Strategic defense and attack for series and parallel reliability systems: reply to rejoinder

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STRATEGIC DEFENSE AND ATTACK FOR SERIES AND PARALLEL RELIABILITY SYSTEMS: REPLY TO REJOINDER

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ABSTRACT
Kovenock and Roberson's (2012ab) replication of Hausken's (2008a) equations and parameter restrictions do not enhance our insight into the defense and attack of reliability systems. This reply intends to fill the remaining understanding gaps.

Suggestions of hand tricks, misleading, and appeals to something else do not belong in academic discourse. The correct quote from p. 857 expresses maximization versus minimization of reliability and does not appeal do anything beyond itself. Von Neumann and Morgenstern (1944) cannot be accused of "sloppy analysis and ad hoc assumptions" because they do not use the Nash (1951) equilibrium concept. Hausken (2008a) determines the first order conditions and parameter restrictions correctly. Kovenock and Roberson (2012a) interpret Hausken (2008a) as having used the Nash equilibrium concept. Hausken (2012) accepts the interpretation. Hausken's (2008b) subsequent paper uses the Nash equilibrium concept. If Hausken's (2008a) paper were to be rewritten, the Nash equilibrium concept might well be used. Future research may choose to interpret Hausken (2008a) as having used the Nash equilibrium concept if that makes the paper easier to understand. The suggestion that Kovenock and Roberson's (2012a) "critique is misguided" because Hausken "never mentioned the word "equilibrium"" is incorrect logical reasoning. The critique is not misguided because of that. The critique is empty because it replicates what Hausken (2008a) has already provided.

Regarding the prose after equation (17), Hausken (2012) writes that "For the event of negative utilities Hausken (2008a) provides one possible solution based on one of many possible assumptions about what players may do when a Nash equilibrium does not exist in pure strategies." Hausken (2008a) should have used another term such as "corner assumption" instead of "corner solution", and thus agreement exists on this point.

Agreement also exists on the parenthetical nonsense issue. It is valid of the authors to apply "the representation of Nash equilibrium as a fixed point of the best-response correspondence so that…..", as long as we now know that they don't invoke sequential games. This point has no relevance for the issue.

Hausken (2012) writes twice that "The necessary conditions have been provided by Hausken (2008a) as $u \geq 0$ and $U \geq 0$." Hausken (2012) and Kovenock and Roberson (2012a) thus agree that necessary, and not sufficient, conditions for Nash equilibrium have been provided. Hausken's (2012) reply contains two errata. The first is related to the "Introduction, paragraph 2" where the term "conditions" should be replaced with "necessary conditions". The second is the two confusing sentences quoted by Kovenock and Roberson (2012b). I do not know how the two sentences made their way into the reply, but "necessary conditions" should not be confused with "necessary and sufficient conditions". Only "necessary conditions" for Nash equilibrium have been provided.

Whether or not it is "immediate to appeal to existence of mixed strategy equilibria in games with discontinuous payoffs" depends on which parts of the literature we are discussing. Kovenock and Roberson (2012a) refer to "many of the models in this literature", and Kovenock and Roberson (2012b) refer to "Hausken's contributions". Since I do not know what Kovenock and Roberson refer to, I cannot address this issue.

We agree that Hirshleifer's (1995) model and Hausken's (2008a) model are different. We also agree that a parameter restriction valid for one model cannot justify a parameter restriction for a different model. Both models imply that the decisiveness parameter cannot be too large. We may well conclude that Hirshleifer's (1995) model is irrelevant for Hausken's (2008a) model.

Regarding even resource distribution, Kovenock and Roberson (2012b) refer to "many of the related articles in this literature" without specifying whether or not Hausken has authored these "related articles". Hausken (2012a) was guessing that Kovenock and Roberson referred to for example Hausken and Levitin's (2009) paper, which indeed builds on Hausken (2008), or referred to a number of other papers by Hausken using a similar approach. Hausken and Levitin (2009) do not use the Nash equilibrium concept, and analyze correctly a minmax game with sequential moves where the defender moves first and the attacker moves second.

If the "Three examples of arguments for even resource distribution" provided by Hausken (2012) do not address the issue, and if Kovenock and Roberson (2012ab) do not refer to e.g. Hausken and Levitin's (2009) paper, then I do not know which paper(s) Kovenock and Roberson (2012ab) refer to. Yes, I agree that the issue is with "the existence of a pure-strategy equilibrium with an even allocation of forces", so presumably the reference is to a paper that uses the equilibrium concept. If Kovenock and Roberson (2012b) interpret the reference to Zermelo (1913) and Selten (1965) as misleading or obvious, they may ignore it. The suggestion that Hausken designs statements to be misleading does not satisfy scientific standards. I do not know why misleading would be interesting or relevant. A more plausible explanation is that Hausken (2008a) has not succeeded writing the paper sufficiently clearly. Hopefully this exchange has illuminated the issue.

Differentiating Hausken's (2008a:863) first order conditions in equation (14) for the series system gives the second order conditions

$$\begin{aligned}
\frac{\partial^2 u}{\partial t_i^2} &= \frac{-r m_i T_i^{2m_i} t_i^{m_i-2}\left(1-m_i+(1+m_i)\left(\frac{C_i/R}{c_i/r}\right)^{m_i}\right)}{(t_i^{m_i}+T_i^{m_i})^3}\prod_{\substack{j=1\\ j\neq i}}^{n}\frac{t_j^{m_j}}{t_j^{m_j}+T_j^{m_j}} < 0, \\
\frac{\partial^2 U}{\partial T_i^2} &= \frac{-R m_i t_i^{m_i} T_i^{2m_i-2}\left(1+m_i+(1-m_i)\left(\frac{C_i/R}{c_i/r}\right)^{m_i}\right)}{(t_i^{m_i}+T_i^{m_i})^3}\prod_{\substack{j=1\\ j\neq i}}^{n}\frac{t_j^{m_j}}{t_j^{m_j}+T_j^{m_j}} < 0
\end{aligned} \tag{1}$$

where we have inserted $t_i = \frac{C_i/R}{c_i/r}T_i$ from Hausken's (2008a:862) equation (9) into the two large brackets. The two inequalities in (1) are satisfied as negative when

$$\left(\frac{m_i-1}{m_i+1}\right)^{1/m_i} < \left(\frac{C_i/R}{c_i/r}\right)^{m_i} < \left(\frac{m_i+1}{m_i-1}\right)^{1/m_i} \tag{2}$$

which is equivalent to the requirement shown by Hausken (2008b:1744) in equation (11) for a more general reliability system. Differentiating Hausken's (2008a:870) first order conditions in equation (41) for the parallel system thus gives second order conditions which imply the same requirement.

To summarize, Kovenock and Roberson (2012ab) have replicated Hausken's (2008a) equations and parameter restrictions. Agreement exists that necessary conditions have been provided.

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