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Abstract:

In this paper, we apply an Overlapping Generations (OLG) model with endogenous fertility and a pay as you go (PAYG) pension system to find out what are the economic consequences of different policy measures to increase the number of children. Especially, we take into account the introduction of a child dependent PAYG pension system, child allowances financed by a labor income tax, and a reduction of the child rearing costs. Some authors have shown that in small open economies with exogenous growth it is possible to increase the fertility without harming any generation. Here we show that this is impossible in a model with endogenous growth.

Keywords: Fertility, endogenous growth, pay-as-you-go pension, child allowances

JEL classification: H55, J13, D10

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1- Introduction

In most developed countries, we observe that the fertility rate is insufficient to keep the population constant. This leads to an ageing of the society, which causes in the view of the governments and economists many problems with respect to the health care system, the pension system, the labor market and rural areas. Especially, they fear that the pension systems will become unsustainable in the next 30 to 50 years. Many economists argue that the mostly existing pay-as-you-go (PAYG) pension systems should be abolished and substituted by a capital funded pension system. However, the problem is that a transition from a PAYG system to a capital funded pension system will imply a double burden for one generation or more generations; some generations have to save for their pension and to finance the pensions of the living retired generation. It seems to be politically difficult to enforce as well as to justify such a transition. Therefore, governments would like to implement policy measures to increase the fertility rate so that the number of citizens will be at minimum constant. Here, we take it as given that it is a political desire to increase the fertility rate. Additionally, we take a PAYG pension system as granted. We make this assumption because in most countries exists a form of this pension scheme. In a standard OLG model without endogenous growth suchlike policy measures exist for small open economies, as it was shown by Abio et al. (2004), Kolmar (1997), Fenge and Meier (2005, 2009), van Groezen et al. (2003). These measures are applied in some form in most European countries. In this paper, we want to investigate, if these results also hold in an endogenous growth model.

We use widely well-accepted assumptions from the literature to analyze three kinds of policy options to increase the fertility. The first two are to lower the child rearing costs, and the third is to change the pension system in a way that the pensions of parents depend on the number of their children. On the producer side of the economy, we use an AK-production function of the Romer-type instead of a standard neoclassical production function. On the consumer side, we use a standard OLG model of the Samuelson (1958) with endogenous population growth without any altruism. Then we will compare all three policy options with respect to the induced welfare effects.

The paper is divided into 7 sections. In section 2, we give an overview of the fertility behavior in some countries. In section 3, we introduce the basic model and derive the equilibrium values. In section 4, we will introduce a tax-child allowance mechanism, and we will derive the influence of it on the equilibrium values. In section 5, we will do the same with the introduction of a child factor into the PAYG pension system. In section 6, we will compare the measures, and we will discuss the advantages and disadvantages of a PAYG pension system and capital-funded pension system in general. In section 7, we will conclude our results.

2- Some facts about fertility behavior[1]

In all developed countries, we observe that the total fertility rates are below their sustainable level, which would be theoretically two children per female on average. In 94 countries, the total fertility rate is below two children per female. Surprisingly, the lowest fertility rates will be observed in developed Asian countries, Macau (0.92 children/female), Hong Kong (1.07), Singapore (1.11), Taiwan (1.15), Japan (1.21) and South Korea (1.23). One obvious reason for this could be that these countries have the highest population densities related to the rest of the world. According to the United Nations (2005) Macau has the highest population density (18,534 persons per square kilometer) worldwide, ranked third is Singapore (7,148) followed by Hong Kong (6,349). Also, the territorial Asian states are ranked very high; Taiwan (639) is ranked 16, South Korea (487) 23 and Japan (337) is ranked

[1] The numbers in this section are taken from the CIA World Fact Book 2012, if not otherwise noted. https://www.cia.gov/library/publications/the-world-factbook/geos/xx.html

38 in the world. Without any doubt, this characteristic leads to relative high housing costs and therefore indirectly to relative high child rearing costs, because children need space. However, this could not the only reason for the low fertility rate, for example, the Netherlands has a population density of 403 persons per square kilometer with the rank 30 worldwide, but the total fertility amounts to 1.98 children per female. In Asian countries, it is also a fact, that the overall expenditures for education are relative high, however in addition the private expenditures are very high, where the latter costs increase the individual child rearing costs. According to the OECD[2], Korean parents spend 2.8% of the GDP for the education of their children; these are the relative highest private expenditures in the OECD, in Japan the parents spend 1.7% of the GDP for education, only the relative private expenditures for education in Korea, the USA (2.1%) and Chile (2.7%) are higher in the OECD. On the other hand, the explicit relative total expenditures for pre-primary education are low in Japan and Korea.

The relative total expenditures for pre-primary education are the second-lowest in Korea (0.18% of GDP) and the third lowest in Japan (0.21%) in the OECD. Only Australia spends less with 0.08% of the GDP. This leads to low female labor market participation rates in these countries and high implicit child-rearing costs. In some sense, the choice of parents how much they invest in the education of their children is restricted by Asian culture and habit, and these costs must be interpreted as given by the society. It seems to be that Asian parents' willingness to pay for the education of their offspring is much higher than in non-Asian OECD countries. Additionally, Asian educational systems usually initiate winner-take-all competitions to measure the relative success of the students, which also increase tendentially the private expenditures for education.[3]

2 See OECD (2011).
3 See regarding winner take all contests Frank and Cook (1995).

Additionally, both countries have a kind of a PAYG pension system, which also decreases the fertility rate.[4]

3- The basic model of endogenous growth and endogenous fertility

We assume a closed economy, with only one good, which can be consumed or invested. Let us assume that the production is given by a simple AK production function, as Romer (1986), Barro & Sala-i-Martin (1991), Frankel (1962) introduced it. In the literature on endogenous growth and fertility this function was, for example, used by Wigger (1999), Saint-Paul (1992), Zhang & Zhang (1995, 1998, 2001), Zaigui (2007, 2005), Stauvermann (1996).

$$Y_t = F\left(K_t, L_t, \bar{k}_t\right) = AK_t. \qquad (1)$$

The variable Y_t represents the production, the technology parameter A is positive, K_t is the capital stock, L_t the labor force and $\bar{k}_t$ the economy wide capital intensity, which is exogenous for an individual firm. The subscript *t* indicates period *t*. Following Stauvermann (1997, 2002) we are able to derive the interest factor R_t and the wage rate w_t;

$$R_t = \alpha A \qquad (2)$$

and

$$w_t = (1-\alpha)Ak_t. \qquad (3)$$

The parameter α lies between zero and one. It is assumed without a loss of generality that the depreciation rate of capital per period is one.

The utility of a representative individual is given by the following log-linear utility function, which is commonly used in OLG-models;[5]

$$U_t = ln\, c_t^1 + q ln\, c_{t+1}^2 + v \ln\left(N_t\right), \qquad (4)$$

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4 See Shinkawa (2005) for Japan and Kim & Kim (2005) for Korea.

5 See for example Varvarigos & Zakaria (2010), Wigger (1999), Yang (2005, 2007) or Fanti & Gori (2008, 2010a, 2010b).

The utility U_t depends on the consumption in the first period of life c_t^1, the consumption in the second period of life c_{t+1}^2, and the number of children $N_t = 1 + n_t$. The parameter q represents the individual discount factor, where $0 < q \leq 1$. The positive parameter v reflects the preference to get descendants. The budget constraint is given by:

$$c_t^1 + \frac{c_{t+1}^2}{R_{t+1}} - (1 - d - bN_t)w_t - \frac{p_{t+1}}{R_{t+1}} = 0, \quad (5)$$

where

$$p_{t+1} = \frac{dw_{t+1}L_t(1+n_t)}{L_t} = dw_tN_tg_t. \quad (6)$$

The variable g_t is interpreted as the per-capita growth factor of the wage rate or labor productivity. The parameter d is the fixed contribution rate of the pension system, the parameter b times the wage rate represents the costs to rear a quality-child. This means parents want only to raise a child with a certain level of well-being and education. The level of the well-being and education is often a result of societal developments, which are influenced by culture and history. Additionally, these levels are taken as given by the parents. The assumption here is similar to the specification of Srinivasan (1995), van Groezen et al (2003), Wigger (1999) and Fenge & Meier (2005, 2009).

Using the utility function (4) and the budget constraint (5), we get the following Lagrangian function, which we will maximize:

$$L(c_t^1, c_{t+1}^2, \mu) = \ln c_t^1 + q\ln c_{t+1}^2 + v\ln(1 + n_t) - \mu\left(c_t^1 + \frac{c_{t+1}^2}{R_{t+1}} - (1 - d - bN_t)w_t - \frac{p_{t+1}}{R_{t+1}}\right).$$

The resulting first order conditions are:

$$\frac{1}{c_t^1} - \mu = 0, \quad (7)$$

$$\frac{q}{c_{t+1}^2} - \frac{\mu}{R_{t+1}} = 0, \quad (8)$$

$$\frac{v}{N_t} - \mu bw_t = 0, \quad (9)$$

$$c_t^1 + \frac{c_{t+1}^2}{R_{t+1}} - (1 - d - bN_t)w_t - \frac{p_{t+1}}{R_{t+1}} = 0. \quad (10)$$

From (7) and (8) we get;

$$c_{t+1}^2 = qR_{t+1}c_t^1, \tag{11}$$

and from (7) and (9);

$$N_t = \frac{vc_t^1}{bw_t}. \tag{12}$$

Substitute equations (11) and (12) in (13); and after some simple reformulations, we get for the consumption in the first period of life:

$$c_t^1 = \frac{(1-d)w_t + \frac{p_{t+1}}{R_{t+1}}}{(1+q+v)}. \tag{13}$$

Therefore, the consumption in the second period of life is given by:

$$c_{t+1}^2 = \frac{q\big((1-d)w_t R_{t+1} + p_{t+1}\big)}{(1+q+v)}. \tag{14}$$

And finally the fertility factor N_t becomes to:

$$N_t = \frac{v\left((1-d)w_t + \frac{p_{t+1}}{R_{t+1}}\right)}{(1+q+v)bw_t}. \tag{15}$$

In the next step we take into account the capital market equilibrium condition $S_t = K_{t+1}$, where we use equations (13) and (14). We know that aggregate savings S_t are given by the difference between the aggregate wage sum minus the aggregate consumption of the young generation, the child rearing costs and contributions to the pension system. After some reformulation by using equations (2) and (3), we get the fertility factor:

$$N_t = \frac{v\big(G_t d + \alpha A(1-d)\big)}{\alpha Ab(1+q+v)} \geq 0 \tag{16}$$

The variable G_t is the growth factor of the economy, where $G_t = N_t g_t$. Given this and the capital market equilibrium condition, we are able to calculate the growth factor of the capital stock:

$$G_t^* = \frac{K_{t+1}}{K_t} = \frac{\alpha qA(1-\alpha)(1-d)}{\alpha(1+q+v) + d(1+v)(1-\alpha)}. \tag{17}$$

Substituting (17) into (16) delivers the number of children in the equilibrium:

$$N_t^* = \frac{v\left(\alpha+d(1-2\alpha)-(1-\alpha)d^2\right)}{b\left(\alpha(1+q+v)+d(1+v)(1-\alpha)\right)}. \tag{18}$$

As a result, the per-capita growth factor of capital, the wage rate and consumption in both periods are given by:

$$g_t^* = \frac{G_t^*}{N_t^*} = \frac{\alpha bqA(1-\alpha)}{v(\alpha(1-d)+d)}. \tag{19}$$

Now, we determined all relevant variables of this simple growth model. One important feature of the model is that the growth rate of the economy is only influenced by the contribution rate of the pension scheme d, the preference parameters v and q and the distributional parameter α. Before we begin with analysis, we should note that a decrease of the per capita growth factor always imply a decrease of the welfare in the long run. Therefore, it is not necessary to derive explicit reactions of the utility function.

If the contribution rate d increases, the following results can be expected:

$$\frac{\partial G_t^*}{\partial d} = -\frac{\alpha qA(1-\alpha)(1+\alpha q1+v)}{\left(\alpha(1+q+v)+d(1+v)(1-\alpha)\right)^2} < 0, \tag{20}$$

$$\frac{\partial N_t^*}{\partial d} = -\frac{v\left[2d(1-\alpha)b\left(\alpha(1+q+v)+d(1+v)(1-\alpha)\right)+\left(\alpha+d(1-2\alpha)-(1-\alpha)d^2\right)b(1+v)(1-\alpha)\right]}{\left(b\left(\alpha(1+q+v)+d(1+v)(1-\alpha)\right)\right)^2} < 0, \tag{21}$$

$$\frac{\partial g_t^*}{\partial d} = -\frac{\alpha bqA(1-\alpha)^2}{[v(\alpha(1-d)+d)]^2} < 0. \tag{22}$$

If the contribution rate increases, all growth variables will decrease, because of the lower disposal income, which will decrease the number of children and the savings. This result is not very surprising and well known in the literature. If the pension system is introduced, the resulting equilibrium is not dynamically efficient, because of the positive externality induced by the capital accumulation. A Pareto improvement is possible.[6] However, if we would increase the welfare, a side effect would be a reduction of the number of children. The reason is that a Pareto improvement is only possible if the savings will increase.

[6] A Pareto improvement can be realized by taxing the young and this tax revenue must be saved and it must realize an interest factor which exceeds αA (Stauvermann, 1996). Please note we only take the utility of individuals into account, which will be indeed be born.

Nevertheless, this causes a decline of the fertility factor. However, the objective here is to increase the number of children. The simplest way to do it is to lower the child rearing costs. Of course, that would mean that the actual educational system is inefficient. For example, it can be argued that day-care centers, pre-kindergarten, and kindergarten have a comparative cost advantage compared to home-based child care of every mother. As noted above this educational sector is, despite the relative huge aggregate educational expenditures in Korea and Japan, less developed in these countries. Even that many parents assume that a collective education in day-care centers, pre-kindergartens, and kindergartens is a disadvantage for children; Havnets, Tarjei, & Mogstad (2011) have shown in a remarkable study that this is not the case. However, the results are the following. By differentiation of equations (17), (18), and (19) with respect to the child-rearing costs *b* we get the following results.

$$\frac{\partial g_t^*}{\partial b} = \frac{\alpha(1-\alpha)qA}{v(\alpha(1-d)+d)} > 0. \tag{23}$$

$$\frac{\partial N_t^*}{\partial b} = -\frac{v(1-d)\big(d+\alpha(1-d)\big)}{b^2\big(\alpha(1+q+v)+d(1+v)(1-\alpha)\big)} < 0. \tag{24}$$

$$\frac{\partial G_t^*}{\partial b} = 0. \tag{25}$$

If the child-rearing costs increase, the number of children will decrease, because of the higher opportunity costs of children. As a result, the per-capita growth rate will increase, because of the lower number of children. The effect on the aggregate growth rate is zero. This means that the parents keep the aggregate expenditures for children bN_t constant. This effect is a consequence of the additive separability of the utility function.

Proposition 1: An increase in the child rearing costs will lead to an increase of the per-capita growth rates and a decrease of the fertility.

On the other hand, that means if the child rearing costs can be lowered, then the later born generations are worse off, because their wages will decrease, even that the future production will grow with an unchanged rate, but it must be shared by more persons.

4- A tax-child allowance mechanism

Some governments introduced a subsidy for rearing children, or sometimes it is also called child allowance or family allowance. The similar mechanism was analyzed by van Groezen, et al. (2003), and Fenge and Meier (2005, 2009) and Wigger (1999), but their models differ from our approach. However, because Groezen et al. (2003) and Fenge and Meier (2005, 2009) use a model without endogenous growth, they applied a lump sum tax and lump sum subsidy.

The aim of this policy is to increase the number of children. Here, the expenditures will be financed by a labor income tax. We assume that the government taxes the wages with the tax rate τ and pay a subsidy of mbw_t per child, where $0<m<1$. The resulting government constraint is then $\tau w_t L_t = mw_t bN_t L_t$ or simplified:

$$\tau = mbN_t \tag{26}$$

After substituting (26), the new budget constraint will be:

$$c_t^1 + \frac{c_{t+1}^2}{R_{t+1}} - (1 - d - \tau - b(1-m)N_t)w_t - \frac{p_{t+1}}{R_{t+1}} = 0. \tag{27}$$

Using this budget restriction, the first order condition (12) will change to;

$$\frac{v}{N_t} - \mu b(1-m)w_t = 0. \tag{28}$$

The first order condition (10) will change to (27). If we now repeat the procedures from section 3, we get the following results for the growth variables:

$$G_t^S = \frac{\alpha qA(1-\alpha)(1-d)(1-m)}{\alpha\big((1+v-m)(1-d)+q(1-m)\big)+d(1+v-m)}. \tag{29}$$

$$N_t^S = \frac{v(1-d)\big((1-d)\alpha+d\big)}{b\big((1+v-m)(\alpha(1-d)+d)+q(1-m)\big)}. \tag{30}$$

$$g_t^S = \frac{\alpha bqA(1-\alpha)(1-m)}{v(\alpha(1-d)+d)}\,. \tag{31}$$

Now, we can differentiate (31), (32) and (33) with respect to child allowance *m*.

$$\frac{\partial G_t^S}{\partial m} = -\frac{\alpha qA(1-\alpha)(1-d)\big((1-d)\alpha+d\big)}{\big[\alpha\big((1+v-m)(1-d)+q(1-m)\big)+d(1+v-m)\big]^2} < 0. \tag{32}$$

$$\frac{\partial N_t^S}{\partial m} = \frac{v(\alpha(1+q)+d(1-\alpha))\left((1-d)d+\alpha(1-d)^2\right)}{b\left((1+v-m)(\alpha(1-d)+d)+q(1-m)\right)^2} > 0. \tag{33}$$

$$\frac{\partial g_t^S}{\partial m} = -\frac{\alpha bqA(1-\alpha)}{v(\alpha(1-d)+d)} < 0. \tag{34}$$

The outcome of an increase or introduction of such a child allowance, financed by a labor income tax, will lead to a higher fertility rate, but it will lower the growth rate of the national income and the national income per capita.

Proposition 2: The aggregate and per-capita growth rate will decrease if a child allowance and the corresponding labor income tax increase. However, the number of children will increase.

The child rearing costs will decrease, and this induces the parents to get more children. The aggregate growth rate decreases, because the tax reduces the disposal income and this leads to a decrease of the savings, on the one hand. On the other hand, the per-capita growth will decline because of the decreased savings, and the increased number of children.

5- The child factor

A third possibility to increase the number of children is to introduce a child factor in the existing PAYG pension scheme. That means a part of the pension of parents depends directly on the number of children. Fenge & Meier (2005, 2009) introduced this mechanism.

This means equation (7) has to be changed to $p_{t+1} = p_{t+1}^u + p_{t+1}^c$. The first part of the pension is the usual pension p_{t+1}^u, which follows the rule:

$$p_{t+1}^u = (1-h)\frac{dw_{t+1}L_tN_t}{L_t} = (1-h)dw_tN_tg_t, \tag{35}$$

where $0 \leq h \leq 1$ holds (h is the child factor), and the children dependent part of the pension p_{t+1}^c follows the rule:

$$N_tp_{t+1}^c = hN_t\frac{dw_{t+1}L_t}{L_t} = hN_tdw_tg_t. \tag{36}$$

From the view of parents, the wage rate in the following period is given; they know only, that they can increase their pension by increasing the number of children. This pension reform leads to a new budget restriction:

$$c_t^1 + \frac{c_{t+1}^2}{R_{t+1}} - (1 - d - bN_t)w_t - \frac{p_{t+1}^u + N_t p_{t+1}^c}{R_{t+1}} = 0. \tag{37}$$

If we now maximize the utility function (4) with respect to the budget constraint (37), only the first order conditions (12) and (13) will change. Equation (13) must be substituted by (37) and instead of first order condition (9) we get:

$$\frac{v}{N_t} - \mu\left(bw_t - \frac{hdw_t g_t}{R_{t+1}}\right) = 0. \tag{38}$$

After some calculations, we will get the results for the growth variables:

$$G_t^c = \frac{(1-\alpha)(1-d)q\alpha A}{\alpha(1+v+q)+d(1+v+qh)(1-\alpha)}. \tag{39}$$

$$N_t^c = \frac{(1-d)[(1-\alpha)(v+qh)d+\alpha v]}{b[(1-\alpha)(1+v+qh)+\alpha(1+v+q)]}. \tag{40}$$

$$g_t^c = \frac{(1-\alpha)qb\alpha A}{\alpha v+d(v+qh)(1-\alpha)}. \tag{41}$$

Now, we can differentiate these three equations with respect to the share *h*:

$$\frac{\partial G_t^c}{\partial h} = -\frac{(1-\alpha)^2(1-d)q^2\alpha dA}{[\alpha(1+v+q)+d(1+v+qh)(1-\alpha)]^2} < 0. \tag{42}$$

$$\frac{\partial N_t^c}{\partial h} = \frac{(1-d)(1-\alpha)dq[(1-q-d)\alpha+d]}{b[(1-\alpha)(1+v+qh)+\alpha(1+v+q)]^2} > 0. \tag{43}$$

$$\frac{\partial g_t^c}{\partial h} = -\frac{(1-\alpha)^2q^2db\alpha A}{[\alpha v+d(v+qh)(1-\alpha)]^2} < 0. \tag{44}$$

Proposition 3: The introduction of a child factor in the existing PAYG pension system increases the number of children and decreases the per-capita growth rates and the aggregate growth rates in an endogenous growth model.

If a part of the pension is dependent on the number of children, the parents have an incentive to get more of them. This induces higher expenditures for child-rearing and results in a reduction of savings. Both effects, the higher number of children and the lower savings leads to a decrease of the per-capita growth rate, and the reduced savings lowers the aggregate growth rate. However, in the next section, we compare both policy measures.

6- A comparison

To compare the tax-child allowance policy from section 4 and the child factor policy from section 5 we determine the policy parameters so that the population will become sustainable, which means the population will become constant. We start with the tax-child allowance mechanism. Solving equation (30) for *m* so that $N_t = 1$ gives:

$$m(1) = \frac{v(\alpha(1-d)+d)(1-d-b)-b(\alpha(1-d-q)+d)}{-b[\alpha(1+q-d)+d]}. \tag{45}$$

Substituting (45) into the growth rate (31) leads to the corresponding equilibrium growth factor:

$$g_t^S(1) = \frac{\alpha q A(1-\alpha)(1-d-2b)}{2[\alpha(1+q-d)+d]}. \tag{46}$$

At next, we calculate the values for the child-factor mechanism. Now we calculate the *h*, so that also with child factor policy $N_t = 1$ holds. Using equation (40) we get for *h*:

$$h(1) = \frac{b(\alpha(1-d-q)+d)-v(\alpha(1-d)+d)(1-d-b)}{dq(1-\alpha)(1-d-b)}. \tag{47}$$

Substituting $h(1)$ in the equation (41), leads to the corresponding per-capita growth factor:

$$g_t^c(1) = \frac{\alpha q A(1-\alpha)(1-d-2b)}{2[\alpha(1+q-d)+d]}. \tag{48}$$

If we compare now equation (46) with (48), then we see that they are equal, this leads to the conclusion that both mechanisms deliver the same results with respect to the per-capita growth rate. Because of the fact that the population growth rate equals zero, then the aggregate growth rate and the savings will be changed in the same way by both mechanisms. Consequently, the negative welfare effects are also identical. In so far policy makers are free to choose between both mechanisms. However, now it is clear, that all policy measures, which increase the fertility have strong negative impacts for growth and hence the welfare.

Proposition 4: The child factor mechanism and tax subsidy mechanism have identical negative effects on the growth rates, if the number of children is given.

However, one implication should be clear, in this model both mechanisms will lead to welfare losses. The simple reason is, if the per-capita growth rates decrease the utility of subsequent generations will also decrease.

Nevertheless, some forms of both mechanisms were introduced in Germany and they are still in use. The effect on the fertility is positive but nearly zero. But the mechanisms are very costly, in 2010 the German government paid on average around EUR 7,250 (approx. $ 10,000) per child and year for mechanisms which correspond with the mechanisms above. These costs exclude all expenditures for schools, kindergartens and universities.[7]

Because of the fact that similar mechanisms have been applied by Fenge & Meier (2005, 2009) and van Groezen, Leers & Meijdam (2003) in a small open economy with a neoclassical standard production function, we should compare our results with theirs. They come to the conclusion that both mechanisms can be welfare-enhancing. However, they took a small open economy into account. However, implicitly the financial means to finance the introduction of the proposed mechanisms in their models are borrowed from abroad. This leads then to an increasing current account deficit, which must be seen as problematic.

7- Conclusion

One general result is that the tax subsidy mechanism or child allowances are a perfect substitute to a child dependent PAYG pension system. Even that both mechanisms increase the fertility rate, it must be stated that the aggregate growth rate of production and the per capita growth rates will decrease as a consequence. Even if we would analyze an open

[7] Authors' calculations, which are based on official numbers from the government. According to the actual government of Germany, parents bear only 25% of the total child rearing and educational costs.

economy in the way Wigger and Irmen (2006) have done it, the results would only change quantitatively but not qualitatively.

However, one main reason, why governments of countries with a PAYG pension system think that they should fight against a demographic change is caused by the expectation that a worse ratio between the working and retired population makes it impossible to finance suitable pensions, without burdening the working population over proportionally. However, it is a political decision what suitable means, but according to our model, the pensions will increase with the aggregate growth rate as long as the fertility rate falls not to zero, even that the ration between average pension payment and average labor income will decrease. The fear that a PAYG pension system will collapse is not justified and nobody knows if the population will not stabilize in the future. Anyway, governments should be very careful before they introduce one of the proposed mechanisms to increase the fertility, because the costs of these mechanisms could be much higher than to increase slightly the contribution rate by a few per cent.

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