Estimation of Inefficiency using a Firm-specific Frontier Model

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Abstract

It has been argued that the deterministic frontier approach in inefficiency measurement has a major limitation as inefficiency is mixed with measurement error (statistical noise) in this approach. The result is that inefficiency is contaminated with noise. Later stochastic frontier approach improves the situation with allowing a statistical noise in the model which captures all other factors other than inefficiency. The stochastic frontier model has been used for inefficiency analysis despite its complicated form and estimation procedure. This paper introduced an extra parameter which estimates the amount of proportion that an error component shares in the observational error. An EM estimation approach is used for estimation of the model and a test procedure is developed to test the significance of presence of the error component in the observational error.

Keywords: stochastic frontier model, skew-normal distribution, identification, EM algorithm, Monte Carlo simulation.

JEL Classification: C15, C21, C51

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1 Introduction

In early twentieth century, Cobb and Douglas (1928) introduced the econometric estimation of production function for estimating the economic efficiency of a firm using given inputs and a technology. They used the ordinary least square (OLS) method of estimation to estimate the production function which requires the observations to lie around it. This assumption, however, contradicted the theoretical definition of production function which refers to the maximum (frontier) output attainable using a given inputs and a technology. Next thirty years econometric analysis of production function ignored this frontier property of the production function and was primarily based on the estimated 'average' production function.

Winsten (1957) was perhaps first to attempt estimation of the frontier production function using Corrected OLS (COLS) method. In this method, the intercept of the OLS estimated 'average' production is adjusted so that all the observations lie below the estimated production function. Aigner and Chu (1968) suggested the estimation of production function using linear and quadratic programming technique with the frontier restriction i.e. the residuals are to be positive. However, this approach has two main drawbacks. Firstly, it is deterministic as there is no stochastic specification and, hence, one cannot compute the error margin of the estimates and ii) The estimates were found to be very sensitive to outliers. Timmer (1971) suggested an iterative approach to overcome these problems where at each stage a new deterministic frontier is estimated after deleting those data points with respect to which the estimates at the previous stage were found sensitive and the process is continued until the deterministic frontier function stabilizes. Richmond (1974) improved upon the COLS estimates to make them unbiased and consistent.

Schmidt (1976) estimated the deterministic frontier model with a statistical sense by the maximum likelihood method assuming error with a one-sided distribution like exponential and half-normal. The resulting estimates under these distributional assumptions are equivalent to the linear and the quadratic programming estimators of Aigner and Chu (1968). Later, Greene (1980) estimated another deterministic frontier model assuming errors are gamma variables.

Although the deterministic frontier approach of Aigner-Chu-Schmidt estimates the frontier function respecting its frontier property, an obvious limitation of this approach is that in this approach one cannot isolate the effect of inefficiency from that of the random noise as both are lumped together in the disturbance term of the model. Also, it violates one of the regularity conditions required for application of ML method viz. the support of the distribution of *y* must be independent of the parameter vector. In this approach the regularity condition is violated. One can, however, apply the MOLS method of Richmond (1974) which is a combination of the OLS and MOM, for estimation of the parameters of the deterministic frontier.

The stochastic frontier approach introduced by Aigner, Lovell and Schmidt (1977), Meeusen and van den Broeck (1977) and Bettese and Corra (1977) overcomes the limitations of the deterministic frontier approach by decomposing the disturbance term into two random components representing the "random noise" and the "inefficiency". While the decomposition enables one to separate out the effects of random noise from the inefficiency and makes the support of the distribution of output independent of the parameter space, the concept of stochastic frontier ensures the frontier restriction on the observed outcomes. The stochastic frontier model is the extension of deterministic frontier model with added stochastic noise. However, sensitivity of the stochastic frontier model depends on mis-specification and amount of statistical noise and inefficiency in composite disturbance.

Ruggiero (1999) examined the performance of deterministic and stochastic frontier models using Monte Carlo simulation experiments. The analysis revealed that the deterministic frontier model was more consistent than stochastic frontier model. Also, deterministic frontier model outperformed the stochastic frontier model which concludes that the stochastic frontier model does not decompose the stochastic noise and inefficiency correctly. Measurement error leads to bigger biases in the stochastic frontier model than it does in the deterministic model. This suggests that the main criticism against the deterministic models is hypocritical.

The purpose of this paper is to provide a more general frontier model which is specific to each firm. An extra binary random variable is introduced to decide whether a deterministic frontier or stochastic frontier model is appropriate for each firm.

The rest of the paper is organized as follows. Section 2 presents the more general stochastic frontier model and derives the distribution of the observational error. Section 3 presents the estimation procedure to estimate the parameters of the model. In section 4, a Monte Carlo experiment is constructed to compare the performance and the results of the analysis and their implications are reported. In section 5 we report and analyze the results of an empirical application of the firm-specific frontier model. The major conclusions emerging from this study are noted in the final section.

2 A Firm-specific Frontier Model

Let y_i and x_i be the output and vector of non-stochastic inputs of the ith firm respectively indexed by the production function $f(.)$ and ε_i be the random error. Then a firm-specific frontier model for the ith firm can be presented as

$$y_i = f(x_i, \beta) + \varepsilon_i; \quad \varepsilon_i = J_i v_i + u_i \ , \ i = 1, ..., n \qquad (2.1)$$

where β is a vector of unknown parameters to be estimated. The random error ε_i is composed of two unobservable stochastic terms viz. v_i, the statistical noise and $u_i \geq 0$, the technical inefficiency and J_i is a unobservable binary random variable that defines whether the frontier model is stochastic frontier or deterministic frontier for ith firm. If $J_i = 0$, the frontier model becomes deterministic frontier model and if $J_i = 1$, the frontier model becomes stochastic frontier model.

The distribution of error component v_i can be assumed to be normal i.e. $v_i \sim N(0, \sigma_v^2)$ and the distribution of error component u_i can be assumed to be half-normal (ALS, 1977) or Exponential (Stevenson, 1980) or Gamma (Greene, 1990) with $u_i \geq 0$. We retain these distributional assumptions regarding the error components in this paper. Also it is assumed that $J_i \sim Bern(\pi)$.

The density function of ε_i can be found as:

Let $\varphi_1(\varepsilon_i)$ is the density function of ε_i when $J_i = 0$ and $\varphi_2(\varepsilon_i)$ is the density function of ε_i when $J_i = 1$. Then, the density function of ε_i is given by

$$f(\varepsilon_i) = \pi\varphi_2(\varepsilon_i) + (1-\pi)\varphi_1(\varepsilon_i) \, . \qquad (2.2)$$

The density function of ε_i can be considered as a two component mixture model. The EM algorithm can be most reliable approach to estimate the parameters of the model using the density function of ε_i.

2.1 Estimating firm-specific inefficiency

Though the primary objective of the frontier model is to estimate the unknown parameter vector $\delta = (\beta, \sigma_v^2, \sigma_u^2, \pi)'$, the ultimate objective of the frontier model is the estimation of firm specific inefficiency, *u*. In this section we discuss the approach of estimating the firm-specific inefficiency in the frontier model presented in the above section. The natural estimators for firm-specific inefficiency in the firm-specific frontier model are the Jondrow et al. (1982) proposed conditional mean or mode of u given which is given by

$$E(u_i \mid \varepsilon_i) = g(\varepsilon_i, \delta) \qquad (2.3)$$

which is a function of the unknown parameter vector and the observational error and can be estimated using the estimated value of parameter vector and the observed data. This measure of inefficiency given in (2.3) can be computed using the conditional distribution of u given ε. In our model the conditional distribution of u given ε can be derived as follow:

The conditional distribution function of u given ε can be derived as

$$\begin{aligned} P(U \le u \mid \varepsilon) &= P(U \le u \mid \varepsilon, J=0)P(J=0) + P(U \le u \mid \varepsilon, J=1)P(J=1) \\ &= P(U \le u \mid u)\pi + P(U \le u \mid u+v)(1-\pi) \\ &= \pi F(u) + (1-\pi)F(u \mid u+v) \end{aligned}$$

Then the conditional distribution of u given ε can be derived by differentiating the above equation by u as

$$f(u \mid \varepsilon) = \pi f(u) + (1-\pi) f(u \mid u+v)$$

Therefore the Jondrow, et al. (1982) measure of firm specific inefficiency is given by

$$E(u \mid \varepsilon) = \pi E(u) + (1-\pi)E(u \mid u+v) \qquad (2.4)$$

In ALS (1977) the error components v_i and u_i are assumed to be distributed as normal and half-normal respectively i.e. $v \sim N(0, \sigma_v^2)$ and $u \sim N^+(0, \sigma_u^2)$ with $u_i \ge 0$.

Under these assumptions, when $J_i = 0$, $\varepsilon_i = u_i$ and the density function of ε_i is given by

$$f(\varepsilon_i) = \varphi_1(\varepsilon_i) = \frac{2}{\sqrt{2\pi}\sigma_u} \exp\left(-\frac{1}{2\sigma_u^2}\varepsilon_i^2\right) \qquad (2.5)$$

and similarly when $J_i = 1$, $\varepsilon_i = v_i + u_i$ the density function of ε_i is given by

$$f(\varepsilon_i) = \varphi_2(\varepsilon_i) = \frac{2}{\sqrt{2\pi}\sqrt{\sigma_u^2 + \sigma_v^2}} \Phi\left(\frac{\sigma_u}{\sigma_v} \frac{\varepsilon_i}{\sigma_u^2 + \sigma_v^2}\right) \exp\left(-\frac{1}{2} \frac{\varepsilon_i^2}{\sigma_u^2 + \sigma_v^2}\right) \qquad (2.6)$$

which is a skew-normal density (Azzalini, 1985).

Under these specific assumptions, the Jondrow, et al. (1982) measure of firm specific inefficiency is given by

$$E(u_i \mid \varepsilon_i) = \pi\sqrt{\frac{2}{\pi}}\sigma_u + (1-\pi)\left[\frac{\sigma\lambda}{1+\lambda^2}\left\{\frac{\phi(\varepsilon_i\lambda/\sigma)}{\Phi(-\varepsilon_i\lambda/\sigma)} - \frac{\varepsilon_i\lambda}{\sigma}\right\}\right] \qquad (2.7)$$

where

$$\lambda = \sigma_u / \sigma_v$$

$\sigma = \sqrt{\sigma_u^2 + \sigma_v^2}$

Now, the posterior estimate of probability of a firm being stochastic is given by

$$E(J_i \mid \varepsilon_i) = P(J_i \mid \varepsilon_i) = \frac{P(J_i = 1, \varepsilon_i)}{P(\varepsilon_i)}$$

$$= \frac{P(\varepsilon_i \mid J_i = 1) P(J_i = 1)}{P(\varepsilon_i)}$$

$$= \frac{\pi \varphi_2(\varepsilon_i)}{\pi \varphi_2(\varepsilon_i) + (1-\pi)\varphi_1(\varepsilon_i)} = \gamma_i(\varepsilon_i, \delta) \qquad (2.8)$$

This $\gamma_i(\varepsilon_i, \delta)$ can be termed as the responsibility of randomness of ith firm. The estimates in equation (2.8) can be estimated using the maximum likelihood estimator of the parameter vector δ, $\hat{\delta}$ and the estimated residuals $\hat{\varepsilon}_i$. These posterior estimates of inefficiency are firm specific and provide us the probability of a firm's randomness given the input vectors and a technology.

3 EM Estimation of the Model

The log-likelihood of the model is given by

$$\log L(\delta; y) = \sum_i \log[\pi \varphi_2(y_i, \delta) + (1-\pi)\varphi_1(y_i, \delta)]$$

The maximum likelihood estimators can be found by solving $\partial \log L(\delta; y)/\partial \delta = 0$. Trying to maximize $\log L(\delta; y)$ directly for the estimation of parameters is quite difficult since those equations are nonlinear and no analytic solutions can be found. So numerical procedure like iterative optimization methods often be used to get successive approximation of the solution. In that case The EM algorithm is applied to estimate the parameters of the model. The model is recasted into a missing data framework in order to implement the EM algorithm. The unobserved binary variable J can be treated as the missing data and the observable output *y* can be treated as observed data. Then the complete data is given by (*y*, *J*). The density function of J_i is given by $f(J_i; \pi) = \pi^{J_i}(1-\pi)^{1-J_i}$.

Now see the joint density of the observed data ε and unobserved data J:

$$f(\varepsilon, J) = f(J) f(\varepsilon \mid J) = \prod_{i=1}^{n} (1-\pi)\varphi_1(\varepsilon_i)^{1-J_i} \;\; \pi\varphi_2(\varepsilon_i)^{J_i} \qquad (3.1)$$

Then from (3.1) the joint density of the observed data y and unobserved data J can be obtained by the transformation $\varepsilon = y - x'\beta$ and is given by

$$f(y,J)=f(J)f(y\mid J)=\prod_{i=1}^{n}(1-\pi)\varphi_1(y_i)^{1-J_i}\ \pi\varphi_2(y_i)^{J_i}$$

The complete log-likelihood function of the complete data (y , J) is given by

$$\log L(\delta;y,J)=\sum_i(1-J_i)\log[(1-\pi)\varphi_1(y_i)]+\sum_i J_i\log[\pi\varphi_2(y_i)]$$

Since, the values of J_i is unknown, so we want to use the expected value of $J_i\mid y_i$ to substitute each J_i in above. We have

$$E(J_i\mid y_i)=P(J_i\mid y_i)=\frac{P(J_i=1,y_i)}{P(y_i)}$$

$$=\frac{P(y_i\mid J_i=1)P(J_i=1)}{P(y_i)}$$

$$=\frac{\pi\varphi_2(y_i)}{\pi\varphi_2(y_i)+(1-\pi)\varphi_1(y_i)}=\gamma_i(\delta)$$

The expected value of $J_i\mid y_i$ is called the responsibility of the model for ith observation, denoted as $\gamma_i(\delta)$:

$$\gamma_i(\delta)=E(J_i\mid y_i,\delta)$$

Then the *Q*-function, which is the expected value of complete log-likelihood with respect to the conditional distribution of *J* given *y*, is given by

$$Q(\delta)=E_{J|y}[\log L(\delta;y,J)]=\sum_i E_{J|y}(1-J_i)\log[(1-\pi)\varphi_1(y_i)]+\sum_i E_{J|y}(J_i)\log[\pi\varphi_2(y_i)]$$

$$=\sum_i(1-\gamma_i(\delta))\log[(1-\pi)\varphi_1(y_i)]+\sum_i\gamma_i(\delta)\log[\pi\varphi_2(y_i)]$$

$$=\log(1-\pi)\sum_i(1-\gamma_i(\delta))+\sum_i(1-\gamma_i(\delta))\log\varphi_1(y_i)+\log\pi\sum_i\gamma_i(\delta)+\sum_i\gamma_i(\delta)\log\varphi_2(y_i)$$

(3.2)

Under the assumptions of ALS (1977), the density functions of ε_i under the assumption of deterministic frontier model and stochastic frontier model are presented in (2.5) and (2.6) respectively.

Then, the log-likelihood function under the assumption of deterministic frontier model is given by

$$\log\varphi_1(y_i)=c-\frac{1}{2}\log\sigma_u^2-\frac{1}{2\sigma_u^2}(y-x_i'\beta)^2 \tag{3.3}$$

and the log-likelihood function under the assumption of stochastic frontier model is given by

$$\log\varphi_2(y_i;\delta) = c - \frac{1}{2}\log(\sigma_u^2+\sigma_v^2) + \log\Phi\left(\frac{\sigma_u}{\sigma_v}\frac{y - x_i'\beta}{\sqrt{\sigma_u^2+\sigma_v^2}}\right) - \frac{1}{2(\sigma_u^2+\sigma_v^2)}(y - x_i'\beta)^2 \quad (3.4)$$

Therefore, from (3.2) and using (3.3), (3.4) the *Q*-function is given by

$$Q(\delta) = \log(1-\pi)\sum_i(1-\gamma_i(\delta)) + \sum_i(1-\gamma_i(\delta))\left[c - \frac{1}{2}\log\sigma_u^2 - \frac{1}{2\sigma_u^2}(y - x_i'\beta)^2\right] + \log\pi\sum_i\gamma_i(\delta)$$

$$+\sum_i\gamma_i(\delta)\left[c - \frac{1}{2}\log(\sigma_u^2+\sigma_v^2) + \log\Phi\left(\frac{\sigma_u}{\sigma_v}\frac{y - x_i'\beta}{\sqrt{\sigma_u^2+\sigma_v^2}}\right) - \frac{1}{2(\sigma_u^2+\sigma_v^2)}(y - x_i'\beta)^2\right]$$

4 Monte Carlo Evidence

A Monte Carlo simulation experiment is carried out to study the finite sample properties of the estimators under the generalized SFM. The model that is used for the experiment is a cost frontier model with one output produced by one input which is given below:

$$y_i = \beta_0 + \beta_1 x_i + J_i v_i + u_i \quad (4.1)$$

where $J \sim Bern(\pi)$ $u \sim N^+(0,\sigma_u^2)$ and $v \sim (0,\sigma_v^2)$. Let $\eta = (\beta_0, \beta_1, \sigma_v, \sigma_u, \pi)'$. A random sample on *y* can be generated using a simulation procedure from the distribution of ε by the transformation $\varepsilon_i = y_i - \beta_0 - \beta_1 x_i$ for a given value of $\eta = \eta_0$. The density function of ε_i can be considered as a two component mixture model. The first component is normal distribution with right truncation and a random sample from this distribution can be generated by inverse method. The second component is the skew-normal distribution and a composition method of marginal-conditional can be used to generate a random sample from this distribution. Therefore, a random sample of size *n* on *y* can be generated using the above algorithm using a specified value of π. The composition method of marginal-conditional can be derived with a given η_0, as a single observation, say *i*th observation, of u, is first generated by the marginal distribution of u where u is distributed as half-normal variate. Given the value of u_i as obtained, *i*th observation on ε can be found by the conditional distribution of ε given u which is normal and then *i*th observation on *y* using the relation $y_i = \beta_{00} + \beta_{10} x_i + \varepsilon_i$. The above two steps are repeated *n* times to generate a random sample of size *n* on *y*.

Fixing the parameter vector at $\eta_0 = (1, 0.1, 0.1, 0.5, 0.5)$, the Monte Carlo simulation carries out for each values of η_0. Sample size of *n* is generated using the above algorithm and it is repeated 100 times. The cross sectional sample of size n = 100, 200, 300, 400, 500 are considered for the study.

Under this Monte Carlo experiment, the parameter vector η, is estimated using the EM algorithm, describe above, to compare the small sample behavior of the parameter vector η. The result is reported in table 1. The analysis provides the mean, SD and RMSE of the estimators.

The complete data expected log-likelihood equations were solved by the BHHH algorithm fixing the tolerance at 0.001. For each replication the EM method converged at reasonably fast rate taking about three minutes in a PENTIUM-4 processor. The mean, SE and the root mean square error (RMSE) of the estimates for each experiment, obtained from their generated sampling distributions, are reported in Table-1. It is seen that for lower values of n the performance of the EM estimates as measured by standard error and the RMSE is poor and the number of iterations for convergence moderately high. However, the performance improves, as n is increased from 150 to 400 when estimates stabilize. Interestingly the number of iterations required for convergence of the EM algorithm decreases by almost one-third as n is increased from 150 to 500. At n=500, the small sample error of the EM estimates are between 6 to 10 per cent. Given the fact that the single equation estimates of the SFM suffer from simultaneous equation bias, the small sample performances of EM estimates of our model is reasonably good.

5 Empirical Analysis

We have used the US electricity utility industry data (Greene 1990, Table-3) to illustrate the method. The model to be fit is a cost function rather than a production function, given by

$$\ln(\cos t/P_f) = \beta_0 + \beta_1 \ln(Q) + \beta_2 \ln^2(Q) + \beta_3 \ln(P_l/P_f) + \beta_4 \ln(P_k/P_f) + u + J\,v$$

where Q is the output, a function of labor (l), capital (k), fuel (f), and P_l, P_k and P_f are their respective factor prices; and $u \sim N^+(0,\sigma_u^2)$, $v \sim N(0,\sigma_v^2)$; u truncated at zero. It may be noted that the change in the expression for ε requires that $\varepsilon + u$ should now be replaced by $\varepsilon - u$ in the above derivations.

We have estimated two models. The SFM with independent error components is estimated using maximum likelihood method with BHHH algorithm. The other firm-specific frontier model discussed above with independent error components is estimated using the illustrated EM algorithm. The estimators with their asymptotic variance–covariance matrix are given in the Table-2. It can be seen from the Table-2 that the regression coefficients of the uncorrelated components SFM have significantly higher asymptotic variance than the firm-specific frontier model which is estimated by EM algorithm. Also, the estimated value of the parameter π is 0.817 which is statistically significant and suggests that almost 81% firms prefers stochastic frontier model whereas 19% firms prefers deterministic frontier model. Fig. 1 presents measures of firm-specific cost inefficiency in the firm-specific frontier model. Fig. 2 presents the estimated posterior probability of randomness.

6 Conclusions

In this paper we have proposed a firm-specific frontier model where each firm is open to choose between stochastic and deterministic frontier model. This generalized model is estimated using EM estimation method. It is seen that the EM method does not face the problems like divergence, instability and low. The results of Monte Carlo simulation experiments show fairly good small sample properties of the EM estimates. Application of the model to the cross-section data of 123 US electricity firms shows stochastic frontier model is preferable to almost 81% firms and deterministic frontier model is preferable to remaining 19% firms.

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Table 1: Results of the Monte Carlo simulation experiments

$$\eta = (\beta_0, \beta_1, \sigma_v, \sigma_u, \pi) = (1, 0.1, 0.1, 0.1, 0.5)$$

n	EM Estimate		
	Mean	SD	RMSE
100	1.224	0.134	0.202
	0.079	0.141	0.135
	0.136	0.135	0.143
	0.146	0.218	0.227
	0.594	0.245	0.291
200	1.158	0.121	0.159
	0.087	0.127	0.133
	0.138	0.118	0.121
	0.135	0.183	0.224
	0.557	0.228	0.247
300	1.113	0.096	0.129
	0.091	0.098	0.111
	0.121	0.097	0.112
	0.129	0.123	0.149
	0.546	0.185	0.213
400	1.109	0.087	0.114
	0.093	0.082	0.092
	0.118	0.092	0.104
	0.116	0.111	0.128
	0.532	0.145	0.167
500	1.110	0.082	0.109
	0.095	0.076	0.087
	0.107	0.089	0.102
	0.112	0.092	0.114
	0.528	0.118	0.120

Table 2: Estimates of the parameters

Parameter of the model	Estimate of the parameter of the model	
	Generalized Model	SF Model
β_0	-7.349 (0.296)	-7.390 (0.341)
β_1	0.401 (0.016)	0.405 (0.018)
β_2	0.029 (0.057)	0.031 (0.065)
β_3	0.240 (0.051)	0.244 (0.048)
β_4	0.058 (0.083)	0.061 (0.063)
σ_u	0.149 (0.022)	0.151 (0.027)
σ_v	0.108 (0.026)	0.111 (0.018)
π	0.817 (0.136)	0

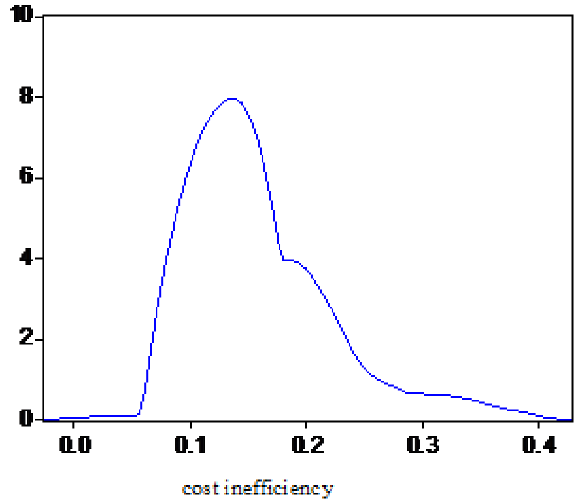

Fig 1: Kernel density of estimated cost inefficiency

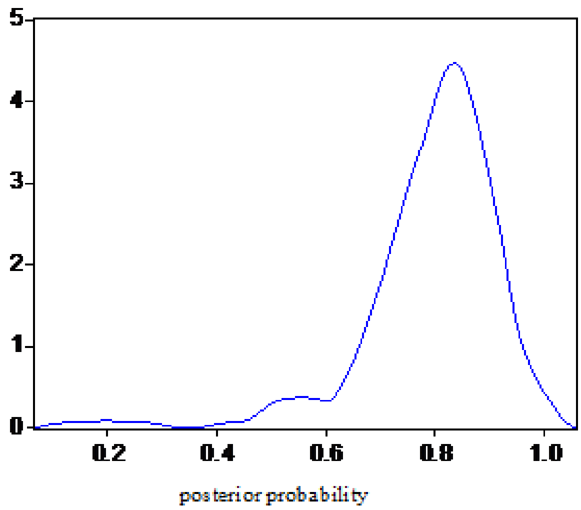

Fig 2: Kernel density of estimated posterior probability

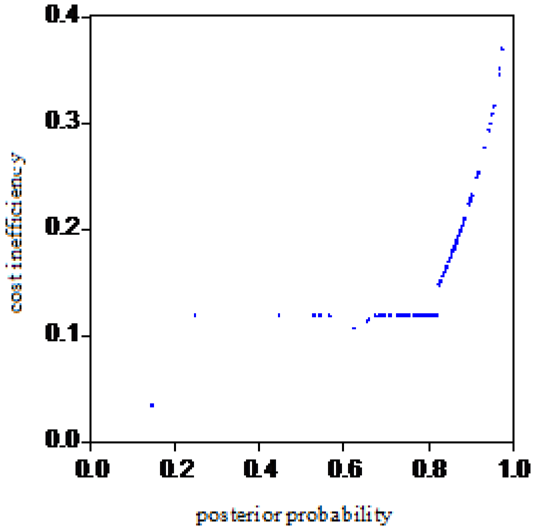

Fig 3: Scatter plot of estimated cost inefficiency and posterior probability