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The Puzzle of Privately-Imposed Price Limits: Are the Limits Imposed by Financial Exchanges Effective?*

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Abstract

Some of the world's largest futures exchanges impose daily limits on the price movements of individual contracts. Using data from three of the most active US commodity futures contracts, we show that these price restrictions are largely ineffective because traders are able to take similar positions using other contracts. When price limits become binding on the futures market, the associated (but unrestricted) options market becomes the price discovery market: much of the trading that would have occurred on the futures market migrates to the options market, and options prices accurately predict the (unconstrained) futures price the next day. We also show that the presence of options mitigates the effect of price limits on information revelation by documenting that futures markets reflect more accurate information on days following limit hits when the associated options were trading on the previous day. Overall, our evidence suggests that price limits in US futures markets have little effect on prices when options markets exist.

Keywords

Price limits, Regulatory evasion, Put-call parity, Satellite market, Price discovery.

JEL classification

G100, G130.

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■ 1. Introduction

Generally, price controls are thought to prevent markets from clearing, and lead to inefficient allocation and/or rent-dissipation. Certainly, empirical examples abound of price controls having these effects (see e.g., Frech and Lee, 1987, on gasoline, Glaeser and Luttmer, 2003, on rental housing). Perhaps surprisingly, price limits on financial exchanges are fairly common in developed countries. These limits often take the form of an exchange prohibiting trading of individual securities or contracts at prices that differ from the previous day's price by more than some fixed amount, or imposing halts on trading a stock if its price moves by some pre-determined amount.

In contrast to the physical products examples described above, in many instances these restrictions are apparently voluntarily imposed by private, for-profit exchange organizations on market participants. The decision by private organizations to adopt rules that limit price movements suggests that such rules might have effects in financial markets that differ from those realized in physical product markets (like housing and gasoline). That is, one generally anticipates that such organizations choose rules that maximize their profits, and as such, rules that serve to restrict voluntary transactions that use the exchange's services would not seem to be in an exchange's interest. Several models have been proposed to explain how price limits might serve the exchange's interest. Brennan (1986) proposed a model in which price limits can increase a private exchange's profit by limiting its exposure to opportunistic behavior by some traders. Specifically, by concealing the equilibrium price, price limits reduce the incentive of traders to renege on their mark-to-market margin obligations when a large adverse price movement occurs¹. Alternatively, it has been suggested these restrictions serve to prevent large price movements (volatility) that are unrelated to fundamentals (see, e.g., Kim and Yang, 2004).

Implicit in these explanations of price limits is that the restrictions are effective: they limit the ability of individuals to trade at prices outside of certain bounds. If there are alternative trading venues, transaction prices on the alternative venue could lead to the same prices as would have occurred on the primary venue absent the restraint, but the trades instead take place elsewhere. If that is the case, then price limits will have no effect on information production (if one takes the Brennan view of price limits), or the ability of "speculators" to move prices from their fundamental values (if one views large price movements as typically reflecting something other than fundamentals).

In this paper, we perform several empirical analyses to examine the efficacy of alternative markets in accommodating traders who are unable to trade on the primary

¹ Shanker and Balakrishnan (2005) provide a methodology for establishing optimal price limits and margins in the context of Brennan's model.

exchange due to a price limit. While price limits and similar restrictions have not generally been imposed in US equity markets, price limits have long been in place on U.S. futures markets². As such, futures markets provide an environment in which to examine several related questions on the consequences of price limits. In this paper, we look at three heavily traded U.S. futures contracts (lean hogs, live cattle and pork bellies) on the Chicago Mercantile Exchange (CME) that have price limits, and the associated options contracts, that do not. In sum, our analysis suggests that the options market largely replaces the futures market as a price discovery venue when price limits are encountered on the futures market.

This paper looks at two types of evidence to examine the effects of price limits. In the first part of the paper, we look at daily price and volume data. This allows us to examine several questions. The most basic is whether price limits dampen price movements or, alternatively, delay price discovery. In the latter case, we would expect that limit hit days would be characterized by continuations, whereby the opening futures price on the day following a limit hit would be higher (lower) than the closing price on the limit hit day when the price has reached its upper (lower) limit. In contrast, if limits do dampen price movements (rather than merely delay them), then the opening price on the day following a limit hit should exhibit a reversal, i.e., move in the opposite direction of the limit hit. Consistent with the premise that price limits interfere with price discovery, we find that continuations are more common than reversals for all three contracts.

Given this finding, we then examine whether the change between the limit-constrained futures price on the limit hit day and the opening price on the following day is predictable. To do so, we examine the closing prices for the put and call options associated with a futures contract on days for which the futures market closed at the limit price. Specifically, we take advantage of the arbitrage relationship between futures and options to evaluate the futures price implied by the closing price for the options. We find that the difference between the implied and the constrained futures prices provides a strong indication of change in the futures price that occurs when trading resumes on the next day. For every \$1 deviation between the implied and constrained futures price, there is a change in the next day opening price on the futures market between 50 cents and \$1 in the same direction.

Our third inquiry is how price limits affect futures and options trading volume. For the contracts in our sample, we find that options trading volume increases dramatically when price limits are hit on the futures market, while futures trading fell. In this

² As noted above, some equity markets do impose trading halts, which have a similar effect to price limits. For example, the NYSE and Nasdaq markets both impose temporary trading halts when significant information is expected. As Christie, Corwin and Harris (2002) note, one ostensive reason for this policy is to allow more equal access to information.

sense, options trading replaces, at least in part, futures market trading on days in which price limits are hit.

This finding is consistent with the premise that price limits result in the shift of trading volume to an alternative trading venue when price limits are encountered, where the alternative venue depends on the characteristics of the market that is price constrained. The commodities we study have high storage costs. As such, the associated options markets are likely to be the lowest transaction-cost way of replicating a futures market position. For other kinds of financial markets, the lowest-cost alternative trading instrument may be different. Indeed, Hall, Kofman and Manaster (2006) show that for the low storage-cost commodity in their study, a significant amount of volume shifts to maturities of the same commodity that are not subject to the price limit. Similarly, Berkman and Steenbeek (1998) show that when the prices for stocks trading on the Tokyo Stock Exchange reach a price limit, trading volume in that stock shifts to a non-Japanese exchange that did not have a binding price limit³.

The observed changes in options markets when futures limit hits occur motivates the analysis in the second part of the paper, in which we investigate intra-day liquidity and volatility in the aftermath of price limit hits. A key contribution of this paper is that we exploit the institutionally-created variation in whether futures options are offered on specific dates. This allows us to investigate whether the presence of options on limit days has an impact on the underlying futures market the day after a limit hit. Specifically, we take advantage of exogenous variation in whether options contracts are traded on the dates of the limit hits to examine how the presence of options affects spreads and volatility following limit hits. Intuitively, the occurrence of a limit hit should exacerbate informational asymmetries on the following day, as less information is incorporated into the price on the limit hit day. Consequently, if adverse selection leads to higher bid-ask spreads (as in Glosten, 1987), one would expect greater spreads in early trading on days following limit hits. Given our finding that the presence of options leads to greater price discovery on the limit hit day, this deleterious effect of limit hits on spreads should be mitigated when options are traded. Similar logic suggests that price would make a smoother adjustment to its new equilibrium on the day following a limit hit when more information is available. Consistent with this intuition we find that, while having options trading alongside the associated futures reduces bid-ask spreads and intra-day volatility on the futures market for all days, this reduction is particularly large on days following limit hits⁴.

³ That is, in the futures market they study (coffee futures), price limits are imposed on distant-maturity contract, but not on the closest-maturity contract.

⁴ A limit hit might also increase other components of the bid/ask spread, especially inventory management costs. Since the presence of options could reduce inventory management costs, our findings do not identify the cause of the reduction in spreads and volatility.

This evidence provides further support for the premise that that price limits primarily shift price discovery to options markets, rather than conceal or prevent futures price changes. Several extant theoretical models provide rationales for how delaying price discovery can be socially productive. For example, as noted, Brennan (1986) shows that, by restricting price discovery, price limits help protect exchanges from strategic default by traders. In Brennan's model, traders will have an incentive to default on their obligation to the exchange if they know the one-day price change is sufficiently large. By concealing that information, the exchange can reduce the number of defaults. Chowdry and Nanda (1998) point to the potential that a large price change could lead to some investors facing binding borrowing constraints. This in turn leads them to liquidate their positions, thereby exacerbating the price change. Kodres and O'Brien (1994) show that price limits can provide insurance when traders have ex-ante risky positions in incomplete markets.

Because information revelation does shift to the options market when limits are hit, it follows that the presence of the options market reduces the effectiveness of futures price limits in achieving these kinds of policy objectives. For example, in the context of the Brennan model, if option prices predict unrestricted futures prices well, then traders will base their decisions on the futures price implied by the options prices. As such, price limits will not have the effect on trader defaults that the Brennan model would suggest⁵.

The evidence on the effectiveness of options markets makes the decision by for-profit exchanges to impose price limits something of a puzzle; why impose a policy that is readily circumvented? One potential explanation is that forcing traders to use alternative contracts does raise trading costs (although not as much as if the options markets did not exist), and thereby reduces trading volume and price discovery. Such a reduction may be desirable for one of the reasons suggested by the models described above.

Alternatively, it may be that the limits are not voluntarily chosen, but rather reflect indirect pressure from regulatory authorities. Certainly, there is evidence that regulatory authorities attempt to reduce price movement through price limits and similar restrictions. For example, the U.S. Securities and Exchange Commission (SEC) recently adopted additional restrictions on short sales of any security whose price has fallen significantly, with the apparent premise being that the restriction will limit price reductions⁶. These proposed changes would be in addition to existing exchange rules

⁵ To the extent that price limits lead to a reduction in price discovery, even in the presence of options, then trader defaults will be lower due to the presence of price limits (Chou et al., 2005). That is, Chou et al. shows that price limits can reduce defaults, as long as the correlation between futures and options prices is imperfect, with the likelihood of default increasing in the correlation.

⁶ See, e.g., Michael Mackenzie "SEC's New Short Selling Rules 'Threaten Liquidity'" Financial Times, March, 2, 2010.

that serve as “circuit breakers” by temporarily halting trading in all securities when aggregate equity prices decline substantially, and rules that call for trading halts in advance of the revelation of significant firm-specific information. Although the government’s objective in trying to influence price is unclear, the evidence suggests that such policies will be ineffective unless similar restrictions are imposed across potential alternative trading venues.

■ 2. Background

2.1. Price Limits on the Chicago Mercantile Exchange

The use of daily price limits by US futures exchanges dates back to at least the 1920s. One event that may have precipitated their imposition was the federal government’s reaction to a large run-up in wheat futures prices in 1925. Shortly after the wheat price increase, the Secretary of Agriculture threatened to revoke the designation of the Chicago Board of Trade (CBoT) as a contract market if it did not adopt a number of changes, including giving the exchange’s board of directors the power to limit daily price changes⁷. Faced with this threat, the CBoT did alter their rules, granting this authority to the board of directors. Daily price limits did not become permanent on the CBoT’s grain contracts until 10 years later, however.

The three contracts in our study began trading on the CME in the 1960s, and all three had price limits in place at their introduction. Options on these three futures contracts began trading in the 1980s, and none have ever been subject to daily price limits.

This history leaves some ambiguity in place as to the “voluntary” nature of the price limits. That is, while the price limits are not the result of explicit government regulation, there is some history of the federal government using its regulatory powers to pressure exchanges to impose price limits.

While there is no explicit U.S. government policy mandating the use of price limits in futures markets, exchanges have had limits in place on some contracts for many years. These limits are primarily in place for agricultural commodities, such as grains (traded on the CBoT during our sample period), and livestock and related products (traded on the CME). As a percentage of product price, the limits were considerably tighter for the CME products than for the CBoT products. As a result, limit hits are considerably more common on the CME. Consequently, in order to study the effect of limit hits, we look at contracts on the CME; specifically, three of the most actively traded

⁷ See, *letter from the Secretary of Agriculture to the US Senate, dated 6/28/1926. Senate Document #135.*

contracts on the exchange, among those contracts that had price limits in place. In total, there were 1049 limit hits on those three products during the sample period⁸.

2.2. Previous Research

One major difference between models of price limits in which price limits serve to control volatility, and those in which they merely delay price discovery concerns the price movement that occurs once the limit is no longer binding. This typically involves looking at daily price movements following a limit hit. The evidence in that respect is mixed. Ma et al. (1989) look at episodes of limit hits on four U.S. futures contracts (corn, soybeans, silver and treasury bonds) over a 10-year period. They find that, on average, the price on the day following the market closing at the limit moves in the same direction as the limit hit for three of the four products. Park (2000) focuses on the issue of continuations vs. reversals following limit hits, for the four largest-volume agricultural products on the Chicago Board of Trade (CBoT) over a 12-year period⁹. He concludes that continuations are more likely than reversals¹⁰. Chen (1998) studies 14 U.S. futures contracts that had price limits across various categories (including grains, financials, metals, fiber and livestock), for sample periods ranging from 11 to 21 years. He finds that, on average, there were continuations for all 14 products¹¹. Veld-Merkoulova (2003) studies 7 agricultural products and likewise finds that price on the day following a limit hit tends to move in the same direction as the limit hits. Our results are consistent with Chen's and Veld-Merkoulova's; we find that all three products are characterized by continuations being at least three times as common as reversals. In addition, because of the large number of observations in our sample, the statistical significance of these differences is greater than in previous work.

Several studies examine the relationships between futures and options prices and volumes. There are two basic approaches to estimating implied futures prices from options prices on limit hit days. One approach, which we describe in detail in Section III, is to use the put/call parity relationship to estimate implied futures price from options prices on limit hit days. Evans and Mahoney (1996) use this approach to determine a "synthetic" futures price, which forms the basis of their empirical work. Using cotton futures and options prices, they informally show that the synthetic futures

⁸ In contrast, there were only 51 limit hits across the 4 largest-volume price-limit-constrained contracts on the CBoT during that same period.

⁹ We examined these same four products in an earlier draft, but omit those results here because the low number of limit hits for those products does not allow us to use them for the rest of our statistical analyses.

¹⁰ Kuserk and Locke (1996) find that pork belly prices continued to drift in the direction of the limit hit during the first non-limit day.

¹¹ Several papers analyze price limits on stock exchanges. Kim and Rhee (1997) examine the effects of stock-specific price limits on the Tokyo Stock Exchange. They compare stocks that experience limit hits to those that have large price changes that are somewhat smaller than the limit (e.g., 90% of the maximum allowable change). They find that continuations are statistically significantly more likely for the limit hit stocks than the near-hit stocks. Phylaktis et al. (1999) similarly find evidence that price limits impede price discovery in their sample of 10 stocks on the Athens Stock Exchange.

price provides an accurate estimate of the actual futures price on non-limit hit days, and hence is likely a good estimate of the unobserved equilibrium futures price on limit hit days as well. Their findings, however, are based on a single month of data in which futures price limits were hit on one or more futures contracts on 12 of the 20 trading days; further, the futures price did not close at the limit on all of those days. Our findings reinforce their's; using a much larger sample of 1049 limit hits in three futures markets over a 7-year period, we demonstrate that the synthetic price has substantial power in predicting price movements.

A second approach is to obtain implied volatility from the options data, which allows one to estimate futures prices from options prices. Hall et al. (2006) use this approach to study coffee futures and options prices. They find that the difference between the implied futures prices and the closing (or settlement) price is almost always in the same direction of the price limit (e.g., when the futures price last trades at the upper limit, the implied futures price is above the settlement price).

Whether one uses put-call parity, or estimates volatility in order to measure implied futures prices, the underlying logic in both cases is that a price limit in one market causes price discovery to shift to less constrained markets. Another method to determine if discovery shifts is to examine trading volume on limit hit days. When a futures market is lock limited (i.e., closes at the limit), traders can (perhaps at higher cost) replicate their desired futures market position through trades in related markets. Subrahmanyam (1994) develops a formal model in which a “circuit breaker” (an extreme kind of price limit) on one market causes trading volume to shift to other markets. Berkman and Steenbeerk (1998) show that such a shift occurred for the Nikkei futures contract, where price limit hits on the Osaka Exchange caused volume to shift to the Singapore exchange.

Both Evans and Mahoney (1996) and Hall et al. (2006) examine the question of whether trading volume shifts from the contract that has an effective price limit to another contract that does not. Evans and Mahoney measure the presence of price limits by calculating the percentage of the trading day for the limit is binding. They find that futures volume falls with the percentage of the trading day for which price is at the limit, while (delta-adjusted) options volume rises. These two effects were of similar magnitude, so that total trading volume in a specific maturity's contract was essentially independent of the presence of price limits.

For the product studied by Hall et al. (2006), there were two related markets that had no price limits; the options market associated with the price-limited futures market, and the futures market for the closest-maturity contract. Hence, they consider not only the movement from futures to options induced by the price limit on coffee futures, but

also the movement from distant month contracts (which have price limits) to nearby month contracts (which do not). They find that when there is a limit hit on a distant futures contract, the number of trades increases for the nearby contract, although they do not find a statistically significant decrease in trades on the coffee futures contract that experiences the limit hit. In contrast to Evans and Mahoney, Hall et al. do not find any significant migration from futures to options when limit hits occur¹².

■ 3. Methodology

As discussed in Section 2, theory provides a link between options prices and futures prices, and therefore provides an estimate of the unconstrained futures price when the futures markets has a price limit in effect, but the options market does not. We noted there that two basic approaches have been taken with respect to using options price to estimate futures price in these circumstances.

One approach is based on put-call parity. For a European-style option (i.e., one that can only be exercised on its expiration date), an arbitrage condition implies that the difference between the price on day t of a call option (C) and the price of a put option (P) with the same strike price (X) and same expiration date (T) must equal

$$C - P = (F_t - X)e^{-r(T-t)} \quad (1)$$

The logic is that a portfolio consisting of a call option and $Xe^{-r(T-t)}$ in cash in period t is worth $\max(F_T, X)$ at period T , and one consisting of a put option, a long position in the futures and cash of $F_t e^{-r(T-t)}$ at period t will also be worth $\max(F_T, X)$ in period T . Since they are worth the same in period T , they must also be worth the same in period t ; i.e., $C + Xe^{-r(T-t)} = P + F_t e^{-r(T-t)}$.

One implication of this relationship is that for European-style options, the put and call prices imply a specific value for F_t ;

$$F_t = X + (C - P)e^{r(T-t)} \quad (1')$$

This relationship means that if the futures contract is not traded and the options are, then one can infer the price of a European futures contract from the options prices. In fact, the futures options we study are American-style (i.e., can be exercised at any

¹² Veld-Merkoulova (2003) finds that for the most of the products she studies, there seems to be little volume shift between maturities when a limit hit occurs on some, but not all, maturities. Specifically, she finds that for only one of the 14 cases she examines (14 = 7 products times 2 directions) does a limit hit only on the second-closest-to-maturity contract increase trading volume on the nearest-to-maturity contract. However, she does not test if volume shifts to options when limit hits occur.

time prior to expiration). The potential for early exercise means that futures prices are not necessarily characterized by equation (1'), but instead by two inequalities.

$$C - P + X e^{-r(T-t)} \leq F_t \leq (X + C - P) e^{-r(T-t)} \quad (2)$$

As with European-style options, the logic for these inequalities is that if either of the inequalities is not satisfied, then there is profitable arbitrage opportunity (abstracting from transactions costs). Note, that the F_t implied by the European option arbitrage condition always lies in this range.

An alternative means of using the options prices to infer futures prices was implemented by Hall et al. (2006) and Egelkraut et al. (2007). Hall et al. (2006) recognize that futures options in the market they study are indeed American in nature. As such, the early exercise premium may be non-trivial which may, in turn, invalidate the use of the European put-call parity relationship. To account for this possibility, Hall et al. (2006) employ the Barone-Adesi and Whaley (1987) approximation for American options to help recover implied futures prices. In recovering the implied futures price of a limit-constrained 'distant' contract, the authors note that because the nearby futures is unconstrained by price limits, the nearby futures price may be used in the option pricing formula to obtain the implied volatility. The implied volatility measure from the nearby contract can then be used in the 'distant' month option formula, and the implied distant futures price can then be recovered.

Egelkraut et al. (2007) also directly estimate volatility and use it to calculate implied futures prices. In contrast to Hall et al. (2006), they estimate the implied futures price and the implied volatility simultaneously, rather than in a two-step procedure.

The evidence suggests that all three approaches yield a better estimate of the opening price on the day following a limit hit than the settlement price on the limit hit day. At the same time, all are subject to errors of some kind. For example, taking the put-call parity relationship that applies to European options and using it to price American options creates model error in the estimation.

More subtly, there are model error issues with respect to estimating volatility and then determining futures price from those estimates. For example, the Hall et al. (2006) approach assumes that implied volatility recovered from the nearby contract is indeed a correct measure of volatility for distant maturity contracts (i.e., they are the same)¹³. Moreover, while the model they employ may be theoretically correct, it is still nonetheless

¹³ This assumption, in turn, requires non-stochastic interest rates. If the interest rate is stochastic, then the volatility of the nearby contract will exceed that of the distant maturity contract. Proof available from authors.

a function of parameters that are estimated with error. Accordingly, as the authors note, since the model is an approximation of the true arbitrage-free equilibrium price, model errors are to be expected. Similarly, as Egelkraut et al. (2007) note, their procedure may dilute information and introduce further error (although Hall et al. (2006) suggest that their improved implied volatility measure outweighs this additional bias).

Recognizing that any approach to recover implied futures prices is not without error, in this paper we simply employ a European futures option pricing model, but recognize that such a model may either overestimate or underestimate a futures price. However, as shown by Ramaswamy and Sundaresan (1985) and Barone-Adesi and Whaley (1987) the associated pricing error is small and minimized for options with a strike price close to the current market price (*at the money*), since the early exercise feature of American options is unlikely to be valuable for the put and call owners of at-the-money options. As such, equation (1') might serve as a reasonable approximation for the futures price¹⁴. In Section 4 we present evidence on the efficacy of equation (1') in predicting futures prices.

■ 4. Analysis of Interday Changes

For the major U.S. commodity futures exchanges, futures price limits are in place primarily for agricultural products. During our sample period, price limits tended to be tighter (as a percentage of product price) on the Chicago Mercantile Exchange (CME) than on other futures exchanges, and hence CME products tended to have the most limit hits. For this reason, analysis of the CME products has the advantage of allowing for more power in the statistical tests. Accordingly, this study focuses on the three most heavily traded agricultural contracts (during the sample period) on the CME. All three had daily price movement limits in effect for futures contracts for the entire sample period (January 2, 1998 through February 23, 2005).

4.1. Data

The daily data used in this study are derived from the US Commodity Futures Trading Commission's (CFTC's) Integrated Surveillance System database, which includes price data on both futures and futures options on these contracts. Specifically, it includes opening and settlement prices for each date, as well as daily high and low prices, and trading volume and open interest for each futures contract tradable option.

¹⁴ In addition, the early exercise feature will tend to be most valuable when markets are relatively illiquid, which tends to happen for options with strike prices that are far from the futures price. Martin and Overdahl (1994) show that early exercise is more likely for options in equity markets that have strike prices far from current market price.

● **Table 1. Limit Hits**

Product	Initial Limit ^a	Mean Price	Number of Limit Hits	Mean Change on Day $t + 1$	% continuation ^b	
					Overall ^c	For largest open interest contract
Pork Bellies	\$3	\$73	488	\$0.94	79.31%	80.33%
Lean Hogs	\$2	\$57.31	295	\$0.44	74.92%	77.21%
Live Cattle	\$1.50	\$73.19	266	\$0.76	77.06%	81.97%

This table reports initial price limits, mean prices, number of limit hits, the mean change in price after a limit hit, overall continuation rate and continuation rate for the maturity with the largest open interest for pork bellies, lean hogs and live cattle.

a. As noted in text, the daily price limit for all products except lean hogs changed over the sample period.

b. Mean change in the same direction as the limit hit (e.g., price reduction following a down limit day).

c. These percentages exclude days in which there was no change on $t+1$.

The relevant limits on futures prices at the beginning of the sample period are listed in the first column of numbers in Table 1. For live cattle and pork bellies, the price limit changed during the sample period. Specifically, the daily price limit on pork bellies would be *expandable* on days following a price limit hit (increasing to \$4.50 on the following day). Live cattle price limits were expandable between October, 2003 and February, 2004; increasing to \$3 following a limit hit and to \$5 if there was a limit hit at \$3. Between February, 2004 and the end of the sample, the limit was \$3. In contrast to futures prices, there are no price limits on the CME for the options contracts on the three futures products we study.

We define a limit hit as occurring when the settlement price on day t is different from the settlement price on day $t-1$ by the limit amount. This procedure excludes days in which a price limit is hit during trading on day t , but price subsequently moves in the opposite direction (limit bounce day), so that trading that is unconstrained by the price limit occurs before the trading day ends. Focusing on days in which the market closes at a limit allow us to directly measure the price at the first point at which price is no longer constrained by the price limit (i.e., the opening price on the following day). In contrast, if the futures price hits a limit in the middle of a trading day, then the next trade is constrained in that it may not take place outside the limit and hence futures prices can only move in one direction at that point¹⁵. That is, we do not directly observe subsequent futures price changes on limit bounce days if they would have been continuations¹⁶.

¹⁵ *In principle one way to overcome this limitation might be to use observations on options prices to estimate an implied futures price. This potential solution, however, may be complicated by the difficulty of finding trades in both the put and the call at the same strike price that are close together in time. By contrast, the settlement prices used here are the prevailing prices at the close, and hence are less likely to be asynchronous.*

¹⁶ *For example, if there is a price limit hit at noon (and the trading day ends at 3:00 p.m.), no potential trades at prices outside the limit will be executed. Hence, if a trade occurs at 2:30 within the limit, we cannot determine if a continuation would have occurred in the interim, but for the limit.*

4.2. Price Continuations vs. Reversals

Central to the discussion of the costs and benefits of price limits is the question of whether, on the one hand, they simply delay price discovery, or on the other hand, they allow a break that reduces the speculative activity (unrelated to fundamentals) that drives the price change. In the former case, we would expect that the price change between the closing price and the opening price on the following day would move in the same direction as the limit hit. In contrast, if price limits serve to prevent movements due to speculation, then the price change on the day following a limit hit would move in the opposite direction of the limit.

Table 1 provides summary statistics regarding the episodes of price limits in these three markets. Limit hits occurred between 266 and 498 times for the three products studied. Most limit hits were followed by continuations; in the three categories, between 75 and 79% of limit hits were followed by continuations. Typically, if there was a limit hit on a day, there would be a hit on more than one contract maturity. The average limit-hit day had limit hits on futures contracts on three or more maturities (out of the 6-8 maturities trading each day). As shown in the last column, the percentage of continuations is similar when one restricts the analysis to the limit hit contract with the largest open interest on each date.

We formally test the hypothesis that most price limit hits are followed by continuations. Specifically, the direction of price movement between the limit hit day and the following day is a binomial, and hence the distribution of the proportion of continuations is a binomial with mean p and variance of the mean equal to $p(1-p)/n$, where p is the probability of a continuation. Under the null hypothesis that continuations and reversals are equally likely, the standard deviation of the proportion of continuations is $\sqrt{(.25/n)}$. Thus, for all three contracts, the standard deviation of the proportion of continuations under the null hypothesis is less than 0.03. We therefore reject the null hypothesis in favor of the alternative hypothesis that continuations are more likely than reversals at the 1% level of significance for all three products. This result undermines the premise that price limits serve to mitigate the tendency for prices to move for reasons unrelated to fundamentals.

In the next section, we further evaluate the predictability of the price movement between the limit hit day and the following day for these three contracts.

■ 5. Put-Call Parity and Price Discovery

5.1. The Predictive Power of Put-Call Parity

As noted in Section 3, we use the put-call parity relationship for European-style options to obtain an estimate of the (unobservable) futures price on limit hit days. Even

though this procedure yields some model error, the evidence suggests that this error is likely to be small. That is, the question of interest is whether one can obtain good predictions for futures prices from put and call prices.

● **Table 2. Implied and Actual Futures Prices on Non Limit-Hit Days**

Panel A. Differences between futures prices and implied price from European put-call parity condition

	Average difference $F_t - (C_t - P_t)e^{r(T-t)} - X_t$	Average absolute difference $ F_t - (C_t - P_t)e^{r(T-t)} - X_t $	Absolute difference as a % of average option price $((C_t + P_t)/2)$	Absolute difference as a % of average upper bound – lower bound $((C_t - P_t)(1 - e^{-r(T-t)}) + 2X_t e^{-r(T-t)})$
Live Cattle	-.0018	\$.0094	.53%	.89%
Lean Hogs	\$.0013	\$.0076	.26%	.58%
Pork Bellies	\$.009	\$.0148	.35%	1.92%

Panel B. Regression of actual change in futures price against implied change

	α	β	\bar{R}^2	Number of observations
Live Cattle	0.074*** (0.0006)	0.9977*** (0.0019)	0.987	7140
Lean Hogs	0.0013*** (0.0002)	1.0002*** (0.0002)	.9997	6540
Pork Bellies	0.0009 0.0023)	0.9949*** (0.0026)	.997	1689

This table illustrates various measures of the predictive power of the put-call parity condition for days when a limit hit does not occur for the three commodities markets. The first column of panel A shows the average difference between the futures settlement price (observed) and the predicted futures price given by put-call parity $((C_t - P_t)e^{r(T-t)} + X_t)$ and the second shows its average absolute value. Columns (3) and (4) present the absolute difference expressed as a percent of average options prices, and the absolute difference expressed as a percent of the average difference between the upper and lower bound of the no-arbitrage condition for American options, respectively.

Panel B displays regression results of the actual change in futures price against the change implied by the no-arbitrage condition for European options: $F_t - F_{t-1} = \alpha + \beta[(C_t - P_t)e^{r(T-t)} + X_t - F_{t-1}]$. Standard errors are in parentheses and the adjusted R^2 s and standard errors are estimated using the Newey-West (1987) procedure. *, ** and *** denotes significance at 10, 5 and 1 percent level, respectively.

Table 2 presents statistics on the predictive power of equation (1') for the three futures markets. Panel A shows the mean difference between the actual futures price at the close and the futures price predicted by the formula for European options for days on which the price limit is not hit. It suggests that the model error introduced by using the put-call parity condition for European options to predict futures price is small, so that the futures price predicted by the condition is likely to be a good estimate of the actual futures price. Despite the sometimes large range of values consistent with the no-arbitrage condition for American-style options, the actual value is typically extremely close to the price predicted by the no-arbitrage condition for European-style options. For live cattle for example, on non limit-hit days the average difference be-

tween the value predicted by the no-arbitrage condition for European options and the actual futures price is about -0.18 cents, while the average absolute difference between the values is about 0.94 cents. 0.94 cents represents 0.53 % of the option prices $((put+call)/2)$, or 0.89% of the range implied by the no-arbitrage condition for American options (i.e., $(C_t - P_t)(e^{r(T-t)} - 1) + 2Xe^{-r(T-t)}$).

A second way of evaluating the goodness of fit of the European option formula is to compare the change in the futures settlement price to the change implied by the put and call prices using the relationship in (1'). Panel B displays the results of a regression of the form

$$F_t - F_{t-1} = \alpha + \beta [(C_t - P_t)e^{r(T-t)} + X - F_{t-1}] + \varepsilon_t$$

where F_t is the settlement price for the futures contract, and C_t and P_t are the closing prices on the call and put options at strike price X on day t .

As shown there, the slope coefficients for all three products are very close to 1, and the adjusted R^2 s all exceed .98. These results suggest that the implied futures price is a reliable predictor of the actual futures price on days without limit hits. Given the relationships in panels A and B, we would expect that the put-call parity relationship for European options would likewise provide reliable predictions for futures prices when the futures price cannot be observed (i.e., on limit hit days)¹⁷. The next section empirically examines the predictive power of put-call parity on limit-hit days.

5.2. Put-Call Parity and Futures Price Changes

Since the futures price implied by the put-call parity condition seems to be a good predictor of the market-clearing futures settlement price when the settlement price is observable (i.e., on non limit-hit days), we would likewise expect it to be a good estimate when the market-clearing futures price is not observable. Hence, we next examine the reliability of the put-call parity condition in predicting the opening futures price on the day following a limit hit. That is, we examine the relationship between the deviation from put-call parity on limit hit days on the one hand and the change in price between the (limit-constrained) closing price and the next day open on the other¹⁸.

¹⁷ While Evans and Mahoney do not present statistics on the fit of the futures price implied by the European parity condition and the actual futures price, the graphical evidence they present suggests that the implied price is likewise a good fit for that product on limit hit days.

¹⁸ We are interested in the opening price because it represents the first observation of a futures price when there are no constraints on the futures price, except when the following day also had a limit hit. For this reason, we drop all observations for which the following day opened at the day $t+1$ price limit. That is, when the day following a limit hit opens at the price limit, the opening price on day $t+1$ does not fully reflect all of the information contained in the deviation from parity on day t .

Our regressions take the form:

$$F_{t+1} - F_t = \alpha + \beta (C - P + (X - F_t)e^{\sigma(T-t)}) + \varepsilon_t$$

where F_t is the (constrained) settlement price on the limit hit day, and F_{t+1} is the opening price on day $t+1$. The expression on the right-hand side is a measure of the deviation from put-call parity on the limit hit day. A positive sign on β is consistent with the view that price limits prevent futures prices from reaching their equilibrium levels. Alternatively, if price limits serve as “circuit breakers” and affect the ultimate equilibrium, then the deviation would not be a good predictor of future price movements, and β could be zero or even negative.

As noted above, on days in which one futures contract price hits a limit, other futures contracts on that same commodity, but with other maturities, typically experience limit hits as well. Examining multiple futures contracts on the same commodity on a given date would lead to correlation across observations, and would supply little new information. Accordingly, we restrict our analysis to a single contract maturity on any date on which there is a limit-hit on the commodity. Specifically, among all contracts for a commodity that had a limit hit on a given date, we only examine subsequent price changes for the contract with the greatest open interest on that date (the results are robust, however, to alternative choices of the specific contract to be examined).

Similarly, we use only one set of option prices to determine the implied futures price for each limit hit, namely, the options with the strike price closest to the futures price. These options tend to have the greatest liquidity, which should minimize problems associated with thin markets, such as the non-synchronicity of put and call prices. In addition, choosing options close to the money reduces the difference between the values of European and American-style options (see, e.g., Ramaswamy and Sundaresan, 1985 and Barone-Adesi and Whaley, 1987).

To further minimize problems associated with non-synchronicity of prices, we dropped all observations for which either the call or put trading volume was zero on day t . This reduced the number of observations by about 20%, but increased the slopes and adjusted R^2 s, consistent with the premise that put and call settlement prices are less accurate on such days.

● **Table 3. Regression Results on Limit Hit Days**

	Live Cattle		Lean Hogs		Pork Bellies	
	largest OI contract	all contracts	largest OI contracts	all contracts	largest OI contracts	all contracts
Intercept	.205** (.096)	.236** (.09)	.333*** (.064)	.375*** (.056)	.647*** (.187)	.645*** (.151)
Deviation from parity	.520*** (.133)	.520*** (.131)	.604*** (.127)	.309** (.137)	.976*** (.266)	.999*** (.238)
\bar{R}^2	.18	.158	.173	.051	.267	.290
# of obs.	51	78	85	134	54	69

The dependent variable is the price change from the closing settlement price on day t to the opening price on day $t+1$ ($F_{0,t+1} - F_t$). Robust standard errors, estimated using the Newey-West (1987) procedure, are in parentheses. *, ** and *** denotes significance at 10, 5 and 1 percent level, respectively.

The results of these regressions are presented in Table 3. The findings are fairly similar across products. The price change from closing on day t to opening on day $t+1$ bears a positive and significant relationship to the deviation from put-call parity. β is between .5 and 1 for all 3 commodities (based on largest Open Interest (OI) contracts), and is significantly different from zero at the 1% level¹⁹. The deviation from parity explains a substantial portion of the variation in price changes; between 18 and 26.7%. On the other hand, because the adjusted R^2 s are below .3, and 2 of the 3 slopes coefficients are significantly less than 1, these results suggest that model error is important or that considerable new information occurs after the closing on the limit hit day, or that the put and call prices imperfectly reflect the information available on the limit hit day (or some combination of these three).

These findings are robust to relaxing the sample restrictions. For example, expanding the sample to include every futures contract with a limit hit on each limit-hit date (rather than the maturity with the largest open interest), increases the number of observations by 1/3 to 1/2, but the estimated β s and adjusted R^2 s are extremely similar for two out of the three commodities (see columns 2, 4 and 6 of Table 3). Similarly, looking at the limit-hit contract with the shortest time to expiration (rather than the contract with the greatest open interest) does not change the estimates dramatically.

5.3. Price Discovery and Options

The preceding results suggest that the existence of a functioning options market mitigates the price-discovery reducing effects of price limits. In this subsection, we provide additional evidence of the shift. First, we document the extent to which price changes

¹⁹ For limit hit days, we multiplied both the deviation from parity and the price change by -1 for observations with down limit hits. This avoids the “dumbbell” regression problem, since without this adjustment, the data would consist of two groupings; those with positive deviations and (typically, see table 1) positive price changes, and those with negative values for both. Without the adjustment, the regression line would essentially connect the centers of these two groups.

are larger on days following limit hits, and the extent to which those changes are predictable. Second, we show the extent to which trading volume shifts to options markets when price limits are hit.

● **Table 4. Average Price Changes (Absolute Values)**

	Live Cattle	Lean Hogs	Pork Bellies
Actual Change – non LH days	.199 (.238)	.316 (.294)	.502 (.519)
Actual Change – LH days	.578 (.624)	.476 (.425)	1.205 (.887)
Implied Change – non LH days	.198 (.234)	.316 (.294)	.499 (.510)
Implied Change – LH days	.476 (.560)	.435 (.445)	.913 (.703)

Actual change reflects the difference between closing settlement price and the opening price on the following day. The implied change is determined from put-call parity. Standard errors are in parenthesis.

Table 4 shows the absolute changes between the settlement price on limit hit day t and the opening price on day $t+1$. The first row lists the average absolute change between the settlement price and the next day's opening price on days without a limit hit. It can be seen as a measure of the average amount of information revealed between trading sessions. The second row lists the average absolute change between the settlement price on limit hit days and the next day's opening price. The price changes following limit-hit days are between 1.5 and 3 times larger than on non-limit hit days. This difference can be interpreted as some combination of incomplete price discovery due to the price limit and a greater degree of new overnight information on limit hit days. The third row contains the average absolute difference between the implied settlement prices on non-limit hit days (that is, $(C-P)e^{(T-t)+X}$), and the opening prices on the following days. As one would expect, these changes are very similar to the changes reported in the first row. The fourth row contains the average absolute difference between the implied futures settlement prices on limit hit days, and the opening prices on the following days. Note that the changes relative to the implied futures prices are smaller than the changes relative to the (limit constrained) futures settlement prices, suggesting that the existence of the options market undermines the price-concealing effect of the price limit. This is especially true for pork bellies; the average absolute price change between the limit hit day and the next day for pork bellies was \$1.205, which is more than twice the average change on non limit hit days (suggesting the limit does reduce price discovery), but the average absolute change in the implied price on limit hit days is 91 cents.

These data suggest that the existence of an options market mitigates, but does not eliminate, the information-dampening effect that Brennan ascribed to price limits. That is,

the existence of options provides enhanced predictability of the following day's price on a limit hit day. As Chou et al. (2005) show, Brennan's analysis can still hold when the options market provides information about futures prices, as long as the information is of imperfect quality. For example, their simulation suggests that if the correlation between the implied gap on limit hit days (i.e., the difference between the implied futures price and limit-constrained price) and the price change at the next day's opening (i.e., the difference between the next day's opening price and the limit-constrained price) is .75, then price limits reduce the number of strategic trader defaults by more than half, while allowing exchanges to require smaller margins. The greater the correlation, the less effective the price limits are at preventing strategic defaults. The correlation between the implied futures price and the following day's opening futures price in our sample ranges from .39 to .52 (adjusted R^2 between .154 and .267, as shown in Table 3), suggesting that price limits do significantly hamper price discovery. Moreover, these limits hamper price discovery most for deferred expirations, since the correlation tends to rise as time to maturity falls²⁰. Brennan notes that the cost of the delayed price discovery is lower when contracts are relatively far from expiration, so that price limits tend to work efficiently, in that they reduce defaults most dramatically when price discovery is relatively unimportant. Even granting that price limits can reduce defaults in the presence of price limits, however, one still has to wonder why the exchange allows an unrestricted options market at all.

One way of viewing these results is that they suggest price limits inhibit trading on futures markets. Specifically, price limits mean that traders are restricted in their ability to offer better prices in order to attract counterparties; hence, one would expect futures trading volume to fall when price limits are hit. At the same time, however, it seems plausible that some of that reduction in volume could effectively shift to the related options market. Intuitively, the logic of put-call parity flows from the observation that any futures position can be replicated using an appropriate combination of options and borrowing/ lending. Hence, when price limits are hit, one would expect that traders that cannot find counterparties in the futures market would instead take positions in the options market for that same commodity. Such a shift in volume would be consistent with the observed shift in the price discovery venue.

Ultimately, it is an empirical question whether futures volume drops, and in the affirmative, whether some of the futures trading volume shifts to the options market. For this reason, we next examine the effect of price limit hits on trading volume for both futures and for the related options. In estimating the extent of "migration" from futures to options, we control for two factors other than the presence of a price limit that could affect volume. First, as both Mahoney and Evans and Hall et al. note, previous research suggests

²⁰ Part of this is reduced model error, as the upper and lower bounds on the put-call pricing of American options converge to the value of the European option over time. Empirically, in our sample, the adjusted R^2 of the regressions of next day's opening price on implied futures prices increases to between .5 and .6 during the month prior to options expiration.

that there is a positive association between volume and new information, as reflected in the price change (Martell and Wolf, 1987, Lo and Wang, 2002). In addition, we would anticipate that volume will vary over course of a contract, generally increasing as the contract approaches expiration. This leads to the following specification.

$$\begin{aligned}
 Fut_Vol &= \alpha_0 + \alpha_1 |F_t - F_{t-1}| + \alpha_2 Limit + \alpha_3 Days Left + u \\
 Opt_Vol &= \beta_0 + \beta_1 |F_t - F_{t-1}| + \beta_2 Limit + \beta_3 Days Left + u
 \end{aligned}
 \tag{3}$$

where F_t is the futures settlement price on day t , and Opt_Vol is the delta-weighted combined volume of the put and call options on all of the options on that contract. The options are delta-weighted in the sense that the volume of each put and each call is multiplied by the absolute value of the delta (the slope of the value of option with respect to the futures' price) associated with that option²¹. *Limit* is a dummy variable equal to one on limit hit days, and *Days Left* is the number of days until contract expiration²².

One potential issue with this specification is that F_t is by definition truncated on limit hit days, so that some of the observed effect of price limits on volume could be due to the direct effect of a higher (unobservable) market-clearing price (on up limit days, or lower on down limit days), rather than the limit per se. To account for this possibility, we also estimated a version of these equations where we replace²³ F_t with $(C_t - P_t) / e^{-r(T-t)} + X$. As shown below, these two specifications yield quite similar results.

● **Table 5. Volume Effects of Price Limits**

A. Live Cattle

	Delta Adjusted Options Volume		Futures Volume	
Dependent Variable Mean	159.894		2770.601	
$F_t - F_{t-1}$	55.903*** (8.074)		1543.789*** (95.502)	
$(C_t - P_t) / e^{-r(T-t)} + X$		42.524*** (8.998)		825.743*** (304.165)
Limit	157.094 *** (38.748)	140.808*** (39.712)	-1136.058*** (286.245)	-1006.558** (413.189)
Days Left	-1.518*** (.034)	-1.525*** (.034)	-25.549*** (.277)	-33.019*** (.419)
Constant	325.511*** (6.521)	331.67*** (7.00)	6464.598*** (71.829)	7512.579*** (159.608)
Obs.	6695	6589	10136	7253
\bar{R}^2	0.308	.307	.663	.646

²¹ The delta can be thought as the number of options one would have to buy to replicate a single futures' contract.

²² The analysis includes only those dates on which there is a non-missing observation for the volume of the options contract with the strike price closest to the futures' price. The reason we exclude observations with missing volume for this options contract is that the total volume on these dates will be measured with a significant bias.

²³ Evans and Mahoney likewise include the synthetic prices as an explanatory variable.

B. Lean Hogs

	Delta Adjusted Options Volume		Futures Volume	
Dependent Variable Mean	48.901		1511.54	
$F_t - F_{t-1}$	9.9933*** (2.089)		614.27*** (39.80)	
$(C_t - P_t) \exp(r(T-t) + \text{Strike}_t - F_{t-1})$	10.364*** (2.075)		639.20*** (56.72)	
Limit	59.085*** (12.866)	55.094 *** (12.573)	-151.07 (143.19)	-171.16 (200.45)
Days Left	-6.05*** (.0153)	-6.05 *** (.015)	-14.43*** (.198)	-17.74*** (.269)
Constant	101.076*** (2.554)	100.848*** (2.5189)	3330.59*** (46.01)	3638.81*** (56.29)
Obs.	5558		11008	
\bar{R}^2	.295		.502	

C. Pork Bellies

	Delta Adjusted Options Volume		Futures Volume	
Dependent variable mean	4,3403		541.653	
$F_t - F_{t-1}$	1.347*** (.365)		131.253*** (12.5255)	
$(C_t - P_t) \exp(r(T-t) + \text{Strike}_t - F_{t-1})$	1.716*** (.553)		120.71*** (11.613)	
Limit	6.4966** (2.493)	4.867* (2.559)	-195.87*** (39.579)	19.850 (57.639)
Days Left	-.037*** (.004)	-.036*** (.003)	-3.22*** (.124)	-3.117*** (.113)
Constant	5.074*** (.579)	4.650*** (.695)	643.833*** (17.76)	646.579*** (16.04)
Obs.	2137		3049	
\bar{R}^2	.065		.211	

Robust standard errors, estimated using the Newey-West (1987) procedure, in parentheses. *, ** and *** denotes significance at 10, 5 and 1 percent level, respectively.

Table 5 presents results from determining the extent of migration from futures to options on limit hit days. The following regressions were ran: $Fut_Vol = \alpha_0 + \alpha_1 |F_t - F_{t-1}| + \alpha_2 Limit + \alpha_3 Days\ Left$, and $Opt_Vol = \beta_0 + \beta_1 |F_t - F_{t-1}| + \beta_2 Limit + \beta_3 Days\ Left$, where F_t is the futures settlement price on day t , and Opt_Vol is the delta-weighted combined volume of the put and call options on all of the options on that contract. $Limit$ is a dummy variable equal to one on limit days, and $Days\ Left$ represents the number of days to expiration. The change in futures price (on right hand side) is also replaced with $(C_t - P_t) / \exp(r(F_{t+1}) + \text{Strike}_t - F_{t-1})$ to account for the fact that F_t is, by definition, truncated on limit days.

The results of these regressions are reported in Table 5. For all three products, options trading volume increases on limit hit days. The effect is both statistically and economically large. The top row of Table 5 lists the mean value of the dependent variable (volume). Comparing the coefficients on the limit hit variable to the dependent variable's mean, we find that for all 3 products, delta-weighted options volume essentially doubles when futures price limit are hit. The estimated effects

on futures is somewhat mixed, but generally supports the premise that limits reduce futures trading. Taking live cattle as an example, the coefficient estimates are statistically significant and suggest that futures volume falls by approximately 40% on limit hit days. The one exception to the general finding that futures volume fell was the estimate for pork bellies when the price change is measured as the implied change, rather than the actual. The coefficient estimate in this case is small (less than 4%), and has low statistical reliability.

Our estimates of the effect of price changes are consistent with previous research that suggests volume for futures trading is positively associated with the magnitude of the price change. Finally, we find that the effect of days until expiration is negative; futures and options volume generally rises as expiration nears, although due to the offsetting of contracts, it typically begins to fall over time towards the very end of the contract cycle (the last few weeks before expiration)²⁴.

Our results are more similar to Evans and Mahoney's findings relative to Hall et al.²⁵ Like Evans and Mahoney, we find that when price limits are hit in the futures market, futures trading volume falls, and options volume increases. Unlike Evans and Mahoney, however, we find that the options-volume increase does not fully offset the decrease in futures volume. One reason why we find different results than Hall et al. may be the nature of the underlying products. A key difference between the commodities in our study and those in other studies (crops such as cotton or coffee) is that storage is considerably less expensive for the latter (Kolb and Overdahl, 2006)²⁵. When a commodity is expensive to store, it becomes more difficult to arbitrage price discrepancies across maturities. In fact, for the commodity futures in our sample, it is not uncommon for the prices of different contract months to move in opposite directions on a trading day. Hence, in contrast to low storage-cost commodities, a trader wishing to hedge a physical position corresponding to a particular month's contract in a high storage-cost commodity would likely retain significant risk if she tried to do so by taking a position in a different month's contract. Consequently, when limit hits occur in the livestock products that we examine, one would expect to see greater migration from the futures market to the options market than between futures contracts with different maturities²⁶.

²⁴ Specifying a functional form in which days until contract expiration has a non-linear form yields very similar estimates for the effect of the price limit on volume as those reported in Table 5.

²⁵ Livestock is generally considered the most difficult asset to store. For example, cattle held in storage might continue to grow such beyond the size stipulated in the futures specifications. Animals that are too young or too old have diminished value for consumption. These characteristics of livestock mean that inter-contract price differences tend to be more erratic than other commodities.

²⁶ Another possible reason for differing results is data related: whereas we have data on the number of contracts traded, Hall et al. (2006) only examine the effect of price limits on the number of trades made (i.e., they have no data on average trade size).

The overall conclusion from this analysis is that price limits do shift trading volume from futures to options. This result complements the price analysis above, in that both suggest limit hits can shift price discovery from the futures to the options market. Still, while the percentage changes in options trading due to limit hits are large, the absolute changes are insufficient to make up for the reduction in futures volume. For example, for live cattle, our estimates suggest that about 1/7 of the reduction in futures trading resulting from price limits was offset by increased options trading. Hence, it appears that price limits do raise the cost of establishing a position. This suggests that, while the options market allows for enhanced price discovery on limit hit days, its presence does not fully offset the effect of the limit on the futures market.

■ 6. Effects of Options on Intraday Futures Market Trading the Day after a Limit Hit

The above results, based on daily changes indicate, that options play an important price discovery role when price limits are hit on the futures market. In this section, we use intraday data to investigate whether the presence of options has an effect on the futures market once the price limit is no longer in effect.

Previous research suggests that options can affect the microstructure of the market for the underlying product by providing additional information. For example, Grossman (1988) shows that the price of a traded option can convey information that would otherwise be unobservable, thereby affecting trading on the underlying product. Empirical research has generally found that after the introduction of options, prices tend to reflect new information more quickly, and bid-ask spreads narrow, perhaps due to a reduction in adverse selection (see Mayhew, 2000, for a summary). Since price limits reduce available information, it seems plausible that these effects would be particularly large on days following limit hits. That is, price discovery is inhibited when a price limit is hit, and hence information asymmetry is likely to be greater on the following days. We would also expect limit hits to increase inventory management costs for futures contracts, which likewise leads to higher bid-ask spreads. Options might mitigate that effect, since options can allow traders to hedge their futures positions. It follows that if the presence of options reduces the informational asymmetry or allows greater hedging, then the presence of options would lead to lower spreads and volatility on the day following a limit hit.

6.1. Creating the Sample

In our data, futures options are traded for some, but not all of the days on which limit hits occur. The reason that some trading days had futures trading but no associated options trading is that CME contract specifications result in shorter time periods be-

tween the first and last trading day for options than for the underlying futures, especially for live cattle and pork bellies. For example, live cattle futures begin trading 14 to 15 months before contract expiration, while the options typically begin trading two months later. Similarly, the option contracts expire about two weeks before the associated futures contracts. In addition, there are some short-duration futures contracts that have no associated options. For these reasons, there were no options contracts traded for some of the days on which futures contracts experienced limit hits.

● **Table 6a. Microstructure of Market the Day After a Limit Hit: Live Cattle**

Panel i.

	Limit Hit Days (Futures-Only)	Limit Hit Days (Futures and Options)		
	N=20	N=104	Wilcoxon Z-test	Pr>Z
Standard deviation (volatility) of prices	0.1764	0.1548	1.0902	0.1378
Absolute value of the change in price	0.8445	0.4938	1.1719	0.1206
Spread	0.3375	0.3034	1.3631	0.0864

Panel ii.

	Down Limit Hit Days (Futures-Only)	Down Limit Hit Days (Futures and Options)		
	N=7	N=50	Wilcoxon Z-test	Pr>Z
Standard deviation (volatility) of prices	0.2058	0.1852	0.9404	0.1735
Absolute value of the change in price	1.448	0.571	1.5925	0.0556
Spread	0.457	0.395	1.0339	0.1506

Panel iii.

	Up Limit Hit Days (Futures-Only)	Up Limit Hit Days (Futures and Options)		
	N=13	N=54	Wilcoxon Z-test	Pr>Z
Standard deviation (volatility) of prices	0.1617	0.1249	0.9889	0.161
Absolute value of the change in price	0.519	0.422	0.4836	0.3143
Spread	0.273	0.219	1.183	0.1184

Panel iv.

	Non-Limit Hit Days (Futures-Only)	Non-Limit Hit Days (Futures and Options)		
	N=16	N=77	Wilcoxon Z-test	Pr>Z
Standard deviation (volatility) of prices	0.0895	0.0983	0.5127	0.3041
Absolute value of the change in price	0.323	0.336	0.5616	0.2872
Spread	0.153	0.161	0.0255	0.4898

● **Table 6b. Microstructure of Market the Day After a Limit Hit: Pork Bellies**

Panel i.

	Limit Hit Days (Futures-Only)	Limit Hit Days (Futures and Options)	Wilcoxon Z-test	Pr>Z
	N=26	N=200		
Standard deviation (volatility) of prices	0.3869	0.2424	3.108	0.0009
Absolute value of the change in price	1.135	1.0669	0.8283	0.2037
Spread	0.824	0.548	2.3957	0.0083

Panel ii.

	Down Limit Hit Days (Futures-Only)	Down Limit Hit Days (Futures and Options)	Wilcoxon Z-test	Pr>Z
	N=16	N=80		
Standard deviation (volatility) of prices	0.3806	0.2347	3.0996	0.001
Absolute value of the change in price	1.25	1.151	0.9585	0.1689
Spread	0.866	0.531	2.4651	0.0068

Panel iii.

	Up Limit Hit Days (Futures-Only)	Up Limit Hit Days (Futures and Options)	Wilcoxon Z-test	Pr>Z
	N=8	N=120		
Standard deviation (volatility) of prices	0.4012	0.2474	0.9479	0.1716
Absolute value of the change in price	0.905	1.011	-0.123	0.451
Spread	0.822	0.56	0.98	0.1635

Panel iv.

	Non-Limit Hit Days (Futures-Only)	Non-Limit Hit Days (Futures and Options)	Wilcoxon Z-test	Pr>Z
	N=21	N=163		
Standard deviation (volatility) of prices	0.2927	0.1966	1.0889	0.1381
Absolute value of the change in price	0.812	0.618	0.8319	0.2027
Spread	0.565	0.44	0.9931	0.1603

Tables 6a and 6b present results comparing the underlying market microstructure (bid ask spread, volatility etc) following a limit hit day when options are trading versus to when they are not trading. Panel i presents results for all limit days, panel ii presents results for down limit days, panel iii for up limit days and panel iv presents the results for the control group (non limit hit days, futures only and also futures and options).

As shown in Table 6a and 6b there were no futures options available on about 16% and 13% of the days with limit hits for live cattle and pork bellies, respectively²⁷. Unfortunately, because the first and last trading dates for options and futures typically were the same for lean hogs, there were very few limit hit days on which there were no options traded for lean hogs. Hence, our analysis of the effect of options on spreads and volume is limited to pork bellies and live cattle.

To analyze the effect of options on futures market microstructure following limit hits, we collected intraday data on bid, ask and trade prices for dates following limit hits. Some of these dates had tradable options on the limit hit days, and some did not²⁸. The database contains the price and time of each “transaction”, type of contract (futures or option), the maturity of each contract, as well as information on whether the transaction is a bid/ask order or an actual trade.

To control for other differences between the futures market on days tradable options were present and those when only futures were trading, we created a control group. This allows us to control for effects of time to maturity, seasonality etc. To construct a control group, we examine a day that is similar to each sample date, but did not have a limit hit. To select the non-limit-hit days, the following procedure was used: for each limit hit day, we determine which futures contract was affected by the limit. The control (non-limit hit) date is then determined by the condition that it has to be same date and for the same futures contract one year later. The only exception to this rule occurs if the one-year ahead date also has a limit hit. In this case we use the same date in previous year as our control day²⁹. As a result, each sample observation has a corresponding observation in the control group with the same time to maturity, seasonality and date of trading, albeit a year apart.

6.2. The Effect of Options on Futures Trading

The exogenous variation in the presence of options provides a natural experiment for evaluating the effect of options on price discovery. Specifically, we compare several measures of performance on days with and without tradable options for these two products. First, we examine price volatility within the first few minutes of trading. If options provide information about future movements in the futures price on limit hit days (as Table 3 suggests), then more information will be available on days following limit hits when options are traded than when they are not. Consequently, we would anticipate that the futures price will converge to its equilibrium level on the day following a limit hit more smoothly when options are present, as reflected in lower intraday volatility on the day following a limit hit. Second, we examine the bid-ask spread. By a similar logic, the availability of better information reduces adverse selection, and therefore should reduce the spread. As such, we would expect lower spreads in the presence of options. We also measure the price change between the previous day’s settlement and average price during the first five minutes of trading. We would expect these changes to be larger on limit hit

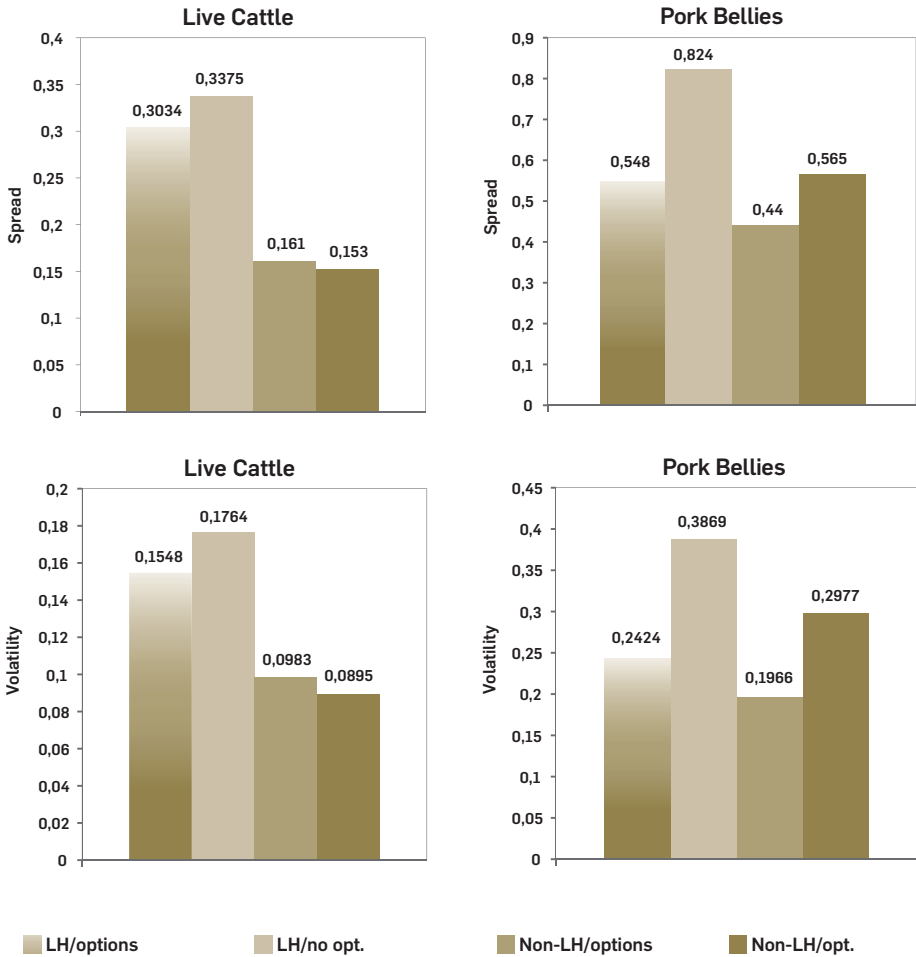
²⁷ In results not reporting here, we show that the continuation rate for following limit hit is quite similar whether or not options are traded. This suggests that the presence of options has no impact on the futures price that emerges once price limits are no longer in effect.

²⁸ The intraday transaction data were kindly provided to us by the CME.

²⁹ If both of these dates were limit hit days, we use the same date two years ahead.

days. A further question is whether average changes on limit hit days are different in the presence of options (e.g., if “overshooting” of the equilibrium price is more common absent options).

● **Figure 1. Comparisons on Spreads and Volatility for Live Cattle and Pork Bellies the Day after a Limit Hit (LH).**



³⁰ The following results are based on non consecutive limit hit days (i.e., we focus on the last limit day of what might be a series of limit hit days). However, the results are similar when consecutive limit hit days are included.

Our results are presented in Table 6a for live cattle and 6b for pork bellies, and some key statistics are presented graphically for both³⁰ in Figure 1. Panel i in the tables presents the results for days following all limit days, Panel ii presents the results for days following down limit days, Panel iii shows results for days following up limit days and Panel iv presents the results for the control (non limit hit days, futures only and then futures and options). For both products, up-limit days are somewhat more common than down-limit days. In each panel, we compare three variables across the two treatments: volatility, absolute price change and bid-ask spread. Volatility is measured as the standard deviation of transaction prices during the first five minutes of trading on the day following the limit hit or control day. The absolute change in price is the absolute difference between the settlement price on the limit hit (or control) day and the arithmetic average transaction price during the first five minutes on the following day. The spread is defined as the difference between the maximum ask price and minimum bid price during the first five minutes of trading on the day following the limit hit (or control) day³¹. We use the Wilcoxon Z-test to test the statistical difference between the variables, and hence do not have to make any distributional assumptions.

In general, the findings presented in Tables 6a and 6b suggest that, as expected, the standard deviation of prices (volatility) and bid-ask spreads are higher when futures are traded but options are not than is the case when both futures and options are traded³². These differences are statistically different for days following both overall limit hit days and down limit days for pork bellies, but not so for live cattle. For each commodity, the absolute price change following the limit hit day is larger for futures-only days than when both the futures and options are traded, so that large price changes are more common absent options.

Panel (i) in Tables 6a and 6b compares limit hit days with futures-only trading and those with both futures and options trading. For both products, we find a statistically smaller bid-ask spread in the presence of options on days following limit-hits. These differences are displayed visually in the upper two panels of Figure 1. Volatility is also lower following limit hits when options are trading (see Table 6b Panel (i) or bottom half of Figure 1). This effect is statistically significant for pork bellies; volatility falls from 0.3869 in their absence to 0.2424 when they are trading (p-value of 0.0009; see).

Panel (ii) in Tables 6a and 6b provides a comparison for down limit hits. As was the case for days following all limit hits, both futures price volatility and bid-ask spreads

³¹ A five-minute window was used to approximate contemporaneous prices. We recognize that in some cases, bids and asks observed during a five-minute window may not be economically contemporaneous. To reflect this possibility, we screen out cases in which this algorithm is clearly inappropriate (e.g., when the spread is negative).

³² The number of limit hit days of futures trading in Tables 6a and 6b is smaller than in Table 1 due to: 1) In Tables 6a and 6b, the analysis is done using only non-consecutive limit hit days as opposed to all limit hit days; 2) We include only cases where we have positive bid-ask spreads on the day following the limit hit, i.e., if the spread is missing or negative, we exclude the case from our analysis.

are lower in the presence of options. These effects are larger and of greater statistical significance for pork bellies than live cattle. For up limit hit days, Panel (iii) of Table 6a and 6b shows qualitatively similar results to the down limit hit days. For example, volatility is lower in the presence of options trading for both live cattle and pork bellies, albeit not statistically lower.

Panel (iv) of Tables 6a and 6b (see the right two columns in Figure 1) present our results for the control group (days not preceded by limit hits). In this case, for live cattle (Table 6a) we find that, although it is not statistically different, the volatility is actually higher in the presence of options than without options trading. This is the opposite of what we observed on limit hit days. Similarly, for the control days, the spread for live cattle is higher in the presence of options trading. For pork bellies we find somewhat similar results to that found on limit days. For example volatility and the spread are lower in the presence of options, although unlike limit hit days, they are not statistically significantly so on the control days. These results for pork bellies suggest, however, that we cannot ascribe the entire difference between futures-only and futures and options on days following limit hits solely to the effect of options.

6.3. Limit-Hits vs. Control Days

To analyze the effect of options trading on days following limit hits, in Tables 7a and 7b, we directly compare these changes in trading cost and volatility on days following limit hits to those same measures on control days. The first three variables (rows) of these tables display similar information to that presented in Tables 6a and 6b (but we make a different comparison). As shown there, all three measures are statistically significantly higher on days following limit hits (as expected) than on control days: Limit hits are followed by larger price changes, more volatile price movements, and larger bid-ask spreads than the control days.

● **Table 7a. Direct Comparisons of Limit vs. Non Limit Hit Days: Live Cattle**

	Limit Hit Days (Futures-Only)	Non-Limit Hit Days (Futures and Options)	Wilcoxon Z-test	Pr>Z
	N=104	N=77		
Standard deviation (volatility) of prices	0.1548	0.0983	5.2982	0.0001
Absolute value of the change in price	0.4938	0.336	-3.8418	0.0001
Spread	0.3034	0.161	-3.2561	0.0006
Standard deviation (volatility) difference	-0.022	0.0088	3.9131	0.0001
Absolute Price Change Difference	-0.351	0.013	7.1464	0.0001
Spread Difference	-0.034	0.0079	3.9714	0.0001

● **Table 7b. Direct Comparisons of Limit vs. Non Limit Hit Days: Pork Bellies**

	Limit Hit Days (Futures-Only)	Non-Limit Hit Days (Futures and Options)		
	N=200	N=163	Wilcoxon Z-test	Pr>Z
Standard deviation (volatility) of prices	0.2424	0.1966	-3.7210	0.0001
Absolute value of the change in price	1.0669	0.618	-5.8443	0.0001
Spread	0.548	0.440	-2.4928	0.0063
Standard deviation (volatility) difference	-0.144	-0.096	4.7083	0.0001
Absolute Price Change Difference	-0.068	-0.194	-0.2888	0.3864
Spread Difference	-0.276	-0.125	5.1621	0.0001

Tables 7a and 7b directly compare the changes in trading cost and volatility on limit days to the control days. The first three rows are repeated from tables 6a and 6b but direct comparisons are made. The last three rows assess the impact of options on limit days. To this end we define a variable θ_{Gmi} equal to the difference between the i^{th} observation on measure m (where m = volatility, absolute price change or spread) when options are traded on a day of type G (where G is either the day following a limit hit or a control day) and the mean value of measure m for group G on days where options are not traded. E.g., θ_{LVI} is equal to the 1st observation on volatility for limit hit days when options are traded minus the mean value of volatility for limit hit days when options are not traded. The Wilcoxon test statistic evaluates whether $\theta_{Lmi} = \theta_{Cmi}$ for each m .

The primary question of interest is whether the presence of options reduces the effect of limit hits on the next day's trading costs, as measured by bid-ask spread and volatility. To examine this, we compare the effects of options on limit hit days to their effects on the control days in the next three rows in a type of difference-in-difference experiment. To make this comparison, we constructed a variable θ_{Gmi} . The variable is equal to the difference between the i^{th} observation on measure m (e.g., spread) when options are traded on a day of type G (where G is either a day following a limit hit or a control day) and the mean value of measure m for group G on days where options are not traded. For example, θ_{LVI} is equal to the 1st observation on volatility for days following limit hits when options are traded, minus the mean value of volatility when options are not traded for days following limit hit days. We then compare the distribution of θ_{LVi} to the distribution of θ_{CVi} using a Wilcoxon test for equality. We do this test for the 3 measures; volatility, spread, and absolute price change.

For both products, the effect of options on both volatility and spread is statistically significantly larger on days following limit hits (that is, both measures fall more with options), with a p-value of $< .0001$. For example, the pork bellies mean spread is \$0.548 for limit hit days (futures and options) while it is \$0.824 on futures-only limit hit days, see Table 6b, Panel (i). This indicates that spreads are \$.276 (about 1/3) lower when options are present on days following limit hits. For the control group, the mean spread is \$0.44 in the presence of options trading while it is \$0.565, see Table 6b, Panel (iv), on futures-only days, so that spreads are on average \$0.125 (about 22%) lower when options are present on control days. This indicates that the reduction in next-day spreads associated with the presence of options is \$0.151 larger

when price limits are in effect. A statistical test of the proposition that the difference in differences is larger when limit hits occur is presented in Table 7. The Wilcoxon test statistic of 5.16 indicates that the reduction in spreads associated with options is statistically significantly larger when price limits are in effect. Similarly, the reduction in volatility for pork bellies is .144 (37%) when options are available on limit hit day and .096 (33%) when options are available on the control days. Similar to the difference in the reduction in spreads, the reduction in volatility is statistically significantly larger on limit hit days (Wilcoxon *Z*-test statistics of 4.71). The results for spreads and volatility provide further evidence that options are particularly important in providing price discovery when limits are hit. Finally, the effect of options on the absolute price change is larger on days following limit hits, although it is only statistically significant for live cattle. This is consistent with “overshooting” following limit hits, especially when options are not present.

■ 7. Conclusion

We show that the primary effect of price limits in the futures markets we examine is to change the means by which information becomes incorporated into futures prices, rather than affecting futures price determination. That is, the evidence suggests that when price limits are binding in a futures market, reliable information about changes in futures prices is contained in the prices of the associated options. This is reflected both in the power of deviations from put-call parity in predicting futures price changes, and in the migration of trading volume from the futures market to the options market on limit hit days. Further, we exploit variation in the presence of options on these futures contracts to show that when options trading is available, options trading results in reduced spreads and lower price volatility on days following limit hits. This finding provides further evidence that the options market effectively replaces the futures market as a trading venue when price limits become binding in the futures market.

From the standpoint of positive economics, one research question is evaluating how traders react to a binding price limit. For the high storage-cost products analyzed here, the evidence is consistent with our hypothesis that price discovery would move to the associated options market. For other exchange-traded products, the price discovery market may be something other than the options market (e.g., other contract months, as Hall et al. found for low storage-cost commodities, or foreign markets as Beekman and Steenbeek found for equities). An avenue for further research might examine whether there is a relationship between storage costs and the size of the options market, and in particular on whether the trading shifts that occur when price limits become binding on a futures market can be predicted by the storage costs (or other economic characteristics) of the product.

From a policy perspective, the key question is why certain exchanges choose to set price limits. Existing models of why exchanges might choose to impose price limits are premised on the limits actually restricting futures price movements. The evidence, however, suggest that the limits have little effect on either price formation or available information in futures markets. Without an understanding of why exchanges choose to impose price limits, it is difficult to justify regulatory expansion of restrictions on price movements on financial exchanges.

■ References

- Barone-Adesi, G. and Whaley, R. (1987). Efficient Analytic Approximation of American Option Values, *Journal of Finance*, **42**, pp. 301-20.
- Berkman, H. and Steenbeek, O.W. (1998). Daily Price Limits and Trading in Nikkei Futures, *Journal of Futures Markets*, **18**, pp. 265-279.
- Brennan, M. (1986). A Theory of Price Limits in Futures Markets, *Journal of Financial Economics*, **16**, pp. 213-33.
- Chen, H. (1998). Price Limits, Overreaction, and Price Resolution in Futures Markets, *Journal of Futures Markets*, **18**, pp. 243-63.
- Christie W., Corwin, S. and Harris, J. (2002). Nasdaq Trading Halts: The Impact of Market Mechansims on Price, Trading Activity, and Execution Costs, *Journal of Finance*, **57**, pp. 1443-1478.
- Chou, P-H., Lin, M-C. and Yu, M-T. (2005). Risk Aversion and Price Limits in Futures Markets, *Finance Research Letters*, **2**, pp. 173-184.
- Chowdry, B and V. Nanda (1998). Leverage and Market Stability: The Role of Margin Rules and Default Risk, *Journal of Business*, **71**, pp. 179-210.
- Egelkraut, T., García, P. and Sherrick, B. (2007). Options-Based Forecasts of Futures Prices in the Presence of Limits Moves, *Applied Economics*, **39**, pp. 145-152.
- Evans, J. and Mahoney, J. (1996). The Effect of Daily Price Limits on Cotton Futures and Options Trading. WP. New York: Federal Reserve Bank of New York.
- Frech, H.E. III and Lee, W. C. (1987). The Welfare Cost of Rationing by Queuing across Markets: Theory and Estimates from the U.S. Gasoline Crises, *Quarterly Journal of Economics*, **102**, pp. 97-108.
- Glaeser, E. and Luttmer, E. F. (2003). The Misallocation of Housing under Rent Control, *American Economic Review*, **93**, pp. 1027-1046.
- Glosten, L. (1987). Components of the Bid-Ask Spread, and the Statistical Properties of Transaction Prices, *The Journal of Finance*, **42**, pp. 1293-1307.

- Grossman, S. (1988). An Analysis of the Implications for Stock and Futures Price Volatility of Program Trading and Dynamic Hedging Strategies, *Journal of Business*, **61**, pp. 275-298.
- Hall, A., Kofman, P. and Manaster, S. (2006). Migration of Price Discovery in Semiregulated Derivatives Markets, *The Journal of Futures Markets*, **26**, pp. 209-241.
- Kim, K. and Rhee, S. (1997). Price Limit Performance: Evidence from the Tokyo Stock Exchange, *The Journal of Finance*, **52**, pp. 885-899.
- Kim, Y. and Yang, J. (2004). What Makes Circuit Breakers Attractive to Financial Markets: A Survey Financial Markets, *Institutions and Instruments*, **13**, pp. 109-146.
- Kolb, R.W. and Overdahl, J.A. (2006). *Understanding Futures Markets*, Blackwell Publishing, New York.
- Kodres, L. and O'Brien, D. (1994). The Existence of Pareto-Superior Price Limits, *American Economic Review*, **84**, pp. 919-933.
- Kuserk, G. and Locke, P. (1996). Market Making With Price Limits, *Journal of Futures Markets*, **16**, pp. 677-696.
- Lo, A. and Wang, J. (2002). Trading Volume: Definitions, Data Analysis and Implications of Portfolio Theory, *Review of Financial Studies*, **13**, pp. 257-300.
- Ma, C., Rao, R. and Sears, R. (1989). Volatility, Price Resolution, and the Effectiveness of Price Limits, *Journal of Financial Services Research*, **3**, pp. 165-199.
- Martell, T. and Wolf, A. (1987). Determinants of Trading Volume in Futures Markets, *Journal of Futures Markets*, **7**, pp. 233-244.
- Martin, P. and Overdahl, J. (1994). The Exercise of Equity Options: Theory and Empirical Evidence, *Journal of Derivatives*, **2**, pp. 38-51.
- Mayhew, S. (2000). The Impact of Derivatives on Cash Markets: What Have We Learned?, Working paper, U.S. Securities and Exchange Commission.
- Newey, W and West, K. (1987). A Simple Positive Semi-Definite, Heteroskeasticity and Autocorrelation Consistent Covariance Matrix, *Econometrica*, **51**, pp. 76-89.
- Park, C. (2000). Examining Futures Price Changes and Volatility on Trading Days After a Limit-Lock Day, *The Journal of Futures Markets*, **20**, pp. 445-466.
- Phylaktis, K., Kavusannos, M. and Manalis, G. (1999). Price Limit and Stock Market Volatility in the Athens Stock Exchange, *European Financial Management*, **5**, pp. 69-84.
- Ramaswamy, K. and Sundaresan, S.M. (1985). The Valuation of Options on Futures Contracts. *Journal of Finance*, **40**, pp. 33-59.
- Shanker, L. and Balakrishnan, N. (2005). Optimal Clearing Margin, Capital and Price Limits for Futures Clearinghouses, *Journal of Banking and Finance*, **29**, pp. 1611-1630.
- Subrahmanyam, A. (1994). Circuit Breakers and Market Volatility: A Theoretical Perspective, *Journal of Finance*, **49**, pp. 237-254.
- Veld-Merkoulova, Y. (2003). Price Limits in Futures Markets: Effects on the Price Discovery Process and Volatility, *International Review of Financial Analysis*, **12**, pp. 311-328.

