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City Silhouette, World Climate

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Abstract: Global emissions of CO_2 need to fall lest climate change will accelerate. Any effective climate policy must raise the price of carbon consumption. From an urban perspective, one desirable effect of a carbon tax would be to induce households to move closer to where they work. This paper shows that: If the initial distribution of commuting distances (the city silhouette) is skewed towards the periphery then a carbon tax will leave resident landlords better off – even if these landlords need to shoulder those extra commuting costs themselves, too. If resident landlords are decisive then this insight provides an urban silhouette based explanation of why some governments appear so much more willing to confront their citizens with the true cost of emitting CO_2 than others. More briefly, the paper suggests a connection between urban form and climate politics.

Keywords: Urban Silhouette, Climate Policy, Political Economy, Carbon Tax
JEL-Classifications: R12, Q54, H41

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1 Introduction

Global emissions of carbon dioxide need to fall lest climate change will accelerate. Any effective climate policy must raise the price of carbon consumption (Schelling (1992), Stern (2006), Tol (2009)). From an urban perspective, one desirable effect of such a price increase would be to induce households to move closer to where they work. Then society's aggregate commuting would fall. Cities would become more "compact" (Glaeser (2011)), and commuting related carbon dioxide emissions would recede. This in turn would help slow down the ongoing rise of world temperatures.

Only, will countries *want to* raise the price of consuming carbon? There are various well-known reasons for why voters do not seem too keen on pricing these greenhouse gas emissions. Many of those countries predicted to suffer from climate change most also tend to be much poorer (Desmet/Rossi-Hansberg (2012)). These countries are loth to contribute even the little in policy that most richer and safer countries currently offer, a position that reflects the fact that fighting climate change has all those ingredients of a global public good. Many countries may simply be hoping to free-ride on their neighbors' efforts when conveniently postponing any serious efforts of their own.

With this in mind it is all the more surprising to find that there does exist a small subset of countries that are placing a substantial price tag on carbon consumption. Unless we are willing to endow these countries' voters with stronger climate change aversion their motivations must remain mysterious. This paper is an attempt to contribute to explaining these voters' motivation, and thus to explaining the existing cross-sectional variation in mitigation. The explanation offered here is virtually built around countries' urban silhouettes. It departs from the simple fact that urban rents must rise in all but the most peripheral locations once a federal carbon tax drives the cost of commuting up.

Urban tenants will always deeply resent the attendant rise in travel cost and rent. But resident landlords (as a group, or class) may actually *benefit* from this rise *even if* these resident landlords, too, need to face up to those higher travel to work costs. As our recurrent theme, we identify a surprisingly simple (not just necessary but also sufficient, as well as easily verifiable) condition under which the resident landlord class benefit from the carbon tax. Specifically we show that the landlord class benefit in countries where the representative city's distribution of commuting distances is skewed towards the periphery. The opposite is true if this distribution is skewed towards the center.

This result we may even show to be independent of how landlords and tenants are assigned to city rings. If the landlord class are decisive then a country's climate policy can be pinned down to the skewness of that country's distribution of commuting distances or, more graphically, to the skew of its urban silhouettes. As one illustrative application note that Europe – a region not as rich as the US and one that seems to be threatened less by the impending change in its regional climate – currently appears to engage carbon dioxide emissions stronger. From this paper's perspective this is no oddity. Intuitively, the European distribution of urban commuting distances is skewed towards the periphery while in the US it must be skewed towards the center. We do not need to resort to these

ad hoc assertions of transatlantic differences in climate change beliefs that are frequently made.

One first refinement of the paper's central result obtains as we allow for housing stock adjustment. When forming its view on the long run effects of the carbon tax the landlord class continue to consult the initial urban silhouette. Only, now the landlord class also consult the ways in which this silhouette can be expected to change. After all, as the carbon tax makes urban residents flock to more central parts of the city new developments there replace those more peripheral properties left behind. More distant properties can no longer be sustained either because residents would need to be paid to live there or because landlords would need to be paid to maintain them.

This adjustment of the housing stock looks a lot like the "compactification" many environmentalists hope for. But can we be confident of its support among landlords? Landlord support of the tax, so we find, is stronger (i) the more skewed is the initial silhouette towards the periphery, (ii) the closer is new development to the CBD, (iii) the less expensive is construction. Put differently, landlord disenchantment with the tax is stronger the more difficult remaking the city is. For example, building height restrictions prevent property development precisely where it becomes more lucrative most, i.e. near the CBD. Similarly, green building ordinances may actually put up the cost of remaking the city, and may thus be at cross-purposes with the green objective of the carbon tax.

A second extension of the basic model relaxes our initial understanding of urban land ownership. As the number of owner-occupiers (individuals owning but their own property) grow rental incomes gradually fade away. The more owner-occupiers there are the less rent there is to collect. Remaining members of the landlord class will be less and less enthusiastic about the carbon tax (because they receive less and less compensation for those rising commuting cost) while members of the growing class of owner-occupiers never warm to the idea of a carbon tax in the first place. It also is the higher share of owner-occupiers in the US that may help explain why the US are less enthusiastic about raising carbon taxes than is the EU.

From an applied perspective, the paper fits into the debate on the role of urban compactness for climate change. Some authors have suggested that shifting residents closer to the city center has them emit less carbon dioxide (e.g. Glaeser/Kahn (2010), Glaeser (2011)). Others dispute this idea, arguing that society's carbon dioxide emissions depend not just on within-city commuting but on inter-city transporting, too (Gaigné/Riou/Thisse (2012)). Now, it may be true that more compact urban systems need not necessarily consume less carbon, holding everything else equal. Yet a federation of compacter cities may also choose higher tax rates. Everything else may not be equal. A federation of more compact cities may well consume less carbon once the climate policy response is taken into account, too.

From a theoretical perspective, the paper provides a novel alternative to the traditional urban modeling framework according to which residents either (i) are tenants to landlords that are absentee or (ii) are tenants to themselves, sharing their aggregate land rent by virtue of public land ownership (e.g. Fujita (2008)). This paper splits the citizenry into two distinct resident classes instead, one a tenant class, the other a class of resident landlords.

This scenario seems to address better many contentious issues of local political economy. A society of homeowners only has no reason to embrace a carbon tax; whereas in a society of tenants only it is difficult to see absentee landlords exerting the strong influence on local politics so often attributed to them (e.g. Wildasin (1984)). We emphasize that this alternative framework comes with the modeling benefit of never having to worry about where to locate members of either class.

In also operating on aggregate rents and aggregate commuting costs the paper may be assigned to the literature following Arnott/Stiglitz (1982). Yet note that these authors' focus is on how aggregate rents and commuting costs relate to each other, and to society's welfare. This paper's focus, in contrast, is on how commuting costs and rental incomes shape the incentives of resident landlords as to whether to embrace the carbon tax. Put differently, here it is not the ratio between aggregate land rent and aggregate commuting cost but the difference between these two urban aggregates that is our concern, and how this difference relates to society's political economy.

Further predecessors to this paper are Borck/Wrede (2005) and Borck/Wrede (2007) on the political economy of commuting subsidies. Much as a climate tax raises urban travel costs do subsidies to commuting reduce it, and hence the urban principles guiding voters' behavior should largely be similar. Only, of course, a carbon tax commands extra (landlord and tenant) support from those urban residents truly concerned about the environment and the climate. Such extra support is not to be expected in the case of a commuting subsidy. Moreover, the focus in Borck/Wrede (2005) is on the dual antagonisms between (i) landlords and tenants and (ii) rich and poor. Political decisions depend on whether landlords are resident or absentee, and on where rich and poor households reside.

In contrast, this paper's focus is on the antagonism between landlords and tenants only. Political decisions here depend on the city's physical contours, and never on where exactly landlords and tenants reside. The paper comes in seven sections. The second section gives a one page starter on the central theme's flavor. Section 3 presents a very standard closed-city framework that then is augmented by a class of resident landlords. Section 4 explores the resident landlord class' incentives to (i) embrace the carbon tax and to (ii) discourage shopping and employment sprawl. Section 5 allows for city-wide housing stock adjustment in response to the carbon tax, to properly address the potential environmental benefits to be had from it. Section 6 allows for two extensions.

Most countries' cities will not exhibit an even and exclusive landlord-tenant split, and correspondingly section 6 accounts for owner-occupiers, too. While owner-occupiers' presence attenuates the silhouette-policy link dear to this paper's previous sections accounting for this presence should broaden the paper's scope. Section 6 also briefly addresses the uses of an open (rather than closed) city framework. Section 7 concludes.

2 The Idea Illustrated, and one Night Time Silhouette

We illustrate the paper's central theme by way of a simple linear-city example. Consider three "rings" at ever greater distances from the central business district (CBD). Then

let introducing a carbon tax raise the cost of commuting from ring 1 to the CBD by 0 Euro, from ring 2 to the CBD by 2 Euro, and from ring 3 to the CBD by 4 Euro. Once adjustments have taken place, rent in ring 1 must have risen by 4 Euro, and rent in ring 2 must have risen by 2 Euro because it is only these changes that just offset the commuting cost advantage those locations command by being closer to the CBD than the urban fringe.

Now consider six units of (equally sized) housing. Three of these units are to be found in ring 1, two in ring 2, and one unit of housing is peripheral, in ring 3. Let half of the society, also referred to as its three landlords, own all six units of housing. We now evaluate the costs/benefits attached to different allocations of homeowner-tenant matches. One first potential allocation is $(\{1,3\}, \{1,2\}, \{1,2\})$, where the interpretation of, say, $\{1,3\}$, is that a landlord residing in ring 1 herself has her tenant live in ring 3.

In this allocation, the first match $\{1,3\}$ loses nothing in commuting costs but also gains nothing in rent. The second and third match $\{1,2\}$ both lose 0 in commuting costs yet gain 2 in rent. Adding up yields a net aggregate landlord gain of 4. Intuitively, this net aggregate gain's being positive reflects the fact that the distribution of commuting distances is skewed towards the periphery. There simply are more of those valuable, attractive housing units in the center. If, alternatively, this distribution were not skewed at all (or even skewed towards the center) such that housing units were the same in every ring (or even less in the center) then landlords' aggregate net benefit would be zero (negative).

Suppose the three units of housing in ring 1 were stacked on top of each other, composing a building of three stories. Likewise, the two units of housing in ring 2 could form a two storey house while the single unit of housing in ring 3 is the "bungalow". From the distance an observer would immediately make out the skew of the urban silhouette lit up against the night time sky (rather than recognize a skewed distribution of commuting distances). This discussion points us to a connection not commonly made. Its relevance is that it permits us to treat the city's silhouette as expressing the interests of the resident landlord class, and possibly even the climate policy chosen by the local government.

Less intuitively even, picking a completely different spatial allocation does not alter the aggregate landlord benefit we found. Nothing in our discussion is specific to the particular landlord-tenant allocation analyzed above. For example, quite a different landlord-tenant allocation, of $\{(2,3),(1,2),(1,1)\}$, also yields an identical net aggregate landlord gain of 4, as in fact does any other allocation. This is easily verified by going through all (of the few) conceivable allocations here. As the subsequent sections also demonstrate, the insignificance of which particular spatial landlord-tenant allocation is chosen generalizes to any number of rings or dwellings – as does the significance of the urban silhouette.

3 Landlords, Tenants, Silhouettes

Consider a circular city extending from the CBD out to $\tilde{r}$. No specific assumption is made with respect to the city's shape. While our discussion best fits the notion of a circular

city, other (simpler or more irregular) shapes are justifiable, too. The city is split into n rings spaced equally far apart from each other. If distance from the CBD is r and if $\widetilde{r}$ is the urban boundary then the first of these rings extends from the CBD to $\widetilde{r}/n$, the second ring from $\widetilde{r}/n$ to $2\widetilde{r}/n$, and so forth. The number of dwellings within ring i is S_i, and is inherited from the past.

Landlords are resident, not absentee. Half of all urban dwellings $S = \sum_i S_i$ are populated by these landlords, the other half are inhabited by these landlords' tenants. The number S is even. There is no restriction on, or necessary pattern of, where landlords and tenants live. All households commute to the city center where they earn the exogenous urban wage w. For a resident in ring i, round trip commuting costs are tr_i, where r_i is the distance from the midpoint between ring i's outer and inner annulus to the CBD. For the moment we assume that within-ring-travel is costless, but further below we make rings arbitrarily small so that this assumption becomes less and less restrictive (section 5). Everyone consumes one unit of housing.

The city is closed, and is representative of every of the urban system's cities (Wheaton (1974), Brueckner (1987)). The increase in the carbon tax prominent in this paper will occur in all cities alike so that no inter city migration is to be expected. At the same time, competition among tenants for the best location within the city implies that tenant net income, i.e. income remaining after commuting cost and rent have been deducted, must be the same on every tenant-occupied plot, implying $q(r) + tr = t\widetilde{r} + q(\widetilde{r})$. Throwing in $q(\widetilde{r}) = 0$ (peripheral residents do not need to compete, given the abundance of land just one step beyond the urban fringe) joint with the assumption that all residents are perfectly mobile makes urban rent follow the following simple variant of Muth's condition,

$$q(r) = t\,(\widetilde{r} - r). \tag{1}$$

Rent in location r equals the commuting cost savings from living in r rather than $\widetilde{r}$.

Now consider a landlord who resides in ring i yet also rents out housing in ring j. This is a "match" $\{i, j\}$. For a landlord involved in such a match, utility is $v_{ij} = w - tr_i + q_j$. Dropping the wage for convenience, the full $n \times n$ matrix of landlord utilities residing in i and renting out in j becomes

$$V = \begin{pmatrix} -tr_1 + t(\widetilde{r} - r_1) & \dots & -tr_1 + t(\widetilde{r} - r_n) \\ & \vdots & \vdots \\ -tr_n + t(\widetilde{r} - r_1) & \dots & -tr_n + t(\widetilde{r} - r_n) \end{pmatrix} \tag{2}$$

Having a landlord and his/her tenant exchange locations does not affect that landlord's payoff, to the extent that $v_{ij} = v_{ji}$ for all i and j. So V as the matrix of landlord utilities is symmetric.[2] Moreover, V's counter diagonal (comprising all the elements on the diagonal stretching from the bottom left corner to the top right hand corner) consists of zeros only

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[2]Landlords not only will not want to switch locations with their corresponding tenants. Also, in view of tenant mobility manifesting itself according to (1) landlords have no incentive to rent out their own dwelling to become tenant elsewhere either. A landlord moving out of his owner-occupied dwelling in ring i to become tenant in j gains $tr_i + q_i$ in income yet also expends an extra, and equal sized, $tr_j + q_j$.

because $r_i + r_{n+1-i} = \widetilde{r}$. Put differently, matches for which row index i and column index j sum to $n + 1$ represent those perfect hedges for which the landlord's rental income is always just offset by her or his travel cost.

To V corresponds quite naturally a second matrix M of identical dimensions collecting the frequencies with which the various matches occur. In this "match matrix" the entry m_{ij} represents the number of times the match $\{i, j\}$ applies. The aggregate surplus accruing to the landlord class W_L may then be simply computed as

$$W_L = \iota'(M \circ V)\iota, \tag{3}$$

where $\circ$ is the entry wise (or Hadamard) product while ι is a commensurate (i.e. $n \times 1$) vector of ones. Now, in applications we are unlikely to be informed about the precise structure of landlord-tenant matches. Fortunately, these – unobservable – matches are intimately related to the – observable – housing units they are housed by. Before we explore this relationship we briefly define two urban aggregates that turn out to ease welfare accounting in the urban economy. The expression $t \sum_{i=1}^{n} S_i r_i$ represents aggregate urban travel or commuting cost (ATC), while the expression $t \sum_{i=1}^{n} S_i(\widetilde{r} - r_i)$ captures what henceforth we may refer to as aggregate land rent (ALR). Proposition 1 now states this section's result.

Proposition 1 (Urban Form and Political Economy)
(i) (Landlord Welfare): Landlord class welfare W_L is dependent on urban form, and is given by any of the following three formulas:

$$t\sum_{i=1}^{n}\left((\widetilde{r}/2) - r_i\right)S_i = t\sum_{i=1}^{n/2}\left((\widetilde{r}/2) - r_i\right)\left(S_i - S_{n+1-i}\right) = \left(ALR - ATC\right)\Big/2. \tag{4}$$

(ii) (Tenant Welfare): Tenant class welfare W_T is independent of urban form, and is given by either $-St\widetilde{r}/2$ or $-(ALR + ATC)/2$.
(iii) (Spatial Invariance): Both landlord and tenant welfare are invariant w.r.t. how landlords and tenants are allocated to rings 1 through n.

(The formal proof of this proposition and most other propositions is delegated to the Appendix.) To appreciate the simplicity of the formulas in Proposition 1's Part (i) note that the number of potential landlord-tenant matches that could possibly be housed by the existing structure of dwellings $(S_1, \dots S_n)$ is bound to be very large. Yet even so landlord and tenant welfare are entirely independent of how landlords and tenants are allocated to these rings or to the dwellings in them. For instance, we certainly do not restrict landlords to reside in the more attractive central parts of the city. What matters is the spatial distribution of housing units, not the specifics of individual matches (even as, of course, individual matches need to be consistent with the overall urban form.)

Here is an intuitive explanation of the first two formulas in (4). Even if (at first appearance) a very special case, for the moment consider a city in which the S_i are decreasing in i. Such a city is illustrated below, in the upper panel of Figure 1(a). Now assign landlords

and tenants to rings 1 through n by making use of the two following rules: (i) First, all S_n landlords to ring n live in ring 1, all S_{n-1} landlords to ring $n-1$ reside in ring 2, etc. And (ii), housing in ring i not occupied yet by the demands of rule (i) (and shown as the upper parts of those bars in the lower panel of Figure 1(a)) is equally shared between remaining tenants and their respective landlords. This special case makes for a particularly simple description of landlord welfare.

First, none of the landlords described by (i) receives any match benefit because for these landlords' matches indices i and j sum to $n+1$. And second, this in turn implies that all of those $(S_i - S_{n+1-i})/2$ landlords in rings $i = 1, \ldots, n/2$ addressed by rule (ii) (rather than by rule (i)) receive a utility of $(t\widetilde{r} - 2tr_i)$ each. Aggregating all landlord utilities across the first $n/2$ rings just yields the second expression given in (4). Generalizing this special case to an arbitrary allocation of landlords and tenants would proceed from the idea that interchanging landlord and tenant for any given match will not affect the match's benefit to the landlord. (Showing how an arbitrary allocation may be transformed into the one just discussed may be the basis for a general proof. The proof in the Appendix in fact proceeds from a somewhat different angle.)

Proposition 1's Part (i) offers yet another description of landlord welfare. According to the last expression in (4), landlord welfare W_L may also be expressed in terms of the two prominent aggregates ALR and ATC. Specifically, landlord welfare W_L may also be written as the difference $(ALR - ATC)/2$ even as of course, it might be emphasized, ALR includes (imputed) rent payments never received, just as ATC also includes commuting costs never incurred, by the landlord class. ALR and ATC allow for some straightforward urban welfare accounting. For instance, $ALR + ATC = St\widetilde{r}$. Geometrically, the sum of aggregate transport costs and aggregate land rent can be represented as a cylinder exhibiting a circular base area filled with S units of housing and exhibiting height $t\widetilde{r}$ (Mohring (1961)).

In their absentee landlord framework, Arnott/Stiglitz (1981) focus on whether changes in ALR rent adequately monitor changes in ATC. Arnott/Stiglitz look at the – ratio – of these urban aggregates and that ratio's role for society welfare, and find that except for simple cases that ratio not necessarily is one or one half. In our resident landlord framework in contrast, our interest is in the – difference – between these aggregates and this difference's implications for interest group welfare. In any case, the warning spelt out in Arnott/Stiglitz (1981) must be heeded in our framework, too, i.e. it should not be taken for granted that ATC necessarily is equal to, or twice the size of, ALR. We cannot, and in any case need not, expect a simple relationship to govern the relationship between ALR and ATC.[3]

Proposition 1's Part (ii) takes to the issue of aggregate tenant welfare. Whichever way tenant welfare is expressed, the city's spatial structure plays no role in it. Moreover, we now may easily affirm that utilitarian city welfare W reduces to $W_L + W_T = -ATC$. Once

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[3]For example, if S_i is the same in every ring i then indeed it is true that $ATC = ALR$. This is then true because then the average commuting distance $(\sum_{i=1}^{n} r_i)/n$ is $\widetilde{r}/2$. Yet even in the simple linear city this no longer is true if the S_i are not identical across rings.

we net out those redistributional rent payments ALR societal welfare of course is nothing but (aggregate wage income, omitted here for brevity) minus aggregate commuting costs. Finally, Proposition 1's Part (iii) generalizes the spatial invariance theme introduced in the previous section to any finite number of rings n and dwellings S. As emphasized above, the practical importance of this is to free us of having to model resident landlord or tenant location. Any subsequent discussion of either local political economy or overall welfare does in no way depend on the specific intra-city allocation of landlords and tenants.

Let us take a closer look at the first expression for landlord welfare in (4), ignoring t for the moment and referring to $\widetilde{r}/2$ as "midtown". Then the first expression may effectively be said to sum over weighted deviations of all commuting distances from the midtown commuting distance $(\widetilde{r}/2 - r_i)$, where weights are given by ring specific housing (or population) shares S_i/S. If we define rings between the CBD and midtown to constitute the city's "center" and rings stretching from midtown to the urban fringe to make up the city's "periphery" then the expression $W_L/t = S \sum_{i=1}^{n}((\widetilde{r}/2) - r_i)(S_i/S)$ could be thought of as a measure of the extent to which the distribution of urban commuting distances lacks symmetry about midtown.

Put differently, we may employ W_L/t as an indicator of the city's skew. If $W_L/t > 0$ we say that the city is skewed towards the periphery whereas if $W_L/t < 0$ it exhibits a skew towards the center.[4] To complement this discussion we also investigate the second expression for landlord welfare in (4), again ignoring t for the moment and referring to $(S_i - S_{n+1-i})$ as "ring differences". For example, for $i = 1$ this ring difference is the excess of housing in the innermost ring 1 over housing in the outermost ring n. Or, for $i = 2$ the corresponding ring difference is the excess of housing in the second innermost ring (ring 2) over corresponding housing in the second outermost ring (ring $n - 1$).

But then W_L/t or $\sum_{i=1}^{n/2}((\widetilde{r}/2) - r_i)(S_i - S_{n+1-i})$ may also be said to sum over weighted ring differences, with weights now represented by those deviations of commuting distances from the midtown commuting distance. Note that all the weights entering the previous sum are positive. Now suppose this sum is found to be positive. Then at least one ring difference need not only be strictly positive but must also be sufficiently positive to offset the effect of all those other ring differences assumed negative. From the distant observer's perspective, the single outstanding positive ring difference suggests an overall urban skew towards the periphery (even as it is single).[5] A related discussion applies to the case where more than one ring difference is positive, or where urban-skewness W_L/t is negative.

With the concept of urban skew at hand we now evaluate the different distributions of commuting distances, or cities, on display in Figure 1. Data on these distributions could have been collected from simple linear cities of width one. In this case these distributions do coincide with the cities' respective *silhouettes*. Alternatively, these data may come from cities of any shape. For these cities the notion of a one to one correspondence between the distribution of commuting distances and the city's night time silhouette clearly will

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[4]W_L/t merely resembles, yet is not equal to, the first central moment, given that in W_L the mean distance $\sum_{i=1}^{n} r_i(S_i/S)$ is replaced by midtown distance $\widetilde{r}/2$.

[5]Admittedly, the notion of skew need no longer be too intuitive for more general distributions than those considered in Figure 1. This is a feature shared with more standard definitions of skewness.
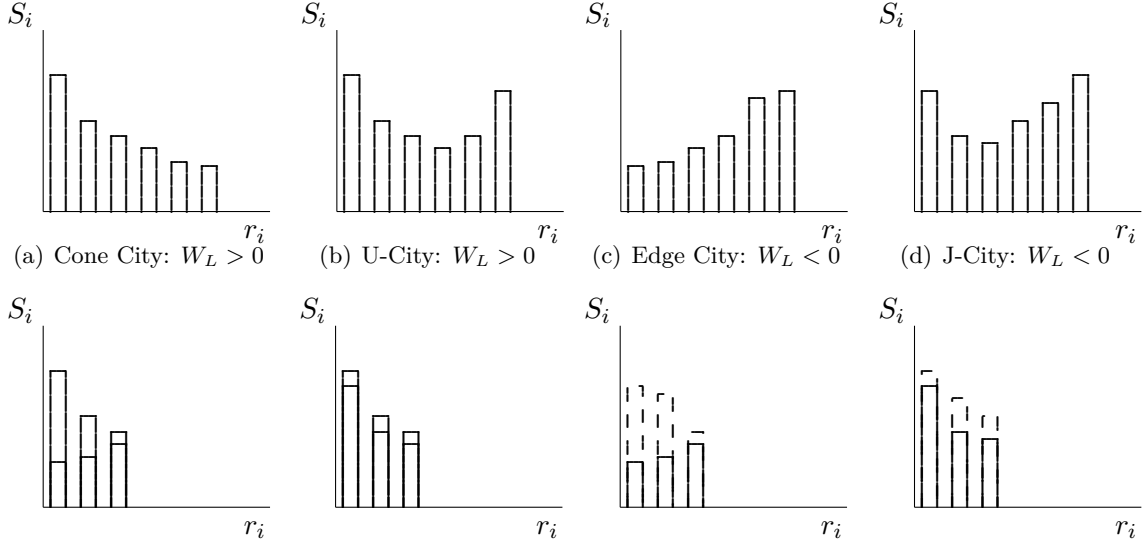
Figure 1: City Silhouette and Urban Skew

be less exact. Nonetheless in these cases also will we continue to refer to the distribution of commuting distances as the city silhouette. Now, four different urban silhouettes are shown in Figure 1 for the case of a six-ring city.

The first two-figure panel in (a) shows ring i housing S_i in a city skewed towards the periphery. In (a) the top picture has the city silhouette as it reveals itself to the distant observer while the bottom picture matches up the left part of that silhouette with the mirror image of its right part, with a vertical line passing through $\tilde{r}/2$ as the axis of symmetry (not shown). The bottom picture thus also indicates the excess of interior ring housing over outer ring housing, an excess that prominently features in (4) and that defies symmetry about midtown. A similar interpretation applies to the two figures in panel (b). In contrast, the two figure pairs in (c) and (d) exhibit cities skewed towards the center.[6] Bottom pictures in either panel indicate the deficits of inner ring housing when compared with matching outer ring housing.

Figure 1's silhouettes (a) through (d) indicate what could be referred to as a cone city, U-city, edge city and J-city, respectively. This categorization may be helpful in assessing the sign of W_L, as may be the monotonicity of S_i with respect to r_i. In a six ring city, for instance, a sufficient (though not a necessary) condition for W_L to be positive is $S_1 > \ldots > S_6$, as is true of the cone city in panel (a). Another sufficient condition (though again not a necessary one) combines inequalities $S_1 > S_6$, $S_2 > S_5$, $S_3 > S_4$, as exemplified by the U-city in panel (b). Of course, Figure 1 must not distract from the fact that many other silhouettes are conceivable, and that ring differences need (and typically will) not share the same sign.

Before turning to the carbon tax next, let us briefly explore the effect of non-linear trans-

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[6]In a linear city it is difficult to see ring housing numbers rise towards the periphery, given that rents are decreasing in distance to the CBD. At the same time it is conceivable to have ring housing stocks increase in r in a circular city context. This is why Figures 1(c) and (d) more appropriately capture the circular city.

port costs. Replace tr_i by $f(r_i)$ and $t\tilde{r}$ by $f(\tilde{r})$, where f merely is some function increasing in t, and observe that the matrix of landlord utilities V remains symmetric even as commuting costs are non-linear: $v_{ij} = -f(r_i) + (f(\tilde{r}) - f(r_j)) = -f(r_j) + (f(\tilde{r}) - f(r_i)) = v_{ji}$. Owing to this symmetry, the derivation of W_L that is central to Proposition 1 goes through virtually unchanged. Landlord welfare in the non-linear transport cost case becomes $W_L = \sum_{i=1}^{n}(f(\tilde{r})/2 - f(r_i))S_i$. Otherwise, while so far corresponding rings S_i and S_{n+1-i} received equal weight in the expression for landlord welfare, with non-linear commuting cost this no longer is the case. We expect the principles of subsequent analysis to remain unchanged, if not more difficult to expound.

4   Enter the Carbon Tax

Effectively any climate policy that comes in the disguise of a carbon tax also is a tax on carbon intensive commuting by car. Let us assume that unit commuting costs increase from t_0 to $t_1 = t_0 + \Delta t$, with Δt equal to the tax. Adjustments simultaneously unfold in every city throughout the urban system, and so the overall allocation of households remains unchanged. In Figure 2 the urban rent gradient rotates upwards, from the rent gradient's initial position (at vertical intercept $t_0\tilde{r}$ and horizontal intercept $\tilde{r}$) up to its new position (with vertical intercept $t_1\tilde{r}$ and identical horizontal intercept $\tilde{r}$).[7]

Rent in the CBD rises by the very extent to which the carbon tax drives up commuting costs at the urban fringe, or $\Delta t\,\tilde{r}$. Moreover, at any distance $r \in [0, \tilde{r})$ do rents rise, if to a lesser degree. It is at the urban fringe only that rent does not rise at all. This reflects the fact that being closer to the CBD now has even greater value. To illustrate consider a landlord resident in ring 5 and renting out in ring 1. As the carbon tax drives commuting costs up this landlord certainly suffers from the larger costs of commuting. For example, for a landlord in the fifth ring this loss equals the average height of lightly shaded trapezoid A_5'.

However, this landlord also benefits from risen rental income thanks to owning another property in ring 1. In Figure 2 this gain equals the average height of the lightly shaded trapezoid A_1. This latter gain more than offsets the landlord's loss from having to pay more for commuting. At the same time, of course, the benefit to the landlord comes at her tenant's expense. Generally tenants do not only suffer from those extra commuting costs but also suffer from having to pay those even greater premia for CBD-access that their housing commands vis-à-vis peripheral housing, at $\tilde{r}$. Following (1) we have $q(r) + tr = t\tilde{r}$, and hence each tenant's welfare loss is easily seen to equal $\Delta t\,\tilde{r}$, irrespective of her or his location.

Every tenant will always oppose this carbon tax. But landlord opposition against the carbon tax is far from equivocal. While the landlord we just considered in the example is better off other landlords surely are worse off. For instance, landlords owning two neighboring properties in the sixth ring clearly suffer. In spite of such landlord heterogeneity in

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[7]While in this section focus is on rents at distances r_i, $i = 1, \ldots, n$, the diagram allows for subsequent sections' analysis of rents at any distance between the CBD and the periphery, too.
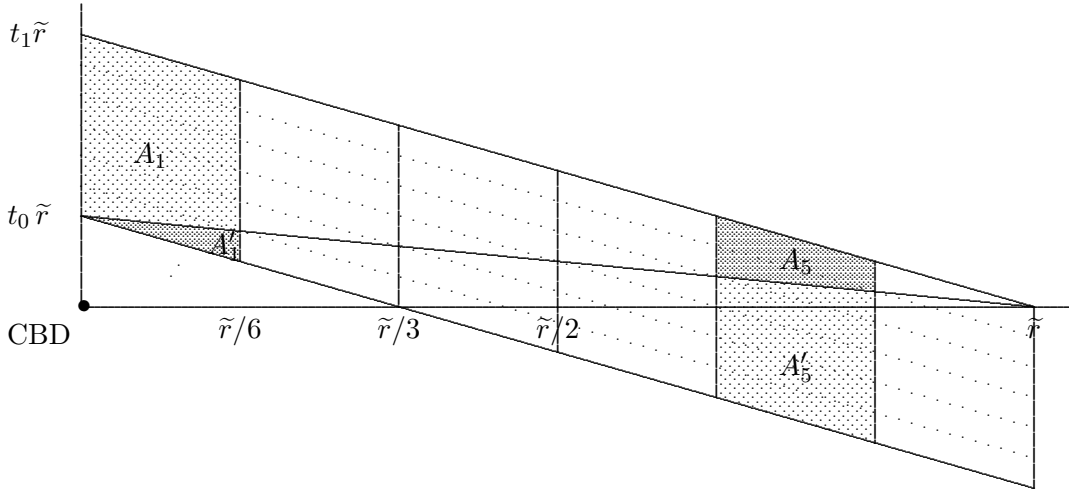
Figure 2: Welfare changes under the carbon tax

political interests our focus is the landlord class' welfare – the change of which is unambiguous, as we see next. We assume that a positive welfare gain for the class begets joint landlord action because landlords losing out under the policy not just can but also will be compensated by those who stand to benefit. We also suggest that at least one tenant will be swayed by the carbon tax's beneficial effect on the climate, identified below.[8]

This section's focus is on the short run, defined by a time horizon within which neither development nor decay take place. In this scenario the city boundary $\widetilde{r}$ cannot adjust. (For example, suppose the city shrank. Housing at the urban fringe would be lost without being replaced by any extra dwellings in the remaining city's interior. Some households would become homeless, contradicting equilibrium in the housing market.) But with the urban boundary given landlord class welfare reduces to a simple linear function of t. Taking the first derivative of the second expression in (4) we see the impact of a one Euro increase in the carbon tax to be

$$\frac{dW_L}{dt} = \sum_{i=1}^{n/2} \left((\widetilde{r}/2) - r_i \right) \left(S_i - S_{n+1-i} \right), \tag{5}$$

or simply W_L/t.

Will the carbon tax raise (reduce) W_L so that the landlord class will pursue an increase (a reduction, or no increase at least) in this tax? As is clear from (5), the answer to this question entirely depends on the city silhouette's skew. Cities with positive skew, and only those, will witness the landlord class welcome the carbon tax (Proposition 2, Part (i)). At

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[8]By itself, our treatment of landlords as a coherent interest group is nothing but an assumption that ultimately can only be justified by its explanatory power in explaining cross-sectional or time series variation. We return to this below.

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the same time, tenant class welfare simply is $-(S/2)t\tilde{r}$, and from this it is clear that the tenant class will always oppose the carbon tax (Proposition 2, Part (ii)). While the urban silhouette's skew informs landlord class interests it does not play into the tenant class' interests. Urban silhouettes have an influence on the intensity of landlord support of, or resentment against, climate policy but they have no influence on the intensity of tenant interests.

Finally, Proposition 2's Part (iii) claims the landlord class welfare gain and policy preference to carry over into the city's – and hence the federation's – actual policy choice. Landlords represent half of the electorate. At least one tenant may be won over by a side payment from (and easily afforded by) the landlord class when it comes to inaugurate the tax. No counter offers come from any of the co-tenants because none of these tenants can afford them. Alternatively, at least one tenant may be thought to be sufficiently environmentally motivated, placing the mere climate policy effort above the extra in his or her cost of living. This is true even as this policy effort has no direct climate benefit (as opposed to section 5, where aggregate commuting actually falls).[9]

Proposition 2 (Carbon Tax, Silhouette Skew, and the Carbon Tax: Short Run)
(i) (Landlord Class Support): The landlord class benefit from (lose under) the carbon tax and hence support its implementation if (and only if) the urban silhouette's skew is towards the periphery (center).
(ii) (Tenant Class Opposition): The tenant class are always worse off under, and hence oppose, the carbon tax.
(iii) (Climate Policy): The representative city supports (opposes) the carbon tax if (and only if) the urban silhouette is skewed towards the periphery (center).

Literally Proposition 2 suggests that a country's climate policy depends on its cities' silhouettes, all else equal. This may contribute to understanding better why the US do not, while Europe does, embrace carbon dioxide mitigation even as the US seem haunted more by extreme weather events often feared to be related to climate change and also are considerably richer. With the notable exception of New York and Chicago, intuitively the stylized US urban silhouette conforms more closely to the top row pictures (c) and (d) in Figure 1; whereas stylized European cities exhibit silhouettes closer to that Figure's top row pictures (a) and (b).

We briefly reinterpret Gaigné/Riou/Thisse (2012) from within this paper's framework. Gaigné/Riou/Thisse may be seen to argue that countries consisting of positive skew cities need not necessarily emit less carbon dioxide than countries made up of negative skew cities, if between city transport costs in the former type of cities exceed inter city transport costs in the latter and assuming that carbon tax levels are the same. However, Proposition 2 points to the very real possibility that carbon tax levels may not be identical. For example, cone city countries will exhibit higher carbon taxes than J-city-systems.

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[9]Even if carbon dioxide production levels remain the same revenues from the carbon tax might be used to prevent carbon dioxide's emission into the atmosphere.

Proposition 1 may even be another starting point for understanding the dynamics of sprawl, and the adverse climate effects that come along with it. Consider a country consisting of cities that exhibit negatively skewed urban silhouettes only (e.g., as illustrated by Figure 1's panel (c) or (d)). Resident landlord classes in this country might be tempted to convert currently negative welfare into W_L being positive (e.g., as illustrated by Figure 1's panel (a) or (b)). How could this be done? For the past three or four decades, suburbanization has not just had households migrate to the urban boundary. Local governments have also permitted, or even encouraged, employers and shops to relocate to that boundary, too (for the US on this see Kahn (2006)).

In a linear city context such sweeping suburbanization may be interpreted as a switch of the central business district from where it has earned its name out to the city fringe. The tenant class are not affected by this switch, their welfare equalling $-St\tilde{r}/2$ before and after that shift. But the resident landlord class become unambiguously better off. In a city exhibiting a negative skew initially, relocating the CBD to what then actually is the urban fringe converts its initial negative skew into one that is positive. If initial landlord welfare $W_L' = t\sum_{i=1}^{n/2}((\tilde{r}/2) - r_i)(S_i - S_{n+1-i})$ is negative then shifting the central business district to the urban fringe has the i-th ring become the $(n + 1 - i)$-th ring instead. Then new landlord welfare W_L'' becomes $W_L'' = t\sum_{i=1}^{n/2}((\tilde{r}/2) - r_i)(S_{n+1-i} - S_i)$ which is easily seen to be $-W_L'$ and hence is positive. For the landlord class there is a welfare gain of $-2W_L'$. The greater is the initial negative skew the more tempting is relocating the CBD.

This is a shift of the CBD which increases landlord welfare by reducing ATC only (leaving $St\tilde{r}/2$ unchanged). The initial negative skew of the urban silhouette may offer an explanation of why some countries have suburbanized much quicker than others. These countries' cities simply had a negative skew to begin with (Proposition 3, Part (i)). However, relocating the CBD must appeal to the landlord class even in cases where this raises ATC and hence is detrimental to the climate. In the context of the circular city CBD relocation is likely to not just shift but also disperse the CBD (along the ring road, say). This dispersal may be read as a concomitant rise in $\tilde{r}$. Oddly, this rise even adds to (rather than taking away from) the landlord incentive of dispersing the CBD. Far from introducing this paper's carbon tax, countries made up of negative skew cities may actively pursue CBD relocation even as this results in sprawl and rising emissions (Proposition 3, Part (ii)).

Proposition 3 (Silhouette, Center Relocation, Sprawl)
Consider countries whose urban silhouettes are skewed towards the center (periphery).
(i) (Political Economy of Sprawl): These countries encourage (discourage) a relocation of the CBD towards the urban fringe. There is suburbanization of shopping and employment.
(ii) (Silhouette and Commuting): These countries may (never) encourage a CBD relocation to, and a CBD dispersal at, the urban fringe even if aggregate commuting goes up.

We have not yet allowed emissions to respond to the tax. The following section remedies this by permitting the city to be "compactified". To address emission abatement we introduce the aggregate travelling distance $\sum_{i=1}^{n} S_i r_i = ATC/t$, or ACD. We suggest that carbon dioxide emissions E are a strictly increasing and differentiable function of

aggregate commuting, hence $E'(ACD) > 0$. Thus emissions will only fall if the distribution of commuting distances (our silhouette) adjusts favorably. Such an adjustment, and the circumstances under which it is to happen, is the central theme of the following section.

5 Carbon Tax, Silhouette Adjustment, Urban Greenness

Before turning to the analysis of the long run, let us reduce ring width first. We capture housing by a housing shape function $\Phi(r)$, indicating the number of housing units available between the CBD and r units of distance out. Then we can approximate the number of dwellings in ring i of width Δr, S_i, by $\Phi'(r_i)\Delta r \equiv \phi(r_i)\Delta r$. In the context of a circular city, $\phi(r_i)$ indicates available housing in a one-unit-width ring r_i units of distance away from the CBD. Landlord class welfare W_L in (19) can then be written as $t \sum_{i=1}^{n} (\widetilde{r}/2 - r_i)(\phi(r_i)\Delta r)$.

So far we have assumed that all commuting from ring i is via node r_i, and that within-ring commuting is costless. To remedy this we reduce ring width. As we let n tend to infinity the landlord class welfare W_L converges to

$$ t \int_{0}^{\widetilde{r}} \left(\widetilde{r}/2 - r\right) \phi(r)\, dr \qquad \text{or} \qquad St\left(\widetilde{r}/2 - \rho\right), \tag{6} $$

where $\rho = \int_{0}^{\widetilde{r}} (\phi(r)/S)\, r\, dr$ is the mean commuting distance. As before (Proposition 1, Part (i)), the first expression in (6) again is one possible indicator of the silhouette's skew about midtown. Alternatively, the second expression in (6) suggests to actually employ $(\widetilde{r}/2 - \rho)$, the deviation between midtown distance and mean distance, as our formula for the skew of the distribution of urban commuting distances. We continue to assume that cities inherit their silhouettes from their past – silhouettes that are relevant for their inclination to raise commuting costs. But we now add that changes in these silhouettes are possible if triggered by changes in rent. In fact, ϕ depends on distance precisely because ϕ must depend on rent. Naturally we suggest that increases (decreases) in rent drive ring i housing up (down), i.e. $\phi_q > 0$.

Of course rent in turn is a function of distance and tax, according to arbitrage condition (1). For brevity, whenever we write $\phi(t, r)$ below then $\phi(q(t, r))$ is meant. Besides requiring equation (1) to hold, a closed city equilibrium also posits that the urban population fit into the city area, or $S = \int_{0}^{\widetilde{r}} \phi(t, r)\, dr$. In the short run a carbon tax t does not affect the city's boundary $\widetilde{r}$. Now in the long run it does. Specifically, it must fall. (Suppose this were not so. Since rent at distance r is $t(\widetilde{r} - r)$ rent would rise at almost every distance. But then housing would rise at most distances from the CBD, too, and would nowhere fall. Aggregate housing supply would grow, contradicting the closed city presumption of the housing stock's full employment.) Climate policy makes the city contract, or $d\widetilde{r}/dt < 0$.

Knowledge of this response in turn permits us to discuss the resulting changes in rent further. Differentiating (1) with respect to t reveals that

$$ \frac{dq}{dt} = (\widetilde{r} - r) + t\frac{d\widetilde{r}}{dt}. \tag{7} $$

On the one hand, the carbon tax pulls down rents close to the urban boundary. This is because there the non-negative term $(\widetilde{r} - r)$ is close to zero whereas $t(d\widetilde{r}/dt)$ is strictly negative. This is a reflection of the fact that peripheral locations become less attractive. On the other hand, a higher tax drives up rents close to the city center.[10] Also note the cutoff $\widehat{r}$ implied by (7), i.e. $\widehat{r} = \widetilde{r} + t(d\widetilde{r}/dt)$. For distances beyond (below) $\widehat{r}$, rents fall (rise).

Implicitly differentiating $S = \int_0^{\widetilde{r}} \phi(t, r)\, dr$ leaves us with

$$\frac{d\widetilde{r}}{dt} = -\frac{1}{\phi(t, \widetilde{r})} \int_0^{\widetilde{r}} (d\phi/dt)\, dr, \tag{8}$$

where $d\phi/dt = \phi_q\,(dq/dt)$ is the local housing response to both the exogenous change in t as well as to the implied change in $\widetilde{r}$. While we drop the argument it should be kept in mind that the function $d\phi/dt$ depends on r. Since the boundary response on the l.h.s. of (8) is strictly negative, as argued above, the r.h.s. of (8) must be strictly negative, too. But then $\int_0^{\widetilde{r}} (d\phi/dt)\, dr$ must be strictly positive. An equivalent restatement of this is that more new apartments are constructed in what is the post-tax city than are lost to decay within it. This is very intuitive because the post tax city's structure somehow must make up for the housing lost to abandon outside it (Proposition 4, Part (i) below).

We evaluate the impact of a one Euro (increase in the) carbon tax for landlord welfare by consulting the derivative of the first expression in (6) with respect to t while also observing the responses of urban boundary and housing shape.[11] By Leibniz' rule,

$$\begin{aligned}
\frac{dW_L}{dt} &= \int_0^{\widetilde{r}} (\widetilde{r}/2 - r)\, \phi(r)\, dr \\
&+ t \int_0^{\widetilde{r}} (\widetilde{r}/2 - r)\, (d\phi/dt)\, dr \quad - \quad t\,\phi(\widetilde{r})\left(\widetilde{r}/2\right)\left(d\widetilde{r}/dt\right) \\
&+ t \int_0^{\widetilde{r}} \left(d\widetilde{r}/dt\right)\left(\phi(r)/2\right) dr\,.
\end{aligned} \tag{9}$$

The single term on the first line of (9) just is the urban silhouette's skew as discussed in our previous analysis (see text surrounding (5)). The stronger the distribution of commuting distances is skewed towards the periphery the greater is the landlord class' welfare gain from a carbon tax. Let us assume that the initial silhouette is skewed towards the periphery throughout what follows. Such a city will pursue a carbon tax in the short run context. But will it also do so in the context of the long run? To clarify this we turn to the remaining terms on the two following lines of (9).

Following (7) the carbon tax has rents and building stock (i) go up near the city center (at distances between 0 and $\widehat{r}$) as well as (ii) go down close to the new urban boundary

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[10]Rents close to the city cannot fall lest rents would have to fall throughout the city, causing the housing stock to contract while population remains unchanged, thereby violating the closed city equilibrium condition.

[11]We ignore marginal construction cost in what follows. Modeling the cost of constructing those extra units identified below requires a more detailed description of the initial urban equilibrium than we are willing to supply for our current purposes. Instead we loosely suggest that one time marginal construction costs pale against those recurrent benefits set out below.

(at distances beyond $\hat{r}$). Correspondingly the first term on the second line of (9) outlines the effect on landlord welfare of (i) new housing construction near the CBD and of (ii) surplus housing decay near the fringe. The net effect of this may be positive or negative. The second term on the same line captures the effect of abandoned housing on landlord welfare. This latter effect is always positive. Moreover, the sum of the two effects on the second line of (9) is unambiguous, and strictly positive (Proposition 4, proof of Part (i)). This is an intuitive result in view of the fact that city residents move to rings more likely to receive a positive weight in the formula for landlord class welfare.

Finally, the single term on the third line of (9) represents landlords' reassessment of the city's ring differences. This effect is negative. As the city shrinks in area, all ring differences receive but a lower weight. Now, while (9) supplies a breakdown of the individual effects of the carbon tax on landlord welfare, a more compact picture emerges when taking the derivative of the second expression in (6) instead, i.e. of $W_L = St \cdot (\tilde{r}/2 - \rho)$ or even $W_L = St \cdot Skew$. Then

$$\frac{dW_L}{dt} \;=\; S \cdot Skew \;+\; St \;\cdot\; \frac{d\,Skew}{dt} \tag{10}$$

This simple derivative traces the landlord class welfare change back to both (i) the initial silhouette's skew and (ii) also the implied change in that skew. A carbon tax that reduces skewness may still raise landlord welfare as long as the loss in skewness is not too large (Proposition 4, Part (ii)). Also, we easily see under which conditions the long run landlord class welfare change (10) will exceed that in the short run. Even though we cannot predict the change in skewness generally (because both $\tilde{r}$ and ρ fall as t goes up) this change is certainly measurable in applications.

Proposition 4 (Silhouette, Compactification, Mitigation: Long Run)
Introducing a carbon tax has the following long run effects per Euro of tax:
(i) (Structural Change): The city boundary $\tilde{r}$ recedes. New housing developments within the contracted city make up for abandoned housing outside, and decaying housing inside the, new boundary.
(ii) (Landlord Class Welfare): The landlord class benefit from the carbon tax if (and only if) the elasticity of the silhouette's skewness with respect to the tax is greater than minus one, i.e.

$$\frac{d\left(\tilde{r}/2 - \rho\right)}{dt} \frac{t}{\left(\tilde{r}/2 - \rho\right)} \;>\; -1. \tag{11}$$

(iii) (Tenant Class Welfare): Tenant class welfare falls.
(iv) (Urban Greenness) Aggregate commuting ACD falls, and so do the city's carbon dioxide emissions E.

Proposition 4 adds that tenant class welfare falls (Part (iii)). This is because the implied fall in $\tilde{r}$ is not enough to offset the initial rise in t. The product of these two variables, effectively capturing a tenant's cost of living, $t\tilde{r}$, rises still. Finally, Proposition 4 also

states that carbon dioxide emissions fall. In contrast to what we observe in the short run (section 4), a carbon tax now indeed reduces emissions, by reducing the commuting that produces them. More formally, $ACD = \int_0^{\widetilde{r}} \phi(r) r\, dr = S\rho$ responds to a one-unit increase in the carbon tax as follows:

$$\frac{d\,(ACD)}{dt} = \int_0^{\widetilde{r}} (d\phi/dt)\, r\, dr + \phi(\widetilde{r})\, \widetilde{r}\, (d\widetilde{r}/dt) < 0. \tag{12}$$

As the inequality in (12) reports, this latter response is strictly negative (Proposition 4, Part (iv)). Intuitively, shifting housing (and hence commuters) closer to the CBD, as is achieved by the carbon tax, reduces average and hence aggregate commuting. This aggregate commutes response is landlord policies' green contribution to fighting climate change.

6   Extensions: Owner-Occupiers and Open Cities

Owner-Occupiers: This far landlords have been individuals owning exactly two dwellings. We now introduce owner-occupiers (owning but the dwelling they live in) by breaking up one of the $S/2$ landlord-tenant matches. A note on terminology: we reserve the term homeowners to addressing both landlords and owner-occupiers. Breaking up a landlord-tenant match creates two fresh owner-occupiers: one who was landlord, plus another who was tenant, before being separated from one another. The ex-tenant is an inhabitant of ring i. Then not only do the homeowner class (i.e. the ex-landlord) lose the rent $q(r_i)$ they would have collected prior to separation. Also the homeowner class now include an extra member (i.e. the ex-tenant) contributing nothing but $-tr_i$ to class welfare.

In sum, and in comparison to the pure landlord class case, the new homeowner class have a benefit that is smaller by $q(r_i) + tr_i$, or using (1), just by $t\widetilde{r}$. The addition of two extra homeowners (and the simultaneous one individual reduction of landlords and tenants) has the homeowner class welfare retreat to $W_L - t\widetilde{r}$. More generally, in the case of H homeowners overall and H_i homeowners in ring i, homeowner class welfare W_H becomes

$$t\sum_{i=1}^n \left(\widetilde{r}/2 - r_i\right)\left(S_i - H_i\right) - t\sum_{i=1}^n H_i r_i = W_L - \left(H/2\right)t\widetilde{r}. \tag{13}$$

The first expression in (13) covers the short run (section 3) while the second also fits into the long run (section 5). As this latter expression documents, our earlier spatial invariance insight (Proposition 1, Part (iii)) also extends to more general land ownership constellations (Proposition 5, Part (i)). The more prominent the share of owner-occupiers in society is, the less emphasis do the homeowner class place on the silhouette's skew when making its decision on the carbon tax (Proposition 5, Part (ii)). According to (13), and as has been established earlier, if no housing is inhabited by owner-occupiers ($H_i = 0$) then homeowner class welfare is $(ALR - ATC)/2$. Alternatively, if all housing is occupied by owner-occupiers ($H_i = S_i$) then homeowner class welfare apparently reduces to $-ATC < (ALR - ATC)/2$.

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Proposition 5 (Owner-Occupiers and the Diminishing Role of the Silhouette)
(i) (Spatial Invariance): Homeowner class welfare W_H is independent of which urban ring landlords, tenants and even owner-occupiers are assigned to.
(ii) (Owner-Occupiers and Silhouette): While homeowner class welfare W_H is dependent on the urban silhouette, too, this silhouette's weight is decreasing in the owner-occupier share.

Few (if any) countries will exhibit an exclusive 50:50 landlord-tenant urban split. Proposition 5 plays an important role for generalizing our previous results to urban systems in which city constituencies are not evenly split, or where owner-occupiers enter, too. According to Proposition 5 the urban silhouette has a diminished role in such systems' cities. The extent to which the silhouette has a role in shaping local decision depends on (interacts with) the prevalence of owner-occupiership. Roughly, silhouettes play a role if owner-occupiership is small. But they play no role if owner-occupiership is pervasive. This observation need not take away from the paper's scope. Instead it may actually help testing for the silhouette's role in empirical analysis.

In the short run context the homeowner class welfare gain from a one Euro increase in the tax $W_L/t - (H/2)\widetilde{r}$ is linearly decreasing in H. From this we easily identify a homeowner threshold $\widetilde{H} = 2W_L/t\widetilde{r}$ at which the homeowner class's welfare gain from the carbon tax is zero. Graphing this threshold against W_L/t collects combinations of homeowner numbers H and urban skewness W_L/t for which the homeowning class remain indifferent to the carbon tax. Pairs $(W_L/t, \widetilde{H})$ "to the right" of this upward sloping locus may represent European countries with strong owner-occupiership yet also endowed with cities skewed towards the periphery. Italy and Spain may be examples here. These countries embrace the carbon tax. Pairs $(W_L/t, \widetilde{H})$ "to the left" of that locus may represent countries with a similar (or even higher) share of owner-occupiers yet also endowed with silhouettes skewed towards the center.

Open Cities: Many cities pursue local climate policies of their own, charging ahead with climate action plans or various policies such as bus and bicycle lanes, local green building ordinances, regulations on solar panels, etc. (Millard-Ball (2012)). Many of these instruments also have an impact on the cost of commuting, and hence on urban rents. Analysis of these instruments benefits from this paper's framework once we allow for migration across cities.[12] While we postpone a thorough analysis to a follow-up paper it may seem reasonable enough to hypothesize now and here that a city is more likely to embrace commuting cost increasing policies if (and only if) its silhouette is sufficiently positively skewed. This should be true even as introducing a carbon tax in a subset of cities only inevitably implies outmigration and tenant loss. Such an inter-city hypothesis will also more readily be testable than this paper's inter-country one.[13]

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[12]Section 5's modeling extension prepares the ground for this.

[13]It is also the more educated cities that introduce ambitious climate action plans (Millard-Ball (2012)). College towns in particular (i.e. towns in which the landlord-tenant dichotomy is particularly relevant) seem to frequently be reserving increasing parts of their road capacity to bus or bicycle use. Should these lanes contribute to increasing average commuting costs then – from this paper's perspective – reallocating road capacity to them may as much be motivated by college town landlords' growing collection of rent as by college town student residents' stronger environmental preferences.

7 Conclusions

We offer an explanation of the existing variation in greenhouse gas mitigation policies. This explanation is based on the variation in the skewness of a representative city's distribution of commuting distances (i.e. the skew of its "silhouette"). The resident landlord class benefit from introducing a carbon tax if – and only if – the urban silhouette is skewed towards the periphery. Ultimately we offer an urban form based theory of the political economy of the carbon tax.

In fact the city silhouette proves the sole model determinant of the landlord class's public choice irrespective of how we assign landlords and tenants to the city's rings. Further, the silhouette plays a role in local politics even if we account for the existence of mere owner-occupiers, too. Only, of course, the more prevalent owner-occupiers the less important the urban silhouette's role for climate politics. At the same time the silhouette retains its importance when placed into a long run context. In the long run the landlord class's benefit from the carbon tax can, and needs to, be expressed not merely in terms of the initial silhouette's skew but also in terms of how this silhouette skew adjusts over time.

Allowing for long term adjustment is an important theme of its own right. "Compactifying" the city may be desirable because it reduces mean commuting (and maybe even because, according to this paper, more compact cities are more likely to embrace the carbon tax). Only, many supposedly green policies appear to make "compactification" more difficult and costly. Building height requirements in the city center come to mind here, as do green building ordinances. Creating a consistent set of green policies truly seems a daunting task.

Extensions to the model could allow for also redistributing carbon tax revenues to households, for permitting landlords to own more than two dwellings, for inter city migration, or for landlords in part also being absentee. We expect none of of these amendments to fundamentally change the paper's results, however.

8 Literature

Arnott, R. and J. Stiglitz (1982) Aggregate Land Rent and Aggregate Transport Costs, Economic Journal 91: 331-347.

Borck, R. and M. Wrede (2005) Political Economy of Commuting Subsidies, Journal of Urban Economics 57: 478-499.

Borck, R. and M. Wrede (2007) Commuting Subsidies with Two Transport Nodes, Journal of Urban Economics 63: 841-848.

Brueckner, J. (1987) The Structure of Urban Equilibrium, in: E. Mills (ed.) Handbook of Regional and Urban Economics Vol. 2, North-Holland:

Desmet, K. and E. Rossi-Hansberg (2012) On the Spatial Impact of Global Warming.

Fujita, M. (1989) Urban Economic Theory: Land Use and City Size, Cambridge University Press.

Gaigné, C., St. Riou, and J.F. Thisse (2012) Are Compact Cities Environmentally Friendly?, Journal of Urban Economics 72.2: 123-136.

Glaeser, E. (2011) Triumph of the City, Macmillan.

Glaeser, E. and M. Kahn (2010) The Greenness of Cities. Carbon Dioxide Emissions and Urban Development, Journal of Urban Economics 67.3. 404-418.

Kahn, M. (2006) Green Cites. Urban Growth and the Environment, Washington: Brookings.

Larson, W., L. Feng and L. Yezer (2012) Energy Footprint of the City: Effects of Urban Land use and Transportation Policies, Journal of Urban Economics 72.2: 147-159.

Millard-Ball, A. (2012) Do City Climate Plans Reduce Emissions?, Journal of Urban Economics 71.3: 289-311.

Mohring, H. (1961) Land Values and the Measurement of Highway Benefits, Journal of Political Economy 69.3: 236-249.

Schelling, Th. (1992) Some Economics of Global Warming, American Economic Review 82.1: 1-14.

Stern, N. (2006) Review on the Economics of Climate Change.

Tol, R. (2009) The Economic Effects of Climate Change, Journal of Economic Perspectives 23.2: 29-51.

Wheaton, W. (1974) A Comparative Static Analysis of Urban Spatial Structure, Journal of Economic Theory 9: 223-237.

Wildasin, (1984) Urban Public Finance, Chur.

9 Appendix

Lemma (Properties of housing matrix U):

(i) (Symmetry): The housing matrix U is symmetric.

(ii) (Housing Supply in Ring i): Housing supply in ring i, S_i, is given by $\sum_{j=1}^{n} u_{ij}$ or $\sum_{j=1}^{n} u_{ji}$.

(iii) (Equivalent Landlord Benefit Representation): Landlords' welfare W_L can be represented as the expression on either side of:

$$\iota'\left(M \circ V\right)\iota \;=\; \iota'\left(U \circ V\right)\iota/2.$$

Proof of the Lemma:

(i) (Symmetry): Note that $U' = (M + M')' = M' + M = U$. $\square$

(ii) (Housing Supply in Ring i): By definition of U, $u_{ij} = (m_{ij} + m_{ji})$. Hence summing over all entries in U's row i gives $\sum_j u_{ij}$ or $\sum_j m_{ij} + \sum_j m_{ji}$. This latter sum is the sum of all entries in M's column i and row i, respectively, and hence total housing in i. Finally, summing over all entries in U's column i also gives total housing in i because U is symmetric (cf. Property (i)). $\square$

(iii) (Equivalent Landlord Benefit Representation): Let us define $D = M \circ V$. Now consider $d_{ij} = m_{ij}\, v_{ij}$ and $d_{ji} = m_{ji}\, v_{ji}$. Calculating B entails summing over all entries in D and hence including $d_{ij} + d_{ji}$ in this sum. But

$$
\begin{aligned}
d_{ij} + d_{ji} &= (m_{ij}\, v_{ij}) + (m_{ji}\, v_{ji}) \\
&= (m_{ij} + m_{ji})\, v_{ij} \\
&= u_{ij}\, v_{ij} \\
&= (u_{ij} + u_{ji})\, v_{ij}/2.
\end{aligned}
\tag{14}
$$

Here the first equality reflects our definition of D, the second equality exploits the fact that V is symmetric, the third equality enters the definition of U in (9), and the fourth equality makes use of U's symmetry.

B obtains when summing over all of these $d_{ij} + d_{ji}$, i.e. when computing the expression on the l.h.s. of (9). Alternatively, B also obtains when summing over all of those $(u_{ij} + u_{ji})\, v_{ij}/2$, i.e. when computing the expression on the r.h.s. of (9). $\square$

Proof of Proposition 1:

(i) (Landlord Welfare) Consider M's first column first. This column has all those landlords renting out to tenants in ring 1, irrespective of which ring these landlords live in themselves. Next consider M's first row. This row collects all those landlords living in ring 1. Surely then summing row 1 and column 1 gives the total number of residents (be they landlords or tenants) in the first ring. More generally, summing over row i and column i gives the total number of residents in ring i. We suggest the construction of a matrix of urban housing, U,

$$U \;=\; M + M',$$

a matrix that, by design, has the three properties detailed in the Lemma (at the beginning of the Appendix).

Now, V may be decomposed into the simple sum of two even more strongly patterned matrices:

$$
V = \begin{pmatrix} -tr_1 & \dots & -tr_1 \\ \vdots & & \vdots \\ -tr_n & \dots & -tr_n \end{pmatrix} + \begin{pmatrix} t(\widetilde{r} - r_1) & \dots & t(\widetilde{r} - r_n) \\ \vdots & & \vdots \\ t(\widetilde{r} - r_1) & \dots & t(\widetilde{r} - r_n) \end{pmatrix} \tag{15}
$$

where the first (commuting costs) matrix is labeled V_1 and the second (rents) matrix is referred to as V_2. The value of this decomposition lies in representing V as the sum of two matrices that either have identical rows (in the case of V_1) or identical columns (as with V_2).

With this decomposition joint with employing the Lemma's Property (iii) we may alternatively rewrite landlords' welfare (3) via

$$
\begin{aligned}
2\,W_L &= \iota' \left(U \circ (V_1 + V_2) \right) \iota \\
&= \iota' \left(U \circ V_1 \ + \ U \circ V_2 \right) \iota \\
&= \iota' \left(U \circ V_1 \right) \iota \ + \ \iota' \left(U \circ V_2 \right) \iota.
\end{aligned} \tag{16}
$$

Here the second equality makes use of the distributive law for Hadamard products and the third equality conforms to the standard rules of conventional matrix multiplication.

We analyze the sum on the r.h.s. of the last equality in two steps. First consider the second term here. In it the expression $\iota'(U \circ V_2)$ is nothing but a $1 \times n$ vector exhibiting $t(\widetilde{r}-r_1) \cdot \sum_i u_{i1}$ in column 1, $t(\widetilde{r}-r_2) \cdot \sum_j u_{i2}$ in column 2, $t(\widetilde{r}-r_3) \cdot \sum_j u_{i3}$ in column 3, etc. Yet these products in turn reduce to $t(\widetilde{r} - r_1)S_1$, $t(\widetilde{r} - r_2)S_2$, $t(\widetilde{r} - r_3)$ etc. respectively because the sum of all entries in column i of U just represents total housing in ring i (Lemma, Property (ii)). We conclude that the second term on the last line of (16) is

$$
\iota' \left(U \circ V_2 \right) \iota \ = \ \sum_{i=1}^{n} t(\widetilde{r} - r_i)\, S_i. \tag{17}
$$

Incidentally, this expression also captures aggregate land rent (ALR).

We can trace through a similar argument when analyzing the first term on the r.h.s. of the last equality in (16). There the expression $(U \circ V_1)\,\iota$ is nothing but the $n \times 1$ vector containing $(-tr_1) \cdot \sum_j u_{1j}$ in row 1, $(-tr_2) \cdot \sum_j u_{2j}$ in row 2, $(-tr_3) \cdot \sum_j u_{3j}$ in row 3 etc. As before, these latter sums may be rewritten as $(-tr_1)S_1$, $(-tr_2)S_2$, $(-tr_3)S_3$, etc. Thus we have shown that

$$
\iota' \left(U \circ V_1 \right) \iota \ = \ \sum_{i=1}^{n} (-tr_i)\, S_i, \tag{18}
$$

This latter expression also is (the negative of) aggregate transport or commuting costs (ATC).

We join equations (16), (17) and (18), and collect terms. This yields

$$
W_L = t \sum_{i=1}^{n} \left((\widetilde{r}/2) - r_i \right) S_i, \tag{19}
$$

as claimed in the Proposition. $\square$

Proof of Proposition 3:

(ii) (Silhouette and Commuting): Consider a shift of the CBD to the urban fringe that comes along with a simultaneous increase of $\widetilde{r}$, reflecting the idea that a CBD on the urban fringe also represents a dispersal of the city. (A more detailed analysis is postponed to a later paper.)

Making use of the fact that $(\widetilde{r}/2 - r_i) = (\widetilde{r}/2n)(n+1-2i)$ we may write initially negative landlord class welfare as

$$W_L' \;=\; (t\widetilde{r}'/2n) \sum_{i=1}^{n/2} (n+1-2i)\,(S_i - S_{n+1-i}). \tag{20}$$

Shifting the central business district now not just reverses the ring order (as in Part (i)) but also increases $\widetilde{r}$ to $\widetilde{r}'$. Then from (20) it is obvious that new landlord welfare increases to even more than $-W_L' > 0$.

At the same time, aggregate commuting may increase. If $\widetilde{r}$ did not change at all then the rise in landlord welfare due to CBD location would entirely be attributable to the decrease in ATC. This decrease in fact equals $\Delta ATC = -2W_L'$. Yet ATC also is a strictly increasing function of $\widetilde{r}$. There exists a level of $\widetilde{r}$, $\widetilde{r}''$ say, for which $ATC'' > ATC'$. $\square$

Proof of Proposition 4:

(i) (Structural Change): We first show that $d\widetilde{r}/dt < 0$. Suppose $d\widetilde{r}/dt \geq 0$ instead. On the one hand $dq/dt > 0$ at any non-zero distance from the CBD, given (7), On the other hand $\phi_q > 0$, as assumed. But then $d\phi/dt = (\phi_q)(dq/dt) > 0$ at any non-zero distance from the CBD, too. But then

$$\int_0^{\widetilde{r}} (d\phi/dt)\,dr \;+\; \phi(\widetilde{r})\,(d\widetilde{r}/dt) \;>\; 0, \tag{21}$$

given that the first term is positive while the second term is non-negative. Slightly rearranging this inequality reveals that it contradicts (8). We conclude that $d\widetilde{r}/dt < 0$.

Next we show that the sum of the two terms on the second line of (9) is positive. We observe that

$$0 \;<\; -\phi(\widetilde{r})(d\widetilde{r}/dt) \;=\; \int_0^{\widetilde{r}} (d\phi/dt)dr \;=\; \int_0^{\widehat{r}} (d\phi/dt)dr \;+\; \int_{\widehat{r}}^{\widetilde{r}} (d\phi/dt)dr, \tag{22}$$

The inequality in (22) follows from $(d\widetilde{r}/dt) < 0$, established above. The first equality in (22) is a restatement of (8). The second equality decomposes the overall housing adjustment within the remaining city into partial housing decay in peripheral areas (the second integral on the r.h.s. of the last equality in (22), with negative integrand) and new property development in more central areas (the first, and positive, integral on the r.h.s. of that last equality, with positive integrand).

We multiply the positive sum on the r.h.s. of the last equality of (22) by $(\widetilde{r} - \widehat{r})$, a positive number itself. This gives

$$\begin{aligned}
0 \;&<\; \int_0^{\widehat{r}} (\widetilde{r} - \widehat{r})\,(d\phi/dt)\,dr \;+\; \int_{\widehat{r}}^{\widetilde{r}} (\widetilde{r} - \widehat{r})\,(d\phi/dt)\,dr \\
&<\; \int_0^{\widehat{r}} (\widetilde{r} - r)\,(d\phi/dt)\,dr \;+\; \int_{\widehat{r}}^{\widetilde{r}} (\widetilde{r} - r)\,(d\phi/dt)\,dr \\
&=\; \int_0^{\widetilde{r}} (\widetilde{r} - r)\,(d\phi/dt)\,dr
\end{aligned} \tag{23}$$

Substituting variable weights $(\tilde{r} - r)$ for the fixed weights $(\tilde{r} - \hat{r})$ magnifies the first (new property development) integral yet shrinks the second (existing property decay) integral. Thus the difference between the two integrals must become stronger, which explains the second inequality in (23).

After substituting (8) into the last term in (23) we get the sum of the two terms on the second line of (9) (net property development):

$$0 \; < \; t \int_0^{\tilde{r}} (\tilde{r} - r)\,(d\phi/dt)\,dr \; = \; t \int_0^{\tilde{r}} \left(\tilde{r}/2 - r\right)(d\phi/dt)\,dr \; - \; t\,\phi(\tilde{r})\left(\tilde{r}/2\right)\left(d\tilde{r}/dt\right)$$

Hence we have shown that the two terms on the second line of (9) sum to a positive number. The effect of net property development on landlord welfare is always positive. $\square$

(iii) (Tenant Welfare) We first note that equation (8) may more explicitly also be spelt out as

$$0 \; = \; \int_0^{\tilde{r}} \phi_q(\tilde{r} - r)\,dr \; + \; \left(\int_0^{\tilde{r}} \phi_q\,t\,dr \; + \; \phi(\tilde{r})\right)\frac{d\tilde{r}}{dt} \tag{24}$$

But then

$$\begin{aligned}
\frac{d(t\tilde{r})}{dr} \; &= \; \tilde{r} \; + \; t\,\frac{d\tilde{r}}{dt} \\
&= \; \left(\tilde{r}\,\phi(\tilde{r}) + t\int_0^{\tilde{r}} \phi_q\,r\,dr\right)\Big/\left(\int_0^{\tilde{r}} \phi_q\,t\,dr \; + \; \phi(\tilde{r})\right).
\end{aligned} \tag{25}$$

Here the first equality merely reproduces (7), while the second follows from making use of the solution for $(d\tilde{r}/dt)$ implied by (24). With this we conclude that each tenant's utility, equal to $-(q(r) + tr)$ or $-t\tilde{r}$, falls as t rises. Moreover, the tenant class welfare change is independent of urban form. $\square$

(iv) (Urban Greenness) Consider the two terms on the r.h.s. of (12). Now,

$$\begin{aligned}
&\int_0^{\tilde{r}} (d\phi/dt)\,r\,dr \; + \; \phi(\tilde{r})\,\tilde{r}\,(d\tilde{r}/dt) \\
< \quad &\int_0^{\hat{r}} (d\phi/dt)\,\hat{r}\,dr \; + \; \int_{\hat{r}}^{\tilde{r}} (d\phi/dt)\,\hat{r}\,dr \; + \; \phi(\tilde{r})\,\hat{r}\,(d\tilde{r}/dt) \\
= \quad &\hat{r}\left(\int_0^{\tilde{r}} (d\phi/dt)\,dr \; + \; \phi(\tilde{r})\,(d\tilde{r}/dt)\right) \; = \; 0
\end{aligned} \tag{26}$$

As to the inequality, note that $\hat{r}$ puts a smaller (larger) weight on the positive (negative) integrand in the first (second) integral than does the previous weight r. Hence the sum of the resulting two integrals must be larger. Moreover, by replacing $\tilde{r}$ by the smaller $\hat{r}$ also reduces the weight of the last, negative, term. The first equality is straightforward, while the second one follows from employing (8). $\square$