On Keynes's Z-function

Francesco Strati

Academy of Advanced Scientific Studies (RC)

29. November 2012

On Keynes's Z-function

Francesco Strati
Academy of Advanced Scientific Studies
Via Filippini, 71
francesco.strati@aass.it
Reggio Calabria

Abstract

This paper is intended to give a general, but rigorous view about what is the Z-function and what are the hidden relations of the Keynes's General Theory. In Section 1 I shall depict the concept of probability and that of the weight of argument, in Section 2 I shall introduce quite an important definitions such as the Z-function is different from the Z^*-curve, and some paramount notions. The Section 4 is intended to grasp the importance of the chapters 20-21 of the General Theory, whereas in Section 5 I shall comment, very quickly, some properties of Z^* in a topological view.

Contents

1 The meaning of the expectation **2**

2 The tools **4**

3 Subtle relations in the General Theory **8**

4 A very brief topological comment on Z^* **9**

1 The meaning of the expectation

The Keynes's General Theory is not a difficult book itself and it is not mathematically bad-written. I reckon that this book is a fair son of a genius mind, a difficulty might stem from its unconventional geniality, it cannot be read as an everyday book about macroeconomics puzzles, it has to be read as a book brimful of lurking intuitions. It is because of this that there are several interpretations of his thoughts. In order to comprehend the Keynes's masterpiece it is obviously needed at least a weak knowledge about his less famous dissertation *A Treatise on Probability*. Althought this is the only Keynes's work concerning probability matters, it is fundamental in order to soundly understand the later Keynes's works. The General Theory (GT) is uncertainty soaked and the notions of knowledge and rational belief play a central role in the entrepreneur-consumer decision making: the core of the GT. It has always been stressed the important role of the demand and thus of the propensity to consume, liquidity-preference et cetera, and it was left in the beckground the role of the supply side. This is quite an interesting lack in economic analysis inasmuch supply matters play a paramount role in order to reach the point of effective demand and hence to reach a level of employment. This is reached by combining several expectations of prices and profits, and thus the point of effective demand gives rise to an employment level. It is fascinating to say that there is a bright combination between psychological and real world, rather Keynes gives birth to a model where psychological puzzels rule real matters and viceversa. I daresay that a psychological framework of an entrepreneur hinges on his belief and that of the environment, namely pessimism or optimism. The belief is rational and thus this is not a subjective *caprice*

> If a man believes something for a reason which is preposterous or for no reason at all, and what he believes turns out to be true for some reason not known to him, he cannot be said to believe it rationally, although he believes it and it is in fact true. On the other hand, a man may rationally believe a proposition to be probable when it is in fact false. The distinction between rational belief and mere belief, is not the same as the distinction between true beliefs and false beliefs. The highest degree of rational belief, which is termed certain rational beliefs, corresponds to knowledge Keynes J.M. (1921).

It is thus important to understand that by the term expectation it is intended a *rational* expectation, hence the Keynes's view of decision making underpins the notion of effective demand. The decision making is of the form already seen in the *Treatise* and a central role is assumed by its weight of argument (ω) (see Strati F. (2012), Strati F. (2010)). The probability relation is a/h; our premisses consist of any set of propositions h and our conclusion consist of any set of propositions a, it is a particular kind of relation $a\mathcal{R}b$, it can be thought of as a sub-relation of $\mathcal{R}$. It is important to know that that h is the subjective part, whereas a/h is objective, rather, as a matter of definition, the Keynes's probability is objective but only once the subjective h has been fulfilled. A decision is make taking into account a kind of knowledge: the *acquaintance*, together with the intuition, is a direct knowledge *of* an object; a certain belief. In Keynes's world (and our one) very few events can be solved by using *direct* knowledge and thus it is important the notion of *indirect* knowledge. In summary, there are two kinds of knowledge: by things and by truth, the former is given by acquaintance whereas the latter by a proposition. When we percieve something or understand it we can talk about direct acuaintance, albeit we do not have a knowledge yet. We might have an experience such as a senstation of an object, but we need an intuition about that object. Given that, this object u is linked with our h and thus we know h because of our acquaintance of u, in few words this is the so called indirect knowledge, hence h hinges on u. In order to soundly comprehend these puzzles see Strati F. (2012). The weight of argument plays a significant role in this kind of knowledge:

i $\omega = 0$ total uncertainty (the case of the famous *Animal Spirit*);

ii $\omega < 1$ but > 0 one faces uncertainty (it is the general case in the GT);

iii $\omega = 1$ complete knowledge.

I daresay that ω is the completeness of information available, rather in the case of $\omega = 1$ there is a direct knowledge *of* an object and thus it is known directly by acquaintance, hence it is certain. It is the case of the *classical* quantitative theory of money and the movement of money and prices. When $\omega = 0$ there is no information *about* an object, hence there will not be a rational decision making. The case $\omega < 1$ is the most important, rather there is a certain level of knowledge *about* an object but it is not complete, there exists something that we do not know and thus we cannot control, however it is the general case of the General Theory. For now it is important to understand that the weight of argument is the level of information available (evidence), of course it is different from the concept of probability

> The weight, to speak metaphorically, measures the sum of the favourable and unfavourable evidence, the probability measures the difference Keynes J.M. (1921).

The notion of ω encompasses the important role played by the information in order to build up a good estimation of the probability a/h. Thus the information about u infers the information about h given ω. The point of effective demand is the matched $Z = D$ functions, and thus the expected profits and the expected prices play a paramount role so as to establish the level of employment needed. It is straightforward the importance of ω, a reliability of an expectation is underpinned by the level of ω and thus the weight of argument, the level of information available, is crucial in the decision making in order to forecast prices and profits for a wanted production (its costs) and a wanted employment.
When Keynes said that the probability (its great part) cannot be represented by a number, he intended that the propability must be represented by an interval (see Brady M. E., Gorga C. (2009)). The $\omega \in [0, 1]$, whereas the uncertainty $u \in [1, 0]$ (different from the u used above), it is plain that the uncertainty is the *inverse* of the weight of argument $0 \leq \omega \leq 1$ and $1 \leq u \leq 0$. When $u = 0$ then $\omega = 1$, thus $u = f(\omega)$ Brady M. E., Gorga C. (2009). The Keynes's concepts have been misunderstood and his theory is treated as if the $\omega = 1$ would be the general case of the GT and thus there is only risk and not uncertainty. This implies that $L_2 = 0$ ($M_2 = 0$), the propensity to hoard will be zero, $M = M_1$, the only transaction-money will be treated, so $u = 0$ and $\omega = 1$.
The Keynes's concept of decision making is well depicted by the *coefficient of weight-risk* denoted by c:

$$c = p\left[\frac{1}{(1+q)}\right]\left[\frac{2\omega}{(1+\omega)}\right] = \frac{2p\omega}{(1+q)(1+\omega)} \tag{1}$$

in Keynes' words

> If A is the amount of good which may result, p its probability ($p + q = 1$), and E the value of the 'mathematical expectation' so that $E = pA$, then the risk is R, where $R = p(A - E) = p(1 - p)A = pqA = qE$. This may be put in another way: E

measures the net immediate sacrifice which should be made in the hope of obtaining A; q is the probability that this sacrifice will be made in vain; so that qE is the risk Keynes J.M. (1921) p.315.

Furthermore it is plain that the aim of a decision maker is to maximize cA. If the $\omega = 1$ then

$$c = p\left[\frac{1}{(1+q)}\right] \cdot 1 = \frac{p}{(1+q)}$$

and thus there will be only the risk calculation in c, but it is a special case of the GT.
In this section we have seen the important and, alas, underestimated theory behind the concept of expectation, the core notion of the GT mechanism. In the following sections I shall use the notion of ω or c in order to clarify their importance in the GT, they are the soul of the expectations treated in the General Theory. In particual I shall depict their meaning in the paramount context of the elasticities of the chapters 20 and 21 of the GT.

2 The tools

Once I have explained, albeit broadly, the concept underpins the Keynes's expectation, it is now time to know the tools of the heart of the GT.

Equilibrium In Hayes M. (2008) it is clarifed the interesting concept of the Keynes's equilibrium, Hayes divides it into four different moments; in his words:

- *a* a state of balance between countervailing forces or processes, notably supply and demand;
- *b* a state in which no party has both reason and power to change their position;
- *c* a state in which all parties make their preferred choices;
- *d* a state in which expectations are fulfilled.

Following the Marshall approach, Keynes stressed the a, b-equilibrium, this is intended in its "mechanical sense of the balancing of supply and demand through price competition in perfect markets: a state of rest, however temporary" Hayes M. (2007). Furthermore, the competitive equilibrium is given by the choices of entrepreneurs, consumers and investors, and thus the state of rest and the choices will be metched. It is important to know that the equilibrium is in rest, but at a given moment, because it might change its position from day to day. The *time* plays a central role in the notion of equilibrium and it is of utmost importance in defining what kind of time is used in the GT. The Keynes' shortest unit of time is the day, the elder Marshall's market and short periods, as much the difference between realised and expected prices and between income and effective demand; between something that has just happened (thus known) and something that will happen (unknown). An entrepreneur decides on production and employment in two kinds of period: the *day* and the *period of production* made of several *days*. The day is "the shortest interval after which the firm is free to revise its decision as to how much employment to offer" Keynes J.M. (1936), but an employment adjustment will be done at the end of the period of production. That is to say, each day an entrepreneur maximises his expected profit and thus he adjusts the level of employment in his firm, but the actual adjustment could be done at the end of the production. In each day (that might be or not the calendar day) there is a $Z_n = D_n$, that is to say as much D_n^* (effective demand) as days, each D_n^* is the daily profit maximisation and psychological adjustment of employment given the expectations "of the market prices they will receive for the heterogeneous finished output that will emerge at the end of the various production periods" Hayes M. (2007). Hence the supply curve, denoted by Z^*. I am talking in a microeconomic perspective so as to clarify the notions, but of course they have to be seen through a macroeconomic view as we shall see later on.

Income and Effective Demand Hayes has talked about the *Swedish method* or *ex ante-ex post* method. It is used, for instance, by using the keynesian cross where the output is given when the expectations of income (which is *ex ante*) are fulfulled (*ex post*). In this case there is an (a, d)-equilibrium and, of course, it is not the case of an (a, b)-equilibrium. According to Hayes, Keynes did not care about the fulfillment of the expectations and the point of effective demand is by no means a *Swedish* point of equilibrium. Moreover, this model has only one point of equilibrium whereas Keynes intended several points of equilibrium from day to day. The swedish method is a very huge mistake in defining Keynes's approach and, alas, it is the most famous expression of Keynes in the academic world but it is not a Keynes's view at all; it is such a shame for that brilliant mind. Thus, following Hayes, this equilibrium condition is

$$Y^* = D^* = D$$

where Y^* is the income in equilibrium, D^* the effective demand and D the aggregate demand function. It is a very simply view because, in a general fashion, all of them are different. A more interesting equilibrium point could be

$$Y^* = D^* = Z^*(= (D = Z))$$

where Z^* is the aggregate supply curve, Z the aggregate supply function. It is worth noting that there is a subtle but fundamental distinction, noted in Heller C. (2009), from Z^* to Z which is clear by the equality $Z^* = (Z = D)$ (it will be treated in the next section). It is important to say that the expected income and the realised income are different $Y \neq Y^*$, furthermore the employment, and thus the production, is given by the expectations of profits and prices that are quite a different things from the espected income Y^*. Thus $Y \neq Y^*$ and $D^* \neq Y^*$, moreover $D^* \neq D$ but $D^* = Z^*$, thus

$$Y \neq Y^* \neq D^* = Z^*.$$

The point of effective demand is a bunch of *expectation* points and cannot be the same thing of an expected value or an expected point. The employment is *always* in equilibrium from day to day, the quantum entity of the GT.

What about Z and D? In Keynes' words

> Let Z be the aggregate supply price of the output from employing N men, the relationship between Z and N being written $Z = \psi(N)$ which can be called the *aggragate supply function*... let D be the proceeds which entrepreneurs expect to receive from the employment of N men, the relationship between D and N being written $D = f(N)$, which can be called the *aggregate demand function*.

The aggregate Z-function relates the expected proceeds (value added, namely the expected profits and the factor costs) from heterogeneous output to the employment, this can be homogeneous if one refers to it by using monetary wage measure. The population's level of income is of utmost importance in defining the level of employment, rather $Y = D_1 + D_2 = C + I = PO$, where Y is the realised aggregate demand, D_1 the propensity to consume, D_2 the propensity to invest, P the current price and O the finished output. We call $D = \rho O$ the expected aggregate demand demand where ρ is the expected price and O the wanted level of output. The propensity to consume, as the reader knows, grows slower than the income, and thus in order to obtain the full employment it is needed to fill the gap between income and actual consume by the investment. It is now sensible the importance of those *psychological factors*. The propensity to consume is denoted by $\chi(N)$ and the equilibrium point is $D_1 + D_2 = D^* = \psi(N)$, where $\psi(N)$ is the aggregate supply function and $C + I \in \rho D$, that is to say they are intended to be in their *expected* meaning. Now, so as to fill the gap, $D_2 = \psi(N) - \chi(N)$ and hence $\psi(N) = D$, and we shall obtain the point of effective demand of full employment, but it is of course a hypothetical point, generally D_2 does not fill the gap, it is a very strong property.
Now it is important the difference $Z^* \neq Z$, rather "the slip of pen" of Keynes (see Heller C. (2009)) is that he defined the function of employment as the inverse of the aggregate supply *function* Z. It is not so because it is the inverse of the aggregate supply *curve* Z^*. The latter is given by the connection between the aggregate production function $O = \phi(N)$ and the $Z = \psi(N)$, where the employment N implies a production O. In order to give an equality among the concept of supply curves of Marshall and Keynes it should be added the user costs, but so as to emphasize those notions belonged to Keynes only, the aggregate supply *curve* is

$$\rho = \frac{Z}{O}$$

and thus

$$\rho O = Z$$

but this means that

$$Z = \rho O = D$$

therefore

$$D = \rho O.$$

This is the expected aggregate demand I have defined above, it is important the difference: $Y = PO = C + I$ realised and $D = \rho O$ expected. Now, the inverse of the function of employment cannot be the Z-function because we know that the Z is made of factor costs (WN in this paper, where W denotes the wages and N the employment) and the profits Π, that is to say $Z = WN + \Pi$. These are some factors by which a level of employment can be obtained, it is just one side of the puzzle. The employment is given by the point of effective demand D^* where $Z = D(= \rho O)$, and thus the aggregate cure Z^*. We state that: the inverse of the function of employment is the aggregate supply curve. Thus the linearity (following the infamous footnote on pp.55-6 of the GT) is given by Z^* and obviously not by Z. Given the function of employment $N = F(Z^*)$, the derivative

$$\frac{dZ^*}{dN} = w$$

and thus

$$\frac{dN}{dZ^*} = \frac{1}{w}$$

where w is the nominal wage and hence Z^* is linear with a slope given by the reciprocal of the nominal wage. The problem was that $Z_w = \frac{Z}{w}$ cannot be w and $\frac{1}{w}$ at the same time, whereas it is possible by using the Z^* curve.

The point of equilibrium Following Brady M. E., Gorga C. (2009), the mathematical framework behind the linearity of the employment function is the clue that Keynes's thoughts were clear and mathematically elegant. The $D = Z$ is $\rho O = WN + \Pi$ and thus dividing for w, so as to have the quantity in terms of money-wage, $\rho_w O = N + \Pi_w$. It is the same to write $\rho_w O - \Pi_w = N$ (namely $D_w - \Pi_w = N$). The derivative with respect to N

$$\frac{d\rho_w O}{dN} - \frac{d\Pi_w}{dN} = \frac{dN}{dN},$$

thus

$$\frac{d\rho O_w}{dN} - \frac{d\Pi_w}{dN} = 1.$$

In calculating the product derivative for the first term on the left hand side

$$\frac{\rho_w dO}{dN} + \frac{d\rho_w O}{dN} - \frac{d\Pi_w}{dN} = 1 \tag{2}$$

but given that Keynes on p. 283 of the GT writes that $Od\rho_w = d\Pi_w$, namely

$$\begin{aligned} Od\rho_w &= dD_w - \rho_w dO \\ &= dD_w - (\text{primary cost})dO \\ &= d\Pi_w. \end{aligned}$$

Now, the equation(2) can be written as

$$\frac{\rho_w dO}{dN} + \frac{d\Pi_w}{dN} - \frac{d\Pi_w}{dN} = 1$$

hence

$$\frac{\rho_w dO}{dN} = 1$$

and thus $\rho_w \phi'(N) = 1$, it means that the marginal product of labour is equal to one, that is to say each additional worker produces one unit of additional output, hence $\phi''(N) = 0$. It is worth noting that $\rho_w dO = dN$ given that

$$\rho_w \frac{dO}{dN} = \frac{\rho}{w}\frac{dO}{dN} = 1$$

thus

$$\frac{dO}{dN} = \frac{w}{\rho}.$$

Now, given that $d\Pi_w = d(D_w - N) = dD_w - dN$ then $\int d\Pi_w = \int dD_w - \int dN$ therefore $\Pi_w = D_w - N$, this equation times w gives $\Pi = D - WN$ namely $\Pi = \rho O - WN$. Adding WN to both sides we obtain

$$\Pi + WN = \rho O \;\; \textit{or} \;\; Z = D \tag{3}$$

that is the point of effective demand. It is worth noting that the expected profit hinges on either the prices are greater, equal or less than the actual prices

3 Subtle relations in the General Theory

The chapters 20-21 of the GT are of utmost importance, albeit have been always skipped by economists. The whole Keynes theory is formally depicted through these two chapters in terms of *expectational elasticities* in wage units and thus this analysis is paramount in order to comprehend the theory of supply curve/function. It will be clear the relation between the notion of elasticity and the subject of probability we have seen in section 1.

Elasticities Given the employment function $N = F(D^*)$, it is important to know that it is possible to add up all individual employment functions and thus

$$F(D_w) = N = \sum N_n = \sum F_n(D_w).$$

There are six expectational elasticities:

e_e is the elasticity of employment, the variation of the employed workers with respect to the costs of the firm's production

$$e_e = \frac{dN}{dD_w} \cdot \frac{D_w}{N}$$

e_o the elasticity of production, how much the production of a firm grows when this firm has to face a particular effective demand

$$e_o = \frac{dO}{dD_w} \cdot \frac{D_w}{O}$$

e_p the monetary prices elasiticy, their elasticity with respect to variations of the effective demand in terms of money

$$e_p = \frac{dp}{dD} \cdot \frac{D}{p}$$

e_w the elasticity of the monetary wages with respect to variations of the effective demand in terms of money

$$e_w = \frac{dw}{dD} \cdot \frac{D}{w}$$

e_d is the elasticity of the effective demand with respect to variations in the quantity of money

$$e_d = \frac{dD}{dM} \cdot \frac{M}{D}$$

e is the elasticity of the monetary prices with respect to the quantity of money

$$e = \frac{dp}{dM} \cdot \frac{M}{p}$$

In chapter 10 of the GT, Keynes used realized results $Y = PO$, thus N is the level of employment, N_1 aggregate employment in the consumption goods industries, and N_2 aggregate employment in the investment industries, it is straightforward that $N = N_1 + N_2$, but in this chapter the theory is treated as realised, thus $\rho = P^*$, where P^* is the exact value of P.

In chapters 20-21 Keynes is referring to his theory in an expectational view and thus this approach is soundly different from the methodology used in the chapter of multiplier (ch.10). Moreover he gave a dynamical theory by means of elasticities so as to depict the relations which are the heart of the GT.
If we denote by M the quantity of money, by V its velocity, then

$$MV = Y = PO \ \text{ if } \ \omega = 1, \ M_2 = L_2 = 0, \ M = M_1, \ [e = 1, e_d = 1];$$

$$MV = D = \rho O \ \text{ if } \ \omega < 1, \ M_2 = L_2 > 0, \ M = M_1 + M_2, \ [e_d < 1].$$

If $e_d = 1$, people hold a constant proportion of their income in terms of money and there is no propensity to hoard Brady M. E., Gorga C. (2009), rather there is no uncertainty $\omega = 1$ and thus there is not any speculative motive, the $M = M_1$ (transaction money demand) and hence $e = 1$, that is to say the quantity of money moves proportionally to the variations of prices: the quantitative theory of money. Furthermore, given that $\rho_w \psi'(N) = 1$, we have that $w/\rho = \psi'(N)$, namely the marginal product of labour (mpl) is equal to the marginal factor costs, thus

$$\frac{w}{\rho} = \frac{\text{mpl}}{e} \ \text{ and } \ \frac{w}{\rho} = \frac{\text{mpl}}{\omega}$$

this tells us that if $e = 1$ and thus $\omega = 1$ there will be another *classical* property that is just a special case in the Keynes's theory.
If $e_d < 1$ there is a degree of the propensity to hoard, whereas if $e = 0$ it will be a maximum level of the propensity to hoard and the velocity V will be zero. It is now plain the importance of these chapters in the Keynes's formal approach, each elasticity is linked with the other. In the case $e_d = 1$ the population holds a constant portion of money and thus $L_2 = 0$ and $u = 0$. Now, $e = e_p \cdot e_d$ thus if $e_d = 1$ we shall obtain $e = 1$ because $e_p = 1$ too, rather if the effective demand is moving in the same proportion of M, then the movements of the prices hinge on the movements of D^* in terms of money, therefore the prices move proportionally to the variation of D^* that is the same variation of M. Also $e_w = 1$ because the movement of money wages are in terms of D^* and thus in terms of e, hence it is the most important elasticity.
The weight of argument is hidden behind the e-elasticity level, rather its values are always the same, this relation ensure the use of probability (treated in the *Treatise*) in the GT. It is plain that the movements of D^* by e_d are of utmost importance in order to define all the elasticities such as e_w, e_p, e_e et cetera.
An important relation is that of the production function $O = \phi(N)$, where its elasticity is

$$\frac{\frac{dO}{dN}}{\frac{O}{N}} = \frac{\frac{N}{O}}{\frac{dO}{dN}}$$

where $e'_o = e_o/e_e$ that is to say $e_e e'_o = e_o$, thus when $e_e e_o = 0$, then $e_d = 1$ because there will be full employment given that D^* does not imply movement of N. Furthermore if $e_e e_o = 1$ there will be constant returns, hence decreasing returns which grow proportionally to the growth of N.

4 A very brief topological comment on Z^*

We say that Z^* and $N = F(Z^*)$ are homeomorphic, and hence they share the same topological properties. We know that a Keynes's day is the quantum entity in his theory and thus the

smallest, but it is important to understand that although a day is intended quantum-like, the D^* of that day hinges on a sequence of information popped up during that day. There is a sequence of information called *filters* $\mathfrak{F}$, every filter is a convergent sequence of daily events. For instance, if there is a production period made of 4 days: $\mathfrak{F}_1 \subset \mathfrak{F}_2 \subset \mathfrak{F}_3 \subset \mathfrak{F}$. It is so because the empty set does not belong to $\mathfrak{F}$ and every intersection and union of elements in $\mathfrak{F}$ belongs to it. Thus we have a filter of neighborhoods of $\mathfrak{F}(x)$ (a topology $\mathfrak{T}$) where x is the point of D^*. In each day the closure of a set belonged to a day $\bar{D}$ has to encompass the point x of D^* of this day, and every set (a day) is dense in $\mathfrak{F}(x)$. We say that $\mathfrak{F}$ is finer than $\mathfrak{F}(x)$, and thus it is convergent because it becomes smaller and smaller while it is reaching the point x.
The inverse means the continuity, rather if $x \in D^*$ and the neighborhood U of the image $f(x) \in N$, the *preimage* of U, the set

$$f^{-1}(U) = \{x \in D^*; f(x) \in U\}$$

is a neighborhood of x. It is stressed that the inverse of N is Z^* in a topological point of view. Thus given a (daily) sequence of $\{x_1, x_2, \dots\}$ which converges to x in $Z_n = D_n$, so the sequence $\{f(x_1), f(x_2), \dots\}$ converges to $f(x)$ in N.
Another important property is that each set is a Hausdorff space, thus given that the last day is $(X, \mathfrak{T})$, a subspace of it is Hausdorff as well. If this subspace is $(K, \mathfrak{T}_k)$ (if X is today, K is yesterday, or two days ago et cetera), then the two points $a, b \in K \subset X$ and $a \neq b$. These points are those which build the filter of that day up, and it is important to bear in mind. Furthermore, if there exist two subspaces $A, B \in \mathfrak{T}$ where $a \in A$ and $b \in B$, so $A \cap B = \emptyset$, thus $(K, \mathfrak{T})$ is Hausdorff q.e.d.
Given that for every day we have a value for D^*, the Z^*-curve is made of Hausdorff spaces, because for each point x_n of D^*_n, if $x_k \in G$ and $x_n \in B$ then $G \cap B = \emptyset$. The Z^*-curve is also compact, rather given the finite closed sets D^*_n, $Z^* = \{D^*_1 \cup D^*_2, \dots\}$ there exists a $Y \subset \cup_n D^*_n$, then x and $f(x)$ are homeomorphic (as I have already said).

References

Brady M. E., Gorga C. (2009). Integrating the Formal, Technical, Mathematical Foundations of Keyness D-Z Model of the Theory of Effective Demand Into Keyness Decision Theory: Toward a New (and Final?) Interpretation of the General Theory. *International Journal of Applied Economics and Econometrics* **17(3)**, 195-235.

Brady M. E. (2011). Keynes, Mathematics and Probability: A Reappraisal. *International Journal of Applied Economics and Econometrics* **20(2)**, 324-357.

Hayes M. (2007). Keynes's Z-function, heterogeneous output and marginal productivity. *Cambridge Journal of Economics* **31(5)**, 741-753.

Hayes M. (2008). *The Economics of Keynes: A New Guide to the General Theory*, Edward Elgar Publishing Ltd.

Hayes M. (2007). The Point of Effective Demand. *Review of Political Economy* **19(1)**, 55-80.

Heller C. (2009). Keynes's slip of the pen: aggregate supply curve vs employment function *MPRA* Paper no. 12837.

Keynes J.M. (1921). *A Treatise on Probability*, McMillan.

Keynes J.M. (1936). *The General Theory of Employment, Interest, and Money.*, Harcourt, Brace and Company.

Strati F. (2012). Keynes's probability: an introduction to the theory of logical groups. *PKSG*, PKWP1205.

Strati F. (2010). The weight of argument: A suggested interpretation. *PKSG*, PKWP1002.