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# A New Keynesian Triangle Phillips Curve

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## Abstract

We propose a solution to address the observed negative sign on the marginal cost variable in new Keynesian Phillips curve estimations. Our solution is based on an elaborate specification of the cost function faced by firms and the formulation of a reduced-form production function which is characterised by non-linear input-output relations. The resultant Phillips curve features the standard hybrid expectational term, labour share, output gap, speed-limit effects and supply shock variables. In general, GMM estimations of the model for developed and emerging markets yield a positive and significant coefficient on the labour share and the output gap. We conclude that supply shock variables are essential to the empirical validity of the cost-based Phillips curve.

Keywords: new Keynesian Phillips curve, marginal cost, supply shocks.

## 1. Introduction

The new Keynesian Phillips curve is part of the core elements of modern dynamic macro-models (e.g. Smets and Wouters (2003), Amato and Laubach (2003), Christiano et al.(2005), Gali et al.(2011)). The strength of the new Keynesian Phillips curve is that it is derived from microfoundations. Therefore, the parameters that characterise it have a clear structural interpretation. However, the empirical performance of the new Keynesian Phillips is still a matter of debate. Gali et al.(2001, 2005) argue that the

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new Keynesian Phillips curve provides an adequate account of inflation dynamics, whereas Rudd and Whelan (2005a, 2007) argue that the backward-looking Phillips curve better explains inflation dynamics. Cogley and Sbordone (2008) find that allowing for time-variation in trend inflation makes the backward-looking component of the hybrid new Keynesian Phillips curve statistically insignificant, thereby supporting the evidence provided by Gali et al.(2001, 2005).

The problem of parameter identification has also been a subject of discussion in new Keynesian Phillips curve literature. Ma (2002), Bardsen (2004), Mavroeidis (2005), Nason and Smith (2008) and Martins and Gabriel (2009) point out that the new Keynesian Phillips curve suffers from weak identification. A related issue is the choice of instruments. Fair (2008) argues that the instruments that are used to estimate the new Keynesian Phillips curve, such as higher lags of inflation, output gap, commodity prices, etc., are invalid because "the lagged values are not part of the model and so theoretically are not appropriate to use". The lack of a robust criterion that should guide the choice of instruments remains a major problem that may be responsible for conflicting results in the literature, despite the proposals by Andrews (1999), Donald and Newey (2001), Kapetanios (2006) and Hwang and Kim (2012).

Another problem with the new Keynesian Phillips curve is that the coefficient of the forcing variable, real marginal cost, tends to be insignificant and in some cases, carries a wrong sign when estimated. Rudd and Whelan (2007) find that the sign on the forcing variable is either not statistically significant or is negative in the case of the US. Mazumder (2010, 2011) proposes an alternative, procyclical measure of marginal cost, and still finds that the new Keynesian Phillips curve fails to explain inflation dynamics. Estimates of the new Keynesian Phillips curve for Australia by Abbas and Sgro (2011) produce similar findings. Similarly, Vašíček (2011) finds that alternative measures of real marginal cost tend to be insignificant and sometimes carry the wrong sign for some transitional economies.

The contribution of this paper is to present a more elaborate specification of marginal cost than has been used in the literature. In this sense, we build on the work by Petrella and Santoro (2012), who find micro-economic evidence in support of the new Keynesian Phillips curve in the case of US manufacturing firms. These authors formulate a production function with

raw material inputs and labour as factors of production. Their resultant real marginal cost is a linear combination of the firm-level labour share and relative input prices. They conclude that this measure of real marginal cost produces dynamic properties that are in line with new Keynesian theory.

This paper also provides the theoretical basis for the new Keynesian Phillips curve formulation that is proposed by Mehra (2004). We extend Petrella and Santoro (2012) to the macroeconomic level in the following way. We exploit non-linear input-output relationships as suggested by Batini et al.(2005) to formulate a reduced-form production function. The non-linearity in input-output relations, coupled with adjustment costs, leads us to a new Keynesian Phillips curve that features the output gap, speed-limit effects, the labour share and "supply shock" variables. This formulation can be interpreted as the "new Keynesian Triangle Phillips curve" because it features an expectational element, excess demand pressure and "supply shock" variables, as in Gordon (2011).

Our formulation achieves three objectives. Firstly, it directly constructs a procyclical measure of real marginal cost, thereby addressing part of the empirical problems of the new Keynesian Phillips curve as pointed out by Mazumder (2010, 2011). Secondly, at the empirical level, it bridges the gap between the "left fork" and the "right fork", i.e. between the triangle Phillips curve literature and the new Keynesian approach (see Gordon, 2011) by formulating a Phillips curve that has baseline new-Keynesian features whilst at the same time exhibiting variables that are found in the triangle Phillips curve approach. Thirdly we show that Gali et al.'s (2001) statement about the redundancy of supply shocks may be unjustified, because the empirical validity of the new Keynesian Phillips curve depends critically on the significance of supply shock variables.

The paper is structured as follows: Section 2 derives the new Keynesian Triangle Phillips curve. Section 3 presents the empirical results and section 4 is the conclusion.

## **2. Theoretical framework**

As pointed out by Fuhrer et al.(2010) and Ascari et al. (2011), there are two ways to derive the new Keynesian Phillips curve. One way, due to

Rotemberg (1982), is based on quadratic price adjustment costs. The other way, due to Calvo (1983), assumes that at each point in time a fraction of firms re-sets prices with a constant, exogenously determined probability. In this paper, we use the hybrid, Calvo-style, new Keynesian Phillips curve that is proposed by Gali and Gertler (1999) and Gali et.al. (2001) of the following form:

$$\pi_t = \gamma_f E_t \pi_{t+1} + \gamma_b \pi_{t-1} + \lambda \widehat{mc}_t \quad (1)$$

where  $\gamma_f$ ,  $\gamma_b$  and  $\lambda$  are non-linear combinations of the discount factor, the fraction of firms that re-sets prices and the fraction of firms that optimise. Gali and Gertler (1999) and subsequent authors assumed procyclicality of marginal cost so that  $\widehat{mc}_t = \kappa \widehat{y}_t$ , where  $\kappa > 0$ . However the output gap was soon found to be a poor proxy of marginal cost (see Gali et al., 2001). Consequently, by assuming a simple production function with labour as the only input, Gali et al.(2001) found that the labour share is a better proxy of marginal cost. However the findings by Rudd and Whelan (2001, 2005a, 2007) cast serious doubt on the usefulness of the labour share as a proxy of real marginal cost, and thus put the new Keynesian approach into question.

Our contribution is to provide an elaborate specification of marginal cost by building on the work by Petrella and Santoro (2012). To do so we assume, along the lines of Batini et al.(2005), that firms exhibit non-linear input requirements in production such that :  $X_{it} = Y_t^{\delta_i}$ , where  $X_{it}$  is the amount of non-labour input  $i$  required in production and  $\delta_i > 0$  is the input requirement coefficient. In addition we assume no substitution between labour and non-labour inputs. With fixed capital normalised to 1, we can write the production function as:

$$Y_t = A_t L_t^\alpha \left[ \prod_{i=1}^n Y_t^{\theta_i \delta_i} \right]^\varphi, \quad (2)$$

where  $A_t$  is the state of technology,  $L_t$  is the level of employment and,  $0 < \alpha < 1$ , and  $\theta_i$  is the elasticity of output with respect to input  $i$ . The reduced-form expression for eq.(2) is given by:

$$Y_t = A_t' L_t^\sigma, \quad (3)$$

where  $\phi = \sum_{i=1}^n \theta_i \delta_i$ ,  $\sigma = \frac{\alpha}{1-\phi}$  and  $A_t' = A_t^{\frac{1}{1-\phi}}$ . Using eq.(3), real total cost faced by the firm can be written as follows:

$$TC_t = \frac{W_t Y_t^{\frac{1}{\sigma}}}{A_t^{\frac{1}{\sigma}} P_t} + \sum_{i=1}^n \frac{P_{it}}{P_t} Y_t^{\delta_i}, \quad (4)$$

where  $P_{it}$  is the price of non-labour input  $i$ ,  $P_t$  is the aggregate price level and  $W_t$  is the nominal wage. Let  $p_{it}$  denote the real price of non-labour input  $i$ . We can write real marginal cost as:

$$MC_t = \frac{W_t Y_t^{\frac{1-\sigma}{\sigma}}}{\sigma A_t^{\frac{1}{\sigma}} P_t} + \sum_{i=1}^n \delta_i p_{it} Y_t^{\delta_i - 1}, \quad (5)$$

Linearising eq.(5) around the steady state we get the following relationship:

$$\widehat{m}c_t = \frac{S_0}{MC_0 \sigma} \widehat{s}_t + \sum_{i=1}^n \frac{\delta_i p_{i0} Y_0^{\delta_i - 1} (\delta_i - 1)}{MC_0} \widehat{y}_t + \sum_{i=1}^n \frac{\delta_i p_{i0} Y_0^{\delta_i - 1}}{MC_0} \widehat{p}_{it}. \quad (6)$$

We can then insert eq.(6) into eq.(1) to get the following extended version of the new Keynesian Phillips curve:

$$\pi_t = \gamma_f E_t \pi_{t+1} + \gamma_b \pi_{t-1} + \lambda \vartheta_s \widehat{s}_t + \lambda \vartheta_y \widehat{y}_t + \lambda \sum_{i=1}^n \vartheta_{ip} \widehat{p}_{it}, \quad (7)$$

where:

$$\vartheta_s = \frac{S_0}{MC_0 \sigma}, \vartheta_y = \sum_{i=1}^n \vartheta_{ip} (\delta_i - 1) \text{ and } \vartheta_{ip} = \frac{\delta_i p_{i0} Y_0^{\delta_i - 1}}{MC_0}.$$

Eq.(7) can be viewed as an extension of the baseline framework of Gali and Gertler (1999). It builds on Petrella and Santoro (2012) in the sense that, besides the labour share and relative input prices, the output gap enters the Phillips curve as well. Because of the presence of the expectations, excess demand pressure and "supply shock" variables, we refer to eq.(7) as the "new Keynesian Triangle Phillips curve". The significance of the output gap in driving inflation depends entirely on the relevance of relative input prices in the determination of production costs. Thus, we are able to provide a structural interpretation of the finding by Mehra (2004), that the omission of "supply shocks" makes the output gap statistically insignificant in new Keynesian Phillips curve estimations.

Based on eq.(7), we are able to provide a structural interpretation of the sign of the output gap. Batini et al.(2005) assume that  $\delta_i > 1$ . They justify the convexity of the non-linear input-output relation on the grounds that at high levels of output, inefficiencies in production increase at an increasing rate because firms tend to draft old machines into the production line, which use more inputs than new machines. However it is possible, especially if production technology exhibits significant economies of scale, for inefficiencies to increase at a decreasing rate at high levels of output. In this case  $\delta_i < 1$ , which delivers a negative sign on the output gap. Furthermore, if the input-output relation is linear, i.e.  $\delta_i = 1$ , then the output gap parameter would be zero.

If the assumption that input-output relations are convex holds, eq.(7) provides a straightforward way in which a procyclical measure of real marginal cost can be constructed. In this sense, eq.(7) also extends the work by Mazumder (2010), although in a different direction. Mazumder proposes a procyclical measure based on Bils (1987). However, when this measure is used, the new Keynesian Phillips curve collapses. The measure that we propose in eq.(7) explicitly features the output gap which, by definition is procyclical. If our assumptions about production technology are correct, then it means that the sign problem in new Keynesian Phillips curve estimations may reflect misspecification. Secondly, our theoretical formulation suggests that supply shocks and the level output gap have to be jointly significant if our assumptions hold empirically.

Some scholars, e.g. Mehra (2004) and Fuhrer et al.(2010), find that speed-limit effects play a significant role in driving inflation over and above the

level effects of excess demand pressure. In the framework presented above, we can introduce speed-limit effects in the basic new Keynesian model by assuming that firms face output adjustment costs in addition to production costs. This assumption is analogous to the standard investment adjustment cost found in DSGE literature, e.g. Smets and Wouters (2003) and Christiano et al.(2005). Therefore we specify output adjustment costs as follows:

$$AdjC_t = \left( \frac{Y_t}{Y_{t-1}} \right)^\omega Y_{t-1}, \quad (8)$$

where  $\omega > 0$  is the adjustment cost parameter. Log-linearising the marginal output adjustment cost and incorporating it in eq.(4) the resultant marginal cost, the new Keynesian Triangle Phillips curve becomes:

$$\pi_t = \gamma_f E_t \pi_{t+1} + \gamma_b \pi_{t-1} + \psi_s \hat{s}_t + \psi_y \hat{y}_t + \psi_{\Delta y} \Delta \hat{y}_t + \sum_{i=1}^n \psi_{ip} \hat{p}_{it}, \quad (9)$$

where  $\psi_s = \lambda \vartheta_s$ ,  $\psi_y = \lambda \vartheta_y$ ,  $\psi_{\Delta y} = \lambda \left( \frac{\omega(\omega-1)G_0^{\omega-1}}{MC_0} \right)$ ,  $\psi_{ip} = \lambda \vartheta_{ip}$  and  $G_0$  is the gross steady state growth rate of output. Eq.(9) can be viewed as a fully specified new Keynesian Phillips curve where real marginal cost includes the labour share as in the baseline framework of Galí and Gertler (1999). The speed-limit variable adds further procyclicality to marginal cost and thus assists in resolving the negative sign problem, as pointed out by Mehra (2004). Our theoretical formulation therefore suggests that whilst level of the output gap and "supply shock" variables have to be jointly significant, the significance of the speed-limit effect does not depend on production technology. The significance of supply shock variables is a necessary condition for the significance of the output gap but it is not sufficient, since the input-output relation may be linear or concave.

Estimations of the standard hybrid model generally produce serially correlated residuals. Bardsen et al. (2004) view this as a sign of misspecification. Zhang and Clovis (2010) derive a new Keynesian Phillips curve under the assumption that backward-looking agents may take more than one period to respond to actual inflation. In the light of this extension, we can extend eq.(9) as follows:



$$\pi_t = \gamma_f E_t \pi_{t+1} + \gamma_b \pi_{t-1} + \phi(L) \pi_{t-1} + \psi_s \widehat{s}_t + \psi_y \widehat{y}_t + \psi_{\Delta y} \Delta \widehat{y}_t + \sum_{i=1}^n \psi_{ip} \widehat{p}_{it}, \quad (10)$$

where  $\phi(L)$  is a lag operator. Eq.(10) further closes the gap between the traditional triangle Phillips curve of Gordon (1997, 2011) and the new Keynesian approach in that eq.(10) admits long lags of inflation. As Zhang and Clovis (2010) demonstrate, the parameters in eq.(10) remain structurally interpretable. From a reduced-form perspective, the only difference between the two Phillips curve approaches is that the new Keynesian Phillips curve explicitly features the forward-looking term. We estimate both eqs.(9) and (10) in this paper.

### 3. Reduced-form evidence

#### 3.1 Instrument choice and the endogeneity of the labour share

One of the problems faced by an econometrician who attempts to estimate the new Keynesian Phillips curve is the choice of instruments since simple OLS is inconsistent. A number of studies, e.g. Andrews (1999) and Donald and Newey (2001) propose methods to select the set of valid instruments. Donald and Newey (2001) in particular, propose that the optimal number of instruments should minimise the mean square error (MSE) of the first-stage regression. However, as pointed out by Kapetanios (2006), in the context of a large set of instruments, it is not clear how to order the instruments in order to choose the ones that should be included in the estimation. He proposes simulated annealing as a procedure to select instruments.

A related method is the  $L_2$ -boosting method proposed by Hwang and Kim (2012). The strategy followed by these authors involves the sequential inclusion of instrumental variables, starting with the one that has the highest explanatory power, i.e. the one that delivers the lowest first-stage MSE. We applied a similar procedure to select instruments. In the first stage regression, we chose the lags of variables in such a way that the MSE is minimised. We then used the resultant instruments to conduct the GMM estimation. The results were not encouraging. In other words, we found that instruments that deliver the minimum MSE do not necessarily produce the best GMM results.

The method we use in this paper begins by specifying high lags of instruments and then run GMM estimation on the basis of these lags. We then sequentially reduce the number of lags per instrumental variable up to the point where further reduction produces insignificance in the GMM parameters. Thus, our method is a version of the general-to-specific approach applied by Scheufele (2010).

One of the issues that is not mentioned in the literature in relation to new Keynesian Phillips curve estimations is the endogeneity of the labour share. This point is also raised by Gordon (2011). To illustrate, we note that  $s_t = \widehat{w}_t - \widehat{p}_t + s_{t-1}$ , where  $\widehat{w}_t$  is the nominal unit labour cost inflation rate. It follows from this that the labour share is negatively related to the price inflation rate, assuming partial indexation of nominal unit labour cost to prices. One way to deal with this problem is just to use  $s_{t-1}$  instead of  $s_t$  in the Phillips curve. However, the presence of  $\widehat{p}_{t-1}$  on the right hand side of the Phillips curve creates multicollinearity with  $\widehat{s}_{t-1}$  since by definition,  $s_{t-1} = \widehat{w}_{t-1} - \widehat{p}_{t-1} + s_{t-2}$ . In addition, in so far as  $\widehat{p}_t$  is strongly correlated with  $\widehat{p}_{t-1}$ , then it follows that even  $s_{t-1}$  may produce a counter-intuitive sign in the Phillips curve. Therefore instead of using  $\widehat{s}_t$  in the estimation, we use  $\widehat{s}_{t-2}$ .

### 3.2 Data and empirical results

We estimate eqs.(7) and (9) for six developed and six emerging markets. The developed markets comprise: the United States, United Kingdom, Canada, France, Germany and Australia. The six emerging markets comprise: Brazil, Mexico, Poland, Turkey, South Korea and South Africa. Data is drawn from the International Financial Statistics database and where there are gaps, we used the OECD database and country statistical offices. The data is quarterly with a sample from 1975:1–2012:2 for developed economies. For emerging markets the data starts from 1995–2012:2.

Inflation is measured using the CPI, since many central banks are concerned with this measure of prices in their policy decisions. Following Gordon (1997) and Mehra (2004) supply shock variables are consumer prices for energy, food and the import price deflator, all drawn from the OECD database. Real output is measured by real GDP. The labour share is calculated as the ratio of real employee compensation to real GDP where data is available. In some

countries, e.g. France, Brazil, and Mexico, real unit labour cost is used. Percentage deviations from trend are derived using the HP-filter.

Following Abbas and Sgro (2011), we report both GMM and Two-Stage-Least Squares (2SLS) estimations to check for the robustness of our results to estimation technique. In addition, since our theory allows for the de-coupling of speed-limit effects from the overall structure without significantly affecting the parameters of the model, we also report results for the case where there are speed-limit effects (eq.9) and where these are absent (eq.7), to check whether our formulation is robust to speed-limit effects.

As noted by Bardsen et al.(2004), Mavroeidis (2004, 2005), Nason and Smith (2008), and Martins and Gabriel (2009) among others, the new Keynesian Phillips curve is vulnerable to identification problems. We thus report three statistics to test for identification. The first statistic is the standard J-statistic. The second statistic is the first-stage F-statistic, proposed by Staiger and Stock (1997) for the case of a single regressor. However this statistic has been used by some authors even in the case where there are multiple regressors, e.g. Bardsen et al.(2004), Agénor and Bayraktar (2010) and Abbas and Sgro (2011). The requirement is that the first-stage F-statistic exceeds 10 for the model to be identified. The third statistic is the Anderson-Rubin (AR) statistic, which has been applied by Dufour et al.(2006) and Nason and Smith (2008). Instead of testing for the individual parameters, we conduct the test jointly for all the estimated parameters.

Table 1 displays the instruments used for each of the countries.

Table 1: Lags for instrumental variables

	$\pi_t$	$\hat{s}_t$	$\hat{y}_t$	$\hat{p}_{mt}$	$\hat{p}_{ft}$	$\hat{p}_{et}$	$\hat{w}_t$
GMM Estimation							
Australia	20	7	20	20	20	20	1
Canada	16	5	1		4	3	
Germany	2	8	7	9			4
France	24	1	12		1	1	1
United Kingdom	19	4	4	5			20
United States	24	2	2	7			12
Brazil	24	5	4			2	2
Mexico	16	9	2	9	9	9	9
Poland	12	8	12	8	2	12	1
South Africa	24	6	12	8	12		1
South Korea	16	5	12	8	12	12	4
Turkey	16	1	2		1	4	1
2SLS Estimation							
Australia	20	2	20	16	12	20	4
Canada	16	1	1		1	2	1
Germany	2	1	4	1			2
France	24	1	12		1	1	1
United Kingdom	19	4	4	5			20
United States	20	1	1	2			1
Brazil	4	1	4			1	2
Mexico	16	9	2	9	9	9	10
Poland	12	8	12	2	2	4	1
South Africa	1	2	5	1	4		1
South Korea	16	5	12	8	12	12	4
Turkey	2	20	1		1	8	1

Notes:  $\hat{p}_{mt}$  is real import price,  $\hat{p}_{et}$  is real energy price,  $\hat{w}_t$  is unit labour cost.

Table 2 provides the results for developed markets. Except for France, all the developed markets exhibit a positive sign for the output gap. The labour share is consistently positive. Supply shock variables are significant, thereby providing the necessary, though not sufficient, basis for the significance of the output gap, consistent with the theory. On average forward-looking behaviour is as important as backward-looking behavior. For the GMM results, the J-statistic suggests that all the estimations pass the over-identification test.

The first-stage F-statistic is also above the threshold of 10, which suggests that the model is not weakly identified.

The more powerful and identification-robust AR statistic shows that the hybrid new Keynesian Phillips curve suffers from weak identification, except for Australia. However, as noted by Nason and Smith (2008), the AR statistic may lack power, especially when there are many instruments and where there is overidentification. In the context of our study, this is not a problem, since the test finds weak identification. In addition the standard hybrid model exhibits significant serial correlation in the residuals, except for the UK. The 2SLS estimations are not as efficient as the GMM estimations because of the relatively higher standard errors. However qualitatively the results are the same.

Table 3 reports results for emerging markets. Except for Brazil, we obtain positive and significant parameters for the output gap. Except for Turkey, we also obtain positive and significant parameters for the labour share. The Turkish case constitutes an empirical rejection of the new Keynesian model, since it is not theoretically plausible to have the labour share negatively affecting inflation. Across the economies, the J-statistic suggests that the model passes the test for over-identification. The first-stage F-statistic also suggests that there are no identification problems, except for Brazil. However the more powerful AR-test suggests that there are identification problems for Mexico, South Africa and Turkey. Here too, we observe the presence of significant serial correlation in the residuals.

Table 2: Estimates of the Phillips curve (Advanced Economies) (Eq.9)

	Australia		Canada		Germany		France		US		UK	
	GMM	2SLS	GMM	2SLS	GMM	2SLS	GMM	2SLS	GMM	2SLS	GMM	2SLS
$\gamma_f$	0.49* (0.00)	0.49* (0.04)	0.41* (0.04)	0.37* (0.09)	0.49* (0.02)	0.50* (0.08)	0.55* (0.02)	0.55* (0.04)	0.51* (0.02)	0.49* (0.05)	0.45* (0.01)	0.45* (0.03)
$\gamma_b$	0.52* (0.00)	0.51* (0.04)	0.60* (0.04)	0.64* (0.09)	0.51* (0.02)	0.50* (0.07)	0.45* (0.02)	0.45* (0.04)	0.49* (0.02)	0.51* (0.05)	0.56* (0.01)	0.56* (0.03)
$\psi_s$	0.02* (0.00)	0.01 (0.04)	0.05** (0.03)	0.09 (0.07)	0.03* (0.01)	0.07** (0.04)	0.08* (0.03)	0.08 (0.05)	0.02* (0.01)	0.01 (0.06)	0.08* (0.01)	0.07* (0.03)
$\psi_y$	0.01* (0.00)	0.00 (0.04)	0.04* (0.02)	0.05 (0.06)	0.03* (0.01)	0.08* (0.04)	-0.02* (0.01)	-0.02 (0.02)	0.04* (0.01)	0.06 (0.04)	0.05* (0.01)	0.05 (0.04)
$\psi_{\Delta y}$	-0.02* (0.01)	-0.05 (0.07)	0.19* (0.08)	0.39* (0.16)	0.06* (0.02)	0.10 (0.07)	0.05** (0.03)	0.05 (0.06)	0.11* (0.03)	0.30* (0.12)	-0.23* (0.03)	-0.25* (0.07)
$\psi_{fp}$	0.02* (0.00)	0.01 (0.03)	0.03* (0.01)	0.27* (0.10)			0.07* (0.01)	0.06* (0.03)				
$\psi_{ep}$	0.04* (0.00)	0.03* (0.01)	0.11* (0.05)	0.06* (0.03)			0.02* (0.00)	0.03* (0.01)				
$\psi_{mp}$	0.01* (0.00)	0.01 (0.01)			0.01* (0.00)	0.00 (0.01)			0.02* (0.00)	0.03* (0.01)	0.03* (0.00)	0.03* (0.01)
$R^2$	0.97	0.97	0.95	0.95	0.96	0.96	0.99	0.99	0.98	0.97	0.99	0.99
$J^\dagger$	1.00	0.22	0.80	0.02	0.90	0.59	0.96	0.00	0.99	0.00	0.99	0.00
$F_1$	15.9	24.0	23.9	58.3	42.0	75.5	138.5	168.4	41.5	61.2	84.8	84.8
$AR^\dagger$	0.85	0.10	0.01	0.08	0.61	0.96	0.00	0.00	0.00	0.00	0.00	0.00
$DW$	2.88	2.90	2.50	2.15	2.82	2.68	2.51	2.50	2.57	2.22	2.26	2.26
$LM^\dagger$	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.18	0.22
$ARCH^\dagger$	0.00	0.00	0.15	0.69	0.10	0.20	0.17	0.11	0.00	0.00	0.00	0.00
$JB^\dagger$	0.00	0.00	0.86	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Note: Std errors in parentheses, \*Significant at 5%, <sup>†</sup>Probability,  $F_1$  first-stage F-stat, ARCH(4) test for heteroskedasticity.

Table 3: Phillips curves estimations (Emerging markets) (Eq.9)

	Brazil		Mexico		Poland		S.Africa		S.Korea		Turkey	
	GMM	2SLS	GMM	2SLS	GMM	2SLS	GMM	2SLS	GMM	2SLS	GMM	2SLS
$\gamma_f$	0.52* (0.01)	0.57* (0.10)	0.51* (0.01)	0.50* (0.06)	0.58* (0.01)	0.60* (0.07)	0.51* (0.01)	0.26* (0.11)	0.46* (0.01)	0.45* (0.07)	0.49* (0.03)	0.50* (0.06)
$\gamma_b$	0.49* (0.01)	0.43* (0.10)	0.49* (0.01)	0.50* (0.05)	0.43* (0.01)	0.41* (0.06)	0.49* (0.01)	0.74* (0.11)	0.54* (0.01)	0.54* (0.07)	0.52* (0.03)	0.50* (0.05)
$\psi_s$	0.02* (0.01)	-0.00 (0.09)	0.09* (0.01)	0.09 (0.06)	0.003 (0.001)	0.00 (0.03)	0.03* (0.01)	0.09 (0.09)	0.03* (0.00)	0.03 (0.04)	-0.17* (0.00)	-0.09 (0.10)
$\psi_y$	-0.08* (0.01)	-0.36* (0.16)	0.02* (0.00)	0.02 (0.03)	0.02* (0.00)	0.01 (0.05)	0.05* (0.02)	0.29* (0.13)	0.04* (0.00)	0.04 (0.04)	0.05* (0.02)	0.04 (0.05)
$\psi_{\Delta y}$	-0.02* (0.01)	-0.36* (0.16)	-0.07* (0.01)	-0.07 (0.05)	-0.15* (0.01)	-0.25* (0.11)	-0.17* (0.04)	0.69** (0.40)	-0.06* (0.00)	-0.06 (0.07)	-0.03* (0.01)	-0.00 (0.03)
$\psi_{fp}$			0.24* (0.01)	0.24* (0.07)	0.14* (0.00)	0.11** (0.06)	0.02* (0.01)	0.13** (0.07)	0.05* (0.00)	0.05 (0.05)	0.26* (0.04)	0.21* (0.11)
$\psi_{ep}$	0.01* (0.00)	0.03* (0.01)	0.11* (0.01)	0.11* (0.05)	0.13* (0.00)	0.17* (0.08)			0.04* (0.00)	0.04* (0.02)	0.09* (0.02)	0.09* (0.05)
$\psi_{mp}$			-0.01* (0.00)	-0.01 (0.03)	-0.02* (0.00)	-0.02 (0.03)	0.02* (0.01)	0.01 (0.04)	0.01* (0.00)	0.01 (0.02)		
$R^2$	0.95	0.89	0.98	0.98	0.98	0.98	0.96	0.93	0.85	0.85	0.98	0.99
$J^\dagger$	1.00	0.95	1.00	0.47	1.00	0.23	1.00	0.91	1.00	0.48	0.89	0.15
$F_1^\dagger$	2.02	4.86	1198	184	25.0	25.0	13.0	18.3	21.9	21.9	46.3	69.2
$AR^\dagger$	0.14	0.99	0.06	0.06	0.18	0.13	0.02	0.99	0.19	0.19	0.00	0.02
$DW$	2.03	2.23	2.53	2.53	2.52	2.46	2.34	1.90	2.81	2.81	2.75	2.45
$LM^\dagger$	0.00	0.61	0.06	0.05	0.00	0.09	0.00	0.00	0.00	0.00	0.00	0.00
$ARCH^\dagger$	0.05	0.93	0.40	0.38	0.80	0.89	0.00	0.00	0.05	0.30	0.23	0.69
$JB^\dagger$	1.00	0.54	0.41	0.50	0.39	0.28	0.78	0.00	0.00	0.64	0.00	0.61

Note: Std errors in parentheses, \*Significant at 5%, †Probability,  $F_1$  first-stage F-stat, ARCH(4) test for heteroskedasticity.

The above results show that our formulation resolves the perverse sign in new Keynesian Phillips curve literature. However, we note that the standard hybrid model suffers from weak identification. This problem is pervasive, as documented by Nason and Smith (2008). In addition, we note that the standard hybrid model exhibits significant serial correlation in its residuals which implies that the model may be misspecified.

### 3.3 Estimations with serial correlation extension

We follow Zhang and Clovis (2010), Scheufele (2010) and Abbas and Sgro (2011) by augmenting the standard hybrid model with additional lags of inflation. This has potential to eliminate serial correlation in the residuals. We mention that despite adding higher lags of inflation, we could not eliminate serial correlation. Consequently, we had to supplement these higher lags of inflation with a set of dummy variables in order to eliminate serial correlation.

Table 4 presents the GMM results for eq.(10). The parameter  $\phi(1)$  denotes the sum of coefficients of lags of inflation from the second lag onwards. Two results stand out from Table 4; we could not eliminate serial correlation for Germany and the US despite addition of some dummy variables. In the case of Germany, higher lags lead to the complete collapse of the equation in the sense that no variable becomes significant. In relation to the US, the addition of dummy variables does eliminate serial correlation however the equation also collapses. Nevertheless, the addition of higher lags is justified, since they are statistically significant. The results also imply little evidence for the dominance of forward-looking behaviour for Canada, the US and the UK, in line with the findings by Nason and Smith (2008).

In relation to emerging markets we also observe that the results are in line with theory. Forward-looking behaviour appears to be dominant in Mexico and Poland and not in the rest of the emerging markets under consideration. Brazil is the only country with a perverse sign on the supply shock variable, but nevertheless has a positive sign for the output gap. This means that more reliable supply shock variables are required to validate the Phillips curve in Brazil. For the rest of the emerging market economies, the results remain qualitatively similar as the earlier ones. Lastly, across all the results, the AR-test suggests that there is no weak identification.



Table 4: GMM estimates of the Phillips curve (Eq.10)

Lags	Developed Markets										Emerging Markets							
	Aus.		Can.	Ger.	Fr.	US	UK	Br.	Mex.	Pol.	S.Afr.	S.Kor.	Turk.					
	17	5	5	4	22	5	8	22	20	5	16	4						
$\gamma_f$	0.51* (0.01)	0.46* (0.06)	0.50* (0.03)	0.60* (0.02)	0.42* (0.02)	0.40* (0.02)	0.46* (0.01)	0.54* (0.03)	0.65* (0.04)	0.48* (0.01)	0.42* (0.02)	0.41* (0.04)						
$\gamma_b$	0.42* (0.01)	0.63* (0.08)	0.53* (0.05)	0.45* (0.05)	0.87* (0.05)	0.65* (0.03)	0.98* (0.04)	0.41* (0.04)	0.44* (0.07)	0.64* (0.07)	0.59* (0.02)	0.69* (0.07)						
$\phi(1)$	0.11* (0.01)	-0.16* (0.08)	-0.06** (0.04)	-0.05** (0.03)	-0.29* (0.04)	-0.03** (0.02)	-0.42* (0.08)	0.05* (0.02)	0.05* (0.02)	-0.12* (0.02)	-0.01 (0.02)	-0.08* (0.06)						
$\psi_s$	0.04* (0.01)	0.08* (0.04)	0.05* (0.02)	0.20* (0.04)	0.03 (0.02)	0.10* (0.01)	0.00 (0.01)	0.20* (0.02)	0.06* (0.02)	0.00* (0.01)	0.05* (0.01)	0.04 (0.09)						
$\psi_y$	0.02* (0.01)	0.05** (0.03)	0.03* (0.01)	-0.03* (0.01)	0.04* (0.02)	0.05* (0.02)	0.03* (0.01)	0.10* (0.02)	0.03** (0.02)	0.03* (0.01)	0.04* (0.01)	-0.06* (0.03)						
$\psi_{\Delta y}$	0.01 (0.01)	0.22 (0.11)	0.05 (0.03)	0.01 (0.04)	0.09* (0.04)	-0.13* (0.04)	0.05* (0.02)	-0.09* (0.01)	-0.15* (0.05)	-0.22* (0.03)	-0.14* (0.01)	0.05* (0.02)						
$\psi_{fp}$	0.05* (0.01)	0.16* (0.05)	0.05* (0.02)	0.08* (0.02)				0.49* (0.02)	0.14* (0.02)	0.02* (0.01)	0.07* (0.06)	0.00 (0.06)						
$\psi_{ep}$	0.05* (0.00)	0.04* (0.02)		0.03* (0.01)			-0.01* (0.00)	0.15* (0.02)	0.19* (0.02)	0.04* (0.01)	0.04* (0.01)	0.09* (0.03)						
$\psi_{mp}$	0.01* (0.00)		0.01* (0.00)		0.01* (0.00)	0.02* (0.00)		-0.05* (0.01)	-0.04* (0.02)	0.00 (0.00)	0.00 (0.00)							
$R^2$	0.99	0.97	0.98	0.99	0.98	0.99	0.98	0.99	0.99	0.99	0.95	0.99						
$J^\dagger$	1.00	0.75	0.79	0.75	0.99	0.86	0.96	0.92	0.17	0.99	0.99	0.69						
$AR^\dagger$	0.99	0.51	0.99	0.14	1.00	0.90	0.85	0.70	0.71	0.71	0.57	0.76						
$DW$	2.40	2.32	2.67	2.02	2.57	2.81	2.52	2.32	2.34	2.43	2.52	1.95						
$LM^\dagger$	0.20	0.16	0.00	0.28	0.00	0.33	0.13	0.18	0.71	0.07	0.21	0.81						
$ARCH^\dagger$	0.65	0.04	0.07	0.81	0.00	0.17	0.32	0.83	0.06	0.16	0.48	0.09						
$JB^\dagger$	0.57	0.26	0.57	0.88	0.22	0.00	0.58	0.48	0.08	0.37	0.70	0.47						

## 4. Conclusion

The perverse sign of the forcing variable in new Keynesian Phillips curve estimations has been viewed as proof of the rejection of the model by the data. Gali et al.(2001) suggest using the labour share instead of the output gap, on the grounds that the output gap delivers the wrong sign. However Rudd and Whelan (2005b) finds that the labour share does not play a significant role in driving inflation. Mazumder (2010, 2011) also finds that the new Keynesian model exhibits the wrong sign even when there is a procyclical measure of marginal cost in the case of the US. In the Euro area, Lawless and Whelan (2011) also find consistently negative signs of the labour share in sector-level data. In the context of emerging markets, Agénor and Bayraktar (2010) do not find a significant impact of the output gap on inflation. Similarly Vašíček (2011) finds the forcing variable to be insignificant and has the wrong sign in the context of transitional economies.

In this paper we assumed a non-linear input-output technical relation as suggested by Batini et al.(2005) and on that basis derived a more elaborate measure of marginal cost. Our formulation can be viewed as an extension of the work by Petrella and Santoro (2012) in that we conduct the analysis at a macroeconomic level and include excess demand pressure in our formulation. The resultant Phillips curve comes very close to the traditional triangle Phillips curve in that it features the forward and backward looking expectational element, the output gap and speed-limit effects to capture demand pressure, the labour share and relative input prices to capture supply shock variables.

Our formulation resolves the sign problem that plagues the new Keynesian model and, for the case where the output gap exhibits a negative sign, we are able to provide a structural interpretation based on the non-linearity of the technical input-output relation. We also test whether our results hold in the case where higher lags of inflation are admitted in the model, following Zhang and Clovis (2010). We find that indeed, the problem of the perverse sign on the labour share and the output gap is largely resolved. We therefore conclude that the inclusion of supply shock variables is important to render parameters of the new Keynesian model of inflation plausible.

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