An Asymptotically Non-Scale Endogenous Growth Model

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Abstract

This paper presents an endogenous growth model in which the economy grows without either scale effects or population growth. The key mechanisms are an increase in uncompensated knowledge spillovers from an increased number of firms and substitution between investments in capital and technology. The model indicates that an increase in population does not make investments in technology more attractive than those in capital because of increased uncompensated knowledge spillovers as a result of both Marshall-Arrow-Romer and Jacobs externalities. Scale effects are generated by the non-rivalness of technology, but they are cancelled out by increases in the amount of uncompensated knowledge spillovers that are also generated by the non-rivalness of technology.

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1 INTRODUCTION

Scale effects have been a central issue in the study of endogenous growth. Early endogenous growth models (e.g. Romer, 1986, 1987; Lucas, 1988) commonly included scale effects. However, the existence of scale effects in present-day economies is not supported by empirical evidence (Jones, 1995a). The source of scale effects lies in the assumption of a linear relation between capital (K_t) and technology (A_t). Given a Harrod-neutral production function such that $y_t = A_t^{\alpha} k_t^{1-\alpha}$, the familiar optimal growth path is

$$\frac{\dot{c}_t}{c_t} = \varepsilon^{-1}\left[(1-\alpha)\left(\frac{A_t}{k_t}\right)^{\alpha} - n_t - \theta\right] ,$$

where $y_t = \frac{Y_t}{L_t}$, $k_t = \frac{K_t}{L_t}$, and $c_t = \frac{C_t}{L_t}$; Y_t (≥ 0) is output, K_t (≥ 0) is capital input, L_t (≥ 0) is labor input, A_t is technology, C_t (≥ 0) is consumption, and $n_t\left(= \frac{\dot{L}_t}{L_t}\right)$ is the population growth rate in period t. In addition, θ is the rate of time preference, ε is the coefficient of relative risk aversion, and α is a constant. Hence, if $\frac{A_t}{k_t}\left(= \frac{A_t L_t}{K_t}\right)$ and n_t are both constant, the growth rate of consumption is constant; that is, the economy can proceed on a balanced growth path.

To make $\frac{A_t}{k_t}$ constant, it is necessary that

$$\frac{\dot{K}_t}{K_t} - \frac{\dot{L}_t}{L_t} = \varphi_1 \frac{\dot{A}_t}{A_t} ,$$

where φ_1 is a constant. The simplest solution to construct a model that satisfies $\frac{\dot{K}_t}{K_t} - \frac{\dot{L}_t}{L_t} = \varphi_1 \frac{\dot{A}_t}{A_t}$ is to assume that there is a linear relation between K_t and A_t and that $\frac{\dot{L}_t}{L_t} = 0$. Early endogenous growth models such as the familiar "*AK*" model adopted this strategy (e.g., Romer, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992).[1] Assuming a linear relation between A_t and $K_t\left(= k_t L_t\right)$ means that

$$\frac{A_t}{k_t} = \frac{\varphi_2 K_t}{k_t} = \varphi_2 L_t ,$$

where φ_2 is a constant. Hence, L_t plays an important role for growth because, as L_t increases, $\frac{\dot{c}_t}{c_t}$ also increases. This relationship is known as scale effects.

Jones (1995b) adopts a completely different strategy (see also Kortum, 1997; Segerstrom, 1998; Eicher and Turnovsky, 1999), which focuses on the relation between L_t and

[1] Early human-capital-based endogenous growth models also belong to this category of models.

A_t instead of that between K_t and A_t. A linear relation between $\frac{\dot{A}_t}{A_t}$ and $\frac{\dot{L}_t}{L_t}$ is assumed, such that $\frac{\dot{A}_t}{A_t} = \varphi_3 \frac{\dot{L}_t}{L_t}$, where φ_3 is a constant. Only the case in which

$$\frac{\dot{K}_t}{K_t} = (1 + \varphi_1 \varphi_3) \frac{\dot{L}_t}{L_t} = \frac{\dot{Y}_t}{Y_t} \tag{1}$$

is selected to be relevant because only this case simultaneously satisfies the relation $\frac{\dot{K}_t}{K_t} - \frac{\dot{L}_t}{L_t} = \varphi_1 \frac{\dot{A}_t}{A_t}$ and achieves a balanced growth path. This model can eliminate scale effects because there is no linear relation between K_t and A_t. Instead, the population growth rate $\frac{\dot{L}_t}{L_t}$ plays a crucial role, as equation (1) clearly exhibits. In this sense, Jones's (1995b) model still does not appear to be successful as a model of endogenous growth.

To eliminate the influence of population growth, Young (1998), Peretto (1998), Aghion and Howitt (1998), and Dinopoulos and Thompson (1998) propose a third approach. They assume a relation between $\frac{\dot{A}_t}{A_t}$ and L_t such that $\frac{\dot{A}_t}{A_t} = \varphi_4 L_t^{1-\varphi_5}$, where φ_4 and φ_5 are constants. Hence,

$$\frac{\dot{K}_t}{K_t} = \frac{\dot{L}_t}{L_t} + \varphi_1 \varphi_4 L_t^{1-\varphi_5} = \frac{\dot{Y}_t}{Y_t}$$

if the relation $\frac{\dot{K}_t}{K_t} - \frac{\dot{L}_t}{L_t} = \varphi_1 \frac{\dot{A}_t}{A_t}$ holds and the economy is on a balanced growth path. Therefore, if $\varphi_5 = 1$, the economy grows at the constant rate $\varphi_1\varphi_4$ even if $\frac{\dot{L}_t}{L_t} = 0$; that is, the influence of population growth and scale effects can both be eliminated. However, Jones (1999) shows that this model crucially depends on a very special assumption, that $\varphi_5 = 1$.

Peretto and Smulders (2002) take a fourth approach. They assume that A_tL_t and K_t are positively linked instead of A_t and K_t, and

$$\lim_{L_t \to \infty} \frac{A_t}{k_t} = \lim_{L_t \to \infty} \frac{A_t L_t}{K_t} = \varphi_6 \quad ,$$

where φ_6 is a constant. Hence, the scale effects asymptotically vanish. In addition, population growth is unnecessary for economic growth unlike in the non-scale model developed by Jones (1995b).

The model developed in this paper superficially has this same feature, but the mechanism through which scale effects vanish is fundamentally different. The key assumption in Peretto and Smulders (2002) is that uncompensated knowledge spillovers diminish as the number of firms (and thus the population) increases. However, this assumption is problematic because the concepts of Marshall-Arrow-Romer (MAR) externalities (Marshall, 1890; Arrow, 1962; Romer, 1986) and Jacobs externalities (Jacobs, 1969) both predict that, if the number of firms increases, uncompensated knowledge spillovers will also increase. Hence, the key assumption of Peretto and Smulders (2002) contradicts the theory of knowledge spillover. This problem arises primarily because they neglect Jacobs externalities and focus only on the negative side of MAR externalities; that is, as the number of sectors increases, knowledge spillovers will work less effectively. Many empirical studies support the existence of Jacobs externalities (e.g., Glaeser et al., 1992; Chen, 2002; Stel and Nieuwenhuijsen, 2002), and neglecting them will heavily bias the structure of model.

The model in this paper, in contrast to that of Peretto and Smulders (2002), is consistent with knowledge spillover theory because uncompensated knowledge spillovers are assumed to increase when the number of firms increases. This opposite interpretation of the effect of knowledge spillovers could potentially make scale effects much worse, but it does not because of substitution between investments in capital and technology. Because of the non-rivalness of technology, identical technologies can be simultaneously utilized at any production site, and thus $\frac{\partial Y_t}{\partial A_t}$ increases as L_t increases. In contrast, because of the rivalness of capital, capital can be used only by workers at the production sites where the capital is installed. Thus, $\frac{\partial Y_t}{\partial K_t}$ is unchanged even if L_t increases. Therefore, firms will invest more in technology than in capital as population increases because returns on investing in technology become more attractive than those on capital. However, at the same time, uncompensated knowledge spillovers increase with an increase in population because of the non-rivalness of technology. The model presented in this paper indicates that an increase in population does not necessarily make investments in technology more attractive than those in capital because of increased uncompensated knowledge spillovers. Therefore, the non-rivalness of technology generates scale effects, but it simultaneously cancels them out with an inevitable increase in uncompensated knowledge spillovers. As a result, scale effects disappear, and an increase in population does not accelerate the growth rate. The model can eliminate both scale effects and the influence of population growth.

The paper is organized as follows. In Section 2, the production of technology and uncompensated knowledge spillovers are examined, and an endogenous growth model that incorporates substitution between investments in capital and technology is constructed. Section 3 shows that scale effects asymptotically diminish as population increases and shows that population growth is unnecessary for economic growth in the model. Concluding remarks are offered in Section 4.

2 THE MODEL

2.1 Production of technologies

Outputs Y_t are the sum of consumption C_t, the increase in capital, and the increase in technology such that

$$Y_t = C_t + \dot{K}_t + v\dot{A}_t \quad .$$

Thus,

$$\dot{k}_t = y_t - c_t - \frac{v\dot{A}_t}{L_t} - n_t k_t \quad ,$$

where $v(>0)$ is a constant, and a unit of K_t and v^{-1} of a unit of A_t are equivalent; that is, they are produced using the same quantities of inputs (capital, labor, and technology). This means that technologies are produced with capital, labor, and technology in the same way as consumer goods and services and capital. Unlike most idea-based growth models, no special mechanism is required for the production of technology because endogenous balanced growth (i.e., constant $\frac{A_t}{k_t}$) is not materialized by any special property of the production function of technology but by uncompensated knowledge spillovers and arbitrage between investments in capital and technology.

Because balanced growth paths are the focal point of this paper, Harrod-neutral technical progress is assumed.[2] Hence, the production function is $Y_t = K_t^{1-\alpha}(A_t L_t)^{\alpha}$; thus,

$$y_t = A_t^{\alpha} k_t^{1-\alpha} \quad .$$

It is assumed for simplicity that the population growth rate (n_t) is constant and not negative such that $n_t = n \geq 0$.

2.2 Substitution between investments in K_t and A_t

For any period,

$$m = \frac{M_t}{L_t} \quad , \tag{2}$$

where M_t is the number of firms (which are assumed to be identical) and m (> 0) is a constant. Equation (2) presents a natural assumption that the population and number of firms are proportional to each other. Equation (2) therefore indicates that any firm consists of the same number of employee regardless of L_t. Note that, unlike the arguments in Young (1998), Peretto (1998), Aghion and Howitt (1998), and Dinopoulos and Thompson (1998), M_t is not implicitly assumed to be proportional to the number of sectors or researchers in the economy (see also Jones, 1999). Equation (2) merely indicates that the average number of employees per firm in an economy is independent of the population. Hence, M_t is not essential for the amount of production of A_t. As will be shown by equations (3) and (4), production of A_t does not depend on the number of researchers but on investments in technology. In contrast, M_t plays an important role in the amount of uncompensated knowledge spillovers.

The constant m implicitly indicates that the size of a firm is, on average, unchanged even if the population increases. This assumption can be justified by Coase (1937) who argued that the size of a firm is limited by the overload of administrative information. In addition, Williamson (1967) argued that there can be efficiency losses in larger firms (see also Grossman and Hart, 1986 and Moore, 1992). Their arguments equally imply that there is an optimal firm size that is determined by factors that are basically independent of population.

Next, for any period,

$$\frac{\partial Y_t}{\partial K_t} = \frac{\varpi}{M_t^{1-\rho}} \frac{\partial Y_t}{\partial (vA_t)} \quad ; \tag{3}$$

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[2] As is well known, only Harrod-neutral technological progress matches the stylized facts presented by Kaldor (1961). As Barro and Sala-i-Martin (1995) argue, technological progress must take the labor-augmenting form in the production function if the models are to display a steady state.

thus,

$$\frac{\partial y_t}{\partial k_t} = \frac{\varpi L_t^{\rho}}{m^{1-\rho} v} \frac{\partial y_t}{\partial A_t} \tag{4}$$

is always kept, where $\varpi(>1)$ and $\rho(0 \le \rho < 1)$ are constants. The parameter ρ describes the effect of uncompensated knowledge spillovers, and the parameter ϖ indicates the effect of patent protection. With patents, incomes are distributed not only to capital and labor but also to technology. For simplicity, the patent period is assumed to be indefinite, and no capital depreciation is assumed. An extended model with a finite patent period and capital depreciation is examined in Section 3.5.

Equations (3) and (4) indicate that returns on investing in capital and technology for the investing firm are kept equal. The driving force behind the equations is that firms exploit all opportunities and select the most profitable investments at all times. Through arbitrage, this behavior leads to equal returns on investments in capital and technology. With substitution between investments in capital and technology, the model exhibits endogenous balanced growth. Because $\frac{\varpi}{mv}\frac{\partial y_t}{\partial A_t} = \frac{\partial y_t}{\partial k_t} \Leftrightarrow \frac{\varpi L_t^{\rho}\alpha}{m^{1-\rho}v} A_t^{\alpha-1} k_t^{1-\alpha} = (1-\alpha) A_t^{\alpha} k_t^{-\alpha}$, $A_t = \frac{\varpi L_t^{\rho}\alpha}{m^{1-\rho}v(1-\alpha)} k_t$ by equations (2) and (3), which lucidly indicates that $\frac{A_t}{k_t}$ = constant, and the model can therefore show balanced endogenous growth.

2.3 Uncompensated knowledge spillovers

Equations (3) and (4) also indicate that the investing firm cannot obtain all of the returns on its investment in technology. That is, although investment in technology increases Y_t, the investing firm's returns are only a fraction of the increase in Y_t, such that $\frac{\varpi}{M_t^{1-\rho}} \frac{\partial Y_t}{\partial (vA_t)}$, because knowledge spills over to other firms without compensation and other firms possess complementary technologies.

Broadly speaking, there are two types of uncompensated knowledge spillovers: intra-sectoral knowledge spillovers (MAR externalities: Marshall, 1890; Arrow, 1962; Romer, 1986) and inter-sectoral knowledge spillovers (Jacobs externalities: Jacobs, 1969). MAR theory assumes that knowledge spillovers between homogenous firms are the most effective and that spillovers will primarily emerge within sectors. As a result, uncompensated knowledge spillovers will be more active if the number of firms within a sector is larger. On the other hand, Jacobs (1969) argues that knowledge spillovers are most effective among firms that practice different activities and that diversification (i.e., a variety of sectors) is more important in influencing spillovers. As a result, uncompensated knowledge spillovers will be more active if the number of sectors in the economy is larger. If all sectors have the same number of firms, an increase in the number of firms in the economy results in more knowledge spillovers in any case, as a result of either MAR or Jacobs externalities.

As uncompensated knowledge spillovers increase, the investing firm's returns on investment in technology decrease. $\frac{\partial Y_t}{\partial A_t}$ indicates the total increase in Y_t in the economy by an increase in A_t, which consists of increases in both outputs of the firm that invested in the new technologies and outputs of other firms that utilize the newly invented technologies, regardless of whether the firms obtained the technologies by compensating the originating firm or through uncompensated knowledge spillovers. If the number of firms increases and uncompensated knowledge spillovers increase, the compensated fraction in $\frac{\partial Y_t}{\partial A_t}$ that the investing firm can obtain becomes smaller, as do its returns on the investment in technology. The parameter ρ describes the magnitude of this effect. If $\rho = 0$, the investing firm's returns are reduced at the same rate as the increase of the number of firms. $0 < \rho < 1$ indicates that the investing firm's returns diminish as the number of firms increase but not to the same extent as when $\rho = 0$.

Both types of externalities predict that uncompensated knowledge spillovers will increase as the number of firms increases, and scale effects have not actually been observed (Jones, 1995a), which implies that scale effects are almost canceled out by the effects of MAR and Jacobs externalities. Thus, the value of ρ is quite likely to be very small. From the point of view of a firm's behavior, a very small ρ appears to be quite natural. Because firms intrinsically seek profit opportunities, newly established firms work as hard as existing firms to profit from knowledge spillovers. An increase in the number of firms therefore indicates that more firms are trying to obtain the investing firm's technologies.

Because of the non-rivalness of technology, all firms can equally benefit from uncompensated knowledge spillovers, regardless of the number of firms. Because the size of firms is independent of population and thus constant as argued in Section 2.2, each firm's ability to utilize the knowledge that has spilled over from each of the other firms will not be reduced by an increase in population. In addition, competition over technologies will increase as the number of firms increases, and any firm will completely exploit all opportunities to utilize uncompensated knowledge spillovers as competition increases.[3] Hence, it is quite likely that the probability that a firm can utilize a unit of new technologies developed by each of the other firms without compensation will be kept constant even if the population and the number of firms increase. As a result, uncompensated knowledge spillovers will increase eventually to the point that they increase at the same rate as the increase in the number of firms.

The investing firm's fraction of $\frac{\partial Y_t}{\partial A_t}$ that it can obtain will thereby be reduced at the same rate as the increase in the number of firms, which means that ρ will naturally decrease to zero as a result of firms' profit-seeking behavior. Based on $\rho = 0$,

$$\frac{\partial Y_t}{\partial K_t} = \frac{\varpi}{M_t} \frac{\partial Y_t}{\partial (vA_t)} \tag{5}$$

by equations (3) and (4); thus,

$$\frac{\partial y_t}{\partial k_t} = \frac{\varpi}{mv} \frac{\partial y_t}{\partial A_t} \tag{6}$$

is always maintained.

Complementary technologies also reduce the fraction of $\frac{\partial Y_t}{\partial A_t}$ that the investing firm can obtain. If a new technology is effective only if it is combined with other technologies, the returns on investment in the new technology will belong not only to the investing firm but also to the firms that possess the other technologies. For example, an innovation in computer software technology generated by a software company increases the sales and profits of

[3] Moreover, a larger number of firms indicates that firms are more specialized. More specialized and formerly neglected technologies may become valuable to the larger number of specialized firms. Hence, knowledge spillovers will increase.

computer hardware companies. The economy's productivity increases because of the innovation but the increased incomes are attributed not only to the firm that generated the innovation but also to the firms that possess complementary technologies. A part of $\frac{\partial Y_t}{\partial A_t}$ leaks to these firms, and the leaked income is a kind of rent revenue that unexpectedly became obtainable because of the original firm's innovation. Most new technologies will have complementary technologies. Because of both complementary technologies and uncompensated knowledge spillovers, the fraction of $\frac{\partial Y_t}{\partial A_t}$ that an investing firm can obtain on average will be very small; that is, ϖ will be far smaller than M_t except when M_t is very small.[4]

2.4 The optimization problem

Because $A_t = \frac{\varpi\alpha}{mv(1-\alpha)}k_t$, then $y_t = \left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}k_t$ and $\dot{A}_t = \frac{\varpi}{mv}\dot{k}_t\left(\frac{\alpha}{1-\alpha}\right)$. Hence, $\dot{k}_t = y_t - c_t - \frac{v\dot{A}_t}{L_t} - n_t k_t = \left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}k_t - c_t - \frac{\varpi}{mL_t}\dot{k}_t\left(\frac{\alpha}{1-\alpha}\right) - n_t k_t$ and

$$\dot{k}_t = \frac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\varpi\alpha}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}k_t - c_t - n_t k_t\right] . \tag{7}$$

As a whole, the optimization problem of the representative household is to maximize the expected utility

$$E\int_0^{\infty} u(c_t)\exp(-\theta t)dt$$

subject to equation (7) where $u(\bullet)$ is a constant relative risk aversion (CRRA) utility function and E is the expectation operator.

3 AN ASYMPTOTICALLY NON-SCALE BALANCED GROWTH PATH

3.1 Growth rate and transversality condition

Let Hamiltonian H be

$$H = u(c_t)\exp(-\theta t) + \lambda_t \frac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\varpi\alpha}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}k_t - c_t - n_t k_t\right],$$

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[4] If M_t is very small, the value of ϖ will be far smaller than that for sufficiently large M_t because the number of firms that can benefit from an innovation is constrained owing to the very small M_t. The very small number of firms indicates that the economy is not sufficiently sophisticated, and thereby the benefit of an innovation cannot be fully realized. This constraint can be modeled as $\varpi = \tilde{\varpi}\left[1-\left(1-\tilde{\varpi}^{-1}\right)^{M_t}\right]$, where $\tilde{\varpi}(\geq 1)$ is a constant. Nevertheless, for sufficiently large M_t (i.e., in sufficiently sophisticated economies), the constraint is removed such that $\lim_{M_t\to\infty}\tilde{\varpi}\left[1-\left(1-\tilde{\varpi}^{-1}\right)^{M_t}\right] = \tilde{\varpi} = \varpi$.

where λ_t is a costate variable. The optimality conditions for the optimization problem shown in the previous section are

$$\frac{\partial u(c_t)}{\partial c_t}\exp(-\theta t)=\lambda_t\frac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\varpi\alpha} \tag{8}$$

$$\dot{\lambda}_t=-\frac{\partial H}{\partial k_t} \tag{9}$$

$$\dot{k}_t=\frac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\varpi\alpha}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}k_t-c_t-n_t k_t\right] \tag{10}$$

$$\lim_{t\to\infty}\lambda_t k_t=0 \quad . \tag{11}$$

By equation (9),

$$\dot{\lambda}_t=-\lambda_t\frac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\varpi\alpha}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-n_t\right] \quad . \tag{12}$$

Hence, by equations (8) and (12), the growth rate of consumption is

$$\frac{\dot{c}_t}{c_t}=\varepsilon^{-1}\left\{\frac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\varpi\alpha}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-n_t\right]-\theta\right\},$$

where $\varepsilon=-\frac{c_t u''}{u'}$. Note that usually $\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-n_t>0$, so this is the case examined in this paper.

By equation (10), $\frac{\dot{k}_t}{k_t}=\frac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\varpi\alpha}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-n_t-\frac{c_t}{k_t}\right]$, and by equation (12), $\frac{\dot{\lambda}_t}{\lambda_t}=-\frac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\varpi\alpha}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-n_t\right]$. Hence,

$$\frac{\dot{\lambda}_t}{\lambda_t}+\frac{\dot{k}_t}{k_t}=-\frac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\varpi\alpha}\left(\frac{c_t}{k_t}\right) \quad .$$

Therefore, if $\frac{c_t}{k_t}>0$ for any period, then $\frac{\dot{\lambda}_t}{\lambda_t}+\frac{\dot{k}_t}{k_t}<0$, and transversality condition (11) is satisfied. Conversely, if $\frac{c_t}{k_t}=0$ for any period after a certain period, the transversality condition is not satisfied.

3.2 Balanced growth path

There is a balanced growth path on which all the optimality conditions are satisfied.

Lemma: If and only if $\lim_{t\to\infty}\frac{\dot{c}_t}{c_t}=\lim_{t\to\infty}\frac{\dot{k}_t}{k_t}$, all the conditions (equations [7]–[10]) are satisfied.

Proof: (Step 1) $\lim_{t\to\infty}\frac{\dot{c}_t}{c_t}=\varepsilon^{-1}\left\{\frac{m\lim_{t\to\infty}L_t(1-\alpha)\left[\left(\frac{\varpi\alpha}{m\nu}\right)^{\alpha}(1-\alpha)^{-\alpha}-n_t\right]-\alpha n_t}{m\lim_{t\to\infty}L_t(1-\alpha)+\varpi\alpha}-\theta\right\}=$ $\varepsilon^{-1}\left[\left(\frac{\varpi\alpha}{m\nu}\right)^{\alpha}(1-\alpha)^{-\alpha}-n-\theta\right]$. Therefore, $\lim_{t\to\infty}\frac{\dot{c}_t}{c_t}=\text{constant}$. On the other hand, $\lim_{t\to\infty}\frac{\dot{k}_t}{k_t}=$ $\frac{m\lim_{t\to\infty}L_t(1-\alpha)}{m\lim_{t\to\infty}L_t(1-\alpha)+\varpi\alpha}\left[\left(\frac{\varpi\alpha}{m\nu}\right)^{\alpha}(1-\alpha)^{-\alpha}-n-\lim_{t\to\infty}\frac{c_t}{k_t}\right]$.

(Step 2) If $\lim_{t\to\infty}\frac{\dot{k}_t}{k_t}>\lim_{t\to\infty}\frac{\dot{c}_t}{c_t}$, then $\frac{c_t}{k_t}$ diminishes as time passes because $\lim_{t\to\infty}\frac{\dot{c}_t}{c_t}$ = constant by (Step 1) while $\lim_{t\to\infty}\frac{\dot{k}_t}{k_t}$ increases by (Step 1). Thus, eventually $\frac{c_t}{k_t}$ diminishes to zero, and as shown in Section 3.1, transversality condition (11) is not satisfied.

If $\lim_{t\to\infty}\frac{\dot{k}_t}{k_t}<\lim_{t\to\infty}\frac{\dot{c}_t}{c_t}$, then $\frac{c_t}{k_t}$ increases indefinitely as time passes because $\lim_{t\to\infty}\frac{\dot{c}_t}{c_t}$ = constant by (Step 1) while $\lim_{t\to\infty}\frac{\dot{k}_t}{k_t}$ diminishes and eventually becomes negative by (Step 1). Hence, k_t decreases and eventually equation (10) is violated because $k_t\geq 0$.

On the other hand, if $\lim_{t\to\infty}\frac{\dot{c}_t}{c_t}=\lim_{t\to\infty}\frac{\dot{k}_t}{k_t}$, then $\lim_{t\to\infty}\frac{c_t}{k_t}$ is constant; thus, $\lim_{t\to\infty}\frac{\dot{k}_t}{k_t}$ and $\lim_{t\to\infty}\frac{\dot{c}_t}{c_t}$ are identical and constant because $\lim_{t\to\infty}\frac{\dot{c}_t}{c_t}$ = constant by (Step 1). ■

Rational households will set an initial consumption that leads to the growth path that satisfies all the conditions. The Lemma therefore indicates that, given an initial A_0 and k_0, rational households will set the initial consumption c_0 so as to achieve the growth path that satisfies $\lim_{t\to\infty}\frac{\dot{c}_t}{c_t}=\lim_{t\to\infty}\frac{\dot{k}_t}{k_t}$, while firms will adjust k_t so as to achieve $\frac{\partial Y_t}{\partial K_t}=\frac{\varpi}{M_t}\frac{\partial Y_t}{\partial(\nu A_t)}$.[5] With this household behavior, the growth rates of technology, per capita output, consumption, and capital converge at the same rate.

Proposition: If all of the optimality conditions (equations [7]–[10]) are satisfied,

[5] Arbitrage conditions (3) and (4) indicate that until $A_t=\frac{\varpi\alpha}{m\nu(1-\alpha)}k_t$ is achieved, no investment is made in technology if $A_0>\frac{\varpi\alpha}{m\nu(1-\alpha)}k_0$ and in capital if $A_0<\frac{\varpi\alpha}{m\nu(1-\alpha)}k_0$.

$$\lim_{t\to\infty}\frac{\dot{y}_t}{y_t}=\lim_{t\to\infty}\frac{\dot{A}_t}{A_t}=\lim_{t\to\infty}\frac{\dot{c}_t}{c_t}=\lim_{t\to\infty}\frac{\dot{k}_t}{k_t} \quad .$$

Proof: Because $y_t = A_t^{\alpha} k_t^{1-\alpha}$, $\dot{y}_t = \left(\frac{A_t}{k_t}\right)^{\alpha}\left[(1-\alpha)\dot{k}_t + \alpha\frac{k_t}{A_t}\dot{A}_t\right]$. Since $\dot{A}_t = \frac{\varpi}{mv}\dot{k}_t\left(\frac{\alpha}{1-\alpha}\right)$, then $\dot{y}_t = \dot{k}_t\left(\frac{A_t}{k_t}\right)^{\alpha}\left[(1-\alpha)+\frac{\varpi\alpha^2}{mv(1-\alpha)}\frac{k_t}{A_t}\right]$; thus, $\frac{\dot{y}_t}{y_t} = \frac{\dot{k}_t}{k_t}\left[(1-\alpha)+\frac{\varpi\alpha^2}{mv(1-\alpha)}\frac{k_t}{A_t}\right]$. Since $A_t = \frac{\varpi\alpha}{mv(1-\alpha)}k_t$, $\frac{\dot{y}_t}{y_t} = \frac{\dot{k}_t}{k_t}[(1-\alpha)+\alpha] = \frac{\dot{k}_t}{k_t}$. Therefore, $\lim_{t\to\infty}\frac{\dot{y}_t}{y_t}=\lim_{t\to\infty}\frac{\dot{c}_t}{c_t}=\lim_{t\to\infty}\frac{\dot{k}_t}{k_t}$. In addition, since $\dot{y}_t = \dot{A}_t\left(\frac{A_t}{k_t}\right)^{\alpha}\left[\frac{mv(1-\alpha)^2}{\varpi\alpha}+\alpha\frac{k_t}{A_t}\right]$ by $y_t = A_t^{\alpha}k_t^{1-\alpha}$ and $\dot{A}_t = \frac{\varpi\alpha}{mv(1-\alpha)}\dot{k}_t$, then $\frac{\dot{y}_t}{y_t} = \frac{\dot{A}_t}{k_t}\frac{mv(1-\alpha)^2}{\varpi\alpha}+\alpha\frac{\dot{A}_t}{A_t}$. Because $\dot{A}_t = \frac{\varpi\alpha}{mv(1-\alpha)}\dot{k}_t$, then $\frac{\dot{y}_t}{y_t} = (1-\alpha)\frac{\dot{k}_t}{k_t}+\alpha\frac{\dot{A}_t}{A_t}$. Thereby, $\frac{\dot{y}_t}{y_t} = \frac{\dot{k}_t}{k_t} = (1-\alpha)\frac{\dot{k}_t}{k_t}+\alpha\frac{\dot{A}_t}{A_t}$ and $\lim_{t\to\infty}\frac{\dot{k}_t}{k_t}=\lim_{t\to\infty}\frac{\dot{A}_t}{A_t}=\lim_{t\to\infty}\frac{\dot{A}_t}{A_t}$ = constant. Hence, by the Lemma, $\lim_{t\to\infty}\frac{\dot{y}_t}{y_t}=\lim_{t\to\infty}\frac{\dot{A}_t}{A_t}=\lim_{t\to\infty}\frac{\dot{c}_t}{c_t}=\lim_{t\to\infty}\frac{\dot{k}_t}{k_t}$ if all the optimality conditions are satisfied. ■

By Proposition and Lemma, the balanced growth path is

$$\lim_{t\to\infty}\frac{\dot{y}_t}{y_t}=\lim_{t\to\infty}\frac{\dot{A}_t}{A_t}=\lim_{t\to\infty}\frac{\dot{c}_t}{c_t}=\lim_{t\to\infty}\frac{\dot{k}_t}{k_t}=\varepsilon^{-1}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-n-\theta\right] \quad . \qquad (13)$$

This balanced growth path can be seen as a natural extension of the steady state in the conventional Ramsey growth model with exogenous technology.

3.3 *Vanishing scale effects*

In contrast to the arguments in Peretto and Smulders (2002), uncompensated knowledge spillovers increase when the population increases as shown in Section 2.3. This nature of uncompensated knowledge spillovers is consistent with the theories of MAR and Jacobs externalities. They could potentially make scale effects much worse, but they do not. The balanced growth path shown in equation (13) is not a function of L_t.

Although knowledge spillovers increase as the population increases, their effects are simultaneously cancelled out through substitution between investments in capital and technology. If returns on investments in technology become more attractive because of an increase in population, firms will invest more in technology than in capital. Decisions about whether to invest in capital or technology are made by firms that compare returns on investments in capital ($\frac{\partial Y_t}{\partial K_t}$) with those in technology ($\psi\frac{\partial Y_t}{\partial A_t}$), where ψ is a variable that indicates the degree of uncompensated knowledge spillovers, that is, how much a firm that invests in technology can obtain from $\frac{\partial Y_t}{\partial A_t}$ as the return on generating a new technology. By arbitrage, both returns are equalized such that $\frac{\partial Y_t}{\partial K_t} = \psi \frac{\partial Y_t}{\partial A_t}$, as shown in equation (3).

As the size of the population becomes larger, $\frac{\partial Y_t}{\partial A_t}$ increases, because a larger number of workers (L_t) can simultaneously utilize new technologies.[6] The non-rivalness of technology enables this increase in $\frac{\partial Y_t}{\partial A_t}$. Therefore, $\frac{\partial Y_t}{\partial A_t} = L_t \frac{\partial y_t}{\partial A_t}$. Even if $\frac{\partial y_t}{\partial A_t}$ is constant, $\frac{\partial Y_t}{\partial A_t}$ increases as L_t increases. To the contrary, because of the rivalness of capital, it cannot necessarily be used by the larger number of workers when the population increases. Capital inputs can only be used by the workers at the production sites where they are installed; that is, $\frac{\partial Y_t}{\partial K_t} = \frac{\partial y_t}{\partial k_t}$. If $\frac{\partial y_t}{\partial k_t}$ is constant, $\frac{\partial Y_t}{\partial K_t}$ is also constant, even if the size of the population increases.

Therefore, if ψ is constant, familiar scale effects emerge. $\psi \frac{\partial Y_t}{\partial A_t}$ would increase with an increase in population, and $\frac{\partial Y_t}{\partial K_t}$ would not change. Returns on investments in technology would therefore become more attractive than those on investments in capital as the size of the population increases; thus, firms would invest more in technology than in capital. As a result, the growth rate would accelerate with an increase in population.

However, ψ is not constant. As shown in Section 2.3, the theory of knowledge spillover predicts that the amount of uncompensated knowledge spillovers and number of firms are positively correlated. In addition, the number of firms will increase as the population increases, as shown in equation (2). Hence, the theory of knowledge spillover indicates that ψ is not constant but rather a function of population, and it decreases as the population increases. An increase in population increases returns on investments in technology, but at the same time, they decrease the returns on investments in technology because of the increase in uncompensated knowledge spillovers. In other words, the total reward per innovation increases as the population increases, but at the same time, those rewards are shared by an increased number of firms without compensation. Therefore, an increase in population does not necessarily make investments in technology more attractive than those in capital.

An increase in population therefore does not necessarily accelerate the growth rate. As shown in Section 2.3, ρ will be almost zero as a result of firms' profit-seeking behavior. Equations (5) and (6) indicate that, because $\rho = 0$, an increase in $\frac{\partial Y_t}{\partial A_t}$ caused by an increase in L_t is completely cancelled out by an increase in uncompensated knowledge spillovers represented by $M_t = mL_t$.

However, if the population is small, scale effects still exist. Scale effects are measured by

$$S(L_t) = \varepsilon \frac{\dot{c}_t}{c_t} + \theta \quad ;$$

[6] In addition, an increase in population also indicates that a larger number of households can consume products produced by utilizing a unit of new technology.

that is, by the population related part of $\frac{\dot{c}_t}{c_t}$. In the model,

$$S(L_t) = \frac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\varpi\alpha}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - n\right] \quad . \tag{14}$$

If $\frac{dS(L_t)}{dL_t} > 0$, scale effects exist, and if $\frac{dS(L_t)}{dL_t} = 0$, no scale effect exists. When the population is small, $\frac{dS(L_t)}{dL_t} > 0$ and scale effects exist. However, scale effects vanish asymptotically as population increases such that

$$\lim_{L_t \to \infty} S(L_t) = \left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - n \quad , \tag{15}$$

and

$$\lim_{L_t \to \infty} \frac{dS(L_t)}{dL_t} = 0 \quad .$$

As the population increases, scale effects asymptotically disappear. An economy with a sufficiently large population therefore can grow without scale effects.

Equations (14) and (15) indicate that scale effects are economically important if the size of population is very small (i.e., the number of firms is very small), which implies that scale effects played a crucial role in early human history. Conversely, in present-day industrialized economies, scale effects have been observed to have no influence on growth (Jones, 1995a) because these economies are integrated with the world economy and have a large total population.

3.4 Growth without population increase

The model also indicates that population growth is not necessary for economic growth. If $n_t = 0$ for any period,

$$\frac{\dot{c}_t}{c_t} = \varepsilon^{-1}\left\{\frac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\varpi\alpha}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}\right] - \theta\right\} = \text{constant}$$

because L_t = constant. Clearly, $\frac{\dot{c}_t}{c_t}$ is irrelevant to n_t and is positive even though $n_t = 0$.[7] This result is important because it indicates that the economy can grow endogenously and indefinitely at a constant rate without population growth, which contrasts with the non-scale model shown in Jones (1995b).

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[7] As mentioned above, usually $\left(\frac{\varpi L_t^{\rho}\alpha}{m^{1-\rho}v}\right)^{\alpha}(1-\alpha)^{-\alpha} - n > 0$, so this is the only case examined in this paper.

3.5 Extension to a finite patent period

In the previous sections, for simplicity, an indefinite patent period was assumed and capital depreciation was not taken into consideration. In this section, these assumptions are relaxed. Let $\chi(>0)$ be the length of the patent period and $\delta(>0)$ be the rate of capital depreciation. After the patent period of a technology ends, the price of the technology is zero and the returns on investment in that technology are also zero indefinitely. Thereby, after the end of patent period, the increased income generated by use of the technology is only distributed to owners of capital and labor, not to the owner of the technology. Hence, the total return on investment in technology to the investing firm during the patent period is

$$\chi \frac{\varpi}{mv} \frac{\partial y_t}{\partial A_t} \quad .$$

Next, because capital depreciates by δ every period, the total return on investment in capital to the investing firm during the entire period is

$$\frac{\partial y_t}{\partial k_t} \int_0^\infty e^{-\delta s} ds = \delta^{-1} \frac{\partial y_t}{\partial k_t} \quad .$$

Through the arbitrage between investments in capital and technology,

$$\delta^{-1} \frac{\partial y_t}{\partial k_t} = \chi \frac{\varpi}{mv} \frac{\partial y_t}{\partial A_t} \quad ;$$

thus,

$$\frac{\partial y_t}{\partial k_t} = \delta \chi \frac{\varpi}{mv} \frac{\partial y_t}{\partial A_t} \quad . \tag{16}$$

Therefore, in an economy with a finite patent period and capital depreciation, equation (4) is replaced with equation (16). Equation (16) clearly shows that the original model's conclusion still holds with a finite patent period and capital depreciation. In many countries, the patent period is 20 or more years (i.e., $\chi \geq 20$), and the useful life of capital is usually about 20 years (a depreciation rate of about 0.05). For χ = 20 and δ = 0.05, $\delta\chi$ = 1, which means that equation (4) and (16) will be practically identical for reasonable patent periods and depreciation rates. In this situation, it appears reasonable to assume for simplicity that the patent period is indefinite and the rate of capital depreciation is zero.

Although 20 years have been used as the patent period in many countries, there may be other possibilities. Because

$$\lim_{L \to \infty} \frac{\dot{c}_t}{c_t} = \varepsilon^{-1} \left[\left(\delta \chi \frac{\varpi \alpha}{mv} \right)^{\alpha} (1-\alpha)^{-\alpha} - \delta - n - \theta \right] \quad ,$$

as the patent period χ increases, the growth rate of consumption increases if the population is sufficiently large. This result suggests that the patent period should be indefinite. However, this is not the case because

$$\lim_{\chi\to\infty}\frac{\dot{c}_t}{c_t}=\lim_{\chi\to\infty}\varepsilon^{-1}\left\{\frac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\delta\chi\varpi\,\alpha}\left[\left(\delta\chi\frac{\varpi\,\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-n\right]-\theta\right\}=-\frac{\theta}{\varepsilon}$$

for $L<\infty$. If the patent period is very long, the growth rate of consumption becomes negative because firms will restrain their accumulation of capital because investments in technology will be much more lucrative as compared with those in capital. Equation (16) indicates that, as $\chi\to\infty$, firms will become extremely tempted to invest in technology rather than in capital and eventually no investment in capital will be made. Therefore, the patent period should be finite to achieve high growth rates.

The optimal length of the patent period depends on the parameter values. In addition, technological obsolescence may also have to be considered because, in many industrial countries, a technology is often replaced with other technologies or demands shift to other goods and services that use other technologies in a period that is shorter than the patent period. If we also consider obsolescence, equation (16) can be replaced with

$$\delta^{-1}\frac{\partial y_t}{\partial k_t}=\frac{\varpi}{mv}\frac{\partial y_t}{\partial A_t}\int_0^{\chi}\exp(-\mu t)dt\quad;$$

thus

$$\frac{\partial y_t}{\partial k_t}=\frac{\delta}{\mu}\left[1-\exp(-\mu\chi)\right]\frac{\varpi}{mv}\frac{\partial y_t}{\partial A_t}\quad,$$

where μ is the obsolescence rate. For example, if μ = 0.1, then $\mu^{-1}\left[1-\exp(-\mu\chi)\right]$ = 8.65 for χ = 20, 9.50 for χ = 30 and 9.82 for χ = 40. If μ = 0.15, then $\mu^{-1}\left[1-\exp(-\mu\chi)\right]$ = 9.50 for χ = 20, 9.89 for χ = 30 and 9.98 for χ = 40. Hence, the value of $\mu^{-1}\left[1-\exp(-\mu\chi)\right]$ is almost identical if χ > 20, which implies that an approximate 20-year patent period is sufficiently long and practically reasonable.

4 CONCLUDING REMARKS

Early endogenous growth models (e.g. Romer, 1986, 1987; Lucas, 1988) employed scale effects. Jones (1995b) presents a different type of endogenous growth model that eliminates scale effects, but the population growth rate plays a crucial role for economic growth. Models developed by Young (1998), Peretto (1998), Aghion and Howitt (1998), and Dinopoulos and Thompson (1998) eliminate the influence of population growth as well as scale effects, but Jones (1999) argues that those models crucially depend on a very special assumption. Using a fourth approach, Peretto and Smulders (2002) assume that A_tL_t (instead of A_t) and K_t are positively linked; thus, scale effects asymptotically vanish.

The model developed in this paper superficially has the same feature as the model developed in Peretto and Smulders (2002), but the mechanism through which scale effects vanish is fundamentally different. The concepts of MAR and Jacobs externalities both predict uncompensated knowledge spillovers will increase as the number of firms increases, and to be consistent with the theory of knowledge spillover, uncompensated knowledge spillovers increase when the number of firms increases in the model presented in this paper. Even though the direction of the effect of knowledge spillovers is reversed, scale effects still diminish as they do in Peretto and Smulders' (2002) model because of increased uncompensated knowledge spillovers and substitution between investments in capital and technology.

Because of the non-rivalness of technology, $\frac{\partial Y_t}{\partial A_t}$ increases as L_t increases. In contrast, because of the rivalness of capitals, $\frac{\partial Y_t}{\partial K_t}$ is unchanged even if L_t increases. This difference makes investments in technology more attractive when L_t increases. However, at the same time, an increase in population increases the amount of uncompensated knowledge spillovers, which makes investments in technology less attractive. Therefore, firms do not necessarily invest more in technology than in capital when L_t increases. That is, the non-rivalness of technology generates scale effects but simultaneously cancels them out as the amount of uncompensated knowledge spillovers increases. As a result, scale effects disappear, and an increase in population does not necessarily accelerate the growth rate. By combining the theory of knowledge spillover and substitution between investments in capital and technology, an asymptotically non-scale endogenous growth model that can eliminate both scale effects and the influence of population growth was constructed.

Asymptotically diminishing scale effects indicate that, if a population is very small, scale effects greatly influence growth, but if it is sufficiently large, scale effects vanish. This result suggests that scale effects were a crucial factor for economic growth in the early history of civilizations, but they are no longer important in present-day industrialized economies.

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