Sharing a polluted river network

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ABSTRACT: A polluted river network is populated with agents (e.g., firms, villages, municipalities, or countries) located upstream and downstream. This river network must be cleaned, the costs of which must be shared among the agents. We model this problem as a cost sharing problem on a tree network. Based on the two theories in international disputes, namely the Absolute Territorial Sovereignty (ATS) and the Unlimitted Territorial Integrity (UTI), we propose three different cost sharing methods for the problem. They are the *Local Responsibility Sharing (LRS)*, the *Upstream Equal Sharing (UES)*, and the *Downstream Equal Sharing (DES)*, respectively. The LRS and the UES generalize Ni and Wang ("Sharing a polluted river", Games Econ. Behav., 60 (2007), 176-186) but the DES is new. The DES is based on a new interpretation of the UTI. We provide axiomatic characterizations for the three methods. We also show that they coincide with the *Shapley values* of the three different games that can be defined for the problem. Moreover, we show that they are in the *cores* of the three games, respectively. Our methods can shed light on pollution abatement of a river network with multiple sovereignties.

JEL classification: C71, D61, D62.

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1 Introduction

Since ancient times, control of water resources has been the cause of many wars and conflicts. Accounting for more than 50% of the land area of the Earth, more than 200 river basins are shared by two or more sovereign nations. Unfortunately, the majority of this invaluable resource is polluted. One example is the Ganges-Brahmaputra basin, an international river basin shared by India, Bangladesh and Nepal. The Great Lakes is another example, consisting of a group of five[1] large lakes in North America and shared by Canada and the United States. In many of these shared waters, pollution has become an increasing threat. To deal with this issue, international cooperation is needed.

Thus far, the studies on international waters have been focusing on water sharing. To name a few, they include Barrett (1994), Kilgour and Dinar (1996, 2001), Ambec and Sprumont (2002), Ambec (2008), Ambec and Ehlers (2008a,b), Ansink and Ruijs (2008), Marchiori (2010), and Wang (2011). Recently, there have been a number of papers considering the water pollution problem. Examples include Weber (2001), Hung and Shaw (2005), Ni and Wang (2007).

This paper considers the cost sharing problem of a polluted river network.[2] Suppose that a number of agents (e.g., firms, villages, municipalities, or countries) are connected in a river network. Some agents are located upstream and some downstream. In using the river network, agents may generate pollutants such as industrial chemicals, pesticides from agricultural run-off, or sewage. Consequently, a cost is incurred to each link of the network. These costs can be the costs that the agents must spend in order to clean up the polluted water to meet certain environmental standards. Or they can be the costs needed to maintain the water quality of the river network. Whatever these costs might be, they must be shared among the agents.

While it is clear that all the agents in the polluted river network should share the costs of cleaning up the network since these costs are incurred by their *joint* uses (or abuses) of the river network, it is not immediately clear how to assign these costs to each individual agent. While, the rights to use the

[1]Many people argue that there are six by adding Lake St. Clair.

[2]This work is motivated by a real life example in Northern China, where a lake was polluted by the upstream users of the river system. See Subsection 3.4 for the detail.

river come with responsibilities and responsibilities should be in proportion[3] to their rights (uses) of the network, it is often hard if not impossible to identify precisely how much each agent uses the network and thus contributes to the pollution. Even these agents' pollutant emissions can be identified, the complex interactions (e.g. chemical reactions) of the pollutants in flowing water make it difficult to estimate their actual impacts on pollution costs.

In the simplest two-agent case (i.e., one upstream and one downstream), when property rights on water are clearly assigned and the upstream agent's pollutant emission is clearly identified, the Coase theorem (Coase, 1960) implies that the two agents can always resolve their problem through bilateral bargaining. In practice, however, most problems involve more than two agents. Moreover, property rights on most international river systems are not well-defined. For instance, in an international river system, downstream countries may argue that even the upstream countries can do whatever they want to the water they control but they shouldn't alter the nature of the river system to the disadvantage of the downstream countries.[4] However, it is not clear or a simple matter to determine to what extent these rights actually mean for the water pollution problem. Therefore, it would be much more difficult to reach an agreement through multilateral bargaining.

To deal with this challenge, we focus on the problem of what a fair allocation of the pollution costs should be.[5] We base our theory on the two well-known theories in international disputes. They are the theory of *Absolute Territorial Sovereignty* (ATS) and the theory of *Unlimited Territorial Integrity* (UTI).[6] In fact, Ambec and Sprumont (2002) have applied these two theories in a water sharing problem. Later, Ni and Wang (2007) use them in a water pollution sharing problem. Our model differs from Ni and Wang (2007) in two respects. First, we consider a more general tree network

[3]Aristotle said that equals should be treated equally and unequals unequally in proportion to their inequality.

[4]See the Unlimited Territorial Integrity theory below.

[5]We ignore the strategic reactions of the agents under a given allocation of the pollution costs, which will affect their polluting behavior and then the actual costs. We do not deal with the efficiency issue of a cost sharing method in our current model.

[6]The ATS theory says that *a country has absolute sovereignty over the area of any river basin within its territory.* The UTI theory, on the other hand, says that *a country shouldn't alter the natural conditions within its own territory to the disadvantage of a neighboring country.* For more discussions on the ATS and the UTI, see Godana (1985), Kilgour and Dinar (1996).

than their line-tree model. Second, we propose two different but equally compelling interpretations of the UTI theory. We show that these two different interpretations lead to two entirely different cost sharing methods. More importantly, the new interpretation of the UTI opens door to explore new cost sharing methods.[7]

Unlike in the water sharing problem, these two theories, each stand alone or together, still allow a lot of flexibility in choosing cost sharing methods in our polluted river network problem. In Ni and Wang (2007), the UTI is interpreted as a downstream responsibility (DR) principle which says that an agent is responsible for the cost of cleaning her own link and partially responsible for the costs of all her downstream links. In our model, we propose another interpretation of the UTI, which is exactly the opposite of the above DR principle. That is, the UTI can also imply that an agent is responsible for the cost of her own link and also partially responsible for the costs of all her upstream links. We call it the Upstream Responsibility principle (UR).[8]

To justify this alternative interpretation, revisit the water sharing problem of Ambec and Sprumont (2002). It is shown that the combination of the ATS and the UTI determines a unique water sharing method called the Downstream Incremental Distribution method (DID). Ambec and Sprumont show that the DID method lexicographically maximizes the welfare of the agents according to the ordering from downstream to upstream. Thus, in a DID distribution the last downstream agent obtains the highest welfare she can possibly achieve, and then follows by the next to the last, and so on. If responsibilities should be in proportion to rights in any problem of distributive justice, then the UTI theory also implies that the downstream agents should bear some of the upstream costs of the network. To say it more directly, since they have benefited from being downstream agents in the water sharing problem, it is compelling to require them to pay part of the upstream environmental costs.[9] In hindsight, downstream agents have

[7]See the Concluding Remarks Section on the potential directions of research.

[8]In fact, between these two opposing interpretations, a range of them can be proposed. For more on this, see Wang (2011) and the Concluding Remarks Section.

[9]We can further reinforce this argument by pointing out that, in many river networks, downstream agents may have used the water resources relatively more intensively than their upstream counterparts. For instance, downstream agents are big industries or cities. But we hesitate to push this argument too much for the reason that other characteristics

certain *derived* upstream responsibilities.

Accordingly, we propose three different cost sharing methods for the problem. The *Local Responsibility Sharing* (LRS) method, which corresponds to the Local Responsibility principle implied by the ATS, assigns costs to agents based on the costs that are associated with their locations. The *Upstream Equal Sharing* (UES) method,[10] which corresponds to the Downstream Responsibility (DR) principle implied by the UTI, assigns costs to agents based on their associated local costs plus the equal sharing of their downstream costs. The *Downstream Equal Sharing* (DES) method is first introduced in this paper. It corresponds to the Upstream Responsibility principle implied by the UTI. The DES method assigns to each agent the associated local cost plus the equal sharing of her upstream costs.

The above three methods are axiomatized, respectively. In Theorem 1, the Local Responsibility Sharing method is characterized by the axioms of *Additivity*, *No Blind Costs*, and *Efficiency.* In Theorem 2, the Upstream Equal Sharing method is characterized by the axioms of *Additivity, Independence of Upstream Costs, Upstream Symmetry, Independence of Irrelevant Costs*, and *Efficiency.* In Theorem 3, the Downstream Equal Sharing method is characterized by the axioms of *Additivity, Independence of Downstream Costs, Downstream Symmetry, Independence of Irrelevant Costs*, and *Efficiency.*

In the characterizations of the three methods, two axioms stand out. Additivity is an axiom used in all three characterizations. The Independence of Irrelevant Costs axiom is a new axiom we introduce in this paper and used in two of the three characterizations (the UES and the DES). As is well-known in the cost sharing literature, Additivity is a classical axiom (Shapley, 1953; Moulin, 2002). It is a structural axiom that allows us to focus on the cost sharing methods that depend additively on costs. The Independence of Irrelevant Costs axiom, on the other hand, is an equity axiom. It says that an agent should not be responsible for any cost that is irrelevant to her.[11] Roughly speaking, we say two agents are irrelevant to each other if they are on two different branches in a tree. The Independence of Irrelevant Costs axiom is indispensable in our model.[12]

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of the agents like city size, population, etc. are not considered in our present model.

[10]Note that the UES is the extension of the UES in Ni and Wang (2007).

[11]We speak interchangeably between an agent and her associated link cost.

[12]Note that in Ni and Wang (2007)'s line-tree model, all agents are relevant to each

We also relate the three methods to the Shapley value (Shapley, 1953). We show that each method is the Shapley value of a special game associated with the problem. The three games are the *stand-alone game*, the *upstream-oriented game*, and the *downstream-oriented game*, respectively. In the stand-alone game, the cost of a coalition is given by the total costs of all the agents in the coalition according to the LR principle. In the upstream-oriented game, the cost of a coalition is given by the total costs of all the agents in the coalition plus all their downstream costs according to the DR principle. In the downstream-oriented game, the cost of a coalition is given by the total costs of all the agents in the coalition plus all their upstream costs according to the UR principle. We show that the LRS, the UES, and the DES are the Shapley values of the above three games, respectively. Moreover, we show that these three games are all *concave* and, therefore, the cost allocations given by these three methods are in the *cores* of the corresponding games.

Finally, we point out that our work is closely related to the growing literature on cost sharing problems in networks. In many network problems, a common feature is that an agent usually uses only a part of the network and different agents use different parts.[13] This is exactly the case in our polluted river network problem, in which each agent is only related to a sub-network of the whole network. Apparently, the associated cost sharing problems are different from the traditional cost sharing models in which a single cost function (or equivalently, a production function) is shared by all users. More specifically, in the traditional models, a user contributes to the total costs by using the production technology jointly owned by all users to obtain her demand of a good or service. In contrast, in network cost problems, a user's contribution to the total costs depends on her location in the network. Thus, each user is related to only a subset of other users.[14] For new developments and research directions on cost sharing in networks, see Moulin (2011).

We organize the paper as follows. In Section 2, we define the model for the cost sharing problem on a tree structure and propose the three methods mentioned above. In Section 3, we introduce a number of axioms and provide the characterizations for the three methods (Theorems 1-3). Meanwhile, we

other.

[13]For example, in a gas pipeline network, a customer usually uses a subset of the pipelines to connect to the source.

[14]A prominent example is the airport landing fee problem (Littlechild and Owen, 1973).

study their relationships with the Shapley value and the core. In Subsection 3.4, we provide a real life example that motivated this study. In Section 4, we conclude our paper with some remarks on potential extensions.

2 The Model

Consider a river network connecting a set of agents, $N = \{1, 2, ..., n\}$, directly or indirectly, to a special agent L, called the lake. The river network is polluted due to the agents' use or abuse of the river network. To clean up or to maintain the river network clean. Certain costs are incurred. We assume that these costs are associated with all the links and are exogenously given. The main question is how these costs should be shared among the agents.

Formally, let $N \cup \{L\}$ be the set of all agents, and let E be the set of links on $N \cup \{L\}$. Assume that $G = \{N \cup \{L\}, E\}$ is a tree network; i.e., G is a connected graph with no cycle of links. For each agent $i \in N \cup \{L\}$, there is a unique path to L, i.e., a sequence of links, $(i, j), (j, k), ..., (l, m)$ in E, where $m = L$ and (i, j) is the first link in the path. We call the link (i, j) agent i's link and the agent j agent i's immediate downstream agent. A cost function on the network G is a mapping $C : E \cup \{L\} \to R_+$, where $C((i, j)) = c_i$ is the cost of agent i's link $(i, j) \in E$, and $C(L)$ is the cost associated with L. Sometimes, we also call c_i agent i's cost. With a slight abuse of notation, denote $C(E) = \sum_{i \in N} c_i$ (the total link costs). A *cost-sharing problem on a river network* is a triple $(N \cup \{L\}, G, C)$. A solution to a problem $(N \cup \{L\}, G, C)$ is a vector $x = (x_1, ..., x_n, x_L) \in R_+^{n+1}$ such that $\sum_i x_i = C(E) + C(L)$, where x_i is the cost share assigned to agent $i (\in N \cup \{L\})$. A method is a mapping x that assigns to each problem $(N \cup \{L\}, G, C)$ a solution $x(N \cup \{L\}, G, C)$. For convenience, we often write C instead of $(N \cup \{L\}, G, C)$, $x(C)$ instead of $x(N \cup \{L\}, G, C)$

For a given tree network G, the upstream-downstream relation among the agents is uniquely determined by the node L. This upstream-downstream relation will play an important role in our model. To represent this relation, we define an upstream-downstream structure for the network.[15] Given G,

[15]This is reminiscent of the permission structure concept used in van den Brink and Gilles (1996).

consider first the following mapping $P : N \cup \{L\} \rightarrow 2^{N\cup\{L\}}$:

$$P(i) \;=\; \{j|\text{there is a path from } j \text{ to } L \text{ such that } i \text{ is } j's \text{ immediate downstream agent in } G\}.$$

Note that the set $P(i)$ consists of all the immediate upstream agents of agent i.

EXAMPLE 1.

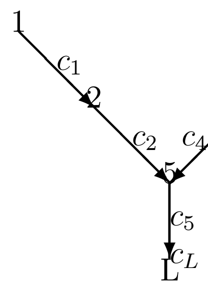

Figure 1

In this example, we have the following P.

$$P(1) = \emptyset, P(2) = \{1\}, P(3) = \emptyset, P(4) = \{3\}, P(5) = \{2, 4\}, P(L) = \{5\}.$$

The *upstream-downstream structure*, denoted as $\hat{P}$, is the *transitive closure* of P. It is defined as a mapping from $N \cup \{L\}$ to $2^{N\cup\{L\}}$ such that for all $i \in N \cup \{L\}$ we have $j \in \hat{P}(i)$ if and only if there exists $h_1, ..., h_m$ in $N \cup \{L\}$ such that $h_1 = i, h_{k+1} \in P(h_k)$ for all $1 \leq k \leq m - 1$, and $h_m = j$. The agents in $\hat{P}(i)$ are called the upstream agents of i in G.

Given $\hat{P}$, its inverse mapping $\hat{P}^{-1}$ is defined by $\hat{P}^{-1}(i) \equiv \{j \in N \cup \{L\} | i \in \hat{P}(j)\}$. We call the agents in $\hat{P}^{-1}(i)$ the downstream agents of i in G. For every $S \subseteq N$, denote $\hat{P}(S) = \cup_{i \in S} \hat{P}(i)$.

Note that for a given network G, it has a unique $\hat{P}$.

EXAMPLE 1 continued: It is easy to check that

$$\hat{P}(1) = \emptyset, \hat{P}(2) = \{1\}, \hat{P}(3) = \emptyset, \hat{P}(4) = \{3\},$$

$$\hat{P}(5) = \{1,2,3,4\}, \hat{P}(L) = \{1,2,3,4,5\}.$$

$$\hat{P}^{-1}(1) = \{2,5,L\}, \hat{P}^{-1}(2) = \{5,L\}, \hat{P}^{-1}(3) = \{4,5,L\},$$

$$\hat{P}^{-1}(4) = \{5,L\}, \hat{P}^{-1}(5) = \{L\}, \hat{P}^{-1}(L) = \emptyset.$$

Given a network G or $\hat{P}$, for any coalition of agents, $S \subseteq N \cup \{L\}$, we can define three different coalitions of agents that are related to S. The *stand-alone* counterpart of S is S itself. The *Upstream-oriented* counterpart of S is the coalition

$$\sigma(S) \equiv S \cup \hat{P}^{-1}(S), \tag{1}$$

while the *Downstream-oriented* counterpart of S is the coalition

$$\alpha(S) \equiv S \cup \hat{P}(S). \tag{2}$$

According to the upstream-downstream structure $\hat{P}$, we can define the following three different cost sharing methods for the problem. First, we begin with a method which can be considered as being based on the Absolute Territorial Sovereignty (ATS) theory. It is the following so-called Local Responsibility Sharing Method (LRS).

Definition 1 *For any $C \in R_+^{n+1}$, The Local Responsibility Sharing method is given by*

$$x_i^{LRS}(C) = c_i, i = 1, ..., n, L. \tag{3}$$

Apparently, the LRS method does not depend on $\hat{P}$. But, the following two methods depend on $\hat{P}$.

Definition 2 *The Upstream Equal Sharing method (UES) is defined by*

$$x_i^{UES}(C) = \sum_{j \in \sigma(\{i\})} \frac{c_j}{|\alpha(\{j\})|}, i = 1, ..., n, L, \tag{4}$$

where $|\alpha(\{j\})|$ is the number of agents in $\alpha(\{j\})$.

In the UES, an agent's cost share is the sum of her cost plus the equal sharing of all her downstream costs. This is the so-called Downstream Responsibility, derived from the Unlimited Territorial Integrity theory. Roughly speaking, downstream agents have the rights on the *quality* of the water they receive. If upstream agents pollute, thus alter the nature of the water flow, upstream agents should be held responsible. In other words, upstream agents have downstream responsibility in sharing their costs. Note that each individual agent's responsibility is dependent on $\hat{P}$.

In contrast, if we require that all the downstream agents are equally responsible for their upstream costs (the Upstream Responsibility principle), we then have the following Downstream Equal Sharing method (DES).

Definition 3 *The Downstream Equal Sharing method is defined by*

$$x_i^{DES}(C) = \sum_{j \in \alpha(\{i\})} \frac{c_j}{|\sigma(\{j\})|}, i = 1, ..., n, L. \tag{5}$$

In the DES, an agent's cost share is the sum of her cost plus the equal sharing of all her upstream costs. This so-called Upstream Responsibility is also derived from the Unlimited Territorial Integrity theory. To see how, think of the dual problem of the polluted river problem, the water sharing problem. In water sharing, downstream agents have the rights on the *quantity* of water they can possibly receive from the upstream under the condition that all the upstream agents have received their maximum welfare levels they can possibly achieve given the water resources they control.[16] As we mentioned in the introduction, in Ambec and Sprumont (2002) this argument gives rise to a water sharing method[17] called the Downstream Incremental Distribution method, which favors the downstream agents. We argue that when it comes to cost sharing (responsibility), the downstream agents should inherit certain proportion of responsibility in sharing the upstream costs. Like the UES, each individual agent's specific responsibility also depends on $\hat{P}$. But unlike the UES, the DES is based on the Upstream Responsibility principle.

[16]This does not mean that upstream agents would use up all the water they control. In fact, they might do better by transferring some water to their downstream counterparts and receiving certain monetary transfers from them. See Ambec and Sprumont (2002).

[17]To be precise, it is a welfare distribution method which corresponds to a water allocation.

Interestingly, the above three methods coincide with the Shapley values of three different games that can be defined for the problem. These games embody different interpretations of the ATS and the UTI theories.

Specifically, for any given problem $(N \cup \{L\}, G, C)$ we define the following three games: the *stand-alone game*, the *Upstream-oriented game* and the *Downstream-oriented game*, respectively.

Definition 4 *Let $(N \cup \{L\}, G, C)$ be a problem. Define the stand-alone game $L^{s.a.}(C)$ as follows:*

$$L^{s.a.}(C)(\emptyset) = 0, \;\; L^{s.a.}(C)(S) = C(S), S \subseteq N \cup \{L\}, \tag{6}$$

where $C(S) = \sum_{i \in S} c_i$ is the total cost of the links associated with S.

Definition 5 *Let $(N \cup \{L\}, G, C)$ be a problem. Define the Upstream-oriented game $L^U(C)$ as follows:*

$$L^U(C)(\emptyset) = 0, \;\; L^U(C)(S) = C(\sigma(S)), S \subseteq N \cup \{L\}. \tag{7}$$

where $\sigma(S)$ is defined in (1)

Definition 6 *Let $(N \cup \{L\}, G, C)$ be a problem. Define the Downstream-oriented game $L^D(C)$ as follows:*

$$L^D(C)(\emptyset) = 0, \;\; L^D(C)(S) = C(\alpha(S)), S \subseteq N \cup \{L\}. \tag{8}$$

where $\alpha(S)$ is defined in (2)

EXAMPLE 2. The Upstream-oriented game and the Downstream-oriented game generated from the problem in EXAMPLE 1 are given below. Note that not all coalitional values are listed.

The **Upstream-oriented Game** $L^U(C)$:

$$L^U(C)(1) = c_1 + c_2 + c_5 + c_L, L^U(C)(3) = c_3 + c_4 + c_5 + c_L,$$

$$L^U(C)(1,2) = c_1 + c_2 + c_5 + c_L, L^U(C)(1,3) = c_1 + c_2 + c_3 + c_4 + c_5 + c_L,$$

$$L^U(C)(2,3) = c_2 + c_3 + c_4 + c_5 + c_L, L^U(C)(2,5) = c_2 + c_5 + c_L,$$

$L^U(C)(1,2,3) = c_1 + c_2 + c_3 + c_4 + c_5 + c_L$, $L^U(C)(1,2,5) = c_1 + c_2 + c_5 + c_L$,
$L^U(C)(1,2,3,4) = c_1+c_2+c_3+c_4+c_5+c_L$, $L^U(C)(1,3,4,5) = c_1+c_2+c_3+c_4+c_5+c_L$,
$L^U(C)(1,2,3,4,5) = c_1+c_2+c_3+c_4+c_5+c_L$, $L^U(C)(1,2,3,4,5,L) = c_1+c_2+c_3+c_4+c_5+c_L$.

The **Downstream-oriented Game** $L^D(C)$:

$$L^D(C)(1) = c_1, L^D(C)(3) = c_3, L^D(C)(5) = c_1 + c_2 + c_3 + c_4 + c_5,$$

$L^D(C)(L) = c_1+c_2+c_3+c_4+c_5+c_L$, $L^D(C)(1,2) = c_1+c_2$, $L^D(C)(1,3) = c_1+c_3$,
$L^D(C)(1,5) = c_1 + c_2 + c_3 + c_4 + c_5$, $L^D(C)(1,L) = c_1 + c_2 + c_3 + c_4 + c_5 + c_L$,
$L^D(C)(1,2,3) = c_1 + c_2 + c_3$, $L^D(C)(2,5,L) = c_1 + c_2 + c_3 + c_4 + c_5 + c_L$,

$L^D(C)(1,2,3,4) = c_1+c_2+c_3+c_4$, $L^D(C)(1,3,4,5,L) = c_1+c_2+c_3+c_4+c_5+c_L$,

$L^D(C)(1,2,3,4,5) = c_1+c_2+c_3+c_4+c_5$, $L^D(C)(1,2,3,4,5,L) = c_1+c_2+c_3+c_4+c_5+c_L$.

In the cooperative game theory, the Shapley value (Shapley, 1953) is the most important solution concept. It is given below.

Definition 7 *For any game $C : 2^N \to R$, the Shapley value is given by*

$$x_i^{Sh}(C) = \sum_{S \ni i} \frac{(n-|S|)!(|S|-1)!}{n!}(C(S) - C(S \setminus \{i\}), i = 1, ..., n. \quad (9)$$

The Shapley value of an agent can be regarded as an average of the marginal cost incurred by the agent to each and every coalition of other agents.

Another important solution concept is the *core*. In the context of cost sharing, a *core allocation* is an allocation which cannot be dominated by an alternative allocation in which some agent or coalition of agents can have their cost shares reduced by themselves and be better off given their stand-alone cost. In other words, once a core allocation is proposed, no coalition of agents has the incentive to change the allocation. The core of a game is the set of all core allocations. In general, a game may have an empty core.

The Shapley value of a game may or may not be in the core of the game when the latter is nonempty. But, Shapley (1971) shows that a *concave* cost game always has a nonempty core and moreover, the Shapley value is in the core. A (cost) game is concave if an agent's marginal cost is non-increasing as the agent joins larger coalitions. Formally, a game $C(\cdot)$ is concave if

$$C(S \cup i) - C(S) \geq C(T \cup i) - C(T), \forall S \subseteq T \subseteq N, i \notin T.$$

In the next section, we will show that the three games defined above are all concave. Thus, their Shapley values are in the core of the corresponding games. Moreover, we will show that the three methods coincide with the Shapley values of the three games, respectively. Therefore, the solutions by the three methods are all core allocations of the corresponding games.

We conclude this section by pointing out that while the LRS and the UES are the extensions of the two corresponding methods in Ni and Wang (2007), the DES is closely related to the well-known airport landing fee solution (Littlechild and Owen, 1973, 1977).[18] This can be shown by considering the following special case where the tree network is a line-tree.

$$1 \xrightarrow{c_1} 2 \xrightarrow{c_2} 3 \xrightarrow{c_3} 4 \xrightarrow{c_4} \text{L}, c_L$$

The Upstream Equal Sharing is the following:

$$\begin{aligned}
x_1^{UES}(C) &= c_1 + \frac{1}{2}c_2 + \frac{1}{3}c_3 + \frac{1}{4}c_4 + \frac{1}{5}c_L, \\
x_2^{UES}(C) &= \frac{1}{2}c_2 + \frac{1}{3}c_3 + \frac{1}{4}c_4 + \frac{1}{5}c_L, \\
x_3^{UES}(C) &= \frac{1}{3}c_3 + \frac{1}{4}c_4 + \frac{1}{5}c_L, \\
x_4^{UES}(C) &= \frac{1}{4}c_4 + \frac{1}{5}c_L, \\
x_L^{UES}(C) &= \frac{1}{5}c_L,
\end{aligned}$$

and it coincides with the UES in Ni and Wang (2007).

[18]Note that the DES is defined for a different problem than the airport landing fee problem.

The Downstream Equal Sharing, on the other hand, coincides with the airport landing fee solution.

$$
\begin{aligned}
x_1^{DES}(C) &= \frac{1}{5}c_1 \\
x_2^{DES}(C) &= \frac{1}{5}c_1 + \frac{1}{4}c_2 \\
x_3^{DES}(C) &= \frac{1}{5}c_1 + \frac{1}{4}c_2 + \frac{1}{3}c_3 \\
x_4^{DES}(C) &= \frac{1}{5}c_1 + \frac{1}{4}c_2 + \frac{1}{3}c_3 + \frac{1}{2}c_4 \\
x_L^{DES}(C) &= \frac{1}{5}c_1 + \frac{1}{4}c_2 + \frac{1}{3}c_3 + \frac{1}{2}c_4 + c_L
\end{aligned}
$$

3 Characterizations of the LRS, the UES and the DES Methods

3.1 A Characterization of the LRS Method

In order to characterize the LRS method, similar to Ni and Wang (2007), we need the following axioms.

Additivity: For any $C^1 = (c_1^1, \ldots, c_n^1, c_L^1) \in R_+^{n+1}$ and $C^2 = (c_1^2, \ldots, c_n^2, c_L^2) \in R_+^{n+1}$, we have $x_j(C^1 + C^2) = x_j(C^1) + x_j(C^2)$ for all $j \in N \cup \{L\}$.

Additivity is a classical axiom in the cooperative game theory (Shapley, 1953) and the cost sharing literature (Moulin, 2002). As a mathematical structural invariance axiom, Additivity itself has no normative content. However, for our pollution cost sharing problem, we can provide the following interpretation.

Imagine that in the river network cost sharing problem, the agents have to share two kinds of costs, namely the pollution costs and the maintenance costs. Additivity says that there is no difference whether the agents share the two costs separately or together.

No Blind Costs: For any $i \in N \cup \{L\}$ and any $C \in R_+^{n+1}$, if $c_i = 0$, then $x_i(C) = 0$.

No Blind Costs says that if an agent's cost is zero, then the agent's cost share should be zero as well. This axiom rules out cross subsidization between the agents. If an agent's cost is zero, it means that there is no pollution (cost) at the agent's location. Therefore, the agent neither pollutes herself and her downstream nor is polluted by her upstream agents. Thus, the agent shouldn't bear any cost.

Efficiency: $\sum_{j=1}^{n+1} x_j = \sum_{j=1}^{n+1} c_j$.

Efficiency is always satisfied by a cost sharing method by definition. Nevertheless, we mention it explicitly in all our characterizations as it is the only axiom that is related to economic efficiency.[19]

Theorem 1 *The Local Responsibility Sharing method is the unique method that satisfies Additivity, No Blind Costs and Efficiency. It coincides with the Shapley value of the stand-alone game of the problem. Moreover, it is in the core of the game.*

The proof of theorem is similar to the proofs of Theorem 1, Propositions 1 and 2 in Ni and Wang (2007). We omit it.

3.2 A Characterization of the UES Method

In this subsection, we provide a characterization of the Upstream Equal Sharing method and show that it coincides with the Shapley value of the Upstream-oriented game generated from the problem. Moreover, it is in the core.

[19] By economic efficiency we mean Pareto efficiency. But note that in this paper Efficiency is just a budget balance condition, i.e., the sum of the cost shares collected from the agents is exactly equal to the total costs. It does not mean that it is a Pareto efficient cost sharing in which the cost sharing method induces the agents reporting their true pollution costs. A complete discussion of this cost sharing game would be too complicated and is beyond the scope of this paper.

Recall that for a given cost function $C = (c_1, c_2, \ldots, c_n, c_L)$, the associated Upstream-oriented game is defined by

$$L^U(C)(S) = C(\sigma(S)) = \sum_{j \in \sigma(S)} c_j, S \subseteq N \cup \{L\},$$

where $\sigma(S) = S \cup \hat{P}^{-1}(S) = S \cup (\bigcup_{j \in S} \hat{P}^{-1}(i))$.

Note that

$$L^U(C)(N \cup \{L\}) = C(\sigma(N \cup \{L\})) = C(N \cup \{L\}).$$

The Upstream Equal Sharing method is repeated below.

$$x_i^{UES}(C) = \sum_{j \in \sigma(\{i\})} \frac{c_j}{|\alpha(\{j\})|}, i = 1, ..., n, L, \tag{10}$$

where $|\alpha(\{j\})|$ is the number of the elements in $\alpha(\{j\})$, and $\alpha(\{j\}) = \{j\} \cup \hat{P}(\{j\}) = \{j\} \cup \hat{P}(j)$.

We introduce the following axioms.

Independence of Upstream Costs: For any $i \in N \cup \{L\}$, any $C, C' \in R_+^{n+1}$ such that $c_l = c'_l, l \in \hat{P}^{-1}(i)$, we have $x_j(C) = x_j(C')$ for all $j \in \hat{P}^{-1}(i)$.

This axiom is based on the Downstream Responsibility principle, which is a responsibility version of the theory of the Unlimited Territorial Integrity as we discussed in the introduction. It says that an agent's cost share only depends on her own pollution cost as well as all her downstream costs, but not on upstream costs for which she is assumed not responsible.

Upstream Symmetry: For any $i \in N \cup \{L\}$, for all $j, k \in \alpha(\{i\}) = \{i\} \cup \hat{P}(i)$, we have

$$x_j(0, ..., 0, c_i, 0, ..., 0) = x_k(0, ..., 0, c_i, 0, ..., 0).$$

The Upstream Symmetry requires that all upstream agents have equal responsibilities for a given downstream cost. We treat all upstream agents equally in terms of responsibility for a given downstream cost because we assume that, first, pollution cannot be washed away easily and, secondly, it is hard to tell exactly how much each upstream agent contributes to the given downstream cost. Moreover, the axiom implies that any agent, no matter how far she is from any of her downstream agents, her responsibility remains the same.

The following Independence of Irrelevant Costs axiom is essential in all our characterizations in this paper. As we discussed before, it imposes an upper bound on an agent's responsibility, i.e., an agent is not responsible for any cost that is irrelevant to her. Formally,

Independence of Irrelevant Costs: For any $i \in N \cup \{L\}$, for all $j \in N \cup \{L\} \setminus (\hat{P}(i) \cup \{i\} \cup \hat{P}^{-1}(i))$, we have

$$x_j(0, ..., 0, c_i, 0, ..., 0) = 0.$$

This is a compelling axiom since if two agents are not related to each other in the river network in terms of the upstream-downstream relationship, each agent wouldn't affect the other in pollution cost sharing. Thus, it is plausible to require that neither agent should be responsible for the cost of the other. This axiom allows us to extend the responsibility theory derived from the theory of the Unlimited Territorial Integrity to a tree network.

Note that in Ni and Wang (2007), each agent is either an upstream or a downstream of all other agents in the linear river model. But in our tree model, the upstream-downstream relation on the agents is a partial order.

Now we are ready to state the following theorem.

Theorem 2 *The Upstream Equal Sharing method is the unique method that satisfies Additivity, Independence of Upstream Costs, Upstream Symmetry, Independence of Irrelevant Costs and Efficiency. Moreover, it coincides with the Shapley value of the Upstream-oriented game of the problem, and it is in the core of the game.*

Proof: We divide the proof into three steps.

Step 1. First, we show that the Shapley value x^{Sh} of the Upstream-oriented game $L^U(C)$ coincides with the UES method x^{UES}.

By the definition of the Upstream-oriented game, for any

$$C^k = (0, \ldots, 0, 1, 0, \ldots, 0),$$

where 1 is the kth component of the $n+1$-dimensional vector C^k, the corresponding Upstream-oriented game is given by

$L^U(C^k)(S) = 0$ if $S \subset N \cup \{L\} \backslash \alpha(\{k\})$ and $L^U(C^k)(S) = 1$ otherwise.

Clearly, all agents in $N \cup \{L\} \backslash \alpha(\{k\})$ are dummy agents and all agents in $\alpha(\{k\})$ are symmetric. Thus the Shapley value of the game $L^U(C^k)$ is given by

$$x_i^{Sh}(L^U(C^k)) = \begin{cases} \frac{1}{|\alpha(\{k\})|}, i \in \alpha(\{k\}) \\ 0, \text{ otherwise} \end{cases} \quad (11)$$

for all $i \in N \cup \{L\}$.

Since the cost vectors, $C^k(k \in \{1, \ldots, n, L\})$, form a basis of R^{n+1}, for any $C \in R_+^{n+1}$, it can be uniquely written as $C = \sum_{k \in N \cup \{L\}} c_k \cdot C^k$. By the definition of the Upstream-oriented game, for $\emptyset \neq S \subseteq N \cup \{L\}$, we have

$$\begin{aligned} L^U(C)(S) &= \sum_{j \in \sigma(S)} c_j \\ &= \sum_{j \in \sigma(S)} (\sum_{k \in N \cup \{L\}} [c_k \cdot C^k]_j) \\ &= \sum_{k \in N \cup \{L\}} c_k \cdot (\sum_{j \in \sigma(S)} [C^k]_j) \\ &= \sum_{k \in N \cup \{L\}} c_k \cdot L^U(C^k)(S) \end{aligned}$$

where $[C]_j$ is the jth component of the vector C. Because the Shapley value satisfies Additivity, we have

$$\begin{aligned} x_i^{Sh}(L^U(C)) &= \sum_{k \in N \cup \{L\}} c_k \cdot x_i^{Sh}(L^U(C^k)) \\ &= \sum_{k \in N \cup \{L\} \backslash \sigma(\{i\})} 0 + \sum_{k \in \sigma(\{i\})} \frac{c_k}{|\alpha(\{k\})|} \\ &= x_i^{UES}(C) \end{aligned}$$

for all $i \in N \cup \{L\}$.

Step 2. We show that the UES method is the unique method satisfying Additivity, Independence of Upstream Costs, Upstream Symmetry, Independence of Irrelevant Costs and Efficiency.

We first show that x^{UES} satisfies these five axioms. Additivity is straightforward. To show that x^{UES} satisfies Independence of Upstream costs, for any $i \in N \cup \{L\}$, any $C, C' \in R_+^{n+1}$ such that $c_l = c'_l, l \in \hat{P}^{-1}(i)$, then for all $j \in \hat{P}^{-1}(i)$, we have

$$\begin{aligned} x_j^{UES}(C) &= \sum_{l \in \sigma(\{j\})} \frac{c_l}{|\alpha(\{l\})|} \\ &= \sum_{l \in \{j\} \cup \hat{P}^{-1}(j)} \frac{c_l}{|\alpha(\{l\})|} \\ &= \sum_{l \in \{j\} \cup \hat{P}^{-1}(j)} \frac{c'_l}{|\alpha(\{l\})|} \\ &= x_j^{UES}(C') \end{aligned}$$

since $j \in \hat{P}^{-1}(i) \Rightarrow \hat{P}^{-1}(j) \subseteq \hat{P}^{-1}(i)$.

To see that x^{UES} satisfies Upstream Symmetry, for any $i \in N \cup \{L\}$, for all $j, k \in \alpha(\{i\}) = \{i\} \cup \hat{P}(i)$, we have

$$\begin{aligned} x_j^{UES}(0, ..., 0, c_i, 0, ..., 0) &= \sum_{l \in \sigma(\{j\})} \frac{c_l}{|\alpha(\{l\})|} \\ &= \sum_{l \in \{j\} \cup \hat{P}^{-1}(j)} \frac{c_l}{|\alpha(\{l\})|} \\ &= \frac{c_i}{|\alpha(i)|} \\ &= \sum_{l \in \{k\} \cup \hat{P}^{-1}(k)} \frac{c_l}{|\alpha(\{l\})|} \\ &= \sum_{l \in \sigma(\{k\})} \frac{c_l}{|\alpha(\{l\})|} \\ &= x_k^{UES}(0, ..., 0, c_i, 0, ..., 0). \end{aligned}$$

We now show that x^{UES} satisfies Independence of Irrelevant Costs. Since $j \in N \cup \{L\} \backslash (\hat{P}(i) \cup \{i\} \cup \hat{P}^{-1}(i))$, we have $i \notin \hat{P}^{-1}(j)$. Otherwise, $i \in \hat{P}^{-1}(j)$ implies $j \in \hat{P}(i)$, a contradiction. Thus, if $C = (0, ..., 0, c_i, 0, ..., 0)$, then $c_l = 0$ for all $l \in \{j\} \cup \hat{P}^{-1}(j) = \sigma(j)$. We therefore have

$$x_j^{UES}(C) = \sum_{l \in \sigma(\{j\})} \frac{c_l}{|\alpha(\{l\})|} = 0.$$

Finally, in Step 1 we have shown that x^{UES} is the Shapley value of the Upstream-oriented game. Since the Shapley value satisfies Efficiency, x^{UES} also satisfies Efficiency.

Now we show that the UES is the *only* method satisfying the five axioms. Suppose that a cost sharing method x satisfies the five axioms. Fix a arbitrary tree consisting of $n+1$ nodes. For any $k \in \{1, 2, \ldots, n, L\}$, let $C^k = (0, 0, \ldots, 0, 1, 0, \ldots, 0)$ where 1 is the kth component of the $n+1$-dimensional vector C^k. By Independence of Upstream Costs, $x_j(C^k) = 0$ for all $j \in \hat{P}^{-1}(k)$. By Independence of Irrelevant Costs, $x_j(C^k) = 0$ for all $j \in N \cup \{L\} \backslash (\hat{P}(k) \cup \sigma(\{k\}))$. By Upstream Symmetry, $x_j(C^k) = x_{j'}(C^k) = \beta \geq 0$ for all $j, j' \in \alpha(\{k\})$. By Efficiency, we have

$$\begin{aligned}
& \sum_{j \in N \cup \{L\}} x_j(C^k) \\
= & \sum_{j \in \hat{P}^{-1}(k)} x_j(C^k) + \sum_{j \in \{k\} \cup \hat{P}(k)} x_j(C^k) + \sum_{j \in N \cup \{L\} \backslash (\hat{P}(k) \cup \{k\} \cup \hat{P}^{-1}(k))} x_j(C^k) \\
= & \ 0 + \sum_{j \in \alpha(\{k\})} \beta + 0 \\
= & \ \beta \times |\alpha(\{k\})| \\
= & \ 1
\end{aligned}$$

therefore $x_j(C^k) = \frac{1}{|\alpha(\{k\})|}$ if $j \in \alpha(\{k\})$ and $x_j(C^k) = 0$ otherwise.

Again, for any $C \in R_+^{n+1}$, it can be uniquely written as $C = (c_1, \ldots, c_n, c_L) = \sum_{k \in N \cup \{L\}} c_k \cdot C^k$. By Additivity, we have

$$\begin{aligned}
x_i(C) &= x_i\Big(\sum_{k \in N \cup \{L\}} c_k \cdot C^k\Big) \\
&= \sum_{k \in N \cup \{L\}} c_k \cdot x_i(C^k) \\
&= \sum_{k \in N \cup \{L\} \backslash (\hat{P}(i) \cup \sigma(\{i\}))} 0 + \sum_{k \in \hat{P}(i)} 0 + \sum_{k \in \sigma(\{i\})} \frac{c_k}{|\alpha(\{k\})|} \\
&= x_i^{UES}(C)
\end{aligned}$$

for all $i \in N \cup \{L\}$.

Step 3. We now show that the Shapley value is in the core. It suffices to show that the Upstream-oriented game $L^U(C)$ is concave. That is, for all

$i \in N \cup \{L\}$, all $S, T \subset (N \cup \{L\}) \backslash \{i\}$ and $S \subset T$, we have

$$L^U(C)(S \cup \{i\}) - L^U(C)(S) \geq L^U(C)(T \cup \{i\}) - L^U(C)(T). \qquad (12)$$

Suppose that $S \subset T$ and $i \notin T$. Let $H_S^U = \sigma(S \cup \{i\}) \backslash \sigma(S)$ and $H_T^U = \sigma(T \cup \{i\}) \backslash \sigma(T)$. We claim that $H_T^U \subseteq H_S^U$.

Since

$$\begin{aligned} H_S^U &= \sigma(S \cup \{i\}) \backslash \sigma(S) \\ &= S \cup \{i\} \cup \hat{P}^{-1}(S) \cup \hat{P}^{-1}(i) \setminus (S \cup \hat{P}^{-1}(S)) \\ &= \{i\} \cup \hat{P}^{-1}(i) \setminus (S \cup \hat{P}^{-1}(S)), \end{aligned}$$

and

$$H_T^U = \{i\} \cup \hat{P}^{-1}(i) \setminus (T \cup \hat{P}^{-1}(T)),$$

and that

$$T \cup \hat{P}^{-1}(T) \supseteq S \cup \hat{P}^{-1}(S),$$

we thus have

$$H_T^U \subseteq H_S^U.$$

Therefore,

$$\begin{aligned} L^U(C)(S \cup \{i\}) - L^U(C)(S) &= \sum_{j \in \sigma(S \cup i)} c_j - \sum_{j \in \sigma(S)} c_j \\ &= \sum_{j \in H_S^U} c_j \\ &\geq \sum_{j \in H_T^U} c_j \\ &= \sum_{j \in \sigma(T \cup i)} c_j - \sum_{j \in \sigma(T)} c_j \\ &= L^U(C)(T \cup \{i\}) - L^U(C)(T). \end{aligned}$$

This completes the proof of the theorem.

3.3 A Characterization of the DES Method

We now provide a characterization of the Downstream Equal Sharing method and show that it coincides with the Shapley value of the Downstream-oriented game of the problem and, moreover, is in the core of the game.

Recall that for a given cost function $C = (c_1, c_2, \ldots, c_n, c_L)$, the Downstream-oriented game is defined by

$$L^D(C)(S) = C(\alpha(S)) = \sum_{j \in \alpha(S)} c_j, S \subseteq N \cup \{L\},$$

where $\alpha(S) = S \cup \hat{P}(S) = S \cup (\bigcup_{j \in S} \hat{P}(i))$.

Note that

$$L^D(C)(N \cup \{L\}) = C(\alpha(N \cup \{L\})) = C(N \cup \{L\}).$$

The Downstream Equal Sharing method is repeated below.

$$x_i^{DES}(C) = \sum_{j \in \alpha(\{i\})} \frac{c_j}{|\sigma(\{j\})|}, i = 1, ..., n, L, \tag{13}$$

where $\sigma(\{j\}) = \{j\} \cup \hat{P}^{-1}(\{j\}) = \{j\} \cup \hat{P}^{-1}(j)$.

To characterize the DES, we need the following two axioms.

Independence of Downstream Costs: For any $i \in N \cup \{L\}$, any $C, C' \in R_+^{n+1}$ such that $c_l = c'_l, l \in \hat{P}(i)$, we have $x_j(C) = x_j(C')$ for all $j \in \hat{P}(i)$.

The Independence of Downstream Costs imposes an upper bound on an agent's cost share, i.e., it wouldn't be dependent on her downstream agents' costs. This axiom, as we discussed in the introduction on the Downstream Responsibility principle, says that an agent's responsibility in cost sharing is directed toward her upstream rather than the downstream. This alternative theory is in contrast to the Upstream Responsibility. Note that this version of the responsibility theory is a *derived* responsibility from the dual problem, namely the water sharing problem (Ambec and Sprumont, 2002).

Downstream Symmetry: For any $i \in N \cup \{L\}$, for all $j, k \in \sigma(\{i\}) = \{i\} \cup \hat{P}^{-1}(i)$, we have

$$x_j(0, ..., 0, c_i, 0, ..., 0) = x_k(0, ..., 0, c_i, 0, ..., 0).$$

Similar to the Upstream Symmetry, the Downstream Symmetry treats all downstream agents equally in terms of responsibility for a given upstream cost. Again this axiom ignores the physical distance between the agents. In other words, all agents, no matter how far they are from any of their given upstream agents, are equally responsible for their costs. This axiom together with the previous axiom, specifies the downstream responsibility.

Now we state the following theorem.

Theorem 3 *The Downstream Equal Sharing method is the unique method that satisfies Additivity, Independence of Downstream Costs, Downstream Symmetry, Independence of Irrelevant Costs and Efficiency. Moreover, it coincides with the Shapley value of the Downstream-oriented game of the problem and, is in the core of the game.*

Proof: The proof is analogous to the proof of Theorem 2. To do that, we only need to replace the Independence of Upstream Costs and the Upstream Symmetry in the proof of Theorem 2 by the Independence of Downstream Costs and the Downstream Symmetry, respectively. It is also straightforward to check that the Downstream Equal Sharing method coincides with the Shapley value of the Downstream-oriented game of the problem and is in the core of the game. We omit the details.

The UES and DES methods provide the upper and lower bounds for an agent's cost shares. Apparently, the upstream agents prefer the DES but the downstream agents prefer the UES. Some compromise between the two solutions should be made. For example, in realistic situations, a weight can be assigned to each agent according to the agent's observable characteristics such as the agent's pollutant emission/abatement level, willingness-to-pay or ability-to-accept, the size of the agent (e.g., population), the distance between the agent to the Lake, etc. Costs can be then allocated according to a weighted average rather than the equal division of the costs among the relevant agents.

3.4 An Example: the Baiyangdian Lake Catchment

In this part, we provide a real life example of the pollution cost sharing problem in China. The two cost sharing methods, the UES and the DES, suggested by our model bring forward two extreme viewpoints between which policy makers can balance their choices in allocating the total costs of the project.

The Baiyangdian Lake, the largest remaining semi-closed freshwater body in the northern China, is one of the most important and vulnerable ecosystems. It lies in the middle reaches of Daqing River Basin and ultimately discharges in Bohai Gulf and the Yellow Sea. All of the lake body is located in Anxin and Xiong counties of Baoding Municipality. The lake has a surface area of 366 km^2 and consists of a series of natural low-lying depressions and reed marshes. There are four large-scale reservoirs upstream of the lake, including Angezhuang, Longmen, Wangkuai, and Xidayang with water flowing into the lake via eight rivers.

Historically, the lake served many environmental and economic purposes and was called as the "kidney" of the northern China. Its strategic position is reflected by its major role in regulating floodwater discharges and water levels, and subsequently moderates waterlogging. The lake has been an economically important source of freshwater fish and reed production as well as a source for drinking water and irrigation.

However, over the last four decades, the size of the lake has decreased by almost half because of controlled water flows, and soil erosion, rising population, expanded agricultural and industrial activities, limited solid and wastewater disposal measures in the upstream and within the lake area.[20] The lake has been a major depository of wastewater discharges, pollutant substances, and sediments. Almost all the subproject urban areas discharge untreated wastewater into the adjacent rivers and eventually to the lake. Major industries within the service areas include tanneries, textile, paper, automobile, fertilizer, chemicals, and machinery plants which are all polluting industries. It is estimated that half of the pollution originates from the upstream, while the other half emanates from the population of more than 200,000 living in the lake's wetlands and surrounding areas.

[20]Indiscriminate logging in the upper watersheds, mining and quarrying, and conversion of forests and wetlands to agricultural/industrial sites, have contributed to massive soil erosion and land degradation which in turn, have caused excessive water obstruction along the tributaries that feed into the lake.

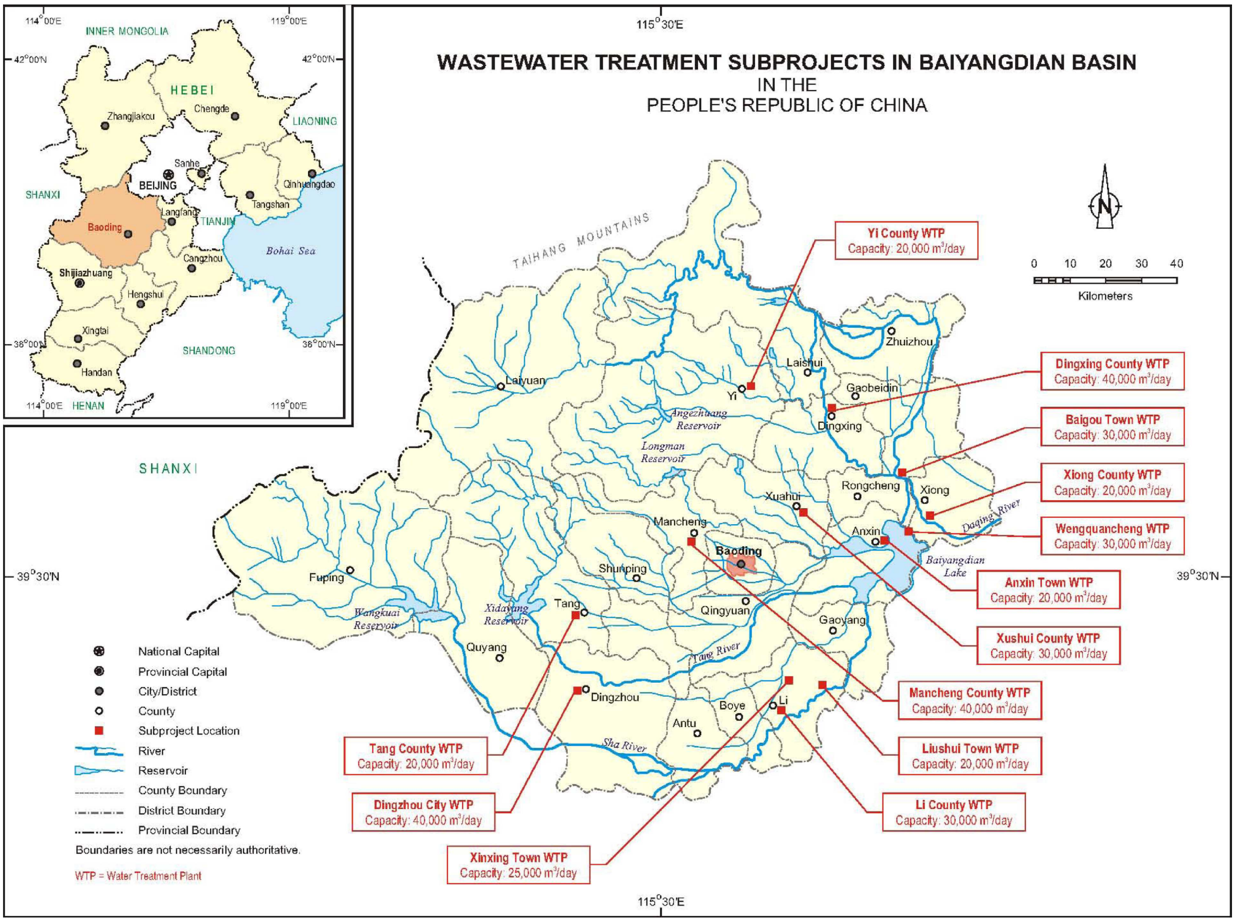

Figure 2. Wastewater treatment plants in Baiyangdian area

Eutrophication of the lake has become a serious problem. Water quality deteriorated from class III to classes IV and V over the past four decades.[21] In 2008, an integrated Project cofinanced by the Asian Development Bank (ADB) and Global Environment Facility (GEF), with a total investment of US$203.7 million, helps to construct 13 wastewater treatment plants in the upstream of Lake Baiyangdian, along with other facilities such as water supply, reforestation, rehabilitation of watershed, flood control etc. The Project reduces the COD by 36,850 tons/year (or 365,000 m^3/day of wastewater)

[21] Class V is considered equivalent to raw sewage according to China's National Ambient Water Quality Standards for Surface Water.

upon completion. It also reduces morbidity rates by 10% and improves sanitation and hygiene for 1.12 million people and maintains a sustainable healthy water level of the lake. The locations of the wastewater treatment plants are shown in Figure 2.

The 13 wastewater treatment plants located in the main counties and townships in the upstream are essential for pollution reduction in Lake Baiyangdian. Other subprojects around the lake such as reforestation and wastershed rehibilitation also contribute to the restoration of the lake's environmental and economic functions. The investment figures for the subprojects are round up and depicted in Figure 3 with an illustration of the tree-structure water flows. The unit is million US$.[22]

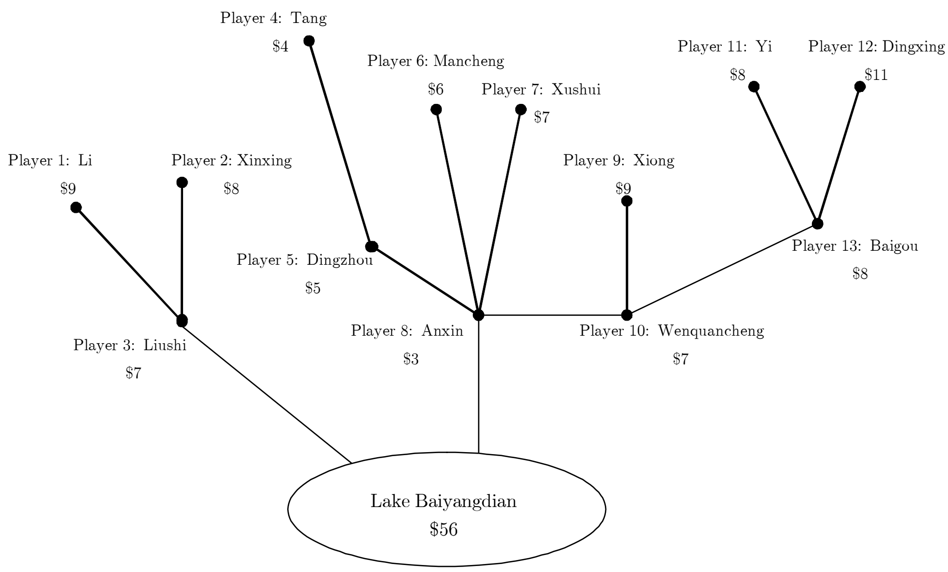

Figure 3. Illustration of the Baiyangdian Lake Project

According to the formulae developed in this paper, we calculate the two cost sharing solutions based on the UES and DES methods, respectively.

[22]Because the operation and maintenance costs can be covered by the wastewater tariff collected from households and industrial users, we confine our cost sharing analysis only on the construction costs.

First, we calculate the UES sharing:

$$
\begin{array}{ll}
x_1^{UES}(C) = c_1 + \frac{1}{3}c_3 + \frac{1}{14}c_L = \frac{46}{3}, & x_2^{UES}(C) = c_2 + \frac{1}{3}c_3 + \frac{1}{14}c_L = \frac{43}{3}, \\
x_3^{UES}(C) = \frac{1}{3}c_3 + \frac{1}{14}c_L = \frac{19}{3}, & x_4^{UES}(C) = c_4 + \frac{c_5}{2} + \frac{c_8}{10} + \frac{c_L}{14} = \frac{168}{10}, \\
x_5^{UES}(C) = \frac{1}{2}c_5 + \frac{1}{10}c_8 + \frac{1}{14}c_L = \frac{108}{10}, & x_6^{UES}(C) = c_6 + \frac{1}{10}c_8 + \frac{1}{14}c_L = \frac{133}{10}, \\
x_7^{UES}(C) = c_7 + \frac{1}{10}c_8 + \frac{1}{14}c_L = \frac{133}{10}, & x_8^{UES}(C) = \frac{1}{10}c_8 + \frac{1}{14}c_L = \frac{43}{10}, \\
x_9^{UES}(C) = c_9 + \frac{c_{10}}{5} + \frac{c_8}{10} + \frac{c_L}{14} = \frac{112}{10}, & x_{10}^{UES}(C) = \frac{1}{5}c_{10} + \frac{1}{10}c_8 + \frac{1}{14}c_L = \frac{62}{10}, \\
x_{11}^{UES}(C) = c_{11} + \frac{1}{3}c_{13} + \frac{1}{5}c_{10} + \frac{1}{10}c_8 + \frac{1}{14}c_L = \frac{203}{15}, & \\
x_{12}^{UES}(C) = c_{12} + \frac{1}{3}c_{13} + \frac{1}{5}c_{10} + \frac{1}{10}c_8 + \frac{1}{14}c_L = \frac{298}{15}, & \\
x_{13}^{UES}(C) = \frac{1}{3}c_{13} + \frac{1}{5}c_{10} + \frac{1}{10}c_8 + \frac{1}{14}c_L = \frac{133}{15}, & \\
x_L^{UES}(C) = \frac{1}{14}c_L = 4. &
\end{array}
$$

Then, we calculate the DES sharing:

$$
\begin{array}{ll}
x_1^{DES}(C) = \frac{1}{3}c_1 = 3, & x_2^{DES}(C) = \frac{1}{3}c_2 = \frac{8}{3}, \\
x_3^{DES}(C) = \frac{1}{3}c_1 + \frac{1}{3}c_2 + \frac{1}{2}c_3 = \frac{55}{6}, & x_4^{DES}(C) = \frac{1}{4}c_4 = \frac{3}{2}, \\
x_5^{DES}(C) = \frac{1}{3}c_5 + \frac{1}{4}c_4 = \frac{35}{6}, & x_6^{DES}(C) = \frac{1}{3}c_6 = 3, \\
x_7^{DES}(C) = \frac{1}{3}c_7 = 3, & \\
x_8^{DES}(C) = \frac{c_8}{2} + \frac{c_5+c_6+c_7+c_{10}}{3} + \frac{c_4+c_9+c_{13}}{4} + \frac{c_{11}+c_{12}}{5} = \frac{1363}{60}, & \\
x_9^{DES}(C) = \frac{1}{4}c_9 = \frac{5}{4}, & \\
x_{10}^{DES}(C) = \frac{1}{5}c_{10} + \frac{1}{5}(c_{11} + c_{12}) + \frac{1}{4}(c_9 + c_{13}) = \frac{563}{60}, & \\
x_{11}^{DES}(C) = \frac{1}{5}c_{11} = \frac{8}{5}, & x_{12}^{DES}(C) = \frac{1}{5}c_{12} = \frac{11}{5}, \\
x_{13}^{DES}(C) = \frac{1}{5}(c_{11} + c_{12}) + \frac{1}{4}c_{13} = \frac{29}{5}, & \\
x_L^{DES}(C) = \frac{c_3+c_8}{2} + \frac{c_1+c_2+c_5+c_6+c_7+c_{10}}{3} + \frac{c_4+c_9+c_{13}}{4} + \frac{c_{11}+c_{12}}{5} + c_L = 87\frac{53}{60}. &
\end{array}
$$

As for the project implementation, Baoding municipal government repaid the investment costs to ADB and GEF. Historically the Baoding municipality hoped that the counties and townships could finance their wastewater treatment plants but it turned out that they did not have an incentive to do so. The reason is twofold, first, the pollutants that an upstream township discharges can be hardly observable or verifiable; second, the counties and townships did not have an idea what is the fair share or "price" they should contribute. It is hoped that in situations where there is no centralized authority such as international water resource negotiations, our methods can provide a framework or guideline.

4 Concluding Remarks

We studied the cost sharing problem in a polluted river network. We proposed three cost sharing methods for the problem and provided their axiomatic characterizations. Our characterizations are based on the two well-known theories in international disputes. We also showed that these three methods are the Shapley values of the three associated games of the problem. Moreover, they are in the cores of the three games, respectively.

Our paper can be extended in a number of directions. For example, between the two opposing versions of responsibility (the UR and the DR), a range of them can be proposed. Accordingly, a set of methods can be generated by considering all the weighted averages of the UES and the DES methods. We can make the weights depending on the weights we assign to each and every agent. In assigning weights to the agents, we can take into account the differences between the agents in, e.g., population, pollutant emission, the location of the agent in the network, etc.

A more challenging but perhaps more useful extension of our work could be to combine the water sharing problem with the water pollution cost sharing problem. In fact, Weber (2001) has considered the problem of allocating water and pollution rights along a river. But, in Weber (2001), there is a single source of water. There are no branches or inflows at various locations of the agents. Also the pollution rights are exogenously assigned by a regulator. Agents engage in trading water rights and pollution rights by playing a noncooperative game. Our approach in this paper is axiomatic and can be considered as the reverse of the Weber's. We begin with ceratin normative principles (axioms). The cost sharing methods determined by the various combinations of the axioms would, in effect, determine or imply implicitly an assignment of property rights.

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