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Team Beats Collusion*

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Abstract

This paper analyzes optimal contracts in a linear hidden-action model with normally distributed returns possessing two moments that are governed jointly by two agents, who can observe each others' effort levels and draft enforceable side-contracts on chosen effort levels and realized returns. After showing that standard constraints, resulting in incentive-contracts, may fail to ensure implementability, we examine (centralized) collusion-proof contracts and (decentralized) team-contracts. We prove that optimal team-contracts provide the highest implementable returns to the principal. In other words, the principal may restrict attention to outsourcing/decentralization without any loss of generality. Moreover, situations in which incentive-contracts are collusion-proof, thus implementable, are fully characterized.

Journal of Economic Literature Classification Numbers: D82; J30; M12.

Keywords: Principal-agent problems, moral hazard, linear contracts, side-contracting, collusion, team, outsourcing, decentralization.

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1 Introduction

The design of managerial contracts with strategic interaction among employees has been one of the major issues in economics of organization. Two significant aspects are (1) whether or not agents can draft state-contingent binding side-contracts among themselves, and, (2) whether or not they are better informed than the principal about effort levels chosen. Naturally, the first of these two holds in any free society.¹ On the other hand, when agents can observe and verify others' actions, they may employ enforceable side-contracts based on their joint effort choices. In this case, considered in the current study, standard incentive constraints are not sufficient to eliminate side contracting opportunities that enable agents not only to insure one another, but also to coordinate their actions. Consequently, this paper revisits the problem of constructing optimal incentives in a two-agent setting when agents can write enforceable side-contracts based on effort levels and realized outcomes. By examining three forms of contracts, namely incentive-contracts, collusion-proof contracts and team-contracts, we provide a through welfare comparison.

In our linear two-agent hidden-action framework, the principal maximizes the expected utility obtained from the returns of the organization. These returns are governed by a normal distribution with a mean and variance that depend on the effort profile agents choose. All parties, including the principal, have exponential utility functions, with given coefficients of absolute risk aversion (CARA, henceforth). And, agents observe each other's effort choices and exploit all feasible collusion opportunities via enforceable side-contracts contingent on effort levels and realized outcomes.²

First, we establish that *incentive-contracts*, contracts that are individually rational and incentive compatible and involve efficient risk sharing among agents, are not necessarily implementable. That is, colluding agents may deviate jointly to an effort profile and a feasible

¹This is because after all agents cannot be prevented access to either the judicial system, or the capital markets in a free trade economy. Thus, they are able to insure one another with the use of state-contingent binding side contracts, even if agents cannot observe others' actions.

²We work with a general model where agents jointly control the mean and the variance in nonrestricted ways, yet, an interesting case happens when effort choices of the first agent, e.g. *the sales manager*, only increases the mean, and those of the second, e.g. *the finance manager*, only decreases the variance.

redistribution of their compensation schemes, in turn, obtaining strictly higher payoffs while making the principal strictly worse off. That is why, optimal contracts free from collusion are obtained by borrowing the collusion constraints from Barlo (2003); who, adopting the ideas and formulations of Laffont and Martimort (2000), works with cooperative game theoretic notions and proves the existence of collusion-proof contracts in multi-agent hidden action models.³ In particular, for a contract to be *collusion-proof*, principal's offer must be in the core of the strategic interaction it induces. That is, the principal's proposed effort profile and state-contingent compensation scheme must be such that, there should not be a non-empty set of agents and another feasible and participatory side contract among them, making each of these agents strictly better off. Next, we concentrate on *team-contracts*, that can be interpreted also as outsourcing or subcontracting or decentralization. Indeed, agents' side-contracting abilities may be beneficial for the principal. This is because with team-work, agents allocating the total share coordinating their choices in order to maximize the sum of their expected utilities implies that their voluntary coordination in effort choices and efficient risk allocation are ensured without the need of incentive compatibility constraints.⁴

This study proves that in this setting among all implementable contracts, team-contracts provide highest returns. In other words, the principal may restrict attention to team-contracts without any loss of generality, meaning that outsourcing or decentralization outperforms other implementable methods of contracting.

In order to reach this conclusion, we first show that the principal always prefers to employ team-contracts rather than collusion-proof contracts. Then, we provide a full characterization of situations under which incentive-contracts are also collusion-proof, hence, implementable. We find that collusion may be ignored when returns are monotone, and

³A large body of literature, some of which includes Demski and Sappington (1984), Demski, Sappington, and Spiller (1988), Mookherjee (1984), Ma, Moore, and Turnbull (1988), Tirole (1986), Laffont and Rochet (1997), Brusco (1997), Faure-Grimaud, Laffont, and Martimort (2002) and Felli and Hortala-Vallve (2011) among many others, has emphasized the significance of collusion in hidden-action models.

⁴Some of the significant studies on team-work include Holmstrom (1982), Holmstrom and Milgrom (1990), Varian (1990), Itoh (1991), Itoh (1993) and Ramakrishnan and Thakor (1991), and indicate that agents' sharing information unobservable to the principal is necessary for him to benefit from team-work. This is because when only returns (not chosen effort levels) are contractible among agents, the principal can offer such contracts (with efficient risk sharing) herself.

implementing the *best effort profile* is optimal with incentive–contracts.⁵ To show that these assumptions are minimal, we present examples proving that collusion makes the principal strictly worse off (i.e. incentive–contracts are not implementable) when any of these assumptions are violated. These results, then, imply our main finding.

Holmstrom and Milgrom (1990) analyze welfare effects of side contracting in a linear agency model and identify situations in which side–contracts are desirable for the principal. Agents are engaged in two–tasks by providing inputs to both. A performance measure is observed for each activity, an indicator of the production function, that depends on input profile chosen by agents, combined with activity specific and possibly correlated error terms. The principal pays to each agent as a function of both performance measures. Under several properties of the production technologies and performance measures, Holmstrom and Milgrom (1990) show that team–contracts (side–trading) are beneficial to the principal when compared with incentive–contracts (no side–trading). These properties include technologically independent production, meaning that production function of each task depends only on the input of one of the agents, and sufficiently low correlation coefficient of the error terms. Hence, their result suggests that cooperation maybe potentially harmful because of interactions in the production function and/or correlated error terms. Along the same lines, Itoh (1993) examines the effects of coalitions through side–contracting based on efforts profiles and side–transfers in a two agent setting.⁶ In that model, each agent is responsible of and governs a separate production process where the outcome of one depends also on the effort level of the other and a noise term, and compensation schemes are contingent on both of the outcomes. He first shows that when each agent governs only his process, and his effort choice affects the other’s returns only through the noise term both of which are stochastically independent, the principal can implement any effort pair with less costs under agents side–contracting than under no side–contracting. In this case, optimal incentive–contracts

⁵The returns are said to be monotone if the mean of the return is increasing and the variance decreasing separately in the effort levels of both of the agents. And, the best effort profile is one that induces the highest mean and the lowest variance, and is not related with costs. It exists whenever returns are monotone.

⁶Itoh (1993)’s notion of coalition–free contracts corresponds to our incentive–contracts, and the principal’s problem with team–work in our paper corresponds to what he refers to as effort–coordination problem.

consists of individual-based schemes. Then, he considers team-work when the principal can observe an aggregate output level, the distribution of which depends on the efforts of the two agents. He finds that the same result holds if the probability distribution over total output satisfies the conditions which make the first-order approach valid and agents are identical with identical effects on probability distributions of output.⁷

Therefore, earlier literature displays that sufficiency conditions under which the principal prefers team-contracts to incentive-contracts, involve separable production functions with either no or weak interactions, or the limitation to identical agents when a richer set of interactions is allowed. Naturally, a model involving team production with agents having different abilities and attitudes towards risk and jointly determining both the mean and the variance of returns in nonrestricted ways, is more appealing. Moreover, the consideration of implementability through the use of collusion-proofness allows us to produce the sharp conclusion that hiring agents as a team is more desirable by the principal, without the need of any additional restrictions. Meanwhile, the full characterization of situations in which incentive-contracts are collusion-proof (hence, implementable), also characterizes cases in which comparisons between incentive-contracts and team-contracts are justified.

In section 2 the model is presented. Section 3 demonstrates an example showing that standard constraints fail to eliminate collusion, and presents collusion constraints. In section 4 we formulate the principal's problem with team-work and in the subsequent section we present our results.

⁷Similar settings are also featured in Itoh (1994) and Hortala-Vallve and Sanchez Villalba (2010). The former analyzes optimal methods to allocate tasks among agents and whether the principal benefits from offering relative performance schemes, in a simplified version of the model of Itoh (1993). On the other hand, the latter displays that with team production, efficiency of organizations maybe improved by partially internalizing externalities between workers via the use of hierarchy, the delegation of authority of contracting rights. Other relevant papers include Baliga and Sjostrom (1998), Macho-Stadler and Perez-Castrillo (1998), and Jelovac and Macho-Stadler (2002).

2 The Model

Ours is a linear two-agent single-task hidden-action model with observable and verifiable returns. The principal possesses an asset that delivers observable and verifiable returns drawn from a normal distribution whose mean and variance are determined by employees' effort choices that the principal cannot observe. Hence, contracts cannot depend on agents' effort profile. On the other hand, the returns from the asset and the return-contingent contracts that the principal offers to agents are all observable and verifiable. We assume that each agent can observe and verify the other's effort choices. Finally, everyone is assumed to have access to capital markets and none of the parties involved are wealth-constrained.

The pioneer work in establishing theoretical justifications for the use of linear contracts obtained from normally distributed returns and exponential utility functions is Holmstrom and Milgrom (1987), and was generalized by Schättler and Sung (1993) and Hellwig and Schmidt (2002). They involve repeated settings with a single agent, and the lack of income effects due to exponential utility functions is employed to obtain the *optimality of linearity*: Among optimal contracts there is one that is linear in aggregate output. Thus, the situation, given by a complicated repeated agency setting, is as if the agent chooses the mean of a normal distribution only once, and the principal is restricted to employ linear sharing rules. Sung (1995) generalizes this result by allowing the single agent to control the variance as well. Barlo and Ozdogan (2011) consider the multi-agent version of this generalization with instantaneous efficient risk sharing and/or collusion possibilities, and prove that the optimality of linearity continues to hold, in turn, justifying the analysis of the current study.

We assume that E_i , the set of effort levels of agent $i = 1, 2$, is a finite and ordered set.⁸ Asset's effort-contingent returns $x \in \mathfrak{R}$ are distributed with $F(x | e)$, such that for all $e = (e_1, e_2) \in E \equiv E_1 \times E_2$, $F(x | e)$ is the normal cumulative distribution given by the mean $\mu(e)$ and the variance $\sigma^2(e)$. We assume that both of the agents are strictly risk averse, yet, our formulation includes cases when the principal is risk neutral. The principal and agents have exponential utility functions with the following CARA figures: R for the

⁸Finite effort set is assumed to abstract from non-fruitful technicalities and to keep numerical programming simple.

principal, and r_i for agent $i = 1, 2$, where $R \geq 0$, and $r_1, r_2 > 0$. Agent i 's private cost of effort, denoted by $c_i(e_i)$ for $e_i \in E_i$, is given in terms of returns. Each agent has an outside employment opportunity resulting in a reserve certainty equivalent of W_i , $i = 1, 2$. Given a contract $(S_i(x))_{i=1,2,x \in \mathfrak{R}}$ which makes both agents accept and exert the effort level $e \in E$, expected utilities of the principal and agent $i = 1, 2$ are:

$$\begin{aligned} Eu_p(S_1, S_2 | e) &= \int -\exp\{-R(x - \bar{S}_x)\} dF(x | e) \\ Eu_i(S_i | e_i, e_{-i}) &= \int -\exp\{-r_i(S_i(x) - c_i(e_i))\} dF(x | e_i, e_{-i}), \end{aligned}$$

where $\bar{S}_x = S_1(x) + S_2(x)$. The attention is restricted to linear contracts of the form $S_i(x) = \gamma_i x + \rho_i$, where $\gamma_i \in \mathfrak{R}_+$ are such that $\sum_{i=1,2} \gamma_i \in [0, 1]$, and $\rho_i \in \mathfrak{R}$, $i = 1, 2$. Notice that these restrictions contain the feasibility requirement making sure that principal's asset cannot be inflated. Then, the certainty equivalent, henceforth to be abbreviated as CE, of agent $i = 1, 2$, when $e \in E$ is:

$$CE_i(\gamma_i, \rho_i | e) = \gamma_i \mu(e) + \rho_i - \frac{r_i \gamma_i^2}{2} \sigma^2(e) - c_i(e_i).$$

Similarly, the CE of the principal is:

$$CE_p(\gamma, \rho | e) = \left(1 - \sum_{i=1}^2 \gamma_i\right) \mu(e) - \frac{R}{2} \left(1 - \sum_{i=1}^2 \gamma_i\right)^2 \sigma^2(e) - \sum_{i=1}^2 \rho_i. \quad (1)$$

In this setting, the individual rationality constraint for agent $i = 1, 2$, who has a given reserve CE of W_i , is:

$$\gamma_i \mu(e) + \rho_i - \frac{r_i \gamma_i^2}{2} \sigma^2(e) - c_i(e_i) \geq W_i. \quad (IR_i)$$

Similarly, the incentive compatibility constraint for agent $i = 1, 2$ is:

$$\gamma_i (\mu(e) - \mu(e'_i, e_{-i})) - \frac{r_i \gamma_i^2}{2} (\sigma^2(e) - \sigma^2(e'_i, e_{-i})) \geq c_i(e_i) - c_i(e'_i), \quad \forall e'_i \in E_i. \quad (IC_i)$$

So far the model described is basically the two-agent single-task version of the one given by Holmstrom and Milgrom (1991).

Since returns from the asset and contracts the principal offers are all observable and verifiable, it is natural to consider efficient risk sharing, and not allowing agents the ability

to insure each other is rather restrictive.⁹ It is worth noting that these side insurance contracts are not contingent upon the effort choices. In the N -agent Brownian model, Barlo and Ozdogan (2011) show that agents' instantaneous insurance abilities lead to the *substitution compatibility constraint*.¹⁰ In the current setting which can be interpreted as being the reduced form of that model, this constraint ensures that the marginal rate of substitution of the first agent between any two states is equal to that of the second agent. Due to CARA utilities and having two agents, substitution compatibility is simplified to:

$$\frac{\gamma_1}{\gamma_2} = \frac{r_2}{r_1}. \quad (SC)$$

Consequently, the *principal's problem* is to maximize (1), subject to (IR_i) , (IC_i) , $i = 1, 2$, and (SC) . Henceforth, we call the set of contracts satisfying (IR_i) , (IC_i) , $i = 1, 2$, and (SC) , the *incentive-contracts*.

3 Collusion

Before presenting the formal execution, we wish to present a numerical example to display that an optimal incentive-contract may be susceptible to collusion.

3.1 Example 1

Let $E_i = \{e_L, e_H\}$, and returns $x \in \mathfrak{R}$ are distributed with the normal cumulative distribution function F with the mean $\mu(e)$ and variance $\sigma^2(e)$, whose particular levels are given in table 1. The private cost of agents' effort choices are: $c_1(e_H) = 1$, $c_1(e_L) = c_2(e_L) = 0$, and $c_2(e_H) = 0.35$. Let $r_i = 10$, $i = 1, 2$, and the principal is risk neutral, i.e. $R = 0$. Moreover,

⁹This is because, even under such a restriction, an alternative way of obtaining such an insurance is as follows: Suppose that markets are complete, and agents have access to them. Then, there exists a portfolio with returns that are equal to the returns that agents wish to obtain via the insurance contract. Thus, by trading that portfolio agents may obtain the returns they desire from the insurance contract.

¹⁰In fact, Theorem 1 of Barlo and Ozdogan (2011), in addition to showing optimality of linearity, pins down the substitution compatibility requirement: With N agents, $N \geq 2$, it is shown that

$$\gamma_i = \frac{\prod_{j \neq i} r_j}{\sum_{i \in N} \prod_{j \neq i} r_j}, \quad i = 1, \dots, N.$$

| | (e_H, e_H) | (e_H, e_L) | (e_L, e_H) | (e_L, e_L) |
|---------------|--------------|--------------|--------------|--------------|
| $\mu(e)$ | 10 | 10 | 5 | 5 |
| $\sigma^2(e)$ | 1 | 2 | 1 | 2 |

Table 1: The mean and variance figures of example 1.

| (e_1, e_2) | $\gamma_1(e)$ | $\gamma_2(e)$ | $\rho_1(e)$ | $\rho_2(e)$ | $CE_p(e)$ |
|--------------|---------------|---------------|-------------|-------------|-----------|
| (e_H, e_H) | 0.264575 | 0.264575 | -0.295751 | -1.44575 | 6.45 |
| (e_H, e_L) | 0.20 | 0.20 | 0.40 | -1.10 | 6.70 |
| (e_L, e_H) | -- | -- | -- | -- | -- |
| (e_L, e_L) | 0 | 0 | 1 | 0.50 | 3.50 |

Table 2: Optimal incentive-contracts of example 1.

the reserve CE figures of agents, are $W_1 = 1$ and $W_2 = 0.5$. It is worthwhile to point out that in this example, effort choices of the first agent only affects the mean, and those of the second agent only the variance. Another interesting feature is that the principal would desire high effort from the second agent only to decrease the risk faced by the first agent.

When optimal incentive-contract is considered (IC_i) and (SC) constraints are:

$$\begin{aligned}
(IC_1) & : \gamma_1 (\mu(e_1, e_2) - \mu(e'_1, e_2)) \geq (c_1(e_1) - c_1(e'_1)), \quad \forall e'_1 \in E, \\
(IC_2) & : -\frac{r_2 \gamma_2^2}{2} (\sigma^2(e_1, e_2) - \sigma^2(e_1, e'_2)) \geq (c_2(e_2) - c_2(e'_2)), \quad \forall e'_2 \in E. \\
(SC) & : \gamma_1 = \gamma_2.
\end{aligned}$$

Consequently, table 2 presents optimal incentive-contracts and corresponding CE figures to the principal when a given effort level $e \in E$ is to be implemented with (IR_i), (IC_i), $i = 1, 2$, and (SC). Thus, the optimal incentive-contract, $(S_i^*)_{i=1,2}$ involving the implementation of the effort profile $e = (e_H, e_L)$, is given by $(\gamma_1^*, \gamma_2^*; \rho_1^*, \rho_2^*) = (0.20, 0.20; 0.40, -1.10)$, and delivers the principal a return of 6.70.¹¹ It should be mentioned that when the effort profile (e_L, e_H) is to be implemented, the incentive compatibility constraint of the first and second

¹¹When the principal is risk averse with a CARA given by $1/2$, the optimal contract involves the same compensation scheme and the same effort profile as the one with a risk neutral principal, $(\gamma_1^*, \gamma_2^*; \rho_1^*, \rho_2^*) = (0.20, 0.20; 0.40, -1.10)$; but delivers a CE of 6.52 to the risk averse principal.

agents result in the set of constraints being empty.¹²

However, S^* is not immune to collusion, hence, is not implementable. That is, there is a feasible side-contract contingent on agents' effort choices, making both strictly better off. Consider S' , involving (e_H, e_H) and $(\gamma'_1, \gamma'_2; \rho'_1, \rho'_2) = (0.20, 0.20; 0.205, -0.905)$. S' is feasible because $\gamma_1^* + \gamma_2^* = \gamma'_1 + \gamma'_2$ and $\gamma'_1, \gamma'_2 \geq 0$ and $\rho'_1 + \rho'_2 = \rho_1^* + \rho_2^*$, and clearly (SC) holds since $\gamma'_i = \gamma_i^*$, $i = 1, 2$. The resulting CE figures are $CE_1(S') = 1.005 > 1 = CE_1(S^*)$, and $CE_2(S') = 0.545 > 0.50 = CE_2(S^*)$, and $CE_P(S') = 6.70 = CE_P(S^*)$. With this side-contract, the first agent agrees to make a side-transfer to the second in return of her high effort choice, resulting in a lower variance of output; and this, in turn, mitigates the amount risk the principal desires the agents to be exposed to.¹³ Therefore, with side-contracting agents are able to sustain the best effort profile, even though the risk neutral principal (or one with sufficiently low CARA) finds it costly to make the second agent exert high effort. It should be pointed out such side-contracting on effort levels enables non-incentive compatible (yet, feasible and participatory) arrangements to be beneficial for the agents.¹⁴

3.2 Collusion Formulation

Borrowing collusion constraints from Barlo (2003) and using the fact that in this study we consider two-agent situations, the resulting constraint is that the principal is restricted to offer individually rational contracts that none of the agents can strictly benefit upon by

¹²To be precise, the (IC_1) for that case calls for $\gamma_1 \leq 0.20$, and (IC_2) for $\gamma_2 \geq \sqrt{0.07} = 0.26458$. Finally, due to (SC) , $\gamma_1 = \gamma_2$, resulting in the constraint set to be empty.

¹³That is, the “sales” person, agent 1, provides incentives to the agent in charge of “finance” for her high effort choice, in order to alleviate the effects of the risk that the principal’s contract is exposing them to.

¹⁴The side-contract S' is not incentive compatible for the second player, because $\gamma'_2 = 0.20$ is strictly lower than $\sqrt{0.07} = 0.264575$. Moreover, in this example S' does not hurt the principal. However, a risk neutral principal (or a risk averse principal with a sufficiently low CARA) may get strictly worse off by side-contracting, when it involves an effort profile which results in a lower mean. In order for agents to benefit from such an arrangement, they should have sufficiently high CARA, and the effort profile agreed upon results in a low enough variance compensating the decrease in the total surplus due to a lower mean. To see this, consider the above example with the only change to the mean of the return associated with the effort profile $e = (e_H, e_H)$ to from 10 to 9.98. Then, the optimal incentive-contract remains the same and delivers a return of 6.70 to the risk neutral principal. On the other hand, the same side contract S' given above is still strictly beneficial to both of the agents and brings about $CE_p(S'') = 6.688 < 6.70 = CE_p(S^*)$.

deviating jointly to a feasible side contract and effort profile. In the current setting it results in the following: $(S_i, e_i)_{i=1,2}$ satisfies the collusion constraint, if there is no feasible $(\tilde{S}_i, \tilde{e}_i)_{i=1,2}$ (i.e. $\sum_{i=1,2} \tilde{S}_i(x) - S_i(x) \leq 0$ for all $x \in \mathfrak{R}$ and $\tilde{e} \in E$), such that the expected utility of every agent when the side-contract $(\tilde{S}_i, \tilde{e}_i)_{i=1,2}$ is used, strictly exceeds that under $(S_i, e_i)_{i=1,2}$ (i.e. $E[u_i(\tilde{S}_i | \tilde{e})] > E[u_i(S_i | e)]$ for all $i = 1, 2$). Barlo (2003) deals with this constraint by formulating the interaction among agents with a utilitarian bargaining game (see Thomson (1981) for more on utilitarian bargaining games) which is induced by the principal's offer who does not know the particular bargaining weights of the agents. He proves that when for all i , u_i is twice differentiable and satisfies assumptions A1 – A3 of Grossman and Hart (1983) $(S_i, e_i)_{i=1,2}$ satisfies the collusion constraint if and only if the following two conditions hold: (1) $E[u_i(S_i | e)] - E[u_i(S_i | \tilde{e})] \geq 0$ for $i = 1, 2$ and all $\tilde{e} \in E$; and, (2) there exists a bargaining weight vector $\theta \in \Delta(\{1, 2\}) = \{\theta' : \theta'_i \in [0, 1] \text{ for all } i, \text{ and } \sum_i \theta'_i = 1\}$ such that $\theta_1 u'_1(S_1(x)) = \theta_2 u'_2(S_2(x))$ for all $x \in \mathfrak{R}$. Adopting these techniques in the instantaneous interaction among agents in the multi-agent Brownian model, Barlo and Ozdogan (2011) prove that among the optimal collusion-proof contracts there is one that is linear in aggregate output, in turn, justifying the following: The set of collusion constraints, denoted by (CC) , can be replaced by (1) Feasibility, i.e. $\gamma_i \in [0, 1]$, $i = 1, 2$, and $\sum_i \gamma_i \leq 1$; $(e_1, e_2) \in E$; (2) (IR_i) holds at γ_i, ρ_i, e_i , $i = 1, 2$; (3) (SC) holds; and (4) for $i = 1, 2$

$$\gamma_i (\mu(e) - \mu(e')) - \frac{r_i \gamma_i^2}{2} (\sigma^2(e) - \sigma^2(e')) \geq c_i(e_i) - c_i(e'_i), \quad \forall e' \in E. \quad (CC_i)$$

A contract satisfying (CC) is said to be *collusion-proof*; and, the *principal's problem under collusion* is to maximize (1) subject to (CC) .

3.2.1 Example 1 under collusion

Table 3 presents the optimal collusion-proof contracts and associated CE figures to the risk neutral principal when a given effort level $e \in E$ is to be implemented.¹⁵ The optimal

¹⁵When (e_H, e_L) and (e_L, e_H) are to be implemented, the set of constraints is empty, i.e. they cannot be sustained with collusion. This is because, when for (e_H, e_L) we have $\gamma_1^2 \leq 0$ from CC_1 considering deviations from (e_H, e_L) to (e_H, e_H) , and $\gamma_1 \geq 0.20$ from CC_1 considering deviations from (e_H, e_L) to (e_L, e_L) . For the implementation of (e_L, e_H) , similarly, we have $5\gamma_2^2 \geq 0.35$ from CC_2 considering a deviation from (e_L, e_H) to (e_L, e_L) , and $\gamma_2 \leq 0$ again from CC_2 when considering a deviation from (e_L, e_H) to (e_H, e_H) .

| (e_1, e_2) | $\gamma_1(e)$ | $\gamma_2(e)$ | $\rho_1(e)$ | $\rho_2(e)$ | $CE_p(e)$ |
|--------------|---------------|---------------|-------------|-------------|-----------|
| (e_H, e_H) | 0.264575 | 0.264575 | -0.295751 | -1.44575 | 6.45 |
| (e_H, e_L) | --- | --- | --- | --- | --- |
| (e_L, e_H) | --- | --- | --- | --- | --- |
| (e_L, e_L) | 0 | 0 | 1 | 0.5 | 3.5 |

Table 3: Optimal collusion-proof contracts of example 1.

collusion-proof contract, given by (S^{**}, e^{**}) involves $e^{**} = (e_H, e_H)$ and $(\gamma_1^{**}, \gamma_2^{**}; \rho_1^{**}, \rho_2^{**}) = (0.264575, 0.264575; -0.295751, -1.44575)$, and delivers the risk neutral principal a return of 6.45.¹⁶ Recall from section 3.1 that the optimal incentive-contract, (S^*, e^*) , is given by $e^* = (e_H, e_L)$ and $(\gamma_1^*, \gamma_2^*; \rho_1^*, \rho_2^*) = (0.20, 0.20; 0.40, -1.10)$, with an associated return of 6.70 to the risk neutral principal. Therefore, collusion makes the principal strictly worse off when compared with incentive-contracts. This is because, $e^* = (e_H, e_L)$ cannot be implemented with collusion (see footnote 15), thus, the principal is obliged to go for (e_H, e_H) .

4 Team-Work

In example 1, it can easily be computed that the CE of a risk averse principal with $R = 1/2$ under the side-contract S' is 6.61, higher than 6.52, the CE associated with the optimal incentive-contract S^* (see footnote 11 for the details). Thus, in general the principal may benefit from side-contracting. This leads to the identification of situations when the principal should hire agents as a team, offering a total share from the return and let agents coordinate effort choices and allocations from the total share themselves. Then, the principal does not need to deal with incentive constraints because the side-contracts, which are not necessarily incentive compatible, are binding.

¹⁶When the principal is risk averse with a CARA of 1/2, the optimal contract involves the same compensation scheme and the same effort profile as the one given for the risk neutral principal, $(\gamma_1^*, \gamma_2^*; \rho_1^*, \rho_2^*) = (0.264575, 0.264575; -0.295751, -1.44575)$; but, delivers a return of 6.39458 to the risk averse principal. Recall that the optimal contract with (IR_i) , (IC_i) , $i = 1, 2$, and (SC) constraints, (S^*, e^*) is given by $e^* = (e_H, e_L)$ and $(\gamma_1^*, \gamma_2^*; \rho_1^*, \rho_2^*) = (0.20, 0.20; 0.40, -1.10)$, with an associated return of 6.52 to the risk averse principal with a CARA of 1/2.

A *team-contract* (T, e^T) consists of $T : \mathfrak{R} \rightarrow \mathfrak{R}$, a linear compensation plan for the team where $T(x) = \gamma^T x + \rho^T$, $\gamma^T \in [0, 1]$ and $\rho^T \in \mathfrak{R}$; and $e^T \in E$. Given a team-contract (T, e^T) , a feasible *within team allocation* $(T_i, e_i)_{i=1,2}$ consists of $T_i : \mathfrak{R} \rightarrow \mathfrak{R}$ with $T_1(x) + T_2(x) \leq T(x)$ and $T_i(x) = \gamma_i^T x + \rho_i^T$ for all $x \in \mathfrak{R}$ (i.e. $\gamma_1^T + \gamma_2^T = \bar{\gamma}^T$, $\gamma_i^T \in [0, 1]$, $i = 1, 2$; and $\rho_1^T + \rho_2^T = \bar{\rho}^T$); and $(e_1, e_2) \in E$. We say that, given a team-contract (T, e^T) , a within team allocation, denoted by $(T_i, e_i)_{i=1,2}$, solves the *team's problem* if it is (1) feasible; (2) satisfies individual rationality, i.e. (IR_i) , $i = 1, 2$; and (3) there is no other feasible and individually rational within team allocation $(\hat{T}_i, \hat{e}_i)_{i=1,2}$ which provides both of the agents strictly higher CE figures. It should be pointed out that incentive constraints are not included because of the perfect enforceability of side-contracting. Following the same steps of Barlo and Ozdogan (2011), as is discussed in section 3.2, it is not difficult to verify that given a team contract (T, e^T) , a within team allocation $(\gamma_i^T, \rho_i^T, \tilde{e}_i^T)_{i=1,2}$ solves the team's problem if and only if (1) it is feasible; and (2) (IR_i) holds for $i = 1, 2$; and (3) (SC) holds; and (4) the *team constraint* defined below (denoted by (TC)) holds:

$$\begin{aligned} (\gamma_1^T + \gamma_2^T) (\mu(\tilde{e}^T) - \mu(e'_1, e'_2)) - \left(\frac{(\gamma_1^T)^2 r_1 + (\gamma_2^T)^2 r_2}{2} \right) (\sigma^2(\tilde{e}^T) - \sigma^2(e'_1, e'_2)) \\ \geq \sum_{i=1,2} (c_i(\tilde{e}_i^T) - c_i(e'_i)), \text{ for all } (e'_1, e'_2) \in E. \end{aligned}$$

Moreover, we say that a team-contract (T, e^T) is *practicable* if there exists a within team allocation $(\gamma_i^T, \rho_i^T, \tilde{e}_i^T)_{i=1,2}$ solving the team's problem and $\tilde{e}^T = e^T$.

The *principal's problem with team-contracts*, then, is

$$\max_{\gamma^T, \rho^T, e^T} (1 - \gamma^T) \mu(e) - \frac{R}{2} (1 - \gamma^T)^2 \sigma^2(e) - \rho^T,$$

subject to (T, e^T) being a practicable team-contract.

4.1 Example 1 with Team-Work

Revisiting example 1 of section 3.1, table 4 presents optimal team-contracts and associated CE figures to the principal when a given effort profile $e \in E$ is to be implemented. The optimal team-contract, $(S_i^{***})_{i=1,2}$ involving the implementation of $e^{***} = (e_H, e_L)$, is

| (e_1, e_2) | $\gamma_1(e)$ | $\gamma_2(e)$ | $\rho_1(e)$ | $\rho_2(e)$ | $CE_p(e)$ |
|--------------|---------------|---------------|-------------|-------------|-----------|
| (e_H, e_H) | 0.187083 | 0.187083 | 0.304171 | -0.845829 | 6.80 |
| (e_H, e_L) | 0.10 | 0.10 | 1.10 | -0.40 | 7.30 |
| (e_L, e_H) | --- | --- | --- | --- | --- |
| (e_L, e_L) | 0 | 0 | 1 | 0.50 | 3.50 |

Table 4: Optimal team-contracts of example 1.

given by $(\gamma_1^{***}, \gamma_2^{***}; \rho_1^{***}, \rho_2^{***}) = (0.10, 0.10; 1.10, -0.40)$, and delivers the principal a CE of 7.30. It is appropriate to remind the reader that the principal obtains 6.45 as the CE achieved at the optimal collusion-proof contract, (S^{**}, e^{**}) involving $e^{**} = (e_H, e_H)$ and $(\gamma_1^{**}, \gamma_2^{**}; \rho_1^{**}, \rho_2^{**}) = (0.264575, 0.264575; -0.295751, -1.44575)$. So, the principal is better off with team-contracts compared with collusion-proof contracts. Similarly, the optimal incentive-contract S^* delivers a CE of 6.70 to the risk neutral principal, and involves the same effort profile $e^* = (e_H, e_L)$, but a different compensation scheme given by $(\gamma_1^*, \gamma_2^*; \rho_1^*, \rho_2^*) = (0.20, 0.20; 0.40, -1.10)$. Thus, in this example the principal strictly prefers team-contracts when compared with incentive-contracts.¹⁷

5 Team Wins

This section presents our findings concerning the comparison of the principal's welfare under the three types of contracts, team-contracts, collusion-proof contracts, and incentive-contracts.

The following displays that the principal prefers team-contracts to collusion-proof contracts.

Proposition 1 *Suppose that a contract is collusion-proof, i.e. satisfies (IR), (SC) and (CC) constraints. Then, it also satisfies team-work constraint (TC).*

¹⁷If the principal were to be risk averse with a CARA 1/2, the optimal team-contract, $(S_i^{***})_{i=1,2}$ involving $e^{***} = (e_H, e_L)$ and $(\gamma_1^{***}, \gamma_2^{***}; \rho_1^{***}, \rho_2^{***}) = (0.10, 0.10; 1.10, -0.40)$, delivering the principal a CE of 6.98, which is strictly higher than 6.39458 attained by the optimal collusion-proof contract S^{**} , and also higher than 6.52 provided by S^* , the optimal incentive-contract. Hence, the conclusions of example 1 with a risk neutral agent do not change when a risk averse principal (with a CARA of 1/2) is considered.

Proof. The result follows from the observation that (TC) is simply obtained from the summation of (CC_1) and (CC_2) . ■

Restricting attention to collusion-proof contracts and incentive-contracts, we provide a full characterization of situations in which the principal can ignore collusion, and consider only incentive-contracts. In other words, cases in which incentive-contracts are implementable are fully characterized.

We need the following for the statement of our results.

Definition 1 *The asset of the principal is said to have monotone returns if $\mu(e_1, e_2)$ is weakly increasing, and $\sigma^2(e_1, e_2)$ weakly decreasing separately in both e_1 and e_2 . Moreover, define the best effort profile, $e \in E$, by $\mu(e) \geq \mu(e')$ and $\sigma^2(e) \leq \sigma^2(e')$ for all $e' \neq e$.*

Monotonicity of returns entails the interesting case where the first agent governs only the mean, i.e. $\mu(e_1, e_2) = \mu(e_1)$ which is weakly increasing in e_1 , and the second only the variance, i.e. $\sigma^2(e_1, e_2) = \sigma^2(e_2)$ which is weakly decreasing in e_2 . On the other hand, in general the best effort profile may not exist. But, because that the set of effort levels is finite, there exists a best effort profile whenever returns are monotone.

Proposition 2 *Suppose that returns are monotone and the optimal incentive-contract involves the best effort profile. Then, any incentive-contract is collusion-proof, hence, optimal incentive-contracts and optimal collusion-proof contracts provide the principal the very same CE. Furthermore, for situations in which any of these conditions are violated, optimal incentive-contracts are not necessarily immune to collusion which may make the principal strictly worse off.*

Proof. By hypothesis there exists a best effort profile in E , and is denoted by e . Moreover, the principal finding it optimal with (IR_i) , (IC_i) , $i = 1, 2$, and (SC) , to make agents choose (e_1, e_2) , implies that (CC_i) , $i = 1, 2$, are satisfied. This is because for $i, j = 1, 2$ and $i \neq j$

$$(CC_i) : \gamma_i (\mu(e) - \mu(e'_1, e'_2)) - \frac{r_i \gamma_i^2}{2} (\sigma^2(e) - \sigma^2(e'_1, e'_2)) \geq c_i(e_i) - c_i(e'_i), \quad \forall e' \in E.$$

$$(IC_i) : \gamma_i (\mu(e) - \mu(e'_i, e_j)) - \frac{r_i \gamma_i^2}{2} (\sigma^2(e) - \sigma^2(e'_i, e_j)) \geq c_i(e_i) - c_i(e'_i), \quad \forall e'_i \in E_i.$$

and the left hand side of (IC_i) is less than the left hand side of (CC_i) for $i = 1, 2$, we conclude that any solution satisfying (IC_i) also satisfies (CC_i) , $i = 1, 2$.

The four examples, presented in the Appendix, consider cases when the hypothesis of Proposition 2 is not satisfied, and concludes the proof. These examples' features are:

1. Returns are monotone, but implementing the optimal incentive–contract does not involve the best effort profile (in Appendix A);
2. Returns are not monotone, but there exists a best effort profile which the principal finds optimal with incentive–contracts (in Appendix B);
3. Returns are not monotone, but there is a best effort profile, yet implementing that effort profile with incentive–contracts is not optimal (in Appendix C);
4. Returns are not monotone, and there is no best effort profile (in Appendix D).

■

At this stage it is useful to remind that our setting features agents who can perfectly observe each other's effort choices and can engage in enforceable side–contracting based on these observations. Thus, incentive–contracts are not necessarily implementable. On the other hand, Proposition 2 describes some set of conditions with which incentive–contracts become implementable, because with these conditions every incentive–contract turns out to be collusion–proof. In other words, collusion can be ignored when returns are monotone and the best effort profile is chosen in the optimal incentive–contract. Proposition 2 also displays the minimality of these conditions. That is, the violation of any single part of them results in optimal incentive–contracts being not implementable, in the sense that a set of colluding agents can strictly benefit by a joint deviation sustained in an enforceable side–contract. Therefore, incentive–contracts have an appeal only when returns are monotone and the best effort profile is chosen in the optimal incentive–contract.

Meanwhile, Proposition 1 establishes that (without the need of any of these conditions) optimal team–contracts provide more CE levels to the principal than those obtained with optimal collusion–proof contracts. But, due to the minimality part of Proposition 2, in situations when optimal team–contracts provide strictly less returns than optimal incentive–contracts (due to the conditions of Proposition 2 not being satisfied) incentive–contracts

are not implementable. This is because, when conditions of Proposition 2 are not satisfied incentive-contracts are either not implementable or provide the same CE to the principal as the optimal collusion-proof contract. To see the specifics, one should revisit example 1 where the condition that the best effort profile needs to be chosen in the optimal incentive-contract is violated. Hence, even when the conditions of Proposition 2 are not satisfied team-contracts must be preferred by the principal.

These observations are summarized in the following theorem, our main result; and, due to the above, is stated without a proof:

Theorem 1 *The maximum implementable certainty equivalent to the principal can be obtained with team-contracts. Hence, the principal may restrict attention to team-contracts without any loss of generality.*

A Example 1

This example is the one given in section 3.1 and displays that collusion cannot be ignored even when monotonicity of returns holds (thus, there is a best effort profile) but the best effort profile, (e_H, e_H) , is not chosen in the optimal incentive-contract.

The optimal collusion-proof contract, discussed in section 3.2.1, involves $e^{**} = (e_H, e_H)$ and $(\gamma_1^{**}, \gamma_2^{**}; \rho_1^{**}, \rho_2^{**}) = (0.264575, 0.264575; -0.295751, -1.44575)$, delivering a risk neutral principal a CE of 6.45. On the other hand, the optimal incentive-contract, $(S_i^*)_{i=1,2}$ involving the implementation of the effort profile $e = (e_H, e_L)$, is given by $(\gamma_1^*, \gamma_2^*; \rho_1^*, \rho_2^*) = (0.20, 0.20; 0.40, -1.10)$, and delivers the risk neutral principal a return of 6.70. Moreover, as displayed in footnote 16 our conclusions do not change if the principal were to be risk averse with a CARA of $1/2$: The specifics of the contracts remain the same, and the principal's CE figures become 6.39458 under collusion and 6.52 with the optimal incentive-contract.

B Example 2

This example demonstrates that collusion cannot be ignored when returns are not monotone, even when there is a best effort profile which the principal finds optimal to implement with

| | (e_H, e_H) | (e_H, e_L) | (e_M, e_H) | (e_M, e_L) | (e_L, e_H) | (e_L, e_L) |
|---------------|--------------|--------------|--------------|--------------|--------------|--------------|
| $\mu(e)$ | 31 | 29 | 30 | 31 | 26.5 | 28.3 |
| $\sigma^2(e)$ | 1/2 | 1 | 1 | 2 | 1/2 | 4/3 |

Table 5: The mean and variance figures of example 2.

| | γ_1 | γ_2 | ρ_1 | ρ_2 | CE_P |
|--------------|------------|------------|----------|----------|---------|
| (e_H, e_H) | 0.289898 | 0.289898 | -6.52673 | -7.26673 | 26.7315 |
| (e_H, e_L) | -- | -- | -- | -- | -- |
| (e_M, e_H) | 0.26411 | 0.26411 | -5.82453 | -6.06453 | 25.8199 |
| (e_M, e_L) | -- | -- | -- | -- | -- |
| (e_L, e_H) | -- | -- | -- | -- | -- |
| (e_L, e_L) | 0.142866 | 0.142866 | -2.90705 | -2.40705 | 24.8476 |

Table 6: Optimal incentive-contracts of example 2.

incentive-contracts. In this example, the mean and variance of the returns depend on effort levels of both agents. Let $E_1 = \{e_L, e_M, e_H\}$, and $E_2 = \{e_L, e_H\}$, and the levels of mean and variance figures be given in table 5. The cost of efforts are, $c_1(e_L) = 0$, $c_1(e_M) = 0.75$, $c_1(e_H) = 1.25$; $c_2(e_L) = 0$, and $c_2(e_H) = 0.01$. Moreover, reserve CE figures are $W_1 = 1$, $W_2 = 1.5$; and CARA levels $R = 2$, $r_i = 10$ for $i = 1, 2$. In this example, the best effort profile is given by (e_H, e_H) , yet agents' effort choices may affect the variables in opposite directions. That is why, returns are not monotone, so Proposition 2 does not hold.

For any given effort profile, optimal incentive-contracts are given in table 6. Note that the principal cannot make agents choose the effort levels (e_H, e_L) , (e_M, e_L) , and (e_L, e_H) . This is because incentive and substitution compatibility constraints for these effort profiles, result in the constraint set of the principal's problem be empty.¹⁸ On the other hand, optimal

¹⁸When (e_H, e_L) is to be implemented with incentive contracts, IC_1 ensuring that agent 1 chooses e_H instead of e_M is $-2\gamma_1 + 5\gamma_1^2 \geq 0.50$, and holds only if $\gamma_1 \geq 0.57417$. But, due to (SC), $\gamma_1 = \gamma_2$, so $\gamma_1 + \gamma_2 > 1$, an impossibility. For, (e_M, e_L) , the IC_1 ensuring agent 1 chooses e_M instead of e_L is $2.70\gamma_1 - 10/3\gamma_1^2 \geq 0.75$, and cannot be satisfied for any $\gamma_1 \in [0, 1/2]$. Finally, for (e_L, e_H) , IC_2 making sure that agent 2 chooses e_H instead of e_L is $-1.80\gamma_2 + 25/5\gamma_2^2 \geq 0.01$, which is satisfied for every $\gamma_2 \in [0.43749, 0.50]$. But, due to (SC), $\gamma_1 = \gamma_2$, and thus, IC_1 guaranteeing that agent 1 does not choose e_M and e_H over e_L (which are given by $-3.50\gamma + 5/2\gamma^2 \geq -0.75$ and $4.50\gamma_1 \leq 1.25$, respectively) cannot be obtained.

| | γ_1 | γ_2 | ρ_1 | ρ_2 | CE_P |
|--------------|------------|------------|----------|----------|---------|
| (e_H, e_H) | 0.312377 | 0.312377 | -7.18975 | -7.92975 | 26.6817 |
| (e_H, e_L) | --- | --- | --- | --- | --- |
| (e_M, e_H) | --- | --- | --- | --- | --- |
| (e_M, e_L) | --- | --- | --- | --- | --- |
| (e_L, e_H) | --- | --- | --- | --- | --- |
| (e_L, e_L) | 0 | 0 | 1 | 1.5 | 24.4667 |

Table 7: Optimal collusion-proof contracts of example 2.

collusion-proof contracts for given effort profiles are presented in table 7. It should be mentioned that because that collusion constraints include incentive compatibility, the effort levels (e_H, e_L) , (e_M, e_L) , and (e_L, e_H) continue to be impossible to be obtained. Moreover, with collusion, the effort profile (e_M, e_H) is added to this list.¹⁹ Without collusion the principal has an optimal level of CE given by 26.7315, which results from the contract involving (e_H, e_H) and $\gamma_1 = \gamma_2 = 0.289898$. But, this level of shares does not suffice to make the agents ignore joint deviations in effort choices. In order to do that, the principal has to increase the share of the project allocated to the agents. This, however, is costly because agents are more risk averse than the principal. Indeed, the optimal collusion-proof contract involves the same effort profile of (e_H, e_H) , but higher shares allocated to the agents, 0.312377. This, in turn, decreases the principal's optimal CE from 26.7315 to 26.6817.

C Example 3

This example involves cases with non-monotone returns and features a best effort profile which is not optimal to be implemented with incentive-contracts. In fact, example 3 is the same as example 2 with only $c_1(e_H)$ changed from 1.25 to 1.75.

Thence, optimal incentive-contracts implementing $e \in E$ are given in table 8. The best

¹⁹When the principal desires the colluding agents to choose the effort level of (e_M, e_H) , CC_2 guaranteeing agent 2 to prefer this effort profile to (e_H, e_H) becomes $-\gamma_2 - 5/2\gamma_2^2 \geq 0$ implying that $\gamma_2 = 0$. But due to (SC) we have $\gamma_1 = \gamma_2$, and $\gamma_1 = 0$ is not compatible with CC_1 that ensures that agent 1 prefers the effort profile (e_M, e_H) to (e_L, e_H) , which is given by $3.50\gamma_1 - 5/2\gamma_1^2 \geq 0.75$.

| | γ_1 | γ_2 | ρ_1 | ρ_2 | CE_P |
|--------------|------------|------------|----------|----------|---------|
| (e_H, e_H) | 0.463325 | 0.463325 | -11.0764 | -12.3164 | 25.664 |
| (e_H, e_L) | --- | --- | --- | --- | --- |
| (e_M, e_H) | 0.26411 | 0.26411 | -5.82453 | -6.06453 | 25.8199 |
| (e_M, e_L) | --- | --- | --- | --- | --- |
| (e_L, e_H) | --- | --- | --- | --- | --- |
| (e_L, e_L) | 0.142858 | 0.142858 | -2.90682 | -2.40682 | 24.8476 |

Table 8: Optimal incentive-contracts of example 3.

| | γ_1 | γ_2 | ρ_1 | ρ_2 | CE_P |
|--------------|------------|------------|----------|----------|---------|
| (e_H, e_H) | 0.463325 | 0.463325 | -11.0764 | -12.3164 | 25.664 |
| (e_H, e_L) | --- | --- | --- | --- | --- |
| (e_M, e_H) | --- | --- | --- | --- | --- |
| (e_M, e_L) | --- | --- | --- | --- | --- |
| (e_L, e_H) | --- | --- | --- | --- | --- |
| (e_L, e_L) | 0 | 0 | 1 | 1.50 | 24.4667 |

Table 9: Optimal collusion-proof contracts of example 3.

effort profile is (e_H, e_H) , but the optimal one for the principal is (e_M, e_H) .²⁰ The optimal collusion-proof contracts implementing $e \in E$ are presented in table 9. Notice that besides (e_H, e_L) , (e_M, e_L) , and (e_L, e_H) collusion additionally render (e_M, e_H) impossible.²¹ The optimal incentive-contract involves (e_M, e_H) , and a share allocation of 0.26411 to each agent. But, these shares fail to eliminate collusion considerations. Indeed, implementing (e_M, e_H) is impossible, and thus, with collusion the principal has to go for (e_H, e_H) allocating 0.463325 portion of the asset to each of agent, in turn, decreasing his CE from 25.8199 to 25.664.

²⁰ (e_H, e_L) , (e_M, e_L) , and (e_L, e_H) cannot be obtained with incentive-contracts. First, note that due to (SC) , $\gamma_1 = \gamma_2$. For (e_H, e_L) , every (IC) constraints can only be satisfied for $\gamma_1 = \gamma_2 \geq 0.83599$, which is not feasible because $\gamma_1 + \gamma_2$ cannot exceed 1. When (e_M, e_L) is to be sustained, IC_1 that guarantees that agent 1 chooses e_M over e_L takes the form of $2.70\gamma_1 - 10/3\gamma_1^2 \geq 0.75$, and this constraint cannot be satisfied for any $\gamma_1 \in [0, 1/2]$. Finally, for (e_L, e_H) , IC_2 implying agent 2 prefer e_H to e_L is $-1.80\gamma_2 + 25/6\gamma_2^2 \geq 0.01$ which requires γ_2 to be greater or equal to 0.43749. But, then the all (IC_1) constraints (which are given by $-3.5\gamma_1 + 5/2\gamma_1^2 \geq -0.75$ and $4.50\gamma_1 \leq 1, 75$) cannot be satisfied for $\gamma_1 = \gamma_2 \in [0.43749, 0.50]$.

²¹The reason is the very same given in footnote 19.

| | (e_H, e_H) | (e_H, e_L) | (e_L, e_H) | (e_L, e_L) |
|---------------|--------------|--------------|--------------|--------------|
| $\mu(e)$ | 30 | 31 | 26.50 | 28.30 |
| $\sigma^2(e)$ | 1 | 2 | 1/2 | 4/3 |

Table 10: The mean and variance figures of example 4.

| (e_1, e_2) | $\gamma_1(e)$ | $\gamma_2(e)$ | $\rho_1(e)$ | $\rho_2(e)$ | $CE_p(e)$ |
|--------------|---------------|---------------|-------------|-------------|-----------|
| (e_H, e_H) | 0.26411 | 0.26411 | -6.32453 | -6.06453 | 26.3199 |
| (e_H, e_L) | -- | -- | -- | -- | -- |
| (e_L, e_H) | -- | -- | -- | -- | -- |
| (e_L, e_L) | 0.142858 | 0.142858 | -3.40682 | -2.40682 | 25.3476 |

Table 11: Optimal incentive-contracts of example 4.

D Example 4

This example shows that collusion cannot be ignored when returns are not monotone and there is no best effort profile. Let $E_1 = \{e_L, e_H\}$, and $E_2 = \{e_L, e_H\}$, and the mean and variance figures are given in table 10. The cost of efforts are, $c_1(e_L) = 0$, $c_1(e_H) = 0.75$; $c_2(e_L) = 0$, and $c_2(e_H) = 0.01$. Moreover, the reserve CE figures are $W_1 = 0.50$, $W_2 = 1.5$; and CARA levels $R = 2$, $r_i = 10$ for $i = 1, 2$.

The optimal incentive-contracts (collusion-proof contracts) implementing $e \in E$ are given in table 11 (and table 12, respectively), showing that collusion decreases the optimal

| (e_1, e_2) | $\gamma_1(e)$ | $\gamma_2(e)$ | $\rho_1(e)$ | $\rho_2(e)$ | $CE_p(e)$ |
|--------------|---------------|---------------|-------------|-------------|-----------|
| (e_H, e_H) | 0.332674 | 0.332674 | -8.17687 | -7.91687 | 26.0213 |
| (e_H, e_L) | -- | -- | -- | -- | -- |
| (e_L, e_H) | -- | -- | -- | -- | -- |
| (e_L, e_L) | 0 | 0 | 0.50 | 1.50 | 24.9667 |

Table 12: Optimal collusion-proof contracts of example 4.

CE of the principal from 26.3199 to 26.0213, even though the associated effort profile remains the same.²²

²²In this case, (e_H, e_L) and (e_L, e_H) cannot be implemented. For (e_H, e_L) , IC_1 implies $2.70\gamma_1 - 10/3\gamma_1^2 \geq$

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0.75, and cannot be satisfied for any $\gamma_1 \in [0, 0.50]$. Considering (e_L, e_H) , IC_1 is $-3.50\gamma_1 + 5/2\gamma_1^2 \geq -0.75$, and holds only if $\gamma_1 \leq 0.26411$. And, IC_2 is $-1.8\gamma_2 + 25/6\gamma_2^2 \geq 0.01$, and holds whenever $\gamma_2 \geq 0.43749$. Thus, due to $\gamma_1 = \gamma_2$ by (SC), they cannot be satisfied simultaneously.

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