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Mihai Daniel Roman

Bucharest Academy of Economic Studies, Bucharest, Romania

15. July 2008

ENTREPRISES BEHAVIOR IN COOPERATIVE AND PUNISHMENT 'S REPEATED NEGOTIATIONS

Mihai Daniel ROMAN
PhD Professor Academy of Economic Studies, Faculty of Cybernetics, Statistics and Economic Informatics, Department of Economic Cybernetics, 010374, 6 Romana Square, Bucharest, Romania

E-mail: romanm@ase.ro

Abstract: *Our paper considers a "negotiation game" between two players which combines the features of two-players alternating offers bargaining and repeated games. Generally, the negotiation game in general admits a large number of equilibriums but some of which involve delay and inefficiency. Thus, complexity and bargaining in tandem may offer an explanation for cooperation and efficiency in repeated games. The Folk Theorem of repeated games is a very used result that shows if players are enough patience then it is possible to obtain a cooperative equilibrium of the infinite repeated game. We proof a new folk theorem for finitely repeated games and also we find new conditions (under stage number and minimum discount factor value) such that players cooperate at least one period in cooperative-punishment repeated games. Finally we present a study-case for Cournot oligopoly situation for n enterprises behavior under finitely and infinitely repeated negotiations. We found for this situation discount factor depends only on players number, not on different player's payoffs.*

Keywords: *Negotiation Game, Repeated Game, Bargaining, Folk theorem, Bounded Rationality, Cournot oligopoly*
JEL Classification: C73, C78, D43, L13

1. Introduction

Our paper considers a "negotiation game" which combines the features of two-players alternating offers bargaining and repeated games. Generally, the negotiation game in general admits a large number of equilibriums but some of which involve delay and inefficiency. Thus, complexity and bargaining in tandem may offer an explanation for cooperation and efficiency in repeated games.

The Folk Theorem of repeated games is a very used result that shows if players are enough patience then it is possible to obtain a cooperative equilibrium of the infinite repeated game. A few contributions on folk theorem shows that the result survives more or less intact when incomplete (Fudenberg and Maskin, 1986) or imperfect public (Fudenberg, Levine, and Maskin, 1994) information is allowed, or when the players have bounded memory (Sabourian, 1998).

These findings are made precise in numerous *folk theorems*[1]. Each folk theorem considers a class of games and identifies a set of payoff vectors each of which can be supported by some equilibrium strategy profile. There are many folk theorems because there are many classes of games and different choices of equilibrium concept. For example, games may be repeated infinitely or only finitely many times. There are many different specifications of the repeated game payoffs. For example, there is the Cesaro limit of the means, the Abel limit (Aumann, 1985), the overtaking criterion (Rubinstein, 1979) as well as the average discounted payoff, which we have adopted. They may be games of complete information or they might be characterized by one of many different specifications of incomplete information. Some folk theorems identify sets of payoff vectors which can be supported by Nash equilibrium; of course, of more interest are those folk theorems which identify payoffs supported by subgame-perfect equilibrium.

Our paper develop Benoit and Krishna's (1985, 1993) idée developing a new folk theorem applied for finite repeated games. Player's strategies are "trigger" strategies. Players start by adopting a cooperative strategy and will play the same strategy as long as the other players also play cooperative strategies. If one player deviates from cooperative strategy (due on greater payoff) starting next stage he will be "punished" and his payoff will be "minmax" payoff.

Also, we found the conditions (the discount tare level) such that is is possible player's cooperation, and the minimum stage number such that at lest one stage our players cooperate.

Finally we present a study-case for Cournot oligopoly situation for n enterprises behavior under finitely and infinitely repeated negotiations, finding the discount factor level such that it is possible to enforce a cooperative behavior.

2. Literature

The Folk theorem gives economic theorists little hope of making any predictions in repeated interactions. However, as the aforementioned examples suggest, it seems that negotiation is often a salient feature of real world repeated interactions, presumably to enforce co-operation and efficient outcomes. Can bargaining be used to isolate equilibrium in repeated games?

Busch and Wen (1994) analyze the following game: in each period, two players bargain - in Rubinstein's alternating - offers protocol over the distribution of a fixed and commonly known periodic surplus. If an offer is accepted, the game ends and each player get his share of the surplus according to the agreement at every period thereafter. After any rejection, but before the game moves to the next period, the players engage in a normal form game to determine their payoffs for the period. The Pareto frontier of the

[1] The strongest folk theorems are of the following loosely stated form: "Any *strictly individually rational* and *feasible* payoff vector of the stage game can be supported as a subgame-perfect equilibrium average payoff of the repeated game." These statements often come with qualifications such as "for discount factors sufficiently close to 1" or, for finitely repeated games, "if repeated sufficiently many times."

disagreement game is contained in the bargaining frontier. The negotiation game generally admits a large number of subgame-perfect equilibrium, as summarized by Busch and Wen in a result that seems to be as the Folk theorem in repeated games.

Considerable effort has gone into introducing considerations that reduce the equilibrium set of a repeated game. For instance, depending on the stage game, the set of equilibrium payoffs is known to shrink by varying degrees when complexity costs are (lexicographically) taken into account (Rubinstein, 1986, Abreu and Rubinstein, 1988, Piccione, 1992, Piccione and Rubinstein, 1993), when strategies and beliefs are restricted to be Turing-computable (Anderlini and Sabourian, 1995, 2001), or when asynchronous choice is allowed (Lagunff and Matsui, 1997).

Obara (2009) proves a folk theorem with private monitoring and communication extending the idea of delayed communication in Compte (1998) to the case where private signals are not correlated.

We should mention that many folk theorem results without communication have been obtained recently. However, most of them assume almost perfect monitoring (Bhaskar and Obara (2002), Ely and Välimäki (2002), Hömer and Olszewski (2006), and Mailath and Morris (2002)).[4] One exception is Matsushima (2004) that allows for noisy private monitoring. However he assumes a certain type of conditional independence of private signals as in Compte (1998). The result of this note may be useful to deal with noisy correlated private signals even without communication, but that is left for future research.

Olson (1965) was among the first to formally pose the puzzle of group formation and cooperation, and this has provoked a large literature seeking to understand group behavior. Thorsten and Lim (2009) introduce two incentive mechanisms to sustain intra-group cooperation with prisoner's dilemma payoffs. They examine three-agent groups where relations may either be triadic one person interacting with two others/or tripartite, where all agents interact. Due to shirking incentives, sustained group cooperation requires a system of endogenous enforcement, based on punishments and reward structure and they found that both can ensure cooperation.

Fudenberg and Levine (2007) proves a Nash-threat folk theorem when players' private signals are highly correlated. Ashkenazi-Golan (2004) assumes that deviations are perfectly observable by at least one player with positive probability and proves a Nash-threat folk theorem. These results, as well as the result of this note, apply to repeated games with two or more players. Finally, McLean, Obara and Postlewaite (2005) prove a folk theorem when private signals are correlated and can be treated like a public signal once aggregated. But this result requires at least three players.

Also, there is an existing literature that seeks to model institutions and social networks in terms of endogenous enforcement. The use of incentive slackness in triadic relations to tie strategies across two party games or domains, has been studied by Aoki (2001); Bernheim & Whinston (1990) while exogenous superior information or enforcement capability among group members compared to non- group members is used in (Fearon & Laitin 1996; Ghatak & Guinnane 1999). Moreover, such an institutional arrangement may itself be endogenous (Okada 1993).

Fong and Surti (2008) study also the infinitely repeated Prisoners' Dilemma with side payments and they found that Pareto dominant equilibrium payoffs are implemented by partial cooperation supported by repeated payments. That seems to confirm folk theorems for infinitely repeated games.

The literature on repeated games with different time preferences is still relatively small. In an important contribution, Lehrer and Pauzner (1999) have studied how players in a repeated game exploit the difference in their time preferences by the intertemporal trade of instantaneous payoffs to enhance efficiency. Their paper provides the key insight that, by letting the impatient player consume more in the near future and the patient player consume more in the farther future, the set of feasible payoff vectors becomes larger than the convex hull of IR stage game payoffs identified by the folk theorem. They demonstrate that, keeping constant the relative patience of the players, as both become arbitrarily patient, they can achieve outcomes in equilibrium that would be infeasible were their time preferences identical.

Benoit and Krishna (1985, 1993) analyze particular folk theorems for finite repeated games. They show that under such hypothesis it is possible to reinforce collusive equilibrium that not require any binding agreements to ensure that players conform. An important example given by Benoit and Krishna show that for constant cost Cournot duopoly with linear demand it is possible to obtain enterprises cooperation if finite repeated game contains enough stages and discount factor is close to 1.

3. The Model

A (one-shot) game, *G*, in normal or strategic form, consists of a set of n players, the strategy sets of the players, and their payoff functions.
Thus, we define $G = (S_1, S_2, \ldots, S_n; U_1, U_2 \ldots, U_n)$, where S_i is player *i*'s strategy space and $U_i : S \to R$ is *i*'s payoff function, where $S = S_1 \times S_2 \times \ldots \times S_n$. The strategy space is represented by player's offers in negotiation process.

We may also write $U_i : S \to R^n$ as the function whose *i*-th component is U_i. We will assume that the strategy spaces are compact sets and that the payoff functions are continuous. *G(T)* denotes the game that results when G is successively played T times (T is a positive integer). Let $\delta_i \in (0, 1)$ be the *i*'th player discount factor ant T enough large (eventually ∞).

For $t = 1, 2, \ldots, T$ if $s_i \in S$ denotes the outcome of the game *G(T)* at time *t*, player *i*'s average payoff in *G(T)* is given by $u_i(s) = \frac{1-\delta_i}{1-\delta_i^{T+1}} \sum_{t=0}^{T} \delta_i^t u_i(s^{*t})$.

A strategy for player *i* in the game *G(T)* is a function s_i which selects, for any history of play, an element of S_i. A Nash equilibrium of G(T) is an n-tuple of strategies s^*, such that for all *i*, and any strategy or for player *i*:

$$u_i(s^*) = \frac{1-\delta_i}{1-\delta_i^{T+1}} \sum_{t=0}^{T} \delta_i^t u_i(s^{*t}) \geq \frac{1-\delta_i}{1-\delta_i^{T+1}} \sum_{t=0}^{T} \delta_i^t u_i(a^t) \quad (\forall) a \in S .$$

Let *N(T)* denote the set of Nash equilibrium outcome paths of *G(T)*. We will assume that *N(1)* is not empty.

Let $\underline{u}_i$ denote player *i*'s minmax payoff and let $m_i \in S_i$ denote a corresponding strategy combination. A payoff vector *u* is said to be individually rational if for all *i*:

$u_i > v_i$. Again, for the game G, consider the set of all payoff vectors which may result from players' choices (the range of the function *U*). The convex hull of this set, denoted by *F*, will be called the feasible region of payoffs. Note that in both *G* and *G(T)*, we are restricting attention to pure strategies only. The effect of this restriction is that minmax payoffs, which will play a significant role in what follows, may be higher than those attainable using mixed strategies.

The notion of a subgame perfect equilibrium is made precise as follows:

Definition: The strategy profile a is a (subgame) perfect equilibrium of *G(T)* if (i) it is a Nash equilibrium of *G(T)*, and (ii) for all *T' < T* and all *T'* period histories h (*T'*), the restriction of *s* to *h(T')* is also a Nash equilibrium of *G(T –T')*.

In our paper we use an alternative offer negotiation game. Each player makes offers at every stage but they don't have the possibility to reject the opponent's offer. We study next two different situations. For the first situation we consider the infinitely repeated negotiation and for the second case, the finitely repeated negotiation.

We suppose there exist in our negotiation game three different types of solutions: minmax equilibrium, corresponding to a punishment situation, a cooperation solution and a deviation situation. The relationships between the payoffs of these three strategies are: deviation payoff is greater than cooperation payoff that is greater than minmax payoff.

First case: the three phases of the game are:

- Cooperation phase (*T'* periods) from $t = 0$ to $t = T' - 1$, with cooperation payoffs;
- Deviation phase – one period – for $t = T'$: with deviation payoff for the player that deviate;
- Punishment phase starts from $T' + 1$ phase and continue all the game for the player that deviate from cooperative strategy

The variables:

- v_i – cooperative payoff;
- v_i^D – deviation payoff;
- $\underline{u}_i$ – minmax payoff/punishment payoff ;
- Relationships: $v_i^D \geq v_i > \underline{u}_i$;
- $\underline{\delta}$ - minimum discount factor to cooperate;
- δ_i - player *i* discount factor.
- a parameter $A = \frac{u_i^D - v_i}{v_i - \underline{u}_i}$ that shows the relative gap between deviation from cooperation payoff and punishment payoff.
- *T* is the number of game stages and *T'* is the stage where player *i* deviates from cooperative phase.

A. Infinitely repeated games

First we consider the situation of infinitely repeated game ($T = \infty$). Game solution of infinitely repeated game result from next theorem:

Theorem 1. Folk Theorem

Let G be a static, finite game of complete information and $G(\infty)$ the infinitely repeated game. Let $\underline{u}_i$ the minmax payoff of G for any player i, so for any payoff vector v so that $v_i > \underline{u}_i, (\forall) i$, there exists a minimum level of discount factor $\underline{\delta} < 1$, such that $(\forall)\delta \in (\underline{\delta}, 1)$ there exists a subgame perfect Nash Equilibrium that achieves v as average payoff. (see proof in Appendix)

This theorem show as also some interesting findings related to player's behavior:

The minimum level of discount factor such that the cooperation strategy depend on relative gain from deviation related on punishment possible to be implemented. Starting on these hypotheses we proof the following results:

- If deviation payoff is close to cooperation payoff then players cooperates in every period of the game;
- If cooperation payoff is close to punishment (minmax) payoff, then cooperative situation is not possible;
- If deviation payoff is very large, then player's cooperation is not possible for any period of the game.

Corollary 1. If there exist a minimum level for discount factor $\underline{\delta}$, then $\underline{\delta} = \dfrac{u_i^D - v_i}{u_i^D - \underline{u}_i}$. (1)

This corollary shows the discount factor depends on deviation payoff, cooperation payoff and punishment payoff.

Corollary 2. If deviation payoff is close to cooperation payoff ($\boldsymbol{u_i^D} \to \boldsymbol{u_i^C}$) then $\underline{\delta} \to 0$ and players cooperates in every period of the game.

Corollary 3. If cooperation payoff is close to punishment payoff ($\boldsymbol{u_i^C} \to \boldsymbol{u_i^P}$), then $\underline{\delta} \to 1$ and cooperative situation is not possible.

Corollary 4. If deviation payoff is very large, ($\boldsymbol{u_i^D} \to \infty$), then $\underline{\delta} \to 1$ and players cooperation is not possible for any period of the game.

B. Finitely repeated games

In the second situation we consider the finitely repeated negotiation game, where T represents the final stage of the game. The strategies and the payoffs situation still are the previous ones. The game phases are:

- Cooperation phase (t' periods) from $t = 0$ to $t = T' - 1$;
- Deviation phase – one period – for $t = Tt'$;
- Punishment phase, for $T - T' - 1$ periods (from $t = T' + 1$ to $t = T$).

The backward induction solution of finite repeated games shows that in every period of the game the players must play and repeat the Nash Equilibrium of stage game. However, a large number of authors show there exists equilibrium of repeated game different from repetition of Nash equilibrium of stage game (see Benoit-Krishna Theorem).

Theorem 2. Benoit-Krishna Theorem

Let G(T) a finite repeated game and s^* a Nash equilibrium for stage game. Let $\hat{s}$ a strategy such that $u(\hat{s}) > u(s^*)$. Then it exists for T enough large, a time limit *T'<T*, such finite repeated game equilibrium is $\hat{s}$ repetition for T' periods and s^* repetition for T – T' periods.

Benoit-Krishna theorem does not show the discount factor limit or the minimum number of game stages such that players cooperate.

We solve this problems extending Benoit-Krishna Theorem.

The first question we answer is: If *T* is enough large, which is the discount factor level starting players became the have a cooperative behavior?

Corollary 5. If the discount factor not exceed $\frac{u_i^D - u_i}{u_i^D - \underline{u}_i}$ then cooperation is not possible.

Corollary 6. There exists $\underline{\delta} \in (0,1)$, solution of the equation:

$$\delta_i^{T-T'+1} - \delta_i \cdot (A+1) + A = 0, \qquad (2)$$

such that for every $\boldsymbol{\delta_i} > \underline{\boldsymbol{\delta}}$ the players cooperate for T' periods.

Corollary 7. If T is very large, then the condition form C1 is satisfied and we retrieve the folk theorem with $\underline{\delta} = \frac{u_i^D - v_i}{u_i^D - \underline{u}_i}$, and $\boldsymbol{\delta_i} > \underline{\boldsymbol{\delta}}$.

If we know players discount factors, which is the necessary number of stages (T) need to played to be possible the cooperative situation?

Corollary 8. The minimum number of stages to can obtain a cooperative game for T' stages is $T > T' - 1 + \frac{\ln[(\underline{\delta} \cdot A + \underline{\delta} - 1)/A])}{\ln \underline{\delta}}$. (3)

Corollaries proof are retrieved in Theorem 3:

Theorem 3 (Roman).

Let G be a static, finite game of complete information and G(T) the finitely repeated game for T stages. Let $\underline{u_i}$ the minmax payoff of G for any player *i*, so for any payoff vector v so that $v_i > \underline{u}_i, (\forall) i, (\exists) \underline{\delta} < 1$, (there exists a minimum level of discount factor $\underline{\delta} < 1$), for T enough large, $(\forall)\delta \in (\underline{\delta},1), (\exists)T' > 0$ there exists a subgame perfect Nash Equilibrium that achieves v as average payoff for the first T' stages and for T-T' stages the Nash equilibrium is the strategy that achieves $\underline{u_i}$ as average payoff.

Proof.

We suppose also there exists a deviation payoff, $v_i^D = \max_a u_i(a) > v_i$. So $v_i^D \geq v_i > \underline{u_i}$. v_i represent the *i*'th player cooperation payoff, and $\underline{u_i}$ represent the punishment payoff.

Player *i* will play v_i for *T'* periods with v_i payoff, then deviate, and his payoff will be $v_i^D = \max_a u_i(a)$, and for the rest of the game (*T-T'-1* stages) all other players will punish player *i* and he will receive minmax payoff $\underline{u_i}$.

If player *i* cooperates for *T* periods then his average cooperation payoff is:

$$u_i^C = \frac{1-\delta_i}{1-\delta_i^{T+1}} \sum_{t=0}^{T} \delta^t v_i \tag{4}$$

If player *i* cooperates for *T '* , then deviates at *T' +1* period and for the rest of the game (*T-T'-1* periods) his payoff will be $\underline{u_i}$ (punishment payoff) then his average deviation payoff is:

$$u_i^D = \frac{1-\delta_i}{1-\delta_i^{T+1}} \left(\sum_{t=0}^{T'-1} \delta^t v_i + \delta^{T'} v_i^D + \sum_{t=T'+1}^{T} \delta^t \underline{u_i} \right) \tag{5}$$

We found two different situations for our game. The first one give us the discount minimum level such that players cooperate (with *T* and *T'* done), and the second one show the minimum number of stages needs to repeat games such that at least *T'* periods our players cooperates (if discount factor is done).

So equilibrium condition such that players cooperate is:

$$\begin{aligned} & u_i^C \geq u_i^D \Leftrightarrow \frac{1-\delta_i}{1-\delta_i^{T+1}} \sum_{t=0}^{T} \delta^t v_i \geq \frac{1-\delta_i}{1-\delta_i^{T+1}} \left(\sum_{t=0}^{T'-1} \delta_i^t v_i + \delta_i^{T'} \cdot v_i^D + \sum_{t=T'+1}^{T} \delta_i^t \underline{u_i} \right) \Leftrightarrow \\ & \Leftrightarrow \delta_i^{T'} (v_i^D - v_i) \leq \sum_{t=T'+1}^{T} \delta_i^t (v_i - \underline{u}_i) \Leftrightarrow \delta_i^{T'} (v_i^D - v_i) \leq (v_i - \underline{u}_i) \frac{1-\delta_i^{T-T'}}{1-\delta_i} \cdot \delta_i^{T'+1} \Leftrightarrow \\ & \Leftrightarrow \frac{1-\delta_i}{\delta_i - \delta_i^{T-T'+1}} \leq \frac{v_i^D - v_i}{v_i - \underline{u}_i} \end{aligned} \tag{6}$$

Case. 1. For given $\delta_i > \underline{\delta}_i$ we find the minimum stage periods such that players cooperate:

$$T > T' - 1 + \frac{\ln[(\delta_i \cdot A + \delta_i - 1)/A])}{\ln \delta_i}, \text{ where } A = \frac{v_i^D - v_i}{v_i - \underline{u}_i} .$$

Case 2. For given *T* and *T'*, $\underline{\delta}_i \in (0,1)$ is solution of equation:

$$A \cdot \underline{\delta}_i^{\,T-T'+1} - \underline{\delta}_i \cdot (A+1) + 1 = 0 \tag{2}$$

Obs. It is easy to show there exists a unique solution of equation (1) in (0,1) interval.

Let $\underline{\delta} = \max_i \underline{\delta}_i$. So there exists a minimum level of discount factor $\underline{\delta} < 1$, such that $(\forall)\delta \in (\underline{\delta},1)$ there exists a subgame perfect Nash Equilibrium that achieves v as average payoff. q.e.d.

4. Study-case: Cournot oligopoly application

We consider the Cournot case of oligopoly with linear demand functions, with n identical enterprises. Let x_i denote the quantities of a homogeneous product produced by enterprise i. *Let* $P(X) = a - b\,X$, (and $b > 1$) be the market clearing price function, where X is the aggregate quantity on the market ($X = \sum_{i=1}^{n} x_i$). More precisely, inverse demand function is $P(X) = \begin{cases} a - b\,X, & \text{for } X < a/b \\ 0, & \text{for } X \geq a/b \end{cases}$.

We assume that the total cost for firm i is $C_i(x_i) = c\,x_i$. For simplicity, there are no fixed costs for firm i and the marginal cost is equal with average cost and constant, c (we assume also $c < a/b$). Following Cournot suppose that the firms choose their quantities simultaneously. Each firm's strategy space can be represented as $S_i = [0,\infty)$, the nonnegative real numbers. In this case a strategy is a quantity choice, x_i . From players rationality principle, neither firm will produce a quantity $x_i > a/b$ (otherwise $P(X) = 0$ and no firm will have a positive profit). The payoff for firm i will be represented by profit function: $\pi_i(x_i, x_{-i}) = P(X) \cdot x_i - C_i(x_i) = P(\sum_{i=1}^{n} x_i) \cdot x_i - c \cdot x_i$.

(where $x_{-i} = (x_1, x_2, ..., x_{i-1}, x_{i+1}, ..., x_{n)}$), quantities vector chosen by other players).

A. One stage game
a. Non-cooperative game situation

We obtain the Cournot-Nash equilibrium solving for each firm the problem:

$$\max_{x_i \in S_i} \pi_i(x_i, x_{-i}^*) = \max_{x_i \in S_i} P(\sum_{i=1}^{n} x_i) \cdot x_i - c \cdot x_i \tag{7}$$

The first order conditions for i's firm optimization problem is both necessary and sufficient (if $x_j^* < a/b - c$, as well be shown to be true), it yields:

$$x_i = \frac{1}{2 \cdot b} \cdot \left(a - c - b \cdot \sum_{j=1, i \neq j}^{n} x_j \right), \; i,j = 1,\ldots,n \tag{8}$$

Solving the linear equation system (8) we obtain Cournot –Nash solution:

$$x_i^* = \frac{1}{n+1} \cdot \frac{a-c}{b} \qquad (9)$$

which is indeed less than *a/b – c*, as assumed.

The *i*'s player payoff is: $\pi_i = \left(\frac{1}{n+1}\right)^2 \cdot \frac{(a-c)^2}{b}$ (10)

for non-cooperative situation, that represents also the minmax payoff.

b. The cooperative situation

We obtain the solution of cooperative situation solving following problem:

$$\max_{X \in S_i} \Pi(X) = \max_{X \in S_i} P(X) \cdot X - c \cdot X \;, \qquad (11)$$

where $X = \sum_{i=1}^{n} x_i$, with equal payoffs for each player.

First order conditions for (10) optimization problem is also both necessary and sufficient and we obtain:

$$X^C = \frac{1}{2} \cdot \frac{a-c}{b} \text{ , so } x_i^C = \frac{1}{2 \cdot n} \cdot \frac{a-c}{b}. \qquad (12)$$

Total payoff is:

$$\Pi^C(X) = \frac{1}{b} \cdot \left(\frac{a-c}{2}\right)^2 \qquad (13)$$

and each firm cooperation payoff will be:

$$\pi^C{}_i(x_i) = \frac{1}{n \cdot b} \cdot \left(\frac{a-c}{2}\right)^2 \qquad (14)$$

We can observe that $\pi_i^C > \pi_i$ for *n > 1* and *b > 1*, so enterprise's payoffs are greater if firm cooperates (that means they form a cartel) rather them adopt a non-cooperative behavior.

c. The deviation situation

There is another situation which one firm deviate from cooperative behavior trying to maximize his payoff (profit). In this case player *i* maximize his payoff for given quantities from cooperative situation: $x_j = \frac{1}{2 \cdot n} \cdot \frac{a-c}{b}, \quad i \neq j, j = 1,\dots,n$:

$$\max_{x_i \in S_i} \pi_i(x_i, x_{-j}^C) = \max_{x_i \in S_i} P(x_i, x_{-j}^C) \cdot x_i - c \cdot x_i = \max_{x_i \in S_i} [a - b \cdot (x_i + \sum_{j=1, j \neq i}^{n} x_j^C)] \cdot x_i - c \cdot x_i \qquad (15)$$

First order conditions for (15) optimization problem is also both necessary and sufficient and we obtain:

$$x_i^D = \frac{3n-1}{4\cdot n}\cdot\frac{a-c}{b} \tag{16}$$

and the deviation payoff is

$$\pi^D{}_i(x_i) = \frac{(n+1)(3n-1)}{16\cdot n^2\cdot b}\cdot(a-c)^2. \tag{17}$$

It is easy to verify that $\pi_i^D((x_i) > \pi_i^C(x_i)$, so it exist temptation to deviate from cooperative situation for any firm *i*.

B. Infinitely repeated game

For infinitely repeated game, the minimum discount factor so that companies cooperate is : $\underline{\delta} = \frac{u_i^D - v_i}{u_i^D - \underline{u}_i}$ (see formula 1), so for our game we obtain:

$$\underline{\delta} = \frac{\pi_i^D - \pi_i^C}{\pi_i^D - \pi_i} = \frac{\frac{(n+1)(3n-1)}{16\cdot n^2\cdot b}\cdot(a-c)^2 - \frac{1}{n\cdot b}\cdot\left(\frac{a-c}{2}\right)^2}{\frac{(n+1)(3n-1)}{16\cdot n^2\cdot b}\cdot(a-c)^2 - \left(\frac{1}{n+1}\right)^2\cdot\frac{(a-c)^2}{b}} = \tag{18}$$

$$= \frac{(3n^2-2n-1)\cdot(3n^2+2n-1)}{(3n^3+5n^2-3n-1)\cdot(3n^3+5n^2+5n-1)} < 1$$

for n > 1.

We can observe that discount factor depends only on firm numbers in oligopoly.

Table 1. Evolution of minimum discount factor depending on enterprieses number

Enterprises number n	Minimum discount factor $\underline{\delta}$
2	0.0535
3	0.0394
4	0.0285
5	0.0212
6	0.0163
7	0.0129
8	0.0104
9	0.0086
10	0.0072

C. Finitely repeated game

If we consider to need at lest 20 stages of cooperation, for $A = \frac{\pi_i^D - \pi_i^C}{\pi_i^C - \pi_i}$ the minimum level for discount factor is (0,1) solutions of equation $\delta_i^{20} - \delta_i \cdot (A+1) + A = 0$ (see Corollary 6) are presented in table 2.

Table 2. Evolution of minimum discount factor depending on enterprises number and for 20 stages of cooperation

Enterprises number n	Minimum discount factor $\underline{\delta}$	A factor value	Stage number
2	0.1127	7.875	20
3	0.1304	6.667	20
4	0.1287	6.771	20
5	0.1220	7.200	20
6	0.1142	7.758	20
7	0.1066	8.381	20
8	0.0996	9.040	20
9	0.0933	9.722	20
10	0.0876	10.419	20

5. Conclusions

In our paper we present the enterprises behavior on repeated negotiations. Based on new folk theorem for finitely repeated games we found conditions such that players cooperate at least some stages even backward induction told as this situation is not an Nash subgame perfect equilibrium. Other findings in our paper are:

- For infinitely repeated negotiations there exists the possibility to implement a cooperative solution if player's discount factor is close to 1 and cooperative payoff are not fare away from deviation payoff and punishment payoffs;
- If deviation payoff is very large or cooperation payoff is close to punishment payoff then it is not possible to obtain a cooperative solution for infinitely repeated negotiations;
- For finitely stages negotiations, first rational solution is to repeat Nash equilibrium of stage game every period (backward induction);
- Another solution for finitely repeated games depends on limited (bounded) rationality of players: they starts with cooperative strategies and continue so on until one of other players deviate from cooperative strategy. Starting on this moment of negotiation, the other players punish deviating player for all periods until negotiations end. In this case it is possible to obtain some cooperative stages of the game but this situation is more complex;
- Even we have the minimum stage number, if players discount factor is smaller like a certain level, cooperation it is not possible;
- If the deviating payoff is enough large, the cooperation also it is not possible for any period of the game;

- If the cooperative payoff is closer to the punishment payoff, then cooperation it is not possible;
- There exists a minimum stages number such that it is possible to implement a cooperative behavior;
- If stage number and cooperation stage number are known then it is possible to find the discount minimum level such that players cooperate.

Our study-case shows that it is possible to reinforce a cooperative behavior between players that play Cournot oligopoly following bounded rationality and trigger strategies. Also, we find that discount factor minimum level does not depend on payoff's levels, only dependency factor is firm number. As long as firm number increases, we obtain a lower level of discount factor and if n tend versus infinity then δ is closer to zero and all players cooperates all game stages.

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Appendix.

Proof of Theorem 1 (Folk Theorem).

Suppose that there exists a pure strategy such that $u(a) = v$ (with $v > \underline{u}$) and every player will play next strategy*: „ I will play a_i at stage 0 and I will continue to play a_i such time previous period all players played a. Anywhere I'll play minmax strategies for the rest of the game."* How it is this possible for player i to improve his payoff playing this strategy?

We suppose also there exists a deviation payoff, $v_i^D = \max_a u_i(a) >$ v_i. So $v_i^D \geq v_i > \underline{u_i}$.

Player *i* will play a_i for *t* periods with v_i payoff, then deviate, and his payoff will be $v_i^D = \max_a u_i(a)$, and for the rest of the game all other players will punish player *i* and he will receive minmax payoff $\underline{u_i}$.

So average deviation payoff at *t* stage is:

$$u_D = (1-\delta_i^t)u_i + \delta^t (1-\delta)\cdot v_i^D + \delta^{t+1}\underline{u}_i$$

This payoff is greater like v_i as long as discount factor δ_i is smaller like a minimum level of discount factor $\underline{\delta}_i$, given by relationship:

$$(1-\underline{\delta}_i)\cdot v_i^D + \underline{\delta}_i \cdot \underline{u}_i = v_i$$

So $\underline{\delta}_i = \dfrac{v_i^D - v_i}{v_i^D - \underline{u}_i}$.

Let $\underline{\delta} = \max_i \underline{\delta}_i$. So there exists a minimum level of discount factor $\underline{\delta} < 1$, such that $(\forall)\delta \in (\underline{\delta},1)$ there exists a subgame perfect Nash Equilibrium that achieves *v* as average payoff.

q.e.d.

Proof of Theorem 2 (Benoit-Krishna Theorem).

To proof this theorem we use players rationality principle, so that our players try to maximize total payoff. If they play s^* strategy T periods then their mean payoff is:

$$v_i(s^*) = \frac{1-\delta_i}{1-\delta_i^{T+1}} \sum_{t=0}^{T} \delta_i^t u_i(s^{*t})$$

If they play for T' periods $\hat{s}$ strategy and Nash equilibrium s^* for T – T' periods then expected payoff is:

$$v_i(s') = \frac{1-\delta_i}{1-\delta_i^{T+1}} \left(\sum_{t=0}^{T'} \delta_i^t u_i(\hat{s}^t) + \sum_{t=T'+1}^{T} \delta_i^t u_i(s^{*t}) \right),\ s' = (\hat{s}^1, \hat{s}^2, \ldots \hat{s}^{T'}, \hat{s}^{T'+1}, \ldots . s^{*T})$$

From hypothesis $u(\hat{s}) > u(s^*)$, that means $u_i(\hat{s}) > u_i(s^*), (\forall)$ for every player i, let be i_1 the player such that it obtain $\min_{i \in I}(u_i(\hat{s}) - u_i(s^*))$. So for player i_1 we have:

$$v_{i_1}(s') - v_i(s^*) = \frac{1-\delta_{i_1}}{1-\delta_{i_1}^{T+1}} \left(\sum_{t=0}^{T'} \delta_{i_1}^t u_{i_1}(\hat{s}^t) + \sum_{t=T'+1}^{T} \delta_{i_1}^t u_{i_1}(s^{*t}) \right) - \frac{1-\delta_{i_1}}{1-\delta_{i_1}^{T+1}} \sum_{t=0}^{T} \delta_{i_1}^t u_{i_1}(s^{*t}) =$$

$$= \frac{1-\delta_{i_1}}{1-\delta_{i_1}^{T+1}} \left(\sum_{t=0}^{T'} \delta_{i_1}^t (u_{i_1}(\hat{s}^t) - u_{i_1}(s^{*t})) \right) > 0 .$$

So, for each player is better to play at least T' periods strategy $\hat{s}$, that is not a Nash equilibrium for stage game.

q.e.d.