Furrow Flow Measurement Accuracy

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Abstract: The primary source of error in properly calibrated, constructed, and installed flow measurement devices is due to reading error or uncertainty. Head reading uncertainty in small V-notch flumes and submerged orifices is measured in the field as ±3mm with no consistent variation with reading. Elapsed time measurement uncertainty for volumetric measurements increases with the square root of the time. The sensitivity of flow measurement uncertainty to head or time reading uncertainty is proportional to the ratio of the device discharge equation exponent to the reading. Furrow flow measurement uncertainty varies with the device and flow rate, but generally exceeds ±5% and often exceeds ±10%. Maintaining uncertainty below ±10% requires flume measurements in the upper 50% of their range, orifice measurements with head readings greater than 13mm, and volumetric measurement elapsed times greater than 4 seconds.

Flow Measurement Uncertainty

Any measurement process involves a degree of uncertainty, sometimes referred to as inaccuracy or potential error. If many measurements are made of a value, they will randomly scatter around the true value if all systematic errors are eliminated or corrected. The standard deviation of the measurements s is a measure of the scatter and is used to determine the uncertainty of the measurement E, or the interval within which the true value lies with a certain confidence level or probability. The uncertainty of a measurement can be calculated by

$$E = \frac{ts}{\sqrt{n}} \qquad (1)$$

where t = the Student's t-statistic (two-tailed), dependent upon the degrees-of-freedom associated with estimating s (usually n-1) and the desired probability level; and n = the number of measurements used to calculate the mean. The t-statistic can be found in basic statistics texts.

Often, the uncertainty of a measurement or device is given in terms of a value implied to be the maximum error. *International Standard #5168* (ISO 1978) defines this "maximum" deviation in flow measurement as the 95% confidence interval.

Bos et al. (1984), Bos (1976), and ISO (1978) discuss flow measurement error sources and their effects. The major sources of uncertainty in head:

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discharge-type open channel flow measurement devices (flumes, weirs, and orifices) are:

1. Uncertainty in the device calibration, or equivalently, uncertainty in the dimensional duplication of the calibrated device, usually limited by calibration standards and construction tolerances.
2. Uncertainty in the duplication of calibration flow conditions, generally limited by standard installation procedures.
3. Uncertainty in the reference elevation of the gage.
4. Uncertainty of the head reading due to gage resolution, water surface waves, and light reflection or meniscus interference.

Source 1 uncertainty will cause a systematic error in a given device at a given flow rate. Sources 2 and 3 can cause systematic errors for a given device setting. Procedure-related source 3 and 4 uncertainties (i.e., using an unlevel surface to level a flume) can also cause systematic errors. Systematic errors must be added to random uncertainties to estimate the total measurement uncertainty. However, if several devices of the same type are used interchangeably, source 1 uncertainty due to dimensional variations becomes a random error source. If the device is reset or the critical factors (leveling, sediment accumulation, etc.) are monitored and corrected for before each reading, source 2 and 3 uncertainties will also tend to be random. If properly calibrated and dimensioned devices are used according to standard procedures, the remaining uncertainty will be primarily the result of sources 3 and 4 and will be random.

In addition to head:discharge-type devices, flow measurements can also be made volumetrically by recording the time required to fill a container of known (calibrated) volume or to add a mass of fluid to a container. For volumetric measurements, the error sources are container calibration (systematic), which should be small, and timing. Timing uncertainties result from imprecise timer starting and stopping and the judgement decision of when the container is full. These random timing errors are considered equivalent to the flow depth or head measurement errors in the other devices.

The relationship between the variance (the square of the standard deviation) of a device head reading and the variance of a flow measurement can be approximated by the first term of a Taylor series expansion (Kendall and Stuart 1963):

$$V_Q = \left(\frac{dQ}{dR}\right)^2 V_R \qquad (2)$$

where V_Q = variance of the flow measurement; V_R = variance of the reading; Q = flow rate, and R = reading. Inserting Eq. 2 into Eq. 1:

$$E_Q = \frac{ts_Q}{\sqrt{n}} = \frac{t\left[\left(\frac{dQ}{dR}\right)^2 V_R\right]^{1/2}}{\sqrt{n}} = \frac{t\left|\frac{dQ}{dR}\right| s_R}{\sqrt{n}} = \left|\frac{dQ}{dR}\right| E_R \qquad (3)$$

where E_Q = uncertainty of the flow measurement; s_Q = standard deviation of the flow measurement = $\sqrt{V_Q}$; s_R = standard deviation of the device reading = $\sqrt{V_R}$; and E_R = uncertainty of the reading.

Consequently, the uncertainty in a flow measurement is proportional to the uncertainty in the device reading. The derivative term, dQ/dR, is also called the sensitivity of the device (Bos 1976; ISO 1978).

The objective of this study is to determine the flow measurement uncertainties under field conditions of three types of furrow flow measurement devices and to determine methods to minimize uncertainty.

Procedure

During three trials conducted on three different days, nine technicians each measured the flow at the head of the same 10 furrows with a 3.78-L (1-gal) bucket, three-holed acrylic orifice plates (Trout 1986a, 1986b) and fiberglass long-throated 60° V-notch flumes (Robinson and Chamberlain 1960; ASAE 1983; Trout 1986c). Measurement device calibrations are given in Table 1. Flow rates into each furrow were preset and held constant during each trial at between 4–60 L/min. All technicians were trained and had some experience in the use of the devices.

The volumetric measurement was done in pairs with one member holding the bucket and the second operating a stopwatch. Time-to-fill readings were recorded to the nearest 0.1 sec. In trials 1 and 3, each individual measured each furrow four times.

The orifice plates were preinstalled and the appropriate orifices selected to produce a submerged flow head loss of between 10–40 mm. Readings of the upstream and downstream water surface elevation difference were made with portable differential point gages (Trout 1986b) to the nearest millimeter. Gage levelness was only visually assessed.

In trials 1 and 3, the V-notch flumes were installed, leveled, and read by each individual. Each individual randomly selected two or three flumes to use from a group of 12. In trials 2 and 3, flumes were preinstalled in the furrows and each individual checked and altered, if necessary, its installation (level, sediment accumulation, water leakage) before taking the reading. Flume flow depth readings were taken from the flume side gage, which has markings at every 2 mm but was estimated to the nearest millimeter, and with a vertical point gage (Trout 1986c) with a 1-mm resolution. Both depth readings were made in the flowing water at the flume gaging point. Side-gage readings were reduced by 1.5 mm, as indicated in Table 1, to correct for a systematic bias caused by the water surface meniscus. The fiberglass flumes are mass produced on precision metal forms.

TABLE 1. Furrow Flow Measurement Device Calibrations

Type (1)	Device (2)	Reading resolution (3)	Discharge equation (Q in L/min) (4)	Sensitivity (dQ/dR) (5)
Volumetric	3.78 L-bucket and stop watch	0.1 sec	$Q = 227R^{-1}$	$227R^{-2}$ $454R^{-1.5}$[c]
Flume[a]	Fiberglass 60° V-furrow flume	1 mm 1 mm	$Q = 0.000543(R-1.5)^{2.63}$ $Q = 0.000767R^{2.63}$	$0.0014R^{1.63}$ $0.0020R^{1.63}$
Orifice[b]	Three-hole acrylic orifice plate	1 mm	$Q = 0.00411D^2R^{0.5}$	$0.0021D^2R^{-0.5}$

[a]With sloping side gage (line 1) and vertical point gage (line 2).
[b]D = orifice diameter (mm), C_d = 0.625.
[c]Square root transformation.

During trial 3, the furrow flow rates were also measured volumetrically with a 15.7-L container. Due to the large size of the container, the measurement is less sensitive to reading uncertainties, and the measurement was considered the "standard" flow rate for the furrow. Deviations of the means of all flow measurements with a device for each furrow from that standard flow rate was used to indicate systematic errors in the devices.

Variance Component Estimation

The VARCOMP procedure of SAS (SAS Institute 1982) was used to estimate the variance components of both the device readings and the predicted flow rates for each trial and each device. Except for cases with multiple measurements per subject x furrow combination, (volumetric trials 1 and 3), the error mean square was not subtracted out of the subject mean square when estimating the subject variance component. Our reasoning is that the subject x furrow variance is the largest component in the error term, and it does not enter into the subject expected mean square.

TABLE 2. Furrow Flow Measurement Device Reading Variance Component Estimates

Device (1)	Component (2)	VARIANCE: Trial 1 (3)	Trial 2 (4)	Trial 3 (5)	Best Estimate (6)	Standard Deviation (7)
(a) V-Furrow Flume						
Preset–side gage	Subject x Furrow	—	0.190	1.236		
	Subject		0.050	0.314		
	Total		0.240	1.550	1.55	1.2
Preset–point gage	Subject x Furrow	—	0.173	0.951		
	Subject		0.133	0.034		
	Total		0.306	0.985	0.99	1.0
Individually set–side gage	Subject x Furrow	1.253	—	3.449		
	Subject	0.259		0.660		
	Total	1.512		4.109	4.11	2.0
Individually set–point gauge	Subject x Furrow	0.913	—	—		
	Subject	0.902				
	Total	1.815			1.82	1.3
(b) Orifice Plates						
—	Subject x Furrow	1.285	1.470	1.475	1.42	
	Subject	0.445	0.377	0.140	0.32	
	Total	1.730	1.847	1.615	1.74	1.3
(c) 3.78 L-Bucket[a]						
—	Furrow–subject–reading	0.001342[b]	—	0.000950[b]	0.00115	
	Furrow–reading	0	—	0	0	
	Subject–Furrow	0.000381	0.00126	0.000245	0.00031	
	Subject–reading	0	—	0	0	
	Reading	0.000029	—	0.000015	0.00002	
	Subject	0.001037	0.00127	0.000261	0.00065	
	Total	0.002789	0.00253	0.001471	0.00213	0.046

[a]After square-root transformation.
[b]Four readings.

This results in a slight upward bias in the subject variance component estimates.

Table 2 lists the various variance components. The subject x furrow component estimates the variance associated with differences among readers between furrows. The subject variance is an estimate of the variance between individual readers. The total variance, which includes both components, is the best estimate of the variance associated with an individual reading, but on a multiple reader basis. When multiple readings were made (volumetric trials 1 and 3), the reading component estimates the variance between the individual readings by one reader.

When variances between tests were homogeneous (as determined by F or Bartlett's test), data were pooled to determine the best combined estimate of the variance components. Otherwise the trial with the largest variance was accepted as a conservative best estimate. Reading standard deviations (square root of the variances), which have the same units as the reading, were calculated from the best estimate of the total variance for each measurement procedure. The reading standard deviations were plotted against the average reading for each furrow to determine if the variance varied with the mean.

RESULTS

Table 2 lists the individual trial reading-variance components, the variance best estimate, and the reading standard deviation for each measurement device. The total reading variance for the V-notch furrow flumes varied from test to test and with the measurement procedure. In trial 2, the square root of the preset flume reading variance was about equal to the reading resolution limits (±0.5mm), indicating that the resolution was the limiting factor. In all other cases, the standard deviations are at least double the resolution limit.

The data indicate that the random variation in head readings of the preset flumes was about 1/3–1/2 less than the reading variations of the individually set flumes. This is expected since all measured variability due to device construction, and part of that due to installation (primarily flume levelness) is eliminated in the preset installations. These construction and installation effects result in systematic errors when the flumes are preset.

Evidence of this systematic error is provided by comparing the deviations of the mean flow rates measured with a device on a furrow with the actual flow in the furrow. These data for trial 3 are plotted in Fig. 1 and summarized in Table 3. Five of the 20 preset flume measurement means exceeded the standard flow by more than the 95% confidence limit, while the confidence interval of all of the individually installed flume measurements included the standard measurement, indicating the preset measurements included some systematic error. The predominately positive deviations indicate that the installer tended to install the flumes sloping toward the front. The root-mean-square (rms) deviation (as defined in Table 3) of the preset side-gage measured flows was about 2 percentage points larger than the rms deviation of the individual installation means, indicating about a 2% systematic error. Thus flow measurement with the previously installed flumes, even when the setting and flume level is rechecked, resulted in a smaller random uncertainty, but at least a 2% systematic

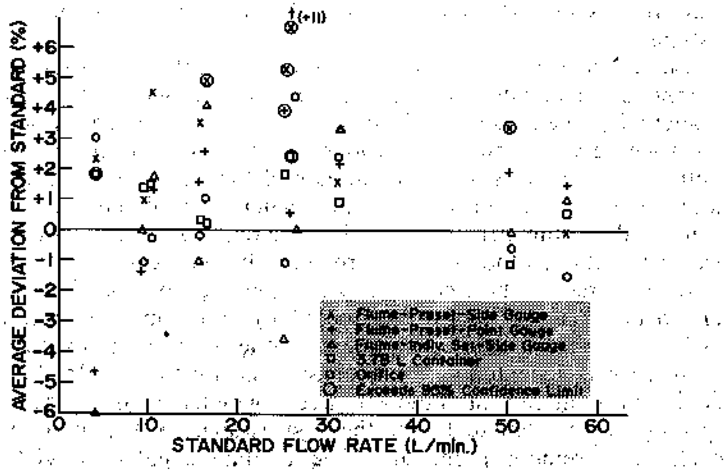

FIG. 1. Deviations of Mean Measured Flow Rates from Standard Measurement in Trial 3

error. The reduction in random uncertainty will be roughly equivalent to the increase in systematic error. For truly independent measurements, the device and installation must be changed.

Table 2 indicates that when the head reading was made on the same flume setting with both the 60°-sloped side gage and the portable point gage, the reading standard deviations were similar. The side-gage readings were expected to be more difficult to judge due to the curved water surface meniscus against the gage. Apparently the portable point gage is sufficiently difficult to use that the advantages of its precision are not always realized in the field.

For both gages, the reading variability s_R tended to increase slightly with the reading, although the relationship was not consistent so no variation of

TABLE 3. Average Deviations of 10 Mean Measured Flow Rates from Standard Flow Rate in Trial 3

Device (1)	Average absolute deviation (2)	Root-mean-squared deviation[a] (3)	Maximum deviation (4)	Average deviation (5)	Number of deviations greater than 95% confidence limit (6)
Flume-preset					
Side gage	3.8	4.7	+11	+3.8	4
Point gage	2.2	2.3	−5	+1.0	1
Flume-individual set					
Side gage	2.1	2.8	−6	0.0	0
3.78-L container	1.1	1.4	+2	+0.9	2
Orifice plate	1.6	2.0	+4	+0.6	0

[a]Root-mean-squared deviation = $[\Sigma(\text{mean measured flow-standard flow})^2/n]^{1/2}$.

s_R with R was assumed. The Froude number at the gaging point varied only from about 0.1 at 4 L/min to about 0.2 at 60 L/min, so the water surface was fairly smooth over the measured range.

Although the total reading variance of one flume test was about four, the remainder were below two, so the reading standard deviation of the V-notch flumes, including effects of flume construction, installation, and gage reading, will be estimated as 1.5 mm.

Orifice-plate differential point-gage head-reading variances were consistent among the trials, and the best estimate standard deviation was 1.3 mm. The reading standard deviations did not vary consistently with measured differential head or orifice size.

Although all orifice plates were preinstalled, Fig. 1 and Table 3 show that little systematic error resulted from orifice-construction or plate-installation differences. None of the means deviated from the standard measurement by more than the 95% confidence limit, although the small positive average deviation (0.6%) indicates a possible slight positive error in the orifice discharge coefficient.

The reading standard deviation for the volumetric method increased at a decreasing rate from about twice the resolution (0.2 sec) at low readings (high flow rates) to about seven times the resolution at high readings. The increase is likely due to the increasing time required to raise the water level in the bucket a depth increment. Consequently, any difference in judgement of when the bucket is full causes greater reading varability at larger time readings. The decreasing rate of increase could result from the reduced judgement differences at lower flow rates (higher readings) due to less water surface turbulence. A square root transformation of the readings was used to stabilize the variances over the reading range (Box et al. 1978), and the variances listed in Table 2 are of the transformed data. The volumetric method sensitivity, dQ/dR, must also be adjusted for the transformed readings.

$$\frac{dQ}{d\sqrt{R}} = \frac{d(227\sqrt{R}^{-2})}{d\sqrt{R}} = 454\sqrt{R}^{-3} = 454R^{-1.5} \quad (4)$$

An estimate of the reading standard deviation s_R (untransformed basis) from $s_{\sqrt{R}}$ would be

$$s_R = 2\sqrt{R} \cdot s_{\sqrt{R}} \quad (5)$$

Thus, s_R is estimated to vary from 0.19 sec at $R = 3.8$ sec ($Q = 60$ L/min) to 0.72 sec at $R = 57$ sec ($Q = 4$ L/min).

Fig. 1 and Table 3 show a small positive bias in the volumetric measurements, possibly resulting from underestimation of bucket fullness. Adjustment for this apparent systematic error brings all mean deviations to within the 95% confidence limit of the standard flow.

When multiple readings are made, the precision of the estimated flow rate is increased. The standard deviation of the mean, $s_{\bar{x}}$, can be estimated by

$$s_{\bar{x}} = \sqrt{\frac{rV_S bV_R + rV_{FS} + V_e}{br}} \quad (6)$$

where V_S = subject variance component estimate; V_R = reading variance component estimate; V_{FS} = furrow x subject variance component estimate; V_e = furrow x reading subject variance component estimate; r = number of readings per subject; and b = number of subjects (readers). For tests 1 and 3, utilizing the mean of four readings for an individual would reduce the standard deviation of the transformed readings from 0.046 to 0.035 or by 24%.

For all devices and flow rates, the device sensitivity times the reading standard deviation predicted the flow rate measurement standard deviation to within ±8%, and in most cases to within ±4%, so the Taylor series approximation used in Eq. 2 appears sufficiently accurate.

Analysis

Flow measurement uncertainty in properly calibrated and used devices is primarily dependent on the uncertainty in the reading and the sensitivity of the device. Thus, to minimize uncertainty, readings must minimize uncertainty relative to the reading, and devices should be chosen and sized to minimize sensitivity in the measured flow range.

Reading uncertainty will be minimized in head:discharge-type devices by: (1) Selecting and designing the device such that Froude numbers at the gaging point will be low and/or taking readings in still water such as in a stilling well; and (2) carefully leveling the device or otherwise establishing the reading datum or translocating the gaging point, via a pressure line, to a stilling well near the device control point (Bos et al. 1984). Both stilling wells and gaging point translocation increase the bulk and cost of measurement devices. A mechanical device such as a point gage can increase the precision of a reading, especially if used in a stilling well. However, the reading variability measurements indicate that, for the small portable point gages used on the flowing water surface, the potential improvement was not realized. From the field measurements made, the source of uncertainty cannot be separated between that due to errors in gage reading and that due to errors in gage datum elevation.

The head loss readings for an orifice plate can be measured in relatively still water near the plate due to the large amount of flow contraction through the orifice. Thus, reading uncertainty should be low due both to the tranquil water surface and to decreased sensitivity to gage leveling. A gage datum one degree out of level relative to the device control point will cause only a 0.7-mm error in a submerged orifice plate reading when the gaging points are 40 mm apart, but a 3-mm error in a furrow flume reading when the gaging point is 200 mm from the flume control point. The more consistent and smaller average measured reading uncertainties of the orifice plate differential head measurements compared to the V-notch flume head measurements reflect these potential advantages.

Eq. 3 shows that the sensitivity of a measurement device, dQ/dR, determines the uncertainty in the flow rate measurement for a given reading measurement uncertainty. The sensitivity will vary with the measurement device and the flow rate measured. For the studied devices and most head:discharge-type flow measurement devices, the discharge relationship can be approximated by

$$Q = aR^u \qquad (7)$$

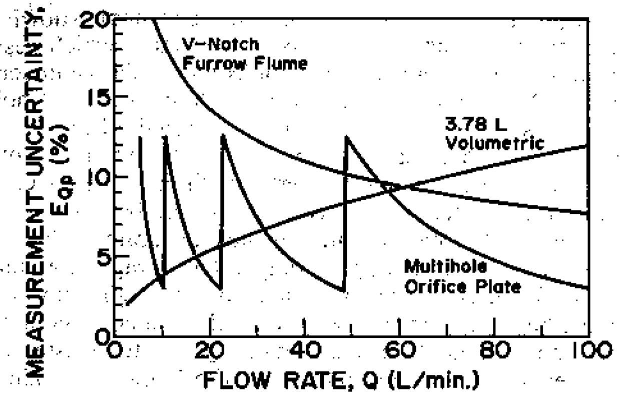

FIG. 2. Relative Flow Measurement Uncertainty for Three Furrow Flow-Measurement Devices

where a and u = the device discharge coefficient and exponent, respectively. The exponent values are about 2.5 for sloping-sided flumes and weirs, 1.5 for rectangular flumes and weirs, 0.5 for orifices, and −1 for volumetric measurements. The coefficient is proportional to the control section cross-sectional area of a device. By differentiating Eq. 7, the sensitivity is given

$$\frac{dQ}{dR} = uaR^{(u-1)} \qquad (8)$$

The sensitivities of the tested devices are listed in Table 1. By dividing Eq. 8 by Eq. 7 and inserting into Eq. 3, an expression for the relative flow measurement uncertainty E_{Qp} is derived

$$E_{Qp} = \frac{E_Q}{Q} = u\left(\frac{E_R}{R}\right) \qquad (9)$$

Thus, the relative uncertainty in the measurement of a flow Q is proportional to both u and the ratio E_R/R.

Fig. 2 shows the measured flow measurement uncertainty of the three studied devices with s_R = 1.5 mm for the flumes, 1.3 mm for the orifices, and $0.046 \cdot 2\sqrt{R}$ for the volumetric readings. Since the derived reading standard deviations were determined with a large number of measurements, $t \approx 2.0$ for the 95% confidence limit, so E_R for a single measurement ($n = 1$) was calculated as $2s_R$. Note that the figure projects uncertainties above the 60-L/min measured range on the assumption that the measured s_R values are valid up to 100 L/min. Fig. 2 shows that projected furrow flow measurement uncertainties vary widely with measurement device and head loss.

Due to the low sensitivity of orifice plates, the head loss through an orifice increases rapidly with flow rate. The head loss range must be limited to prevent excessive backwater conditions upstream due to high head loss and low E_R/R ratios and thus high uncertainty at low head loss (Trout 1983, 1986b). Consequently, when the head loss in the example shown in Fig. 2 exceeds 45 mm, orifice area is increased 2.1 times, which results in a minimum head loss of 10 mm and thus a maximum relative uncertainty of 13%. Trout (1986b) describes the design process for such multihole orifice plates. As Fig. 2 shows, three orifice sizes are required to measure from 10–100 L/min within these head loss constraints. Note that the lower head loss limit establishes the maximum uncertainty limit. If the lower head loss limit were 15 mm, the maximum uncertainty would be 8.5%.

The 3.78-L volumetric measurement is most accurate in the lower flow range but least accurate in the upper range, due to rapid container filling and small time readings. To maintain accuracy at higher flows, a larger container must be used. Sensitivity is inversely related to container size. For example, the uncertainty of the standard flow rate measured with the 15.7-L container would be about half ($\sqrt{3.78/15.7}$) the uncertainty shown in Fig. 2, assuming s_R remains the same. Container shape (surface area-to-volume ratio) affects the depth increase rate and thus s_R.

The V-notch flume is the least accurate device in the lower flow range, primarily due to the high discharge equation exponent value u. Its 95% uncertainty limit is greater than 10% up to 50% of its approximately 100-L/min capacity. Flume sensitivity could be decreased (and thus accuracy increased at a constant s_R) by increasing the head or flow depth by decreasing the throat width or by using a throat with side rather than bottom contractions, and by decreasing the discharge equation exponent u by using a rectangular rather than trapezoidal control section. However, greater flow depths generally require greater head loss for critical flow conditions and thus increase upstream flow depths, and less sensitivity decreases the measurement range.

The furrow flow measurement uncertainties shown in Fig. 2 do not include any systematic error. They apply to properly calibrated and used devices. Device calibrations are often reported to be accurate to within ±2%. The measurements indicate the tested devices met this criterion. Systematic error can be estimated by comparing measurements made with different devices.

Summary

Furrow flow measurement device design or selection will require evaluating the trade-off between required accuracy, desired range, and allowed head loss. Volumetric measurements are potentially accurate but require several centimeters of free-falling water. The required fall height increases with container size and thus range and accuracy. Submerged orifice measurements are potentially accurate due to the low sensitivity, but range is consequently narrow and head measurement is equal to the head loss. Flumes measure a wide range of flows and, with long-throated flumes, required head loss is often less than 20% of the head measurement. However, flume flow measurement is most sensitive to head reading errors, and sensitivity increases with range and decreasing allowable head loss.

Flow measurement accuracy is also dependent upon the accuracy of the head or time measurement. Measured head reading standard deviations with side or point gages on the flowing water surface were in the range of

1–2 mm and thus 95% uncertainty limits were 2–4 mm. Reading uncertainty in head:discharge devices can be reduced with stilling wells, especially if the head is translocated to near the device control point. These improvements increase device bulk, complication, and cost. Head reading uncertainty will also depend upon the training and care of the reader. The readers in this study were trained and closely supervised.

With the measured reading uncertainties, flow measurement uncertainties of the tested devices often exceeded 10%. This is larger than is commonly claimed and strongly affects the uncertainties of infiltration and irrigation evaluation measurements. A companion paper (Trout and Mackey 1988) analyzes the effects on infiltration measurement.

Conclusions

1. Head measurement uncertainty in small head:discharge-type flow measurement devices measured in the field by closely supervised technicians on a relatively smooth flowing water surface was ±3 mm.
2. Volumetric time measurement uncertainty increased with the square root of the container fill time.
3. Flow measurement uncertainty with three tested devices generally exceeded 5% and exceeded 10% over parts of their range.
4. Desired measurement range or allowed head loss constraints must often be limited to achieve desired flow measurement accuracy.

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Appendix II. Notation

The following symbols are used in this paper:

a = coefficient of measuring device discharge equation;
b = number of subjects (readers);
E = measurement uncertainty or one-half confidence interval;
E_{Qp} = relative flow measurement uncertainty;
n = number of measurements used to calculate mean;
Q = flow rate;
R = device reading such as flow depth in flume;
r = number of readings per subject;
s = standard deviation = $\sqrt{V}$;
t = Student's t-statistic (two-tailed);
u = exponent of measuring device discharge equation;
V = variance;
V_e = furrow x reading x subject variance component estimate;
V_{FS} = furrow x subject variance component estimate;
V_R = reading variance component estimate; and
V_S = subject variance component estimate.

Subscripts

R = reading;
Q = flow; and
$\bar{x}$ = mean.