Nontaxable income and necessary consumption: the Rousseau's paradox of fiscal egalitarianism

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Abstract

The traditional concept of a *strict minimum of necessary consumption and nontaxable income* equal for all taxpayers embedded in most current income-tax systems is the result of a paradox of fiscal egalitarianism. The paper shows that substituting the traditional notion of a *strict minimum of nontaxable income* (Surplus Income Tax Method) for a scheme of *growing personal allowances* to meet the amounts of *necessary consumption* required by the different living standards of the taxpayers (Discretionary Income Tax Method) generates an income-tax scheme more progressive than the traditional one. In the paper we also show that this alternative proposal for nontaxable incomes generates an after-tax income distribution less unequal (Lorenz dominance) and superior in terms of social welfare (Atkinson, 1970).

Key Words: nontaxable income, necessary consumption, progressivity, tax burden, income distribution

JEL Classification: D31; D63; H24

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1. INTRODUCTION

In current income tax systems there is a trend towards simplification of taxation. Several reforms introducing flat rate taxes have being adopted (Auerbach, 2006; OCDE, 2006; Banks and Diamond, 2008; Domínguez Martínez and López del Paso, 2008; Domínguez Martínez, 2009; Keen et al**,** 2000). These reforms reduce managerial costs and deterrence effects from taxation. However, fiscal systems based on flat rates taxes or dual income-taxes can be reconciled with the values of equity and progressivity which lie at the heart of taxation principles. Keen et al (2000) have shown that increasing personal allowances leads to decreasing inequality degrees in after-tax income distributions (residual progression) whenever the proportionate reduction in tax payments be greater for the poor than for the rich. This is the case when personal allowances have income elasticity less than one and there are flat rates.

The literature on progressive taxation and its justification on the grounds of minimal equal sacrifice has evolved since the classical formulation by Samuelson (1947) to the current developments that shed light on the links between tax progressivity and the concavity of utility functions (Moyes, 2003, Mitra and Ok, 1997, Young, 1990). On the other hand, the links between tax progressivity and the reduction of the inequality in the after-tax income distribution have been widely studied (Atkinson, 1970, Atkinson and Bourguignon, 1987, Shorrocks, 1983)[1] .

In analyzing the effects of tax reforms on progressivity, Keen et al (2000) study the impact of increasing personal allowances in progressive taxes on the concentration in the tax rates (liability progression) and the unequal distribution of after-tax income (residual income progression). They show that if the degree of progressivity is moderate (log-concavity in prices) the increase in personal allowances would lead to an increase in the equal distribution of income after taxes. What also occurs with increasing personal deductions with income elasticity less than unity if the tax is proportional. However, they do not take into account the drop in revenue associated with increased personal allowances. This is a factor worth exploring for its own sake, especially in the current circumstances. But it also deserves special attention because it affects the very applicability of the results obtained by Keen et al (2000). When there is a minimum

[1] A comprehensive analysis of the link between tax progressivity and inequality can be seen in the classical book by Lambert (1993). The distribution and redistribution of Income. A mathematical analysis, 2nd edition, Manchester University Press, Manchester.

income exemption, things change considerably even with fixed rates where there is strong and sharp progressivity in the vicinity of the exempt amount (see paragraph 3 and Figure 2 below).

In the lower reaches of the income scale near the minimum exemption is likely not to maintain the conditions of applicability of the results of Keen et al (2000) about the progressivity in residual income (after-tax) and, generally not possible to speak of Lorenz dominance without compensating adjustments in the amount of revenue. To draw conclusions in terms of welfare it would be needed income distributions with the same mean (Theorem Atkinson, 1970) considering variations in average income and the degree of inequality under the principle of Shorrocks (1983) based on generalized Lorenz curves. However, this second possibility does not deserve much attention as the first, since it would imply significant declines in tax collection.

The most interesting problem is analyzing the impact of personal allowances over the tax burden distribution under the assumption that the tax collection is held constant. In this line, this work shows the possibilities of extending the values of equity and social welfare in the income tax with fixed rates by means of the introduction of personal exemptions differentiated according to the standards of living of the taxpayers. We compare two methods of taxation (with revenues equal): the classic method (based on a strict minimum for all taxpayers) and the discretionary income method (based on differentiated personal exemptions according to taxpayers' living standards) to show that this proposal not only reduces the inequality in after-tax income distribution but unequivocally achieves social welfare improvements according to a wide range of welfare functions with inequality aversion (Atkinson, 1970).

Personal deductions of nontaxable income have been analyzed by Tanzi (2009) as an important tool to improve equity in taxation. However, the effects of this option in terms of progressivity are rather limited due to the traditional conception of a *strict minimum of nontaxable income* equal for all taxpayers. In this paper we prove that an alternative criterion which substitutes the strict minimum of nontaxable income for a scheme of *growing personal allowances* to meet the amounts of *necessary consumption*[2] according to the living standards associated to the different income levels

[2] See section 2 for a definition of necessary consumption.

of the taxpayers, not only generates less unequal after-tax income distributions, but unambiguously increases income-tax progression and social welfare.

Originally, in the first discussions about personal income taxation it was stated on, that the exemption of necessary consumption expenditures should be increased accordingly with living standards to meet the higher needs of people in the upper income bands. However when this idea was first suggested, it was considered to be unfair because of the commonly accepted principle that basic needs are the same for everyone. Rousseau (1755) strongly argued: *"He who possesses only the common necessaries of life should pay nothing at all, while the tax on him who is in possession of superfluities may justly be extended to everything he has over and above the mere necessaries. To this he will possibly object that, when his rank is taken into account, what may be superfluous to a man of inferior station is necessary for a grandee. But this is false: for a grandee has two legs just like a cow-herd, and, like him again, but one belly."*(Rousseau, 1755, p.46).

The concept of nontaxable necessary consumption expenditure has always been present in the analysis of tax systems. Stuart Mill (1951), one of the greatest exponents of modern economic philosophy, established that according to the principle of equal sacrifice there is a dramatic difference between a tax *"that could be economized of luxury and one that curtails, albeit in very small degree, the necessary to live."* Therefore taxes should *"leave a certain minimum income sufficient to provide basic necessities of life"* (Stuart Mill, 1951, p. 690).

This paper demonstrates that when the exemption threshold is not set at a strict common minimum to all taxpayers but instead it increases with income to take into account the greater expenditure requirements associated to the living standards in the upper strata of income, the distribution of the tax burden becomes more progressive and it is clearly superior in terms of a social welfare function that values equality in the income distribution. This result implies an obvious paradox.

Two forces: first, the fall in revenue since the rich pay less, secondly, the existence of the tax free allowance gives the tax a progressive structure, so the drop in revenues and an increase in the average income makes Lorenz curves the intersect in the lower strata. The proposition of Keen et al (2000) does not apply to the dominance in the lower strata, as the progressivity is rapid in the levels near the strict minimum allowance.

How is it possible that the proposed alternative criterion (growing personal allowances according to living standards) leads to a distribution of the tax burden more progressive and superior in terms of social welfare? When comparing alternative criteria for tax burden distribution a logical condition must be applied: the total amount of tax collection must be the same[3]. Once this condition is realized, the traditional confusion that richest people would pay less taxes with the proposed scheme of growing personal allowances disappears. It can be disentangled by considering two key features in the distribution of a fixed amount of tax collection among all taxpayers: a) When only considering the reductions in the total amounts of tax liabilities in the upper strata of income, the logical condition of equal tax collection is violated; b) on the contrary, when the total amount of tax collection is kept the same, what really matters are the relative shares of personal tax deductions[4] (taxable income) across the range of income levels. These relative shares of personal tax deductions in the proposed alternative tax method are much higher (lower) in the first tranches of income levels than in the upper part of the income distribution**.**

In the paper we show that the technical reasons behind this result come from two main features of what is usually named as necessary consumption: a) necessary consumption grows more slowly than income (technically income elasticity lower than one) and b) necessary consumption increases at a decreasing rate with income. Therefore**,** on the one hand, for low income levels the share of personal deductions with the proposed tax method is much higher than the one resulting from applying a strict minimum of nontaxable income (traditional tax method) equal for all taxpayers implying low tax liabilities. On the other hand, for high income levels the income share of personal deductions with the proposed tax method is higher but in relative terms much closer to the ones resulting from applying the traditional tax method. Therefor**e,** the tax burden distribution with the proposed tax method is more progressive than the one implied by applying the traditional notion of a strict minimum of nontaxable income equal for all taxpayers. In Rousseau's words, "grandees" necessary consumption remains practically constant as income increases and this is the reason why they are punished under this alternative schedule of nontaxable income. This paradox can be termed as *the Rousseau's paradox of fiscal egalitarianism*.

[3] In the current literature (Moyes, 2003), this condition has been embedded in the technical concept of *tax method*.

[4] Strict minimum nontaxable income equal for all tax payers (traditional tax method) and a scheme of growing personal allowances according to living standards (proposed tax method)

The rest of the paper is divided into three sections. Section 2 defines the concept of necessary consumption. Section 3 analyzes the distribution of effective tax rates (according to the criteria pointed out in Musgrave and Tun Thin, 1948) resulting from two alternative schedules of personal deductions, a strict minimum of nontaxable income equal for all taxpayers and a scheme of growing personal allowances of nontaxable income based on notion of necessary consumption. In section 4 both types of nontaxable income schedules are assessed in terms of a social welfare function and it is shown that when the proposed tax method is applied, the resulting after-tax income distribution is less unequal (Lorenz dominance) and superior in terms of social welfare (Atkinson, 1970). Finally, section 5 contains the main conclusions and policy implications.

2. NECESSARY CONSUMPTION

In the classical Adam Smith's book "The wealth of nations" (1776) the cultural dimension of the notion of necessary consumption has been already highlighted in the following sentence: *"not only the goods that are necessary for the preservation of life, but all those that the custom of the country makes improper for respectable people, even the lowest category, to be without them".* (A. Smith, 1776, pp. 870-1). Nowadays, the concept of necessary consumption is fully embedded in the values and consumption habits of the households. It is important to bear in mind that there is not a closed definition of necessary consumption. Priority or basic needs are those that are first met. When income is low and the budgetary constraint is tight the optimal consumer decisions focus on priority or basic needs. When income grows and budgetary constraints are not so severe, other needs equally important but of lower priority are met.

Necessary goods and services are associated with basic priorities (food, beverages, shoes, etc.) which take most of household budgets for low income levels. As income grows households increase the expenditure devoted to basic priorities (increasing the degree of satisfaction of these needs) but also allocate growing amounts of expenditure to the satisfaction of non-basic priorities (cars, trips, vacations and the like). According to household values and current consumption patterns, the demand for necessary goods becomes rigid with respect to income.

From a microeconomic point of view, the concept of necessary consumption can be defined as the expenditure on those goods and services verifying that their income elasticity is lower than one. From a statistical and operational point of view, the amount of income devoted to necessary consumption can be computed from the data on expenditure in the different kinds of goods and services gathered in household budget surveys[5].

Given a set of goods, j=1…m, and denoting by "$x_j(y)$" the expenditure allocated to each type of goods as a function of the personal incomes "y", the subset of necessary goods can be defined as those having an income-elasticity less than one, $\left\{i = 1...n / \varepsilon_{x_i} < 1\right\}$ with $n < m$. ε_{x_i} represents the income-elasticity of the expenditure in good i, x_i[6], given by the following expression:

$$\varepsilon_{x_i} = \frac{dx_i / dy}{x_i / y} < 1 \Leftrightarrow \frac{x_i}{y} > \frac{dx_i}{dy} \Leftrightarrow S_{x_i} > s'_{x_i} \qquad [1]$$

As it can be seen in the right side of expression [1], by rearranging the definition of income elasticity, we get that average expenditure shares in necessary goods, S_{x_i}, are greater than the corresponding marginal expenditure shares, s'_{x_i}. This implies that average expenditure shares in necessary goods, S_{x_i}, are a decreasing function of personal income (*y*). This technical condition captures the intuitive meaning that necessary priorities are first met by those households placed in the bottom part of the range of incomes.

Once the set of necessary goods has been determined $\left\{x_i\right\}_{i=1...n, n\in m}$, necessary consumption, NC(y), can be computed summing over the amounts of household

[5] For a detailed analysis of the microeconomic tools and econometric techniques that allow estimating necessary consumption see Deaton and Mullbauer (1980). More recently an applied analysis to the Pakistan case can be seen in Schamim and Ahmad (2006).

[6] "x_i" represents the expenditure in the good "I" in monetary terms.

expenditure to buy necessary goods and services in the set $\{x_i\}_{i=1\ldots n, n\in m}$ according to the following expression:

$$NC(y) = \sum_{i=1}^{n} x_i(y) \qquad [2]$$

The average share of necessary consumption, $S_{NC}(y)$, can be given by the proportion of necessary consumption on total personal income, $S_{NC}(y) = \frac{NC(y)}{y}$. The marginal share of necessary consumption, $s'_{NC}(y)$, can be given by the proportion of additional income that is spent in necessary goods. Mathematically it is given by differentiating expression [3] with respect to income, $s'_{NC}(y) = \frac{dNC(y)}{dy}$.

The condition on income elasticity that must be satisfied by the expenditures in necessary goods, implies that the average share of necessary consumption on income, $S_{NC}(y)$, is a decreasing function of personal incomes:

$$1 > \varepsilon_{NC} = \frac{s'_{NC}(y)}{S_{NC}(y)} \Rightarrow S_{NC}(y) > s'_{NC}(y) \qquad [3]$$

3. SURPLUS INCOME TAX METHOD VS DISCRETIONARY INCOME TAX METHOD

Let us consider a distribution of income (*y*), being it a continuous variable over the interval [a,m], $y \in [a, m]$, where *a* and m denote respectively the minimum and maximum levels for personal incomes.

Let *f(y)* denotes the density function of personal income distribution, so that the cumulative frequency of taxpayers with earnings lower than or equal to *y* is given by:

$$F(y) = \int_a^y f(y)dy \qquad [4]$$

We can now compare the tax rate structures of the two income tax methods: the traditional one with a strict minimum of nontaxable income equal for all taxpayers and our proposal of growing personal allowances according to the levels of necessary consumption associated to the different living standards.

a) The traditional tax method (Surplus income tax method) is associated to an income tax over the surplus income exceeding the strict minimum of nontaxable income, *a*, at flat tax rate, t'_s, and

b) The alternative proposal (Discretionary income tax method) is associated to an income tax over the discretionary income exceeding necessary consumption, NC(y), at a flat tax rate, t'_d.

In the case of the surplus income tax method, tax liabilities, T_s, and effective rates t_s are given by the expressions:

$$T_s(y) = t_s^{'}(y-a) \quad \text{and} \quad t_s = \frac{t_s^{'}(y-a)}{y} \text{ with } y \geq a \tag{5}$$

The total tax collection of the surplus income tax method is the result of applying the tax rate t'_s to the income surplus (y- a). The marginal tax rate, t'_s , is constant and equal to the flat tax rate, t'_s.

$$\frac{dT_s}{dy} = t_s^{'} \tag{6}$$

A discretionary income tax method with a flat rate, t'_d, provides tax liabilities, T_d and effective rates, t_d, according to the following expressions:

$$T_d(y) = t_d^{'}(y-NC(y)) \quad \text{and} \quad t_d = \frac{t_d^{'}(y-NC(y))}{y} \text{ with } y \geq a \tag{7}$$

In the discretionary income tax method, marginal rates have a more complex structure than those under the surplus income tax method because they depend on the marginal share of necessary consumption, $s'_{NC}(y)$. Marginal rates under the discretionary income tax method are given by the following expression:

$$\frac{dT_d}{dy} = t_d^{'}\left(1 - \frac{dNC(y)}{dy}\right) \text{ where } 0 < \frac{dNC(y)}{dy} < 1 \tag{8}$$

The derivative of necessary consumption with respect to income ranges from 0 $[\lim_{y\to\infty}\frac{dNC(y)}{dy} = 0]$ and 1 $[\lim_{y\to a}\frac{dNC(y)}{dy} = \frac{dNC}{dNC} = 1$ $\frac{dNC(y)}{dy} = \frac{dNC}{dNC} = 1$]. We work with the assumption that for the minimum personal income level, *a*, all income is devoted to high-priority goods[7]. In the extreme case of a taxpayer with the level of minimum income, a, no tax liabilities are generated [t'_d (1-1) = 0]. As the personal income level increases a surplus over the income devoted to necessary consumption emerges. This surplus is going to be termed as "discretionary income", y_d=y-NC(y), and therefore makes the taxable income in the discretionary income tax method.

Marginal shares of necessary consumption are a decreasing function of personal income. Once taxpayers have covered their basic needs, they can devote a greater share of their income to non-priority goods. Therefore tax liabilities under this tax method grow more than proportionally as personal income increases.

Graph 1 shows the evolution of tax liabilities, T_s and T_d respectively under the two tax methods. Tax liabilities T_s under the SITM grow at a constant rate t'_s in the whole range of incomes, in a different way tax liabilities under the DITM grow at the increasing rate given by the expression: t'_d [1-s'_{NC}(y)][8]. In the bottom part of the personal income distribution [a, y*] tax liabilities are lower under the DITM than those under the SITM. The rationale behind this behavior of tax liabilities is twofold: 1) A *taxable income effect*: necessary consumption (the personal allowances under the DITM) is always equal or greater than the strict minimum of nontaxable income, a, under the SITM, therefore taxable incomes under the DITM are always lower than those in the SITM. 2) A *flat rate effect*: Due to the fact that total tax collection must be the same to rightly compare both tax methods, the flat rate, t'_d, in the DITM must be greater than the flat rate, t'_s, in the SITM. The combined action of taxable income and flat rate effects determine the pattern of tax liabilities for both tax methods.

In the bottom part of the personal income distribution the taxable income effect is very low due to the fact that necessary consumption takes a high share of personal incomes[9] implying that even having a greater flat rate tax liabilities under the DITM are lower than those under the SITM. In SITM tax liabilities grow at a constant rate, t'_s, while in the DITM they grow at an increasing rate, t'_d [1-s'_{NC}(y)], consequently there is a threshold of personal income level, y*>a, in which tax liabilities are the same under

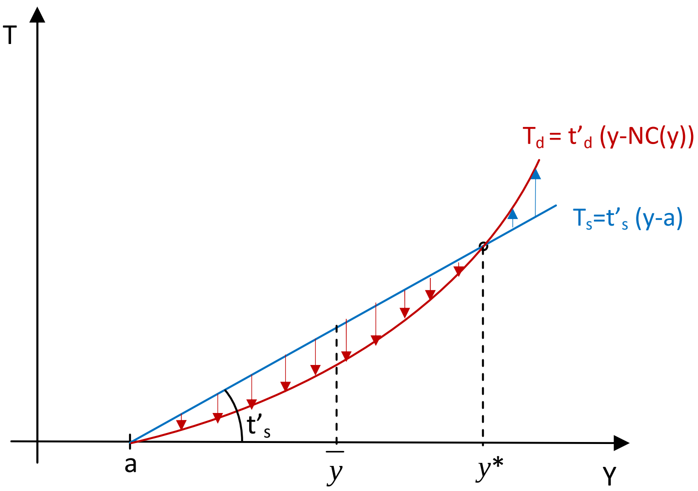
[7] It is important to bear in mind that this assumption does not imply any loss of generality, because of no tax liabilities are generated under the threshold a.

[8] It is important to bear in mind that s'_{NC}(y) is a decreasing function of personal income, y, and its values lay in the interval [1,0) for personal incomes in the interval [a,∞).

[9] Note that the share of necessary consumption over personal income, SNC(y), is a decreasing function of personal income varying monotonously between $\lim_{Y \to a} S_{NC}(Y)=1$ and $\lim_{y \to \infty} S_{NC}(Y)=0$

both methods[10]. Below this threshold, y*, there is a level of personal income, $\bar{y}$, where the difference between tax liabilities, T_s - T_d, reaches a global maximum[11].

Figure 1: Evolution of Tax Liabilities



Following Musgrave and Thin (1948) the degree of progressivity of the two tax methods can be compared by looking at average and marginal tax rates. Average tax rates are considered to be progressive if they increase as income increases. Mathematically, the average tax rate for the SITM is given by the following expression:

$$t_s = \frac{t'_s(y-a)}{y} = t'_s\left(1-\frac{a}{y}\right) \text{ with } y \geq a \quad t_s = \frac{t'_s(y-a)}{y} = t'_s(1-\frac{a}{y}) \text{ with } y \geq a$$

[9]

The average tax rate can be broken down into the flat rate (t'_s) and the factor $\left(1-\frac{a}{y}\right)$ which approaches to 0 for low income levels ($y \to a$) and to 1 for very high income levels. Moreover, the factor $\left(1-\frac{a}{y}\right)$ $\left(1-\frac{a}{y}\right)$ grows quickly as income departs from the minimum of personal income, *a*. This means the average tax rate will grow very fast in the first tranches of income.

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[10] The income threshold (y*) for which both tax methods generate the same tax liabilities is given by the following condition: $y^* \in [a,m] / t'_s(y^*-a) = t'_d(y^*-NC(y^*))$.

[11] $\bar{y}$ is characterized by the condition $t'_s = t'_d\,[1-s'_{NC}(y)]$.

In the case of the DITM, the average tax rate is given by the following expression:

$$t_d = \frac{t_d^{'}(y - NC(y))}{y} = t_d^{'}\left(1 - \frac{NC(y)}{y}\right) \text{with } y \geq NC(y) \geq a \qquad [10]$$

In a similar way to the SITM, there are two factors affecting the average tax rates: the flat rate, $t_d^{'}$, and the factor $\left(1-\frac{NC(y)}{y}\right)$.$\left(1-\frac{NC(y)}{y}\right)$. Comparing this factor with the previous one, it can be easily seen that for low income levels; the factor $\left(1-\frac{NC(y)}{y}\right)$grows more slowly than $\left(1-\frac{a}{y}\right)$ due to the fact that NC(y) is an increasing function of income. Therefore average tax rates grow more slowly under the DITM than under the SITM meaning that in terms of progressivity DITM is preferred than the SITM.

Pigou (1928) suggested another alternative measure of tax progressivity which is based on the variation of average tax rates with respect to income. When this variation is positive the tax is progressive, being more progressive the larger the rate of variation.

Under Pigou's approach, taking first derivatives with respect to income in [12] and [13] we can compare the progressivity degrees of the SITM and the DITM respectively.

In the case of the SITM we obtain the following expression:

$$\frac{dt_s}{dy} = t_s^{'}\frac{a}{y^2} \qquad [11]$$

Expression [15] presents always positive values due to the fact that all its components are positive. Additionally, taking into account that *a* is constant and the denominator is the level of income to the power two, the slope of t_s will decrease rapidly as the level of income increases.

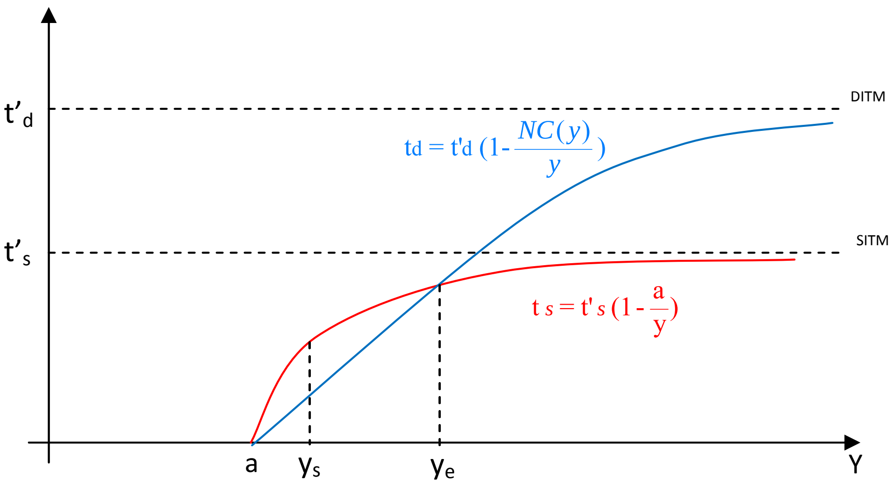
In the case of DITM, we obtain:

$$\frac{\partial t_d}{\partial y} = t_d^{'}\left(\frac{-\partial NC(y)/\partial y \cdot y + NC(y)}{y^2}\right) = t_d^{'}\left(\frac{S_{NC} - s_{NC}^{'}}{y}\right) \qquad [12]$$

Expression [15] presents also positive values due to the fact that $S_{NC}>s'_{NC}$ (an implication from the definition of necessary consumption –income elasticity being lower than one).

An interesting conclusion results from the comparison of expressions [14] and [15]: the DITM is less progressive than the SITM in the lower tranches of income up to the income threshold y_s[12]. Above this threshold the slope of average tax rates in the SITM falls very quickly and its average tax rates approach very soon to the value of its flat rate, t'_s. On the contrary, in the DITM above the income threshold y_s the escalation of its effective tax rates continues with a higher slope approaching asymptotically to its flat rate, t'_d , which is greater than the one in the SITM because of the logical condition of keeping the same amount of total tax collection under both tax methods.

Figure 2: Average Tax Rates under SITM and DITM



As can be seen in Figure 2, average tax rates in the SITM are above those in the DITM up to the income threshold y_e which can be easily determined from expressions [12] and [13]:

$$\frac{t'_s}{t'_d} = \frac{1 - NC(y)}{1 - a} \qquad [13]$$

[12] y_s income threshold satisfies the following condition: $\frac{t'_s}{t'_d} = \frac{S_{NC} - s'_{NC}}{a} y$

4. ASSESSMENT IN TERMS OF SOCIAL WELFARE

After describing and analyzing in terms of progressivity the SITM and DITM tax methods in the previous section, in the present one we undertake the comparative analysis of them **in** terms of social welfare. In order to do such a comparison the analysis will be based on the well-known Atkinson theorem (1970). It will be shown that the DITM based on growing personal allowances for necessary consumption levels according to living standards is preferred in terms of a social welfare function and produces a fairer after-tax income distribution than the SITM based on a strict minimum of nontaxable income equal for all taxpayers.

Atkinson (1970) theorem proves that when we compare two income distributions which have the same average, the one showing a more equal distribution applying Lorenz criterion is clearly superior to the other according to a wide variety of individualistic, symmetric, additively separable and inequality averse social welfare functions.

Let us F(y) denote a cumulative distribution function of income, where *y* is a continuous variable ranging from a minimum of income, *a,* up to a maximum of income, *m*, $y \in [a, m]$. Let f(y) denotes the corresponding density function of the considered income distribution. The total number of taxpayers with an income less or equal than *y* is given by:

$$F(y) = \int_a^y f(y)dy \qquad [14]$$

Total tax collection in the SITM is given by the following expression:

$$TC_s = t_s^{'} \cdot \left[\int_a^m y.f(y)dy - \int_a^m a.f(y)dy \right] = t_s^{'}.SY \qquad [15]$$

Where *SY* refers to the aggregate of surplus income resulting from the sum of incomes exceeding the strict minimum of nontaxable income, *a*, over the whole set of taxpayers.

In the case of the DITM, total tax collection is given by:

$$TC_d = t_d^{'} \cdot \left[\int_a^m y.f(y)dy - \int_a^m NC.f(y)dy \right] = t_d^{'}.DY \qquad [16]$$

Where *DY* refers to the aggregate of discretionary income which is the sum of incomes exceeding necessary consumption, *NC(y)*, over the whole set of taxpayers.

The logical condition for comparison purposes of the two alternative tax methods, namely, the same amount of total tax collection, implies the following relation between them:

$$TC = t_s^{'}.SY = t_d^{'}.DY \Leftrightarrow t_s^{'} = \frac{DY}{SY} t_d^{'} \qquad [17]$$

Taking into account that $DY<SY$, the flat tax rates $t_s^{'}$ and $t_d^{'}$ must verify:

$$t_s^{'} < t_d^{'} \qquad [18]$$

The key condition for applying the Atkinson (1970) theorem, namely, the two income distributions must have the same mean, in our case is guarantee by the logical condition that the two alternative tax methods, SITM and DITM, must collect the same amount of taxes, TC_s (total tax collection under SITM), and TC_d (Total tax collection under DITM). It is also straightforward to check the fulfillment of this condition. Both after-tax income distributions therefore verify the following condition:

$$TC_s = TC_d \Leftrightarrow Y - TC_s = Y - TC_d \qquad [19]$$

Let denote by $F_1(y - T_{s(y)})$ the after-tax cumulative income distribution under SITM and by $F_2(y - T_{d(y)})$ the after-tax cumulative income distribution under DITM, where $T_{s(y)}$ and $T_{d(y)}$ denote respectively the corresponding tax liabilities under both tax methods, SITM and DITM.

In order to apply Atkinson's (1970) theorem, we are going to consider a twice continuously differentiable, additively separable, symmetric and with inequality aversion utility function, *U(y),* to build a social welfare function, W, of individual incomes.

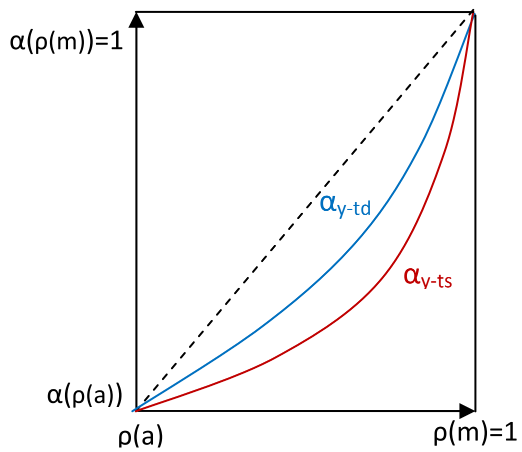
$$W \equiv \int_a^m U(y) f(y) dy, \forall U(y) : \left[U^{'}(y) > 0, U^{"}(y) < 0 \right] \qquad [20]$$

The Atkinson theorem (1970) allows evaluating in terms of social welfare (by means of the function W) both income tax methods: the surplus income tax method, SITM , and the discretionary income tax method, DITM. Applying Atkinson's (1970) results to compare in terms of social welfare the two alternative tax methods under analysis it can be shown that for any social welfare function of the type *W* the after-tax income distribution under the DITM is preferred to the after-tax income distribution under the SITM, because the following condition is satisfied:

$$\int_a^m U(y-T_d)f(y)dy \geq \int_a^m U(y-T_s)f(y)dy \qquad [21]$$

according to Atkinson's (1970) theorem, condition in expression [24] is hold when after-tax income distribution Lorenz curve under the DITM method is less unequal than the after-tax income distribution Lorenz curve under the SITM. As both after-tax income distribution have the same mean (because of the amount of total tax collection is the same), their respective Lorenz curves do not intersect, so the comparison can be based on their proximity to the equal distribution 45º line. In order to carry out this comparison let us define the cumulative shares of population ordered by income levels, $\rho(y)$, and the corresponding cumulative shares of after-tax incomes, $\alpha_{y-T_i}(\rho), i=d,s$, under the alternative discretionary income and surplus income tax methods. Figure 3 shows the shapes of the two Lorenz curves of after-tax income distribution under both tax methods DITM and SITM.

Figure 3: After-tax Income Distributions Lorenz Curves: DITM vs. SITM



For the sake of simplicity, we assume a positive value for the share of individuals earning the strict minimum nontaxable income, $a, \rho(a) > 0$. It is straightforward that $\rho(y)$ is an increasing function of y and therefore at the maximum level of income, m, $\rho(m)=1$.

In terms of the Lorenz curve variables depicted in Figure 3 the DITM is socially preferred to the SITM when the following condition applies:

$$\alpha_{y-T_d}(\rho) \geq \alpha_{y-T_s}(\rho), \rho \in [a,1] \qquad [22]$$

In SITM, which applies a strict minimum of nontaxable income equal for all taxpayers, the cumulative tax liabilities for taxpayers with an income equal or lower than y is given by:

$$T_s(\rho_y) = \int_a^y t_s^{'}(y-a) f(y) dy \qquad [23]$$

Where for the sake of simplicity ρ_y has the same meaning as $\rho(y)$. Therefore for ρ_m, the total tax collection in SITM is given by:

$$TC_s(\rho_m) = \int_a^m t_s^{'}(y-a) f(y) dy = t_s^{'}.SY \qquad [24]$$

In the case of DITM, which applies growing personal allowances of necessary consumption according to living standards, the cumulative tax collection for ρ_y is given by the following expression:

$$T_d(\rho_y) = \int_a^y t_d^{'}(y - NC(y)) f(y) dy \qquad [25]$$

Therefore for ρ_m, the total tax collection for this tax method is:

$$TC_d(\rho_m) = \int_a^m t_d^{'}(y - NC(y)) f(y) dy = t_d^{'}.DY \qquad [26]$$

After-tax income distribution Lorenz curves under the SITM and DITM are computed by substracting the corresponding tax liabilities, T_s and Td, from the before-tax personal income, y. Consequently, their respective expressions are given by:

$$y_{t_s} = y - t'_s(y-a) \qquad [27]$$

$$y_{t_d} = y - t'_d(y - NC(y)) \qquad [28]$$

The cumulative after-tax income distributions under the SITM, $Y_{t_s}(y)$, and DITM, $Y_{t_d}(y)$, are respectively given by the following expressions:

$$Y_{t_s}(y) = \int_a^y \left[y - t'_s(y-a)\right] f(y) dy = \int_a^y y.f(y) dy - t'_s \int_a^y (y-a) f(y) dy \qquad [29]$$

$$Y_{t_d}(y)=\int_a^y\left[y-t'_d(y-NC(y)\right]f(y)dy=\int_a^y y.f(y)dy-t'_d\int_a^y(y-NC(y)f(y)dy \qquad [30]$$

So the corresponding income shares, $\alpha_{t_i}\rho(y), i=s,d$, in the after-tax income distribution Lorenz curves for SITM and DITM are respectively given by:

$$\alpha_{t_s}\rho(y)=\frac{\int_a^y y.f(y)dy-t'_s\int_a^y(y-a)f(y)dy}{\int_a^m y.f(y)dy-t'_s\int_a^m(y-a)f(y)dy}=\frac{\int_a^y y.f(y)dy-t'_s\int_a^y(y-a)f(y)dy}{Y-t'_s.SY} \qquad [31]$$

$$\frac{\int_a^y y f(y)dy-t_s\int_a^y(y-a)f(y)dy}{Y-t_sSY}$$

$$\alpha_{t_d}\rho(y)=\frac{\int_a^y y.f(y)dy-t_d\int_a^y(y-NC(y))f(y)dy}{\int_a^m y.f(y)dy-t_d\int_a^m(y-NC(y))f(y)dy}=\frac{\int_a^y y.f(y)dy-t_d\int_a^y(y-NC(y))f(y)dy}{Y-t_d.DY} \qquad [32]$$

According to Atkinson's (1970) theorem DITM would be preferred to SITM if the following condition is satisfied:

$$\alpha_{t_s}\rho(y)\le\alpha_{t_d}\rho(y) \qquad [33]$$

Or equivalently, taking into account expressions [34] and [35], the next one must be hold:

$$\frac{\int_a^y y.f(y)dy-t'_s\int_a^y(y-a)f(y)dy}{\int_a^y y.f(y)dy-t'_d\int_a^y(y-NC(y))f(y)dy}\le\frac{Y-t'_s.SY}{Y-t'_d.DY} \qquad [34]$$

Proof:

According to the logical condition of keeping the same amount of tax collection under the two tax methods under analysis, we know that:

$$t'_s.SY=t'_d.DY\rightarrow t'_d=\frac{SY}{DY}t'_s \qquad [35]$$

Substituting expression [38] in expression [37] and dividing by the aggregate before-tax income, *Y*, we obtain the following expression:

$$\frac{\int_a^y y.f(y)dy-\frac{t'_s\int_a^y(y-a)f(y)dy}{Y}}{\int_a^y y.f(y)dy-t'_s\frac{SY}{DY}\frac{\int_a^y(y-NC(y))f(y)dy}{Y}}\le\frac{Y-t'_s\frac{SY}{Y}}{Y-t'_s.DY\frac{SY}{DY.Y}} \qquad [36]$$

The right hand side of inequality [39] is equal to 1. Therefore this inequality implies:

$$\frac{\int_a^y (y-a)f(y)dy}{\int_a^y (y-NC(y))f(y)dy} \geq \frac{SY}{DY} \qquad [37]$$

This clearly holds across the whole relevant range of incomes $(a, m]$.

5. CONCLUSION AND POLICY IMPLICATIONS

Under the classical view, the rationale for a strict minimum of nontaxable income (equal for all taxpayers) is only justified as a benefit to those taxpayers in the lowest income brackets to prevent confiscatory taxes. The reason behind this is that low level income earners remain with enough after-tax income to consume what is considered as the strict minimum "necessary for living".

In this paper we show that the classical notion of a strict minimum of nontaxable income equal for all taxpayers to a great extent lacks of a logical justification in terms of progressivity and social welfare. This rather surprising result could be termed as a *Rousseaunian paradox on fiscal egalitarianism.*

In this paper we have defined, discussed and compared the traditional surplus income tax method (SITM) with our proposal of an alternative discretionary income tax method, DITM, based on growing personal allowances for necessary consumption according to living standards. We have proved that DITM is a more progressive tax method (according with the usual criteria in Musgrave-Thin, 1948, and Pigou,1928) not only benefiting lower income individuals but also reducing inequality and improving the after-tax income distribution in terms of social welfare (Atkinson, 1970).

Our results can have important policy implications. The most straightforward one is that under our proposal of discretionary income taxation it is possible to reach progressivity in personal income taxation and improving equity in the after-tax income distribution by means of pure flat rate schemes. Future research avenues to explore these possibilities in different fiscal and socio-economic backgrounds can lead to very interesting results.

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