Imperfect information in a quality-competitive hospital market. A comment on Gravelle and Sivey

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Abstract

I show that the equilibrium derived in Gravelle and Sivey (2010) cannot hold for rational consumers. I then partially characterize the continuum of possible equilibria for rational consumers.

1 Introduction

In Gravelle and Sivey (2010) (GS from now on) the authors study a model of quality choice where two firms (hospitals) compete for consumers (patients). Prices are fixed by a regulator, so hospitals compete in qualities. Patients receive noisy but unbiased signals of true qualities and decide where to purchase the service.

This note explains that GS implicitly assume that patients are irrational. For example, Propositions 1,2, and 3 can only be derived for irrational consumers.[1] The main reason for this is the following. GS argue that patients should visit the hospital that they receive the highest signal of quality from. This contradicts basic rationality if hospitals in equilibrium choose different qualities, as they do for asymmetric hospitals. In this case, patients should visit the hospital that is supposed to choose highest quality, regardless of their signals. For all equilibrium signals, patients receive no information from signals and can safely ignore them. This phenomena is closely related to the literature on imperfect observability and first-mover advantage (Bagwell (1995) and Maggi (1999)).

I start with the description of the model. Then I show how rationality cannot describe the behavior of patients in GS. Then I partially characterize the continuum of equilibria that exist for rational consumers.

2 Model

Here I reproduce the model from GS.

There are two hospitals, H and L, with quality levels q_H and q_L, respectively. All patients demand one unit of hospital care, and have to chose from which hospital to purchase it.

[1] Parts of these propositions hold for rational consumers whenever $\delta_L = \delta_H$, but in this case propositions describe one equilibrium out of a continuum.

A patient receives a signal $\tilde{q}_j = q_j + \epsilon_l$ for each hospital. Signal ϵ_j is distributed uniformly over $(-1/2v, 1/2v)$ where errors are independently distributed between hospitals.

Hospitals cannot change price, as they are fixed by a regulator, and only choose qualities. The cost function of hospital j if it produces D^j units of quality q_j is given by

$$c_j(q_j, D^j) = cD^j + \frac{1}{2}\delta_j q_j^2,$$

where δ_j is a hospital-specific cost parameter of quality. Assume that $\delta_L \geq \delta_H > 0$.

The timing is the following: hospitals simultaneously choose q_H and q_L, consumers receive signals based on these quality choices and decide which hospital to visit. The game ends after one period.

The solution concept use by GS is a stable [*sic*] Nash Equilibrium (NE). It shall become evident soon that NE is not powerful enough for this game of both imperfect and incomplete information. This is because patients do not observe quality choices of hospitals, but they possess signals that hospitals do not know. Thus, it is natural to use, the more restrictive, Perfect Bayesian Equilibrium (PBE) as the solution concept. By doing so I will be able to restrict out-of-equilibrium play by patients. To some extent, GS also use PBE as they require patients to form expectations of (beliefs on) quality using their signals and make optimal decisions given their beliefs. It is the formation of these expectations as described by GS that is inconsistent with rationality. I illustrate this point next.

3 Irrational consumers

To solve the model GS makes the following claim:

"A patient has no prior information about hospital quality and so her expectation of hospital quality after receiving information on quality is $E[q_j|\tilde{q}_H, \tilde{q}_L] = \tilde{q}_j$, $j = H, L$."

I argue next that this statement cannot describe a decision process of a rational consumer. To see the core of the problem, assume $\delta_L > (p-c)v^2$ and $\delta_L > \delta_H$. GS claim that, in this case, Nash Equilibrium exists, is unique, and in equilibrium $q_H^* > q_L^*$ (see Proposition 1).

Now consider a patient who receives a signal $\tilde{q}_j \in [q_j^* - 1/2v, q_j^* + 1/2v]$. According to GS this patient should believe that hospital j's quality is $\tilde{q}_j$. But in fact, if the patient is rational, for any $\tilde{q}_j \in [q_j^* - 1/2v, q_j^* + 1/2v]$ she should expect the quality of hospital j to be q_j^* and not $\tilde{q}_j$.[2] This is because a rational patient should believe hospital j to play according to the PBE if her signal $\tilde{q}_j$ does not indicate a deviation. Thus a rational patient would completely discard her signal instead of using it as her estimate of q_j. Instead, all rational patients should visit hospital H which in equilibrium provides higher quality. In contrast, according to GS, a subset of patients will visit hospital L because for these patients $\tilde{q}_L > \tilde{q}_H$.

Based on the above, the equilibrium derived in GS cannot be correct for rational patients. Only bounded-rational patients who cannot compute the equilibrium and thus

[2]The probability of $\tilde{q}_j = q_j^*$ is zero.

use their own signal as the best guess for hospitals' qualities would use their signal instead of their rational expectation of equilibrium quality.

By, what appears to be, a coincidence, for one case GS do find an equilibrium that is consistent with patients' rationality. When $\delta_L = \delta_H$, equilibrium qualities are equal, so patients are indifferent between hospitals and thus can follow any strategy in choosing between them. In particular, as GS require, Consumers can base their choices on which signal is the highest. Moreover, when one of the hospitals, say hospital j, deviates to a quality $q_j > q^*$, some patients receive signals in the interval $(q^* + 1/2v, q_j + 1/2v]$ which should never arrive in equilibrium. In PBE, rational patients with such signals have to interpret their signal as proof of $q_j > q^*$ and should visit hospital j.[3] This is indeed what GS's requirement that patients follow the highest signal achieves. Patients with signals in $(q^* + 1/2v, q_j + 1/2v]$ always receive higher signal from hospital j and thus always visit j. So, for $\delta_L = \delta_H$, GS find an equilibrium of the model. However, there are more equilibria (e.g. indifferent consumers can be split equally between hospitals instead of following the highest signal). In the following section, I partially characterize possible equilibria of the model and solve for the equilibrium where indifferent patients ignore equilibrium signals.

4 Equilibrium with rational patients

In this section I partially characterize equilibria of the game for rational consumers.

Assume patients expect hospitals to choose qualities q_H^* and q_L^* in equilibrium. From the discussion of the previous section, it is clear that if both hospitals are to set positive quality, they should receive positive demand. This in turn requires that hospitals choose equal qualities.

Lemma 1. *If $q_H^* > 0$ and $q_L^* > 0$, then $q_H^* = q_L^*$*

Proof. Assume the opposite, so that $q_j^* > 0$ for $j = H, L$, and $q_H^* \neq q_L^*$. Without loss of generality, assume that $q_H^* > q_L^*$. In PBE all patients have to visit hospital H, and thus L can set $q_L = 0$ and increase its profit. □

As a result of Lemma 1 there is a continuum of pure strategy equilibria in this model. The reason is that consumers are indifferent between hospitals so they can base their decisions on signals they receive. To determine equilibrium qualities one needs to consider hospitals' deviations. The profitability of such deviations will depend on how patients react to signals, and thus equilibrium qualities can be rather arbitrary.

Lemma 1 leaves three possibilities. Either both hospitals choose the same non-zero quality q^*, or one hospital chooses positive quality while the other one chooses zero quality, or both hospitals choose zero quality. I discard the last possibility in the following lemma.

Lemma 2. *If $q_H^* = q_L^* = 0$ can never hold in equilibrium*

Proof. Assume the opposite so that $q_H^* = q_L^* = 0$. There should be a hospital J that does not receive all the patients in equilibrium. Now consider a deviation by hospital j, to a

[3]Similar argument applies to deviations by j to lower quality.

small but positive quality ϵ. Due to this deviation, for hospital j some patients receive signals in the interval $(1/2v, 1/2v+\epsilon]$. These signals are not on the equilibrium path, thus in PBE these patients should form their beliefs on q_j. Belief $q_j = 0$ is not consistent with received signals. Whatever the beliefs, they should indicate that q_j is positive, and since the other hospital has not deviated and no patient receives an out-of-equilibrium signal for it, all patients who receive signals in $(1/2v, 1/2v+\epsilon]$ should visit j. The number of such patients is linear in ϵ, while the cost of quality is quadratic in ϵ, so there will be small enough ϵ such that hospital j's deviation is profitable. □

We are left with two types of equilibria. Either both hospitals choose the same positive quality, or one chooses zero quality and receives no patients. The latter can be constructed, but is uninteresting because one of the hospitals is inactive.[4]

So the only natural case to consider is where both hospitals choose equal quality. Denote this common quality by q^*. If qualities are equal, patients are indifferent in equilibrium between the two hospitals. This allows patients to be allocated between hospitals in an arbitrary way for all signals on the equilibrium path.[5] This gives rise to a continuum of possible q^*. This is because for each hospital, the choice of q should maximize profits, but because for signals in the interval $[q^* - 1/2v, q^* + 1/2v]$ patients' can base their choice on signals in an arbitrary way, demand response to a deviation in quality is also arbitrary. This means that various levels of q^* can be sustained in equilibrium. Hence, GS provide one possible equilibrium for $\delta_L = \delta_H$. To illustrate equilibrium multiplicity, I will provide another, arguably more standard, equilibrium of the model with symmetric hospitals. It seems to be very difficult, if not impossible, to construct such an equilibrium for asymmetric (in cost) hospitals. In the next section it will become apparent that an equilibrium for asymmetric hospitals does not exist if patients ignore equilibrium signals.

4.1 An equilibrium for symmetric hospitals

Next I will solve for an equilibrium with symmetric hospitals where equilibrium signals are ignored.[6] As is common, I assume that when receiving two uninformative signals, patients are equally likely to visit one of the hospitals.

In order to find q^*, consider what happens if hospital j deviates to a quality $q_j \neq q^*$. If $q_j > q^*$, then some patients will receive signals in the interval $(q^* + 1/2v, q_j + 1/2v]$, that should never be observed in equilibrium. These patients should conclude that hospital j has deviated from its equilibrium strategy. If so, patients with signals in the interval $(q^* + 1/2v, q_j + 1/2v]$ have to update their beliefs about quality of hospital j. The only way to do so is to believe that hospital j has chosen a quality higher than q^*. So these patients will buy from hospital j with probability one. The opposite holds for $q_j < q^*$ for patients who receive signals in the interval $[q_j - 1/2v, q^* - 1/2v)$, who surely buy from the other hospital.

[4]For example, if hospitals have a fixed cost, then the hospital with zero quality would leave the market.

[5]For off-equilibrium signals patients may be required to chose one of the hospitals.

[6]In GS equilibrium for symmetric hospitals, patients follow the highest signal even though it does not change their belief about quality.

Given the above, if hospital j chooses a quality q_j, its demand is given by

$$D^j(q_j, q^*) = \begin{cases} 0 & \text{if } q_j \in [0, q^* - 1/v) \\ \frac{1+v(q_j - q^*)}{2} & \text{if } q_j \in [q^* - 1/v, q^* + 1/v] \\ 1 & \text{if } q_j \in (q^* + 1/v, \infty] \end{cases}$$

Using the above, the profit is

$$\Pi^j(q_j, q^*) = (p - c)D^j(q_j, q^*) - \frac{1}{2}\delta_j q_j^2.$$

Since in equilibrium we should have $q_j = q^*$, q^* should satisfy the following first order condition

$$q^* = \frac{(p-c)v}{2\delta_j}. \tag{1}$$

From the above, it is immediately obvious that because q^* is the same for the two hospitals, an equilibrium where patients are equally likely to visit either hospital regardless of their signals cannot be constructed for $\delta_L > \delta_H$. Thus, if one where to construct an equilibrium for this case, patients should respond to signals in such a way that both hospitals find q^* to be optimal. Given that their costs differ, such a construction if inherently very difficult, if not impossible.

The condition in (1) is necessary but not sufficient. It can be the case that q^* is so large that hospital j may deviate to $q_j = 0$. This is a profitable deviation if $(p-c)v^2 > 4\delta$, in which case $\Pi^j(p^*, p^*) < 0$. So q^* in (1) cannot be an equilibrium quality, but given that (1) is necessary for equilibrium, there can be no pure strategy PBE.

Proposition 1. *Pure strategy PBE does not exist if $(p - c)v^2 > 4\delta$.*

Proof. In a pure strategy PBE condition (1) has to hold, or otherwise hospital j can increase profits by setting q_j different from q^*. If $(p - c)v^2 > 4\delta$, then equilibrium profit of hospital j is negative. Then j can set $q_j = 0$ and guarantee itself a zero profit. Thus if $(p - c)v^2 > 4\delta$ pure strategy symmetric PBE does not exist. □

Next I characterize pure strategy symmetric PBE if it exists.

Proposition 2. *If $\delta_H = \delta_L = \delta$ and $(p - c)v^2 \leq 4\delta$, in the pure strategy PBE*

$$q_H^* = q_L^* = \frac{(p-c)v}{2\delta}$$

As in GS, equilibrium quality increases with the markup, decreases in v and δ. One has to be very cautious with such comparative statics, however. Since the game has multiple equilibria, unless one is very confident in the choice of particular equilibrium, comparative statics cannot be performed. This is because when a parameter changes, patients and hospitals can switch to a different type of equilibrium, thus the direction of comparative statics may be arbitrary (e.g. quality may be increasing or decreasing in v depending on which equilibria are selected as v changes).

5 Discussion

The idea that noisy information may be discarded by rational decision-makers is not new. Bagwell (1995) and Maggi (1999) have shown that when a player (e.g. Stackelberg follower) observes another player's (Stackelberg leader's) move with noise, the leader loses the first-mover advantage. This is because the follower never uses noisy signal and thus reacts to what she thinks the leader did and not to what the leader actually did. In fact, Maggi (1999) in footnote 3 writes:

"Also, consider the simplest problem of moral hazard in the provision of product quality. If consumers cannot observe the quality of the product before purchasing (and there are no repeat purchases or warranties), firms have incentive to provide low quality, and a "lemons" problem arises (see Tirole (1988) for a survey of this literature). The irrelevance result implies that even if consumers do observe quality before purchasing, but with a slight noise, the (pure-strategy) equilibrium outcome is the same as if they did not observe quality at all, hence the lemons problem may not be resolved."

Maggi's reasoning relies on the independence of signal's support from quality choice. For example, if signals have unbounded support (e.g. Normal distribution), then Maggi's conjecture is correct. To avoid this problem, Shelegia (2011), who uses Normally distributed signals in a model of quality choice similar to GS, assumes that quality realizations differ across consumers, so even if consumer knows firm's quality choice, signals are still informative about her idiosyncratic realization of quality. Shelegia (2011) proceeds to show that equilibrium quality increases in the precision of consumers' signals. In GS, signals have bounded support (uniform), so Maggi's conjecture does not hold, and that is why for symmetric firms positive quality can be sustained. It is however, still true that GS's equilibrium is only valid for symmetric firms, or otherwise some rational consumers make wrong choices based on irrelevant information.

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