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# Economic Decisions In Producing Irrigated Grain Sorghum On the Northern High Plains of Texas

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## Foreword

American farmers have a wealth of research data available to them. Much of the data is concerned with technical problems of increasing yields, improving quality and reducing costs. In nearly all instances the improved practices can be applied by a farmer only at a cost. The farmer must decide if the improvement in output, quality or efficiency is enough to justify the extra cost, or more likely, must decide to what extent these practices should be applied. The extent to which any single agricultural input should be used cannot be determined without considering the use and effect of other inputs and the costs of all inputs and the price of the product.

This publication is concerned with the amounts of nitrogen and irrigation water to use in the production of grain sorghum on the Northern High Plains of Texas. There is a strong interacting effect between these two inputs; each is dependent on the presence of the other for much of the yield response from its use. Few studies have attempted to deal with both of these inputs as simultaneous variables.

The experiment from which the data for this report were obtained was designed to study some of the physiological phenomena of production. While the data are not sufficient for a complete economic analysis and are not typical of commercial production situations, they are the best data of this type available. The primary objective of this publication is to illustrate the decision-making process necessary for determining the optimum combination of inputs for any price situation. Estimates of the amounts of nitrogen and water to apply for any given production situation must remain a minor objective since the responses have not been verified for commercial production conditions. To transfer the results of this analysis to a farm situation would require knowledge of how the response on a graded irrigation system would compare to the response on level experimental plots; how the narrower row spacing in the experiment affected responses; and how closer control of planting time, irrigating, harvesting and weeds than is possible on commercial applications will affect results.

The report is organized into three sections. The first section is an outline of the economic procedures used in the illustration. It is a reference for those not familiar with techniques of economic analysis. The reader may wish to scan this section rapidly and refer to it as needed to clarify later sections. The second section uses data from research plots to illustrate the type of decisions that a farmer should make to obtain maximum profit. The final section deals with limitations on applying experimental results to actual farming situations.

# Economic Decisions in Producing Irrigated Grain Sorghum on the Northern High Plains of Texas

James S. Wehrly, Willis H. Sletten and Marvin E. Jensen*

## Types of Economic Decisions Applicable to Irrigation Farming

PROSPECTS FOR HIGHER PRICES for farm products in the next few years are not good while prices paid by farmers for production items are expected to continue rising. For farmers to maintain a profitable business in the cost-price squeeze, crops must be produced as economically as possible. To obtain maximum net returns it is necessary to use variable inputs at levels which yield maximum returns for the expenditures on the inputs used. Managers of irrigated farms in the High Plains areas of Texas have two major inputs, irrigation water and nitrogen fertilizer, which can be varied to give maximum profit for a wide range of possible price situations.

### MAXIMUM PROFIT LEVEL OF A SINGLE INPUT

For maximum profit, an input should be used at the level where the return from the last unit used is just enough to pay the cost of that unit. The level at which an input should be used will depend on (1) the cost of the input, (2) the value of the product and (3) the amount that an added unit of the input increases yield.

The amount that an added unit of the input will increase output depends on (1) the physical and biological limitations of the plant, soil and environment, (2) the amount of the input being used and (3) the amounts of other inputs being used. A farmer's control over the first item is limited to such choices as variety, tillage practices, timing of operations and insect and disease control measures. In many instances he has little relevant choice in this area since the possible savings from using an alternative practice are negligible compared to the yield loss from not using the best practice. These factors are often disposed of under the nebulous term "level of technology," which is usually assumed to be fixed for any given production situation. In the cases of irrigation water and fertilizer, a farmer has possibilities for varying the inputs to get the optimum response for the relevant combination of input and product prices.

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The lower curve in Figure 1 represents the expected yield response for different levels of nitrogen when the level of water application remains unchanged. The upper curve represents the expected yield response when a higher level of water is used. At low levels of nitrogen application the yield increase for each additional unit of nitrogen added is relatively large. The yield response from each additional unit becomes progressively smaller as the level of nitrogen is increased. At the lower level of water application an added unit of nitrogen, increasing the application from 2 to 3 units, produces an additional 450 pounds yield. The same amount of nitrogen added, increasing the application from 10 units to 11 units, produces only 50 pounds additional yield. It is physiologically possible, although in actual situations it may require unreasonably high nitrogen applications, to actually reduce the yield by adding too much nitrogen as illustrated by applications of 13 or more units with the lower level of water application.

Increasing the nitrogen application from 2 to 3 units, which produced an additional 450 pounds of grain at the lower water level, would produce an additional 550 pounds of grain at the higher water level. At the higher level, increasing the application from 10 units to 11 units increases the yield 150 pounds as compared to 50 pounds at the lower water level.

## contents

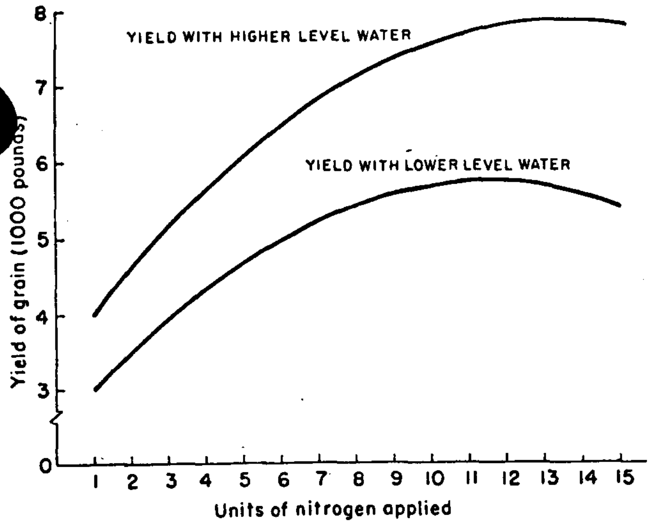



**Figure 1. Hypothetical yield response to nitrogen.**

Determination of the most profitable level of nitrogen application requires knowledge of nitrogen and grain prices, as well as the production responses illustrated above. For maximum profit, the amount of nitrogen used should be increased until the last unit used produces just enough yield increase to pay for the unit of nitrogen. If nitrogen costs \$6 per unit and grain is worth \$2 per hundred pounds, it would require a yield increase of 300 pounds to pay for 1 unit of nitrogen. For these prices the most profitable rate of nitrogen application would be 6 units if water were applied at the lower level, and 8 units if water were applied at higher level. If the price of grain should fall to \$1.50, it would require 400 pounds yield increase to pay for a unit of nitrogen. This would change the most profitable nitrogen application to 4 units and 6 units for lower and higher water applications, respectively. If the cost of nitrogen were \$3 per unit instead of \$6, and the price of grain remained at \$1.50, it would require 200 pounds yield increase to pay for a unit of nitrogen. The maximum profit applications for these prices would be 8 units with the lower level of water and 10 units with the higher level.

This method of finding the maximum profit level of a single input may be expressed as a mathematical equation:

$$\frac{\Delta Y}{\Delta N} = \frac{P_N}{P_Y}$$

$\Delta Y$ and $\Delta N$ are the changes in the amounts of yield and nitrogen, respectively, and $P_Y$ and $P_N$ are prices for grain and nitrogen, respectively.

The discussion above assumed that water would be applied at one of two arbitrary levels.

Water, as well as nitrogen, is an economic variable in irrigation farming. It is available at a cost, and for maximum profit it should be applied at a level where the added yield from additional water is just enough to pay for the additional water.

The same general type of analysis used for nitrogen could be used for water, where in Figure 1 the horizontal axis would measure units of water applied, and the two curves would show yield response with different levels of nitrogen use.

## SUBSTITUTION OF ONE INPUT FOR ANOTHER

In most production situations it is possible to substitute one input for another, within limits, without changing the level of production. When two or more inputs are economic variables in a production process, the maximum profit decision requires determining the relative amounts of each of the inputs to use as well as the total amount of all inputs. The relative amounts, or combination of inputs is determined by the relative prices of the inputs and the technical substitution possibilities. The total amount of all inputs used is determined by (1) the relationship between the input prices and the product price and (2) the yield response from the inputs used in the most economical combination.

The method of selecting the maximum profit combination of inputs is illustrated in Figure 2. The three curves in this figure, technically known as iso-product contours, may be thought of as level contour lines around a hill. The vertical height on the hill represents the yield. The three iso-product contours represent hypothetical yields of 6,000, 7,000 and 7,500 pounds per acre. In studying a figure of this type it should be remembered that the contours shown are only a few arbitrarily-selected yield levels out of a large number possible. Iso-product contours are a graphical representation of technical substitution possibilities.

Any point on a contour gives a theoretical combination of inputs that might be used to produce that particular output level. Figure 2 shows that a 6,000 pound yield could be obtained with 1 unit of nitrogen and 20 units of water, 3 units of nitrogen and 13 units of water, 6 units of nitrogen and 11 units of water, or any one of the many other combinations from other points on the contour.

Near the ends, the contours tend to become nearly parallel to the coordinate axes. This indicates that inputs at these levels are beyond the practical range of substitution. Further decreases in the amount of one input cannot be compensated for by increases in the amounts of other inputs if yield levels are to be maintained.

The maximum profit combination of inputs for any given level of output is at the point where the rate of substitution between the two inputs is equal to the inverse price ratio. Mathematically, the relationship may be expressed:

$$\frac{|\Delta N|}{|\Delta W|} = \frac{P_W}{P_N}$$

where $|\Delta N|$ and $|\Delta W|$ indicate the absolute value of changes in the amounts of nitrogen and water, and $P_N$ and $P_W$ indicate the prices of the two inputs. On Figure 2 this point is illustrated as the point where the iso-product contour is tangent to the appropriate iso-cost line for the prices of the inputs. Iso-cost lines are illustrated by lines ab and ac in Figure 2. The lines show the combinations of inputs that can be purchased for a given expenditure, with prices fixed at given levels. Line ab is an iso-cost line for equal prices for units of water and nitrogen. If the price is \$2 per unit the line represents an expenditure of \$20. For \$20 one can purchase 10 units of water, 8 units of water and 2 units of nitrogen, 4 units of water and 6 units of nitrogen, 10 units of nitrogen, or any one of the many other combinations indicated by other points on the line. Line ac is an iso-cost line for a situation in which a unit of nitrogen costs twice as much as a unit of water, or \$4 if the water price is \$2. The lines a'b' and a"c" are iso-cost lines of the same families of lines as ab and ac, respectively.

If the price per unit is the same for nitrogen and water, the maximum profit combination for producing a 6,000-pound yield is shown by point $P_1$, the point of tangency between the iso-product contour and line a'b'. This combination is approximately 3.6 units of nitrogen and 12.8 units of water. If the price of a unit of nitrogen were twice the price of a unit of water, the maximum profit combination would be 2.6 units of nitrogen and 14.2 units of water as indicated by point $P_4$. As price ratios change it becomes more profitable to use more of the relatively cheaper input and less of the relatively more expensive input. The maximum profit combinations for the two price situations are indicated by points $P_2$ and $P_5$ for the 7,000-pound yield, and by $P_3$ and $P_6$ for the 7,500-pound yield.

It should be noted that the increase in one input to compensate for a decrease in another input is for a *fixed yield level.* The technique is used to determine the most economical combination of inputs for producing a given yield. In the following section, in which these principles are illustrated, it is shown that an increase in the price of one of the inputs decreases the most profitable level of production. As a result, the level of application of both inputs is decreased for maximum profit production, even though the level of the one input is increased for the most efficient production of a given yield level.

The lines $P_1P_2P_3$ and $P_4P_5P_6$ in Figure 2 are known as expansion paths. When the prices of the inputs are known, the level of production is determined by the relative prices of the inputs and the product. If the price per unit is the same for water and nitrogen, production will be expanded along the line of maximum-profit combinations $P_1P_2P_3$ as the price of the product rises relative to the cost of the inputs. For a very low product-input price ratio, production will be near $P_1$. As the price ratio increases, the maximum profit level of production moves toward $P_3$.

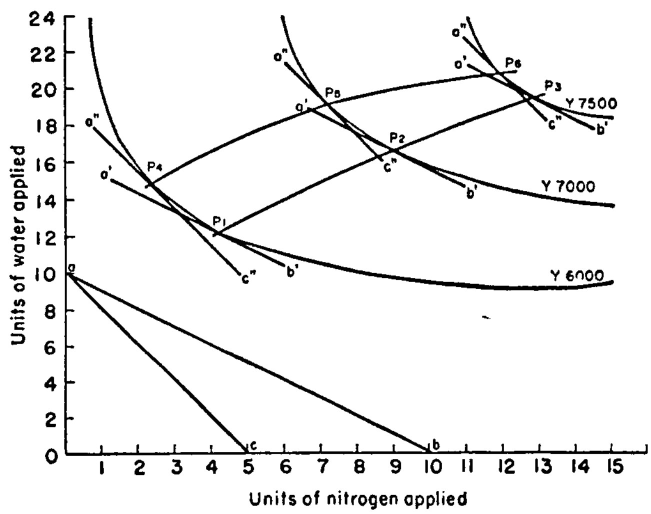



**Figure 2. Hypothetical iso-product contours, iso-cost lines and expansion paths for water and nitrogen inputs.**

## MAXIMUM PROFIT COMBINATIONS WITH MORE THAN ONE VARIABLE INPUT

To apply the previously discussed graphical method to actual data to determine maximum-profit combinations would be an extremely cumbersome trial and error process. The same results can be obtained in a precise manner by using differential calculus.

The first step in obtaining these maximum profit combinations is to fit a regression equation that gives a numerical estimate of the yield response from each of the inputs and the interaction between these inputs, stated as:

$$Y = f(W_2, W_3, N)$$

which is read, yield is a function of water applied in period 2, water applied in period 3 and nitrogen. Each variable may appear more than once in the equation in different forms to express the response in realistic terms and to show the interaction between variables.

The second step is to obtain a partial derivative of the regression equation with respect to each of the input variables. In the case of the nitrogen variable the partial derivative is indicated by symbols:

$$\frac{\partial Y}{\partial N}$$

which may be interpreted as the change in yield caused by a change in nitrogen, when the change in nitrogen is infinitely small and there is no change in the amounts of water applied. This might be compared to data in Figure 1, where the change in yield caused by increasing the nitrogen level from 3 units to 4 units is 400 pounds with the low level of water. If one unit of nitrogen equals 20 pounds, the average change in yield over this range caused by a change of 1 pound of nitrogen is $\frac{400}{20} = 20$. By measuring the yield change over ½ unit of nitrogen, ¼ unit of nitrogen and over progressively smaller increments of nitrogen, one approaches the concept of the derivative. When the equation contains terms to estimate the interaction between inputs, the numerical value of the derivative will be an estimate of the change in yield caused by the variable at any given level of application of the other variables.

The final step in determining the maximum profit combination of inputs is to set the derivative equal to the inverse price ratio, as:

$$\frac{\partial Y}{\partial N} = \frac{P_N}{P_Y}$$

This is equivalent to finding the rate of application at which the added yield just pays for the added nitrogen in the discussion following Figure 1. However, the added yield will depend on the rate at which other inputs are used. The rate at which the other inputs are used will depend on the relationship of their prices to the price of grain and on the amount of nitrogen used. Hence, the maximum profit combination must be found by finding simultaneous solutions for the values, of $W_2$, $W_3$, and N from the following set of simultaneous equations:

$$\frac{\partial Y}{\partial W_2} = \frac{P_W}{P_Y}$$

$$\frac{\partial Y}{\partial W_3} = \frac{P_W}{P_Y}$$

$$\frac{\partial Y}{\partial N} = \frac{P_N}{P_Y}$$

These values of $W_2$, $W_3$ and N may be substituted into the original regression equation to estimate yields for the maximum profit combinations of inputs.[1]

TABLE 1. MOISTURE AND FERTILIZER TREATMENTS USED

A.—Moisture treatments

| Treatment number | Irrigation in addition to preplanting irrigation |
|---|---|
| $M_1$ | None |
| $M_2$ | One irrigation about 1 week before boot stage. |
| $M_3$ | Irrigated when weighted mean soil moisture tension approached 9 atmospheres. |
| $M_4$ | Irrigated when weighted mean soil moisture tension approached 4 atmospheres. |
| $M_5$ | Irrigated when weighted mean soil moisture tension approached 1½ atmospheres. |
| $M_6$ | First irrigation when soil moisture tension approached 9 atmospheres, and second irrigation when soil moisture tension approached 4 atmospheres. |

B.—Fertilizer treatments*

| Treatment number | Nitrogen application, pounds N per acre | Phosphorus application, pounds $P_2O_5$ per acre |
|---|---|---|
| $F_1$ | 240 | 0 |
| $F_2$ | 0 | 30 |
| $F_3$ | 60 | 30 |
| $F_4$ | 120 | 30 |
| $F_5$ | 240 | 30 |
| $F_6$ | 240 | 60 |

*No fertilizer was applied in 1957 because response to fertilizer did not occur in 1956, the first year under irrigation. Response to residual nitrogen occurred in 1957.

# Illustration of Economic Decisions From Experimental Data

## SOURCE OF DATA

The data on which this report is based are from the experiment, "Irrigation Water Management, Consumptive Water Use, and Fertilizer Studies on Irrigated Grain Sorghum."[2] The experimental design was a split plot, a randomized complete block, with four replications. It included six moisture treatments and six fertilizer treatments. Each year, plots were given a pre-planting irrigation sufficient to wet the soil to a depth of 6 feet. The remainder of the moisture treatments and the fertilizer treatments are summarized in Table 1. The experiment was conducted on level plots with borders to contain the irrigation water. RS 610 sorghum was seeded in mid-June each year, with 20-inch row spacings.

The total irrigation water applied yearly varied from 6 to 17.5 inches on the different moisture treatments. Rainfall during the growing season ranged from 6 to 16 inches during the 3 years. Yields ranged from 2,058 to 7,904 pounds per acre. During the 3-year experiment, the last irrigation was applied early in September each year and there was little rainfall during the latter part of the month. September 10 could be considered as the latest date for water application. The time for application of irrigation water was determined by measuring soil moisture tension in the crop root zone. This method gave reasonable

[1]For a more complete discussion of these methods see: Heady, Earl O., *Economics of Agricultural Production and Resource Use*, chapters 5 and 6, Prentice Hall, Inc., Englewood Cliffs, N. J., 1952, and Heady, Earl O., John T. Pesek, and William G. Brown, *Crop Response Surfaces and Economic Optimum in Fertilizer Use*, Research Bulletin 424, Agricultural Experiment Station, Iowa State College, Ames, Iowa, 1955.

[2]Research conducted at the USDA Southwestern Great Plains Research Center, Bushland, Texas, 1957-59, Experiment Number Tex-A-7. A complete description of the experiment is included in Jensen, Marvin E. and Willis H. Sletten, *Evapotranspiration and Soil Moisture-Fertilizer Interrelations in the Southern High Plains with Irrigated Grain Sorghum*, forthcoming USDA Conservation Research Report.

assurance that the water applications were distributed in accordance with plant use.

Phosphorus failed to show a significant effect in the regression analysis and was dropped from the equation. For all practical purposes, the preplant irrigations on all plots were equal, hence, there was no basis for attempting to estimate a yield effect from preplant irrigation. The growing season, after the seedling stage, was divided into two periods for measurement of water applications. The first, called "plant development period" (month of July), included the time from the seedling stage until the boot stage. The second period, the "grain development period" (August 1-September 10), extended from the boot stage to the soft dough stage.

## THE REGRESSION EQUATION

The experimental data were analyzed using multiple regression analysis. The estimated equation is:

$$\hat{Y} = -7071 + 3700\sqrt{W_2} - 792\,W_2 + 5087\sqrt{W_3} - 673\,W_3 - 219\sqrt{N} - 6.8\,N - 293\sqrt{W_2W_3} + 69\sqrt{W_2N} + 100\sqrt{W_3N}$$

In the regression equation:

$\hat{Y}$ = Estimated yield of sorghum in pounds per acre.

$W_2$ = Inches of irrigation water applied plus inches of rainfall during July.

$W_3$ = Inches of irrigation water applied plus inches of rainfall during August and September.

$N$ = Nitrogen application in pounds per acre.

All coefficients in the equation are statistically significant at the 5 percent level. The level of significance is a measure of the statistical reliability of the estimate; the smaller the percentage figure the more reliable the estimate. The $\bar{R}^2$ value for the equation is 0.814. which indicates that 81 percent of the variation in yield is statistically explained by the variations in water and nitrogen applications.

The positive coefficient for the square root term of both the water variables, with a negative coefficient on the linear term, indicates that the general shape of the response curve for water is similar to the theoretical illustration in Figure 1, with the yield response from additional inputs of water becoming progressively smaller as more water is used. The negative coefficient on the cross-product term of the two water input variables, $\sqrt{W_2 W_3}$, indicates that the response from an increment of water in one period will be greater if the water application in the other period is smaller.

It may be noted that the coefficients for both the square root and the linear form of the nitrogen variable are negative. Consideration of these coefficients separately from the rest of the equation would lead to the conclusion that nitrogen depresses yields. However, the water-nitrogen cross-product terms $\sqrt{W_2 N}$ and $\sqrt{W_3 N}$ have positive coefficients, indicating that nitrogen in combination with water increases yields.

## MAXIMUM PROFIT WATER AND NITROGEN COMBINATIONS

Maximum profit combinations of water and nitrogen for different combinations of water, nitrogen and grain sorghum were estimated by solving sets of simultaneous equations. These combinations for 75 different price situations are summarized in Table 2. In some instances, where prices of nitrogen and water are low relative to the price of sorghum, the water and nitrogen applications and the estimated yields for the maximum profit combinations are above any from the experimental data. These estimates are made on the assumption that the mathematical function is valid for all levels of production. Predictions from functions of this type have least error when values for inputs and yield are near the average for the original data. Probability and potential magnitude of errors tend to increase as the values used depart from the averages of the data. Field observations suggest that predictions from this equation tend to be too high as production levels increase beyond the range of the data. To a large extent, this overestimation appears to be caused by an overestimation of the response to nitrogen resulting in excessively large amounts of nitrogen in the optimum combinations. This tendency toward overestimation for favorable price situations should be kept in mind when making practical applications of the data.

Marginal prices should be used to select the optimum combination of inputs from the table. These prices are the costs per unit that must be paid to increase the applications of the inputs by small amounts or the amount per unit that can be saved by decreasing the applications by small amounts. These costs should include the costs of applying the inputs in the field.[3] Since the prices of farm products to an individual farmer are normally not dependent on the amounts he sells, the net farm prices are relevant prices to use in this example.

To illustrate the use of Table 2, assume prices of $1.70 per hundredweight for sorghum, 8 cents per

[3]In case of water, the relevant price would be the costs a farmer would incur by pumping an additional inch of water. This would include the "out-of-pocket" pumping costs such as fuel, depreciation and maintenance due to use of the equipment, labor, etc. If water is considered an exhaustible resource, the depletion allowance should be included in this cost.

The above costs are relevant only if the farmer does not have alternative uses for the water which would yield a greater return than the pumping costs. When the supply of water is limited relative to its possible use, the relevant price is the amount it could return in the most profitable of these alternate uses. These alternate uses may include "other" acres of sorghum as well as other crops.

pound for nitrogen and $1.00 per acre-inch for irrigation water. For a sorghum price of $1.70, the horizontal section second from the bottom of the table is used. For 8-cent nitrogen the center column is used. From the group of figures in the center of the $1.70 sorghum price section of the table select the line for which the price of water is $1. This line gives the maximum profit combination for the price situation assumed. This is 4.3 inches of water during July ($W_2$), 16.5 inches of water during August and September ($W_3$) and 210 pounds of nitrogen per acre. From this combination, a yield of 7,700 pounds per acre would be expected. If the rainfall during these 3 months is 7 inches (approximate average for the USDA Southwestern Great Plains Research Center, Bushland) the income above the cost of water and nitrogen is $94.40 per acre.

The statistical reliability of these estimates is indicated by the standard error of the estimate which had a value of 690 pounds per acre for this equation. If the experiment were to be continued for a large number of trials, the yield for any particular combination of inputs would be expected to be within 690 pounds of these estimates two-thirds of the time. Thus, a yield between 7,010 and 8,390 pounds per acre would be expected two-thirds of the time using the optimum combination of inputs for the price situation assumed above. Nine out of 10 years the yield would be expected to fall within a range of 6,320 to 9,080 pounds per acre. The expected income above water and nitrogen costs would vary between $82.70 and $106.20 two-thirds of the time and between $71.00 and $117.90, 9 years out of 10.

It is logical to ask, "What is the cost of being slightly off the optimum combination?" To answer this question, assume that a farmer uses the combination directly above the optimum combination for the assumed prices. This would be a total water input 2.3 inches greater than the optimum input and a nitrogen input 40 pounds greater than optimum. The expected yield would be 7,990 pounds per acre and the income above water and nitrogen costs $93.70.

**TABLE 2. MAXIMUM PROFIT COMBINATIONS OF WATER AND NITROGEN, ESTIMATED YIELDS, AND ESTIMATED INCOME OVER WATER AND NITROGEN COSTS FOR PRODUCTION OF IRRIGATED GRAIN SORGHUM ON THE TEXAS HIGH PLAINS UNDER DIFFERENT PRICE SITUATIONS**

| Price of sorghum per 100 pounds | Price of water per acre inch | Price per pound of nitrogen | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 6 cents | | | | | 8 cents | | | | | 10 cents | | | | |
| | | $W_2$ | $W_3$ | N | Yield | Income | $W_2$ | $W_3$ | N | Yield | Income | $W_2$ | $W_3$ | N | Yield | Income |
| $0.80 | $0.50 | 3.8 | 14.4 | 105 | 6930 | $40.50 | 3.5 | 13.3 | 70 | 6530 | $38.80 | 3.4 | 12.7 | 50 | 6240 | $37.60 |
| | 1.00 | 3.3* | 11.6 | 75 | 6430 | 33.10 | 3.1 | 10.9 | 50 | 6070 | 31.60 | 3.0 | 10.5 | 35 | 5840 | 30.80 |
| | 1.50 | 3.0 | 9.6 | 55 | 5900 | 26.50 | 3.0* | 9.1 | 35 | 5650 | 25.60 | 3.0* | 8.8 | 25 | 5480 | 25.00 |
| | 2.00 | 3.0* | 8.1 | 40 | 5470 | 21.10 | 3.0* | 7.7 | 30 | 5260 | 20.40 | 3.0* | 7.5 | 20 | 5110 | 19.90 |
| | 3.00 | 3.0* | 5.9 | 25 | 4680 | 12.20 | 3.0* | 5.7 | 20 | 4540 | 11.80 | 3.0* | 5.6 | 15 | 4370 | 11.00 |
| $1.10 | $0.50 | 4.3 | 16.6 | 185 | 7570 | $62.30 | 3.9 | 15.3 | 120 | 7110 | $59.31 | 3.7 | 14.5 | 85 | 6850 | $58.00 |
| | 1.00 | 3.8 | 14.0 | 140 | 7130 | 53.20 | 3.5 | 13.0 | 95 | 6730 | 50.90 | 3.4 | 12.4 | 70 | 6440 | 49.20 |
| | 1.50 | 3.4 | 12.0 | 110 | 6680 | 45.40 | 3.2 | 11.2 | 75 | 6330 | 43.60 | 3.1 | 10.8 | 55 | 6090 | 42.40 |
| | 2.00 | 3.0* | 10.4 | 85 | 6220 | 37.00 | 3.0* | 9.8 | 60 | 5960 | 37.30 | 3.0* | 9.4 | 45 | 5770 | 36.30 |
| | 3.00 | 3.0* | 8.0 | 55 | 5660 | 29.00 | 3.0* | 7.6 | 40 | 53.40 | 26.70 | 3.0* | 7.4 | 30 | 5190 | 26.00 |
| $1.40 | $0.50 | 4.7 | 18.5 | 270 | 8080 | $85.90 | 4.3 | 17.0 | 185 | 7600 | $81.40 | 4.1 | 16.0 | 135 | 7239 | $78.20 |
| | 1.00 | 4.2 | 16.1 | 215 | 7700 | 75.60 | 3.9 | 14.9 | 150 | 7260 | 71.90 | 3.7 | 14.0 | 110 | 6940 | 69.30 |
| | 1.50 | 3.8 | 14.0 | 175 | 7310 | 66.50 | 3.6 | 13.1 | 120 | 6920 | 63.50 | 3.4 | 12.4 | 90 | 6620 | 61.40 |
| | 2.00 | 3.5 | 12.4 | 140 | 6920 | 58.60 | 3.3 | 11.6 | 100 | 6570 | 56.10 | 3.2 | 11.1 | 75 | 6310 | 54.40 |
| | 3.00 | 3.0 | 9.8 | 95 | 6160 | 44.30 | 3.0* | 9.3 | 70 | 5940 | 43.60 | 3.0* | 9.6 | 55 | 5890 | 42.30 |
| $1.70 | $0.50 | 5.1 | 20.3 | 360 | 8510 | $110.80 | 4.6 | 18.5 | 250 | 7990 | $104.80 | 4.3 | 17.3 | 185 | 7590 | $100.40 |
| | 1.00 | 4.6 | 18.0 | 300 | 8170 | 99.40 | 4.3 | 16.5 | 210 | 7700 | 94.40 | 4.0 | 15.5 | 155 | 7330 | 90.80 |
| | 1.50 | 4.3 | 15.9 | 250 | 7820 | 89.30 | 3.9 | 14.7 | 175 | 7380 | 85.00 | 3.7 | 13.9 | 130 | 7060 | 82.00 |
| | 2.00 | 3.9 | 14.1 | 210 | 7450 | 80.20 | 3.7 | 13.1 | 145 | 7060 | 76.70 | 3.5 | 12.5 | 110 | 6760 | 74.10 |
| | 3.00 | 3.4 | 11.5 | 150 | 6780 | 64.70 | 3.2 | 10.9 | 105 | 6460 | 62.30 | 3.0* | 10.4 | 80 | 6200 | 60.37 |
| $2.00 | $0.50 | 5.4 | 21.8 | 440 | 8890 | $138.20 | 4.9 | 19.9 | 315 | 8320 | $129.40 | 4.6 | 18.5 | 235 | 7920 | $123.90 |
| | 1.00 | 5.0 | 19.5 | 375 | 8520 | 124.40 | 4.6 | 17.9 | 270 | 8050 | 118.00 | 4.3 | 16.7 | 205 | 7680 | 113.30 |
| | 1.50 | 4.6 | 17.5 | 320 | 8210 | 113.30 | 4.3 | 16.1 | 230 | 7770 | 107.90 | 4.0 | 15.2 | 175 | 7420 | 103.70 |
| | 2.00 | 4.3 | 15.8 | 275 | 7890 | 103.20 | 4.0 | 14.7 | 200 | 7490 | 98.60 | 3.8 | 13.9 | 150 | 7170 | 95.00 |
| | 3.00 | 3.7 | 13.1 | 200 | 7270 | 85.90 | 3.5 | 12.3 | 150 | 6920 | 82.40 | 3.3 | 11.7 | 115 | 6660 | 79.70 |

*Maximum profit estimate less than average rainfall.

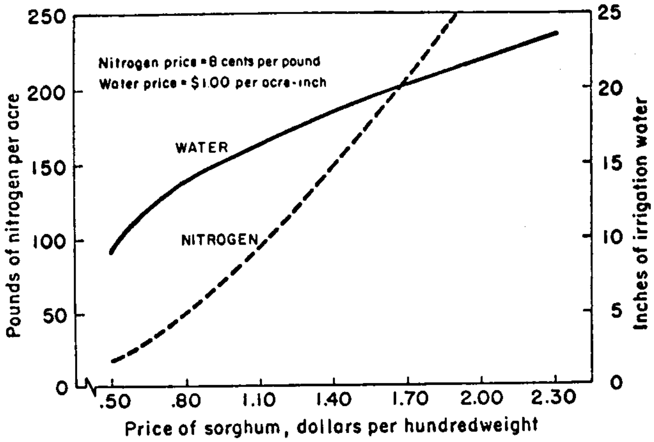
$W_2$ = Inches of rainfall plus irrigation water applied during July.

$W_3$ = Inches of rainfall plus irrigation water applied during August and September.

N = Pounds per acre nitrogen applied.

Yield estimated from regression equation for amounts of water and nitrogen in maximum profit combination.

Income is the gross income above the amount needed to pay for the irrigation water and the nitrogen. The cost of water includes the cost of a 6-inch preplant irrigation, but does not include a charge for the amount of water expected as rainfall.



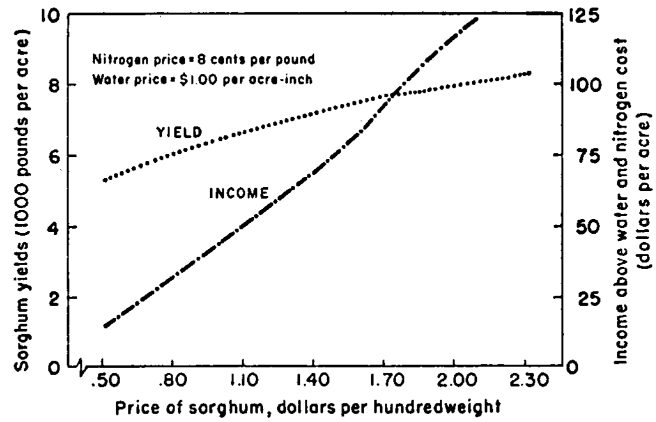




Figure 3. Effect of changing sorghum prices on optimum applications of nitrogen and water, and on yield and income.

Thus, the cost of this non-optimum combination would be $0.70 per acre (*i.e.*, $94.40 — $93.70). By moving to the combination below the optimum combination, the farmer would use 2.2 inches less water and 35 pounds less nitrogen. The cost of this non-optimum combination, in income foregone, is $0.60 per acre.

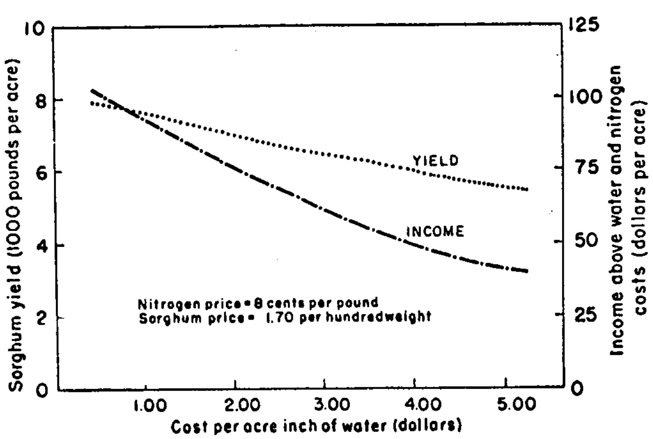
Maximum money income on every acre may not be the objective of all farmers. Some farmers may wish to produce outstanding yields on a small part of their acreage if it can be done at a reasonable cost. If a farmer should try for the 8,890-pound yield (optimal combination of inputs for $2 sorghum, 6 cent nitrogen and 50 cent water) the income above water and nitrogen cost would be $89.60 with prices at the levels assumed for the previous example. His net cost of producing this higher than optimum yield would be $4.80 per acre.

## CHANGING OPTIMUM COMBINATIONS WITH CHANGING PRICES

It can be noted by comparing the different input combinations in Table 2 that the relevant input combination varies systematically with changes in prices and costs. These variations are shown graphically in Figures 3, 4 and 5.

Figure 3 illustrates the changes in the optimum amounts of water and nitrogen, and the resulting changes in yield and income, with changes in the price of sorghum. The optimum water input increases from 9 inches to 24 inches as the price of sorghum increases from $0.50 to $2.30 per hundredweight. For the same price changes, the optimum nitrogen application increases from 15 pounds per acre to more than the 250-pound range of the graph. Over the range of the estimates, an increase of $0.30 in the price of sorghum was associated with an average increase of 32 percent in nitrogen application and a 12 percent increase in water application.

Figures 4 and 5 illustrate the changes in optimum water and nitrogen inputs and changes in yield and income levels, associated with changes in the prices of these inputs. Because of the strong interaction between these two inputs the optimum quantities of both decrease when the price of either increases. For the range of the nitrogen price variation, a 1 cent increase in the price was associated with an average decrease of 13 percent in the optimum amount of water and 17 percent in the optimum amount of nitrogen. With water price varying from 50 cents to $5 per acre-inch, the optimum water input decreased an average of 9 percent with each 50 cent increase in water price, while the optimum nitrogen application



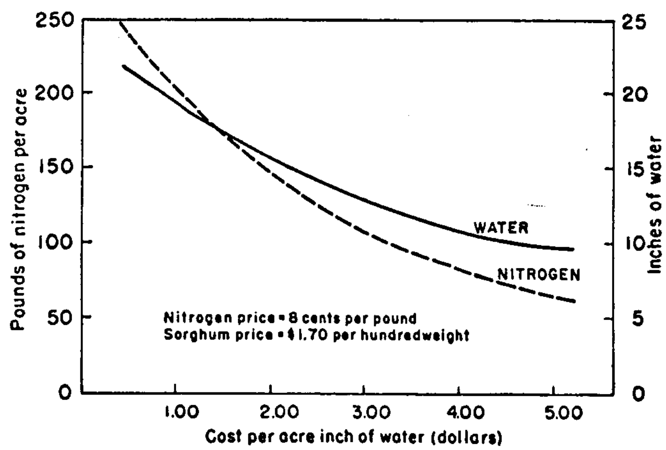




Figure 4. Effect of changing water prices on optimum applications of water and nitrogen and on yield and income.

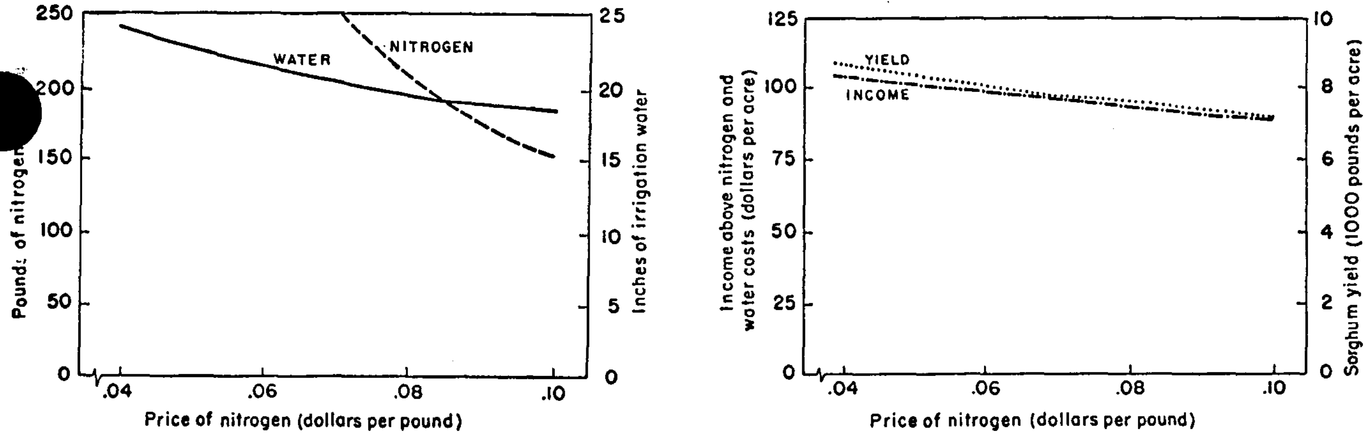



**Figure 5. Effect of changing nitrogen prices on optimum applications of nitrogen and water and on yield and income.**

decreased 17 percent with a 50 cent increase in the price of water. Nitrogen use was sensitive to price changes, and production practices were affected relatively little by changes in the price of nitrogen as compared to changes resulting from changes in water and sorghum prices.

## SUBSTITUTION OF WATER AND NITROGEN

Figure 6 shows iso-product contours fitted to the experimental data for three arbitrarily chosen levels of production, and expansion paths for changing price levels of sorghum and water. The lower expansion path, labeled $E_2$, shows the maximum profit combinations of water and nitrogen when nitrogen is priced at $0.08 per pound and water is priced at $1 per acre-inch. For any given level of production, the maximum profit combination of inputs is determined by the intersection of the expansion path and the iso-product contour. For example, with these prices for water and nitrogen, the most economical combination for production of a 7,000-pound yield is approximately 13½ inches of water and 135 pounds of nitrogen.

When the prices of the inputs are known, the most economical level of production is determined by the price of the product. The prices along line $E_2$ show the level of production that will give maximum profits for the price of sorghum. If the price of sorghum is 50 cents per hundred pounds, the level of production to maximize profits (or minimize losses) is less than 6,000 pounds per acre. If the price of sorghum is $1.70, maximum profits can be made with production of more than 7,500 pounds per acre.

Expansion path $E_1$ shows how production should be adjusted when the prices of nitrogen and sorghum are 8 cents per pound and $1.70 per hundredweight, respectively, and the price of water is variable. If the price of water were approximately $1.50, the maximum profit level of production would be 7,500 pounds of sorghum per acre. This production level should be achieved with ¾ inch less water and 15 pounds more nitrogen than the maximum profit combination for the same level of production with water priced at $1. It is important to note that this shift from water to nitrogen is for a *given level of production.* The maximum profit level of production will use less nitrogen and less water with a change in the price of water from $1 to $1.50.

## TIME OF WATER APPLICATIONS

Figure 7 illustrates the possibilities of substituting water in one time period for water in another time period. At the 6,000-pound level of production, if the July application is reduced from 4 to 3 inches, about ¾ inch of water in August will substitute for 1 inch in July. When the July application is reduced from 3 to 2 inches, it requires an additional 1¼ inches in August to maintain the yield. If the July application is further reduced from 2 to 1 inch, an additional 3½ inches will be needed in August to maintain yield.

At the 7,000-pound yield level, reducing the July application from 4 to 3 inches can be compensated for by increasing the August application by 1 inch. A further reduction from 3 to 2 inches in July requires about 2½ inches additional water in August

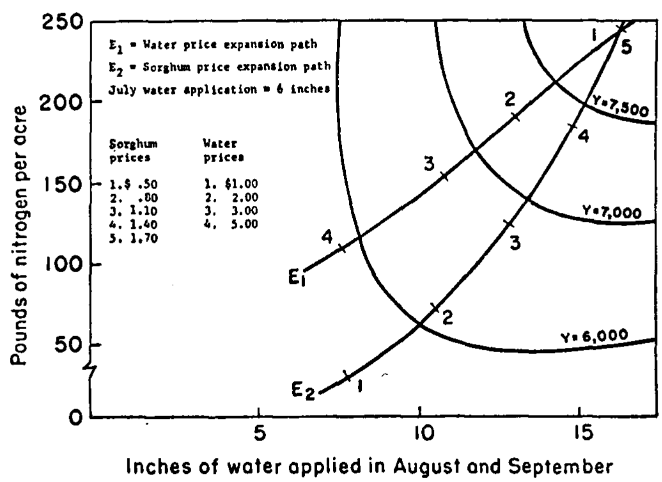



**Figure 6. Iso-product contour map for August-September water applications and nitrogen, and expansion paths for varying prices of water and sorghum in the production of irrigated grain sorghum on the Texas High Plains.**

to maintain the yield, while another ½ inch decrease in July water application below 2 inches requires an additional 4 inches of water in August.

Expansion path $E_1$ indicates the maximum profit combinations of water in the two time periods when adequate amounts of water are available during both periods at a price of $1 per acre-inch. On some farms the supply of water is limited relative to its need during some critical periods. This may be the case during the month of August when the sorghum crop is using water at its peak rate.[4] If, for example, a farm well has only enough capacity to apply one 6-inch irrigation to the sorghum during this critical period, how much additional water should be applied during July? The amount of water to use in July for maximum profit is shown by expansion path $E_2$. For all prices of sorghum, the August-September water application will be 10 inches (6 inches irrigation plus 4 inches expected rainfall). When 200 pounds of nitrogen have been applied, the July water applications will vary from 3.6 inches for 50 cent sorghum to 5 inches for $2 sorghum. The yields for this price range will vary from 6,610 to 6,770 pounds per acre, and the incomes above cost of water and nitrogen will range from $4.45 to $105.44 per acre.

In some cases it may be possible for a farmer with limited supplies of irrigation water to make additional water available at a higher price during critical periods. This might be done by pumping water into a reservoir during periods of low water use, by transporting water from another well on a different part of his farm or, in rare instances, by purchasing water from a source off his farm. Expansion path $E_3$ illustrates the combinations of water in the two time periods that will maximize profits when irrigation water in the August-September period costs $3 per acre-inch, all other irrigation water costs $1 per acre-inch, and 200 pounds per acre of nitrogen have been applied. The portion of $E_3$ that lies to the left of $E_2$ is irrelevant if the limited amount of water

[4]Jensen, M. E. and J. T. Musick, Irrigating Grain Sorghums, Leaflet No. 511, U. S. Department of Agriculture, Washington, D. C., 1962, p. 3.

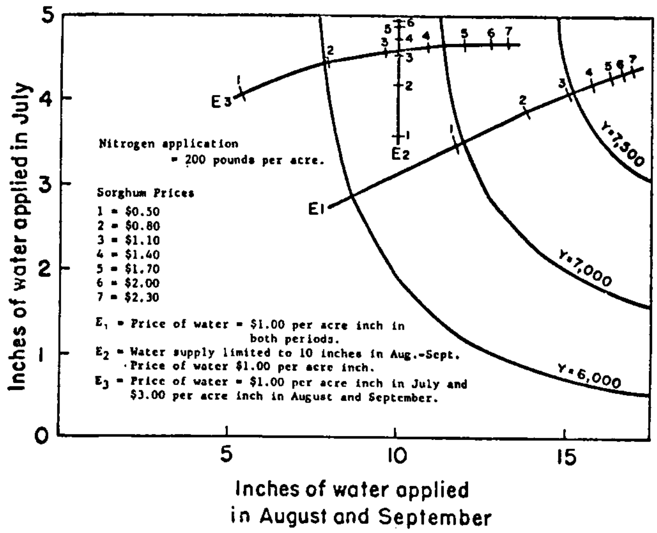



Figure 7. Iso-product contour map for July water applications, August-September water applications and expansion paths for varying conditions of water supply in the production of irrigated grain sorghum on the Texas High Plains.

is available at the lower price. For the conditions shown in Figure 7, it will be profitable to purchase additional irrigation water, costing $3 per acre-inch, when the price of sorghum is above $1.15 per 100 pounds.

A more realistic comparison of these alternatives can be made from the data in Table 3. Here the amounts of both nitrogen and water are allowed to vary. The first section of the table shows the optimum nitrogen and water inputs and the estimated yield and income when the August-September water supply is limited to 10 inches and all irrigation water costs $1 per acre-inch. The second section of the table gives the same information for a situation in which adequate water is available in the later period at a cost of $3 per acre-inch for irrigation water. The column at the right of the table shows the income when only the amount of August-September water in excess of 10 inches is charged at the rate of $3 per acre-inch, and all other irrigation water is charged

TABLE 3. MAXIMUM PROFIT COMBINATIONS OF WATER AND NITROGEN, ESTIMATED YIELDS, AND INCOME ABOVE COST OF WATER AND NITROGEN FOR DIFFERENT CONDITIONS OF WATER SUPPLY AND VARYING PRICES OF SORGHUM

| Price of sorghum | Aug.-Sept. water limited to 10 inches | | | | | Aug.-Sept. water priced at $3 per acre-inch | | | | | 10 inches of Aug.-Sept. water available at $1 per acre-inch. Additional water available at $3 per acre-inch. Income above W and N cost |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $W_2$ | $W_3$ | N | Yield | Income above W and N cost | $W_2$ | $W_3$ | N | Yield | Income above W and N cost | |
| $0.50 | 3.0[1] | 8.2[2] | 15 | 5170 | $14.30 | 3.0[1] | 4.0[2] | 5 | 3630 | $11.73 | |
| 0.80 | 3.1 | 10.0 | 45 | 5880 | 31.50 | 3.2 | 6.2 | 20 | 4750 | 23.80 | |
| 1.10 | 3.6 | 10.0 | 65 | 6130 | 49.50 | 3.6 | 7.6 | 45 | 5450 | 39.00 | |
| 1.40 | 4.0 | 10.0 | 90 | 6310 | 68.20 | 4.0 | 9.3 | 80 | 6091 | 55.90 | |
| 1.70 | 4.2 | 10.0 | 110 | 6440 | 87.40 | 4.3 | 10.9 | 125 | 6660 | 75.30 | $ 87.30 |
| 2.00 | 4.4 | 10.0 | 125 | 6520 | 106.80 | 4.5 | 12.3 | 170 | 7110 | 96.30 | 108.30 |
| 2.30 | 4.7 | 10.0 | 145 | 6610 | 126.60 | 4.8 | 13.6 | 215 | 7490 | 118.30 | 130.30 |

[1]Maximum profit level calculated from equation is below average rainfall.
[2]At this price level it is not profitable to use the ten inches of water that are available.

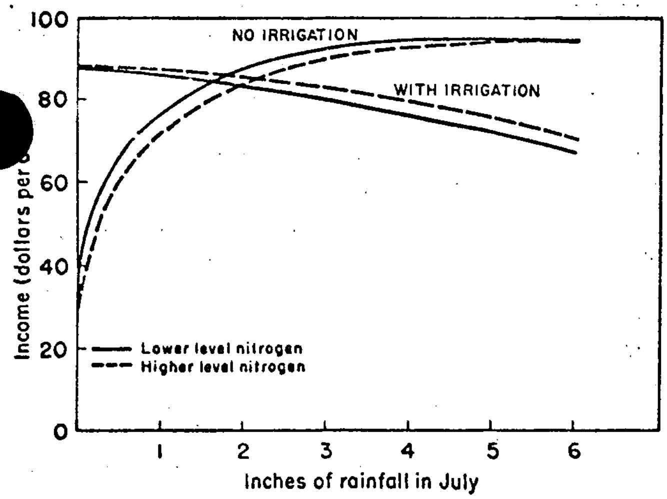



**Figure 8. Effect of July water application on income above cost of water and nitrogen.**

at $1 per acre-inch. For the experimental conditions it would be profitable to purchase additional water, at $3 per acre-inch, when the price of sorghum is higher than $1.70 per hundredweight.

### JULY WATER APPLICATIONS

The maximum profit combination of water and nitrogen in Table 2 for 8-cent nitrogen, $1 water and $1.70 sorghum calls for 4.3 inches of water in July. The average July rainfall at Bushland is approximately 3 inches, leaving 1.3 inches to be applied as irrigation water in an average year. Under normal farm conditions it is impossible to cover a field with so small an application. The farmer's alternatives may be to depend on rainfall or to apply 4 or more inches of irrigation water.

The results of these alternatives are illustrated in Figure 8. For this illustration, it was assumed that 6 inches of irrigation water are required to give uniform coverage of the field. Nitrogen is commonly applied to the field before planting. Therefore, the amount of nitrogen is no longer a variable at the time the July water application is made. It was assumed that the farmer would base his decision on the nitrogen application on the mode, or most frequently occurring amount of rainfall, which is approximately 2 inches. The maximum profit application of nitrogen would be 165 pounds per acre if the July water application is 2 inches of rainfall, or 310 pounds per acre if 6 inches of irrigation water is applied.

Maximum profits will be achieved with the lower level of nitrogen without supplemental irrigation whenever the July rainfall is between 1.8 inches and 5.5 inches, Figure 8. At the Bushland Station, the July rainfall has been within this range 12 years out of 24, or one-half of the time. The potential income gains from the higher nitrogen rate during the other 12 years are smaller than the potential losses during the years when rainfall is within the 1.8 - 5.5-inch range. Hence, a farmer's income over a period of years would be higher using the lower nitrogen rate. With the lower rate of nitrogen he should plan to irrigate during July whenever it appears that the monthly rainfall will be below 1.6 inches.

## Application to Farm Conditions

The preceding analysis of experimental data was intended primarily as an illustration of the factors to be considered by a farmer attempting to earn a maximum income for irrigated sorghum. Direct applications of the empirical results to a particular farm situation should be made with caution. However, some generalized recommendations can be made if recognition is made of the differences between the conditions under which the experimental data were collected and farm conditions, and the limitations of this type of analysis.

All of the experimental treatments received a preplant irrigation of approximately 6 inches. Hence, any findings from this analysis can be applied only to situations with comparable moisture in the soil at planting time. Additional research is needed before recommendations can be made concerning the most profitable level of preplant irrigation.

The estimated maximum profit yield levels from the experimental situation are probably somewhat higher than the yield levels that would be most profitable on a commercial farm. The experiments were conducted on small, level, diked plots with closer control than would be practical on a farm situation. The sorghum was planted in 20-inch rows, which is not a general practice on farms. The efficiency of application of irrigation water on the small, diked plots probably was higher than can be achieved on a commercial graded-furrow system, and the distribution more uniform than is possible with graded furrows. Further research, with an experimental design similar to farm conditions, is needed to bridge the gap between our current research and farm applications.

An analysis of this type is necessarily limited to making recommendations based on an estimate of "average" responses. These average responses are estimated with some error, as is indicated by the measures of statistical reliability. The 3 years for which data were collected may or may not be typical of any year for which recommendations might be made.

Some of the optimum combinations calculated by the analysis are beyond the range of the original data. Recommendations based on such projections must be made with caution. In this example, caution is particularly applicable in the case of nitrogen. The yield response to nitrogen estimated by the equation appears to be much higher than is generally observed in the field. Recommendations based on this analysis should reduce the nitrogen application substantially, possibly as much as 50 percent at the higher rates of application and lesser amounts at the lower rates.

The regression equation provided a basis for estimating the extent to which one input could be substituted for another. It is impossible for a mathematical function of this type to consider all of the biological factors that may be important in an actual production situation. The equation showed rather limited possibilities of substituting water applications before the boot stage for water applications after the boot stage. The storage capacity of the soil would give some possibility for this type of substitution, but not for substituting post-boot watering for pre-boot watering. The substitution possibilities estimated from the equation may be an average response that underestimates the possibility of substituting pre-boot water for post-boot water, and overestimates the possibility of substituting in the other direction. Additional information is needed on the ability of the sorghum plant to recover from moisture stress during the pre-boot stage before specific recommendations can be made for the pre-boot watering. Water stress during this pre-boot period may delay maturity, encourage sucker growth and reduce yields, even though adequate moisture is available during later stages. Excess water during this period may encourage conditions favorable to lodging, especially if it is followed by low water applications in the following period. The rapid income decline with July water application below 2 inches in Figure 8 indicates that a moderate amount of water during this period is essential for profitable production. The decreasing incomes with irrigation, shown as amounts of water increase, suggest that any irrigation made during this period should be as light as is practical to get coverage.

This analysis has not attempted to deal with the problem of timing of irrigation other than between the two rather broad periods. To obtain results comparable to those of the analysis, it is essential that the time of water applications within these two periods be such that water will be available when needed for plant growth. This would suggest that most of the pre-boot applications should be relatively late in the period. Early in the period, when the plants are small, the water needs should be supplied adequately from the preplant irrigation. The greater part of the post-boot application should be relatively early in the period to make the water available during the period of peak water use by the plants. Under farm conditions, as much as 2 weeks may be required to irrigate all of the sorghum crop on the farm. Additional research is needed to estimate the production losses from irrigating a few days before or after the optimum time, as well as determining the optimum time to apply water.

Throughout the analysis it has been assumed that the farmer's objective was maximum monetary income on a particular acre of sorghum. It is more realistic to assume that his objective is maximum total farm income. Using this objective requires considering the competition between feasible enterprises for available resources. To some extent this competition for resources can be reflected in the prices charged for the resources, but finding a maximum profit combination for the entire farm is beyond the range of this type of analysis.

The price a farmer should charge for the water pumped from his own well cannot be determined with a high degree of accuracy. Hughes and Magee have estimated the pumping costs north of the Canadian River in the 50 to 60 cents per acre-inch price range for a well producing 500 gallons per minute, and in the 40 to 50 cent range for a well producing 750 gallons per minute.[5] These costs should be considered as the lower limits of the costs a farmer should use. A depreciation allowance for eventual replacement of equipment and a depletion allowance for the water used from the underground reservoir should be added to these costs. Also, the water cost should be for water actually applied to the field. If it is necessary to run "tail water" to get satisfactory water distribution, the cost of this extra water must be added to the cost for water actually applied to the field.

However, pumping costs are relevant only if a farm has adequate water to irrigate all crops to the maximum profit level. If the water supply is limited and the farm has alternative uses for water, such as other crops or other fields of sorghum, the field of sorghum receiving water should be charged for the water at a rate equal to the amount that water would have increased yields on the most productive of the other fields.

[5]Hughes, William F., and A. C. Magee, *Production Practices and Specified Costs of Producing Wheat and Grain Sorghum on Irrigated Farms, Upper Texas Panhandle, 1960-61*, MP-656, Texas Agricultural Experiment Station, College Station, Texas, May, 1963.