Measuring and Testing Tail Dependence and Contagion Risk between Major Stock Markets

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Abstract

In this paper, three copula GARCH models i.e. Gaussian, Student-t, and Clayton are used to estimate and test the tail dependence measured by Kendall's tau between six stock indices. Since the contagion risk spreads from large markets to small markets, the tail dependence is studied for smaller Taiwanese and South Korean stock markets, i.e. Taiex and Kospi against four larger stock markets, i.e. S&P500, Nikkei, MSCI China, and MSCI Europe. The vector autoregression result indicates that S&P500 and MSCI China indeed impact mostly and significantly to the other four stock markets. However, the tail dependence of both Taiex and Kospi against S&P500 and MSCI Chia are lower due to unilateral impacts from US and China. Using Clayton copula GARCH, the threshold tests of Kendall's tau between most stock markets except China are significant during both subprime and Greek debt crises. The tests of Student-t copula GARCH estimated Kendall's taus are only acceptable for subprime crisis but not for Greek debt crisis. Thus, Clayton copula GARCH is found appropriate to estimate Kendall's taus as tested by threshold regression.

Keywords: contagion risk, tail dependence, copula GARCH, threshold test

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1. Introduction

It is noted that the traditional measure of correlation is often used to measure the linear co-movement between the factors of risk. It lacks to describe the multivariate distribution between underlying assets. Also, the multivariate normal distribution only limits to a linear approach to describe the multivariate distribution and it cannot specify the multivariate dependence i.e. the structural dependence of marginal distribution either. To overcome the drawback of the linear multi-normal method, a new structural dependence called copula is used recently to describe the multivariate distribution and structural dependence in many financial respects including credit risk of bond portfolios, default risk of mortgages, and contagion risk of financial markets. The multi-dimensional distribution is constructed by simply combing individual marginal distributions and one proper copula according to Sklar's theorem (1959). As a consequence, the structural tail dependence can be estimated accordingly. In fact, the conditional tail dependence between global stock markets are essential to analyze the contagion risk, this paper hence develops a copula GARCH model to study the conditional structural tail dependence and a threshold regression to test its significance. The conditional tail dependence would expose to what extent a large shock of one stock market affects another one in certain context particularly when stock markets crash together.

The remainder of the paper proceeds as follows. In section two, the documents of risk related models are organized and discussed. In section three, the data sample, copula GARCH model, and tests of tail dependence are described in detail. In section four, the copula GARCH is estimated using moving windows technique to compute series of dynamic conditional correlations and Kendall’s taus. Then, the tests of Kendall’s taus are performed by threshold regression. The final section concludes the important remarks.

2. Literature Review

The contagion risk is studied by not only structural correlation but structural tail dependence among multivariate random processes. As shown by Embrechts et al. (2001), the

Pearson correlation is too restricted to describe the linear co-movements of two random processes. However, the copulas (e.g. Joe 1997, Nelson 1999) have the advantages to measure the conditional time-varying concordance and tail dependence and thus have been widely and successfully used to study the contagion risk.

It is noticed that the skewness Student-t but not the linear Gaussian copula can measure tail dependence. However, the stock returns drop more than rise in the size of movements (Ang and Chen, 2002) while the correlation of stock returns is generally higher in a high volatility than in a low volatility state (Ang and Bekaert, 1999). This phenomenon is called asymmetric effect that cannot be caught by symmetric elliptical copulas such as Gaussian and Student-t copulas. Thus, the Archimedean copulas including Clayton (1978), Frank (1979) and Gumbel (1960) copulas are considered to be more plausible to model the asymmetric tail dependence.

The empirical evidences reported that the properties of time-varying volatility of stock returns including volatility asymmetry, clustering, persistence, and leptokurtosis exist in stock returns. To catch the conditional heteroskedasticity volatility, the ARCH model was developed by Engle (1982) and extended by Bollerslev (1986) to create the GARCH model. To date, several GARCH type models were proposed to capture the volatility asymmetry such as the exponential GARCH (EGARCH) model by Nelson (1991), the asymmetric GARCH (AGARCH) by Engle and Ng (1993), the GJR-GARCH by Glosten et al. (1993), the power ARCH by Ding et al. (1993) etc.

For the conditional variances and covariances model of multivariate assets, multivariate GARCH (MGARCH) has been used in Bollerslev, Engle, and Wooldridge (1988), Ng (1991), and Hansson and Hordahl (1998). It was applied to explain the spillover effects of contagion in Tse and Tsui (2002) and Bae (2003) et al. An alternative of MGARCH is the use of copula GARCH proposed by Patton (2001) and Jondeau and Rockinger (2002). Later, Jondeau and Rockinger (2006), Patton (2006), and Hu (2006) applied different copulas in GARCH model to study the tail dependence between financial markets.

3. Data and Methodology

In this paper, a copula GARCH framework is proposed. It combines both advantages of the GJR GARCH and the fit copula to study the multivariate distribution, tail dependence, and concordance for the contagion risks of Taiex and Kospi vs. other major stock price indices. Expectedly, GJR GARCH incorporating suitable copula can reveal volatility, correlation, fat tail between stock indices when stock markets crash together.

3.1 Data

Taiwan and South Korea both have export-oriented markets and get more open as well as competitive to global markets. They have the similar industrial structure focusing on consumer electronics production. To date, the stock markets of Taiwan and South Korea are strong correlated with those of US, Japan, China, and Europe. Thus, the six indices of Taiex, Kospi, S&P500, Nikkei, MSCI all China index[2], and MSCI Europe Index[3] are considered in analyzing the contagion risk regarding Taiwanese as well as South Korean stock markets.

3.2 The vector autoregression model

Since the correlation exists between the returns of the five stock price indices, the vector autoregression (VAR) is used to catch the first order effect of return process. The return of stock price index at time t is written as $r_t = \ln(P_t / P_{t-1})$ and the vector autoregression of the five stock returns denoted by $\mathbf{r}_t$ at time t is written in standard form with p lags as

$$\mathbf{r}_t = A_0 + \sum_{i=1}^{p-1} A_i \mathbf{r}_{t-i} + \boldsymbol{\varepsilon}_t, \tag{1}$$

where $\boldsymbol{\varepsilon}_t$ is the vector of error terms at time t which has the properties of conditional time-varying volatility.

Using VAR, the Granger causality can be tested to discover if causality exists between stock price indices. In fact, Taiex is susceptible to S&P 500, MSCI China, MSCI Europe or Nikkei. Kospi is expected to have the same property.

3.3 Asymmetric GARCH models

3.3.2 GARCH

For a specific stock price index, its daily price returns are assumed as the series of error

[2] The MSCI China Index consists of a range of country, composite and non-domestic indices for the Chinese market, intended for both international and domestic investors, including Qualified Domestic Institutional Investors (QDII) and Qualified Foreign Institutional Investors (QFII) licensees. (http://www.msci.com)

[3] The MSCI Europe Index is a free float-adjusted market capitalization weighted index that is designed to measure the equity market performance of the developed markets in Europe. The MSCI Europe Index consists of the following 16 developed market country indices: Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, and the United Kingdom. (http://www.msci.com)

term ε_t that has a time-varying volatility process. According to Engle (1983), the multiplicative conditional heteroscedastic model of ε_t is

$$\varepsilon_t = \xi_t \sqrt{h_t} \ , \tag{2}$$

where F_{t-1} is filtration at time t-1 and $\xi_t \mid F_{t-1} \sim N(0,1)$. Thus, the general autoregression conditional heteroscedastic GARCH(p,q) model is written as

$$h_t = c + \alpha_1 \varepsilon_{t-1}^2 + ... + \alpha_q \varepsilon_{t-q}^2 + \beta_1 h_{t-1} + ... + \beta_p h_{t-p} . \tag{3}$$

However, the empirical h_t might have leverage effect or volatility asymmetry (i.e. bad news has a higher impact on stock prices than good news). Therefore, two asymmetric effect adjusted methods are provided as follows:

(1) GJR GARCH

The GJR model (Glosten, Jagannathan, and Runkle, 1993) uses an indicator of negative returns to capture the leverage effect between negative stock price changes and volatility. Thus, the conditional heteroscedastic GARCH equation is rewritten as

$$h_t = c + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1} + \gamma \varepsilon_{t-1}^2 I_{t-1} , \tag{4}$$

where I_{t-1} = 1 if ε_{t-1} < 0 and I_{t-1} = 0 otherwise. The leverage effect exists if γ>0 for GJR.

(2) Student-t GARCH

On the other hand, the asymmetric effect also exhibits fat-tail property. Thus, the residual ε_t in Equation (2) is considered to follow a t distribution as

$$f(\varepsilon_t) = \frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})} \frac{1}{\sqrt{\pi\nu}} (1 + \frac{\varepsilon_t^{\ 2}}{\nu})^{-\frac{\nu+1}{2}} , \tag{5}$$

where ν is the degree of freedom. The t distribution is used to catch the skewness effect and build a likelihood function.

3.4 Copula GARCH framework

For a single stock price index, the previous section has described how to model the adjusted GARCH model considering nonlinear effects such as fat tail or skewness. For bivariate stock price indices such as Taiex against S&P500 or Taiex against MSCI China etc., the copulas are used to obtain the structural dependence between each marginal distribution of stock price index described previously.

3.4.1 Bivariate distribution and copulas

The copulas introduced by Nelson (1999) and applied by Patton (2006) can decompose a multi-dimension distribution into a few marginal distributions and a structural dependence, i.e. copula. According to (Sklar, 1959), suppose that F is a multivariate distribution function in the unit hypercube [0,1] with marginal uniform function $F_i(x_i)$ for i=1,…,m. Then there exists an m-dimensional copula $C(F_1(x_1),...,F_m(x_m))$ such that for x ∈ Rn, $C(F_1(x_1),...,F_m(x_m)) = F(x_1,...,x_m)$. and the copula function $\mathrm{C}:[0,1]^{\mathrm{m}} \rightarrow [0,1]$.

In this context, the residual $\varepsilon_{i,t}$ in Equation (2) is equivalent to x_i. For an example of the bivariate stock markets of Taiex against S&P500, the copula for $\varepsilon_{i,t}$ with i=1,2 referring to Taiex and S&P500 respectively, can be written as

$$\begin{aligned} & C(F_1(\varepsilon_{1,t}),F_2(\varepsilon_{2,t})) = \Pr(U_1 \le F_1(\varepsilon_{1,t}),U_2 \le F_2(\varepsilon_{2,t})) \\ & = \Pr(F^{-1}(U_1) \le \varepsilon_{1,t}, F^{-1}(U_2) \le \varepsilon_{2,t}) = F(\varepsilon_{1,t},\varepsilon_{2,t}), \end{aligned} \tag{6}$$

where U is a standard uniform random variable. If $F_1(\varepsilon_{1,t})$ and are all continuous, C is uniquely determined on $F_1(\varepsilon_{1,t}) \times F_2(\varepsilon_{2,t})$. Conversely, if C is copula with marginal $F_1(\varepsilon_{1,t})$ and.., then F is a bivariate distribution. To obtain the density of F distribution , i.e. $f(\varepsilon_{1,t},\varepsilon_{2,t})$, just take the derivative of F as

$$\begin{aligned} & f(\varepsilon_{1,t},\varepsilon_{2,t}) = \frac{\partial^2 F(\varepsilon_{1,t},\varepsilon_{2,t})}{\partial \varepsilon_{1,t} \partial \varepsilon_{2,t}} = \frac{\partial^2 C(F_1(\varepsilon_{1,t}),F_2(\varepsilon_{2,t}))}{\partial F_1(\varepsilon_{1,t}) \partial F_2(\varepsilon_{2,t})} \times \prod_i^2 \frac{\partial F_i(\varepsilon_{i,t})}{\partial \varepsilon_{i,t}} \\ & = c(u_{1,t},u_{2,t}) \times \prod_i^2 f_i(\varepsilon_{i,t}), \end{aligned} \tag{7}$$

where u means a number in random variable U, $c(u_{1,t},u_{2,t})$ is the copula density function, and $u_{i,t}$ is the marginal density function of x_i for i=1,2. To catch the leptokurtosis effect, $f_i(\varepsilon_{i,t})$ can be considered as a skewed t-distribution in Equation (5).

Therefore, it is apprentice that the joint probability function of multivariable can be separated into the product of a structural dependence i.e. copula and a few of marginal probability functions. Since the marginal probability functions bare no information at all about dependence between variables, the structural dependence between variables definitely embeds in the copula. That's why copula is described as structural dependence.

3.4.2 Elliptical and Archimedean copulas

There are several candidate copulas common used in modeling. The elliptical copulas including Gaussian and Student-t copulas have linear correlation and symmetric shape in copula function. Using the Sklar's theorem, the distribution function of Gaussian denoted by C_N can be constructed from the Gaussian bivariate distribution as

$$C_N(u_1,u_2;\rho)=\int_{\infty}^{\Phi^{-1}(u_1)}\int_{\infty}^{\Phi^{-1}(u_2)}\frac{1}{2\pi\sqrt{(1-\rho^2)}}\exp\left[-\frac{x^2-2\rho xy+y^2}{2(1-\rho^2)}\right]dxdy \tag{8}$$

where ρ is the Pearson correlation that expresses a linear correlation between random variable x and y, and Φ expresses a cumulative univariate standard normal distribution. Similarly, the distribution function of Student-t copula denoted by C_{St} is given by

$$C_{St}(u_1,u_2;v,\rho)=\int_{\infty}^{t_v^{-1}(u_1)}\int_{\infty}^{t_v^{-1}(u_2)}\frac{1}{2\pi\sqrt{(1-\rho^2)}}\left\{1+\frac{x^2-2\rho xy+y^2}{v(1-\rho^2)}\right\}^{-\frac{v+2}{2}}dxdy, \tag{9}$$

where t_v^{-1} is the inverse univariate t distribution and v is its degree of freedom.

The Archimedean copulas are typically nonlinear including Clayton, Frank and Gumbel. The Gumbel copula features intensive density to the right tail (rising together), and the Frank copula features symmetry without skewness. Whereas, the Clayton copula features intensive density to the left tail (dropping together) and it is lower tail dependent and upper tail independent. Hence, the Clayton copula is much more applicable in analyzing tail behavior for contagion risk. In this paper it is applied and written as

$$C_{Cl}(u_1,u_2;\theta)=\left\{u_1^{-\theta}+u_2^{-\theta}-1\right\}^{-\frac{1}{\theta}},\theta\in(0,\infty). \tag{10}$$

3.4.4 Maximum likelihood estimation of copula GARCH

The maximum likelihood estimation (MLE) is adopted to estimate the parameters contained in each marginal function $f_i(\cdot)$ as in Equation (5) and the copula function $\tilde{c}(\cdot)$ as in Equation (7) for multivariate GARCH model. Suppose that both ϕ_i for i=1,2 and ψ are a constant parameter vector in the ith marginal density function $f_i(\cdot)$ and the copula function $\tilde{c}(\cdot)$ respectively. For a structural dependence between two stock indices, the conditional log-likelihood function for $\varepsilon_{i,t}$ with i=1,2 (i.e. bivariate series of residuals) can be written as

$$\mathrm{L}(\phi,\psi)=\sum_{t=1}^{T}\ln\tilde{c}\,(u_{1,t},u_{2,t},\phi\mid\varepsilon_{t-1};\psi)+\sum_{t=1}^{T}\sum_{i=1}^{2}\ln f_i\,(\varepsilon_{i,t},\phi_i\mid\varepsilon_{t-1}) \tag{11}$$

where ϕ is equal to $[\phi_1,\phi_2]'$ and T denotes the sample size i.e. the number of observations. For the ith series GARCH GJR model, its parameter vector are $\phi_i=[c_i,\alpha_i,\beta_i,\gamma_i]$. Note that $u_{i,t}$ can be estimation by empirical CDF of $\varepsilon_{i,t}$.

The detailed log likelihood of Gaussian, Student-t, and Clayton are given in Appendix A. To estimate the dynamic covariance Q_t and correlation R_t between series, the dynamic conditional correlation (DCC) approach of Engle (2002) is applied in the paper as

$$\begin{aligned} Q_t &= (1 - w_1 - w_2)\bar{Q} + w_1 \boldsymbol{\varepsilon}_{t-1}^{'} \cdot \boldsymbol{\varepsilon}_{t-1} + w_2 Q_{t-1} \\ R_t &= \tilde{Q}_t^{-1} Q_t \tilde{Q}_t^{-1} \end{aligned} \tag{12}$$

where $\bar{Q}$ is the sample variance covariance of $\boldsymbol{\varepsilon}_t$, $\tilde{Q}_t$ is the square root of Q_t with zero off-diagonal elements. Thus, for a two series Clayton GARCH model, the copula parameter vector is $\psi = [w_1, w_2, \theta]$.

To maximize Equation (11), it is complicated to solve out an analytic solution. Thus, an appropriate numerical method such as a two-stage procedure is used as follows. First, ϕ_i is solved by

$$\hat{\phi}_i = \arg\max_{\phi_i} \sum_{t=1}^{n_j} \sum_{i=1}^{2} \ln f_i \ (\varepsilon_{i,t}, \phi_i \mid \varepsilon_{t-1}) . \tag{13}$$

Next, $\hat{\phi}_i$ is used to solve for ψ . So Equation (11) is rewritten as

$$\begin{aligned} \hat{\psi} &= \arg\max_{\psi} \mathrm{L}(\hat{\phi}, \psi) \\ &= \sum_{t=1}^{n_j} \ln \tilde{c} \ (u_{1,t}, u_{2,t}, \hat{\phi} \mid \varepsilon_{t-1}; \psi) + \sum_{t=1}^{n_j} \sum_{i=1}^{2} \ln f_i \ (\varepsilon_{i,t}, \hat{\phi}_i \mid \varepsilon_{t-1}). \end{aligned} \tag{14}$$

Then, the two-stage procedure is iteratedly to solve for optimal $\hat{\phi}_i$ and $\hat{\psi}$. This method is proved to be useful in Patton (2001). Given the estimation of MLE, the tail dependence can be measured consequently.

3.5 Measurement of the tail dependence using Kendall's tau

Several measures of asymmetric dependence can be used for analyzing contagion risk such as tail dependence and exceedance correlation. The advanced studies can be found in Longin and Solnik (2001) and Ang and Chen (2002).

Unlike the simple correlation estimating the linear co-moment of two random variables, the Kendall's tau denoted by ρ_τ measures the dependence between two random variables as

$$\rho_\tau = E[sign\{(X_1 - X_2)(Y_1 - Y_2)\}], \tag{15}$$

where (X_1, Y_1) and (X_2, Y_2) are two pairs of independent and equally distributed random variables and *sign* is a sign function.

In this context, the residual series $\varepsilon_{i,t}$ for i=1,2 as in Equation (2) is equivalent to X and Y

random variables. The Kendall's tau ρ_τ for $\varepsilon_{i,t}$ is given by Schweizer and Wolff (1981) in terms of copula as

$$\rho_\tau = 4\int\int_{[0,1]^2} \tilde{C}\ (u_1, u_2)\tilde{c}\ (u_1, u_2)du_1 du_2 - 1. \tag{16}$$

Note that ρ_τ depends only on the copula function but not the multivariate distribution. The Spearman's correlation[4] ρ_s i.e. the correlation coefficient of copula is given by

$$\rho_s = 12\int\int_{[0,1]^2} u_1 u_2 d\tilde{C}(u_1, u_2) - 3. \tag{17}$$

Note that ρ_s depends only on the marginal distributions.

The Kendall's tau ρ_τ and Spearman's rank correlation ρ_s for elliptical and Clayton copulas are displayed in Table 1. Since there is no Spearman's ρ_s for Clayton, the Kendall's ρ_τ is used in this paper to measure tail dependence.

Table 1 Kendall's ρ_τ and Spearman's ρ_s

Copulas	ρ_τ	ρ_s
Gaussian	$\frac{2}{\pi}\arcsin(\rho)$	$\frac{6}{\pi}\arcsin(\frac{\rho}{2}) \approx \rho$
Student-t	$\frac{2}{\pi}\arcsin(\rho)$	-
Clayton	$\theta/(\theta+2)$	-

Note: θ is the parameter of Clayton copula.

3.5 Test of the tail dependence using threshold regression

The threshold regression is performed to find and test a threshold value to classify the dynamic tail dependence into different states. It is useful to inspect if the tail dependence should have different states and if the threshold value indeed exists, then what it is.

Originally, the threshold autoregressive (TAR) developed by Tong and Lim (1980) is applied to find the threshold value to classify the nonlinear financial process into several regression states. It can explain the behavior of nonlinear process such as shift of returns trend, switch of volatility or heteroscedasticity of volatility. Specifically, it intends to uses several

[4] See Embrechts et al. (2002) for relation between ρ, ρ_τ, and ρ_s.

piecewise autoregression to approximate the nonlinear process of financial series.

Suppose that ρ_{it} denotes the *i*th series of six stock price indices at time *t* and its threshold variable is $\rho_{t-d} \in \Omega_j$ for $j = 1, 2, ..., l-1$ (Ω_j is the set of *j*th state and l is the number of states). Then, the *j*th threshold autoregression equation is expressed as

$$\rho_{i,t} = \rho_i + \sum_{h=1}^{k} b_{i,h}^{(j)} \rho_{i,t-h} \text{ as } L_{j-1} < \rho_{i,t-d} < L_j , \tag{18}$$

where *k* is the order of autoregression, *d* is the lag of threshold ($d \leqq k$). $L_1, ..., L_{l-1}$ are the threshold values that divide $\rho_{i,t}$ into *l* states of equations as the Equation (18).

Because volatility is evidenced to have high and low states, the tail dependence between stock price indices should be assumed to have high and low states as well. Hence, the number of states for threshold regression is set to two, i.e. the number of *L* is one. Hence, if the threshold values are tested significant, it indicates that the tail dependence should have high and low states that cannot be explained by only one equation.

The most influential contagion risk is when the volatility and tail dependence are both in a high state. At that time, it is really interesting to analyze the contagion risk from larger markets such as US and China to smaller markets such as Taiwan and South Korea.

4. Empirical Result

4.1 Data Description

The data of six stock price indices are collected from the Taiwan Economic Journal (TEJ) database and sampled from 01/23/2003 to 05/08/2013 at a daily frequency to acquire more information of stock price changes. The in-sample period is set from 01/23/2003 to 07/23/2007 totaling 917 observations to estimate the model parameters and forecast the structural tail dependence i.e. Kendall’s taus for next day. The out-of-sample period is set from 07/24/2007 to 05/08/2013 covering the durations of two major risk events: subprime disaster in early 2008 and Greek debt crisis in early 2010. Repeatedly moving the in-sample period window one day forward for the out-of-sample data, therefore total 1,126 moving windows are proceeded to estimate copula models and dynamic correlations. Consequently, 1,126 dynamic correlation are obtained for the test of threshold effect.

With entire period, the data description of daily returns for six indices are reported in Table 2. It is clear that most stock price indices exhibit left-skewness and high peakness and the linear tests of Jarqe-Beta are rejected according to Table 2 Panel A. This indicates that the data have lepkurtosis and fat tail property which affect structural tail dependence. The static Pearson's correlation and test are reported in Table 2 Panel B. It shows that Taiex and Kospi are the highest correlated to each other while they are lower correlated to S&P 500 and MSCI China. Nevertheless, they suffer contagion risk undoubtedly due to US financial crises.

4.2 VAR and Granger Causality Tests between Six Indices

To investigate the relationships among the six indices, we perform the vector autoregression (VAR) as in Equation (1) and Granger causality between indices. Before doing so, the lag length used for VAR is decided according to Doan (2005) who computes *LR* test between the shorter i.e. restricted lag length VAR model and longer i.e. unrestricted VAR models with penalty for the number of regressors.

Table 2 Data Description

Panel A. Moments of data

	Taiex	Kospi	S&P500	Nikkei	MSCI China	MSCI Europe
Minimum	-0.06912	-0.11172	-0.09470	-0.12111	-0.07858	-0.12836
Maximum	0.06099	0.11284	0.10246	0.09494	0.09955	0.14044
Mean	0.00016	0.00056	0.00005	0.00009	0.00025	0.00061
Mean=0 Test	(0.28834)	(0.04405)	(0.43227)	(0.39448)	(0.21946)	(0.07208)
Stdev	0.01306	0.01487	0.01299	0.01525	0.01475	0.01886
Skewness	-0.40023	-0.48922	-0.52323	-0.77047	-0.12284	0.09988
Skewness=0 Test	(0.00002)	(0.00000)	(0.00000)	(0.44469)	(0.00000)	(0.00133)
Kurtosis	5.90268	9.20682	11.71228	10.47918	8.24636	9.52032
Krutosis=0 Test	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)
Jarque-Bera Test	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)

Note: The numbers in parenthesis are *p* values.

Panel B. Correlation of data

	Taiex	Kospi	S&P500	Nikkei	MSCI China	MSCI Europe
Taiex	1	0.700795	0.143824	0.582406	0.334388	0.597319
Kospi	0.700795	1	0.210984	0.665411	0.372328	0.642056
S&P500	0.143824	0.210984	1	0.129285	0.565915	0.184
Nikkei	0.582406	0.665411	0.129285	1	0.34801	0.59664
MSCI China	0.334388	0.372328	0.565915	0.34801	1	0.401019
MSCI Europe	0.597319	0.642056	0.184	0.59664	0.401019	1

Note: The tests of correlation=0 are all rejected significantly.

As presented in Table 3 panel A, the optimal lag length of VAR model would be 8 and 1 according to AIC and SBC respectively. Considering the swift changes of stock price indices, the optimal lag length is selected as 1 to keep model parsimonious. In Table 3 panel B, the coefficients of VAR model reveal that in comparison, Taiex and Kospi are affected mostly and significantly by one day lag S&P500(0.3087 and 0.3326 for Taiex and Kospi respectively) and one day lag MSCI China (0.0829 and 0.1246 for Taiex and Kospi respectively). Also, S&P500 significantly influences the other five indices and MSCI China significantly influences most of stock price indices except S&P500. However, MSCI China is affected by S&P500 but not reversely while S&P500 is affected by MSCI Europe.

The granger causality tests test if the information of lags of X variable cause a better prediction of Y variable. The test is a F test that uses Equation (1) as the unrestricted model and the restricted model is set up without the terms of the lags of X variables. According to Table 3 panel C, Taiex Granger causes MSCI Europe and Kospi Granger causes Nikkei. Nikkei doesn't Granger causes any index. Similarly to VAR model reveal, S&P500 and MSCI China could Granger cause most of indices. However, MSCI China doesn't Granger cause S&P500.

4.2 Measurement and test of the structural tail dependence

Using three copula GARCH models i.e. Gaussian, Student-t, and Clayton incorporating moving windows technique through 1,126 out-of-sample data, it follows that 1,126 dynamic conditional correlations estimated by Gaussian and Student-t copula GARCH models and tail dependence i.e. Kendall taus estimated by Clayton copula GARCH are acquired.

The estimation results are listed in Table 4. For the Gaussian copula, the GARCH GJR parameters β and γ are strong significantly. For the Student-t copula, the degree freedom v of t distribution is strong significant in addition to strong significant parameters β and γ and similarly for the Clayton copula GARCH. This indicates that the fail tail effect from v test and asymmetric effect from γ test are evidenced here. The copula parameter w_0 is tested significantly for the Kendall's tau.

The dynamic conditional correlations estimated by Gaussian and Student-t copula denoted by ρ_G and ρ_{St} can be transformed to Kendall's taus $\rho_{\tau,G}$ and $\rho_{\tau,\mathrm{St}}$ respectively according to Table 1. As a result, there are two estimated series of dynamic conditional correlations: ρ_G and ρ_{St}, and three estimated series of Kendall's taus: $\rho_{\tau,G}$, $\rho_{\tau,\mathrm{St}}$, and $\rho_{\tau,C}$.

Table 5 presents the data description for Taiex and Kospi against the other four major stock indices. On average, both Taiex and Kospi against S&P and China have the lower Kendall taus. It appears that the impacts of financial risk evolve from large markets such as US and China to smaller markets such as Taiwan and South Korea but not vice versa. Also, Kospi has the higher

Kentall's tau than Taiex against the other four indices. It implies that South Korean market is more global or larger than Taiwanese's.

Figure 1 plots the five estimated series of dynamic correlations. It is obvious that the dynamic conditional correlations ρ_G and ρ_{St} and Kendall's taus $\rho_{\tau,G}$, $\rho_{\tau,\mathrm{St}}$, and $\rho_{\tau,C}$ are closer when Taiex is against S&P500. On the other hand, the dynamic conditional correlations are apparently higher when Taiex is against the other four major indices. This is reasonable because the relationship of dynamic conditional correlation and Kendall' tau according to Table 1 is $2/\pi \cdot \arcsin(\rho)$ that is a upward slope arcsin function between 0 and 1. Figure 2 shows the same phenomenon for Kospi.

Table 3 VAR Analysis Results

Panel A. Lag selection

Lag Length	1	2	3	4	5	6	7	8	9	10	11	12
AIC	-74217.7	-74298.9	-74287.8	-74298.6	-74294.0	-74286.9	-74306.4	**-74307.5**	-74296.9	-74281.7	-74278.6	-74268.0
SBC	**-73982.1**	-73861.8	-73649.7	-73459.9	-73255.2	-73048.4	-72868.6	-72670.9	-72462.0	-72248.9	-72048.3	-71840.8

Panel B. Coefficients of VAR model

	Taiex	Kospi	S&P500	Nikkei	MSCI China	MSCI Europe
Taiex(-1)	-0.0182	-0.0156	0.0218	-0.0548*	0.007**	-0.1044***
	(0.0305)	(0.0347)	(0.0322)	(0.0335)	(0.0353)	(0.0436)
Kospi(-1)	-0.0459*	-0.0252	0.0274	-0.0677**	-0.0256	0.047
	(0.0297)	(0.0338)	(0.0314)	(0.0326)	(0.0343)	(0.0425)
S&P500(-1)	0.3087***	0.3326***	-0.1025***	0.4469***	0.4128***	0.477***
	(0.0254)	(0.0290)	(0.0269)	(0.0279)	(0.0294)	(0.0364)
Nikkei(-1)	0.0018	-0.0592**	0.0344*	-0.0389*	0.0045	-0.0478*
	(0.0253)	(0.0288)	(0.0268)	(0.0278)	(0.0293)	(0.0362)
MSCI China(-1)	0.0829***	0.1246***	0.0089	0.1816***	-0.2315***	0.1202***
	(0.0242)	(0.0276)	(0.0256)	(0.0266)	(0.0280)	(0.0347)
MSCI Europe(-1)	0.0213	-0.0248	-0.062***	-0.0258	0.042**	-0.026
	(0.0204)	(0.0232)	(0.0216)	(0.0224)	(0.0236)	(0.0292)
R-squared	0.1287	0.1258	0.0118	0.2276	0.0870	0.1428
Akaike AIC	-5.9709	-5.7108	-5.8590	-5.7840	-5.6799	-5.2541
Schwarz SC	-5.9544	-5.6943	-5.8425	-5.7675	-5.6634	-5.2376
	0.0131	0.0149	0.0130	0.0152	0.0148	0.0189
Log likelihood		37335.24				
Akaike criterion		-36.53				
Schwarz criterion		-36.43				

Notes: *, **, and *** denotes significance at 10%, 5% and 1% level respectively. The numbers in parenthesis are standard errors. The same representation is used for the rest tables.

Panel C. Granger causality tests (*p* value)

Y \ X	Taiex	Kospi	S&P500	Nikkei	MSCI China	MSCI Europe
Taiex	0.56854	0.12030	0.00000***	0.95113	0.00071***	0.30133
Kospi	0.73786	0.39758	0.00000***	0.04403	0.00001***	0.27657
S&P500	0.49853	0.40359	0.00013***	0.19203	0.70029	0.00407***
Nikkei	0.10215	0.04044**	0.00000***	0.16007	0.00000***	0.24056

MSCI China	0.84570	0.42256	0.00000***	0.85309	0.00000***	0.07364*
MSCI Europe	0.01844**	0.28228	0.00000***	0.20170	0.00063***	0.35127

Note: The granger causality test tests if each of X variables in columns Granger causes one of Y variables in a specific row. The test statistic is $F = \frac{(SSE_R - SSE_U)/s}{SSE_U / T - k}$ where SSE_R and SSE_U is the sum of square errors for restricted and unrestricted model respectively, *s* is the number of restricted parameters, *T* is the sample size, and *k* is the number of regressors.

Table 4 Copula GARCH Estimations

Panel A. Gaussian copula

Stage 1 Estimation of Multivariate GARCH						
	Taiex	Kospi	S&P500	Nikkei	MSCI China	MSCI Europe
c	-0.013	-0.096***	-0.0299	0.0086	-0.0022	-0.0232
	(0.064)	(0.039)	(0.034)	0.0790	(0.023)	0.0390
α	0.0000	0.0000	0.0000	0.000***	0.0000	0.000*
	(0.0000)	(0.0000)	(0.0000)	0.0000	(0.0000)	0.0000
β	0.0477***	0.0573***	0.0571***	0.1708***	0.0712***	0.0734***
	(0.015)	(0.019)	(0.023)	0.0510	(0.018)	0.0150
γ	0.943***	0.9233***	0.9345***	0.7900***	0.8998***	0.9213***
	(0.015)	(0.022)	(0.027)	0.0480	(0.022)	0.0120
AIC	-5735.95	-6417.97	-5628.43	-5106.33	-6155.66	-4941.18
SBC	-5707.03	-6389.05	-5599.51	-5077.40	-6126.73	-4912.25
LogL	2873.98	3214.99	2820.22	2559.16	3083.83	2476.59
Stage 2 Estimation of Multivariate Copula						
w_1	0.015**					
	(0.009)					
w_2	0.0000					
	(0.012)					
AIC	-1550.46					
SBC	-1540.81					
LogL	777.23					

Notes: The two-stage procedure of estimation is used to estimate elliptical copula GARCH model for 6 stock price indices together. The numbers in parenthesis are standard errors.

Panel B. Student-t copula

Stage 1 Estimation of Multivariate GARCH						
	Taiex	Kospi	S&P500	Nikkei	MSCI China	MSCI Europe
c	0.000	0.000**	0.000*	0.000**	0.000***	0.000
	(0.000)	(0.000)	(0.000)	0.000	(0.000)	(0.000)
α	0.0126	0.0001	0.0391	0.0257	0.0409*	0.0501*
	(0.027)	(0.035)	(0.031)	0.028	(0.025)	(0.033)
β	0.8629***	0.8728***	0.9229***	0.842***	0.9082***	0.886***
	(0.014)	(0.041)	(0.028)	0.033	(0.026)	(0.052)
γ	0.102***	0.125**	0.0579**	0.1512***	0.0699**	0.0741**
	(0.044)	(0.06)	(0.028)	(0.063)	(0.038)	(0.039)
v	5.6757***	9.2302***	9.8539**	9.7874***	10.3405***	10.888***
	(1.096)	(3.195)	(5.729)	(3.392)	(2.011)	(3.234)
AIC	-5781.92	-5473.17	6454.49	-5139.77	-6170.56	-4957.97
SBC	-5743.35	-5434.60	-6415.92	-5101.20	-6131.99	-4919.40
LogL	2898.96	2744.59	3235.25	2577.89	3093.28	2486.98
Stage 2 Estimation of Multivariate Copula						
v_c	19.7661***					
	(4.35)					
w_1	0.0143*					
	(0.01)					
w_2	0.000					

	(0.000)
AIC	-1541.69
SBC	-1527.23
LogL	773.85

Note: The degree of freedom v of t distribution is estimated in stage 1 and the degree of freedom v_c of Student-t is estimated in stage 2.

Table 4 Continued

Panel C. Clayton copula

Stage 1 Estimation of Univariate GARCH for Bivariate Copula								
	Taiex-S&P500		Taiex-MSCI China		Kospi-S&P500		Kospi-MSCI China	
c	0.000***	0.000**	0.000	0.000**	0.000*	0.000**	0.000*	0.000**
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
α	0.023	0.0381**	0.023	0.041**	0.0235	0.0391*	0.0235	0.0409**
	(0.021)	(0.023)	(0.021)	(0.025)	(0.031)	(0.027)	(0.031)	(0.023)
β	0.860***	0.912***	0.860***	0.904***	0.849***	0.912***	0.849***	0.886***
	(0.012)	(0.027)	(0.012)	(0.026)	(0.072)	(0.028)	(0.072)	(0.042)
γ	0.0948***	0.0696**	0.0948***	0.0723**	0.1259*	0.0792**	0.1259*	0.0805**
	(0.036)	(0.031)	(0.036)	(0.039)	(0.098)	(0.038)	(0.098)	(0.048)
v	5.293***	9.491***	5.293***	10.818***	8.296***	9.416**	8.296***	10.285***
	(0.802)	(3.749)	(0.802)	(2.345)	(3.036)	(5.030)	(3.036)	(1.834)
AIC	-5773.51	-6451.95	-5773.51	-6163.28	-5457.38	-6452.53	-5457.38	-6165.81
BIC	-5734.94	-6413.38	-5734.94	-6124.71	-5418.81	-6413.96	-5418.81	-6127.24
LogL	2894.75	3233.97	2894.75	3089.64	2736.69	3234.26	2736.69	3090.91
Stage 2 Estimation of Bivariate Clayton Copula								
w_0	-2.141***		-1.619***		-2.081***		-1.294***	
	(0.522)		(0.385)		(0.551)		(0.199)	
w_1	0.8411		1.929***		-0.2697		-0.1558	
	(1.279)		(0.665)		(1.000)		(0.812)	
w_2	0.2308		0.3546**		-0.1196		-0.1099	
	(0.245)		(0.159)		(0.372)		(0.273)	
	-6.542		-54.785		-22.590		-87.454	
	7.922		-40.321		-8.127		-72.990	
	6.271		30.392		14.295		46.727	

Notes: At present, the bivaratie but not three-variate above Clayton copula GARCH can be estimated. Thus, there are 15 combinations of bivariate copula GARCH for six indices. Four important combination only are excerpted here. Clayton Kendall ρ_τ is estimated using Patton(2006) as $\rho_\tau = \Lambda(w_0 + w_1\rho_{\tau-1} + w_2 \mid u_{1,t-i} - u_{2,t-i} \mid)$ where Λ is the logistic probability transformation function.

Table 5 Dynamic Correlations Statistical Description

		Ta-Ko	Ta-S&P	Ta-Ni	Ta- Ch	Ta- Eu	Ko-S&P	Ko-Ni	Ko-Ch	Ko- Eu
ρ_G	Mean	0.677	0.160	0.596	0.326	0.594	0.220	0.686	0.352	0.617
	Stdev	0.056	0.057	0.055	0.053	0.063	0.051	0.059	0.043	0.054
ρ_t	Mean	0.674	0.153	0.587	0.322	0.591	0.215	0.678	0.359	0.611
	Stdev	0.051	0.053	0.053	0.049	0.062	0.046	0.058	0.039	0.051
$\rho_{\tau,G}$	Mean	0.475	0.103	0.408	0.211	0.407	0.142	0.483	0.229	0.425
	Stdev	0.048	0.037	0.044	0.036	0.050	0.033	0.049	0.029	0.044
$\rho_{\tau,t}$	Mean	0.472	0.098	0.400	0.209	0.404	0.138	0.476	0.234	0.419
	Stdev	0.044	0.034	0.042	0.033	0.049	0.030	0.048	0.026	0.041
$\rho_{\tau,C}$	Mean	0.396	0.065	0.339	0.162	0.336	0.090	0.415	0.184	0.365
	Stdev	0.059	0.026	0.059	0.036	0.059	0.038	0.076	0.013	0.061

Note: Ta, Ko, S&P, Ni, Ch, and Eu are short for Taiex, Kospi, S&P500, Nikkei, MSCI China, and MSCI Europe respectively and the representations are the same for following tables.

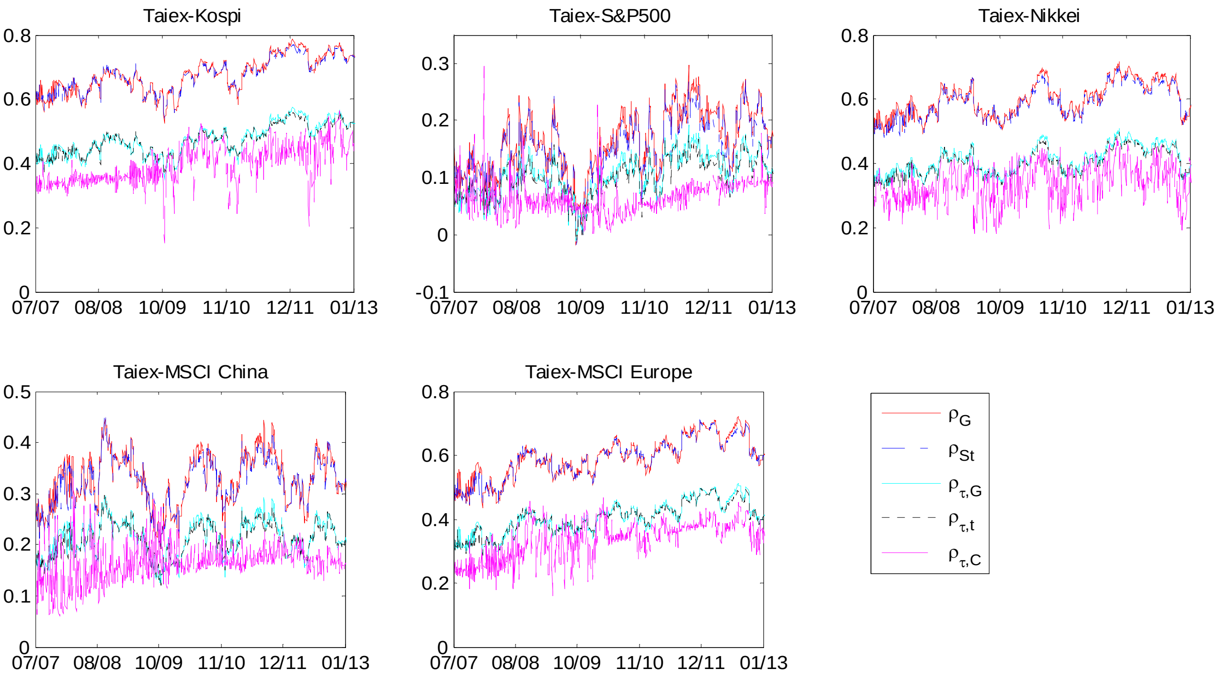

Figure 1 The Dynamic Correlation Changes of Taiex Versus the Major Indices

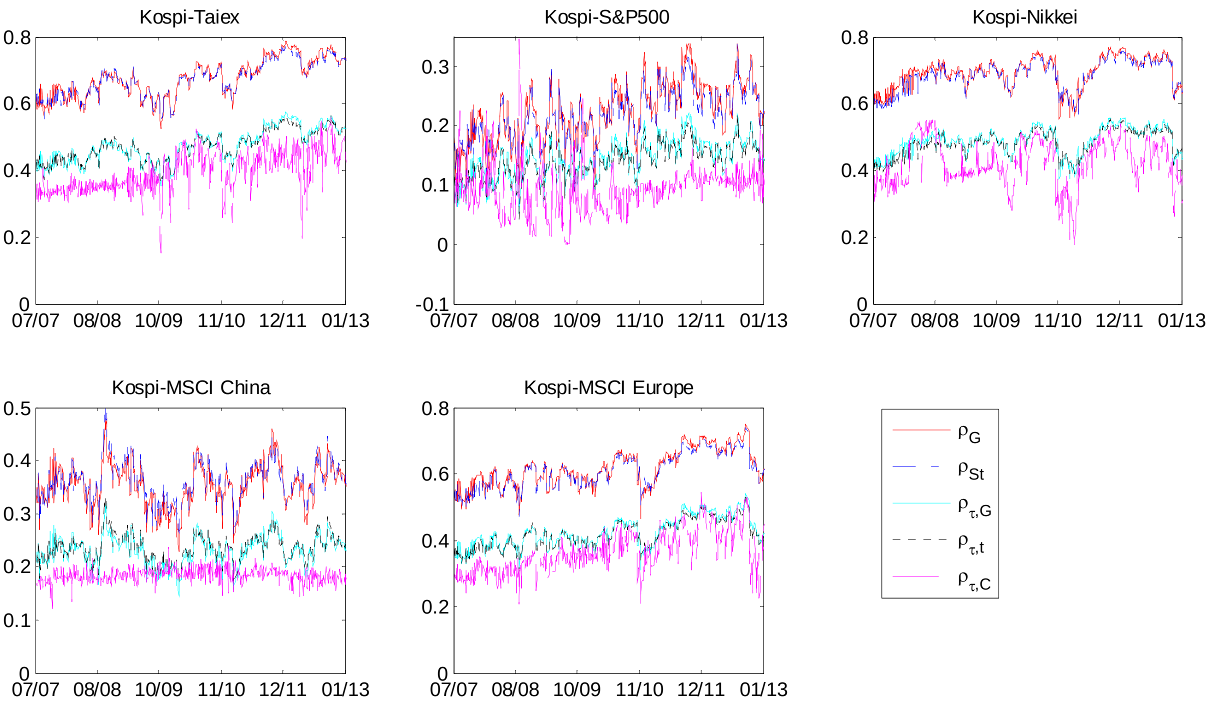

Figure 2 The Dynamic Correlation Changes of Taiex Versus the Major Indices

4.3 Threshold regression test of dynamic correlations

Undoubtedly, the subprime risk has provoked tremendous global contagion risk. Not only

US stock markets crashes but Taiwanese, South Korean, European, Japanese, and Chinese stock markets have dropping sharply during late 2008. Nevertheless, the Greek debt crisis also cause panic selling globally. Thus, the threshold regression test is used to test if the crisis covered period exists different states of tail dependence to be classified by threshold values. For threshold test, the period of subprime crisis is set as 07/24/2007~01/23/2009 and the period of Greek debt crisis is set as 12/01/2009~03/01/2011. Since the copula GARCH evidences that fail tail and asymmetric effected are strong significant in six indices, the threshold test is used to test Kendall's taus of Student-t and Clayton copula regardless of Gaussian copula to investigate if there exists significant two states of tail dependence and one threshold value.

Table 6 reports the results of threshold regression test for subprime crisis. Both threshold tests of Student-t and Clayton Kendall's taus: $\rho_{\tau,\mathrm{St}}$ and $\rho_{\tau,C}$ are consistent in most ways. They reveal that the threshold values are significant for Taiex against Kospi, Nikkei, and MSCI Europe as well as Kospi against Nikkei, MSCI Europe. Taiex against S&P500 has the lowest threshold values that are 0.1032 and 0.0709 according to Student-t and Clayton copulas respectively and similarly for Kospi against S&P500. Also, both Taiex and Kospi against MSCI China have lower threshold values. On the other hand, tests of Student-t copula shows that Taiex against S&P500 is significant but Taiex against MSCI China is not. This result however is contrary to Clayton copula.

Table7 reports the results of threshold regression test for Greek debt crisis. The threshold test of Student-t copula estimated Kendall's taus shows that there are no threshold effects during Greek debt crisis. However, test of Clayton's Kendall's taus reveal that Taiex against S&P500 as well as MSCI Europe are significant but Kospi against S&P500 as well as Nikkei are significant. Actually, the result of threshold test of Clayton's Kendall's taus appears more acceptable and reasonable. Most Taiwan's financial sectors such as Bubon Bank, Mega Holdings, China Development Financial Holdings etc. own bonds directly in European countries. Hence, due to Greek debt crisis, Taiwanese stock market has suffered deeply and Taiex has tumbled 36% from 9,025 on 01/03/2011 to 6,633 on 12/19/2011. On the other hand, South Korean companies have developed very quickly to become larger enough such as Samsung, LG, Hyundai, etc. to be competitive and comparable with Japanese large companies such as Sony, Panasonic, Toyota, etc. As a consequence, Taiex is more related to MSCI Europe than Kospi while Kospi is more connected to Nikkei than Taiex.

Table 6 Threshold Tests of Kendall's Taus during Subprime Crisis

Panel A. Test of Student-t copula estimated $\rho_{\tau,\mathrm{St}}$

	Ta-KO	Ta-S&P	Ta-Ni	Ta-Ch	Ta-Eu	Ko-S&P	Ko-Ni	Ko-Ch	Ko-Eu
Threshold Values									
	0.4331	0.1032	0.3564	0.1941	0.3185	0.1286	0.4413	0.2627	0.3569
Threshold Value Tests									
SupLM	29.0230	36.0035	35.7369	21.0416	38.5570	22.6806	46.5083	19.5261	35.2873
p value	0.0136	0.0004	0.0004	0.1696	0.0000	0.1004	0.0000	0.2276	0.0004
ExpLM	9.6481	13.1349	12.7296	6.6129	14.2421	8.0663	16.7372	5.3422	12.1634
p value	0.0144	0.0000	0.0008	0.1292	0.0000	0.0452	0.0000	0.3036	0.0012
AveLM	10.1200	11.8865	12.1485	8.0884	13.7495	11.4488	13.7101	7.9416	12.4128
p value	0.0536	0.0152	0.0136	0.1876	0.0056	0.0196	0.0024	0.2428	0.0108

Notes: SupLM, ExpLM, and AveLM provided by Hansen(1996) stand for supremum, exponential average, and average *LM* tests respectively. They are computed using 2,500 draws and similarly for rest tables

Panel B. Tests of Clayton copula estimated $\rho_{\tau,C}$

	Ta-KO	Ta-S&P	Ta-Ni	Ta-Ch	Ta-Eu	Ko-S&P	Ko-Ni	Ko-Ch	Ko-Eu
Threshold Values									
	0.3397	0.0709	0.2962	0.1382	0.3294	0.1044	0.4421	0.1690	0.2836
Threshold Value Tests									
SupLM	25.4602	20.7925	26.9083	24.3868	48.0635	26.1868	49.3766	26.4483	41.1254
p value	0.0440	0.2392	0.0228	0.0700	0.0000	0.0420	0.0000	0.0372	0.0000
ExpLM	8.4130	6.3722	8.9997	8.9209	19.1632	8.6465	19.9729	7.3826	15.9502
p value	0.0328	0.2124	0.0200	0.0244	0.0000	0.0360	0.0000	0.0936	0.0000
AveLM	12.1247	7.1254	12.3605	13.6479	20.1612	9.9495	12.8823	8.2551	20.3441
p value	0.0036	0.4064	0.0060	0.0004	0.0000	0.0668	0.0032	0.1408	0.0000

Table 7 Threshold Tests of Kendall's Taus during Greek Debt Crisis

Panel A. Test of Student-t copula estimated $\rho_{\tau,\mathrm{St}}$

	Ta-KO	Ta-S&P	Ta-Ni	Ta-Ch	Ta-Eu	Ko-S&P	Ko-Ni	Ko-Ch	Ko-Eu
Threshold Values									
	0.4557	0.0759	0.3886	0.1786	0.4004	0.1252	0.4728	0.2184	0.3909
Threshold Value Tests									
SupLM	11.5784	16.2283	20.0838	14.0868	16.2053	13.9524	20.3050	12.2855	21.8931
p value	0.7488	0.4516	0.1484	0.5768	0.4324	0.6668	0.0944	0.7488	0.0724
ExpLM	3.5353	4.4406	5.0761	2.6685	4.8220	2.8143	5.4566	2.7181	6.1270
p value	0.5636	0.4716	0.2688	0.8732	0.3696	0.8924	0.1448	0.8412	0.1076
AveLM	5.5650	4.8279	6.4592	3.5299	6.9620	4.1192	6.3397	4.0711	7.7871
p value	0.5188	0.7928	0.3848	0.9532	0.3652	0.9140	0.2992	0.8484	0.1280

Panel B. Test of Clayton copula estimated $\rho_{\tau,C}$

	Ta-KO	Ta-S&P	Ta-Ni	Ta-Ch	Ta-Eu	Ko-S&P	Ko-Ni	Ko-Ch	Ko-Eu
Threshold Values									
	0.3722	0.0440	0.3586	0.1548	0.3066	0.0500	0.4690	0.1976	0.4158
Threshold Value Tests									
SupLM	18.1500	27.8680	17.0850	18.9560	27.9220	28.3400	34.9150	23.6290	24.6060
p value	0.2684	0.0184	0.4104	0.2636	0.0116	0.0132	0.0012	0.0492	0.0452
ExpLM	5.1920	9.7380	4.3650	6.0070	9.0160	7.7320	12.1570	7.8520	7.8090
p value	0.2576	0.0124	0.4912	0.1708	0.0156	0.0440	0.0020	0.0296	0.0432
AveLM	7.9460	13.9080	5.9820	8.5410	11.9970	9.6980	12.8240	7.5230	9.0750
p value	0.1452	0.0016	0.4644	0.0592	0.0132	0.0324	0.0064	0.1712	0.0780

5. Conclusions

Using copula GJR GARCH with t distribution, the fat tail and asymmetric effects are evidenced in Taiex, Kospi, S&P500, Nikkei, MSCI China, and MSCI Europe totaling six indices. Therefore, in practice Gaussian copula indeed should be replaced by Student-t copula. The tail dependence of the six indices is measured by Kendall' tau using either multivariate copula such as Gaussian and Student-t or bivariate copula such as Clayton.

The results of estimation report that both Taiex and Kospi against US and MSCI China have the lower Kendall's taus and this is reasonable due to the unilateral impacts from larger US and China markets. However, Taiex is affected mostly by S&P500 according to the coefficient of VAR that is 0.3087. It suggests us that the Kendall's tau should be combined with volatility measure to interpret contagion risk.

The result of threshold test of Kendall's taus estimated by both Student-t and Clayton copulas reports that subprime crisis indeed causes different states of tail dependence except MSCI China whose market is not so open to global investors. Although Student-t copula estimated Kendall's taus are tested acceptably, it fails to test the threshold effects due to Greek debt crisis. On the contrary, Clayton copula estimated Kendall's taus can be tested well. In practice, it is advantageous to use Clayton copula to estimate and test Kendall's taus.

Appendix A. The Log Likelihoods of Gaussian, Student-t, and Clayton Copulas

(1)The Log likelihood of Gaussian copula is

$$L(\boldsymbol{\varepsilon}_t; R) = 1/2\sum_{t=1}^{T}(\log|\mathbf{R}| + \boldsymbol{\varepsilon}_t^{'}(\mathbf{R}^{-1} - I)\boldsymbol{\varepsilon}_t) \qquad \text{(A.1)}$$

where $\boldsymbol{\varepsilon}_t = (\phi^{-1}(\mathrm{u}_{1,t}),...,\phi^{-1}(\mathrm{u}_{p,t}))$ that is the vector of the transformed standardized residuals and R is the correlation matrix of $\boldsymbol{\varepsilon}_t$ and p is the number of residual series.

(2)The Log likelihood of Student-t copula

$$L(\boldsymbol{\varepsilon}_t; \mathbf{R}, v) = \text{-}T\log\frac{\Gamma(\frac{v+p}{2})}{\Gamma(\frac{v}{2})} \text{-}\, pT\log\frac{\Gamma(\frac{v+p}{2})}{\Gamma(\frac{v}{2})} \text{-}\frac{v+p}{2}\sum_{t=1}^{T}\log(1+\frac{\boldsymbol{\varepsilon}_t^{'}\mathbf{R}^{-1}\boldsymbol{\varepsilon}_t}{v})$$
$$\text{-}\sum_{t=1}^{T}\log|\mathbf{R}| + \frac{v+1}{2}\sum_{t=1}^{T}\sum_{i=1}^{p}\log(1+\frac{\varepsilon_{i,t}^2}{v}) \qquad \text{(A.2)}$$

where v is the degree of freedom.

(3)The Log likelihood of Clayton copula

$$L(\mathbf{u}_t;\theta)=\sum_{t=1}^{T}(\log(1+\theta)(u_{1,t}\cdot u_{2,t})^{-1-\theta}(u_{1t}^{-\theta}+u_{2,t}^{-\theta}-1)^{-2-\frac{1}{\theta}}) \tag{A.3}$$

where $\theta=\frac{2\rho_\tau}{1-\rho_\tau}$,and ρ_τ is Kendall's tau.

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