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Proposal of indexes**

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On the Measurement of Sociopolitical Consensus in Direct

Democracies: Proposal of Indexes

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Abstract

We investigate the measurement of sociopolitical consensus in direct democracies by means of three novel Consensus Indexes. They are special cases of Approval Consensus Measures, a tool from social choice that evaluates the degree of cohesiveness in a fixed group of agents that vote on a list of issues. We perform a basic dynamic analysis of the Swiss votes in popular initiatives, in periods marked by the main reforms and political crisis along those years. We provide novel quantitative arguments to validate the hypotheses that those reorganizations had an impact in terms of sociopolitical consensus during the last decade in comparison with prior stages. In addition we study the cumulative consensus in Italian referendums since 1974. We also investigate to what

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extent our indexes reflect the existence of periods in the development of referendums as a constitutional praxis.

Keywords: consensus indexes, sociopolitical consensus, direct democracy.

JEL classification: H79, D79.

1. Introduction

This paper investigates the measurement of sociopolitical consensus by means of a novel tool from social choice theory named Approval Consensus Measures. These measures evaluate the degree of cohesiveness in a group of agents that vote on a list of issues. Direct democracies, with their tradition about popular votes, provide a natural field of application where data proliferate. In a direct democracy the participation of the citizenship is not limited to electing the highest state bodies, but also the citizens can determine whether and if so which specific issues should be submitted to a decision of the People. Henceforth we propose three novel Consensus Indexes that take advantage of the particular structure of the popular votes (whether or not the population is fixed, and the information about the ballots).

Despite the importance of the topic, little evidence of comparable indicators exists to fill the current gap in knowledge regarding the measurement of sociopolitical consensus. The informational basis of our analysis is the result of popular votes in a political entity. In doing this we are inspired by Lipsitz (1968), who claims: “Political consensus involves kinds of agreements that are politically relevant. [...] Two other views of consensus [...] have been developed and employed in different political contexts as well. The first of these is consensus as legitimacy: approval of the existing government and/or its directives. Understood in this sense, consensus is seen to be one of the key elements in governmental stability.” Building on the latter sense of the concept we propose to use

data on the approval of political directives by citizens in order to produce comparable indicators of sociopolitical consensus. Firstly we present a general tool of analysis to deal with different models. Then we propose indexes that take advantage of the information provided by those qualitatively different formal models. Finally we use them to perform an empirical analysis with data about the popular initiatives in Switzerland in the 1991-2012 period, and about Italian referendums since the 1947 constitution. The first analysis has a fixed population of agents (the cantons) while the second one has variable population (the individual voters).

Our purpose should not be mistaken with other analyses of indexes for the purpose of comparing countries on consensus vs. majoritarian scales (cf., Lijphart, 1984, 1999). These are useful for example, to evaluate the macroeconomic consequences of different types of institutions. According to Lijphart's criteria and other arguments from comparative democracy research, Switzerland, together with Belgium, are widely seen as the most typical cases of consensus democracies (Lijphart, 1999, pp. 33, 34ff., 249), a concept with a precise meaning in political science. These considerations make the Swiss example especially attractive and challenging.

1.1 Empirical analysis of Switzerland's data

In Switzerland there are two different ways to consult the people, depending on the nature of the issue: the popular initiative, and the referendum. Swiss *referendums* are popular votes called to challenge a piece of legislation already approved by the Federal Assembly. If any person or group opposed to the new law manages to collect 50,000 signatures within 100 days of the official publication of the proposed legislation, the voters as a whole are given the chance to decide. The authorities are obliged to hold a referendum if the legislation involves an amendment to the constitution initiated by the

government, or any proposal for Switzerland to sign a major international agreement that cannot be rescinded. In addition, any Swiss citizen has the right to propose new legislation by launching an initiative (a *popular initiative*), although normally initiatives come from pressure groups rather than individuals. If they gather over 100,000 signatures within 18 months in support of the proposal, it must be put to a nation-wide vote. Initiatives have been held on a variety of issues, although they were intended to deal with constitutional matters exclusively.

Since popular votes do not always proceed from the Parliament or the Government, but they directly come from the citizens, they are regarded as the driving force of direct democracy. We focus on this element as an evidence for the measurement of sociopolitical consensus.

In the case of an initiative or a mandatory referendum in Switzerland, there must be a "double majority" for it to pass: both a majority of the people as a whole, and a majority of the cantons, must approve it.

Our target is the investigation of the coherence of the votes across cantons, that is, we build on the data about which issues were approved or rejected in each of the cantons. To that purpose we use information from the Swiss Federal Statistical Office. Besides, in order to perform a basic dynamic analysis we have studied the Swiss cantonal votes in periods marked by the main reforms and political crisis along those years. The federal constitution was completely amended in 1999. For the first time, the cantons used the possibility of calling a nationwide vote in order to challenge a decision of the Parliament in 2003. The result was that a new fiscal equalization scheme was approved in 2004, which meant the greatest reform to Swiss federalism since the federal state was created. Later on, Linder and Iff (2011, p. 77) speak of a short "crisis of consensus democracy" in 2008. Around that time, Vatter (2008, p. 3) had studied whether "(t)he

considerable political changes and institutional reforms of the last decade lead to expectations that (...) Switzerland no longer corresponds to the extreme example of a consensus democracy". In a similar vein we intend to provide novel quantitative arguments to examine the hypotheses that those reorganizations had an impact in terms of sociopolitical consensus. The results show evidences that the same phenomenon has happened during the last decade in comparison with prior stages.

1.2 Empirical analysis of Italy's data

In Italy there are two kinds of legally binding referendums, and there are consultative referendums too. Since the approval of the legislation that regulates them in the early 1970s, a total of 66 popular referendums, 2 constitutional referendums, and 1 consultative referendum were held. Popular referendums are called only to decide if an existing law must be totally or partially abolished (*consultazione referendarie abrogative*). Constitutional referendums are confirmatory (the voters decide on whether they want to keep or abolish the law) but contrarily to popular referendums, they are not subject to any quorum.

Constitutional experts concur with the distinction of periods in the development of abrogative referendums as a constitutional praxis. Building on De Vergottini (2006), we distinguish four periods. From 1974 (when the first referendum after the 1947 constitution was held) to 1985, there were 9 popular referendums on fundamental individual rights. In all cases the citizens decided against repealing the existing laws, and the electorate replicates the distribution of positions in the parliament. From 1987 to 1995, 30 popular referendums were held, of which 10 ended in approval (i.e., the respective laws were abrogated). The resource to referendums started to escape from the control of the parliament. From 1997 to 2009, 23 popular referendums were held but

they all were invalidated because quorum was not met. Besides there were 2 constitutional referendums and 1 consultative referendum. Here the distinctive feature is that the resource to abstaining has hampered the role of the proposers of the referendums. Finally, in 2011 four popular referendums were held and they all were approved (i.e., the quorum was met and the four regulations were repealed), which somehow hints a return to the second period. We investigate to what extent our indexes are responsive to the existence of these periods, and also we examine the evolution of the sociopolitical consensus along time, understood as the coherence in the expression of the votes in past referendums.

2. Consensus: Conceptualization and proposal of indexes under dichotomous opinions

2.1 Notations

When we are concerned with a fixed group of N individual or fictitious agents (like cantons or states in a federal system) who express a dichotomous opinion (like yes-no) on k issues, and we know their responses or votes, we can gather them in a binary matrix M . This means a matrix where each cell has either a 1 or a 0, two digits that symbolize each of the possible answers or results. Agents are given by rows, and issues by columns. Therefore if we look at row 1 then we have the opinions of agent 1 on each of the possible issues. Likewise, if we look at column 1 then we have the opinions of all agents on the first issue. Following Alcantud *et al.* (2013a), these matrices are called **approval profiles**. We adopt the voting terminology for convenience although the model adapts e.g., to polls or surveys as well.

Example 1. For this illustrative toy example we refer to some results in the traditionally called “half-cantons”². Each of these six Swiss cantons has only one seat on the Council of States - there are two seats for each of the other cantons-, and half a cantonal vote in popular referendums about constitutional amendments.

We consider the results of the six “half-cantons” in three popular initiatives voted on March 11, 2012. The first initiative wanted to limit the building of secondary residencies in each municipality in Switzerland, and it was accepted. The second initiative aimed at establishing a new tax instrument to stimulate housing savings in order to promote first homeownership or to finance works of energy-saving measures and environment protection. The third initiative was called “six holiday weeks for everybody”. The latter two initiatives were rejected. Table 1 collects the relevant aspects of the results. According to our convention, a 1 means that the half-canton approved the initiative, and a 0 means that it was rejected.

Table 1

Cantonal results of three popular initiatives voted on March 11, 2012 in the half-cantons (ballot number: 555, 556 and 557). Source: <http://www.atlas.bfs.admin.ch/>

	I	II	III
Appenzell Ausserrhoden	1	0	0
Appenzell Innerrhoden	0	0	0
Basel-Stadt	1	0	0
Basel-Landschaft	1	1	0
Obwalden	0	0	0
Nidwalden	0	0	0
Number of half-cantons	6	6	6

²Appenzell Ausserrhoden and Appenzell Innerrhoden, Basel-Stadt and Basel-Landschaft, Obwalden and Nidwalden.

Following the notation just explained above, these results can be reduced to the following 6x3 matrix:

$$M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

However on occasions we do not know the exact configuration of each ballot, but aggregate gross data are available. It can also happen that the population of voters varies with the issues. Consider the following example:

Example 2. Three referendums on reforming the electoral law were held in Italy on 21-22 June 2009. The referendums were not valid because they did not reach the necessary quorum of 50% voters. Table 2 collects the relevant aspects of the results.

Table 2

Results of the 2009 Italian electoral law referendum. Source: Italian Secretary of State, Archivio Storico delle Elezioni (<http://elezionistorico.interno.it/>)

	I	II	III
For (percentage)	77.63	77.68	87.00
Against (percentage)	22.37	22.32	13.00
Valid votes	10,372,226	10,362,230	10,908,329

Cases like Example 2 suggest that it is convenient to produce indicators solely on the basis of the relative support to each of the issues. This imposes less restrictions than the appeal to approval profiles: we only need what we call **relative-approval profiles**, that is, k -dimensional vectors $V = (r_1^1, \dots, r_1^k)$ that capture the proportion of 1-opinions on

each of the k issues. Thus the outcome of Example 2 boils down to $V = (0.7763, 0.7768, 0.87)$. Obviously, both $r_1^j = 0$ and $r_1^j = 1$ indicate unanimity on issue j . Approval profiles can be transformed into relative-approval profiles easily: r_1^j is simply the sum of the elements in column j divided by N . The outcome of Example 1 could be simplified as $V = (1/2, 1/6, 0)$.

Populated relative-approval profiles provide an intermediate model in between approval profiles and relative-approval profiles. They permit to incorporate the information on the size of the population that gave their opinion on the respective issues. Formally, they are $2 \times k$ matrices P whose cell $(1, j)$ contains r_1^j , the relative support to issue j , and cell $(2, j)$ contains the number of people that voted on that issue, denoted N^j . With each P we associate $v = N^1 + \dots + N^k$, that is, the total number of votes cast by the voters on the different issues.

The information in Example 2 addresses to the populated relative-approval profile

$$P = \begin{pmatrix} 77.63 & 77.68 & 87.00 \\ 10,372,226 & 10,362,230 & 10,908,329 \end{pmatrix} = \begin{pmatrix} r_1^1 & r_1^2 & r_1^3 \\ N^1 & N^2 & N^3 \end{pmatrix}$$

According to our notation, $v = 10,372,226 + 10,362,230 + 10,908,329 = 31,642,785$

The information in Example 1 addresses to the populated relative-approval profile

$$P = \begin{pmatrix} 3 & 1 & 0 \\ 6 & 6 & 6 \end{pmatrix}$$

According to the Merriam-Webster dictionary, consensus of opinion can be conceived of as group solidarity in sentiment and belief. Henceforth there must be groups with larger solidarity in opinions than others. How do we quantify their respective solidarity in beliefs? We study some proposals in the remaining of this section.

Assigning an index with each approval profile seems a suitable way to measure consensus in cases like the one described above. Because it seems natural to conceive of

'full consensus or its fractions' such number should lie in the interval $[0,1]$. Alcantud *et al.* (2013a) argues that a minimum set of properties must be requested, which do not guarantee good performance by themselves. These properties are: (i) an index of 1 is equivalent to unanimity on all issues, (ii) the index is independent of the names of the agents (anonymity), and (iii) the index is independent of the heading of the questions (neutrality). An index that verifies these constraints is called an **approval consensus measure** or ACM for simplicity.

We proceed to describe some proposals of indexes that use information from the various degrees of refinement that the modelizations of the dichotomous group opinions convey.

2.2 Proposal of indexes: earlier literature

2.2.1 The simple approval consensus measure

The Oxford Advanced Learner's Dictionary defines consensus as “an opinion that all members of a group agree with”. If we adhere to this conception then a suitable measure of the consensus in a society during a certain period, is the proportion of alternatives they unanimously agreed with when they voted along such period. This basic approach produces a naive example of a consensus measure, namely the simple measure C_1 studied by Alcantud *et al.* (2013a). It measures the consensus in a relative-approval profile by the proportion of alternatives that have received a unanimous answer. In practice, this is the number of issues on which there is unanimity (i.e., there is either a 0 or a 1 in the corresponding cell) divided by the total number of alternatives (i.e., the dimension of the vector). We refer to this indicator as the **simple consensus index**. In

the situation of Example 1 we have $C_1(V) = \frac{1}{3}$.

2.2.2 The pairwise approval consensus measure

In no society, however consensual, is the consensus ever universally shared (Shils, 1968). Consequently, Alcantud *et al.* (2013b) builds on indexes that better capture the complexity of our framework. The first one, namely the pairwise ACM or C_p , has an intuitive description: it evaluates the consensus by the probability that a randomly selected issue has received the same vote, be it approval or disapproval, by two randomly chosen agents. To this purpose, we must use the information on the corresponding populated relative-approval profile P in order to compute the following expression:

$$C_p(P) = 1 - \frac{1}{k} \sum_{j=1}^k \frac{r_1^j N^j (N^j - r_1^j N^j)}{C_{N^j}^2} = 1 - \frac{2}{k} \sum_{j=1}^k \frac{r_1^j N^j (1 - r_1^j)}{N^j - 1}.$$

We refer to this indicator as the **pairwise consensus index**. When each N^j is sufficiently large this expression can be approximated by

$$C_p(P) \approx 1 - \frac{2}{k} \sum_{j=1}^k r_1^j (1 - r_1^j)$$

because in such case $\frac{N^j}{N^j - 1} \approx 1$ for each j . Therefore if we only know information on the relative-approval profile V of the society, but we also know that the number of votes on each issue is large (as is the case of Example 2), then the latter expression produces a nice approximation of the real appraisal of the consensus by the PACM.

Remark 1. Because $V(r_1^j) = r_1^j (1 - r_1^j) \frac{N^j}{N^j - 1}$ is an inverted parabola with vertex in

$$r_1^j = \frac{1}{2} \Rightarrow V(r_1^j) \leq V\left(\frac{1}{2}\right), \text{ for each } j.$$

Therefore,

$$C_p(P) = 1 - \frac{2}{k} \sum_{j=1}^k V(r_1^j) \geq 1 - \frac{2}{k} \sum_{j=1}^k \frac{N^j}{4(N^j - 1)} = 1 - \frac{1}{2k} \sum_{j=1}^k \frac{N^j}{(N^j - 1)}.$$

For large N^j s, like in the Italian case, $C_p(P)$ is bounded below by an amount that is only slightly smaller than $\frac{1}{2}$.

2.3 New indexes of consensus with unknown or variable population

The simple measure C_1 applies to these cases too: unanimous verdicts on an issue are identified by either 0 or 1 in the corresponding position of the relative-approval profile. For example, if $V = (1, 0.6, 0, 0.3, 0.1)$ then $C_1(V) = 2/5$. But if we seek more meaningful indexes then we must observe that C_p , the PACM, is unsuitable for this more general framework because the evaluation by C_p is affected by the size of the population (v., Alcantud *et al.*, 2013b). To overcome this difficulty, these authors propose a technical modification of the PACM, namely the Modified PACM or MPACM. For the purpose of comparing consensual performance these two indexes are equivalent. However we do not know of any intuitive meaning for the MPACM index that permits to derive qualitative arguments. For these reasons we here propose two new indexes that are both intuitively easy to interpret and designed for data with variable or unknown population.

Definition 1. The Majoritarian ACM, μ_m , evaluates the consensus in a relative-approval profile V by the probability that the vote of a randomly chosen agent on a randomly selected issue is supported by a majority of the agents. We say that an opinion on an issue is supported by a majority of the agents if at least half of them have such opinion on the issue, be it approval or disapproval. Formally:

$$\mu_m(V) = \frac{1}{k} \sum_{j=1}^{j=k} \max\{r_1^j, 1 - r_1^j\}$$

We refer to this indicator as the **majoritarian consensus index**.

If the information on the populated relative-approval profile of the society is available, then we can use it in order to give a more accurate assessment of the consensus in the following terms:

Definition 2. The Weighted Majoritarian ACM, μ_w , evaluates the consensus in a populated relative-approval profile P by the probability that a randomly chosen vote (among the universe of all votes by any agent on any issue) coincides with the vote cast by a majority of the agents on the same issue. Formally:

$$\mu_w(P) = \frac{1}{v} \sum_{j=1}^{j=k} (N^j \cdot \max\{r_1^j, 1 - r_1^j\}) = \frac{1}{v} \sum_{j=1}^{j=k} \max\{N^j \cdot r_1^j, N^j \cdot (1 - r_1^j)\}$$

We refer to this indicator as the **weighted majoritarian consensus index**. It is clear that when all the N^j coincide, the majoritarian consensus index and the weighted majoritarian consensus index must coincide too.

Remark 2. It is simple to check that the values of the majoritarian and the weighted majoritarian consensus indexes are bounded below by 0.5.

To put some examples, when we compute the value of the Majoritarian index μ_m for

$V = (1, 0.6, 0, 0.3, 0.1)$ we obtain $\mu_m(V) = \frac{1+0.6+1+0.7+0.9}{5} = 0.84$. In the situation

of Example 1 we had $V = (1/2, 1/6, 0)$, thus $\mu_m(V) = \frac{1/2 + 5/6 + 1}{3} \approx 0.7778$. Let us

now consider Example 2, i.e.,

$$V = (0.7763, 0.7768, 0.87) \text{ and } P = \begin{pmatrix} 77.63 & 77.68 & 87.00 \\ 10,372,226 & 10,362,230 & 10,908,329 \end{pmatrix}.$$

Now

$$\mu_m(V) = \frac{0.7763 + 0.7768 + 0.87}{3} = 0.8077$$

and

$$\mu_w(P) = \frac{10,372,226}{v} 0.7763 + \frac{10,362,230}{v} 0.7768 + \frac{10,908,329}{v} 0.87 = 0.8087,$$

where $v = 31,642,785$ is the total number of votes cast.

3. The data

Shils (1968) says: “Abstract or general ethical and political beliefs (“principles”) can enter into consensus insofar as they affect agreement or disagreement on particular issues of legitimacy, distribution, selection, and so on. [...] The beliefs which enter into consensus are not clearly articulated or systematically ordered.” To avoid this indeterminacy, for our analysis of sociopolitical consensus in Switzerland we concentrate on the data about the popular votes in the 1991-2012 period provided by the Swiss Federal Statistical Office (<http://www.bfs.admin.ch>). Table 3 summarizes the number of popular votes per year that produced a unanimous answer across cantons. Also in Appendix A, Table A-1 collects broader information on the cantonal results of each of the 201 votes held in such period: for each particular vote, it gives the number of cantons that approved the issue.

Table 3

Yearly number of popular votes voted in the 1991-2012 period in Switzerland, and yearly number of unanimous answers in such votes.

Year	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	
Unanimous answers	1	4	9	5	2	5	4	6	4	9	9	
Initiatives per year	4	16	16	13	7	9	5	10	10	16	11	
Year	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	Total
Unanimous answers	2	7	6	1	3	0	6	4	4	0	6	97
Initiatives per year	9	11	13	5	6	2	10	8	7	1	12	201

Also with the purpose of estimating sociopolitical consensus in Italy, we use the information detailed in Appendix B, Table B-1.

4. The empirical results: computation of indexes

Table 3 permits to calculate that the simple consensus index for the 1991-2012 period in

the Swiss analysis is $C_1(V) = \frac{97}{201} \approx 0.4826$. To see this we observe that in 97 out of

the 201 votes held, all cantons gave the same answer (be it approval or rejection).

An analysis of the full data on the votes of the cantons for that period (cf., Table A-1, Appendix A) shows that the corresponding pairwise consensus indexes $C_p(P) \approx 0.8350$, and the majoritarian consensus index is $\mu_m(V) \approx 0.8858$. It coincides with the weighted majoritarian consensus index because the number of agents (the cantons) is fixed.

In order to perform a basic dynamical analysis motivated by our question on the possible evolution of sociopolitical consensus in Switzerland, we have subdivided the data in four periods, namely 1991-1998, 1999-2003, 2004-2008 and 2009-2012. In each of these subperiods we have reproduced the analysis above. Table 4 collects the results inclusive of the indicators for the whole period:

Table 4

Value of the indicators in the periods 1991-1998, 1999-2003, 2004-2008, 2009-2012, and the whole 1991-2012, for the data about popular votes in Switzerland by cantons.

	1991-1998	1999-2003	2004-2008	2009-2012	1991-2012
Simple	0.45	0.5439	0.4444	0.5	0.4826
Pairwise	0.8288	0.8561	0.8240	0.8242	0.8350
Majoritarian	0.8827	0.8988	0.8793	0.8764	0.8858

In order to draw consequences on sociopolitical consensus, we believe that the simple measure is too naive and only the more sophisticated pairwise and (weighted) majoritarian indexes should be studied. Both coincide in pointing out a decline after the 2003 year, with restricted indexes well below the values of the whole period.

We have first performed a dynamical analysis, using the cumulative data of the Italian referendums since 1974 (see Table 5), as shown in Figure 1.

Table 5

Value of the indicators using aggregate data since 1974, for the data about Italian referendums.

Aggregate until year	C_p	μ_m	μ_w
1974	0.517150	0.592600	0.592600
1978	0.555131	0.640433	0.640026
1981	0.647496	0.746200	0.745276
1985	0.631522	0.723644	0.721331
1987	0.647915	0.749050	0.744272
1989	0.657337	0.757800	0.754313
1990	0.691798	0.785939	0.774116
1991	0.703563	0.794874	0.783666
1993	0.705660	0.797630	0.790616
1995	0.648020	0.731241	0.733855
1997	0.646888	0.734780	0.735464
1999	0.651099	0.738619	0.738846
2000	0.643755	0.734785	0.736659
2001	0.641875	0.733100	0.735514
2003	0.646063	0.737600	0.737760
2005	0.653283	0.745285	0.741789
2006	0.651222	0.743150	0.739486
2009	0.653161	0.746129	0.740974
2011	0.667719	0.757922	0.755343

Now we can observe that the majoritarian consensus index does not coincide with the weighted majoritarian consensus index because the number of voters varies with the issue.

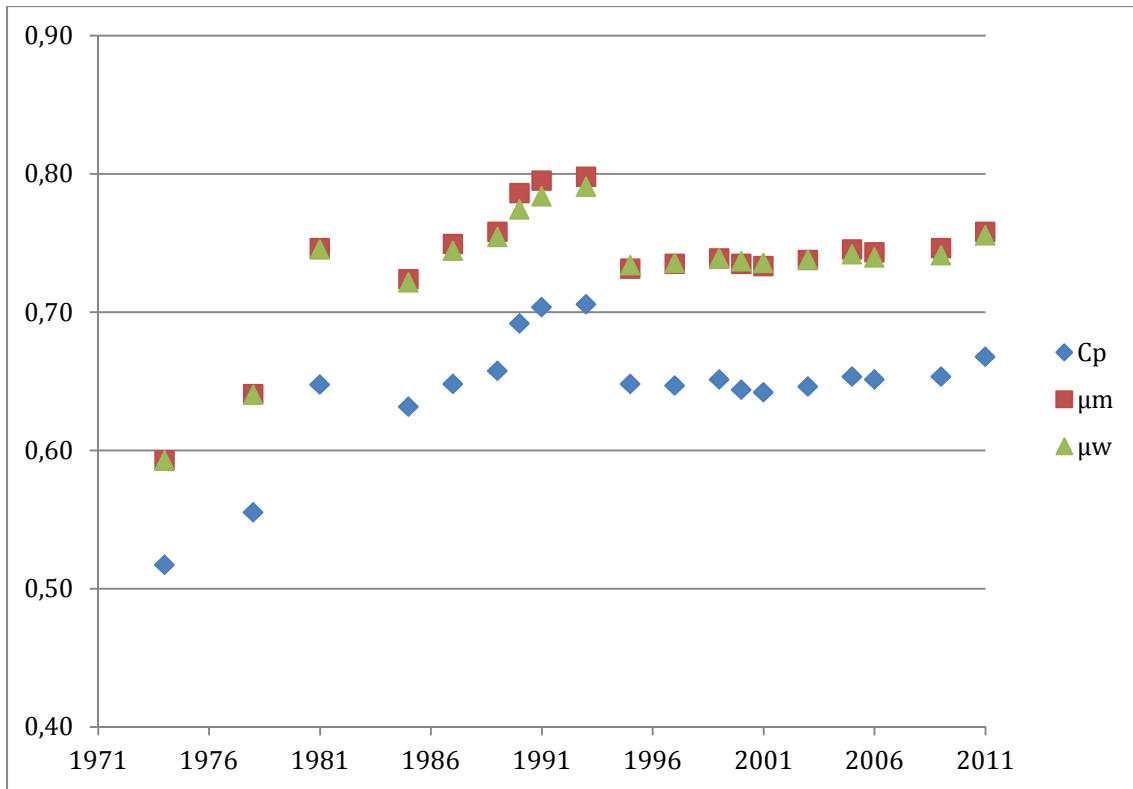


Figure 1. Value of the indicators for the cumulative data from the Italian referendums since 1974. Source: Italian Secretary of State, Archivio Storico delle Elezioni.

We can also perform another dynamical analysis to study the possible evolution of sociopolitical consensus in Italy, distinguishing four periods in the development of abrogative referendums, as we have explained in the introductory section.

Table 6

Value of the indicators in the periods 1974-1985, 1987-1995, 1997-2009, 2011, for the data about Italian referendums.

	1974-1985	1987-1995	1997-2009	2011
Pairwise	0,631522	0,652969	0,660873	0,904280
Majoritarian	0,723644	0,733520	0,768462	0,949550
Weighted	0,721331	0,738170	0,763143	0,949551

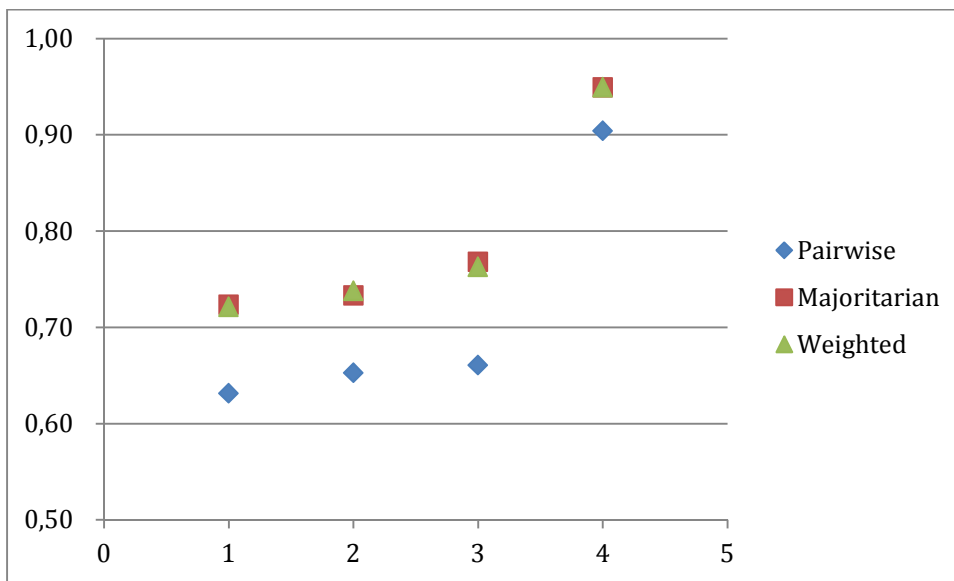


Figure 2. Value of the indicators in the periods 1974-1985, 1987-1995, 1997-2009, 2011, for the data about Italian referendums. Source: Italian Secretary of State, Archivio Storico delle Elezioni.

We observe that the first period produced the smallest consensus across periods, reflecting the controversial nature of the issues put to a vote. The period with the highest consensus (before the current, unended period) corresponds to the years during which the resource to abstention on one side of the electorate corrupted the expressed opinions.

5. Conclusions and final comments

We have introduced some indexes of sociopolitical consensus that rely on the evaluation of the cohesiveness of the agents (either representative, like cantons, or actual voters) in popular votes. They differ in that they use different kinds of information, and that they build on different interpretations of how 'fractions' of full consensus should be interpreted. We show empirical evidence (from two countries with

a tradition for popular votes, namely Switzerland and Italy), that they can be used to assess sociopolitical consensus.

Vatter (2008, p. 31) concludes that "Switzerland is on the way to becoming an average consensus democracy", showing "pronounced elements of assimilation and normalization of the original special case". Our analysis (cf., Table 4) shows that in terms of sociopolitical consensus, there are some evidences that the same phenomenon has happened during the last decade in comparison with prior stages.

With respect to Italy, the division in identifiable periods of the development of referendums has been captured by all these indexes as was expected.

Other applications related with political analysis are hinted in Alcantud *et al.* (2013a). In this paper the predictions about the 2012 presidential elections in USA made by polling agencies are the subject of inspection. Only a toy example is provided, but a more extensive analysis can be performed on the basis of the same argument. With this application we have an example of a possible interpretation of approval consensus measures in terms other than voting situations.

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APPENDIX A

Table A-1. Results of the popular votes in the 1991-2012 period in Switzerland: number of cantons where the issue was approved. Source: Swiss Federal Statistical Office (<http://www.bfs.admin.ch>)

Year	1991	1991	1991	1991
Initiative	1	2	3	4
Yes	26	2	3	22

Year	1992	1992	1992	1992	1992	1992	1992	1992	1992	1992	1992	1992	1992	1992	1992	1992
Initiative	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Yes	1	4	20	20	21	0	25	26	25	23	18	0	1	26	17	8

Year	1993	1993	1993	1993	1993	1993	1993	1993	1993	1993	1993	1993	1993	1993	1993	1993
Initiative	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Yes	18	26	0	8	5	26	26	26	26	26	25	21	25	23	0	0

Year	1994	1994	1994	1994	1994	1994	1994	1994	1994	1994	1994	1994	1994
Initiative	1	2	3	4	5	6	7	8	9	10	11	12	13
Yes	24	26	24	19	26	12	11	4	26	13	13	0	26

Year	1995	1995	1995	1995	1995	1995	1995
Initiative	1	2	3	4	5	6	7
Yes	10	4	3	26	21	0	7

Year	1996	1996	1996	1996	1996	1996	1996	1996	1996
Initiative	1	2	3	4	5	6	7	8	9
Yes	26	26	4	26	17	26	3	12	0

Year	1997	1997	1997	1997	1997
Initiative	1	2	3	4	5
Yes	0	0	26	16	0

Year	1998	1998	1998	1998	1998	1998	1998	1998	1998	1998
Initiative	1	2	3	4	5	6	7	8	9	10
Yes	26	0	0	18	0	5	22	26	0	23

Year	1999	1999	1999	1999	1999	1999	1999	1999	1999	1999
Initiative	1	2	3	4	5	6	7	8	9	10
Yes	24	26	3	21	14	26	26	16	0	6

Year	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
Initiative	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Yes	26	0	0	0	0	24	0	5	3	0	0	6	7	4	0	24

Year	2001	2001	2001	2001	2001	2001	2001	2001	2001	2001	2001
Initiative	1	2	3	4	5	6	7	8	9	10	11
Yes	0	0	0	11	11	26	26	0	0	0	0

Year	2002	2002	2002	2002	2002	2002	2002	2002	2002
Initiative	1	2	3	4	5	6	7	8	9
Yes	13	0	24	0	6	7	11	15	22

Year	2003	2003	2003	2003	2003	2003	2003	2003	2003	2003	2003
Initiative	1	2	3	4	5	6	7	8	9	10	11
Yes	26	26	26	26	1	0	0	3	1	2	0

Year	2004	2004	2004	2004	2004	2004	2004	2004	2004	2004	2004	2004	2004
Initiative	1	2	3	4	5	6	7	8	9	10	11	12	13
Yes	0	0	24	0	0	0	6	7	10	11	23	25	26

Year	2005	2005	2005	2005	2005
Initiative	1	2	3	4	5
Yes	12	19	19	26	7

Year	2006	2006	2006	2006	2006	2006
Initiative	1	2	3	4	5	6
Yes	26	3	26	26	15	25

Year	2007	2007
Initiative	1	2
Yes	2	22

Year	2008	2008	2008	2008	2008	2008	2008	2008	2008	2008
Initiative	1	2	3	4	5	6	7	8	9	10
Yes	0	18	1	0	0	20	4	0	0	26

Year	2009	2009	2009	2009	2009	2009	2009	2009
Initiative	1	2	3	4	5	6	7	8
Yes	22	26	10	13	26	26	0	22

Year	2010	2010	2010	2010	2010	2010	2010
Initiative	1	2	3	4	5	6	7
Yes	26	0	0	18	20	0	4

Year	2011
Initiative	1
Yes	6

Year	2012	2012	2012	2012	2012	2012	2012	2012	2012	2012	2012	2012
Initiative	1	2	3	4	5	6	7	8	9	10	11	12
Yes	15	5	0	26	6	0	0	0	26	10	1	24

APPENDIX B

Table B-1. Results of the Italian referendums in the 1946-2012 period. Source: Italian Secretary of State, Archivio Storico delle Elezioni (<http://elezionistorico.interno.it/>)

Year	1974	1978	1978
Initiative	1	1	2
Yes (%)	40.74	23.54	43.59
No (%)	59.26	76.46	56.41
Valid votes	32,295,858	31,439,425	31,410,378

Year	1981	1981	1981	1981	1981	1985
Initiative	1	2	3	4	5	1
Yes (%)	14.88	22.63	14.08	11.58	32.00	45.68
No (%)	85.12	77.37	85.92	88.42	68.00	54.32
Valid votes	31,161,476	31,445,673	31,418,599	30,984,904	31,625,120	33,845,643

Year	1987	1987	1987	1987	1987	1989
Initiative	1	2	3	4	5	1
Yes (%)	80.21	85.04	80.57	79.71	71.86	88.03
No (%)	19.79	14.96	19.43	20.29	28.14	11.97
Valid votes	25,896,355	26,007,745	26,043,929	25,866,511	26,157,518	33,122,742

Year	1990	1990	1990	1991
Initiative	1	2	3	1
Yes (%)	92.20	92.28	93.51	95.57
No (%)	7.80	7.72	6.49	4.43
Valid votes	19,295,231	19,397,886	19,557,798	28,144,887

Year	1993	1993	1993	1993	1993	1993	1993	1993
Initiative	1	2	3	4	5	6	7	8
Yes (%)	82.57	55.36	90.25	89.80	90.11	82.74	70.23	82.28
No (%)	17.43	44.64	9.75	10.20	9.89	17.26	29.77	17.72
Valid votes	34,412,643	34,785,730	34,598,906	34,571,043	34,663,796	34,971,387	34,638,511	34,672,426

Year	1995	1995	1995	1995	1995	1995
Initiative	1	2	3	4	5	6
Yes (%)	43.07	44.34	43.59	49.97	62.14	64.68
No (%)	56.93	55.66	56.41	50.03	37.86	35.32
Valid votes	26,978,610	27,030,205	26,875,869	24,597,023	24,294,888	24,238,425

Year	1995	1995	1995	1995	1995	1995
Initiative	7	8	9	10	11	12
Yes (%)	63.68	54.90	35.63	56.24	49.40	37.40
No (%)	36.32	45.10	64.37	43.76	50.60	62.60

Valid votes	24,142,229	25,022,962	24,534,037	24,796,712	24,607,219	24,994,779		
Year	1997	1997	1997	1997	1997	1997	1997	1997
Initiative	1	2	3	4	5	6	7	
Yes (%)	74.06	71.69	80.90	83.55	65.52	85.58	66.85	
No (%)	25.94	28.31	19.10	16.45	34.48	14.42	33.15	
Valid votes	12,880,352	13,336,669	13,518,329	12,909,521	12,702,450	13,040,846	12,848,609	

Year	1999	2000	2000	2000	2000	2000	2000	2000
Initiative	1	1	2	3	4	5	6	7
Yes (%)	91.52	71.06	82.02	70.57	69.00	75.22	33.36	61.82
No (%)	8.48	28.94	17.98	29.43	31.00	24.78	66.64	38.18
Valid votes	23,121,888	14,078,269	14,189,487	12,930,715	13,387,954	13,561,179	14,757,427	13,963,498

Year	2001	2003	2003	2005	2005	2005	2005	2006
Initiative	1	1	2	1	2	3	4	1
Yes (%)	64.21	86.74	85.53	88.03	88.78	87.73	77.38	38.71
No (%)	35.79	13.26	14.47	11.97	11.22	12.27	22.62	61.29
Valid votes	16,250,101	12,191,080	12,195,075	12,204,927	12,187,197	12,155,167	12,136,056	25,753,782

Year	2009	2009	2009	2011	2011	2011	2011
Initiative	1	2	3	1	2	3	4
Yes (%)	77.63	77.68	87.00	95.35	95.80	94.05	94.62
No (%)	22.37	22.32	13.00	4.65	4.20	5.95	5.38
Valid votes	10,372,226	10,362,230	10,908,329	27,200,859	27,277,283	27,265,741	27,197,124

Recall that the 0 or 1 values can be attached arbitrarily since they are just labels. This emphasizes the fact that our model applies to cases where a dichotomous choice must be made for all issues, which includes but is not limited to "yes-no" questions (v., Alcantud *et al.*, 2013a).

There was a previous referendum in 1946 where the Italians had to choose the institutional form of the State between "Republic" or "Monarchy". As a result a 54.27% of the voters chose Republic and the rest (45.73%) chose Monarchy. There were 23,437,143 valid votes.