A Simple Model of Bertrand Duopoly with Noisy Prices

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Abstract

We examine a market in which consumers are forced to rely on noisy price signals to select between homogeneous products. The noise originates either from firms' price obfuscation or consumers' bounded information processing capabilities. Standard models and empirical experiments of markets with noise or price obfuscation show that it leads to higher prices detrimental to consumers' welfare. This paper identifies conditions under which an opposite result can be expected. In particular, it shows that a moderate level of noise is beneficial to consumers in a market with a cost leader.

Keywords: noisy pricing; bounded rationality; Bertrand oligopoly; game theory

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1. Introduction

Persistence of price dispersion for homogeneous products is an empirical phenomenon that contradicts textbook microeconomic theory. Some thick markets with dozens of companies offering similar products do not converge to one price. For example, Thompson and Thompson (2006) present evidence for unexplained variation in prices and super-marginal profits for web-hosting companies, Garrod (2007) and Clemons et al. (2002) find similar patterns in data collected from travel agencies and Ancarani and Shankar (2004) for retail industry. This empirical evidence is supported by experimental studies, for example Kalayci and Potters (2011).

One possible explanation for the observed price dispersion is the noise introduced by active price obfuscation by companies. For instance, Baye and Morgan (2004), Clay et al. (2001), Ellison and Ellison (2009) use data from price comparison websites for consumer electronics to argue that retailers actively engage in price obfuscation practices that frustrate consumer search. Such practices include companies' use of fictitious price comparisons or false sale signs to deter consumer search deceptively and profitably. Another source of noise is consumers' inherent inability to reason about multi-dimensional goods and services exhaustively or to predict their future usage, even when no obfuscation efforts are undertaken by companies. Hatton (2005) provides an example from telecommunication markets, discussing how consumers find it difficult to predict their future usage of airtime and other dimensions of service such as voice, data, SMS and MMS.

The literature has investigated different assumptions on market structure. For example, pricing dispersion has been shown to be the equilibrium result of companies operating under monopoly (Rubinstein, 1993), Stackelberg oligopoly (Spector, 2002) or monopolistic competition (Ellison and Wolitzky, 2008; Gabaix and Laibson, 2005; Wilson, 2004) where consumer- or company-induced noise is present on the market. Much less attention was paid to welfare consequences of this issue, especially when the level of noise is exogenous to the market and represents true taste shocks or consumer evaluation errors, similar to Perloff and Salop (1985).

We introduce a model of price competition in a noisy Bertrand duopoly where the total noise in price and demand perception in the market is constant. The model enables interpretation of the source of noise coming from:

(a) companies: active obfuscation,

(b) consumers: boundedly rational or imperfect in reasoning,

(c) environment: external shocks leading to uncertainty, like volatile currency exchange rates or gift cards, where the consumer experience depends on performance of a third party.

In our model, companies are perfectly informed and aware of the noise and consumers use noisy signals to identify the cheapest provider. In Section 2, we elaborate our assumptions on the market structure and decision making of companies and consumers. In Section 3, we use our model to measure the welfare effects of noise.

The welfare impact of noisy pricing on consumers was investigated by models with endogenous noise in which companies choose the level of product complexity or obfuscation. They indicate that noise and obfuscation impact consumers' welfare negatively. Sometimes models with endogenous noise generate unintuitive equilibria. For example, in Laibson and Gabaix (2004) an increase in the number of competitors can increase markups and reduce consumer welfare. Baye and Morgan (forthcoming) show that reductions in search costs may lead to either more or less price dispersion, depending on the market environment. Anderson and De Palma (2002) prove that introducing shoppers who always buy from the cheapest firm may increase market prices. Laibson and Gabaix (2004) assume varying quality of products and endogenous choice of noise in a market where the supplier with the superior product has an incentive to reduce pricing obfuscation. On the empirical side, recent experimental results by Kalayci and Potters (2011) confirm that pricing obfuscation can lead to higher average prices.

In contrast to the theoretical and empirical literature mentioned above, we give a simple example of a model with exogenous noise where certain level of cognitive imperfection can improve consumers' welfare. We trace this counterintuitive result to interaction between the full rationality of companies and the bounded cognition of consumers. Our model is an example of a simple environment in which boundedly rational actors obtain better outcomes than fully rational and fully informed ones.

Proofs of theorems are placed in the Appendix.

2. Bertrand Duopoly with Noisy Consumers

Consider a market with two companies 1 and 2 competing on prices with an undifferentiated product. We assume that the exact final price paid is unknown to the consumer until after he commits to one of companies. Such situation can result from future demand or price uncertainty.

The real-life examples of *demand uncertainty* are choosing a supermarket to make the grocery shopping or selecting a mobile operator. Supermarkets and telecommunications companies offer approximately identical goods in terms of quality and variety and consumers are uncertain about the final market basket they purchase until they enter the shop or sign a telecommunication contract. In this case, companies can introduce noise to consumers' evaluation of their offers by differentiating the prices of individual items.

A prominent example of *price uncertainty* is exchange rate noise when choosing between two suppliers at known spot prices, but when each supplier contracts in a different currency. In this case, consumers cannot determine the cheaper offer at the date of goods delivery when they are signing the contract.

Relevant models of consumer decision-making in noisy environments include simple sampling schemes (Spiegler, 2006), perceptron-like architectures (Rubinstein, 1993), variations of Bayesian updating (Wilson, 2004), costly search processes (Ellison and Wolitzky, 2008; Gabaix and Laibson, 2005) and price difference estimation before purchase (Perloff, 1985). In our model, consumers select the suppliers according to the following procedure:

1. Estimate the difference between prices offered by the companies (*perceived price difference*) and choose the cheaper one. We assume that the perceived price difference is an unbiased estimator of the difference between expected actual prices.
2. Observe the price for the company that has been chosen (*actual price*). This value is drawn independently from the perceived price difference.

In order to decide, the consumer does not have to estimate actual prices for each company. The perceived price difference evaluation is sufficient. For example, in the telecommunications case, the consumer can compare unit prices of a one-minute voice call in mobile operators offers without having to specify the actual demand for it. However, in most cases the perceived price difference is estimated as a difference between expected actual prices from both companies.

We model the estimation of the perceived price difference in the case of demand uncertainty as follows. Assume that a consumer has a set S of possible future demand scenarios with a probability distribution H defined over S. For each demand scheme $s \in S$, the consumer can determine exactly the price $e_i(s)$ he will pay if he chooses to buy from company i. If the consumer tests every $s \in S$ and average the expected prices according to H, he can minimize the expected price. However, we assume that the consumer takes only a finite sample from S drawn according to H, and estimates the prices $e_i(s)$ based on that sample. Such an estimator of the actual price is unbiased; consequently the perceived price difference is unbiased. The actual price is determined by randomizing $s \in S$ according to H using the unit prices of the chosen company.

In the case of price uncertainty, the we assume the same model, but sample price scenarios instead of sampling demand scenarios.

We assume that distribution of perceived price difference between the i-th and $-i$-th companies is Gaussian $N(m_{-i} - m_i, \Sigma)$, where m_i is the expected price of the i-th company and Σ is a measure of the cognitive imprecision of consumers. Companies have no influence on Σ. Consumers and companies are risk neutral and the total demand is inelastic (consumers cannot opt out of the market). Companies maximize their expected revenues by setting expected unit prices m_i given the cost of supplying the product c_i.

Using these assumptions, we obtain the expected profit of the i-th company $V_i\left(m_i, m_{-i}\right)$:

$$V_i = F\left(\frac{m_{-i} - m_i}{\Sigma}\right)(m_i - c_i) \tag{1}$$

where $F(\cdot)$ is the cumulative distribution function of a standard Gaussian. The first part of the formula describes the probability that the consumer chooses i and the second part gives the expected value of the company's profit given that it is chosen.

The next section contains discussion of properties of this model and analysis of Σ influence on welfare of consumers.

3. Influence of Price Noise on Expected Cost

To analyze the model described in Section 2, we apply the approach based on reaction curve calculation, see for example Topkis (1978). We show that there exists a unique pure strategy Nash equilibrium of prices (m_i, m_{-i}):

Theorem 1. *A unique pure strategy Nash equilibrium in prices* (m_i, m_{-i}) *exists.*

Let us denote the equilibrium unit price of i as μ_i. It is a function of c_i, c_{-i}, Σ. Denote the expected profit of i in equilibrium by $P_i = V_i(\mu_i, \mu_{-i})$ and i's market share $S_i = F\left(\frac{\mu_{-i}-\mu_i}{\Sigma}\right)$. Using this notation the expected equilibrium cost to a consumer is equal to:

$$C(c_1, c_2, \Sigma) = \mu_i S_i + \mu_{-i} S_{-i}.$$

Equilibria of the model for different model parameters have simple scale- and shift-invariance properties:

Theorem 2. *For $a > 0$ and arbitrary b we have:*

$$\begin{aligned}
\mu_i(ac_i + b, ac_{-i} + b, a\Sigma) &= a\mu_i(c_i, c_{-i}, \Sigma) + b \\
S_i(ac_i + b, ac_{-i} + b, a\Sigma) &= S_i(c_i, c_{-i}, \Sigma) \\
P_i(ac_i + b, ac_{-i} + b, a\Sigma) &= aP_i(c_i, c_{-i}, \Sigma) \\
C_i(ac_i + b, ac_{-i} + b, a\Sigma) &= aC_i(c_i, c_{-i}, \Sigma) + b.
\end{aligned}$$

Theorem 2 allows us consider only two specific games $(c_1, c_2) = (0, 1)$ and $(c_1, c_2) = (0, 0)$ with varying Σ. Equilibria of all other games can be directly computed from these two. Using this observation one can prove the following:

Theorem 3. *If $c_1 < c_2$ then in equilibrium $\mu_1 < \mu_2$. Moreover, $\frac{d\mu_i}{dc_j} > 0$ and $\frac{d\mu_i}{d\Sigma} < 0 \iff c_i < c_{-i} - 6.683\Sigma$.*

Theorem 3 indicates that an increase in the marginal cost of any company leads to an increase of market prices of both companies. However, an increase in the noise level Σ may result in decreasing the expected price offered by i if the marginal cost is low enough $c_i < 6.683\Sigma$. The following theorem expresses the observation that adding noise to consumer decision-making may have positive welfare consequences:

Theorem 4. *Expected cost for consumers is minimized for $\Sigma \approx 0.126 \|c_2 - c_1\|$.*

Theorem 4 is illustrated in Figure 1. When a company with lower marginal cost is faced with noisy consumers, it will tend to keep its prices low in order to avoid losing market share due to misperception effects. Therefore, in markets with heterogeneous production costs, consumers are better off when the precision of perception expressed by Σ is limited, but non-zero.

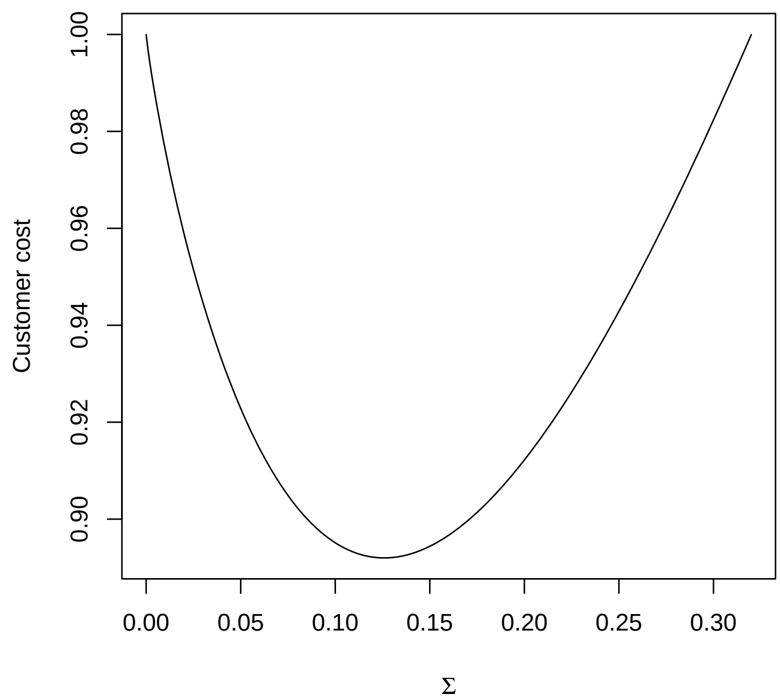

Figure 1: Expected final price paid by consumers on duopoly market with marginal costs $c_1 = 0$ and $c_2 = 1$. Cost to consumer is expressed as a function of noise level Σ.

4. Conclusions

In the paper we developed a simple model of Bertrand duopoly with noisy perception of prices. We assumed that companies control average prices but cannot influence the variance of consumers' perceptions of price difference. This situation can result from uncertainty of the future demand (for example on retail and telecommunication markets) or price uncertainty (for instance in markets with foreign currency contracts).

We assumed that the estimator of price differences used by consumers follows a Gaussian distribution. Under these assumptions, we showed that there exists a unique pure strategy Nash equilibrium of price-setting game between companies. Equilibrium prices increase when either companies' marginal costs increase or when the perception noise increases, provided that the difference in marginal cost between companies is not large. Finally, the level of noise that minimizes average costs for consumers, measured as standard deviation of price difference estimator, is approximately equal to 12.6% of difference between companies' marginal costs.

The results suggest that when fully rational and heterogeneous companies compete in prices, the population of consumers with a degree of bounded perception would be better off than perfectly informed or completely noisy populations. The mechanism relies on the cheaper company lowering its price due to the fear that consumers' randomness can lead consumers to choose its competition. For example, our research explains the rationale behind Wal-Mart *Every Day Low Prices* policy. In noisy markets, the cost leader with lower prices can signal price difference only by setting even lower prices. Wal-Mart's policy aims to reducte pricing noise generated by their competitors' marketing strategies based on active promotion of narrow range of product at very low prices, while keeping other products' prices high.

Appendix A

Proof of Theorem 1. First we show that for a fixed m_{-i}, the optimal response $r_i(m_{-i})$ of player i is given by a unique solution to FOC. Let us calculate it (we denote $F'(x) = f(x)$):

$$\frac{\partial V_i}{\partial m_i} = F\left(\frac{m_{-i} - m_i}{\Sigma}\right) - \frac{f\left(\frac{m_{-i}-m_i}{\Sigma}\right)(m_i - c_i)}{\Sigma} \tag{2}$$

$$\frac{\partial V_i}{\partial m_{-i}} = \frac{f\left(\frac{m_{-i}-m_i}{\Sigma}\right)(m_i - c_i)}{\Sigma} \tag{3}$$

$$\frac{\partial^2 V_i}{\partial m_i^2} = \frac{f\left(\frac{m_{-i}-m_i}{\Sigma}\right)}{\Sigma}\left(\frac{(m_i - m_{-i})(m_i - c_i)}{\Sigma^2} - 2\right) \tag{4}$$

$$\frac{\partial^2 V_i}{\partial m_{-i}^2} = \frac{f\left(\frac{m_{-i}-m_i}{\Sigma}\right)(m_i - m_{-i})(m_i - c_i)}{\Sigma^3} \tag{5}$$

$$\frac{\partial^2 V_i}{\partial m_i \partial m_{-i}} = \frac{f\left(\frac{m_{-i}-m_i}{\Sigma}\right)}{\Sigma}\left(1 - \frac{(m_i - m_{-i})(m_i - c_i)}{\Sigma^2}\right) \tag{6}$$

There exist points $p_l < \min(c_i, m_{-i})$ and $p_h > \max(c_i, m_{-i})$ such that $\frac{\partial^2 V_i}{\partial m_i^2}$ is negative in interval (p_l, p_h) and positive outside interval $[p_l, p_h]$.

Revert to $\frac{\partial V_i}{\partial m_i}$. It is positive for $m_i \leq c_i$. For $m_i > m_{-i}$ we can use the fact that $x < 0 \Rightarrow F(x) < -\frac{f(x)}{x}$:

$$\frac{\partial V_i}{\partial m_i} < \frac{f\left(\frac{m_{-i}-m_i}{\Sigma}\right)}{\frac{m_i-m_{-i}}{\Sigma}} - \frac{f\left(\frac{m_{-i}-m_i}{\Sigma}\right)(m_i - c_i)}{\Sigma}$$

For sufficiently large m_i: $\frac{\Sigma}{m_i-m_{-i}} < \frac{m_i-c_i}{\Sigma}$, therefore $\frac{\partial V_i}{\partial m_i} < 0$.

In the interval $(c_i, +\infty)$, $\frac{\partial V_i}{\partial m_i}$ is first decreasing from some positive value, afterwards it has exactly one local minimum and later increases but does not cross 0. This implies that there exists only one m_i for which $\frac{\partial V_i}{\partial m_i}$ equals 0. Because the second derivative in this point is negative is a unique global maximum of V_i with respect to m_i and therefore the optimal response $r_i(m_{-i})$.

From equation 2 we get that if $m_i = c_i$ then $\frac{\partial V_i}{\partial m_i} > 0$. Let us show that for $m_i = \frac{m_{-i}+c_i}{2} + \sqrt{\left(\frac{m_{-i}-c_i}{2}\right)^2 + \Sigma}$ derivative $\frac{\partial V_i}{\partial m_i}$ is negative:

$$\frac{\partial V_i}{\partial m_i} = F(U - \sqrt{U^2+1}) - f(U - \sqrt{U^2+1})(U + \sqrt{U^2+1})$$

where $U = \frac{m_{-i}-c_i}{2\Sigma}$. Since $U < \sqrt{U^2+1}$:

$$\frac{\partial V_i}{\partial m_i} < -\frac{f(U - \sqrt{U^2+1})}{U - \sqrt{U^2+1}} - f(U - \sqrt{U^2+1})(U + \sqrt{U^2+1}) = 0$$

This fixes the optimal response to the following interval:

$$r_i(m_{-i}) \in I_1 = \left(c_i, \frac{m_{-i}+c_i}{2} + \sqrt{\left(\frac{m_{-i}-c_i}{2}\right)^2 + \Sigma^2}\right)$$

The exact value of the optimal response can be found as a unique solution of equation $\frac{\partial V_i}{\partial m_i} = 0$ in the above interval. Next, we prove that the optimal response $r_i(m_{-i})$ is increasing as m_{-i} increases. For this we use necessary condition $\frac{\partial V_i}{\partial m_i} = 0$ to calculate:

$$\frac{dr_i}{dm_{-i}} = -\frac{\frac{\partial^2 V_i}{\partial m_i \partial m_{-i}}}{\frac{\partial^2 V_i}{\partial m_i^2}} = \frac{\Sigma^2 - (r_i - m_{-i})(r_i - c_i)}{2\Sigma^2 - (r_i - m_{-i})(r_i - c_i)}$$

We know that $r_i > c_i$, as each company needs to remain profitable. If $r_i < m_{-i}$ then the fraction above is positive. Let us analyze the opposite case $r_i > m_{-i}$.

First, observe that if $\Sigma^2 > (r_i - m_{-i})(r_i - c_i)$ then the above fraction is positive. This inequality is met for the following interval:

$$r_i \in I_2 = \frac{m_{-i} + c_i}{2} + (-1; 1)\sqrt{\left(\frac{m_{-i} - c_i}{2}\right)^2 + \Sigma}.$$

We can see that $I_1 \subset I_2$, therefore the condition $\Sigma^2 > (r_i - m_{-i})(r_i - c_i)$ is met in equilibrium.

Summarizing $0 < \frac{dr_i}{dm_{-i}} < 1$, such that the reaction curve for player i is increasing as m_{-i}, but slower than $-i$-th player's price m_{-i}.

Additionally, if $m_i = m_{-i} = \sqrt{\frac{\pi}{2}}\Sigma + c_i$ then $\frac{\partial V_i}{\partial m_i} = 0$. This means that i-th player's reaction curve crosses $m_i = m_{-i}$ line in this point. But as $0 < \frac{dr_i}{dm_{-i}} < 1$ this is the only intersection point. This implies that reaction curves must intersect at least once and at intersection point:

$$r_1, r_2 \in \sqrt{\frac{\pi}{2}}\Sigma + [\min(c_1, c_2), \max(c_1, c_2)]$$

It remains to be shown that the reaction curves intersect in exactly one point. Assume that there were two intersection points $(r_{1,1}, r_{2,1})$ and $(r_{1,2}, r_{2,2})$. But this would imply $\max_{i\in\{1,2\}} \frac{m_{i,1} - m_{i,2}}{m_{-i,1} - m_{-i,2}} \geq 1$ which contradicts the fact that $0 < \frac{dr_i}{dm_{-i}} < 1$.

□

Proof of Theorem 2. We already know that there exists exactly one equilibrium. Thus:

$$\forall i \in \{1,2\} : \frac{\partial V_i}{\partial m_i}(\mu_i, \mu_{-i}) = F\left(\frac{\mu_{-i} - \mu_i}{\Sigma}\right) - \frac{f\left(\frac{\mu_{-i} - \mu_i}{\Sigma}\right)(\mu_i - c_i)}{\Sigma} = 0$$

If we transform c_i by multiplying it by a and adding b and multiply Σ by a, this set of equations is solved by $a\mu_i + b$. Equations for P_i, S_i, C_i are calculated by substituting the transformed μ_i. □

Proof of Theorem 3. In equilibrium $\frac{\partial V_i}{\partial m_i}(\mu_i, \mu_{-i}) = 0$.

Therefore $\frac{\partial V_1}{\partial m_1}(\mu_1, \mu_2) = \frac{\partial V_2}{\partial m_2}(\mu_2, \mu_1)$. By rearranging this condition we get:

$$2F\left(\frac{\mu_2 - \mu_1}{\Sigma}\right) - 1 = f\left(\frac{\mu_2 - \mu_1}{\Sigma}\right)((\mu_1 - c_1) - (\mu_2 - c_2))/\Sigma$$

thus $\mu_2 > \mu_1 \Leftrightarrow \mu_1 - c_1 > \mu_2 - c_2$ and $c_2 > c_1$. Let's calculate all derivatives of profits in equilibrium point (μ_1, μ_2):

$$J = -\begin{bmatrix} \frac{\partial^2 V_i}{\partial m_i^2} & \frac{\partial^2 V_i}{\partial m_i \partial m_{-i}} \\ \frac{\partial^2 V_{-i}}{\partial m_i \partial m_{-i}} & \frac{\partial^2 V_{-i}}{\partial m_{-i}^2} \end{bmatrix}.$$

Using the implicit function theorem:

$$\begin{bmatrix} \frac{d\mu_i}{dc_i} \\ \frac{d\mu_{-i}}{dc_i} \end{bmatrix} = J^{-1} \begin{bmatrix} \frac{\partial^2 V_i}{\partial m_i \partial c_i} \\ \frac{\partial^2 V_{-i}}{\partial m_{-i} \partial c_i} \end{bmatrix}$$

From the proof of Theorem 1 we have $-\frac{\partial^2 V_i}{\partial m_i^2} > \frac{\partial^2 V_i}{\partial m_i \partial m_{-i}} > 0$. Therefore $\det(J) > 0$ and J^{-1} has only positive elements. Furthermore, cross derivatives are as follows:

$$\begin{bmatrix} \frac{\partial^2 V_i}{\partial m_i \partial c_i} \\ \frac{\partial^2 V_{-i}}{\partial m_{-i} \partial c_i} \end{bmatrix} = \begin{bmatrix} f\left(\frac{\mu_{-i} - \mu_i}{\Sigma}\right)/\Sigma \\ 0 \end{bmatrix}$$

and the first term is nonnegative. This implies that prices of both companies increase with an increase of either of marginal costs c_i.

Denote $A = \Sigma^{-1}$. Using the implicit function theorem again and simplifying we have:

$$\frac{d\mu_i}{dA} = \det(J^{-1}) \left(\frac{\partial^2 V_i}{\partial m_i \partial m_{-i}} \frac{\partial^2 V_{-i}}{\partial m_{-i} \partial A} + \frac{\partial^2 V_{-i}}{\partial m_{-i}^2} \frac{\partial^2 V_i}{\partial m_i \partial A} \right)$$

After calculating derivatives and simplifying the expression we get:

$$\frac{d\mu_i}{dA} = \det(J^{-1}) A f(A(\mu_{-i} - \mu_i))$$
$$\left(-(\mu_{-i} - c_{-i}) - 2(\mu_i - c_i) + A^2(\mu_{-i} - \mu_i)^2(\mu_i - c_i) + (\mu_{-i} - \mu_i) \right)$$

From equillibrum condition and equation 2 we have $\mu_i - c_i = \frac{F(A(\mu_{-i} - \mu_i))}{Af(A(\mu_{-i} - \mu_i))}$. Introducing this into the derivative and denoting $x = A(\mu_{-i} - \mu_i)$ we get:

$$\frac{d\mu_i}{dA} = \det(J^{-1}) \left(-1 - F(x)(x^2 - 1) + f(x)x \right)$$

We have proven that $\det(J^{-1}) > 0$. Therefore the sign of the expression depends on $G(x) = -1 - F(x)(x^2 - 1) + f(x)x$. Notice that $G'(x) = 2xF(x)$

and it is positive for $x > 0$. Moreover $G(0) = -\frac{3}{2}$ and for $x < 0$ we have[1] $G(x) < \frac{f(x)}{x} - 1 < -1$. Summing all these conditions we have that there exists $\hat{x}$ such that $x < \hat{x} \Rightarrow \frac{d\mu_i}{dA} < 0$ and $x > \hat{x} \Rightarrow \frac{d\mu_i}{dA} > 0$. A numeric calculation yields $\hat{x} = 1.363$. Using the equilibrium conditions we get:

$$x = \hat{x} \Leftrightarrow c_i = c_{-i} - 6.618\Sigma$$

Now we see that for $c_i = c_{-i}$ we have $x = 0$ so:

$$\frac{d\mu_i}{dA} < 0 \Leftrightarrow c_i < c_{-i} - 6.618\Sigma$$

□

Proof of Theorem 4. Without loss of generality we shall assume that $c_1 < c_2$. The case of $c_1 > c_2$ is symmetric and only companies indices need to be exchanged.

Assume that a certain Σ is the noise level which induces the lowest average consumer prices for a pair (c_1, c_2), that is:

$$\forall \hat{\Sigma} > 0 : C(c_1, c_2, \Sigma) \leq C(c_1, c_2, \hat{\Sigma})$$

According to Theorem 2, this assumption after substituting $a = (c_2 - c_1)^{-1}$ and $b = -c_1$ is equivalent to the following:

$$C(0, 1, \frac{\Sigma}{c_2 - c_1}) \leq C(0, 1, \frac{\hat{\Sigma}}{c_2 - c_1})$$

If $c_1 \neq c_2$ then the optimal Σ is equal to equivalent Σ for a cost pair $(c_1, c_2) = (0, 1)$ multiplied by $|c_2 - c_1|$.

The remaining challenge is to determine the optimal Σ for marginal cost pair $(0, 1)$. A numerical solution to this problem yields $\Sigma = 0.125$ and $C = 0.892$. On Figure 1 we show the shape of C for small Σ. Minimum cannot occur for $\Sigma > 0.25$ because:

1. From Theorem 3 we know that $\Sigma > 0.151 \Rightarrow \frac{d\mu_i}{d\Sigma} > 0$;
2. For $\Sigma = 0.25$ we have $\mu_1 = 0.902$ and $\mu_2 = 1.161$ and they are both greater than 0.892.

This conclusion finishes the proof. □

[1] Again we use the fact that $x < 0 \Rightarrow F(x) < -\frac{f(x)}{x}$.

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