

Salt Movement in Unsaturated Frozen Soil: Principles and Field Observations¹J. W. CARY, R. I. PAPENDICK, AND G. S. CAMPBELL²

ABSTRACT

Soil temperatures, electrical conductivities, and water redistribution were measured at four field sites during a 30-day period in which the soil was never completely thawed. The soil on each site was a silt loam with varying aspects and vegetation covers. Both upward and downward flow of water and solutes were observed. Assuming that liquid water flow in frozen soil is analogous to unsaturated liquid flow in unfrozen soil, led to a simple equation that in general agreed with the field observations. The equation requires knowledge of the soil temperatures, the solute concentrations, and two constants that characterize the soil's water release curve and saturated hydraulic conductivity.

Infiltration and frost heaving are discussed with respect to this simple theory. Water in frozen soil flows from high to low temperatures and from high to low salt concentrations. Consequently, solutes in even very low salt soils are important in decreasing frost heave and increasing infiltration. The liquid flow is so closely coupled with temperature that heat flow must be considered simultaneously in any comprehensive analysis. This coupling, as expressed in the simple liquid flow equation, accounts for the effect of soil water content on frost heave rates and the effects of temperature on maximum heaving pressures.

Additional Index Words: infiltration, heat flow, frost heaving.

Cary, J. W., R. I. Papendick, and G. S. Campbell. 1979. Water and salt movement in unsaturated frozen soil: principles and field observations. *Soil Sci. Soc. Am. J.* 43:3-8.

FROST HEAVING and water percolation in frozen soil are two related phenomena of considerable practical interest. It has been known for many years that water movement occurs in soil when ice is present. There are a number of phenomena involved in this movement that have been studied by Penner (1967), Sutherland and Gaskin (1973), Guymon and Luthin (1974), Miller (1965), Loch and Miller (1975), and Miller et al. (1975). In light of these and other related studies, Cary and Mayland (1972), and Harlan (1973), developed equations describing the flow of water in frozen soil as analogous to the flow of water in unsaturated and unfrozen soil. That is, as a first approximation, they envisioned the films of liquid water in soil with ice present responding to potential gradients in the same way as films of liquid water in unfrozen soil. In support of this hypothesis, Williams and Burt (1974) made direct measurements of the hy-

draulic conductivity in frozen soil and found that it decreased rapidly as the temperature fell, just as does the more familiar relation between water content and conductivity in unfrozen soil.

The liquid water content in soil with ice present depends on the concentration of soluble salts, the temperature, and the shape of the soil water release curve (Cary and Mayland, 1972). Because the liquid water content is such a sensitive function of temperature, the analysis of water flow in frozen soil must be coupled with analysis of heat flow. Several mathematical models have been developed to account for this first-order coupling (Harland, 1973; Guymon and Luthin, 1974; and Groenevelt and Kay, 1974).

Although numerous laboratory studies have been conducted on water flow in frozen soil (Loch and Miller, 1975) and some on simultaneous salt and water flow in frozen soil (Cary and Mayland, 1972), there is relatively little quantitative data from the field on this subject. Campbell et al. (1970) observed the simultaneous rise of nitrate and water toward the soil surface in the field in the winter. Sartz (1969) reported observations of water content changes and frost heaving from an extensive field study, and cited some previously reported results from frozen field conditions. Lacking, however are field data giving simultaneous measurements of temperature and water flow with which the simplified theory may be tested. The object of the study reported here was to investigate the implications of the simplified theory with respect to some field observations of soil temperatures and the simultaneous water and solute redistribution in soil with ice present.

THEORY

Recent advances in our understanding of liquid water contents in frozen soil and in our ability to express the hydraulic conductivity as a function of the soil water release curve leads to relatively simple equations for describing water flow in frozen soil. Campbell (1974) showed that when the soil water release curve is given as

$$\tau = \tau_0 (\theta/\theta_s)^{-b} \quad [1]$$

the hydraulic conductivity may be expressed as

$$k = k_s (\theta/\theta_s)^{a+b} \quad [2]$$

where

τ is the soil water matric potential in cm of H₂O;
 τ_0 is the air entry value, cm H₂O;
 k_s is the saturated hydraulic conductivity, cm/day;
 θ_s is the saturated water content cm³/cm³;
 θ is unsaturated liquid water content, cm³/cm³; and
 b is a constant.

When ice is present at atmospheric pressure, the water potential is closely approximated by the vapor pressure of pure ice (Cary and Mayland, Eq. [2], 1972) so that

$$1.2 \times 10^4 T = \tau + \phi \quad [3]$$

¹Contribution from the Western Region, Science and Education Administration, Federal Research, USDA, in cooperation with the College of Agriculture Research Center, Washington State Univ., Pullman. Scientific Paper No. 5026 of the latter. Received 21 Feb. 1978. Approved 12 Sept. 1978.

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where ϕ is the osmotic potential of the soil solution in cm of H_2O , and 1.2×10^4 is a factor that approximately converts the temperature of ice, T in $^{\circ}C$, to total water potential in cm of water. Combining and rearranging Eq. [1], [2], and [3] gives the unsaturated soil water flow equation written in terms of variables that are important in the frozen system,

$$J_{H_2O} = -k \frac{d(\tau+z)}{dz} = -k_s \left[\frac{1.2 \times 10^4 T - \phi}{\tau_s} \right]^{-2[1+(1/b)]} \left[1.2 \times 10^4 \frac{dT}{dz} - \frac{d\phi}{dz} + 1 \right] \quad [4]$$

where J_{H_2O} is flow in the liquid phase, and z is soil depth. It is assumed that the water content, including ice, is less than saturation, and that water transport due to ion exclusion, liquid ice interface phenomenon, vapor diffusion, and plastic flow of ice are all negligible.

The value for ϕ in Eq. [4] can be obtained from the temperature and ϕ_s which is the osmotic potential of the soil solution at saturation. When solubility limits are not exceeded, $\phi \sim \phi_s (\theta_s/\theta)$ which, combined with Eq. [1] and [3], gives

$$\phi = \phi_s \left[\frac{1.2 \times 10^4 T - \phi}{\tau_s} \right]^{1/b} \quad [5]$$

Eq. [5] converges rapidly to ϕ on iteration with ϕ_s as a starting point when T is less than the freezing point of the soil solution.

Note that the osmotic pressure effect included in Eq. [4] is not the same as that encountered in clays and biological systems with selective semipermeable membranes. In these systems, water flows from regions of low osmotic pressures across the membranes or clay lens to high osmotic pressures. The direction of flow is opposite when ice is present in soil. Solutes increase the amount of liquid phase water, and since the water flows toward the thinner films, it flows away from regions of high osmotic pressure. Ion exclusion effects on water flow in frozen soil were studied by Cary and Mayland (1972) and found to be small compared to other transport mechanisms.

EXPERIMENTAL PROCEDURE

Four sites located on the Palouse Conservation Field Station near Pullman, Washington, were studied. The soil was Palouse silt loam, or a closely related series (Pachic ultic Haploxerolls) formed from loess with deep permeable profiles. In the late fall before freezing, the soil water content was near field capacity in the surface 20 to 30 cm, but contained only 13% by volume water below to a depth of 1.5 m. Two of the sites, one bare and one covered with wheat stubble, were on a south exposure with a 11.5° slope. The third site, with an east aspect and an 8° slope, was covered with short grass. The fourth site, also covered with short grass, was on a north 25.5° slope.

Soil temperatures on the north, east, and straw-covered south slopes were measured daily near midday with thermocouples placed at depths of 1, 8, and 32 cm. Three thermocouples were placed at each depth to give a spatial average of temperature. Soil temperatures on the bare south slope were automatically recorded every four hours at depths of 1, 2, 4, 8, 16, and 32 cm. Additional information on depth of freezing, soil heat flux, and microclimate conditions are reported elsewhere. (Cary et al., 1978).

The soil was sampled at each site several times during the study for gravimetric water content and electrical conductivity. The samples were taken with a small bucket-type orchard auger or with an open-sided stainless steel probe driven into the soil with a heavy hammer. Each sample was formed from a composite of several cores (3 to 20) located within the proximity of 1 meter. Sample depth increments were as shown by the data points in Fig. 5. Electrical conductivity was measured on one-to-one soil-to-water by weight extracts from each sample after drying at $60^{\circ}C$ and grinding.

The soil water release curve and the hydraulic conductivity in the wet range were known from previous laboratory measurements. The following constants needed in Eq. [4] were chosen to best fit these measurements and used for all four study sites: $k_s = 30$ cm H_2O/day ; $\tau_s = -14$ cm H_2O ; $\phi_s = -100$ cm H_2O at saturation; $b = 5$; and $\theta_s = 0.55$.

RESULTS AND DISCUSSION

The soil began to freeze and thaw on a daily basis in early December and by the end of the month some ice remained even on the warmest days. The first half of January had cold, overcast weather that drove the frost layer nearly 70 cm deep on the north slope. This was followed by several days of warm weather and soil thawing from above and below until the frozen layer thickness was reduced to 15 to 25 cm. Another cold period followed, and the frost moved down again until the first few days of February (Cary et al., 1978). The terms "frost" and "frozen" and used here only to indicate the presence of ice.

One of the changes in soil water content on the east slope site is shown in Fig. 1. The water loss below 20 cm is about equal to that gained between the 5- and 20-cm depths. The corresponding change in conductivity of the 1:1 extract suggests this water content change resulted from upward flow in the liquid phase in the frozen soil. That is, solute concentration per unit dry mass of soil increases or decreases as the soil solution moves into or out of that same mass element. The gain in water content above 5 cm was due largely to 0.8 cm of precipitation that occurred during the period and infiltrated as a few cm of the soil surface thawed during the 3rd week of January, as shown by the soil temperatures in Fig. 2. These temperature measurements from the three soil depths were used to find the constants in $T = a_0z^2 + a_1z + a_2$ for each individual day. This function was then used in Eq. [4] to find the fluxes across the 5- and 20-cm depths as shown in Fig. 2. Because ϕ was not known as a function of time and depth, $d\phi/dz$ was taken as zero, though such was obviously not true.

The damping depth of frozen soil is small because of the heat of fusion effects on the apparent heat capacity (Fuchs et al., 1978, and Penner, 1970). Consequently, the diurnal temperature wave did not penetrate much below 15 cm, so calculations of soil water flow made below this depth during midday are also estimates of the average daily flow rates. The cumulative upward flow given by the 20-cm curve in Fig. 2 is about 0.3 cm of water which approximately agrees with the 0.5 cm gain in water content shown in Fig. 1 between the 5- and 20-cm depths. On the other hand, the flow rate shown in Fig. 2 for the 5-cm depth is

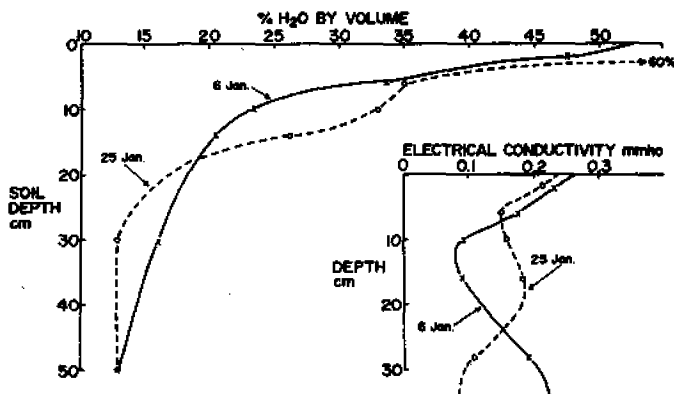


Fig. 1—Water and salt redistribution in frozen soil on grass-covered east slope. The conductivity is for the 1:1 soil water extract.

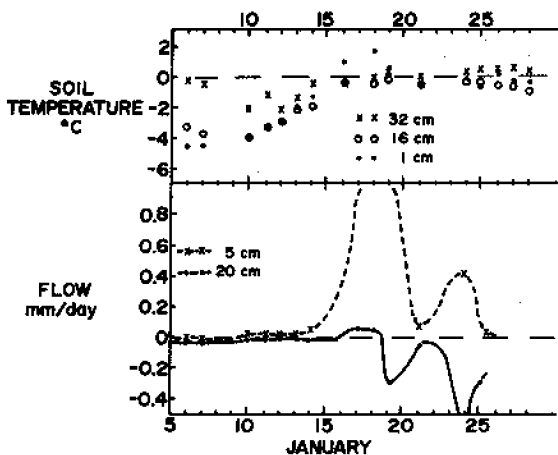


Fig. 2—January soil temperatures for three depths on a grass covered east slope and predicted (Eq. [4]) mid-day water flow for two depths. Downward flow is positive.

only representative of conditions during the warm part of the day because the thermal gradient at that depth reversed most nights, and so would have favored upward water flow. At any rate, Eq. [4] suggests there was significant percolation past the 5-cm depth during the warm part of days when soil temperatures rose to near 0°C, even though ice crystals were still present. The freezing point depression of the soil was <0.05°C (Eq. [3]).

In general, some infiltration into frozen soil may be expected when water is available at the surface and its temperature is warmer than that of the soil below. Table 1 shows the rates of infiltration predicted by Eq. [4] using the constants obtained for this soil, assuming a linear temperature gradient between the surface and 15 cm. Column 3 shows how much the infiltration would be increased if there were a 50 cm of H₂O/cm gradient due to osmotic potential. The infiltration rates in the fourth column illustrate the beneficial effects from a saturated gypsum system as predicted by Eq. [4]. Column 5 shows the decrease in infiltration if there were no osmotic effects. For accurate calculations temperatures must be known to 0.01°C in the region between 0 and -1°C. If the osmotic effects on hydraulic conductivity had not been included in the 20 cm infiltration curve in Fig. 2, the predicted upward flow would have been nearly an

Table 1—Calculated infiltration rates as affected by hypothetical osmotic pressures at the 5-cm depth and temperatures at the 15-cm depth. The surface boundary condition is zero °C and saturation with liquid water.

Temperature at 15 cm °C	$\phi_0 \ddagger = 100 \text{ cm H}_2\text{O}$	$\phi_0 \ddagger = 100 \text{ cm H}_2\text{O}$	$\phi = 500 \text{ cm H}_2\text{O}$	$\phi = 0$
	$\frac{d\phi \ddagger}{dz} = 0$	$\frac{d\phi}{dz} = 50$	(Sat. gypsum)	$\frac{d\phi}{dz} = 0$
Infiltration cm/day				
-0.02	30	30	30	7.8
-0.05	26	57	30	2.1
-0.1	2.9	4.9	30	0.78
-0.3	0.28	0.34	0.61	0.17
-0.5	0.11	0.13	0.16	0.08
-1.0	0.04	0.04	0.04	0.03

† ϕ is the osmotic potential of the soil solution of unsaturated soil.
 ‡ ϕ_0 is the osmotic potential of the soil solution of saturated soil.
 § z is soil depth.

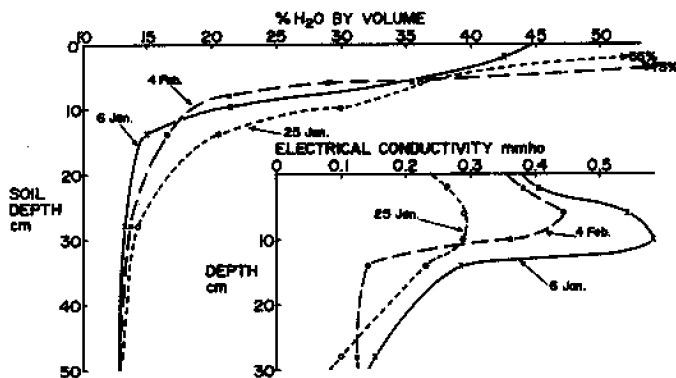


Fig. 3—Water and salt redistribution in frozen soil on north slope. Conductivity is for the 1:1 soil water extract.

order-of-magnitude smaller than observed. Osmotic effects are obviously significant even in this very low salt soil system.

The change in soil water content on the north slope site is shown in Fig. 3, along with changes in the electrical conductivity. The increase in soil water between 6 and 25 January was due to infiltration into frozen soil. During the period of 25 January to 4 February, the net soil water movement was upward past the 5-cm depth and raised the water content at 2 cm to 75% by volume. This interpretation is again verified by the 1:1 soil water extract conductivity curves. There was no significant precipitation during this period. On 4 February electrical conductivity was maximum at the 5-cm depth indicating that frost heave was being initiated in this area as the water content approached saturation. Upward flow of the liquid phase stopped here as the ice crystals forced the soil particles apart and formed a continuous ice layer. The layer then became massive, growing thicker, and salt was concentrated on its lower side as more water froze. Later thawing at the surface created an example of the "ice sandwich" phenomenon (Miller et al., 1975).

Midday flow at 5 cm calculated for the north slope showed some downward percolation during the warm part of most days (Fig. 4). The rates were less than those on the east slopes because the north site was

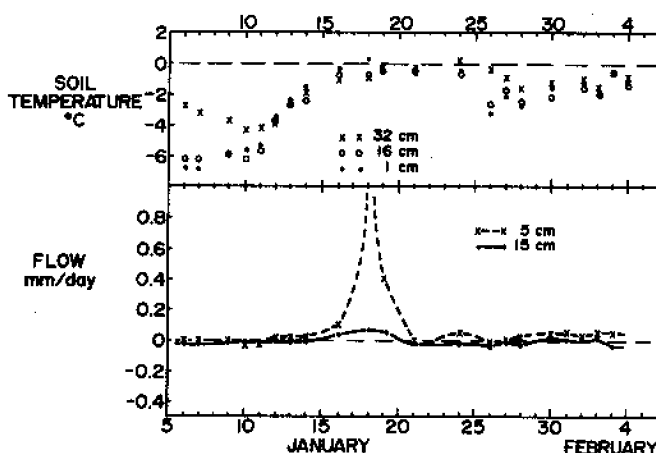


Fig. 4—January and early February soil temperatures for three depths on a north slope and predicted (Eq. [4]) mid-day water flow for two depths. Downward flow is positive.

always shaded from the sun during the study period and remained colder. It is interesting that the soil water redistribution was so different on these two sites, presumably due only to the difference in surface temperatures created by the north and east aspects.

Calculated water flow past the 15-cm depth shown in Fig. 4 indicated a small net downward percolation from 6 to 25 January and a small net upward flux between 25 January and 4 February. These flow directions agree with the water content changes observed in Fig. 3, but the quantities are nearly an order-of-magnitude too small. The discrepancy could be the result of inadequate accuracy in temperature measurements, significant penetration of the diurnal thermal wave below the 15-cm depth, and neglect of the osmotic pressure gradient term in Eq. [4]. Perhaps, though, the greatest uncertainty in the application of this simple approach is the assumption that the frozen and unfrozen hydraulic conductivities are the same function of liquid water content. The formation of ice crystals in the unsaturated soil may cause significant changes in the pore size distribution.

Under some conditions, water flow to the freezing front may be quite rapid (Fig. 5). Detailed water content measurements were made on 23 and 24 February on the south-facing sites. The bare site was dry on the surface, while the one with stubble was still moist. At 1600 hours on the 23rd, the soil temperatures were all above freezing and slightly warmer on the bare site. The following night, freezing water moved rapidly toward the surface of the straw-covered site, even though the soil temperatures did not become as cool as those on the bare soil (Fig. 5). There was almost no redistribution of water in the bare soil though the small differences shown were probably real since each point is composite of 15 core samples. The increase in water content was evidently due to thermally driven upward flow of water in the liquid and vapor phases (Cary, 1966), but not specifically associated with the formation of ice.

The rapid increase of water at the surface of the wetter soil under the straw stubble was caused by the formation of ice and is typical of conditions that lead to frost heave. The flow mechanism obviously depends on soil water content and the simplified theory may be used to explain it in some detail. Consider the penetration of ice into a soil pore. The temperature at the tip of the ice crystal will be very near 0°C , while a short distance behind the tip the temperature will be slightly less. This temperature difference will

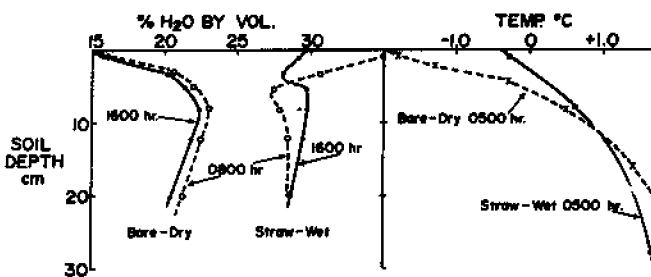


Fig. 5—Overnight water redistribution on bare and straw-covered plots on a south slope. Points represent averages of several samples. Early morning temperature distribution is also shown.

cause a liquid water content gradient adjacent to the ice. The resulting tension gradient will force water toward and along the sides of the growing ice tip. The water tension gradient along the sides of the ice crystal will be continuous and should be approximately constant for a finite distance ahead of the tip of the ice crystal as it penetrates into the unsaturated pore. If a linear temperature gradient is assumed between the tip of the ice crystal and some short distance behind the tip to a point where the temperature is known, the flow of water to the freezing front is given by, $-k (d\tau/dz)$, where Eq. [1] and [2] may be used to find k , and Eq. [3] to find $d\tau/dz$. The result is plotted in Fig. 6 for three hypothetical ice temperature gradients just behind the freezing front for various soil water contents.

The curves in Fig. 6 illustrate the principles underlying the redistribution of water and differences in temperatures shown in Fig. 5. On the bare site, water flow did not respond to ice forming at the surface because the water content was too low. If the temperature 1 cm behind the ice front had fallen to less than -0.1°C , significant upward flow would have developed; but such a condition did not occur because the freezing front moved down into the soil before the temperature fell that low near the growing tips of the ice crystals. In the wetter soil, as the temperatures fell to -0.01°C below the freezing point, significant amounts of water flowed up to the ice front and released latent heat upon the freezing. This latent heat opposed the night chilling of the soil surface, and so the soil temperature did not fall rapidly enough to cause the deeper penetration of ice that occurred in the bare soil. Remember that the temperature and the temperature gradient are essentially the same, 1 cm behind the freezing front.

The change in water content on the wetter straw-covered site was an example of conditions that lead to frost heaving. The maximum heaving pressure in bars in such a low-salt system is approximately 12 times the absolute value of the temperature in $^{\circ}\text{C}$ measured at the bottom of the continuous ice phase or lens where the soil water content, including ice, de-

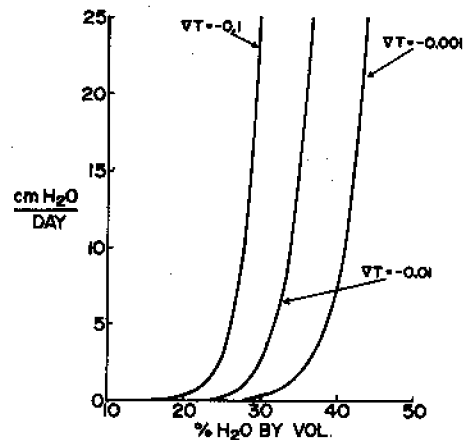


Fig. 6—Calculated water fluxes toward a freezing front as affected by soil water content and the temperature gradient just behind the freezing front. Note that the temperatures 1 cm behind the freezing fronts must also be very near to -0.1 , -0.01 , and -0.001°C .

creases to saturation or slightly below (Hoekstra, 1969). This concept, in light of Fig. 6, shows why the heaving pressure passes through a maximum as the temperature gradient increases as reported by Loch and Miller (1975). Larger temperature gradients create lower temperatures which favor greater heaving pressures, but as the thermal gradient increases, it will reach a point, depending on the soil's salt content and hydraulic conductivity, where the heat flux becomes unstable. At this point the freezing front will override the upward water flow and move deeper into the soil. The heaving pressure will then develop at a greater depth where the temperature is warmer and the soil has reached saturation, producing a new ice lens. Consequently, the heaving pressure falls to that specified by the temperature at the deeper lens as forces around the previous heave location are dissipated by the nonrigid nature of the soil matrix. When an ice lens is confined against expansion or occurs deep enough so that it is subjected to significant overburden, the freezing point will be depressed. This will cause a shift in the temperature profile that must be accounted for in any detailed analysis.

PRACTICAL ASPECTS

Movement of water in frozen soils is significantly affected by soluble salts (Cary and Mayland, 1972), even when the soil solution is very dilute (Table 1). The simple theory suggests that increasing concentrations of solutes oppose frost heave but increase infiltration in frozen soil. For example, a solid phase of gypsum at the surface should significantly increase infiltration into frozen soil under a slowly melting snowpack, compared to the same soil without such a buffering component to maintain soluble salt levels during water flow (Table 1). On the other hand, very soluble salts, such as nitrate and chlorides, would be more effective than gypsum in reducing frost heave. They would reach increasingly high concentrations as soil water freezes out at the bottom of the ice lens. This would increase the liquid water content and so reduce the gradient causing the upward flow of water needed for temperature stability and continued heaving.

It is well known that good drainage or any other management that reduces soil water content will lessen soil frost heaving. Unfortunately, the restricted drainage often results from the frozen soil itself. When soil starts to thaw, melting ice near the surface may cause a saturated zone to develop just above the depth of thaw. Since land is seldom level, this water will seep downslope along the top of the frozen zone. Such seepage may carry away soil nutrients that have previously moved to the surface of the soil during earlier freezing conditions as shown by the 1:1 soil extract conductivity curves in Fig. 1 and 3. At any rate, the seepage during a thaw provides a steady source of water to the soil on the lower parts of the slopes and increases the danger of heaving when the soil starts to freeze downward from the surface during the onset of colder weather. If one wishes to reduce heaving downslope during alternate freeze-thaw weather, a protective vertical band of soil might be formed by packing to reduce the saturated hydraulic conductivity.

The packed band must go as deep as the anticipated depth of frost so in effect it forms a dam against the subsurface flow of water. When the seep water arrives at this boundary of low conductivity, most of it will be forced up to the surface where it can be carried away in an open drain. All these aspects were observed in the field on a north slope during the 3rd week of January when the mild thaw occurred. Some frost heaving was observed where the water seeped out at the bottom of the slope, but where the water was forced to the surface by the packed wheel tracks of a summer access road, the soil downslope did not heave.

If Eq. [4] does, in fact, account for the most important phenomena that cause water flow in unsaturated frozen soil, there are other management practices that may influence infiltration and/or frost heaving in frozen soil. It may be worthwhile to construct a mathematical model of the frozen soil water system so the interacting effects of various management schemes can be studied. It has become apparent that the water movement is so closely coupled to temperature that heat flux must be modeled simultaneously. It now appears that soluble salts are also coupled as a first order effect, even at very low concentrations. If any model is to have extensive practical application to real field situations, it must be based on the simultaneous solutions of a relation like Eq. [4] and those describing the heat flow and salt flow. An appropriate equation for salt flow has been given by Cary and Mayland (1972), and the heat flow equation by Fuchs et al. (1978).

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