled workers in a department $(a>b)$. Then, homogeneous pairing leads to $a$ high-skilled pairs and $b$ low-skilled pairs: thereby, the total team production is $F_{h o m o}^{N}=a G_{h h}^{N}+b G_{l l}^{N}$. In contrast, heterogeneous pairing leads to $2 b$ mixed (heterogeneous) pairs and $a-b$ number of high-skilled pairs: thereby, the total team production is $F_{h e t e r o}^{N}=(a-b) G_{h h}^{N}+2 b G_{h l}^{N}$. Comparing them yields $F_{\text {homo }}^{N}-F_{h e t e r o}^{N}=b G_{h h}^{N}+b G_{l l}^{N}-2 b G_{h l}^{N}=2\left[\left(G_{h h}^{N}+G_{l l}^{N}\right) b / 2-b G_{h l}^{N}\right]$. Therefore, the comparison reduces to $\left(G_{h h}^{N}+G_{l l}^{N}\right) b / 2-b G_{h l}^{N}$, which is the same as the case of half-and-half distributions. The same argument holds in the reverse case, i.e., $a<b$.

[^7]:    ${ }^{8}$ As in footnote 7 , the results in Proposition 4 remain unchanged for any distribution of high- and low-skilled workers in a department.

[^8]:    ${ }^{9}$ Andreoni et al. (2002) also show that the total contributions in the simultaneous game are greater than those in the sequential game in the early round of the experiment, but the results are very similar by the end of the experiment. In contrast, Rivas and Sutter (2011) experimentally show that when each group member can volunteer to contribute before the other members (equivalently to be a leader), the total amount of contributions is greater than when there is no leader at all, i.e., all members contribute simultaneously.
    ${ }^{10}$ In the presence of asymmetric information or uncertainty about the quality of public good, some theoretical studies show the benefit of leadership in the provision of public goods (Hermalin 1998; Verstrlund 2003; Huck and Rey-Biel 2006). In such an environment of asymmetric information, a leader's action can convey relevant information to followers, leading to an efficiency gain in team production. For experimental studies, Potters et al. (2005, 2007) experimentally confirm the benefits of leadership (or those of sequential-move contribution) in such an environment. Although our theoretical model is based on full information, there are some benefits of having leadership in teams: the existence of a leader can prevent workers from consistently reducing their skills.
    ${ }^{11}$ Lazear (1998, 1999) suggests three conditions in which a diverse team can yield productivity gains: (i) the information or skill sets of workers must be disjoint, (ii) the information or skills must be relevant, and (iii) the costs of communication among team members must be insignificant to perform the relevant joint tasks. These requirements relate with complementarity in production among workers within a team. In contrast, the results of our comparison of homogeneous and heterogeneous teams do not come from the complementarities, but from workers' strategic motives to manipulate their skills.

