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# Income inequality and savings: a reassessment of the relationship in cointegrated panels

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# Income inequality and savings: a reassessment of the relationship in cointegrated panels\*

## Abstract

The effect of income on savings and consumption has remained an open empirical issue despite several decades of research. Results obtained in this study indicate that income inequality and private consumption are both  $I(1)$  nonstationary variables that are cointegrated, and inequality has had a negative effect on private consumption in Central-European and Nordic countries. Results for Anglo-Saxon countries are inconclusive. These findings suggest that previous empirical research may have produced biased results on the effect of inequality on savings by assuming that inequality would be a stationary variable.

**JEL Classification:** E21, C22, C23

**Keywords:** Panel cointegration, top 1% income share, private consumption, gross savings.

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# 1 Introduction

The effect of savings on capital accumulation and growth has always been one of the fundamental research topics in economics. According to Smith (1776), increased division of labor raises productivity, but savings govern capital accumulation, which enables production growth. In the 18th century, only rich people saved. Therefore, economic growth was possible only when there were enough rich people in the society. However, according to Keynes (1936), inequality of income would slow down economic growth. Keynes argued that marginal consumption decreases as the income of an individual increases, and thus, aggregate consumption depends on changes in aggregate income. Because demand is the basis of investments, and because inequality lowers aggregate consumption, inequality of income would diminish economic growth. In neo-classical growth models, income distribution determines the level savings and thus the level of capital accumulation (Solow 1956; Kaldor 1957).

In addition to the approach of classical economics, there exist several theories describing the effect of income inequality on savings and consumption. These include the permanent income hypothesis by Friedman (1957), life-cycle hypothesis by Ando and Modigliani (1963), which was augmented with intergenerational transfers by Kotlikoff and Summers (1981), savings under liquidity constraints by Deaton (1991), and political-economy models (e.g. Alesina and Perotti (1994)).

Although theoretical research spans several decades, the effect of income inequality on savings remains an open empirical issue. This is due to the fact that empirical cross-country studies have produced controversial results on the effect of income inequality on savings. In one of the most recent panel econometric studies, Leigh and Posso (2009) estimate the effect of the income share of the top 1% on the percentage value of gross savings of the GDP and find no statistically significant effect of inequality on national savings. Similarly, Schmidt-Hebbel and Servén (2000) find no statistically

significant effect of the percentage value of savings of the GDP using several different measures of income inequality. However, Smith (2001) finds that inequality, measured with Deininger and Squire's (1996) Gini index, has a statistically robust positive effect on the percentage value of savings of the GDP. Cook (1995) finds the same effect in less developed economies. Li and Zou (2004) find that inequality has a negative effect on private savings using Deininger and Squire's (1996) Gini index and the ratio of private savings of GDP.

All the empirical studies summarized above have assumed that income inequality, measured either by the Gini index or by the share of income earned by different income classes, is a stationary variable. However, in the early theoretical literature on income inequality, the income variation was assumed to be driven by a stochastic process (Chambernowne 1953; Mandelbrot 1961). Moreover, Mandelbrot (1961) argued that time-independent, i.e. stationary, income variations are unlikely, and it is possible that the distribution of income will never reach a steady state implying a nonstationary process of income variation. In a recent study with cross-country panel data, Malinen (2011) and Herzer and Vollmer (2011) have obtained results according to which the data generating process of income inequality would be driven by a stochastic trend, indicating that inequality would be an  $I(1)$  nonstationary variable. Previously, for example, Mocan (1999) has obtained similar results.<sup>1</sup> If this assumption of the early theoretical models held in general, it would offer an explanation to the controversy in the previous empirical studies. This is because regressing a stationary variable on an  $I(1)$  variable(s) can lead to a spurious regression (Stewart 2011). In empirical studies, savings is usually measured as a percentage of the GDP. If both logarithmic savings and the logarithmic GDP are  $I(1)$  variables and cointegrated, their difference results, by construction, in a stationary variable, namely savings as a percentage of the GDP. Thus, if inequality were an  $I(1)$  variable and savings as a ratio of the GDP a stationary

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<sup>1</sup>See also Jäntti and Jenkins (2010).

$I(0)$  variable, regressing savings on inequality would give spurious results.

This study uses panel cointegration methods to test the time series properties of the included variables and to estimate the (possible) long-run relation between income inequality and savings. We use data on nine developed economies, and spanning four decades starting from the year 1960. The income share of the top 1%, used to proxy the distribution of income, has been found to track broader measures of income inequality, like the Gini index, very well (Leigh 2007). According to panel unit root tests, the logarithmic income share of the top 1%, logarithmic gross national savings and logarithmic private consumption are all  $I(1)$  nonstationary variables. Income share of the top 1% is also found to be cointegrated with private consumption, which implies that there is a long-run dependency relation between them. The effect of inequality on private consumption is found to be negative in the Nordic and Central-European countries, but for the Anglo-Saxon countries the direction of the effect (positive vs. negative) remains somewhat ambiguous. The results of the panel cointegration tests are inconclusive on possible cointegration between gross savings and the top 1% income share. The real GDP per capita and gross savings as well as the real GDP per capita and private consumption are also found to be cointegrated. This implies that the ratios of savings and private consumption to GDP would be stationary variables and hence previous research is likely to have produced biased results on the effect of inequality on savings and consumption.

The rest of the paper is organized as follows. Section 2 gives the theoretical and empirical background of the study. Section 3 describes the data and presents the results of unit root tests. Section 4 reports the results of cointegration tests and section 5 gives the estimation results. Section 6 concludes.

## 2 Theoretical and empirical considerations

Several theories have been constructed to explain the effect of inequality on savings. In classical economic theory, the form of the individual saving function determines the effect of income inequality on savings. When the saving function is linear or concave, the distribution of income and wealth converge toward equality as the economy grows (Stiglitz 1969). If the saving function is convex, the marginal propensity to save increases with income. According to the permanent income hypothesis, individuals with low income have higher propensity to consume, and small changes in income, or its distribution, do not affect the consumption decisions of households (Friedman 1957). The life-cycle hypothesis argues that, if bequests are luxury, the saving rate should be higher among wealthier individuals (Kotlikoff and Summers 1981). In political-economy models, more unequal income distribution may create demand for more redistribution through taxation and income transfers, and if the saving function of individuals in the economy is convex, i.e. the rich save more, this will diminish aggregate savings through diminished incomes of the rich (Alesina and Perotti 1994).

What is common to all of the theories introduced above is that they generally assume that the individual income process is non-stochastic. However, as pointed out by Stiglitz (1969), the very first (formal) models of income inequality by Chambernowne (1953) and Mandelbrot (1961) were based on stochastic processes. Chambernowne (1953) developed a model assuming that the evolution of income of an individual is determined by his/her income in the previous year and by a stochastic (chance) process. In modern terms, this process would be said to be  $I(1)$  non-stationary.

$I(1)$  non-stationary processes have a infinite memory, i.e., they are highly persistent. Assuming some degree of persistence in the evolution of the income series of an individual is quite intuitive as shocks (e.g., wage raise) to the income process of an individual are likely to have permanent effects on the future incomes of the individual.

Microeconomic literature on household income and consumption behavior adopted the idea of permanent effects affecting the income series of an individual. For instance, Hall and Mishkin (1982) considered a stochastic model of consumption proposed by Muth (1960), where the effect of individual income on consumption was divided into permanent and transitory components. In a recent study on the evolution of consumption and income inequality, Blundell *et al.* (2008) use the same kind of formulation where the income of households varies according to the following function:

$$\log Y_{it} = Z_{it}\vartheta_t + P_{it} + v_{it}, \quad (1)$$

where  $Z_{it}$  is a set of income characteristics of household  $i$  that are observable and known by consumers at time  $t$ ,<sup>2</sup>  $v_{it}$  follows an  $MA(q)$  process, and  $P_{i,t} = P_{i,t-1} + \epsilon_{it}$  with  $\epsilon_{it}$  serially uncorrelated, indicating that the process  $\{P\}$  is  $I(1)$  non-stationary. Several studies in the micro literature tend to find that also empirically the permanent component  $P_{it}$  is a random walk, and hence it can be modeled as an  $I(1)$  nonstationary process (Meghir and Pistaferri 2004; Hall and Mishkin 1982; Blundell *et al.* 2008).

Deaton (1991) studied how liquidity constraints affect national savings, when incomes are driven by a random walk with drift.<sup>3</sup> He argued that the assumption of optimal intertemporal consumption behavior of consumers being restricted by borrowing constraints would help to create a model that could explain the observed patterns of household wealth and the dependency of consumption on income during the life cycle of an individual. According to Deaton, the problem with stochastic life cycle - permanent income models, like the model by Hall and Mishkin (1982), is that they assume substantial wealth accumulation at some point of the life cycle of an individual, which was not supported by the data. Deaton assumed that the labor income of an individual

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<sup>2</sup>These include demographic, education, employment status, ethnic, etc. factors (Blundell *et al.* 2008)

<sup>3</sup>Assuming a drift in income relation is intuitive and necessary, because income tends to grow over time. Thus, just variations around that trend are assumed to be stochastic.



follows an  $AR(1)$  process of the form:

$$\log(y_{t+1}) = \log(y_t) + \log(z_{t+1}) + \delta, \quad (2)$$

where  $y_t$  is labor income,  $z_{t+1}$  is stochastic random variable, and  $\delta > 0$  is a constant. When  $z_{t+1}$  is assumed to be identically and independently distributed, the labor income process,  $\log(y_t)$ , is  $I(1)$  non-stationary and, specifically, it follows a random walk with drift. Deaton found that, when income is a random walk and borrowing constraints are binding, it is undesirable for households to undertake any smoothing of consumption, i.e., they have no incentives to save. This implies that consumption equals income on all income levels. However, he assumed that the interest rate is higher than the consumer's discount rate. If the interest rate equals the consumer's discount rate, the stochastic income process and borrowing constraints lead to the result that the propensity to consume is lower at higher income levels (Seater 1997; Travaglini 2008). In this day and age, the debate on the validity of the life cycle - permanent income hypothesis is still very much ongoing. In a recent paper, Attanasio and Weber (2010) review the literature on intertemporal allocation models and present some modifications to the standard life cycle hypothesis framework to make the model fit the micro data better.

When individual income series are affected by a random walk component, the aggregated time series is likely to be characterized by a random walk (Rossanan and Seater 1995). This applies to the *aggregate* income, but the distribution of income is often measured using some bounded measure, like the Gini index or share of total income. This changes things a bit as any measure that varies within some boundaries like the income share, cannot, by definition, be an  $I(1)$  nonstationary process. This is because the variance of such a series cannot grow to infinity with time which is what happens with random walk series. However, it is possible that the distribution can have a stochastic trend in its other moments, like the mean, skewness, and kurtosis, than variance (White and Granger 2010). Thus, when individual income series that are

affected by a random walk component are aggregated to a bounded distribution, it is likely that this distribution has a stochastic trend in its kurtosis and/or in its skewness. This way the measure of income inequality, being a functional of some income distribution, may exhibit such high levels of persistence that it is better approximated by an  $I(1)$  process than a stationary process.

Thus, the possibility that income inequality is driven by a stochastic trend needs to be taken into account in the analysis of the relationship between inequality and savings. Especially in empirical analysis, this is quite crucial, as the possible  $I(1)$  non-stationarity of the included variables determines the way this relationship can be consistently estimated (see the Introduction). That is why we next turn to testing the time series properties of relevant variables.

### **3 Data and unit root tests**

#### **3.1 Data**

In this study we use the the top 1% income share of population to proxy the income distribution in different countries. Since the work of Piketty (2003), there has been a growing interest towards building long time series of the evolution of top income shares of the population. Measuring the developments in top income shares makes it possible to construct substantially longer time series from the evolution of the distribution of income than would be possible using the Gini index or similar aggregate measures. Top income series are built using national tax data and applying the same method across countries to make the series comparable (Atkinson and Piketty 2007). Leigh (2007) has also demonstrated that the top 1% income share series have a high correlation with other measures of income inequality, like the Gini index. The dataset on the top 1% income shares gathered by Leigh is a primary source of data in this study.

However, after Leigh (2007) there have been additions to the pool of countries for which a historical dataset of the evolution of top incomes is available. Roine and

Waldenström (2011) have used data collected from several different sources in their analysis of common trends and shocks in top income series. Their dataset is extensive, but it, like the dataset by Leigh (2007), has one caveat. Due to the length of some series (starting from the beginning of the 20 century) the observations from all countries are not continuous, and thus they are forced to extrapolate over some observations. This is problematic, because extrapolation of over just one observation may alter the time series properties of the observed variable. From the dataset of Leigh we know that the series of New Zealand has observations missing in 1961, 1974, and 1976. Thus, New Zealand is not included in the datasets that consists of those periods. For the other countries neither Leigh nor Roine and Waldenström explicitly list the observations over which they are extrapolating, and thus we have to check the original data sources to find out about the location of the missing observations. After observing the individual sources of data, we include nine countries in our baseline dataset. These countries are: Australia, Canada, Finland, France, the Netherlands, Norway, Sweden, Switzerland, the United Kingdom and the United States.<sup>4</sup> All these countries have continuous observations from the year 1960 onwards, except Switzerland from which we have only bi-annual average observations (Dell *et al.* 2010). That is, for example, in 1995 and in 1996 the value corresponds to the average share of the top 1% income during 1995-1996. Thus, we do not include Switzerland on any of the unit root tests, but observations from Switzerland are included in the cointegration testing and estimations. Because of the restrictions imposed by some of our testing methods explained later, the nine countries are divided into three groups of countries according to their (assumed) economic models. The groups are: Nordic, Central-European, and Anglo-

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<sup>4</sup>The problematic countries in this dataset are Finland and the Netherlands. In Finland's case the original data, allegedly spanning from 1920 to 2005, could not be checked, but we could check that the data is continuous at least from 1966 onwards. For the Netherlands there are direct observations only from the year 1977 onward and significant part of the previous observations are a result of interpolation and pure estimation. However, panel unit root tests are done also without Nordic countries and the Netherlands, and because extrapolation would have the biggest impact on those tests, the bias created by possible extrapolation over some values of top 1% series in Finland and the Netherlands is diminished.

Saxon countries.<sup>5</sup> In addition, Japan and New Zealand are included in the testing for the time series properties of the top 1% income share data.

The endogenous variables used in this study are the gross savings and the private consumption expenditures. The data on these is obtained from the AMECO database compiled and published by the Directorate-General for Economic and Financial Affairs of the European Commission. AMECO is used, because it has more extensive time series coverage on the variables in question than, for example, the dataset of World Bank. AMECO does not include data on private savings, but because private consumption is the mirror image of private savings, it should not make any difference which of these two variables is used. Both variables are measured in aggregate terms to minimize the effects that a third variable might have on the relation between inequality and savings. Changes, for example, in fertility may have an effect on variables that are measured in per capita terms without affecting the income share of the top 1%. Thus, expressing gross savings and private consumption in per capita terms could add stochastic elements to the time series of those variables unrelated to their relation with income inequality.

The other variables included in the estimation are the real gross domestic product per capita, the dependency rate, and the interest rate. The real GDP per capita is historically thought to proxy the expected lifetime wealth of the residents in a country (Cook 1995). The level of national income may also have a direct effect on gross savings and consumption. The dependency rate is used to control for possible changes in the saving patterns across the life cycle of individuals, and it measures the ratio of the population under 15 to that over 64 years of age. Interest rates may affect the individual propensity to save by affecting the profitability of saving. The year 1960 is the first year included in the dataset but the last year varies from one country to another. The data on depen-

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<sup>5</sup>Canada, United Kingdom and United States are included in the Anglo-Saxon group. France, Netherlands, and Switzerland are included in the Central-European group, and Finland, Norway, and Sweden are included in the group of Nordic countries.

dependency is obtained from the World Development Indicators of the World Bank and the data on interest rates is from the database of the International Monetary Fund.

The data on interest rates varies a little across countries in the dataset. The baseline rate is the discount or the bank rate, i.e. the rate at which central banks lend to deposit banks. However, for some countries, observations of this variable are not available for the whole time period. That is why there are some differences in the indicators of interest rates between the groups of countries. For Finland, Sweden, Norway, and Switzerland, the central bank rate is used. For Canada, the United Kingdom, and the USA, the treasury bill rate is used, which gives the rate at which short-term securities are issued or traded in the market. For France and the Netherlands, the money market rate is used, which gives the rate on short term lending between financial institutions. Although using different indicators of interest rates is not optimal, all these indicators should reflect changes in the interest rate at which consumers borrow from and deposit money to banks.

### 3.2 Unit root tests

To test for possible unit roots, four different panel unit root tests are used. The first two are the so called traditional and the last two the so called second generation panel unit root tests. Traditional panel unit root tests do not allow for cross-sectional dependency while the second generation tests allow for cross-sectional correlation.

The traditional panel unit root tests, by Im *et al.* (2003) and the panel version of the ADF test by Maddala and Wu (1999), are based on the following regression:

$$\Delta y_{it} = \rho_i y_{i,t-1} + \eta_{it} + \alpha_i + \theta_t + \epsilon_{it}, \quad (3)$$

where  $\alpha_i$  are individual constants,  $\eta_{it}$  are individual time trends, and  $\theta_t$  are the common time effects. The tests rely on the assumption that  $E[\epsilon_{it}\epsilon_{js}] = 0 \forall t, s$  and  $i \neq j$ , which is required for calculating common time effects. Thus, if the different series are

correlated, the last assumption is violated.

The second generation tests by Pesaran (2007) and Phillips and Sul (2003) are based on the regression

$$\Delta y_{it} = \rho y_{i,t-1} + \eta_{it} + \alpha_i + \delta_i \theta_t + \epsilon_{it}, \quad (4)$$

where  $\alpha_i$ s are the individual constants,  $\eta_{it}$  are the individual time trends, and  $\theta_t$  is the common time effect, whose coefficients,  $\delta_i$ , are assumed to be non-stochastic, measure the impact of the common time effects of series  $i$ , and  $\epsilon_{it}$  is assumed to be normally distributed with mean zero and covariance of  $\sigma^2$  and independent of  $\epsilon_{js}$  and  $\theta_s$  for all  $i \neq j$  and  $s, t$ . Cross-sectional dependence is allowed through the common time effect, which generates the correlation between cross-sectional units. The matrix  $\delta_i$  gives the non-random factor loading coefficients that determine the extent of the cross-sectional correlation. The null hypothesis in all tests is that  $\rho_i = 0 \forall i$ . The alternative hypotheses are:

$$H_1 : \rho_i < 0, \quad i = 1, 2, \dots, N_1, \quad \rho_i = 0, \quad i = N_1 + 1, N_1 + 2, \dots, N. \quad (5)$$

For consistency of panel unit root tests it is also required that, under the alternative, the fraction of the individual processes that are stationary is non-zero, formally  $\lim_{N \rightarrow \infty} (N_1/N) = \gamma, 0 < \gamma \leq 1$  (Im *et al.* 2003).

Table 1 presents the results of the four panel unit root tests for the top 1% income share. The tests have been applied to three different datasets. The first includes the data on five countries with the longest continuous time series in the dataset by Leigh (2007).<sup>6</sup> The second dataset includes the eight countries on which we have data for all the included variables excluding Switzerland. The third dataset includes observations from seven countries excluding the Nordic countries. This dataset spans from 1983 to

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<sup>6</sup>Countries included in the test are: Australia, Canada, France, Japan, and the USA. All these countries should have continuous observations throughout the period.

2002.<sup>7</sup> The third dataset is used, because, according to Roine and Waldenström (2011), there is a trend break in the series of the top 1% income share in 1991 in the Nordic countries. According to Roine and Waldenström (2011), there are similar trend breaks in the top 1% income share series in the Anglo-Saxon countries, including Australia, Canada, UK and USA (in 1982), in Central European countries, including France, Switzerland, and Netherlands (in 1976), and in Asian countries, including Japan (in 1983). So, to make sure that the results of the panel unit root tests are not driven by structural breaks, the tests are run using only those countries that should not have breaks in their top 1% income share series in the test period. According to all tests in

Table 1: Panel unit root tests of the top 1% income share

variable	period	IPS	ADF	Pesaran	PS
top 1%	1925-1998	3.449 (0.998)	2.270 (0.994)	-0.103 (0.459)	4.200 (0.838)
top 1%	1960-1996	5.256 (0.999)	1.282 (0.998)	-0.348 (0.364)	3.212 (0.976)
top 1%	1983-2002	-0.302 (0.381)	16.670 (0.273)	-0.198 (0.421)	16.636 (0.187)

The tested equation is:  $\Delta y_{it} = \rho_i y_{i,t-1} + \delta_i + \eta_{it} + \theta_i + \epsilon_{it}$ . All variables are tested in logarithms. P-values of the test statistics appear in parentheses. With the exception of the PS test, lag length was determined using Schwarts information criterion (SIC). In PS test the lag length is selected with top-down method. The dataset of 1925-1998 includes 6 countries and 444 observations, the dataset of 1960-1996 includes 9 countries and 333 observations, and the dataset of 1983-2002 includes 6 countries and 121 observations.

all datasets, the logarithmic top 1% income share is a  $I(1)$  nonstationary process. This holds even in the last test, where the Nordic countries are excluded from the test to control for the possible structural breaks present in their top 1% income share series.

Table 2 presents the results of panel unit root tests of the other variables. Because gross savings is expressed in constant Euros and private consumption in purchasing power parities (ppp), two different series of real GDP per capita series are included in the dataset. According to all tests, gross savings, private consumption, the interest rates, and both versions of the real GDP per capita are  $I(1)$  processes. With the ex-

<sup>7</sup>Countries included in the test are: Australia, France, Japan, New Zealand, the United Kingdom, and the USA.

Table 2: Panel unit root tests of the other included variables, 1960-1996

variable	IPS	ADF	Pesaran	PS
gross savings	2.207 (0.986)	2.144 (0.999)	-0.186 (0.426)	4.79 (1.000)
consumption	12.532 (1.000)	0.0645 (1.000)	0.221 (0.587)	7.53 (0.960)
GDP, ppp	11.006 (1.000)	14.039 (1.000)	0.347 (0.636)	5.331 (0.994)
GDP, eur	3.912 (1.000)	19.453 (0.364)	2.420 (0.992)	11.210 (0.796)
dependency	-5.077 (<.001)	82.382 (<.001)	1.005 (0.843)	50.92 (<.001)
interest rate	0.717 (0.764)	13.937 (0.731)	-1.144 (0.126)	19.97 (0.220)

The tested equation is:  $\Delta y_{it} = \rho_i y_{i,t-1} + \delta_i + \eta_i t + \theta_t + \epsilon_{it}$ . All variables are in logarithms. Probabilities of the test statistics appear in parentheses. In all other tests, except in the PS test, lag lengths were determined using Schwarts information criterion (SIC). Testing includes 9 countries and 333 observations.

ception of the test by Pesaran (2007), all tests clearly reject the null hypothesis of the nonstationarity of the dependency rate, indicating that it is a trend-stationary variable.

## 4 Cointegration tests

### 4.1 Testing with the whole data

The methods for testing for cointegration in panel data have developed very rapidly during the first decade of the 21st century. One of the most commonly used cointegration testing methods has been the residual based panel cointegration test by Pedroni (2004). The limitation of Pedroni's test is that it assumes independence of cross-sections, an assumption, which is likely to be violated in econometric cross-country studies.<sup>8</sup> Cross-sectional correlation may bias the results towards rejecting the null of no cointegration (Banerjee *et al.* 2004). To account for this bias, we use a panel cointegration test developed by Banerjee and Carrion-i-Silvestre (2006) that controls for cross-sectional dependency by introducing common factors. The test also controls for possible endogeneity of regressors by including leads and lags of differenced explana-

<sup>8</sup>There are, for example, only few countries that avoided the downturn of 2008 that started from the U.S.



tory variables in estimation (this method is explained more thoroughly in the appendix C explaining the panel DSUR estimator). A more thorough explanation of the test by Banerjee and Carrion-i-Silvestre (2006) can also be found in the appendix A. The test is based on two different test statistics, namely  $Z_{\hat{\rho}_{NT}}(\hat{\lambda})$  and  $Z_{\hat{i}_{NT}}(\hat{\lambda})$  to test for the possible cointegration between two variables. Both statistics are based on the ADF regression, from where the former uses the estimated coefficients of  $\hat{\rho}$  and the latter the associated  $t$ -ratio to compute the test statistic.

Table 3 presents the results of the panel cointegration test by Banerjee and Carrion-i-Silvestre (2006). It reports the results of the cointegration tests between gross savings and the top 1% income share and gross savings and the GDP per capita.<sup>9</sup> In addition, we test for cointegration between private consumption and the top 1% income share and private consumption and GDP per capita.<sup>10</sup> According to the results of the  $Z_{\hat{i}_{NT}}(\hat{\lambda})$  test presented in table 3, gross savings and the income share of top 1% as well as private consumption and top 1% income share are cointegrated at the 5% level. According to the  $Z_{\hat{\rho}_{NT}}(\hat{\lambda})$  test, only gross savings and the top 1% income share are cointegrated. However, Banerjee and Carrion-i-Silvestre (2006) note that the  $Z_{\hat{i}_{NT}}(\hat{\lambda})$  statistic should be preferred over the  $Z_{\hat{\rho}_{NT}}(\hat{\lambda})$  statistic, because the former has considerably better size and power properties especially in small samples. Thus, we rely more on the results of the  $Z_{\hat{i}_{NT}}(\hat{\lambda})$ , and conclude that both gross savings and private consumption seem to be cointegrated with the top 1% income share. The GDP per capita and gross savings as well as GDP per capita and private consumption seem to be cointegrated of order 1.

Table 3 only presents the results of the test without breaks. However, we also conducted tests with breaks, but if trend breaks are allowed for, the results do not change dramatically. This is expected as structural breaks tend to bias the results towards the acceptance of the null (Banerjee and Carrion-i-Silvestre 2006). The results of the tests

<sup>9</sup>GDP per capita in Euros is used in the for gross savings as the gross savings series is also presented in Euros. GDP per capita in ppp is used in the test for private savings for the same reason.

<sup>10</sup>Estimation is done with Gauss. We are grateful to Carrion-i-Silvestre for providing the program code.

Table 3: Banerjee & Carrion-i-Silvestre's cointegration test for gross savings, final consumption and the income share of top 1%

	gross savings	private consumption
<hr/>		
Top 1% income share		
$Z_{i_{NT}}(\hat{\lambda})$	-3.952 (<.0001)	-2.255 (0.0121)
$Z_{\hat{\rho}_{NT}}(\hat{\lambda})$	-3.084 (0.0010)	-0.548 (0.292)
<hr/>		
GDP per capita		
$Z_{i_{NT}}(\hat{\lambda})$	-5.683 (<.0001)	-5.494 (<.0001)
$Z_{\hat{\rho}_{NT}}(\hat{\lambda})$	-3.090 (0.0010)	-6.399 (<.0001)
<hr/>		
countries	9	9
years	1960-1996	1960-1996
observations	333	333
<hr/>		

The tested model includes individual deterministic constants and trends.

including structural breaks are available upon request.

## 4.2 Testing for the cointegration rank

One of the obvious drawbacks of the residual tests, like the ones presented above, is that they cannot identify the number of cointegrating vectors between the variables. If we have just two variables, this is not a problem, because then there cannot be more than one cointegrating vector. However, with three variables, there can be two cointegrating vectors, with four variables three cointegrating vectors, etc. The cointegration rank affects estimators, because some estimators, like the panel DSUR estimator, are based on the single equation approach meaning that a single cointegration relationship is assumed. If there are two or more cointegration relationships between the variables, the asymptotic properties of the estimators derived under the assumption of one cointegration relation are no longer valid. In addition, estimators allowing for multiple cointegrating vectors, like the estimator by Breitung (2005) used in this study, usually assume that the cointegration rank is homogenous across the countries included in the

panel.

To allow for multiple cointegrating vectors, we use the panel trace cointegration test developed by Larsson and Lyhagen (2007) to test for the cointegration rank in models involving several explanatory variables. Their test is based on the likelihood ratio test of Johansen (1995). The general model on which Larsson ja Lyhagen's test is based can be written as

$$\Delta Y_{it} = \mu_i + \Pi Y_{i,t-1} + \sum_{k=1}^{m-1} \Gamma_k \Delta Y_{i,t-k} + \epsilon_{it}, \quad (6)$$

where  $\Pi = \alpha_{ik}\beta_{kj}$ ,  $\Pi$  and  $\Gamma_k$  are of order  $Np \times Np$  and  $\mu_i$  and  $\epsilon_{it}$  are of order  $Np \times 1$ , and  $\epsilon_t = (\epsilon'_{1t}, \epsilon'_{2t}, \dots, \epsilon'_{nt})$  is assumed to be multivariate normally distributed with mean zero and covariance matrix  $\Omega$ .

It is assumed that matrix  $\Pi$  has a reduced rank of  $Nr$ ,  $0 \leq r \leq p$ , and can be decomposed as  $\Pi = \alpha_{ik}\beta'_{kj}$ . The matrix  $\alpha_{ik}$  is assumed to be unrestricted, but  $\beta_{kj} = 0 \forall i \neq j$ . The fact that unrestricted  $\alpha_{ik}$  means that different panel units can be dependent, but because of the restriction on  $\beta_{kj}$ , these dependency relations can only appear in the short run. In other words, cointegrating relations are only allowed *within* the units of the panel. The cointegration rank is estimated by sequentially testing

$$H(r) : \text{rank}(\Pi) \leq Nr \quad (7)$$

against the alternative

$$H(p) : \text{rank}(\Pi) \leq Np \quad (8)$$

as in Johansen (1995). A more detailed explanation of the test is provided in Appendix ??.

A limitation in the panel cointegration test by Larsson and Lyhagen (2007) is that the number of estimated parameters increases rapidly with the number of cross sections. This means that there need to be enough time series observations compared to

cross-sectional units or the parameters of the model cannot be estimated. In our baseline dataset, there are nine countries and 37 time series observations per country, a relation which is far too small for the test. That is why countries are divided into three groups as explained in the section 3.1. Because countries in these groups tend to have similar economic and social structures, it is more likely that they also have homogenous cointegration relations.

Tables 4 and 5 present the results of the test for the cointegration rank by Larsson and Lyhagen (2007) for the three groups of countries for gross savings and private consumption as the dependent variable.<sup>11</sup> All variables are detrended and demeaned before testing. The VAR lag length was determined using the Schwartz information criterion (SIC).

According to the results presented in table 4, the top 1% income share and gross savings appear to be difference stationary variables in the Central European and Anglo-Saxon countries. That is, both are  $I(1)$  variables that are *not* cointegrated. This result contradicts the results presented in Table 3, where gross savings and the top 1% income share were found to be cointegrated. The results of Nordic countries indicate that there would be, at least, two cointegration vectors between top 1% income share and gross savings. When the GDP per capita and the interest rate are added test finds a cointegration rank of two in the Nordic countries.

According to results of table 5, private consumption and the top 1% income share are cointegrated in the Central European and Anglo-Saxon countries. When the GDP per capita and the interest rate are added, the test finds a cointegration rank of two in the Anglo-Saxon countries and cointegration rank of three in the Nordic countries. In the Central European countries only private consumption, the top 1% income share and the GDP per capita were included in the testing, because there were not enough

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<sup>11</sup>All testing is done by Gauss. We are grateful to Johan Lyhagen for providing the Gauss code on his homepage.

Table 4: Panel trace cointegration test for savings and the top 1% income share for the 3 country groups

	Nordic	Central-Europe*	Anglo-Saxon**
<hr/>			
top 1%			
r=0	211.12 (197.64)	292.15 (307.22)	72.32 (92.86)
r≤1	57.52 (50.71)	-	-
<hr/>			
top 1%, GDP & interest			
r=0	469.50 (381.22)	-	-
r≤1	301.53 (283.40)	-	-
r≤2	186.47 (175.66)	-	-
r≤3	113.70 (83.32)	-	-
<hr/>			
countries	3	3	3
years	1960-03	1960-96	1960-00
observations	132	111	123
<hr/>			

All series are detrended and demeaned before testing. \* In the group of Central European countries, only GDP per capita and top 1% income share were included in the test, because there were too few time series observations per country to include a 4 variable. \*\* for Anglo-Saxon countries, GDP per capita in constant Euros was used instead of GDP per capita in purchasing power parities in the test with private consumption. All variables are tested in logarithms. Bartlett corrected critical values are presented in parentheses. Lag lengths were selected using Schwarz information criterion.

Table 5: Panel trace cointegration test for consumption and the top 1% income share for the 3 country groups

	Nordic	Central-Europe*	Anglo-Saxon**
<u>top 1%</u>			
r=0	295.33 (167.08)	294.36 (270.17)	194.75 (171.47)
r≤1	68.33 (51.49)	72.97 (83.63)	38.86 (59.64)
<u>top 1%, GDP &amp; interest</u>			
r=0	410.85 (381.27)	516.43 (470.88)	1137.26 (992.44)
r≤1	281.61 (267.77)	194.42 (214.56)	586.90 (523.78)
r≤2	172.84 (166.60)	-	300.25 (336.24)
r≤3	58.99 (89.81)	-	-
countries	3	3	3
years	1960-03	1960-96	1960-00
observations	132	111	123

All series are detrended and demeaned before testing. \* In the group of Central European countries, only GDP per capita and top 1% income share were included in the test, because there were too few time series observations per country to include a 4 variable. \*\* for Anglo-Saxon countries, GDP per capita in constant Euros was used instead of GDP per capita in purchasing power parities in the test with private consumption. All variables are tested in logarithms. Bartlett corrected critical values are presented in parentheses. Lag lengths were selected using Schwarz information criterion.

time series observations for including four variables. In Central-European countries the cointegration rank among these three variables is found to be one.

The results of Nordic countries presented above thus indicate that there are two stationary cointegration relations between gross savings and the top 1% income share as well as between private consumption and the top 1% income share. In the time series case this would imply that, in both of these tests, the two variables are  $I(0)$  trend-stationary. However, it does not seem likely that some of these series would be trend-stationary in Nordic countries, as all of the panel unit root tests found gross savings, private consumption and the top 1% income share to be  $I(1)$  non-stationary (see section 3.2). If the time series of a variable in three out of nine countries would be trend-stationary, it would be highly unlikely for the tests to present the series as  $I(1)$  non-stationary.

With panel data, the implications of the cointegration rank test are not so straightforward as they are in the case of time series. In model (7), the block matrix elements of  $\Pi$  are given by  $\Pi_{ij} = \sum_{k=1}^N \alpha_{ik} \beta'_{jk}$ , which equal  $\alpha_{ij} \beta'_j$  when  $\beta_{ij} = 0$  for all  $i \neq j$ . However, if  $\beta_{ij} \neq 0$  for  $i \neq j$ , then the block matrix elements of  $\Pi$  are given  $\alpha_{ij} \beta'_{jk}$  and the rank of  $\Pi$  can be larger than the number of variables. That is, because the dimension of  $\Pi$  is  $Np \times Np$ , the number of cross-sectional cointegration relations may increase the rank of the matrix. One likely source of cross-sectional cointegration would be a stationary linear combination of  $I(1)$  non-stationary common factor(s) driving the GDP per capita series in the Nordic countries. As Nordic countries have very similar social structures, and because they are small countries within close proximity to each other, it would be quite natural if a common stochastic trend would affect their GDP series. As GDP per capita and consumption were found to be cointegrated, common stochastic trend driving the GDP series would also affect on the series of savings and private consumption. So, the results of the Nordic countries presented in table 5 are likely to be explained by

cross-sectional cointegration relations affecting the GDP per capita and consumption series.

In conclusion, results of cointegration tests indicate that income inequality, measured with the top 1% income share, and private consumption are cointegrated, but the results of the cointegration tests for gross savings are somewhat inconclusive. It is not quite clear why this is or why the results deviate from the results of the unit root tests. It should be noted, however, that in the case of the test of Banerjee and Carrion-i-Silvestre (2006) we had more than double the number of observations compared to the test of Larsson and Lyhagen (2007). So, even with the Bartlett correction, it is possible that the power of Larsson and Lyhagen's test was not sufficient to reject the null hypothesis of zero rank of the  $\Pi$  matrix.

## 5 Estimation

According to the results presented in the previous section, the cointegration rank among the included variables varies between the groups of countries. Because of this, two different estimation methods are applied to estimate the long-run effect of the income share of the top 1% on the gross savings and private consumption. Panel dynamic seemingly unrelated regressors (DSUR) estimator of Mark *et al.* (2005) is used when there seems to be only one cointegrating vector between the variables, while the two-step maximum-likelihood panel VAR estimator by Breitung (2005) is used when there seem to be two or more cointegrating relations between the variables. The reason for using panel DSUR estimator is that Wagner and Hlouskova (2010) have found that single-equation estimators, like panel DSUR, perform better than the VAR estimator of Breitung (2005), when cross-sections are cross-sectionally correlated and/or cointegrated.<sup>12</sup> Both estimators control for possible endogeneity of the regressors. The

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<sup>12</sup>Results of Wagner and Hlouskova (2010) imply that panel DOLS would perform better than panel VAR by Breitung (2005) in cross-sectionally cointegrated panels. Panel DSUR was not included in testing. However, as panel DSUR is more efficient than panel DOLS when cross-sections are correlated, it is also



panel DSUR estimator controls for endogeneity by including lags and leads of first differences of the explanatory variables in the estimated equation. Panel VAR controls for endogeneity by imposing block-diagonality of the Fisher information matrix with respect to short- and long-run parameters. A more detailed explanation of the estimators used can be found in the Appendix. The possible long-run cross-unit dependency relations in income and consumption series between the Nordic countries found in previous section cannot be controlled with estimators allowing for short-run dependencies and/or cross-sectional correlation. Fortunately, Wagner and Hlouskova (2010) have found that, if there is only a cross-sectionally identical unit-specific cointegrating relationship(s) between the cross-sections, it creates only a small bias in the results of cointegration estimators used here.

The estimated model is:

$$\log(Y_{it}) = \alpha_i + \gamma'_1 \log(GDP_{it}) + \gamma'_2 \log(top1_{it}) + \gamma'_3 (interest_{it}) + \lambda_{it} + u_{it}, \quad (9)$$

where where  $\alpha_i$ 's are individual constant,  $\lambda_{it}$ 's are individual trends, and  $u_{it}$  is a white noise error vector with  $E(u_{it}) = 0$ . Table 6 presents the results of estimation of equation (9) using the panel DSUR and panel VAR estimators.<sup>13</sup> According to the results, the relation between the top 1% income share and gross savings somewhat depends on the included variables. The initial estimate of the cointegrating coefficient of inequality measured with the top 1% income share is negative,<sup>14</sup> but the panel VAR estimate from the model including also the GDP per capita and the interest rate is positive in all country groups. However, the parameter estimate of the top 1% income share is statistically significant only in Nordic countries and on the last estimation of the Anglo-Saxon countries.

Theories describing the relation between income inequality and savings usually

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likely to be more efficient than panel DOLS when cross-sections are cointegrated.

<sup>13</sup>Estimation was conducted with Gauss. Author is grateful to Donggyu Sul and Joerg Breitung for providing the program codes on their homepages.

<sup>14</sup>If first equation is estimated with panel VAR, results are similar to those presented in table 6.

Table 6: Estimates of the long-run elasticity of gross savings with respect to top 1% income share in 3 groups of countries

Dependent variable: log(gross savings)			
	Nordics	Central-Europe	Anglo-Saxon
<hr/>			
Panel DSUR (l&l =2)			
log(top 1%)	-0.0846** (0.0288)	-0.127*** (0.0405)	-0.4462 (0.3706)
<hr/>			
Panel VAR (lags=2;1;1)			
log(top 1%)	0.0799*** (0.0251)	0.3794 (0.2751)	0.2499 (0.1769)
log(GDP)	0.0118*** (0.0015)	0.0946*** (0.0100)	0.0903*** (0.0108)
<hr/>			
Panel VAR (lags=2;1;2)			
log(top 1%)	0.5654* (0.2512)	0.1866 (0.2583)	0.5385* (0.1990)
log(GDP)	0.1374*** (0.0169)	0.0894*** (0.0097)	0.0983*** (0.0105)
log(interest)	-0.3655** (0.1237)	-0.0213 (0.0477)	0.0990 (0.0803)
<hr/>			
countries	3	3	3
years	1960-03	1960-96	1960-00
observations	132	111	123

\* =  $p < .05$ , \*\* =  $p < .01$ , \*\*\* =  $p < .001$ . Standard errors of the parameter estimates are presented in parentheses. Standard errors are estimated using Andrews and Monahan's Pre-whitening method. Inclusion of individual constants means that all estimations are made with fixed effects. Lags gives the lag order of the VAR model. L&l =2 means that first and second leads and lags of first differences of the explanatory variables are used as instruments.

Table 7: Estimates of the long-run elasticity of private consumption with respect to top 1% income share in 3 groups of countries

Dependent variable: log(private consumption)			
	Nordic	Central-Europe	Anglo-Saxon
<hr/>			
Panel DSUR (l&l=2)			
log(top 1%)	-0.1107*** (0.0097)	-0.1219*** (0.0342)	-0.1260*** (0.0325)
<hr/>			
Panel VAR (lags=2;1;2)			
log(top 1%)	-0.1409** (0.0490)	-0.2101*** (0.0530)	0.1130*** (0.0290)
log(GDP)	0.0905*** (0.0039)	0.1002*** (0.0026)	0.0982*** (0.0021)
<hr/>			
Panel VAR (lags=1;1;2)			
log(top 1%)	-0.1349* (0.0512)	-0.1748*** (0.0451)	0.0826*** (0.0262)
log(GDP)	0.0919*** (0.0042)	0.0996*** (0.0024)	0.1007*** (0.0021)
log(interest)	-0.0132 (0.0262)	0.0203 (0.0120)	-0.0383*** (0.0108)
<hr/>			
countries	3	3	3
years	1960-03	1960-96	1960-00
observations	132	111	123

\* = p<.05, \*\* = p<.01, \*\*\* = p<.001. Standard errors of the parameter estimates are presented in parentheses. Standard errors are estimated using Andrews and Monahan's Pre-whitening method. Lags gives the lag order of the VAR model. Individual constants and trends are included in the regressions. L&l =2 means that first and second lags and leads of first differences of explanatory variables are used as instruments.

concentrate on household savings behavior, which makes private savings or consumption a more valid measure to assess the effect of inequality on consumption or savings than gross savings that includes also the government. Table 7 presents the results of panel DSUR and panel VAR estimations where the dependent variable is private consumption.<sup>15</sup> According to the results, the cointegrating coefficient of the GDP per capita has the expected positive sign in all groups of countries, but the interest rate has a statistically significant negative effect only in the group of the Anglo-Saxon countries. The "blurry" estimate of interest rates in Central-European countries is not a

<sup>15</sup>Estimation was conducted with Gauss. Author is grateful to Donggyu Sul and Joerg Breitung for providing the program codes on their homepages.

surprise as these countries had differing indicators of interest rates. Among the Anglo-Saxon and Nordic countries, individual countries had the same indicator of interest rate within each group, but the indicators differed between the groups. In the Anglo-Saxon countries, the treasury bill rate was used, whereas in the Nordic countries, the central bank rate was used. Although it is surprising that the interest rate has no statistically significant effect on the level of private consumption in the Nordic countries, the result may also be explained by different consumption profiles. Bacchetta and Gerlach (1997) found that in the United States and Canada, the changes in credit conditions had a larger impact on consumption than in France or the UK. Humphrey (2004) also shows that credit cards are used more often as a means of payment in Canada and the US than in Europe. Thus, it is likely that changes in the interest rates have a greater effect on private consumption in the Anglo-Saxon than in the European countries.

According to the results of Table 7, the cointegrating coefficient of inequality is negative in all three country groups when the top 1% income share is the only explanatory variable. However, when the GDP per capita is added as an explanatory variable, the cointegrating coefficient of inequality changes to positive in the Anglo-Saxon countries. In the Nordic and Central-European countries the coefficient remains negative and statistically significant, thus indicating that these results are robust.

As such, the results imply that the long-run elasticity of private consumption with respect to inequality would be negative in the Nordic and Central-European countries, but positive in the Anglo-Saxon countries. Although this result is somewhat counter-intuitive, it is, of course, always possible that the individual saving function is convex in the European countries and concave in the Anglo-Saxon countries. However, this contradicts the results of most theories and also some quite recent micro-econometric evidence from the US stating that propensity to save raises with income (Dynan *et al.* 2004). From these reasons some reservations need to be attached to the estimation

results of the Anglo-Saxon countries.<sup>16</sup> It is, for example, possible that the cointegration rank differs among the Anglo-Saxon countries in estimations including the GDP per capita. In this case, the asymptotic properties of the panel VAR estimator derived under the assumption of homogenous cointegration rank are no longer valid. Unfortunately, the time series dimension of individual countries is too small for meaningful country-specific testing of the cointegration rank. Thus, the ambiguity concerning the estimation results of the Anglo-Saxon countries has to be left to be addressed in future studies.

## 6 Conclusions

In this study we assessed the relationship between income inequality and savings using a panel of developed economies. We also tested the assumption presented in early theoretical literature on income variation that the size distribution of income could follow a random walk. According to the results, income inequality, measured with the logarithmic top 1% income share, logarithmic aggregate savings and logarithmic private consumption are driven by stochastic trends. The non-stationarity of the logarithmic top 1% income share implies that the current macroeconomic literature may have taken an erroneous stand by assuming a stationary process of income variation.

The results concerning cointegration between the top 1% income share and gross savings were contradictory as the series were found to be cointegrated by both of the used tests only in the Nordic countries. All tests found the top 1% income share and private consumption to be cointegrated, which indicates that there is a long-run *steady-state* relation between them. The long-run elasticity of private consumption with respect to inequality was found to be negative in the Central-European and Nordic coun-

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<sup>16</sup>To check that this result does not relate to the expansion of credit in the wake of growing inequality, we added the domestic credit claims on the private sector as an explanatory variable for Anglo-Saxon countries. The variable was found to be  $I(1)$  according to all panel unit root tests mentioned in section three. The variable was obtained from IMF database. Adding a measure of credit did not change the main results for Anglo-Saxon countries and it has the expected statistically significant positive effect on private consumption.

tries, but positive in the Anglo-Saxon countries. This result implies that, in the Anglo-Saxon countries, the marginal propensity to save would decrease with income, while in the rest of the countries it would increase. However, estimation results for the Anglo-Saxon countries crucially depend on the inclusion of the GDP per capita series. When the GDP per capita was not included in the estimation, the parameter estimate of the top 1% income share was negative and statistically significant. Thus, it is possible that the inclusion of the GDP per capita series may introduce some form of bias in the estimation of Anglo-Saxon countries which cannot be controlled with current panel estimation methods.

One assumption that needs to be tested in the future is the assumption of homogeneous cointegration rank among the Anglo-Saxon countries. If the cointegration rank has differed among the Anglo-Saxon countries in estimations including the GDP per capita, the estimates will have been biased. In this study, the individual country cointegration relations could not be tested, because testing would have required considerably more time series observations than were available in our dataset. For the Nordic and Central-European countries the observed positive effect of inequality on private savings seems robust. Estimation results were unchanged when either only the top 1% income share or the interest rate and the GDP per capita were also included as explanatory variables. This implies that even if the cointegration rank has differed within the groups of Nordic and Central-European countries in estimations with additional explanatory variables, this has not changed the basic results.

So, although the results for the Anglo-Saxon countries are somewhat inconclusive, the results for the Nordic and Central-European countries clearly indicate that income inequality leads to higher level of private savings. This result is well in-line with the theoretical and micro-econometric evidence. It also implies that the controversy surrounding the results of the previous empirical macroeconomic studies has been likely to

result from miss-specification of the estimated models by assuming stationary income inequality.

## APPENDIX

### Appendix A: Panel cointegration test by Banerjee and Carrion-I-Silvestre (2006)

Panel cointegration test developed by Banerjee and Carrion-i-Silvestre (2006) is based on the normalized bias and the pseudo  $t$ -ratio test statistics by Pedroni (2004). The data generating process behind Pedroni's test statistics is given by:

$$\begin{aligned} y_{it} &= f_i(t) + x'_{it} + e_{it}, \\ \Delta x_{it} &= v_{it}, \end{aligned} \quad (10)$$

$$e_{it} = \rho_i e_{i,t-1} + \epsilon_{it} \zeta_{it} = (\epsilon_{it}, v_{it})',$$

where  $f_i(t)$  includes member specific fixed effects and deterministic trends.

The data generating process is described as a partitioned vector  $z'_{it} \equiv (y_{it}, x_{it})$  where the true process is generated as  $z_{it} = z_{i,t-1} + \zeta_{it}$ ,  $\zeta'_{it} = (\zeta_{it}^y, \zeta_{it}^x)$  (Pedroni 2004).  $\frac{1}{\sqrt{T}} \sum_{t=1}^{[Tr]} \zeta_{it}$  is assumed to converge to a vector Brownian motion with asymptotic covariance of  $\Omega_i$  as  $T \rightarrow \infty$ . The individual process is assumed to be *i.i.d.* so that  $E[\zeta_{it} \zeta'_{js}] = 0$   $\forall s, t, i \neq j$ .

Let  $\hat{e}_{it}$  denote the estimated residuals of obtained from (10) and  $\hat{\Omega}_i$  the consistent estimator of  $\Omega_i$ . The two test statistics can now be defined as :

$$\begin{aligned} \tilde{Z}_{\hat{\rho}_{NT-1}} &\equiv \sum_{i=1}^N \left( \sum_{t=1}^T \hat{e}_{i,t-1}^2 \right)^{-1} \sum_{t=1}^T (\hat{e}_{i,t-1} \Delta \hat{e}_{it} - \hat{\lambda}_i), \\ \tilde{Z}_{t_{NT}^*} &\equiv \sum_{i=1}^N \left( \sum_{t=1}^T \hat{s}_i^{*2} \hat{e}_{i,t-1}^{*2} \right)^{-1/2} \sum_{t=1}^T (\hat{e}_{i,t-1}^* \Delta \hat{e}_{it}^*), \end{aligned}$$

where  $\hat{\lambda}_i = 1/T \sum_{s=1}^{k_i} (1 - s/(k_i + 1)) \sum_{t=s+1}^T \hat{\mu}_{it} \hat{\mu}_{i,t-s}$ ,  $\tilde{\sigma}_{NT}^2 \equiv 1/N \sum_{i=1}^N \hat{L}_{11i}^{-2} \hat{\sigma}_i^2$ ,  $\hat{s}_i^{*2} \equiv 1/t \sum_{t=1}^T \hat{\mu}_{it}^{*2}$ ,  $\hat{s}_{NT}^{*2} \equiv 1/N \sum_{i=1}^N \hat{s}_i^{*2}$ ,  $\hat{L}_{11i}^2 = 1/T \sum_{t=1}^T \hat{\theta}_{it}^2 + 2/T \sum_{s=1}^{k_i} (1 - s/(k_i + 1)) \sum_{t=s+1}^T \hat{\theta}_{it} \hat{\theta}_{i,t-s}$ . The residuals  $\hat{\mu}_{it}$ ,  $\hat{\mu}_{it}^*$  and  $\hat{\theta}_{it}$  are attained from regressions:  $\hat{e}_{it} = \hat{\gamma} \hat{e}_{i,t-1} + \hat{\mu}_{it}$ ,  $\hat{e}_{it} = \hat{\gamma}_i \hat{e}_{i,t-1} + \sum_{k=1}^{K-i} \hat{\gamma}_{ik} \Delta \hat{e}_{i,t-k} + \hat{\mu}_{it}^*$ ,  $\Delta y_{it} = \sum_{m=1}^M \hat{b}_{mi} \Delta x_{mi,t} = \hat{\theta}_{it}$ . (Pedroni 1999, 2004)

The statistics pool the between dimension of the panel and they are constructed by computing the ratio of the corresponding conventional time series statistics and then by computing the standardized sum of the  $N$  time series of the panel. Pedroni (1999,



2004) shows that under the null of no cointegration the asymptotic distributions of the two statistics presented above converge to normal distributions with zero mean and variance of one as  $N$  and  $T$  sequentially converge to infinity.

Banerjee and Carrion-I-Silvestre (2006) extend the model by Pedroni (2004) to include common factors:

$$\begin{aligned}
y_{i,t} &= f_i(t) + x'_{i,t} + u_{i,t}, \\
\Delta x_{i,t} &= v_{i,t}, \\
f_i(t) &= \mu_i + \beta_i t \\
u_{it} &= F'_t \pi_i + e_{it}
\end{aligned} \tag{11}$$

where  $e_{i,t} = \rho_i e_{i,t} + \epsilon_{i,t}$  and  $F'_t$ 's are the common factors which are used to account for the possible cross-sectional dependence.

## **APPENDIX B: Panel trace cointegration test statistic by Larsson and Lyhagen (2007)**

The trace cointegration test by Larsson and Lyhagen (2007) is based on the following model:

$$\Delta Y_t = \mu + \Pi Y_{t-1} + \sum_{k=1}^{m-1} \Gamma_k \Delta Y_{t-k} + \epsilon_t, \tag{12}$$

where  $\mu = (\mu'_1, \mu'_2, \dots, \mu'_N)'$ ,  $\epsilon_t = (\epsilon'_{1t}, \epsilon'_{2t}, \dots, \epsilon'_{nt})$ ,  $Y_{t-1}$  and  $\Delta Y_{t-k}$  are of order  $Np \times 1$ ,  $\Pi$  and  $\Gamma_k$  are  $Np \times Np$ , and  $\epsilon_t$  is assumed to be multivariate normally distributed with mean zero and covariance matrix  $\Omega_{ij}$ .

It is assumed that matrix  $\Pi$  has a reduced rank of  $Nr$ ,  $0 \leq r \leq p$ , which is specified as  $\Pi = \alpha_{ik} \beta'_{kj}$  (Larsson and Lyhagen 2007). Matrices  $\alpha$  and  $\beta$  are both order of  $Np \times Nr$  and the former contains the short-run coefficient and the latter the long-run coefficient. In  $\beta$ ,  $\beta_{ii} \equiv \beta_i$  for each rank of  $r$ . Because of the restriction,  $\beta_{kj} = 0 \forall i \neq j$ , the block matrix elements of  $\Pi$  are given by  $\sum_{k=1}^N \alpha_{ik} \beta'_{kj} = \alpha_{ij} \beta'_j$ .

The cointegration rank is estimated by sequentially testing

$$H(r) : \text{rank}(\Pi) \leq Nr \quad (13)$$

against the alternative

$$H(p) : \text{rank}(\Pi) \leq Np, \quad (14)$$

which is the same method as in Johansen (1995).

Define  $Q_T$  as the maximum likelihood ratio test statistic for the test of  $H(r)$  against  $H(p)$ , and assume that the matrix  $\alpha' \perp \Gamma \beta \perp$  has a full rank and that the roots of the characteristic polynomial

$$A(z) = (1-z)I_{Np} - \alpha\beta'z - \sum_{i=1}^{m-1} \Gamma_k(1-z)z^i \quad (15)$$

lie outside the complex unit circle. Now, if  $r > 0$ ,

$$-2 \log Q_T \xrightarrow{w} U + V, \quad (16)$$

as  $T \rightarrow \infty$ , where  $V$  is  $\chi^2$  with  $N(N-1)r(p-r)$  degrees of freedom independent of  $U$ , and

$$U = \text{tr} \left\{ \int dBF' \left( \int FF' \right)^{-1} \int F dB' \right\}. \quad (17)$$

Larsson and Lyhagen (2007) show that the limit distribution of the test statistic (16) equals the convolution of Dickey-Fuller distribution ( $B$ ) and an independent  $\chi^2$  variate ( $F$ ). The distribution can be simulated by approximating the Wiener process of  $B$  by a random walk.

### APPENDIX C: Panel DSUR and panel VAR estimators

The data generation process in Mark *et al.* (2005) DSUR estimator is of the form

$$y_{it} = \alpha_i + \lambda_{it} + \theta_t + \beta' x_{it} + u_{it}, \quad (18)$$

$$\Delta x_{it} = e_{it} \quad (19)$$

where there are  $n$  cointegrating regression each with  $T$  observations,  $(1 - \beta')$  is the cointegration vector between  $y_{it}$  and  $x_{it}$ , and  $x_{it}$  and  $e_{it}$  are  $k \times 1$  dimensional vectors. Panel DSUR eliminates the possible endogeneity between explanatory variables and the dependent variable by assuming that  $u_{it}$  is correlated at most with  $p_i$  leads and lags of  $\Delta x_{it}$  (Mark *et al.* 2005). The possible endogeneity can be controlled by projecting  $u_{it}$  onto these  $p_i$  leads and lags:

$$u_{it} = \sum_{s=-p_i}^{p_i} \delta'_{i,s} \Delta x_{i,t-s} + u_{it}^* = \delta'_i z_{it} + u_{it}^*. \quad (20)$$

The projection error  $u_{it}^*$  is orthogonal to all leads and lags of  $\Delta x_{it}$  and the estimated equation becomes:

$$y_{it} = \alpha_i + \lambda_{it} + \theta_t + \beta' x_{it} + \delta_i z_{it} + u_{it}^*, \quad (21)$$

where  $\delta'_i z_{it}$  is a vector of projection dimensions. Panel DSUR estimates a long-run covariance matrix that is used in estimation of equation (18). This makes panel DSUR more efficient when cross-sections are dependent. The efficiency of panel DSUR actually improves as the correlation between cross-sections increases. Asymptotics properties of the estimator are based on  $T \rightarrow \infty$  with  $N$  fixed.

Breitung (2005) proposes a panel VAR( $p$ ) model which can be presented as a panel vector error-correction model (VECM) as

$$\Delta y_{it} = \psi_i d_t + \alpha_i \beta'_{y,t-1} + \sum_{j=1}^{p-1} \Gamma_{ij} \Delta y_{i,t-j} + \epsilon_{it}, \quad (22)$$

where  $d_t$  is a vector of deterministic variables and  $\psi_i$  a  $k \times k$  matrix of unknown coefficients,  $\Gamma_{ij}$  is unrestricted matrix, and  $\epsilon_{it}$  is a white noise error vector with  $E(\epsilon_{it}) = 0$  and positive definite covariance matrix  $\Sigma_i = E(\epsilon_{it} \epsilon'_{it})$ . The model is estimated in two stages. First, the models are estimated separately across  $N$  cross-section units. Then cointegration vectors are normalized so that they do not depend on individual specific

parameters. Second, the system is transformed to a pooled regression of the form:

$$\hat{z}_{it} = \beta' y_{i,t-1} + \hat{v}_{it}, \quad (23)$$

where  $\hat{z}_{it} = (\hat{\alpha}'_i \hat{\Sigma}_i^{-1} \hat{\alpha}_i)^{-1} \hat{\alpha}'_i \hat{\Sigma}_i^{-1} \Delta y_{it}$  and  $\hat{v}_{it}$  is defined in similar fashion. The cointegration matrix,  $\beta$ , can now be estimated from (23) using the OLS estimator. It is assumed that the statistical units included in the panel have the same cointegration rank. Consistent estimation is based on sequential limits. Cross-sectional correlation is accounted by using an estimated asymptotic covariance matrix.

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