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23. May 2013

Online at <http://mpa.ub.uni-muenchen.de/47221/>

MPRA Paper No. 47221, posted 27. May 2013 13:31 UTC

# Beliefs and (In)Stability in Normal-Form Games

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In this paper, we use experimental data to study players' stability in normal-form games where subjects have to report beliefs and to choose actions. Subjects saw each of 12 games four times in a regular or isomorphic form spread over two days without feedback. We document a high degree of stability within the same (strategically equivalent) game, although time and changes in the presentation of the game do lead to less stability. To look at stability *across* different games, we adopt the level- $k$  theory, and show that stability of both beliefs and actions is significantly lower. Finally, we estimate a structural model in which players either apply a consistent level of reasoning across strategically different games, or reasoning levels change from game to game. Our results show that approximately 30% of subjects apply a consistent level of reasoning across the 12 games, but that they assign a low level of sophistication to their opponent. The remaining 70% apply different levels of reasoning to different games.

*Key words:* Game theory, Beliefs, Stability, Level- $k$  thinking, Experiment

*JEL codes:* C72, C91, D83

*History:* **May 23, 2013**

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## 1. Introduction

When deciding on an action to take in normal-form games, players must form beliefs about the action(s) that will be taken by their opponent(s). That is, they must have a theory of the mind of their opponents. Several such theories have been proposed in the literature. In static, normal form games, the benchmark is Nash reasoning, which assumes that players form beliefs and perfectly best-respond to them. Moreover, in the equilibrium, beliefs and actions are self-reinforcing in the sense that the observed action profile justifies the underlying beliefs (and vice-versa). However, in many situations, behavior frequently deviates from the Nash benchmark. Because of this, other approaches designed around boundedly rational or error prone decision makers have arisen. Notable examples of those approaches are Quantal Response Equilibrium (McKelvey and Palfrey 1995); Noisy Introspection (Goeree and Holt 2004); Level- $k$  models (Nagel 1995, Stahl and Wilson 1994,

1995, Costa-Gomes et al. 2001) or Cognitive Hierarchy models (Camerer et al. 2004) in which subjects vary according to their strategic reasoning or sophistication, with higher levels of reasoning being represented as more iterations of best-response. There is a now extensive literature about the ability of these models to rationalise observed behaviour in the lab; with Level- $k$  and Cognitive Hierarchy models appearing to be ahead in this horse race (see, e.g, Costa-Gomes and Crawford 2006, Costa-Gomes et al. 2009, Crawford et al. 2013).

Much less, however, is known about the stability of the belief formation process. Such a stability would be a very desirable feature as it would allow out of sample predictions and counterfactual analysis. Lack of stability would mean that agents form beliefs and expectations in largely unpredictable ways, thus rendering many economic policies less reliable due to the greater unpredictability of behaviour. Several degrees of stability are worth looking at. The first, most basic, is stability across identical situations. The second is stability across equivalent – but not identical – strategic situations; and the last, most general and arguably most desirable, is stability across different strategic situations. In this paper, we design an experiment that allows us to look at all three degrees of stability, as well as the stability of subjects across time.

The experimental literature provides mixed evidence about the stability of strategic behaviour. Coming out in favour of stability, Camerer et al. (2004) provide evidence that the distribution of levels of reasoning is stable across games. Stahl and Wilson (1995) is an example of a paper, like ours, that looks at stability of individual players' level of reasoning across games and show that many of their subjects possess a fair degree of stability. Their methodology, however, is likely to overstate stability (Georganas et al. 2010). In contrast, several studies have shown much less stability across games. Most recently, Georganas et al. (2010) show that stability of levels of reasoning is moderate at best, and depends on the class of games being played. They also show that one's performance on quizzes designed to measure strategic reasoning and general intelligence do not predict stability well.

These papers, however, have mostly used action data. In addition to the fact that actions may not reflect underlying beliefs (Costa-Gomes and Weizsäcker 2008), it has been argued (e.g. Manski 2002, 2004) that action data are, by themselves, insufficient to estimate decision rules, and that information on beliefs is crucial. Our study uses both belief and action data, and thus allows for a more direct investigation of the belief formation process. Moreover, these papers rely on specific theories of the mind (mostly level- $k$ ), while we do so only for the third degree of stability.

Agranov et al. (2012) demonstrate that some players are unstable in the sense that they adjust their strategy according to their beliefs about the level of strategic sophistication of their opponent.

This instability is triggered by changes in the information given to subjects about their opponent. Similarly, Georganas et al. (2010) show that some players adjust their strategies when playing against stronger opponents. In this paper, we investigate whether instability occurs without any variation in the information about the opponent.

In our experiment, we chose 12  $3 \times 3$  normal-form games. For each game we had a “regular” and “isomorphic” representation (obtained by adding or subtracting a constant and rearranging the rows and/or columns). Subjects participated over two days, separated either by one day or one week. On each day, they played 24 games, seeing each of the 12 games twice (though possibly under a different frame). Subjects received no feedback until the end of the second day. Therefore, over the two days, a given strategic structure is displayed four times to the subjects, either in its regular or isomorphic versions. As the equilibrium structure of a given game is not affected by the isomorphic transformation, these four instances represent a set of *strategically equivalent games*.

Therefore, we are able to study stability across several interesting dimensions. In particular, we can compare stability both within and across sets of strategically equivalent games, and we can also gain insights into whether the framing of the game or the time between instances of the same game has an impact on stability. We also varied the characteristics of the games subjects played; in particular, four games were dominance solvable games with one Nash equilibrium in pure strategies, four games were not dominance solvable, but also had one Nash equilibrium in pure strategies, and four games had two Nash equilibria in pure strategies. As stated above, one other difference between our study and much of this literature is that we are interested in both the stability of actions and the stability of underlying beliefs. Therefore, in our study, subjects chose an action and stated beliefs about the likely action of their opponent. This allows us to investigate the connections between action and belief stability and may also point to a source for the instability in action choices that have been observed in the literature: namely, to changing beliefs.<sup>1</sup>

Our results indicate a fair degree of belief and action stabilities within the same game. For example, across the four instances subjects saw each set of strategically equivalent games, nearly 50% of the time subjects’ best-response to their beliefs was the same (modulo an isomorphic transformation in the relevant cases) and another 33% of the time it only changed once. Concerning action stability, 38.6% of the time, subjects’ actions never changed (modulo isomorphic transformation) across all four instances of the same game and another 36.9% of the time the action only changed once. Moreover, stability in actions is positively and significantly related to stability in beliefs. We

<sup>1</sup> Although not central to our study, our experimental design also allows us to test the robustness of Costa-Gomes and Weizsäcker (2008) who showed that subjects’ actions and beliefs are inconsistent with each other.

also find that when one’s action changed from one instance to the next instance of the same game, the best-response to her beliefs also changed in a consistent manner. Finally, our results show that changing the frame of the game or separating two instances of the same game across different days increases belief instability by about 10% from the baseline level of variation in beliefs.

The above results concern stability *within* strategically equivalent games (i.e., the first two degrees of stability). However, we also examine stability across strategically *different* games. In order to make comparisons, we need a framework for classifying the decisions of subjects in different games. For our analysis we organize behaviour according to the level- $k$  theory and look at stability in this sense. The level- $k$  model is particularly appealing because it allows us to classify subjects’ actions in different games as equivalent if the actions apply the same depth of strategic reasoning. Our results suggest much less stability with many subjects choosing different levels of reasoning across different games. However, we do document a positive relationship between stability *within* equivalent games and stability *across* different games. That is, subjects who are more stable within equivalent games are also more stable across different games. Beyond this, we find that subjects who report beliefs closer to the centre of the simplex (i.e., uniform or level-1 beliefs) also possess a higher degree of stability across different games, though at a very low level of sophistication.<sup>2</sup>

Our descriptive results suggest that there are at least two different types of subjects: those who are stable across different games and those who are not. To gain more insight into this, we estimate a so-called “mover-stayer” model. In our model, stayers have beliefs which do not change across different games (in the level- $k$  sense), while movers choose one of several possible beliefs for each set of four equivalent games. We find that almost 28.5% of our subjects are stayers. Among these subjects, 99% choose approximately level-1 beliefs (slightly biased towards level-2). The remaining 71.5% of our subjects are movers. Nearly 50% of the time, movers choose a level-2 belief; 20% of the time they state a level-1 belief and another 20% of the time they state a level-3 belief. We also note that the estimated rationality parameter is substantially and significantly higher for stayers, consistent with our earlier finding that subjects who state level-1 beliefs are more stable.

Given the estimates, we are able to compute the posterior probability that a subject is either a mover or a stayer. It turns out that our classification is very precise with the posterior probability being either 0 or 1 that the subject is a mover for the vast majority subjects. With this classification, we show that stayers’ behaviour is significantly more stable than movers. Finally, we show that while most stayers are women, movers are significantly more likely to be men.

<sup>2</sup> While risk aversion may play a role, we do not think that it leads to biased beliefs since subjects best-respond to uniform beliefs at about the same rate as they do to more extreme beliefs. We discuss this in Appendix C.

The rest of the paper proceeds as follows. In Section 2 we provide the details of our experimental design. In Section 3 we provide some descriptive results on belief and action stability, while Section 4 takes a deeper look at the stability of both actions and beliefs and relates stability to other performance measures. Section 5 describes our mover-stayer model and provides the results. Finally, in Section 6 we provide some concluding remarks.

## 2. Experimental Design

### 2.1. Games

Our purpose in this experiment was to look at the stability of both beliefs and actions over time. In order to do this, we designed the 12 games shown in Figure 1. These games were chosen with the following properties: four of them had a unique Nash equilibrium that was in pure strategies (games  $G1$  to  $G4$ ), four of them had a unique Nash equilibrium that could be arrived at through the iterated deletion of dominated strategies (games  $G5$  to  $G8$ ) and four of them had two pure strategy Nash equilibria (games  $G9$  to  $G12$ ). Because we were interested in whether behaviour is sensitive to the frame, we also created 12 isomorphic games by interchanging rows and/or columns and adding or subtracting a constant to the payoffs (cf. Figure 2). Note also that none of the games have any mixed strategy Nash equilibria.<sup>3</sup>

In Figures 1 and 2 we underline the outcomes corresponding to Nash equilibria, and below each payoff matrix in Figure 2 we describe how the game was transformed based on its regular counterpart. For example, in game  $G1'$ ,  $r : (2, 3, 1)$  indicates that row player's first, second and third actions appear in  $G1$  as the second, third and first actions respectively. The notation,  $c : (2, 3, 1)$ , is analogous for column players. Finally, the  $-3$  indicates that all payoffs were reduced by 3 points in game  $G1'$  relative to game  $G1$ . Unless otherwise noted, in our subsequent data analysis, to facilitate the comparison of actions and beliefs between the regular and transformed games, we first apply the inverse transformations so that all games appear in their regular form (i.e., as in Figure 1).

### 2.2. Procedures

Our experiments were run at the Parisian Lab for Experimental Economics (LEEP). Our subjects were recruited among a broad pool of students from the University of Paris 1. The experiment had two treatments. For both treatments, the experiment took place over two days, with identical procedures on each day. The only distinguishing factor between the treatments was the length of

<sup>3</sup> In contrast, many of the games in Costa-Gomes and Weizsäcker (2008) had, in addition to the unique pure strategy equilibrium, two mixed strategy Nash equilibria. Although an analysis of their data did not find any evidence that subjects played any of the mixed strategy equilibria, we wanted our games to be as "clean" as possible and, therefore, made every effort to ensure that there were no mixed strategy equilibria.

Figure 1 Payoff Matrices Of The Regular Games

$G1$	$\ell$	$m$	$r$	$G2$	$\ell$	$m$	$r$	$G3$	$\ell$	$m$	$r$
$T$	75,90	27,31	55,43	$T$	15,89	70,38	75,59	$T$	48,53	47,67	17,60
$M$	90,40	28,35	31,51	$M$	22,44	26,46	38,57	$M$	60,57	31,42	65,33
$B$	63,42	65,86	78,26	$B$	41,85	61,75	21,70	$B$	46,45	54,62	41,73

$G4$	$\ell$	$m$	$r$	$G5$	$\ell$	$m$	$r$	$G6$	$\ell$	$m$	$r$
$T$	73,90	78,28	48,32	$T$	80,10	15,17	22,10	$T$	60,14	20,23	18,19
$M$	69,11	55,80	73,67	$M$	15,24	20,28	25,32	$M$	35,33	25,36	21,44
$B$	24,18	56,46	83,38	$B$	20,35	14,38	21,72	$B$	30,31	22,35	16,55

$G7$	$\ell$	$m$	$r$	$G8$	$\ell$	$m$	$r$	$G9$	$\ell$	$m$	$r$
$T$	78,73	69,23	12,14	$T$	21,67	59,57	85,63	$T$	78,84	27,45	73,29
$M$	67,52	59,61	78,53	$M$	71,76	50,65	74,14	$M$	64,66	77,41	59,77
$B$	16,76	65,87	94,79	$B$	12,10	51,76	77,92	$B$	52,85	77,85	67,36

$G10$	$\ell$	$m$	$r$	$G11$	$\ell$	$m$	$r$	$G12$	$\ell$	$m$	$r$
$T$	22,77	46,64	74,53	$T$	65,41	22,58	35,61	$T$	27,49	13,49	18,34
$M$	47,22	77,70	10,70	$M$	23,31	50,30	88,31	$M$	88,44	13,41	38,48
$B$	59,76	77,35	51,66	$B$	88,70	15,68	88,47	$B$	58,85	11,80	81,88

Figure 2 Payoff Matrices Of The Isomorphic Games

$G1'$	$\ell$	$m$	$r$	$G2'$	$\ell$	$m$	$r$	$G3'$	$\ell$	$m$	$r$
$T$	25,32	28,48	87,37	$T$	39,58	23,45	27,47	$T$	44,76	49,48	57,65
$M$	62,83	75,23	60,39	$M$	22,71	42,86	62,76	$M$	20,63	51,56	50,70
$B$	24,28	52,40	72,87	$B$	76,60	16,90	71,39	$B$	68,36	63,60	34,45
			$(r: (2, 3, 1), c: (2, 3, 1)); -3$				$(r: (2, 3, 1), c: (3, 1, 2)); +1$				$(r: (3, 1, 2), c: (3, 1, 2)); +3$

$G4'$	$\ell$	$m$	$r$	$G5'$	$\ell$	$m$	$r$	$G6'$	$\ell$	$m$	$r$
$T$	57,82	75,69	71,13	$T$	28,35	18,27	23,31	$T$	21,34	15,54	29,30
$M$	58,48	85,40	26,20	$M$	24,75	23,38	17,41	$M$	19,22	17,18	59,13
$B$	80,30	50,34	75,92	$B$	25,13	83,13	18,20	$B$	24,35	20,43	34,32
			$(r: (2, 3, 1), c: (2, 3, 1)); +2$				$(r: (2, 3, 1), c: (3, 1, 2)); +3$				$(r: (3, 1, 2), c: (2, 3, 1)); -1$

$G7'$	$\ell$	$m$	$r$	$G8'$	$\ell$	$m$	$r$	$G9'$	$\ell$	$m$	$r$
$T$	63,85	92,77	14,74	$T$	76,16	73,78	52,67	$T$	64,33	49,82	74,82
$M$	67,21	10,12	76,71	$M$	79,94	14,12	53,78	$M$	70,26	75,81	24,42
$B$	57,59	76,51	65,50	$B$	87,65	23,69	61,59	$B$	56,74	61,63	74,38
			$(r: (3, 1, 2), c: (2, 3, 1)); -2$				$(r: (2, 3, 1), c: (3, 1, 2)); +2$				$(r: (3, 1, 2), c: (3, 1, 2)); -3$

$G10'$	$\ell$	$m$	$r$	$G11'$	$\ell$	$m$	$r$	$G12'$	$\ell$	$m$	$r$
$T$	12,72	49,24	79,72	$T$	16,69	89,48	89,71	$T$	18,34	27,49	13,49
$M$	53,68	61,78	79,37	$M$	23,59	36,62	66,42	$M$	81,88	58,85	11,80
$B$	76,55	24,79	48,66	$B$	51,31	89,32	24,32	$B$	38,48	88,44	13,41
			$(r: (2, 3, 1), c: (3, 1, 2)); +2$				$(r: (3, 1, 2), c: (2, 3, 1)); +1$				$(r: (1, 3, 2), c: (3, 1, 2)); +0$

time between the first and second days. In the first treatment, the experiments took place on two consecutive days, while in the second treatment, the two days were separated by a week. Subjects were informed at the onset of the experiment that they would only be paid if they participated in both days of the experiment. In the treatment with a one-day delay, we had 44 subjects coming on the first day, with 39 returning one day later. In the treatment with a seven-day delay, we had 46 subjects coming on the first day, with 33 returning a week later. When possible, the results will use data from all 90 subjects; however, in Appendix A, we show that behavior (on day 1) is essentially identical whether or not subjects showed up and participated on day 2 — thus attrition does not influence our results. For each subject, we also collected information on gender, level of education and field of study.

On each day, subjects played the games as given in Table 1 (though the order in which the games were played differs from the presentation in the table).<sup>4</sup> Subjects were given the role of either a row player or a column player and kept that role for the entire experiment. For each game that subjects played, they were randomly matched with another subject of the opposite role. In each game, subjects had to complete two tasks: they had to state beliefs about the likely action that their opponent would play; and to choose an action.

**Table 1** Properties of Games

(a) Day 1									(b) Day 2								
Game	# Occ	DS?	Row Player			Column Player			Game	# Occ	DS?	Row Player			Column Player		
			$L_1$	$L_2$	Nash	$L_1$	$L_2$	Nash				$L_1$	$L_2$	Nash	$L_1$	$L_2$	Nash
$G1$	× 2	N	$B$	$M$	$B$	$\ell$	$m$	$m$	$G1$	× 1	N	$B$	$M$	$B$	$\ell$	$m$	$m$
$G2$	× 1	N	$T$	$B$	$B$	$\ell$	$\ell$	$\ell$	$G1'$	× 1	N	$M$	$T$	$M$	$r$	$\ell$	$\ell$
$G2'$	× 1	N	$B$	$M$	$M$	$m$	$m$	$m$	$G2'$	× 2	N	$B$	$M$	$M$	$m$	$m$	$m$
$G3$	× 2	N	$M$	$B$	$M$	$m$	$\ell$	$\ell$	$G3$	× 1	N	$M$	$B$	$M$	$m$	$\ell$	$\ell$
$G4$	× 1	N	$T$	$T$	$T$	$m$	$\ell$	$\ell$	$G3'$	× 1	N	$B$	$T$	$B$	$r$	$m$	$m$
$G4'$	× 1	N	$B$	$B$	$B$	$\ell$	$r$	$r$	$G4'$	× 2	N	$B$	$B$	$B$	$\ell$	$r$	$r$
$G5$	× 2	Y	$T$	$M$	$M$	$r$	$m$	$r$	$G5$	× 1	Y	$T$	$M$	$M$	$r$	$m$	$r$
$G6$	× 1	Y	$T$	$M$	$M$	$r$	$m$	$r$	$G5'$	× 1	Y	$B$	$T$	$T$	$\ell$	$r$	$\ell$
$G6'$	× 1	Y	$M$	$B$	$B$	$m$	$\ell$	$m$	$G6'$	× 2	Y	$M$	$B$	$B$	$m$	$\ell$	$m$
$G7$	× 2	Y	$M$	$T$	$M$	$\ell$	$m$	$\ell$	$G7$	× 1	Y	$M$	$T$	$M$	$\ell$	$m$	$\ell$
$G8$	× 1	Y	$M$	$T$	$M$	$m$	$\ell$	$\ell$	$G7'$	× 1	Y	$B$	$M$	$M$	$r$	$\ell$	$r$
$G8'$	× 1	Y	$T$	$B$	$T$	$r$	$m$	$m$	$G8'$	× 2	Y	$T$	$B$	$T$	$r$	$m$	$m$
$G9$	× 2	N	$M$	$T$	$T, B$	$\ell$	$r$	$\ell, r$	$G9$	× 1	N	$M$	$T$	$T, B$	$\ell$	$r$	$\ell, r$
$G10$	× 1	N	$B$	$T$	$M, B$	$r$	$\ell$	$\ell, m$	$G9'$	× 1	N	$B$	$M$	$M, T$	$m$	$\ell$	$m, r$
$G10'$	× 1	N	$M$	$B$	$M, T$	$\ell$	$m$	$m, r$	$G10'$	× 2	N	$M$	$B$	$M, T$	$\ell$	$m$	$m, r$
$G11$	× 2	N	$B$	$M$	$M, B$	$m$	$\ell$	$\ell, r$	$G11$	× 1	N	$T$	$B$	$M, B$	$\ell$	$r$	$\ell, r$
$G12$	× 1	N	$B$	$M$	$T, B$	$\ell$	$r$	$m, r$	$G11'$	× 1	N	$T$	$B$	$T, B$	$\ell$	$r$	$m, r$
$G12'$	× 1	N	$M$	$B$	$T, M$	$m$	$\ell$	$\ell, r$	$G12'$	× 2	N	$M$	$B$	$T, M$	$m$	$\ell$	$\ell, r$

# Occ: Number of times subjects saw the game on a given day.

DS?: Is the game dominance solvable (Y) or not (N)?

After subjects made all of their choices on both days, they were paid according to their total game payoffs and earnings from their belief statements. Earnings were denoted in experimental currency units and were converted to Euros at the end to the rate of €0.75 for every 100 experimental units. On average, subjects who came on both days earned €19.80.

Because we wanted to create as true as possible a series of one-shot games, and to mitigate any learning effects, subjects did not receive any feedback regarding the action chosen by their opponent or their payoffs for either actions or beliefs until the end of the experiment on the second day. Appendix B checks for any hint of learning in our data and finds none. As can be seen in Table 1, for each of the 12 sets of strategically equivalent games, subjects either saw the exact

<sup>4</sup> More precisely, we randomized the presentation of games within a day with the only constraint being that there must be at least three periods between instances of the same game.



same game twice on day 1 or the game and its isomorphic counterpart on day 1, with the reverse case on day 2. Therefore, subjects saw 4 instances of each game, spread over two days.

### 2.3. Belief Elicitation

For each game, in addition to choosing an action, subjects also stated their beliefs about the probability their opponent would take each of her three possible actions. Consistent with the experimental literature, beliefs were incentivized via a quadratic scoring rule (QSR), which penalizes subjects according to a quadratic loss function depending on how inaccurate their belief statement was. The QSR has the property that for risk-neutral and money-maximizing players it is optimal to report their true beliefs. While the QSR need not be incentive compatible if subjects are risk averse, in Appendix C, we check that risk-aversion does not distort our subjects' belief statements.

## 3. Empirical Strategy and Preliminary Results

### 3.1. Empirical Strategy for Comparing Belief/Action Data Across Games

When analyzing stability across instances of the same game (modulo isomorphic transformations), as noted above, we first apply the inverse transformation so that all games appear in their regular form. However, when we analyze beliefs and actions across different (non-equivalent) games, we cannot simply map the actions or beliefs from one game onto the other game. Therefore, in order to make comparisons we need to be guided by some theory of decision making which can classify chosen actions from two different games as being equivalent in some sense. In this paper, we will use the  $L_k$  theory to enable us to make such cross-game comparisons. The  $L_k$  theory is a boundedly rational theory of human reasoning in games, which can be summarized as follows. First, there are the so-called  $L_0$  players, the only non-strategic players. These players do not respond to beliefs, but instead choose an action randomly in their action space. The other types of players are then characterized by different depths of strategic reasoning. More precisely, the  $L_1$  players select a best-response to  $L_0$  players, the  $L_2$  players select a best-response to  $L_1$  players and so forth. Generally speaking, the  $L_k$  players select a best response to  $L_{k-1}$  players,  $\forall k > 0$ .

Therefore, for  $k > 0$ , all the  $L_k$  players form beliefs and respond to them. In what follows, it will be useful to define  $L_k$  beliefs for these strategic players. To this end we define  $L_1$  beliefs as being located at the centre of the belief simplex; in other words, they are a uniform distribution over the opponent's action space, i.e.  $(1/3, 1/3, 1/3)$ . Then, for  $k > 1$ ,  $L_k$  beliefs put all the probability mass on  $L_{k-1}$  players' best-response. For example, a subject stating an  $L_2$  belief puts all the mass on her opponent playing the  $L_1$  action.

Using the  $L_k$  model to classify beliefs and actions, instability across different games manifests itself by observing subjects choosing a different level of strategic sophistication in different games.

### 3.2. Beliefs and Best-Response Behaviour

**3.2.1. Typology of Beliefs.** Table 2(a) shows the mean and standard deviation of beliefs according to their original labels (i.e., in the form exactly as presented to the subjects). From this, it would seem that subjects are not sensitive to labels when stating their beliefs: none of the average beliefs are significantly different from  $33\frac{1}{3}$ . On the other hand, Table 2(b) shows the mean and standard deviation of beliefs towards the opponent’s  $L_1$ ,  $L_2$ , and “other” action, respectively denoted  $b_{L_1}$ ,  $b_{L_2}$  and  $b_{OA}$  in the table (for games/roles where the opponent’s  $L_1$  and  $L_2$  actions differ). It indicates that beliefs are biased towards the opponent’s  $L_1$  action, and away from the “other” action.<sup>5</sup> The average belief towards the  $L_1$  action is significantly higher than  $33\frac{1}{3}$ , while the belief towards the “other” action is significantly lower than  $33\frac{1}{3}$ .

**Table 2** Summary statistics

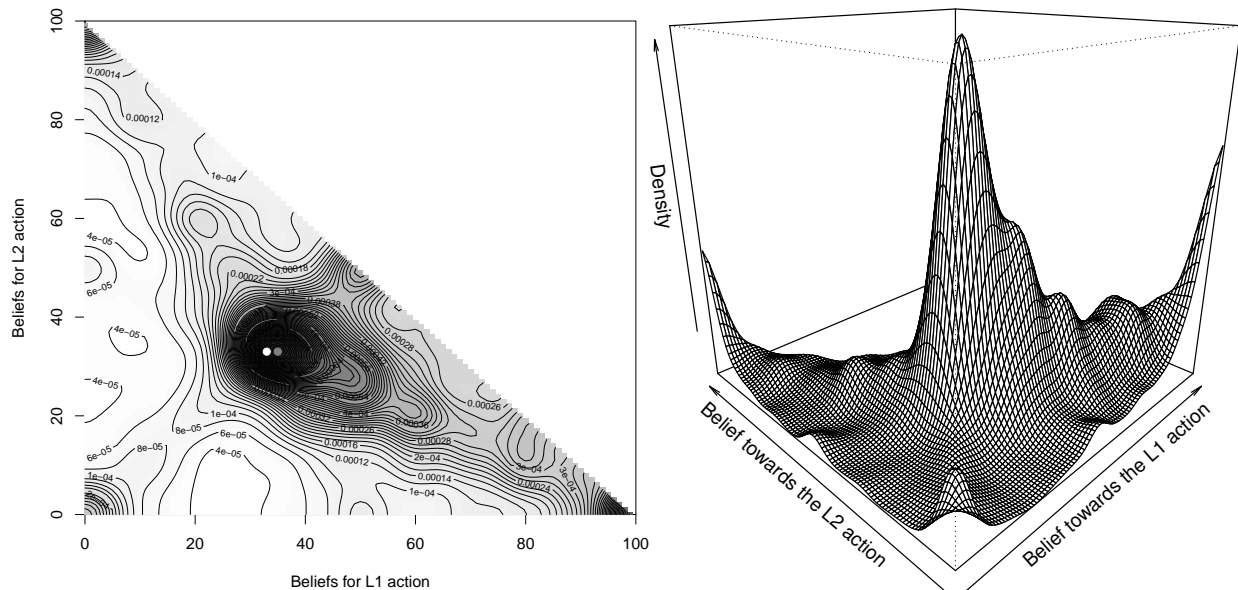
(a) Raw Data				(b) Organized by $L_k$ Theory			
Variable	Mean	Std. Dev.	Std. Err.	Variable	Mean	Std. Dev.	Std. Err.
$b_1$	33.232	26.827	0.717	$b_{L_1}$	44.155	26.546	0.980
$b_2$	33.034	24.646	0.489	$b_{L_2}$	32.398	24.262	0.793
$b_3$	33.735	25.675	0.728	$b_{OA}$	23.447	22.031	0.863
N		3836		N		3512	

Standard errors clustered at the individual level.

Figure 3 shows the density of beliefs in the  $L_k$  simplex using the reflection method described in Haruvy (2002). The white dot indicates  $L_1$  beliefs; while the gray dot indicates the location of the maximum estimated density, which is located at  $(33,35,32)$ . The figure shows that the largest mode is, by far, at  $L_1$  beliefs. A secondary mode can be found at  $L_2$  beliefs. Lower modes also appear at  $(.5, .5, 0)$ ,  $L_3$  beliefs and  $(0, 0, 1)$ . Outside these archetypal beliefs, most of the mass of the distribution can be found around the segment joining  $L_1$  and  $L_2$  beliefs, explaining the higher mean of beliefs towards the opponent’s  $L_1$  action.

We also find that the belief toward the opponent’s Nash action (in the set of games with a single Nash equilibrium) is somewhat high at 39.8%. However, we do not think that players actually use Nash equilibrium when stating their beliefs. Indeed, in our games the Nash action is always either the  $L_1$  or the  $L_2$  action with roughly equal probabilities. If players had the Nash model in mind, then their beliefs towards the  $L_k$  action should be higher when it coincides with the Nash action. To test this prediction, we run OLS regressions clustered at the individual level of the beliefs towards

<sup>5</sup> Note that the “other action” is rarely a Nash action, and when it is, it is one of two Nash equilibrium actions in the multiple equilibria games.

**Figure 3**  $L_k$  beliefs**Table 3** Frequency of Action Choices Organized by Level  $k$  Theory

Action	Frequency	Std. Dev.	N
$L_1$   not $L_2$	0.524	0.500	3512
$L_2$   not $L_1$	0.356	0.474	3512
$L_1$ and $L_2$	0.784	0.412	324
Other Action	0.129	0.356	3836

the  $L_1$  action on a dummy for coincidence between  $L_1$  and Nash.<sup>6</sup> The associated coefficient is 0.068 with a  $p$ -value of 0.962. A similar regression for  $L_2$  beliefs leads to a coefficient of 2.175 and a corresponding  $p$ -value of 0.1. We thus conclude that players do not use the Nash model when forming their beliefs, or do so in a very marginal way.

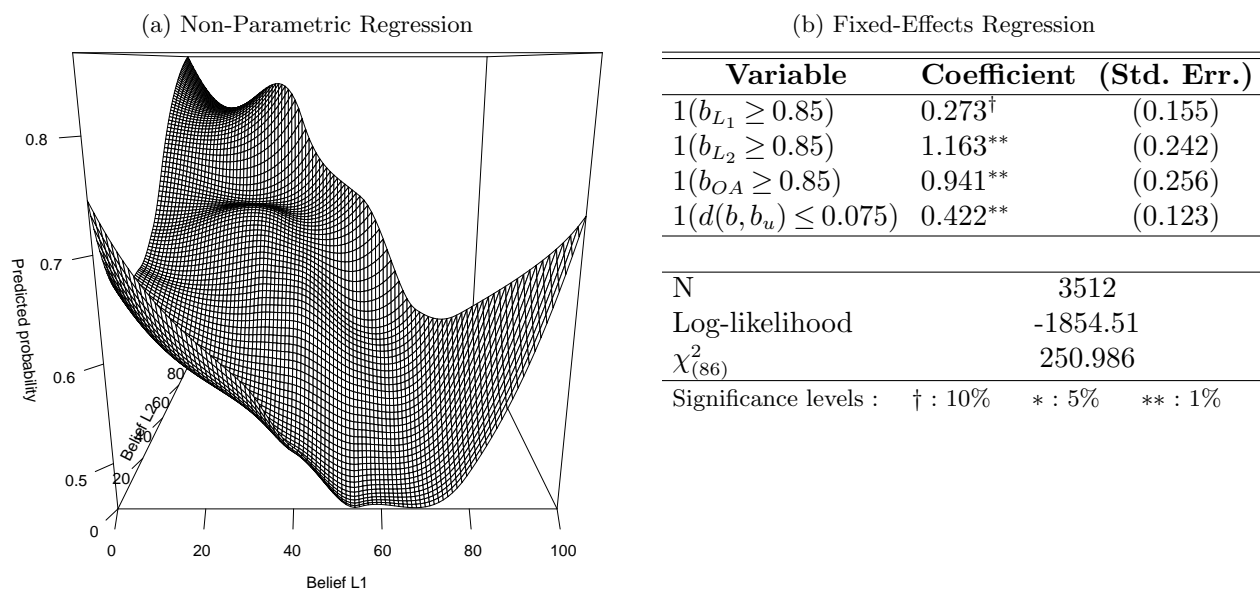
**3.2.2. Typology of Actions.** In Table 3, we show the frequency with which the  $L_1$ ,  $L_2$  and other actions were chosen overall in our experiment. Observe that the  $L_1$  action is chosen 52.4% of the time, with the  $L_2$  action being chosen only 35.6% of the time. Taking the subject average as the unit of independent observation, a paired  $t$ -test easily rejects the null hypothesis that these frequencies are equal ( $p \ll 0.01$ ). Thus it seems that most subjects choose the  $L_1$  action, with fewer subjects choosing the  $L_2$  action and a small frequency of choices which are neither  $L_1$  nor  $L_2$ .

**3.2.3. Best-Response Behaviour.** The overall rate of best-response is 62.6%, which is comparable to the best-response rate reported in Danz et al. (2012), but higher than those reported by Costa-Gomes and Weizsäcker (2008). Note also that unlike Costa-Gomes and Weizsäcker (2008),

<sup>6</sup> We restrict our sample to games with a unique equilibrium and for which the opponent's  $L_1$  and  $L_2$  actions differ.

there are fairly pronounced differences in the best-response rate depending on where a subject's beliefs lie in the simplex. Figure 4(a) shows the results of a bivariate non-parametric regression of the probability to give a best-response on the beliefs towards the opponent's  $L_1$  and  $L_2$  actions. First, it shows that when beliefs lie in a corner of the simplex, subjects exhibit a higher tendency to best-respond to them, with an estimated best-response rate ranging from 0.7 to 0.85. Second, subjects are also more likely to best-respond to beliefs that are near the  $L_1$  beliefs. Finally, the lowest best-response rate is attained close to (0.60, 0.05, 0.35) beliefs.

**Figure 4 Best-response behaviour**



To see this more parametrically, in Figure 4(b) we report the results of a conditional fixed-effects logistic regression of “choosing a best-response” on dummies for having beliefs near the corners of the simplex (i.e., when the belief to the  $L_1$  action, the  $L_2$  action or another action exceeds 0.85) and “being close to the uniform  $L_1$  beliefs” ( $b_u$ ) with a full set of game, day and period dummies.<sup>7</sup> Just as shown in Figure 4(a), having strong beliefs increases the rate of best-response.

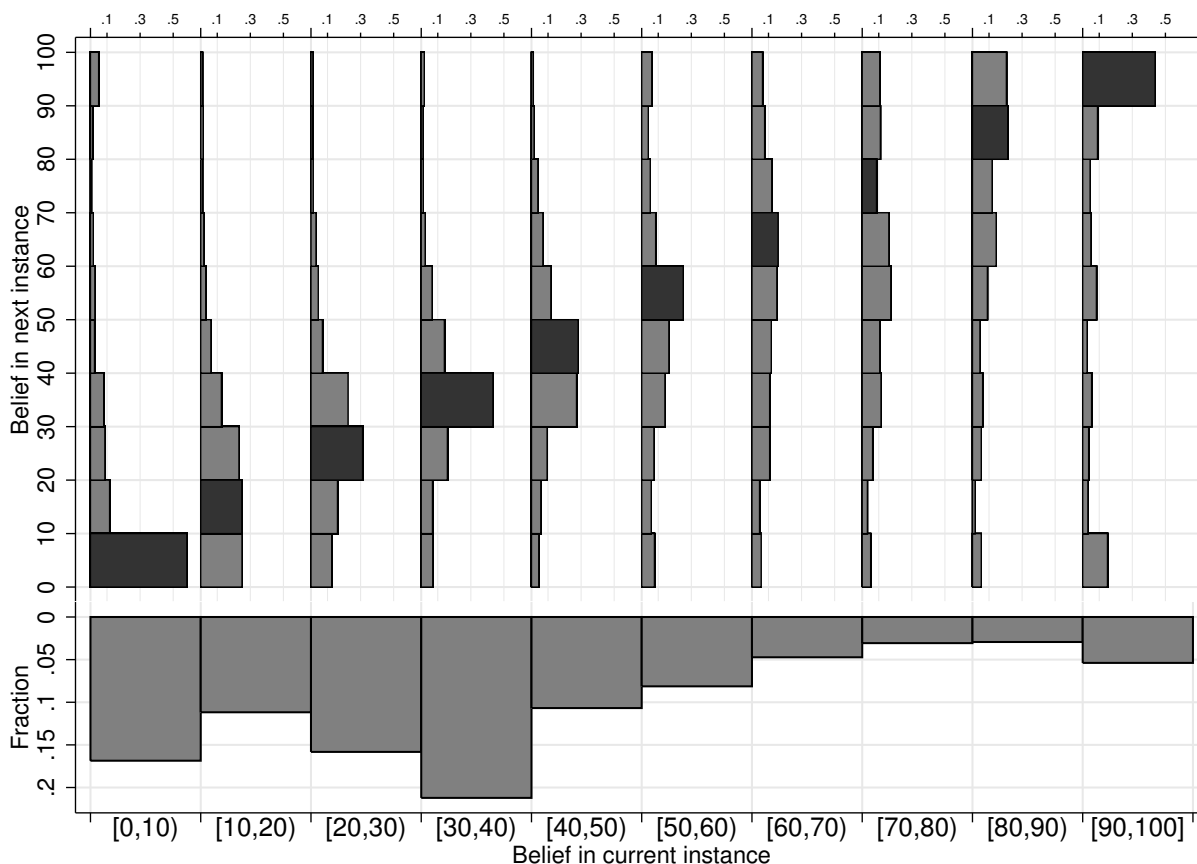
### 3.3. A First Look at Belief and Action Stability

We now turn our attention to both belief and action stabilities. If subjects have a consistent and stable theory of the mind of their opponents, then they should report similar beliefs and choose equivalent actions in different instances of the same game. To get an initial feel for the beliefs

<sup>7</sup> We define closeness to  $L_1$  beliefs based on the Euclidean distance to  $L_1$  beliefs, taking value 1 if the beliefs are contained in a ball of radius 0.075 centred on  $L_1$  beliefs.

data, Figure 5 displays, in its lower panel, the histogram of current beliefs and, in its upper panel, the histograms of the beliefs in the next instance of a strategically equivalent game conditional on current beliefs lying in the corresponding interval.<sup>8</sup> Note that this figure captures our first degrees of stability: within identical or strategically equivalent strategic games.

**Figure 5** Belief stability



It is clear from Figure 5 that stated beliefs are fairly stable across instances of the same game. With the exception of the interval  $[70, 80)$  (which accounts for only 3% of statements), the modal belief interval in the next instance is equal to the same interval in which the belief the current belief lies (shown in a darker shade).

The same pattern can be found for actions. Table 4 shows, for each type of action in the current instance of a game (rows of Table 4), the distribution of actions in the next instance of the same game. As with beliefs, the modal action in the next instance is equal to the action chosen in the current instance. The overall frequency of identical actions being chosen in consecutive instances

<sup>8</sup> In this figure we pool the beliefs towards each of the opponent's three possible actions.

is 0.662, well above random behavior. Table 4 also reveals a clear hierarchy among actions, with  $L_1$  being the most stable, followed by  $L_2$ . The “other” action is the least stable, which may reveal that some of these were errors corrected in the next instance of the same game.

**Table 4** Action stability

	Action in the next instance			Total
	$L_1$ action	$L_2$ action	Other action	
$L_1$ action	0.729	0.209	0.062	1
$L_2$ action	0.306	0.621	0.069	1
Other action	0.273	0.258	0.469	1

**3.3.1. An Index of Belief Stability in Terms of Best-Response.** A rough, but useful way to study the stability of beliefs is to look at the differences between the four best-response sets implied by subjects’ belief statements in the four instances of each game.<sup>9</sup>

Let  $b_g^s$  denote the elicited beliefs of a subject playing the  $s^{th}$  instance of game  $g$ , and denote by  $BR_g(b_g^s)$  the best-response set for these beliefs (recall that we converted all games back to their original frame, so that belief statements are comparable across all four instances of a game). We can distinguish between four different levels of stability across four instances of each game.<sup>10</sup> Specifically, dropping the game subscript,  $g$ , for simplicity:

- (i) For all  $s, t \in \{1, 2, 3, 4\}$   $BR(b^s) = BR(b^t)$ . This is the most stable case in which a subject’s belief statements imply identical best-response sets across all four instances of the game. This is given an index value of 4.
- (ii) There exists a unique instance,  $s$ , such that  $BR(b^s) \neq BR(b^t)$ , while for all  $t, t' \neq s$ ,  $BR(b^t) = BR(b^{t'})$ . In this case, a subject’s statements imply identical best-response sets in 3 out of the 4 instances of the game. This is given an index value of 3.
- (iii) For each instance  $s$ , there is a *unique* instance  $t \neq s$  such that  $BR(b^s) = BR(b^t)$ . That is, over all four instances, there are two different best-response sets, each set occurring twice. This is given an index value of 2.
- (iv) There exists a unique pair  $(s, t)$  such that  $BR(b^s) = BR(b^t)$  and for any other pair of instances  $(t', t'') \neq (s, t)$ ,  $BR(b^{t'}) \neq BR(b^{t''})$ . That is, the subject’s best-response sets coincided in two instances, and differed in every other instance. This is given an index value of 1.

<sup>9</sup> Therefore, results presented here (and subsequently when we look at action stability) excludes those subjects who only participated on the first day. However, in Appendix A we show that there are no significant differences in behavior between subjects who participated on both days and those who only participated on the first day.

<sup>10</sup> Because the best-response set need not be a singleton, it is possible that the best-response sets differ across all four instances; however, this was never observed in our data.

**Table 5** Distribution of the beliefs stability index against random beliefs

index	Game class							
	DSG		MEQ		nDSG		Overall	
	Actual	Random (U)	Actual	Random (U)	Actual	Random (U)	Actual	Random (U)
1	3.93	9.91	2.08	12.05	2.43	10.81	2.80	10.92
2	17.14	16.33	12.50	21.27	16.67	20.81	15.42	19.47
3	36.43	36.71	28.47	38.79	33.68	41.76	32.83	39.09
4	42.50	37.05	56.94	27.89	47.22	26.62	48.95	30.52

Table 5 presents the distribution of the stability index separately for each class of games, as well as pooling across classes. For each class, the first column displays the observed distribution of the index, while the second shows the distribution that would be observed if players chose their beliefs randomly.<sup>11</sup> Overall, subjects have the highest value of our index in half of the games, and err more than once in only about 18% of the games, which indicates fairly high belief stability within the same game. The highest value of the index is markedly more prevalent in the actual data compared to random belief statements, although less so for the set of dominance solvable games.

We next run a series of  $\chi^2$  tests of the empirical distribution of the index against the random statements distribution. The tests are run separately for each game and role so that the observations are independent within each test. The observed distribution of the index is significantly different from the random statements distribution for 19 (resp. 17) out of the 24 comparisons at the 10% (resp. 5%) level of significance.<sup>12</sup> It is also clear from Table 5 that different game classes lead to different degrees of stability. To test this more formally, we define “excess stability” as the difference between the actual value of the index and the expected value under random statements.<sup>13</sup> We then run paired  $t$ -tests at the individual level ( $N = 72$ ) using the individual average excess stability for each game class. MEQ games have the highest excess stability (0.571) followed by nDSG games (0.424) and finally DSG games (0.178). The paired  $t$ -tests reveal that differences between all three pairs of game classes are significant at the 5% level. Thus subjects have significantly more stable beliefs in MEQ games and beliefs are least stable in DSG games.

<sup>11</sup> This distribution was obtained by drawing uniform random beliefs over the simplex for 20000 players (10000 rows and 10000 columns) for 4 occurrences in each set of strategically equivalent games, and by calculating the corresponding index.

<sup>12</sup> We fail to reject at the 10% level of significance in games 2, 3, 6 and 8 (row players) and game 6 (column players), while at the 5% level we also fail to reject the null hypothesis for games 6 (column) and 9 (row).

<sup>13</sup> More precisely, our measure of excess stability for a given set of strategically equivalent games played by a given individual is the difference between the observed index and the expected index under random statements for the corresponding game $\times$ role.

**3.3.2. An Index of Action Stability.** Just as with beliefs, we can construct an index of action stability. The principle of this new index is the same. Specifically, we quantify how many times, across all four instances of the same game, a subject's actions coincided (recall that we converted all games back to their original frame, so that action choices are comparable across all four instances). Let  $x_g^s$  denote the action chosen by a subject in the  $s^{th}$  instance of game  $g$ . There are four different levels of stability. Specifically, dropping the game subscript,  $g$ , for simplicity:

- (i)  $\forall s, t \in \{1, 2, 3, 4\}$ ,  $x^s = x^t$ . This is the most stable case in which a subjects chooses the same action in all four instances. This is given an index value of 4.
- (ii) There exists a unique instance,  $s$ , such that for all  $t \neq s$ ,  $x^s \neq x^t$ , while for all  $t, t' \neq s$ ,  $x^t = x^{t'}$ . In this case, a subject takes the same action in three of four instances. This is given an index value of 3.
- (iii) For each instance  $s$ , there exists a *unique* instance  $t \neq s$  such that  $x^t = x^s$ . That is, over all four instances, a subject chose two different actions and each action was played twice. This is given an index value of 2.
- (iv) There exists a unique pair  $(s, t)$  such that  $x^s = x^t$  and for any other pair of instances  $(t, t') \neq (s, t)$ ,  $x^t \neq x^{t'}$ . This is the least stable case in which a subject chose all three actions, by necessity repeating the same action twice. This is given an index value of 1.

The distribution of the action stability index separated by class of games is given in Table 6.<sup>14</sup>

**Table 6** Distribution of the action stability index

index	Game class				
	DSG	MEQ	nDSG	Overall	Random
1	11.07	4.86	10.42	8.76	44.4
2	17.14	13.19	17.01	15.77	22.2
3	36.43	36.46	37.85	36.92	29.6
4	35.36	45.49	34.72	38.55	3.7

As can be seen, the actual distribution of the index shows substantially more stable behavior than implied by randomness. The  $\chi^2$  tests of equality of the empirical and random uniform distribution of the index give  $p \leq 0.001$  in all 24 comparisons. Again, we can also test whether action stability differs by game class. Paired  $t$ -tests at the individual level reveal that MEQ games are significantly more stable than both DSG ( $p \ll 0.01$ ) and nDSG ( $p \ll 0.01$ ) games, but that DSG and nDSG games do not differ along the stability dimension ( $p = 0.85$ ).

<sup>14</sup> The distribution of the actions stability index under randomness is identical across classes as it does not depend on the payoffs of the game.



**3.3.3. The Relationship Between Belief and Action Stability.** To assess if stability in beliefs is related to stability in actions, we run a fixed-effects regression of stability in actions on stability in beliefs, controlling for a full set of game dummies and clustering the standard errors at the individual level. The estimated coefficient is 0.212, with a  $p$ -value  $< 0.001$ . Running the same fixed effects regression but using our measure of excess stability as an explanatory variable leads to a coefficient of 0.11 with a  $p$ -value of 0.011. Our two measures are thus positively and significantly related: higher belief stability is associated with a higher action stability.

## 4. A Deeper Look at Stability

### 4.1. A Deeper Look at Belief Stability

In this section, we focus on the determinants of belief stability, and on the relation between stability *within* instances of the same game and stability *across* different games. To do so, we compute the Euclidean distance in  $\mathbb{R}^3$  between belief statements. When studying the stability within the same game, we take the distance between belief statements in two consecutive instances of the same game. When studying stability across different games, we compute the distance between belief statements in two consecutive periods in the experiment, which correspond to different games. In the latter case, the coordinates are defined in the  $L_k$  simplex. Both distances are normalized so that the maximum distance attainable from a given belief is given a value of 1.

The average normalized distance between belief statements in two consecutive instances of the same game is 0.276. The first quartile is 0.097, the median is 0.208, and the third quartile is 0.378. Thus, 50% of distances between beliefs are below a fifth of the maximal distance that could be observed, i.e. moving to a (different) vertex of the simplex. Random belief statements over the simplex would lead to an expected distance of 0.456. A  $t$ -test at the individual level reveals that the observed distance between beliefs in instances of the same game is significantly lower than what would have happened under randomness ( $t = -15.34$ ,  $p \ll 0.01$ ).

We now explore how time and framing affects the distance between stated beliefs. Table 7(a) shows the results of paired  $t$ -tests for the equality of normalized distance in beliefs for consecutive instances in the same game. The first cell of Table 7(a) states that the normalized distance in beliefs to equivalent actions when the consecutive instances are played with a 1 day delay is 0.274, while it is 0.242 when the instances are played on the same day. The corresponding  $p$ -value is 0.006. All tests are performed using individual means as the unit of observation. As can be seen from Table 7(a), time has a statistically significant impact on the distance between consecutive belief statements, although its quantitative effect is rather small, with a one week delay adding less than 15% to the average distance compared to games played on the same day.

**Table 7** Belief Stability: The Effects of Time and Framing

(a) The Effect of Time				(b) The Effect of Framing (i.e., isomorphic transformation)					
Time delay	$t = 1$	$t = 7$	$t.d. \geq 1$	Isomorphic	$\Delta = 0$	$\Delta = 1$	$\Delta = 7$	$\Delta \geq 1$	Overall
$\Delta = t$	0.274	0.307	0.292	No	0.242	0.256	0.301	0.280	0.254
$\Delta = 0$	0.242	0.268	0.256	Yes	0.270	0.285	0.311	0.299	0.281
$p$ -values	0.006	0.001	0.000	$p$ -values	0.041	0.405	0.624	0.331	0.024
N	33	39	72	N	72	33	39	72	72

Table 7(b) shows the results of paired  $t$ -tests for the difference in normalized distance for consecutive instances of the same game between pairs that are isomorphic transformations and pairs that are identical. We run the tests on the subsample of subjects that were present on both days of the experiment. The entry in the first row gives the average distance in consecutive beliefs when the instances have identical frames, while the second row gives the distance in beliefs when the instances are isomorphic transformations; the third row give the  $p$ -value of the test.  $\Delta$  refers to the number of days between the two consecutive instances.

While isomorphic transformations have a statistically significant impact when instances are played on the same day, its impact fade when instances are played on different days. The difference in average distances is also rather small (around 10% of the mean distance).

Table 8 presents the results of an OLS regression of normalized Euclidean distance between belief statements to equivalent actions in consecutive instances of the same game on a dummy for isomorphic transformation, dummies for time between statements (and their interactions), as well as dummies for game class. Standard errors are clustered at the individual level. Isomorphic transformation has a small significant impact on the distance, as well as having a 7-day delay between plays. Consistent with our previous results, but surprising nonetheless is the fact that DSG games appear to be the least stable.

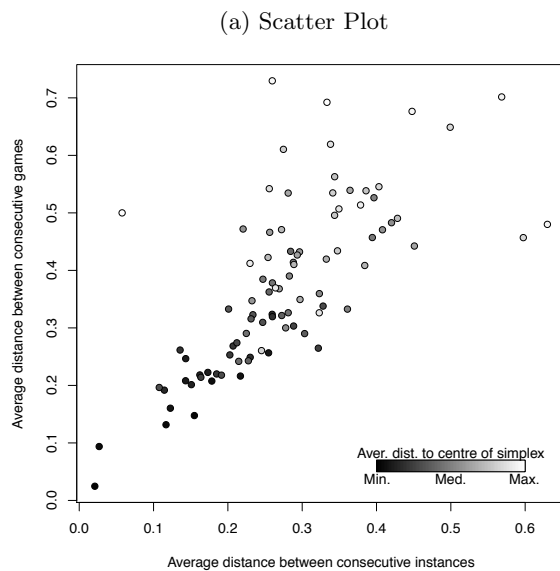
We now investigate stability of  $L_k$  beliefs, and its relation to within game stability. The average normalized distance in the  $L_k$  simplex between belief statements in consecutive games with different strategic structures is 0.374. As was the case for within game stability, this is significantly smaller than the distance implied by random statements ( $t = -5.33$ ,  $p \ll 0.01$ ). Note also that a paired  $t$ -test at the individual level reveals the mean distance between stated beliefs in consecutive instances of the same game are significantly smaller than the mean distance in consecutive games with different strategic structures (0.276 vs 0.374;  $p \ll 0.01$ ). Thus, subjects tend to cluster their beliefs more when facing the same game than when facing a strategically *different* game.

Figure 6(a) shows the scatterplot of average distance between consecutive games with different strategic structures against average distance in consecutive instances of the same game, individuals

**Table 8** Belief stability within sets of strategically equivalent games

Variable	Coefficient	(Std. Err.)
Isomorphic change	0.026*	(0.012)
$\Delta\text{day}=1$	0.020	(0.022)
$\Delta\text{day}=7$	0.047*	(0.023)
Isomorphic change $\times\Delta\text{day}=1$	-0.019	(0.032)
Isomorphic change $\times\Delta\text{day}=7$	-0.005	(0.025)
MEQ	-0.026*	(0.012)
nDSG	-0.040**	(0.011)
Intercept	0.271**	(0.014)
<hr/>		
N	2774	
R <sup>2</sup>	0.011	
F (7,89)	4.042	
<hr/>		
Significance levels : † : 10% * : 5% ** : 1%		

that are closer to the centre of the simplex are shown using a darker shading. Individuals who tend to report beliefs close to the centre of the simplex also tend to have relatively stable beliefs, both within and between sets of strategically equivalent games; and the relation between both stabilities is strong. As the average distance to the centre increases, both kind of stabilities decrease, and the relation between the two gets noisier.

**Figure 6** Within and between stability

(b) Regression Estimates

Variable	Coefficient	(Std. Err.)
Av. dist. cons. inst.	0.348**	(0.102)
Av. dist. centre	0.531**	(0.043)
Intercept	0.056**	(0.020)
<hr/>		
N	90	
R <sup>2</sup>	0.847	
F (2,87)	199.308	
<hr/>		
Significance levels : † : 10% * : 5% ** : 1%		

Figure 6(b) displays the results from an OLS regression of the average distance between consecutive games with different strategic structures on average distance between consecutive instances

of the same game and average distance to the centre of the simplex. Both coefficients are positive and significant, confirming the insight from Figure 6(a).

To investigate further the within-game clustering of beliefs, we compute the overall centroid of beliefs for each individual, as well as for each individual in each set of 12 games. We then compute the Euclidean distance between each belief statement and the overall centroid; and between each belief statement and the corresponding individual centroid in the set of strategically equivalent games. We then compute the mean of the distance to the overall centroid, as well as the mean of the distance to the centroid in the set of identical games. The individual-level average of the normalized distance to the overall centroid is 0.399; and the mean normalized distance to the centroid in the set of strategically equivalent games is 0.186. A paired  $t$ -test at the individual level ( $N=90$ ) rejects the null of equality of the two mean distances at the 1% level ( $t = 16.04$ ;  $p \ll 0.01$ ).

A multivariate ANOVA on the coordinates of the stated beliefs in the  $L_k$  simplex also reveals that the within-individual variance in a set of strategically equivalent games is smaller than the individual-level variance in the coordinates ( $F = 3.39$ ;  $p \ll 0.01$ ).

The results presented point to the fact that beliefs are fairly stable across different instances of the same game, although time and isomorphic transformations have a small but significant impact of belief stability. Moreover, subjects tend to report beliefs that are clustered by sets of strategically equivalent games, with the distance between different games being higher than distances between two instances of the same game. This suggests a model in which individuals pick a focal belief for a given game and make belief statements as trembles around their beliefs. Subjects then may or may not move to other focal beliefs in a different game. In Section 5 we estimate such a model.

#### 4.2. A Deeper Look At Action Stability

Although actions possess a fair bit of stability within strategically equivalent games, we now seek to understand whether there are any underlying causes for instability from one instance to the next. To this end, in Table 9 we report the results of a fixed-effects (conditional) logit regression where the dependent variable takes value 1 if the subject's action changes from one instance to the next. Game fixed effects were included in the estimation, but are not reported in the table.

As can be seen, the biggest determinant of whether a subject changes her action from one instance to the next is a corresponding change in her best-response. This result lends further support to our earlier claim that stability in beliefs and actions are closely related. We also see that subjects who chose a best-response to their beliefs in the last instance are significantly less likely to change their action in the current instance, and that subjects are somewhat less likely to change actions in later instances. Whether the game was presented as an isomorphic transformation from the

**Table 9** Why Do Actions Change Across Instances?

Variable	Coefficient	(Std. Err.)
1(Chose a Best-response)	-0.689**	(0.092)
1( $\Delta$ Best-response)	0.785**	(0.099)
instance	-0.112*	(0.056)
Isomorphic Change	0.077	(0.089)
1( $\Delta$ Day = 1)	0.089	(0.147)
1( $\Delta$ Day = 7)	0.119	(0.125)
N	2762	
LL	-1369.12	
$\chi^2_{(17)}$	182.49	
Significance levels :	† : 10%	* : 5%    ** : 1%

previous instance or the delay from one instance to the next do not seem to have any impact on the likelihood that subjects will change actions. All of these results seem to suggest that, for each game, subjects are searching for the appropriate “model” of behaviour and that once they have found it, their action choices become more stable.

As for beliefs, we can look at whether actions are stable across different games. Table 10 shows, for each type of action in the current game, the distribution of actions in the next game that subjects saw in the actual experiment (which was always strategically different). Comparing with Table 4, we find that actions are substantially less stable across different games than within sets of strategically equivalent games. The overall proportion of actions followed by an action of the same type is 0.495, which is greater than what would have happened under randomness, but below the 0.662 found for consecutive instances of equivalent games. The  $L_1$  action is found to be the most stable. In contrast,  $L_2$  and “other”  $L_k$  actions appear much more unstable as the modal following action is not at the same level.

**Table 10** Stability across games

	Action in the next game			Total
	$L_1$ action	$L_2$ action	Other action	
$L_1$ action	0.607	0.305	0.088	1
$L_2$ action	0.452	0.419	0.129	1
Other action	0.376	0.397	0.227	1

### 4.3. Stability, Accuracy and Best-Response

Do more stable beliefs lead to more predictive power of the opponents’ action? We construct a variable which takes a value of 1 if the best-response set implied by the stated beliefs contains the best-response to the opponent’s action and zero otherwise; and another which takes a value of 1 if

the action taken by the player is a best-response to the opponent's action. We compute the average of these variables over the 4 plays of each of our 12 games and regress these averages on dummies for our stability index, clustering the standard errors at the individual level. Table 11 reports the results and indicates that higher degrees of stability are associated with higher accuracy of stated beliefs, with the lowest degree of stability being associated to a 33.3% — essentially a random belief statement. The same pattern holds for accuracy of actions, except that the lowest stability index is associated with a rate of correct actions below that of random choice. With actions, we also have monotonicity: with higher stability being associated with higher predictive power.

**Table 11 Accuracy and stability**

Variable	Correct Best-response		Correct action	
	Coefficient	(Std. Err.)	Coefficient	(Std. Err.)
Index=2	0.182**	(0.046)	0.143*	(0.055)
Index=3	0.149**	(0.046)	0.183**	(0.053)
Index=4	0.232**	(0.047)	0.255**	(0.052)
Intercept	0.333**	(0.042)	0.281**	(0.051)
<hr/>				
N	856		856	
R <sup>2</sup>	0.024		0.032	
F <sub>(3,71)</sub>	10.35		10.858	
<hr/>				
Significance levels : † : 10% * : 5% ** : 1%				

We now turn to the question of the link between stability and best-response behaviour. We run a fixed-effects regression with clustered standard errors of the average propensity to best-respond in each game on our indices of belief and action stability. We control for a full set of game dummies. The results are displayed in Table 12 and show that the rate of best-response is increasing in both action and belief stability. All in all, the links between stability, best-response and accuracy suggest that stability is related to, if not intelligence, at least a greater focus on the tasks to be performed.

**Table 12 Best-response vs indices**

Variable	Coefficient	(Std. Err.)
Actions stability index	0.097**	(0.011)
Beliefs stability index	0.041**	(0.014)
Intercept	0.226**	(0.053)
<hr/>		
N	856	
R <sup>2</sup>	0.163	
F <sub>(13,71)</sub>	12.15	
<hr/>		
Significance levels : † : 10% * : 5% ** : 1%		

## 5. A Mover-Stayer Model

The descriptive analysis of Section 4.1 highlighted some stylized facts about belief stability within and between sets of similar games. First, beliefs tend to be clustered within similar games, but less so overall. Second, stability within sets of similar games and stability between sets of similar games seemed to be positively correlated. In this section we construct and estimate a more structural model to capture these features. We assume belief statements are really trembles around underlying true beliefs (Costa-Gomes and Weizsäcker 2008) and in which underlying beliefs may vary between strategically different games. The degree to which subjects “tremble” when stating beliefs is allowed to vary with their tendency to switch underlying beliefs between strategically different games.

### 5.1. Model and Results

Each individual  $i$  is a *stayer* ( $s$ ) with probability  $p_s$ , or a *mover* ( $m$ ) with probability  $1 - p_s$ . Stayers’ underlying beliefs are stable from game to game (i.e. they remain on the same level of thinking regardless of the strategic structure of the game). Each stayer’s beliefs is one among  $Q$  possible beliefs in the 2-simplex  $\Delta^2$ . Movers, on the other hand, choose one of the  $Q$  possible beliefs for each set of identical games, and keep the same beliefs for every instance  $h \in \mathcal{G}_g$  in this set, where,  $\mathcal{G}_g$  denotes the set of all games that are strategically equivalent to game  $g$ . Movers may change their underlying beliefs between strategically different games. Stayers choose their beliefs according to the probability distribution  $(p_{1s}, \dots, p_{Qs})$ , while movers choose theirs according to the probability distribution  $(p_{1m}, \dots, p_{Qm})$ ; where  $p_{qt}$  stands for the probability that a type  $t$  player choose the  $q^{th}$  belief;  $p_{qt} \geq 0$  and  $\sum_{q=1}^Q p_{qt} = 1$ ,  $\forall q, t$ ,  $q \in \{1, \dots, Q\}$ ,  $t \in \{m, s\}$ . Each actual belief statement is a tremble around the underlying belief.

The density  $d_{tqj}^i$  of player  $i$ ’s belief statement in game  $g$  when she has the underlying  $q^{th}$  belief and her type is  $t \in \{m, s\}$  is constructed as in Costa-Gomes and Weizsäcker (2008):

$$d_{tqh}^i = \frac{\exp(\lambda_t \nu_g(b_g^i, b_q))}{\int_{s \in \Delta^2} \exp(\lambda_t \nu(s, b_q)) ds}, \quad (1)$$

where  $b_g^i \equiv (b_{g,1}^i, b_{g,2}^i, b_{g,3}^i) \in \Delta^2$  is the belief statement player  $i$  reports in game  $g$  for each of her three actions,  $b_q$  is the  $q^{th}$  belief. The term  $\nu_g(b_g^i, b_q)$  is the expected payoff obtained from the quadratic scoring rule when true beliefs are  $b_q$  and the belief statement is  $b_g^i$ .  $\lambda_t$  is a parameter to be estimated, which captures players’ sensitivity to payoff differences. Specifically, when  $\lambda_t$  tends to 0, type- $t$  players choose their belief reports randomly on the simplex. Conversely, when  $\lambda_t$  tends to  $\infty$ , underlying beliefs are stated without any noise.

The likelihood for the set of belief statements of individual  $i$  is thus:

$$l^i = p_s \sum_q p_{qs} \prod_h d_{sqh}^i + (1 - p_s) \prod_g \sum_q p_{qm} \prod_{h \in \mathcal{G}_g} d_{mqh}^i. \quad (2)$$

We set  $Q = 4$ , and set 3 of the 4 beliefs to be at the vertices of the  $L_k$  simplex:  $b_2 \equiv (1, 0, 0)$ ;  $b_3 \equiv (0, 1, 0)$  and  $b_4 \equiv (0, 0, 1)$ . The first type of belief,  $b_1 \equiv (\mu_1, \mu_2, \mu_3 = 1 - \mu_1 - \mu_2)$  is to be estimated from the data.

We maximise the log-likelihood of our sample using the Nelder-Mead algorithm. We then run one iteration of Newton-Raphson to get the variance-covariance matrix of the estimates using the observed information matrix method. Final standard errors are obtained through the Delta method. Results are displayed in Table 13.

**Table 13** Mover-Stayer model

Description	Parameter	Estimate	(Std. Err.)
Fraction of stayers	$p_s$	0.285**	(0.048)
Movers, frac. interior	$p_{1m}$	0.197**	(0.039)
Movers, frac. $L_2$ beliefs	$p_{2m}$	0.488**	(0.031)
Movers, frac. $L_3$ beliefs	$p_{3m}$	0.217**	(0.021)
Movers, frac. other beliefs	$p_{4m}$	0.098**	(0.006)
Stayers, frac. interior	$p_{1s}$	0.989**	(0.019)
Stayers, frac. $L_2$ beliefs	$p_{2s}$	0.000	(0.006)
Stayers, frac. $L_3$ beliefs	$p_{3s}$	0.001	(0.019)
Stayers, frac. other beliefs	$p_{4s}$	0.000	(0.000)
	$\mu_1$	0.426**	(0.004)
Estimated interior beliefs	$\mu_2$	0.320**	(0.004)
	$\mu_3$	0.253**	(0.003)
	$\lambda_m$	0.263**	(0.012)
Rationality parameters	$\lambda_s$	2.243**	(0.096)
	N	3512 (90 individuals)	
	Log-likelihood	-12238.921	

Significance levels : † : 10% \* : 5% \*\* : 1%

About 28.5% of the individuals are stayers, and they overwhelmingly (99% of them) choose to stay at the beliefs located inside of the  $L_k$  simplex, which we estimate to be at (0.426, 0.320, 0.253), which is very close to the centre of the simplex (i.e., the  $L_1$  belief). Movers, on the other hand hold beliefs that are more evenly distributed among the 4 types, with the  $L_2$  belief having the highest (48%) probability to be chosen, followed by the  $L_3$  belief and inner belief (20% each) and finally the last vertex of the simplex (10%). The overall distribution of underlying belief statements implied by these estimates give the inner belief a lead with 42% of the statements, followed by the  $L_2$  belief with 35%, then the  $L_3$  type with 16%, and lastly the last vertex with 7%. These figures



are consistent with the estimated densities given in Figure 3. Indeed, even though the estimated densities seemed to give a larger difference between  $L_1$  and  $L_2$  beliefs, the fact that stayers (who tend to have their beliefs located in the centre of the simplex) have a higher precision parameter leads to a higher spike in the density than for the other beliefs, which are mainly chosen by movers who have a much lower value of  $\lambda$ .

Note that given that stayers report very nearly  $L_1$  beliefs and the fact that the Quadratic Scoring Rule favors uniform reports for risk-averse people, one might be tempted to argue that stayers are risk averse. However, we argue against this interpretation in Appendix C. A risk averse subject would state beliefs closer to the centre than her true belief. Assuming that she best-responded to her true belief and not to her stated belief, this would create an inconsistency between chosen actions and stated beliefs; that is, a risk averse subject should have a lower rate of best-response. We find no support for this in our data. Instead stayers appear more simply to have quite conservative/unsophisticated beliefs, but generally have the same rate of best-response.

## 5.2. Movers and Stayers

Using Bayes' theorem and the estimated parameters of the mover-stayer model, one can compute the posterior probabilities that a subject belongs to a given type of players. Namely, denoting by  $(b_t^i)_{t=1,\dots,48}$  the vector of subject  $i$ 's stated beliefs across the 48 games played; the probability that  $i$  is a stayer given his observed statements can be written as:<sup>15</sup>

$$\Pr(\text{stayer} | (b_t^i)_{t=1,\dots,48}) = \frac{p_s \sum_q p_{qs} \prod_g d_{sqq}^i}{l^i}. \quad (3)$$

The distribution of the posterior probabilities of being a stayer is heavily concentrated around 0 and 1, as shown by Figure 7.

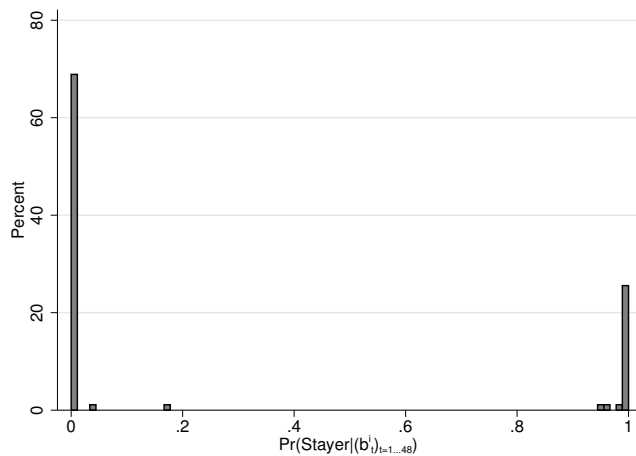
Using these posterior probabilities, we can assign each individual to either the mover or stayer type. More precisely, we define the dummy variable:

$$\text{stayer}_i := \mathbb{I}(\Pr(\text{stayer} | (b_t^i)_{t=1,\dots,48}) > 0.5)$$

Based on this rule, our set of 90 subjects is composed of 27 stayers (30%) and 63 (70%) movers.

Figure 8 shows the density of stated beliefs in the  $L_k$  simplex separating by movers (above) and stayers (below), which confirms that stayers have their beliefs concentrated on  $L_1$  beliefs; while movers' beliefs are more scattered, with highest mass at the  $L_2$  vertex, followed by the centre

<sup>15</sup> For those subjects who did not come back on Day 2, and who consequently played 24 games, we use the vector  $(b_t^i)_{t=1,\dots,24}$ .

**Figure 7** Posterior probabilities

and the other vertices of the simplex. The empirical distribution of stayers' and movers' beliefs is consistent with the estimation results of Table 13. Table 14 summarizes the relation between assigned type/posterior probabilities and several measures of stability. Columns 2 and 3 show the difference in the stability metric between movers and stayers and the corresponding p-value for a t-test of equality. Columns 4 and 5 show the estimated coefficient of an OLS regression of the metric on the posterior probability; and the p-value for the test that the coefficient is 0 (we used robust standard errors). All tests confirm the higher stability of stated beliefs for those classified as stayers by our model.

**Table 14** Stability measures

Metric	By type		By Pr (stayer)	
	$\Delta$	p-value	$\hat{\beta}$	p-value
Av. dist. in successive instances of the same game	-.145	$< 10^{-3}$	-.148	$< 10^{-3}$
Av. dist. in successive games from different sets of games	-.186	$< 10^{-3}$	-.188	$< 10^{-3}$
Av. dist. to the centroid of the set of the same games	-.100	$< 10^{-3}$	-.101	$< 10^{-3}$
Av. dist. to the overall centroid	-.252	$< 10^{-3}$	-.255	$< 10^{-3}$

How do stayers fare relative to movers? Table 15 keeps the same structure as Table 14, but shows results for several measures of performance: actions and beliefs (QSR) payoffs; rate of correct best-response (i.e. the proportion of times their best-response set contained the best-response to the actual opponent's action), rate of correct actions (i.e. the proportion of times their action was a best-response to the actual opponent's action) and rate of best-response. While stayers do not earn significantly more than movers in the action task, they do fare better than movers on the belief task. Movers and stayers have the same rate of best-response, although stayers' beliefs and actions are significantly less often a result of a correct model of the mind of the opponents.

Figure 8 Movers' and stayers' beliefs

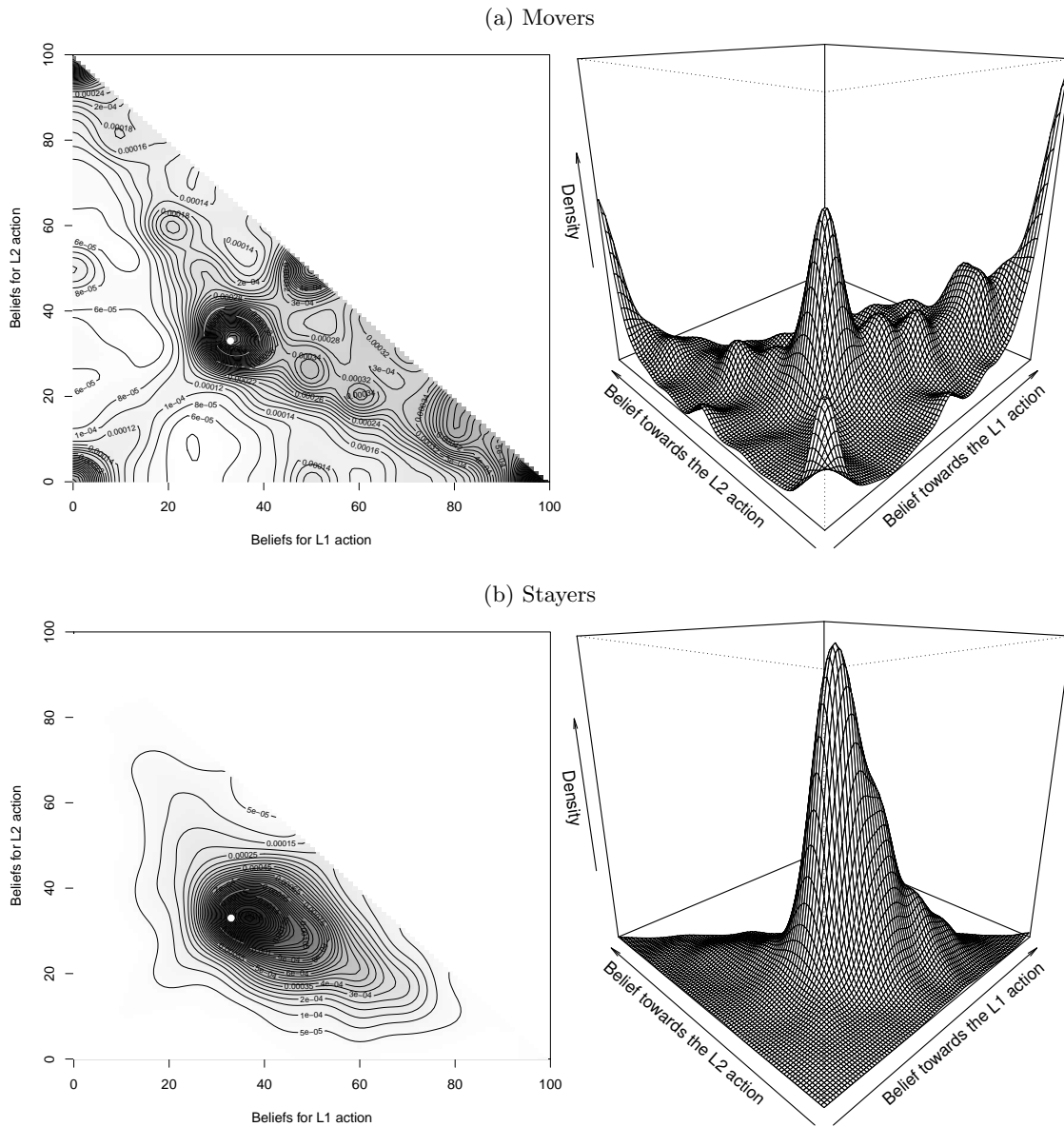


Table 15 Performance

Metric	By type		By Pr (stayer)	
	$\Delta$	p-value	$\hat{\beta}$	p-value
Average action payoff	1.21	0.181	1.290	0.161
Average QSR payoff	.560	$< 10^{-3}$	.567	$< 10^{-3}$
Rate of correct BR	-0.085	0.0015	-0.087	0.001
Rate of correct actions	-0.066	0.017	-0.067	0.016
Rate of best-response	-0.027	0.3936	-0.026	0.412

Finally, stayers are also disproportionately females. While 70% of movers are male, they make up only 33% of stayers. A two-sample t-test reveals that the difference is highly significant ( $t = 3.39$ ;

$p = 0.001$ ). We could not find any significant differences between movers and stayers in terms of year of study ( $t = -0.8256$ ;  $p = 0.413$ ) or row/column role ( $\chi^2 = 0.305$ ;  $p = 0.581$ ), nor does there seem to be any session effects regarding the probability to be a stayer ( $\chi^2 = 0.662$ ;  $p = 0.985$ ).

## 6. Conclusion

Stability of the belief formation process is a very desirable feature as it would allow economists and policymakers to predict how individuals would form expectations in different settings. Moreover, stability is a fundamental assumption, both when setting up theoretical models of strategic thinking and when analyzing subjects' behaviour in experiments. Therefore any evidence that fails to substantiate this assumption reveal a caveat in the theory and/or a bias in experimental measurements. In this paper, we used experimental data on beliefs to explore the extent with which individuals might be classified as stable. In our experiment, subjects saw four instances for each of our 12 normal-form games, spread over two different days (separated in time by one day or one week) and in two different frames. To further limit the possibilities for learning, subjects did not receive any feedback until the end of the second day. Our design also allowed us to look at three progressively more demanding forms of stability: stability within the same game, stability within the same game but framed differently and stability across different games.

Our experimental findings show that stability — especially across different games — is not what one might hope for. Specifically, while stability within instances of the same game is fairly strong, it is much weaker across different games. In any given situation the level- $k$  model accurately describes behaviour, but for different games different levels of reasoning rationalise behaviour. Second, beliefs and actions become more unstable as the time between instances increases and when framed differently. Third, our results showed that belief and action stability are positively related, and that when one's action changes, it is likely due to a change in underlying beliefs. Finally, we also found the counter-intuitive result that dominance solvable games were the least stable, while games with multiple Nash equilibria were most stable.

Motivated by these results, we estimated a so-called *mover-stayer* model of behaviour. In this model, for each set of similar games, subjects are assumed to choose particular beliefs (in the level- $k$  sense) and select a noisy best-response to these beliefs for all four instances in the corresponding set of similar games. The difference is that *stayers* are stable in the sense that they use the same level of reasoning across all 12 sets of similar games, while *movers* may state beliefs consistent with different levels of reasoning in different sets of similar games. Our estimates show a bimodal distribution of subjects according in which approximately 28% of the subjects are stayers, while

the remaining 72% are movers. We find some notable differences between these two groups. First, stayers have a very low level of strategic reasoning, stating beliefs very close to the centre of the belief simplex, while movers often state beliefs consistent with higher levels of reasoning. At the same time, stayers appear to state their beliefs very precisely, whereas movers are more prone to error, and the difference is quantitatively very large. Finally, there are interesting gender differences with most of the subjects classified as stayers being women and a large majority of subjects classified as movers being men.

Overall, our findings suggest that level- $k$  models are best viewed as an *as if* description of how people make choices, but is less reliable to provide a unifying framework of people's guesses of other's behaviour across a diversified range of games.

### Acknowledgments

Kyle Hyndman would like to thank Southern Methodist University, where he began working on this project, for financial support. Antoine Terracol and Jonathan Vaksmann would like to thank the scientific council of the Université du Maine and Paris School of Economics for financial and logistical support. Maxim Frolov is also acknowledged for technical assistance. We also gratefully acknowledge the valuable comments received by Colin Camerer and Andrew Schotter, participants at the Workshop on the Formation and Elicitation of Beliefs in Experiments (Berlin) as well as the 2012 International Economic Science Association Conference.

## Appendix

### A. Attrition in Participation During The Experiment

In our experiment, we had 90 subjects participate on the first day of the experiment. 72 of these subjects returned back for the second day (either the next day or one week later, depending on the treatment), while 18 did not return for the second day. As noted in the text, attrition was a slightly bigger problem in the one-week separation treatment.

Most of the results in our paper include all 90 participants, but results involving stability indexes require that we remove those who only came on the first day. One can thus wonder whether the results are sensitive to attrition. That is, are the results different if we exclude those subjects who only participated on day 1? Table A.1 shows the averages and the  $p$ -values of tests comparing partial (i.e. day 1 only) and full (i.e., days 1 & 2) participants for various measures of individual characteristics, beliefs, actions, stability and payoff. All of the tests use only the data collected on day 1 in order to ensure comparability with respect to potential effects of learning and experience. We compute the individual mean of the variable over the first day and keep only one observation per individual. None of the differences are significant at the 5% level, although we do find a small difference in beliefs towards the  $L_1$  action, but only at the 10% level.

**Table A.1 Summary Statistics: Partial vs. Full Participants**

Variable	Participation		<i>t</i> -test	rank sum
	Day 1 Only	Days 1 & 2		
Fraction of column players	0.61	0.45	0.251	0.249
Fraction of males	0.56	0.60	0.751	0.749
Belief on first action	33.33	33.54	0.914	0.936
Belief on second action	31.94	31.93	0.995	0.739
Belief on third action	34.72	34.52	0.911	0.589
Belief on $L_1$ action	39.57	43.67	0.086	0.075
Belief on $L_2$ action	34.30	32.15	0.290	0.238
Belief on $L_3$ action	26.13	24.18	0.378	0.234
Frequency of best-response	0.62	0.60	0.683	0.561
Frequency of $L_1$ action	0.50	0.51	0.859	0.903
Frequency of $L_2$ action	0.35	0.36	0.924	0.824
Normalised distance (next instance)	0.31	0.26	0.159	0.197
Normalised distance (next game)	0.39	0.38	0.820	0.600
Action consistency	0.64	0.65	0.817	0.693
Average payoff	55.7	55.2	0.695	0.713

In Table A.2 we provide the results of our mover-stayer model on the restricted sample of full participants only. Comparing the results with those of Table 13, we see only very minor differences. When we restrict the sample to full participants only, the fraction of stayers is marginally higher. Among movers, the fraction using the interior belief is a bit lower, while the fraction stating a Level 2 belief is somewhat higher. For stayers, the fraction stating each type of belief is virtually unchanged, as are the estimated interior belief and the rationality parameters. From this we conclude that attrition in our experimental sample over the two days of the experiment does not bias our results in any meaningful way.

## B. Learning

Our experiment was designed to minimize the learning opportunities for subjects: similar games were not played consecutively and subjects were not given any feedback before the end of the second day. In this section we test for any hint that learning nevertheless took place. We run a series of fixed effects regressions of several measures on a given instance in a set of similar games, time, and when appropriate, a set of game dummies. The results are summarised in Table B.1 which shows the coefficient and estimated standard error in parentheses. None of regressions lead to a significant effect of instance and/or time, and we conclude that no significant learning took place.

## C. Risk aversion

A natural concern is whether our results are driven by risk aversion. It is well known that the Quadratic Scoring Rule leads risk-averse subjects to state beliefs that are closer to the  $L_1$  belief  $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$  than their true beliefs. Extremely risk-averse individuals would always report the  $L_1$  belief, and would be classified as stayers by our model. It is thus important to check whether our results are plagued by risk aversion. Although Trautmann and van de Kuilen (2011) find very little differences between the Quadratic Scoring Rule and

**Table A.2** Mover-Stayer model (sample of full participants only)

Description	Parameter	Estimate	(Std. Err.)
Fraction of Stayers	$p_s$	0.302**	(0.056)
Movers, frac. interior	$p_{1m}$	0.173**	(0.042)
Movers, frac. $L_2$ belief	$p_{2m}$	0.523**	(0.034)
Movers, frac. $L_3$ belief	$p_{3m}$	0.211**	(0.022)
Movers, frac. other belief	$p_{4m}$	0.092**	(0.006)
Stayers, frac. interior	$p_{1s}$	0.997**	(0.010)
Stayers, frac. $L_2$ belief	$p_{2s}$	0.000	(0.009)
Stayers, frac. $L_3$ belief	$p_{3s}$	0.001	(0.003)
Stayers, frac. other belief	$p_{4s}$	0.000	(0.000)
	$\mu_1$	0.432**	(0.005)
Estimated interior belief	$\mu_2$	0.320**	(0.004)
	$\mu_3$	0.248**	(0.004)
Rationality parameters	$\lambda_m$	0.266**	(0.013)
	$\lambda_s$	2.238**	(0.122)
	N	3136 (72 individuals)	
	Log-likelihood	-10839.779	

Significance levels : † : 10% \* : 5% \*\* : 1%

**Table B.1** Learning

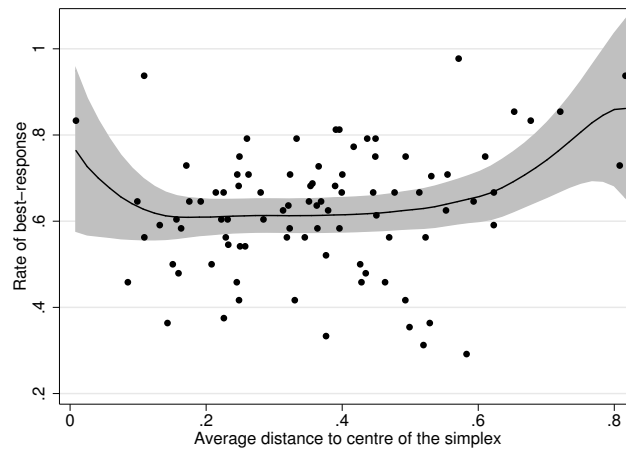
Dependent Variable	Instance	Time	game fixed effects
Giving a best-response	-.042 (.087)	.010 (.007)	Yes
Distance between instances of similar games	-.006 (.005)		Yes
Distance between strategically different games		-.0002 (.0005)	No
Distance to centre	-.0005 (.0006)	.012 (.01)	Yes
Payoff	.011 (.069)	-.097 (.812)	Yes

Significance levels : † : 10% \* : 5% \*\* : 1%

rules such as the Lottery Rule that are not affected by risk aversion, we check in this section whether risk aversion is likely to be a problem in our data.

Because risk-averse individuals would report beliefs that are closer to the centre than their actual beliefs, we would expect that the rate of best-response would steadily decrease as the stated beliefs come closer to the centre of the simplex.

Figure C.1 shows the scatterplot of the individual rate of best-response against the average distance of stated beliefs to the centre of the simplex; together with the results of a locally quadratic regression and the corresponding 95% confidence interval. There is no sign of a steady deterioration of best-response behaviour as subjects state beliefs that are on average closer to the centre. Indeed, the regression line is essentially flat for most (90%) of the less extreme average stated beliefs. In addition to this nonparametric analysis, we run more parametric checks of the stability of best-response behaviour. To do so, we split our 90 subjects in  $K$  groups of roughly equal size according to the quantiles of the average distance to the centre of the simplex. We then run an OLS regression with robust standard errors of the individual rate of best-response on the full set of group dummies and an intercept. We finally test the joint significance of the coefficients of the

**Figure C.1** Rate of best-response vs. distance to centre

dummies. Failing to reject the null that all coefficients are zero indicates that the rate of best-response does not vary with average distance to the centre. Table C.1 reports the results of those F tests for  $K = 1 \dots 10$  and shows that we fail to detect differences in the rate of best-response between as much as 10 different subsamples ordered according to their average distance to the centre. In addition, a simple OLS regression of individual rate of best-response on average distance to the centre gives an estimated coefficient of 0.17 with a p-value of 0.114.

**Table C.1** Testing for risk aversion

K	F-stat	p-value	K	F-stat	p-value
2	0.899	0.346	6	0.933	0.464
3	0.792	0.456	7	1.719	0.127
4	0.481	0.696	8	1.255	0.283
5	0.571	0.684	9	0.633	0.748
			10	1.705	0.101

Finally, subjects classified as stayers by our model do not exhibit a lower rate of best-response than movers. Indeed an OLS regression of individual rate of best-response on stayer status gives an estimated coefficient of  $-0.027$ , with a p-value of 0.391, far from statistical significance.

We thus conclude that our results are unlikely to have arisen because of risk aversion.

#### D. Instructions (translated from French)

*The following instructions were given to subjects on the first day. On the second day, subjects were told that the instructions were exactly analogous and copies of the original (Day 1) instructions remained available for them if they needed some refreshments. In the copy below, the two days were consecutive. The instructions for our treatment with a one-week delay between the two days are represented in [].*

Thank you for participating in this experimental session. During this session, upon the choices you make, you may be able to earn a significant amount of money which will be paid you in private at the end of the



experiment. Your identity and those of the other participants will never be disclosed.

This session contains two parts. The first one takes place today and the second one tomorrow [in seven days] at the same time. The instructions for the second part of the session will be provided tomorrow [in seven days]. The gains you will make during the whole session will be labelled in ECUs (Experimental Currency Units). You will be paid tomorrow [in seven days] according to the gains you would have made in both parts of the session. More precisely, the total amount you will earn in ECUs will be converted into Euros at the following rate.

$$100 \text{ ECUs} = 0.70 \text{ Euros}$$

During this session, you will not be allowed to communicate with other participants. If you have any questions, please raise the hand and the experimenter will publicly answer.

This first part of the session contains 24 repetitions. Your payoff for this part corresponds to the sum of the payoffs you earn during these 24 repetitions.

### Your decision at each period

At the beginning of each period, you will be randomly matched with another participant. Each participant has three alternatives:  $a$ ,  $b$  or  $c$ . You will be paid according to your own decisions and the ones of your partners. These payoffs are indicated in the tables that will appear on your screen at the beginning of each period.

*Example*

	$a$	$b$	$c$
$a$	75,90	27,31	55,43
$b$	90,40	28,35	31,51
$c$	63,42	65,86	78,26

In this table, the first figure in each cell represents your payoff, the second one refers to your partner's one for the current period. For example, if you choose action  $b$  and if your partner choose action  $a$ , you will get 90, while your partner will get 40. To make the reading of these payoffs even easier, your own payoffs will appear in red on your screen while those of your successive partners will appear in blue.

### Your prediction at each period

Prior to choosing a decision at each period, you will be given the opportunity to earn additional money by predicting the decision your pair partner will take at the current period. Thus, at the beginning of each period, you will be asked the following three questions:

- On a scale from 0 to 100, how likely do you think your pair partner will take decision  $a$ ?
- On a scale from 0 to 100, how likely do you think your pair partner will take decision  $b$ ?
- On a scale from 0 to 100, how likely do you think your pair partner will take decision  $c$ ?

For each question you have to key in a number inferior or equal to 0. The sum of these three numbers has to equal 100.

For example, suppose that you think that there is 40% chance that your pair partner will take decision  $a$ , a 35% chance that your pair partner will take decision  $b$  and a 25% chance that your pair partner will take

decision  $c$ . In this case, you will key in 40 in the upper box on the screen and respectively 35 and 25 in the two other boxes. At the end of each period, we will look at the decision actually made by your pair partner and compare his decision to your prediction. We will then pay you for your predictions as follows: Suppose the above example: you entered 40% for decision  $a$ , 35% for decision  $b$  and 25% for decision  $c$ . Suppose now that your pair partner actually chooses  $b$ . In this case, your payoff for your predictions will be:

$$10 - 5(1 - 0.35)^2 - 5(0.40)^2 - 5(0.25)^2.$$

In other words, you will be given a fixed amount of 10 points (in ECUs) from which we will subtract an amount which depends on how inaccurate your predictions was. To do this, when we find out what decision your pair partner has made we will take the number you assigned to that decision, in this example 35% (or 0.35) on  $b$ , subtract it from 100% (or 1), square it and multiply by 5. Next, we will take the numbers assigned to the decisions not made by your pair partner, in this case the 40% (or 0.40) you assigned to  $a$  and the 25% (or 0.25) you assigned to  $c$ , square them and multiply by 5. These three squared numbers will then be subtracted from the 10 ECUs we initially gave you to determine the final payoff associated to your predictions for the current round. Your payoff will be then converted into Euros at the same conversion factor as given above.

Note that since your prediction is made before you know what your partner has actually chosen, the best thing you can do to maximize the expected size of your prediction payoff is to simply state your true beliefs about what you think your pair partner will do. Any other prediction will decrease the amount you can expect to earn as a prediction payoff.

Note also that you can not lose points from making predictions, you can only earn more points. The worst you can do is predicting that your pair partner will take one particular decision with 100% certainty but it turns out that your partner actually takes a different decision. In this case, you will earn 0 point. Similarly, the best you can do is to guess correctly and assign 100% to that decision which turns out to be the actual decision chosen. Here, you will keep the whole 10 points amount that was given to you at the beginning of the current round.

In each round this payoff associated to your prediction will be in addition of what you will make with your decisions. Your payoffs in ECUs for today's part will simply be the sum of all payoffs you will make throughout the 24 periods of this part of the session. However, you will **not** receive any information about your partners' decisions until the end of the whole experiment tomorrow [in seven days].

## E. Screenshots

Figure E.1 Prediction screen

**Estimation de probabilités. Jeu 1**

Veuillez estimer les pourcentages de chance pour que votre partenaire choisisse ses différentes décisions. La somme des pourcentages entrés doit être égale à 100.

Ci-dessous le tableau représentant les gains en fonctions des décisions choisies. Vos gains (en ECU) sont en rouge et ceux de votre partenaire sont en bleu.

		Choix de votre partenaire		
		a	b	c
Votre choix	a	82, 57	48, 58	30, 80
	b	69, 75	40, 85	34, 50
	c	13, 71	20, 26	92, 75

Vos réponses

Quelle est selon vous la probabilité - en pourcentage - pour que votre partenaire choisisse la décision a?

Quelle est selon vous la probabilité - en pourcentage - pour que votre partenaire choisisse la décision b?

Quelle est selon vous la probabilité - en pourcentage - pour que votre partenaire choisisse la décision c?

TOTAL 0  
(reste 100)

Appuyez sur OK pour continuer :

CALC

Figure E.2 Decision screen

**Choix de l'action. Jeu 1**

Veuillez faire votre choix en cliquant sur une des décisions. Vos gains (en ECU) sont en rouge et ceux de votre partenaire sont en bleu.

		Choix de votre partenaire		
		a	b	c
Votre choix	a	82, 57	48, 58	30, 80
	b	69, 75	40, 85	34, 50
	c	13, 71	20, 26	92, 75

Appuyez sur OK pour continuer :

CALC

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