Endogenous R&D and Intellectual Property Laws in Developed and Emerging Economies

Aniruddha Bagchi and Abhra Roy

Kennesaw State University Coles College

June 2011

Endogenous R&D and Intellectual Property Laws in Developed and Emerging Economies

Aniruddha Bagchi[*] [†]
Coles College of Business
Kennesaw State University
Kennesaw, GA 30144

Abhra Roy[‡]
Coles College of Business
Kennesaw State University
Kennesaw, GA 30144

Last Revised: June, 2011

JEL F2, O1, O31, O34

Keywords: Patent Length, Patent Breadth, Productivity

[*]Corresponding Author. Tel. +1-770-423-6720; e-mail abagchi@kennesaw.edu.

[†]This paper was presented at the International Industrial Organization Conference (May, 2008) and we thank the conference participants. We would also like to thank two anonymous referees who helped us in substantially improving the paper.

[‡]E-mail: aroy1@kennesaw.edu

Abstract

The incentive of providing protection of intellectual property has been analyzed, both for an emerging economy as well as for a developed economy. The optimal patent length and the optimal patent breadth within a country are found to be positively related to each other for a fixed structure of laws abroad. Moreover, a country can respond to stronger patent protection abroad by weakening its patent protection under certain circumstances and by strengthening its patent protection under other circumstances. These results depend upon the curvature of the R&D production function. Finally, we investigate the impact of an increase in the willingness-to-pay in the emerging economy and find conditions under which there is an improvement in both patent length as well as patent breadth in the emerging economy.

1 Introduction

There has been a recent literature (summarized in Maskus (2000)) that analyze the determinants of the Intellectual Property (henceforth, IP) laws. This article builds on this literature by examining the incentive of a country in providing IP protection when R&D effort is endogenously determined. We analyze this incentive both for a developed economy as well as for an emerging economy. It is natural to conjecture that in a two-country model, if one country improves its patent protection, then the other country would free ride and reduce its own level of patent protection (Scotchmer (2004a), p. 330). This conjecture generally holds if R&D effort is assumed to be exogenous. However, we show that if R&D effort is endogenized, then such a conclusion can only hold under certain circumstances. Indeed, it is possible to show that there are situations under which an improvement in IP laws in the developed economy can lead to a simultaneous improvement of such laws in the emerging economy. Our analysis can therefore be used to determine conditions under which countries can free ride on each other in framing IP laws, and conditions under which they "cooperate." Interestingly, in our model such "cooperation" can be achieved by countries acting in their best interest, and not because of international treaties.

We also use our model to analyze conditions for convergence of IP laws between developed and emerging economies. From the perspective of IP laws, the most important differences between developed and emerging economies are that emerging economies have lower incomes and a lower level of research capability. One can conjecture that if there is a convergence between the developed and emerging economies in both of these dimensions, then there should be a convergence of their IP laws as well, but in that case it is not clear if the convergence in IP laws is because of the convergence of incomes or because of the convergence of research capabilities. However, the role of each of these dimensions can be isolated if there is a convergence in only one of these dimensions. To do so, we focus on the impact on IP laws if there is convergence in incomes without a commensurate convergence in research capabilities.[1] Therefore, in our framework, the main difference between an emerging and a developed country is that the former has a domestic firm that engages in R&D,

[1] In general, the per capita income and research capabilities are correlated but there need not be a one-to-one relationship between these two variables. Porter and Stern (2001) analyze the relationship between an "innovative capacity index" and GDP per capita (in the year 2000) across countries and find evidence of a strong correlation. However, they also find a lot of variation across countries. For example, it follows from Figure 5 of their article that New Zealand and Israel had approximately the same GDP per capita in the year 2000 but Israel was substantially ahead in terms of innovation capacity.

while the latter has a domestic firm that only imitates a new technology, whenever possible. While we agree that there are many points of difference between an emerging economy and a developed economy, we focus on this difference since it seems to be the most relevant for the purpose of analyzing the different IP regimes that would emerge in equilibrium in either type of economy.[2] Further, throughout the analysis, *IP laws refer to the choice of patent length and breadth.*

In the model, we analyze the solution to the following questions on the structure of IP laws: *(i)* What is the relationship between the optimal patent length and the optimal patent breadth in a country, given a fixed structure of IP laws in other countries? In other words, if a change in circumstances necessitates an improvement in the degree of patent protection in a particular country (given a fixed degree of IP protection abroad), how is this improvement achieved? *(ii)* What is the relationship of the patent regimes across countries? In other words, if one country strengthens its patent laws, should the other country's best response be to weaken its patent laws? *(iii)* What is the impact on the IP laws of an increase in the willingness-to-pay in the emerging economy?

Among these three questions, *(i)* has been studied most extensively in the literature. Patent length and breadth both serve to increase the returns to R&D by introducing different types of distortions in the economy. In particular, the patent length represents the length of time during which the consumers in an economy have to bear the distortionary effects of a monopoly patentee, while the patent breadth represents the excess burden consumers have to bear each period due to such market distortions. Therefore, an economy selects the patent length and breadth that provides a given return to R&D at the minimum possible social cost.

In the developed economy, the domestic firm expends effort in R&D and the profit of this firm is included in the social welfare function. Note that any incentives to this firm by way of IP protection adversely impacts the consumers because of its distortionary effects. Hence, in the developed country, there is a tension between consumers and the domestic firms and the country has to choose its IP laws (patent length and breadth) in a way that balances these conflicting incentives. Gilbert and Shapiro (1990), Klemperer (1990) and Gallini (1992) analyze the tradeoff between patent length and breadth in such economies for a given total reward. However, in these papers, the R&D process has not been explicitly analyzed. DeBrock (1985) endogenizes R&D but only considers patent lengths. However such models do not capture the incentive of emerging

[2] The model is completely general to allow for different levels of willingness-to-pay of consumers or different market structures.

economies to free ride on the IP laws of developed economies. To focus on such effects, we assume that the domestic firm in the emerging economy does not expend effort in R&D but imitates the innovation of the firm located in the developed economy. Therefore, the profit of the innovating firm is not included in the social welfare function of the emerging economy and it might seem that the emerging economy would select weak patent laws. However, in our model, if the emerging economy selects excessively weak IP laws, it reduces the incentives of the firm in the developed economy to expend effort on R&D, and this in turn affects welfare adversely in the emerging economy by the lack of availability of new goods and services to consumers, and by the lack of opportunity to imitate a new product by the domestic firm. Hence, the emerging economy selects its IP laws that balance the interests of its domestic firm and consumers with the interest of the firm in the developed economy.

In our analysis of *(i)*, we find that the optimal patent length in either economy has a positive relationship to its optimal patent breadth. Given the other economy's IP laws, each economy has to determine how much incremental protection to offer the innovating firm (located in the developed economy). Once the desired level of patent protection is determined, the economy achieves this by a combination of suitable patent length and patent breadth. At the optimum solution, the elasticity of the patent length has to equal the elasticity of the excess burden that the economy has to bear due to a positive patent breadth. Further, since the excess burden is increasing in the patent breadth, therefore the patent breadth and the patent length are positively related to each other.

Next, we analyze *(ii),* that is, the relationship of IP regimes across countries. How should a country respond when a competing country improves its degree of patent protection? One would expect that the country should respond by reducing its own degree of patent protection. This is because each country has an incentive to reduce its own market distortions by free-riding on the competitor's IP laws. We show that this free-rider effect is only part of the analysis and indeed when R&D effort is endogenized, there could be a countervailing effect at work that could dampen and even overturn the free-rider effect. To see this, consider the degree of patent protection in each country as an input in the production of R&D effort. It can then be shown using Lemma 2 that depending on the rate of change of the curvature of the R&D production function, the IP laws can be substitutable inputs under certain circumstances (as expected), but more interestingly, can also be complementary inputs under other circumstances. In the latter case (that is, for complementary inputs), a decrease in one input reduces the marginal productivity of the other input. It therefore follows that when the IP laws are complementary inputs, there is a cost associated with free-riding on the competing country's laws and this dampens the incentive to free-ride. Thus, when the IP laws are strongly complementary inputs, the cost of free-riding is too high relative to its benefit and it is possible for a country to strengthen its patent laws in response to an improvement of patent laws in the competing country. The details of this argument is in Section 6.

What is the implication of the above finding? It is commonly observed that the degree of patent protection is weaker in emerging countries than in developed countries. For example, it follows from Table 3 of Park and Wagh (2002) that in the year 2000, the degree of patent protection of emerging countries such as India, China or Brazil was considerably lower compared to the degree of patent protection in the developed economies such as USA, UK or France. In this context, one might want to know if the IP laws in the emerging countries would converge to the IP laws in the developed countries as the emerging countries become richer. Our model shows that an increase in incomes in emerging countries (without an increase in their research capabilities) is not sufficient to guarantee convergence in both aspects of the IP laws. Some related papers such as Chen and Puttitanun (2005), Lai and Qiu (2003), Grossman and Lai (2004) and Yang (1998) index the degree of patent protection by a single parameter. However, patent protection is inherently multi-dimensional. Our work builds on the literature and determines conditions under which the two main aspects of patent protection (length and breadth) move in the same direction and conditions under which they move in opposite directions.[3] Wright (2005) considers both aspects of IP law in a two country setting and is closest to this model. However, the focus of Wright (2005) is on the impact of the curvature of the demand function on the IP laws of the developed as well as of the emerging economy. In contrast, our focus is on the curvature of the R&D production function.

2 Model

It is assumed that there are two countries- 1 and 2. Country 1 is a prototype developed country while country 2 is a prototype emerging country. The term "emerging economy" refers to any country that has a market of reasonable size but is not yet as efficient in R&D as a developed

[3] Note that the results in (*i*) have been derived under the assumption that the structure of IP laws in foreign countries is fixed. In contrast, the analysis in (*iii*) allows the laws in foreign countries to vary as well.

economy. An example of a developed economy would be USA and an example of an emerging economy would be China or India. Each country i $(i = 1, 2)$ has a firm that is denoted by i. These assumptions are identical to the ones made in Zigic (1998) and are also similar to Kim and Lapan (2008).[4] A firm cares for its own profit and the country cares for the welfare which is the sum of consumers' surplus of its own citizens and the profit of the firm that is based in the country. These assumptions need not be taken literally and have been taken for the sake of simplicity. In general, "country 1" may represent the group of developed economies and "country 2" may represent the group of emerging economies. Similarly, "firm i" may represent the relevant industry in country i.

In order to earn profits, firm 1 has to engage in R&D; throughout the analysis, we assume for simplicity that firm 2 does not engage in R&D.[5] These assumptions have also been made in Zigic (1998) and Kim and Lapan (2008). Let R_1 denote the effort that firm 1 expends on R&D. The outcome of R&D is uncertain in the model. If firm 1 expends an effort of R_1, then the probability of success in R&D is denoted by $P(R_1)$. The function $P(\cdot)$ is the "production function" of R&D and the nature of this function characterizes the R&D technology.[6] It is assumed that if firm i does not put in any effort in R&D, its probability of success in R&D is 0, that is, $P(0) = 0$. We also assume that the function $P(\cdot)$ satisfies the following restrictions:

$$P'(\cdot) > 0 \text{ and } P''(\cdot) < 0.$$

Notice that $P'(\cdot)$ is the marginal productivity of R&D and $P''(\cdot)$ is the change in the marginal productivity of R&D due to small changes in R_1.

Suppose the flow of profits to a firm at each instant is π, the patent length is T_i in country i, and the discount rate is normalized to 1. Therefore, the present discounted value of the future profits from the innovation are

$$\int_0^{T_i} \pi e^{-t} dt = \pi \left(1 - e^{-T_i}\right).$$

Notice that because $\left(1 - e^{-T_i}\right)$ and T_i are monotonically related, therefore, we can measure the patent length in country i by

$$\lambda_i \equiv 1 - e^{-T_i}.$$

[4] In Kim and Lapan (2008), there are multiple emerging economies instead of one as is assumed in our model.

[5] What really matters for the results is that firm 1 has a sufficiently higher likelihood of being successful in R&D with the same effort.

[6] Scotchmer ((2004a), p. 54) lists certain well-known papers on the production function approach to R&D.

It follows from the definition that $\lambda_i \in [0,1)$; $i = 1,2$. This notion of patent length is the same as the notion of "discounted time" that is sometimes used in the literature (Scotchmer (2004a), p. 59).[7]

The definition of patent breadth is more problematic as there does not seem to be any unanimity in the literature about either its definition or measurement. In this model, the patent breadth is defined to be the fraction of the technology improvement that does not spill out to the non-innovating firm. Hence, the patent breadth in country i is measured by $\beta_i \equiv (1-\alpha_i)$ where α_i measures the degree of knowledge spillover from the invented product to the imitated product; $\alpha_i \in [0,1]$. A value of $\alpha_i = 0$ means that in country i, the imitator cannot use any of the incremental knowledge embodied in the invention and hence is equivalent to the maximum possible patent breadth. Conversely, a value of $\alpha_i = 1$ means that in country i, the imitator can use all of the incremental knowledge embodied in the invention and hence is equivalent to the minimum possible patent breadth. This definition of patent breadth has been used in Denicolò ((1996), p. 252) and is similar to Klemperer (1990). Further, the degree of spillovers has also been used as a measure of IP protection in Zigic (1998) and Kim and Lapan (2008). Other definitions of patent breadth have been used in the literature. For example, in Gallini (1992), an imitator pays a fixed cost to imitate a new technology and this fixed cost is defined to be the patent breadth. Notice that Gallini's notion of patent breadth is different from the one used in this model.

A successful invention can be patented costlessly in both countries. Further, given knowledge spillovers, a non-patented technology can be imitated perfectly. Hence, firm 1 always patents the innovation in both countries. Moreover, we also assume *national treatment* of IP laws, that is, in each country, the IP laws treat the domestic firm and the foreign firm equally. Below, we determine the patent length λ_i and patent breadth $(1-\alpha_i)$ for country i $(i = 1,2)$ endogenously. In the following examples, we illustrate the notion of patent breadth used in the analysis.

Example 1 *Suppose in an industry, there is a publicly available technology that allows a firm to produce at a marginal cost of μ. There are two firms in the industry. Firm 1 invents a cost-reducing technology that allows it to produce at a marginal cost of $\mu-\theta$. Therefore, the incremental knowledge embodied in firm 1's technology is θ. If the patent breadth in country i is $\beta_i \equiv (1-\alpha_i)$, firm 2 can*

[7] If we had assumed the discount rate to be r instead, then λ_i could take any value between 0 and $\frac{1}{r}$. We structure the discussion using $r = 1$ for simplicity.

reduce its marginal cost by $\alpha_i\theta$ by imitating the technology. Hence, firm 2 can achieve a marginal cost of $\mu - \alpha_i\theta$. ■

Example 2 *Consider a vertically differentiated industry as described in Tirole ((1988), p. 96 and pp. 296-298). There are two firms in the industry. Each consumer purchases at most one unit of the good. There is a publicly available technology that allows a firm to produce at a quality level of 0. Assume that firm 1 develops a technology that improves the quality of its product to $\theta > 0$. Therefore, the incremental knowledge embodied in firm 1's technology is θ. With a patent breadth of $\beta_i \equiv (1 - \alpha_i)$, the maximum quality of the imitated product that firm 2 can produce is $\alpha_i\theta$ in country i ($i = 1, 2$).* ■

Because patent breadth and knowledge spillover have a one-to-one relationship, it is sufficient to determine just one of these variables. Following Denicolò (1996), we focus on the degree of knowledge spillover α_i, in addition to the patent length.

The instantaneous profit of firm i in country j conditional on a successful innovation is

$$\pi_{ij}(\alpha_j)\,; \quad i, j = 1, 2.$$

A higher degree of knowledge spillover results in a reduction in firm 1's profit and this is captured formally as follows:

$$\pi'_{1j}(\alpha_j) < 0. \tag{1}$$

We do not impose any restriction on the sign of $\pi'_{2j}(\alpha_j)$. The net payoff of firm 1 with an R&D effort of R_1 is

$$\Pi_1 = P(R_1)V_1 - R_1, \tag{2}$$

where

$$V_1 = \lambda_1\pi_{11}(\alpha_1) + (1 - \lambda_1)\pi_{11}(1) + \lambda_2\pi_{12}(\alpha_2) + (1 - \lambda_2)\pi_{12}(1) \tag{3}$$

is the gross profit of firm 1 conditional on a successful invention. Note that during the duration of the patent in country 1, the patent breadth is $(1 - \alpha_1)$ and the corresponding flow rate of profit for firm 1 is $\pi_{11}(\alpha_1)$. Hence, the gross payoff of firm 1 in country 1 during the duration of the patent is $\lambda_1\pi_{11}(\alpha_1)$. After the expiry of the patent, the patent breadth in country 1 decreases to 0 and and the corresponding flow rate of profit for firm 1 decreases to $\pi_{11}(1)$. Therefore, the gross payoff of firm 1 in country 1 after the expiry of the patent is $(1-\lambda_1)\pi_{11}(1)$. Similarly, the gross payoff of firm 1 in country 2 is $\lambda_2\pi_{12}(\alpha_2)$ for the duration of the patent and $(1-\lambda_2)\pi_{12}(1)$ after its expiry. In case the R&D effort fails, the gross payoff of everyone (that is both firms and countries) is normalized to 0.[8]

For the discussion below, notice that V_1 can be re-written in the following form:

$$V_1 = \pi_{11}(1) + \pi_{12}(1) - \lambda_1 \int_{\alpha_1}^{1} \pi'_{11}(z)\,dz - \lambda_2 \int_{\alpha_2}^{1} \pi'_{12}(z)\,dz. \tag{4}$$

Analogously, the net payoff of firm 2 is

$$\Pi_2 = P(R_1)\,V_2 \tag{5}$$

where

$$\begin{aligned} V_2 &= \lambda_1\pi_{21}(\alpha_1) + (1-\lambda_1)\pi_{21}(1) + \lambda_2\pi_{22}(\alpha_2) + (1-\lambda_2)\pi_{22}(1) \\ &= \pi_{21}(1) + \pi_{22}(1) - \lambda_1 \int_{\alpha_1}^{1} \pi'_{21}(z)\,dz - \lambda_2 \int_{\alpha_2}^{1} \pi'_{22}(z)\,dz \end{aligned} \tag{6}$$

In the above expression, V_2 is the gross profit of firm 2 conditional on a successful invention. Notice that the gross payoff of firm 2 in country 1 is $\lambda_1\pi_{21}(\alpha_1)$ for the duration of the patent and $(1-\lambda_2)\pi_{12}(1)$ after its expiry. Similarly, gross payoff of firm 2 in country 2 is $\lambda_2\pi_{22}(\alpha_2)$ for the duration of the patent and $(1-\lambda_2)\pi_{22}(1)$ after its expiry.

It is assumed that the consumer surplus in both countries is 0 in the event that R&D is unsuccessful. Let $c_i(\alpha_i)$ be the instantaneous consumer surplus in country i conditional on a successful invention, when the degree of knowledge spillover is α_i. We assume that the instantaneous consumer surplus is increasing in the degree of knowledge spillover α_i, that is,

$$c'_i(\alpha_i) > 0. \tag{7}$$

[8]Observe that we make no assumption about the value of $\pi_{ij}(1)$. In the presence of a competitive fringe, $\pi_{ij}(1)$ could be assumed to be 0, but we do not make any such assumption since the competitive fringe does not play any role in our model.

Then the total consumer surplus in country 1, conditional on a successful invention, is given by

$$\begin{aligned} C_1 &= \lambda_1 c_1(\alpha_1) + (1-\lambda_1) c_1(1) \\ &= c_1(1) - \lambda_1 \int_{\alpha_1}^{1} c_1'(z)\, dz \end{aligned} \tag{8}$$

and in country 2 is given by

$$\begin{aligned} C_2 &= \lambda_2 c_2(\alpha_2) + (1-\lambda_2) c_2(1) \\ &= c_2(1) - \lambda_2 \int_{\alpha_2}^{1} c_2'(z)\, dz. \end{aligned} \tag{9}$$

Below, we demonstrate examples in which the reduced form assumptions on the profit and consumer surplus functions are satisfied.

Example 3 *Consider Example 1. Further, assume that the inverse demand function is given by*

$$p = a - q_1 - q_2.$$

Then, conditional on a successful innovation, the instantaneous profits of firms 1 *and* 2 *are*

$$\pi_{1i}(\alpha_i) = \left(\frac{a-\mu+(2-\alpha_i)\theta}{3}\right)^2 \text{ and } \pi_{2i}(\alpha_i) = \left(\frac{a-\mu-(1-2\alpha_i)\theta}{3}\right)^2$$

respectively. Moreover, conditional on a successful innovation, the instantaneous consumer surplus is given by

$$c(\alpha_i) = \frac{1}{18}\left\{4a^2 - (2\mu-(1+\alpha_i)\theta)^2\right\}.$$

Notice that,

$$\pi_{1i}'(\alpha_i) < 0,\ \pi_{2i}'(\alpha_i) > 0 \text{ and } c'(\alpha_i) > 0.$$

Hence, (1) and (7) are satisfied in the context of this example. ■

Example 4 *Consider Example 2. Further, assume that both firms have a constant marginal cost* *of γ. The preference of a consumer is given by*

$$U = \begin{cases} \theta_j v - p & \textit{if he purchases a good of quality } \theta_j, \\ 0 & \textit{otherwise.} \end{cases}$$

In this formulation, U can be thought of as the consumer's surplus. The "taste for quality" parameter v is uniformly distributed between $[\underline{v}, \overline{v}]$ where $\overline{v} = \underline{v} + 1$. The quality of firm 1's product is θ and the quality of firm 2's product is $\alpha_i \theta$. Then, conditional on a successful innovation, the instantaneous profits of firms 1 and 2 are

$$\pi_{1i}(\alpha_i) = \left(\frac{1 + \overline{v}}{3}\right)^2 (1 - \alpha_i)\,\theta \ \textit{and}\ \pi_{2i}(\alpha_i) = \left(\frac{1 - \underline{v}}{3}\right)^2 (1 - \alpha_i)\,\theta$$

respectively. Further, conditional on a successful innovation, the instantaneous consumer surplus is given by

$$c(\alpha_i) = \frac{1}{2}\theta\left(\overline{v} - \hat{v}\right)^2 + \frac{1}{2}\alpha_i\theta\left(\hat{v} - \underline{v}\right)^2 - \pi_{1i}(\alpha_i) - \pi_{2i}(\alpha_i) - \gamma,$$

where

$$\hat{v} = \frac{\overline{v} + \underline{v}}{3}.$$

Notice that,

$$\pi'_{1i}(\alpha_i) < 0,\ \pi'_{2i}(\alpha_i) < 0 \ \textit{and}\ c'(\alpha_i) > 0.$$

Hence, (1) and (7) are satisfied in the context of this example. ■

Observe that the sign of $\pi'_{2i}(\alpha_i)$ is different in the two examples. However, we have made no assumption about the sign of $\pi'_{2i}(\alpha_i)$ and hence, the assumptions of the model are not violated in these examples. Below, we use the consumer surplus and the profit of the domestic firm to define the welfare of each country. To do so, we define the expected consumer surplus in country i by

$$S_i = P(R_1)\,C_i; \ \ i = 1, 2.$$

The welfare W_i of country i is defined to be

$$W_i = \begin{cases} P(R_1)(C_1 + V_1) - R_1 & \text{for} \quad i = 1, \\ P(R_1)(C_2 + V_2) & \text{for} \quad i = 2. \end{cases}; \tag{10}$$

To analyze the welfare function, first consider country 1. In the event of a successful R&D, the consumers in country 1 enjoy a surplus of C_1 and firm 1 obtains a gross profit of V_1, while in the event of a failure in R&D, the consumers in country 1 as well as firm 1 get a payoff of 0. Hence, the expected gross payoff that accrues to country 1 is $P(R_1)(C_1 + V_1)$. The net payoff of country 1 can be then be determined by subtracting the cost of the R&D effort R_1. Next consider country 2. We assume that firm 2 can imitate firm 1's innovation with certainty by incurring a fixed cost. Such an assumption has been made in some other papers in the literature such as Mukherjee and Pennings (2004). In our framework, the fixed cost of imitation does not play an important role and has been normalized to 0. Hence, the welfare function of country 2 is as given in (10). Notice that country 2 has a stake in the success of R&D since otherwise, the ex post payoff of country 2 is 0 instead of $(C_2 + V_2)$.

In the model, country i maximizes W_i by selecting the patent length λ_i and the patent breadth $(1 - \alpha_i)$; $i = 1, 2$. In country 1, there is a tension between consumers in country 1 (who prefer a shorter patent length) and firm 1 (the innovating firm). The optimal patent length in country 1 therefore balances the tension between the consumers and firm 1. In country 2, the benefits from a strong patent regime does not accrue directly to its citizens and therefore it might seem that country 2 would free-ride on country 1's innovation by selecting excessively weak patent laws. This is however not true in this model because country 2 recognizes that excessively weak patent laws might reduce the incentive of firm 1 to conduct R&D and this in turn adversely affects country 2's welfare. The incentive of each country to provide patent protection is aptly summarized in the following quote by Scotchmer ((2004b), p. 415):

> "To a trade policy negotiator, profit earned abroad is unambiguously a good thing, and the consumers' surplus conferred on foreign consumers does not count at all. There is a domestic interest in capturing profit abroad, and symmetrically, there is a domestic interest in trying to ensure that domestic consumers get access to foreign inventions on competitive terms."

The timeline is similar to Kim and Lapan (2008) and is as follows: In period 1, the two countries simultaneously select their IP laws. Then in period 2, firm 1 determines its R&D effort. Finally, the outcomes of the R&D are known and firms make profits. In the following sections, we determine the subgame perfect Nash equilibrium of the game described above.

3 R&D Effort of Firm 1

In period 2, firm 1 selects its R&D effort that maximizes its net expected profit Π_1. It therefore follows from (2) that the profit maximizing R&D effort, denoted by R_1^*, satisfies the following equation:

$$P'\left(R_1^*\right) = \frac{1}{V_1}. \tag{11}$$

We define the marginal product of R&D as the increase in the probability of success in R&D due to a small change in the effort R_1. Hence, the marginal product in R&D is measured by $P'\left(R_1\right)$. From (11), it follows that at the optimum level of R&D for firm 1, the value of the marginal product of R&D, given by $V_1 P'\left(R_1^*\right)$, is equal to 1, which is the marginal cost of R&D. Also notice that R_1^* is a function of the patent length and the patent breadth of both countries, that is,

$$R_1^* = R_1^*\left(\lambda_1, \alpha_1, \lambda_2, \alpha_2\right).$$

In our analysis, the rate of change of the marginal product in R&D plays a crucial role and is

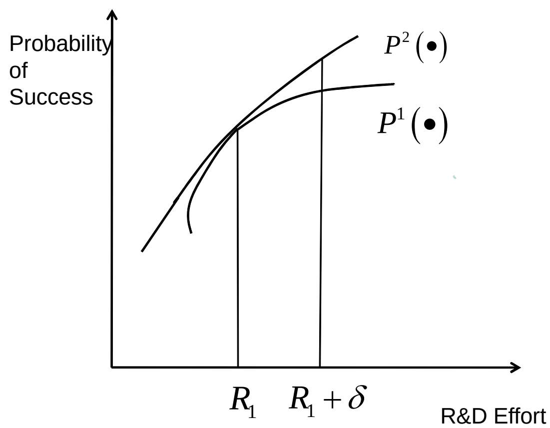

Figure 1: The relationship between curvature and chance of failure. A higher degree of curvature leaves firm 1 exposed to a higher chance of failure in R&D with the same additional effort.

denoted by $\sigma(R_1)$ where $\sigma(R_1)$ is as follows:

$$\sigma(R_1) = -\frac{d}{dR_1}\ln P'(R_1) = -\frac{P''(R_1)}{P'(R_1)}.$$

Notice that $\sigma(\cdot)$ is positively related to the curvature of the R&D production function $P(\cdot)$, that is, an increase in the curvature of $P(\cdot)$ leads to an increase in $\sigma(\cdot)$. The implication is that an increase in R_1 will increase $P(\cdot)$ by a smaller amount, the higher is the curvature of the function $P(\cdot)$. For concreteness, we refer to Figure 1 in which we compare two functions $P^1(\cdot)$ and $P^2(\cdot)$ such that $P^1(\cdot)$ is more curved than $P^2(\cdot)$. Further suppose that at the initial value of R_1, these two functions intersect, that is,

$$P^1(R_1) = P^2(R_1).$$

Then, it must be the case that for a small increase in R_1, say $\delta R_1 > 0$, the following inequality must be satisfied:

$$P^1(R_1 + \delta R_1) < P^2(R_1 + \delta R_1).$$

Therefore, a higher curvature of the function $P(\cdot)$ implies a smaller chance of success in R&D with the same additional effort. Below, we show that it is possible for the curvature of the

R&D production function to decrease "rapidly" with R_1 under certain circumstances, and for the curvature to either increase or decrease slowly under different circumstances.

Example 5 *Suppose the probability of success $P(R_1)$ in R&D is given by the distribution function of the exponential distribution with parameter $\mu > 0$. Therefore,*

$$P(R_1) = 1 - e^{-\mu R_1}; \; \mu > 0,$$

and it follows that

$$\sigma(R_1) = \mu$$

and hence,

$$\sigma'(R_1) = 0.$$

For later reference, also note that

$$\sigma^2(R_1) + \sigma'(R_1) = \mu^2 > 0.$$

Example 6 *Suppose the probability of success $P(R_1)$ in R&D is given by the distribution function of the beta distribution with parameters $(\mu, 1)$; $0 < \mu < 1$. Therefore,*

$$P(R_1) = R_1^{\mu}; \; 0 < \mu < 1 \; \& \; R_1 \in [0, 1],$$

and it follows that

$$\sigma(R_1) = \frac{1 - \mu}{R_1}.$$

For later reference, also note that

$$\sigma^2(R_1) + \sigma'(R_1) = -\frac{(1 - \mu)\mu}{R_1^2} < 0.$$

It will be shown below that one of the important determinants of the intellectual property law in a country is the manner in which the rate of change of the marginal product in R&D changes due to a change in the effort. Using (11) and the implicit function theorem, we derive the following lemma.

Lemma 1 *At the optimum level of R&D effort,*

$$\frac{\partial R_1^*}{\partial \lambda_1} = -\frac{P'(R_1^*)}{\sigma(R_1^*)} \int_{\alpha_1}^{1} \pi'_{11}(z)\, dz > 0, \tag{12}$$

$$\frac{\partial R_1^*}{\partial \lambda_2} = -\frac{P'(R_1^*)}{\sigma(R_1^*)} \int_{\alpha_2}^{1} \pi'_{12}(z)\, dz > 0, \tag{13}$$

$$\frac{\partial R_1^*}{\partial \alpha_1} = \frac{P'(R_1^*)}{\sigma(R_1^*)} \lambda_1 \pi'_{11}(\alpha_1) < 0 \tag{14}$$

and

$$\frac{\partial R_1^*}{\partial \alpha_2} = \frac{P'(R_1^*)}{\sigma(R_1^*)} \lambda_2 \pi'_{12}(\alpha_2) < 0. \tag{15}$$

Proof. See the Appendix. ■

Below, we interpret (12). Notice that

$$\frac{\partial V_1}{\partial \lambda_1} = \int_{\alpha_1}^{1} \pi'_{11}(z)\, dz$$

measures the change in the payoff of firm 1 due to a change in λ_1. Such a change in payoff induces firm 1 to expend more effort in R&D which changes both the marginal product in R&D (as measured by $P'(R_1^*)$) as well as the rate of change of the marginal product in R&D (as measured by $\sigma(R_1^*)$). Therefore,

$$\frac{P'(R_1^*)}{\sigma(R_1^*)}$$

measures the ratio of the marginal product to the rate of change of the marginal product in R&D. Hence, it follows from (12) that in equilibrium, the impact on the R&D effort of firm 1 of a change in λ_1 depends on the change in V_1 and the ratio of the marginal product in R&D to its rate of change. Similar logic applies for (13), (14) and (15). Further, the above inequalities state that other things remaining constant, the R&D effort of firm 1 increases in the patent lengths and patent breadths of the two countries. Therefore, stronger patent protection in either country results in an increase in the optimum effort of firm 1. However, the effectiveness of the IP protection in inducing R&D effort depends on the IP laws in the other country, and we show in the following lemma, that the nature of this relationship is characterized by the value of $\sigma'(R_1)$.

Lemma 2 *At the optimum level of R&D effort,*

$$sign\left(\frac{\partial^2 R_1^*}{\partial\lambda_1\partial\lambda_2}\right) = -sign\left(\sigma^2\left(R_1^*\right) + \sigma'\left(R_1^*\right)\right). \tag{16}$$

$$sign\left(\frac{\partial^2 R_1^*}{\partial\lambda_i\partial\alpha_j}\right) = sign\left(\sigma^2\left(R_1^*\right) + \sigma'\left(R_1^*\right)\right);\, i \neq j. \tag{17}$$

Further,

$$\sigma^2\left(R_1^*\right) + \sigma'\left(R_1^*\right) < 0 \Rightarrow \frac{\partial^2 R_1^*}{\partial\lambda_i\partial\alpha_i} < 0 \text{ and } \frac{\partial^2 R_1^*}{\partial\lambda_i\partial\alpha_i} > 0 \Rightarrow \sigma^2\left(R_1^*\right) + \sigma'\left(R_1^*\right) > 0;\;\; i = 1, 2. \tag{18}$$

Proof. See the Appendix. ■

It follows from (16) that the value of $\sigma'(R_1)$ is a crucial determinant of the effectiveness of a country's IP laws on the optimum amount of R&D effort. In particular, if the rate of change of marginal product in R&D decreases rapidly, that is, if

$$\sigma'\left(R_1^*\right) < -\sigma^2\left(R_1^*\right),$$

then the patent lengths are complementary inputs in the "production" of the invention, while if the rate of change of marginal product in R&D either increases or decreases slowly, that is, if

$$\sigma'\left(R_1^*\right) > -\sigma^2\left(R_1^*\right),$$

then the patent lengths are substitutable inputs. Hence, it follows from Example 6 that if $P(R_1)$ is the distribution function of Beta $(\mu, 1)$, then the patent lengths are complementary inputs, while it follows from Example 5 that if $P(R_1)$ is the distribution function of the exponential distribution, then the patent lengths are substitutable inputs. In the next section, we analyze country 2's problem and determine its choice of IP laws.

4 IP Law in Country 2 (Emerging Country)

We now analyze the factors that determine country 2's IP laws where country 2 refers to the emerging economy. In our analysis, we allow country 2 to select both its patent length λ_2 and its patent breadth $(1-\alpha_2)$. Therefore, country 2 solves the following problem:

$$\underbrace{Max}_{\lambda_2, \alpha_2} \quad W_2 = P\left(R_1^*\right)\left(C_2 + V_2\right), \tag{19}$$

where W_2 refers to country 2's welfare. For the discussion below, we define

$$\phi_i\left(\alpha_i\right) = c_i\left(\alpha_i\right) + \pi_{ii}\left(\alpha_i\right); \, i = 1, 2,$$

as the sum of the instantaneous consumer surplus in country i and the profit of firm i from country i, as a function of the patent breadth. The expression $\phi_i\left(\alpha_i\right)$ is the part of the consumer surplus and producer surplus generated in country i that accrues to country i when the degree of knowledge spillover is α_i. The total welfare of country i derives from $\phi_i\left(\alpha_i\right)$ and the producer surplus of firm i in country j but the latter has no direct impact on the IP laws in country i because it is not within the control of the government of country i.[9] The sign of $\phi_2'(\cdot)$ depends on $c_2'\left(\alpha_2\right)$ as well as on $\pi_{22}'\left(\alpha_2\right)$. Under the assumptions of the model, $c_2'\left(\alpha_2\right)$ is positive but the sign of $\pi_{22}'\left(\alpha_2\right)$ is ambiguous and hence, the sign of $\phi_2'(\cdot)$ is also ambiguous. Notice that $\phi_2'(\cdot) > 0$ in the case of the cost reducing innovation (described is Example 3) as well as the case of the quality enhancing innovation (described in Example 4). Hence, we assume that

$$\phi_2'(\cdot) > 0.$$

An implication of the above assumption is that the optimal patent lengths and the optimal patent breadth of the emerging economy have an interior solution. If, on the other hand, we had assumed that $\phi_2'(\cdot) < 0$, then, it would follow from (21) below that the welfare of country 2 would have been maximized for $\alpha_2 = 0$, that is, the emerging economy would have selected the maximum possible patent breadth. Notice that the term

$$\int_{\alpha_2}^{1} \phi_2'(z)\, dz$$

captures the loss of welfare to country 2 because the government in country 2 selects the degree of knowledge spillover as α_2 instead of 1. This excess burden emanates because a change in the

[9] The producer surplus of firm i in country j is internalized by a global planner but a global planner is not the focus of this section.

degree of knowledge spillover changes consumers' surplus and producers' surplus in country 2 and also because it changes the distribution of the producers' surplus between the domestic and the foreign firm. In the subsequent discussion, we call this the *instantaneous excess burden in country 2*.

From the definition of W_2, it follows that:

$$\frac{\partial W_2}{\partial \lambda_2} = P'\left(R_1^*\right)\left(C_2 + V_2\right)\frac{\partial R_1^*}{\partial \lambda_2} - P\left(R_1^*\right)\int_{\alpha_2}^{1} \phi_2'\left(z\right)dz \tag{20}$$

and

$$\frac{\partial W_2}{\partial \alpha_2} = P'\left(R_1^*\right)\left(C_2 + V_2\right)\frac{\partial R_1^*}{\partial \alpha_2} + P\left(R_1^*\right)\lambda_2\phi_2'\left(\alpha_2\right) \tag{21}$$

In (20) above, the first term on the right hand side is the indirect effect of increasing the patent length while the second term measures the direct effect of increasing the patent length. The two effects capture the two channels through which an increase in patent length affects welfare. If there is an increase in the patent length in country 2, then other things remaining constant, consumers and firm 2 are adversely affected because they have to bear the excess burden for a longer duration and this effect is captured by the direct effect. However, when there is an increase in the patent length in country 2, then other things remaining constant, firm 1 expends a higher degree of effort in R&D and this benefits consumers in country 2 and firm 2 by increasing the chance of a successful invention. This second effect is captured by the indirect effect. The two terms in (21) have an analogous interpretation.

At the optimum, the country 2 selects λ_2 and α_2 to satisfy the following conditions:

$$\frac{\partial W_2}{\partial \lambda_2} = 0, \tag{22}$$

$$\frac{\partial W_2}{\partial \alpha_2} = 0. \tag{23}$$

We now use (22) and (23) to determine the relationship between the optimal λ_2 and the optimal α_2, given a fixed structure of laws in country 1 (the developed economy). To do so, notice that by dividing the two first order conditions, we obtain that

$$\frac{\lambda_2'\left(\alpha_2\right)}{\lambda_2\left(\alpha_2\right)} = -\frac{\phi_2'\left(\alpha_2\right)}{\int_{\alpha_2}^{1}\phi_2'\left(z\right)dz} \tag{24}$$

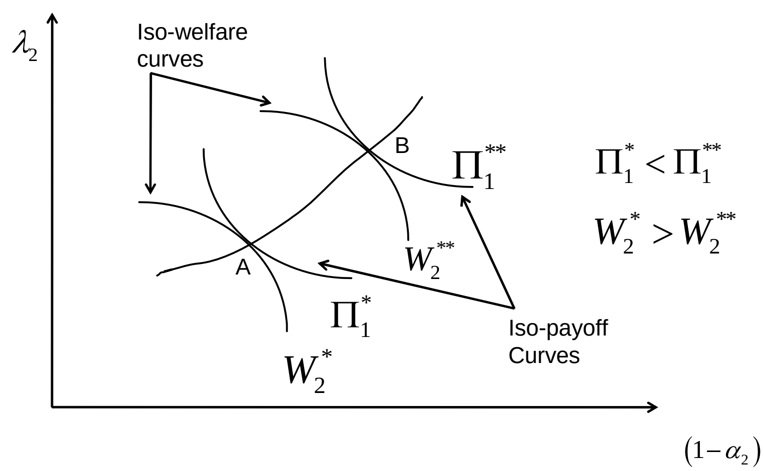

Figure 2: The locus of the points of tangencies between the iso-welfare curves of country 2 and iso-payoff curves of firm 1.

and hence,

$$d \ln \lambda_2 (\alpha_2) = d \ln \left(\int_{\alpha_2}^{1} \phi_2' (z) \, dz \right). \tag{25}$$

Notice from (25) that at the optimum choice of patent length and breadth for country 2, the elasticity of the patent length is equal to the elasticity of the instantaneous excess burden. Since the instantaneous excess burden in country 2 is an increasing function of the patent breadth $(1 - \alpha_2)$, therefore, the optimal patent length is positively related to the optimal patent breadth, *given a fixed structure of IP laws abroad (country* 1*)*. This implies that when a situation warrants that country 2 select a strong (resp., weak) degree of patent protection, country 2 achieves this partly by a long (resp., short) patent length and partly by a broad (resp., narrow) patent breadth. Hence, the "expansion path" of IP laws (depicted by the curve AB in Figure 2) is positively sloped.

In the figure, the iso-payoff curves of firm 1 and the iso-welfare curves of country 2 have been drawn as a function of the patent length and the patent breadth. The iso-payoff curves increase and the iso-welfare curves decrease with an increase in the patent length and the patent breadth. Conditional on a fixed payoff to firm 1, country 2 can maximize its welfare by selecting the patent length and breadth at the point of tangency of the iso-welfare curve and the iso-payoff curve. From (25), we can conclude that the locus of the points of tangencies is positively sloped.

We can also derive the following functional relationship between the optimal values of λ_2 and α_2.

Proposition 1 *Fix λ_1 and α_1. The ratio of the patent length and the instantaneous excess burden of country 2 is a constant and is equal to the ratio of the patent length and the instantaneous excess burden when the patent breadth is set at the maximum possible level. In other words,*

$$\frac{\lambda_2(\alpha_2)}{\int_{\alpha_2}^1 \phi_2'(z)\,dz} = \frac{\lambda_2(0)}{\int_0^1 \phi_2'(z)\,dz}. \tag{26}$$

In the above expression, the ratio on the right hand side depends on the IP law of country 1.

Proof. See the Appendix. ■

It follows from (26) that any change in the degree of protection in country 1 leads to a change in $\lambda_2(0)$ and consequently, for every level of α_2, there is a proportionate change in $\lambda_2(\alpha_2)$. Further, $\lambda_2(1) = 0$ and hence, a change in the degree of protection in country 1 pivots the schedule relating the optimal patent length and the optimal patent breadth in country 2 around the origin. *What will be the direction of such a pivot if there is an increase in the level of IP protection in country 1?* A natural conjecture is that if country 1 increases its degree of IP protection, then it would induce country 2 to free ride on country 1's laws by weakening its patent laws (in which case the schedule AB in Figure 3 would pivot downwards). However, in this model, an additional effect occurs which can pivot the schedule AB upwards under certain circumstances.

To see this, let country 1 increase λ_1. In response, country 2 would like to free ride on country 1's improved patent protection by decreasing its patent length. We call this the *free-rider effect.* If the R&D effort had been exogenous, then the free rider effect would have been the only effect and hence, in such a model, country 2 would have unambiguously reduced its patent length in response to an increase in country 1's patent length. But in our model, R&D effort is endogenous and as a result there is a second effect known as the *productivity effect.* We show below that under certain circumstances, the productivity effect may reinforce the free-rider effect and under other circumstances, it may weaken the free-rider effect. To examine the role of the productivity effect, imagine that the IP laws are inputs in the production of R&D effort. Now consider a situation in which country 1 increases its degree of patent protection and country 2 reduces λ_2 in response.

First, consider the familiar case of $\frac{\partial^2 R_1^*}{\partial \lambda_1 \partial \lambda_2} < 0$, that is, suppose that the patent lengths are

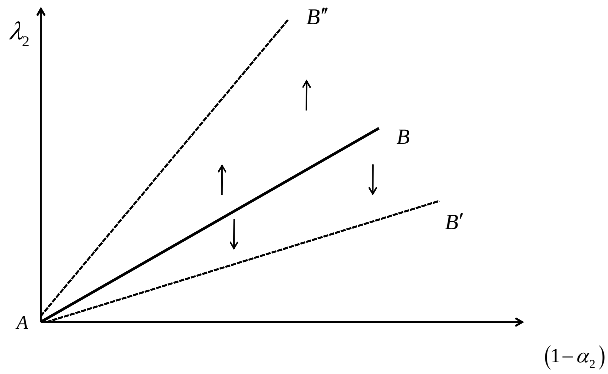

Figure 3: The schedule AB relates the optimal patent length in country 2 with its optimal patent breadth, given a fixed patent length in country 1. An increase in the patent length in country 1 pivots the schedule downwards (resp., upwards) if the patent lengths are substitutable (resp., strongly complementary) inputs.

substitutable inputs in the production of R&D. In this case, when country 2 reduces λ_2, then the marginal productivity of λ_1 increases (that is, $\frac{\partial R_1^*}{\partial \lambda_1}$ increases) and hence, the effectiveness of country 1's law is strengthened. Observe that if the patent lengths are substitutable inputs, then the productivity effect reinforces the free rider effect and thus increases the incentive of country 2 to reduce its level of patent protection. Consequently, the schedule AB in Figure 3 pivots downwards. Therefore, when the patent lengths are substitutable inputs, then the optimal λ_2 is a decreasing function of λ_1. Next, consider the case in which patents are complementary inputs, that is, $\frac{\partial^2 R_1^*}{\partial \lambda_1 \partial \lambda_2} > 0$. In this case, a reduction in λ_2 leads to a reduction in the marginal productivity of λ_1 (that is, $\frac{\partial R_1^*}{\partial \lambda_1}$ goes down) and this weakens the effectiveness of country 1's laws. In this case, the productivity effect imposes a cost associated with a reduction in λ_2 and therefore dampens country 2's incentive to reduce λ_2. If the patent laws are strong complements, then the productivity effect can overturn the free rider effect and hence in this case, country 2 cannot free ride on country 1's improved patent protection. Consequently there is an upward pivot of the schedule relating patent length and breadth in country 2. This argument is presented formally in Proposition 2 below.[10]

Given that λ_2 and α_2 are strictly monotonically related, we can re-write country 2's maximization

[10]Note that in terms of Figure 2, an increase in the degree of patent protection in country 1 changes the R&D effort which can result in the iso-welfare curves becoming flatter in some cases and steeper in others. This is what causes the curve AB to sometimes pivot upwards (and sometimes downwards) in response to stronger IP protection in country 1.

as follows:

$$\underbrace{Max}_{\lambda_2} \quad W_2\left(\lambda_2, \alpha_2\left(\lambda_2\right)\right).$$

The slope of the reduced form welfare function of country 2 is as follows:

$$\frac{dW_2}{d\lambda_2} = \frac{\partial W_2}{\partial \lambda_2} + \frac{\partial W_2}{\partial \alpha_2}\alpha_2'\left(\lambda_2\right)$$

and using (22), (23) and (24), we obtain the following:

$$\frac{dW_2}{d\lambda_2} = P'\left(R_1^*\right)\left(C_2 + V_2\right)\left[\frac{\partial R_1^*}{\partial \lambda_2} + \alpha_2'\left(\lambda_2\right)\frac{\partial R_1^*}{\partial \alpha_2}\right] - 2P\left(R_1^*\right)\int_{\alpha_2}^{1} \phi_2'\left(z\right)dz. \tag{27}$$

At the optimum, country 2 selects λ_2 such that

$$\frac{dW_2}{d\lambda_2} = 0. \tag{28}$$

The solution of the above equation is denoted by $\hat{\lambda}_2\left(\lambda_1\right)$ and is known as the reaction function of country 2 as a function of λ_1. The reaction function of country 2 captures the total impact on the optimal λ_2 due to a change in λ_1, where the total impact includes the direct impact on λ_2 because of the change in λ_1 and an indirect impact on λ_2 because of a change in α_2. We now determine the slope of $\hat{\lambda}_2\left(\lambda_1\right)$ and show that the rate of change of the marginal product in R&D is a crucial determinant of whether the reaction functions are downward or upward sloping, that is, whether patent lengths are strategic substitutes or strategic complements (Bulow et al., 1985).

Proposition 2 *(i) If $\pi_{21}'\left(\alpha_1\right) > 0$ and $\sigma'\left(R_1^*\right) > -\sigma^2\left(R_1^*\right)$ are satisfied, then $\hat{\lambda}_2'\left(\lambda_1\right) < 0$. (ii) If $\hat{\lambda}_2'\left(\lambda_1\right) > 0$, then either $\pi_{21}'\left(\alpha_1\right) < 0$ or $\sigma'\left(R_1^*\right) < -\sigma^2\left(R_1^*\right)$.*

Proof. See the Appendix. ■

The first part of the proposition above states that the reaction function of country 2 is downward sloping if two conditions are satisfied: *(a)* The rate of change of the marginal product in R&D (measured by the curvature of the production function) does not decrease very rapidly with an increase in the R&D effort, and (b) the profit of firm 2 is an increasing function of the degree of knowledge spillover (or a decreasing function of the patent breadth) in country 1. If (a) holds, then Lemma 2 implies that the patent lengths are substitutable inputs and hence, it follows from the discussion above that λ_2 decreases in response to an increase in λ_1. If (b) holds, then firm 2's flow of profits in a country is a decreasing function of its patent breadth. This condition is satisfied for a cost-reducing technology as described in Example 3. Notice that if country 1 increases its patent length, then this reduces the net payoff earned by firm 2 in country 1.[11] Country 2 then compensates for this loss by reducing its own patent length.

The second part of the proposition states the necessary conditions under which the reaction function of country 2 is upward sloping. In particular, if the reaction function of country 2 is upward sloping, then either (c) the rate of change of the marginal product in R&D (measured by the curvature of the production function) must decrease very rapidly with an increase in R&D effort, or (d) the profit of firm 2 is an increasing function of the patent breadth in country 1. If (c) is satisfied, then the patent lengths are complementary inputs and the result follows from the discussion above. If (d) is satisfied, then the profit of firm 2 is an increasing function of the patent breadth of country 1. This condition is satisfied for a quality enhancing technology (as described in Example 4).

5 IP Law in Country 1 (Developed Country)

In this section, we analyze the factors that determine the IP regime in country 1. In period 1, country 1 maximizes its welfare by determining its patent length λ_1 and its patent breadth $(1-\alpha_1)$. Formally, country 1 solves the following problem:

$$\underbrace{Max}_{\lambda_1,\alpha_1} \quad W_1 = P\left(R_1^*\right)\left(C_1+V_1\right) - R_1^*.$$

The instantaneous surplus accruing to the consumers of country 1 and firm 1 from country 1 is defined by

$$\phi_1\left(\alpha_1\right) = c_1\left(\alpha_1\right) + \pi_{11}\left(\alpha_1\right).$$

[11] The gross payoff of firm 2 in country 1 is

$$\lambda_1 \pi_{21}\left(\alpha_1\right) + \left(1-\lambda_1\right)\pi_{21}\left(1\right)$$

which is decreasing in λ_1 whenever

$$\pi_{21}\left(\alpha_1\right) < \pi_{21}\left(1\right).$$

Notice that $\phi_1(\alpha_1)$ is the portion of the welfare of country 1 that originates from country 1. Analogous to the excess burden of country 2, we define the *instantaneous excess burden in country* 1 by the term

$$\int_{\alpha_1}^{1} \phi_1'(z)\, dz.$$

From the definition of W_1, it follows that:

$$\frac{\partial W_1}{\partial \lambda_1} = \left[P'(R_1^*)(C_1+V_1)-1\right]\frac{\partial R_1^*}{\partial \lambda_1} - P(R_1^*)\int_{\alpha_1}^{1} \phi_1'(z)\, dz \tag{29}$$

and using (11), we can re-write (29) as follows:

$$\frac{\partial W_1}{\partial \lambda_1} = P'(R_1^*)\, C_1 \frac{\partial R_1^*}{\partial \lambda_1} - P(R_1^*)\int_{\alpha_1}^{1} \phi_1'(z)\, dz. \tag{30}$$

In the above expression, the term $P(R_1^*)\int_{\alpha_1}^{1} \phi_1'(z)\, dz$ measures the *direct effect* of changing the patent length on the welfare of country 1, while $P'(R_1^*)\, C_1 \frac{\partial R_1^*}{\partial \lambda_1}$ measures the *indirect effect* of changing the patent length by its impact on R&D effort. The direct effect measures the impact of a change in the patent length in country 1 on the excess burden in country 1, while the indirect effect measures the impact on welfare of a change in the patent length due to its effect on the level of R&D effort. Analogous to (30), we also determine that

$$\frac{\partial W_1}{\partial \alpha_1} = P'(R_1^*)\, C_1 \frac{\partial R_1^*}{\partial \alpha_1} + P(R_1^*)\, \lambda_1 \phi_1'(\alpha_1). \tag{31}$$

Notice that if $\phi_1'(\alpha_1) < 0$, then

$$\frac{\partial W_1}{\partial \lambda_1} > 0 \text{ for all } \lambda_1,$$

and

$$\frac{\partial W_1}{\partial \alpha_1} < 0 \text{ for all } \alpha_1.$$

In such a case, the optimal λ_1 would have been 1 and the optimal α_1 would have been 0, that is, country 1 would have chosen the maximum possible level of IP protection. In order to avoid such corner solutions, henceforth, we assume that

$$\phi_1'(\cdot) > 0. \tag{32}$$

This assumption is satisfied for a quality-enhancing technology (as described in Example 4) but not for a cost-reducing technology (as described in Example 3). In the next section, we discuss the impact on the results of a violation of this assumption. Given the above assumption, at the optimum, country 1 selects its patent length λ_1 such that

$$\frac{\partial W_1}{\partial \lambda_1} = 0, \tag{33}$$

and

$$\frac{\partial W_1}{\partial \alpha_1} = 0. \tag{34}$$

Before proceeding, we discuss the impact of introducing endogeneity of R&D effort in the model. This will also allow us to compare the results of this model with the ones obtained in previous work, such as Gilbert and Shapiro (1990) and Klemperer (1990). If R&D had been exogenous, then the first term in the right hand side of (31) would have dropped out. Hence, with exogenous R&D effort,

$$sign\left(\frac{\partial W_1}{\partial \alpha_1}\right) = sign\left(\phi_1'\left(\alpha_1\right)\right).$$

Therefore, if $\phi_1'(\alpha_1) > 0$ for all α_1, then the optimal value of $\alpha_1 = 1$, that is, narrow patents would be optimal. This result is similar to Gilbert and Shapiro (1990) and to some extent, to Klemperer (1990) also. However, with endogenous R&D, the term $P'\left(R_1^*\right) C_1 \frac{\partial R_1^*}{\partial \alpha_1}$ is included in (31) and since this term is negative, therefore, it tends to reduce the optimal value of α_1. Consequently, endogeneity of R&D tends to increase the optimal patent breadth under the assumption that $\phi_1'(\alpha_1) > 0$ for all α_1. In contrast, if $\phi_1'(\alpha_1) < 0$ for all α_1, then broad patents are optimal both when R&D effort is exogenous and when it is endogenous. It also follows from (20) and (21) that if R&D effort is exogenous and if $\phi_2'(\alpha_2) > 0$ for all α_2, then the emerging economy selects $\alpha_2 = 1$ and $\lambda_2 = 0$. Consequently, the emerging economy does not provide any IP protection under these conditions.

Denicolò (1996) considers a model in which the government selects the patent length and breadth to induce a target level of R&D effort. His focus is on finding sufficient conditions under which the optimal patent breadth is at an extremity. Naturally, this paper does not discuss the impact on the IP laws of a change in the target level of R&D effort. After all, if the patent breadth is already at an extremity, then an increase in the target level cannot have any impact on the patent breadth and the adjustment is made entirely through a change in the patent length. Our purpose is to analyze the impact of a change in the IP law of one country on the IP laws in another country. In this setting, the incremental level of inducement in R&D that a country offers depends on the inducement offered by the other country. Because this exercise is more interesting for interior solutions, therefore, we focus on the interior solution.

We now use (33) and (34) to determine the relationship between λ_1 and α_1 when R&D effort is assumed to be endogenous. To do so, notice that by dividing these two equations, it follows that

$$\frac{\lambda_1'(\alpha_1)}{\lambda_1(\alpha_1)} = -\frac{\phi_1'(\alpha_1)}{\int_{\alpha_1}^1 \phi_1'(z)\,dz} < 0 \tag{35}$$

from which we obtain the following:

$$d\ln\lambda_1(\alpha_1) = d\ln\left(\int_{\alpha_1}^1 \phi_1'(z)\,dz\right). \tag{36}$$

It follows from (36) that if (32) is satisfied, then the developed economy also selects its patent length and patent breadth such that the elasticity of the patent length is equal to the elasticity of the excess burden. Since the excess burden is positively related to the patent breadth, therefore, the model predicts that the optimal patent length in country 1 is positively related to the optimal patent breadth even for the developed economy. In the following proposition, we derive the functional relationship between the optimal patent length and the optimal patent breadth of country 1, given a fixed structure of IP laws in country 2.

Proposition 3 *Suppose $\phi_1'(\alpha_1) > 0$ for all α_1. Then, for fixed λ_2 and α_2, the ratio of the patent length and the instantaneous excess burden of country 1 is a constant that is equal to the ratio of the patent length and the instantaneous excess burden when the patent breadth is set at the maximum possible level. In other words,*

$$\frac{\lambda_1(\alpha_1)}{\int_{\alpha_1}^1 \phi_1'(z)\,dz} = \frac{\lambda_1(0)}{\int_0^1 \phi_1'(z)\,dz}. \tag{37}$$

In the above expression, the ratio on the right hand side depends on the IP law of country 2.

Proof. Analogous to the proof of Proposition 1. ■

The above proposition implies that even for developed economies, the optimal patent length is an increasing function of the optimal patent breadth when $\phi_1'(\alpha_1) > 0$, given a fixed structure of IP laws in country 2. Given that λ_1 and α_1 are strictly monotonically related, we can re-write country

1's maximization as follows:

$$\underbrace{Max}_{\lambda_1} \quad W_1\left(\lambda_1, \alpha_1\left(\lambda_1\right)\right).$$

The slope of the reduced form welfare function of country 1 is as follows:

$$\frac{dW_1}{d\lambda_1} = \frac{\partial W_1}{\partial \lambda_1} + \frac{\partial W_1}{\partial \alpha_1}\alpha_1'\left(\lambda_1\right)$$

and using (30), (31) and (35), we obtain the following:

$$\frac{dW_1}{d\lambda_1} = P'\left(R_1^*\right) C_1 \left[\frac{\partial R_1^*}{\partial \lambda_1} + \alpha_1'\left(\lambda_1\right)\frac{\partial R_1^*}{\partial \alpha_1}\right] - 2P\left(R_1^*\right)\int_{\alpha_1}^{1} \phi_1'\left(z\right) dz. \tag{38}$$

At the optimum, country 1 selects λ_1 such that

$$\frac{dW_1}{d\lambda_1} = 0. \tag{39}$$

The solution of the above equation is denoted by $\hat{\lambda}_1\left(\lambda_2\right)$ and is known as the reaction function of country 1 as a function of λ_2. The reaction function of country 1 captures the total impact on the optimal λ_1 due to a change in λ_2, where the total impact includes the direct impact on λ_1 because of the change in λ_2 and an indirect impact on λ_1 because of a change in α_1. We now determine the slope of $\hat{\lambda}_1\left(\lambda_2\right)$ and show that the rate of change of the marginal product in R&D is a crucial determinant of whether the reaction functions are downward or upward sloping, that is, whether patent lengths are strategic substitutes or strategic complements (Bulow et al., 1985).

Proposition 4 *(i) If $\sigma'\left(R_1^*\right) > -\sigma^2\left(R_1^*\right)$ is satisfied, then $\hat{\lambda}_1'\left(\lambda_2\right) < 0$. (ii) If $\hat{\lambda}_1'\left(\lambda_2\right) > 0$, then $\sigma'\left(R_1^*\right) < -\sigma^2\left(R_1^*\right)$.*

The first part of the proposition above states that the reaction function of country 1 is downward sloping if the rate of change of the marginal product in R&D (measured by the curvature of the production function) itself does not decrease too rapidly with an increase in the R&D effort. If $\sigma'\left(R_1^*\right) > -\sigma^2\left(R_1^*\right)$, then Lemma 2 implies that the patent lengths are substitutable inputs and hence, it follows from the discussion in the section above that λ_1 decreases in response to an increase in λ_2. The second part of the proposition states the necessary condition under which the reaction function of country 1 is upward sloping. In particular, if the reaction function of country

1 is upward sloping, then the rate of change of the marginal product in R&D (measured by the curvature of the production function) must decrease very rapidly with an increase in R&D effort.

There may be a concern that in practice, the patent breadth in the developed countries is anyway set at the maximum possible level, and hence for all practical purposes, the only policy lever that governments in developed countries have is the patent length. We can address this concern in two ways that are described below. First, we show that the basic conclusions of the model would not change even if this concern was true in practice. To see this, suppose country 1 sets its patent breadth at the maximum level and only chooses the length. In terms of our model, this is equivalent to saying that α_1 is exogenously set equal to 0, and country 1 effectively only chooses λ_1. In that case, the first order condition of country 1 would be as follows:

$$\frac{\partial W_1}{\partial \lambda_1} = P'\left(R_1^*\right) C_1 \frac{\partial R_1^*}{\partial \lambda_1} - P\left(R_1^*\right) \int_0^1 \phi_1'\left(z\right) dz = 0. \tag{40}$$

Note that (40) is derived from (30) with $\alpha_1 = 0$. Following similar methods as above, it can be shown that $\hat{\lambda}_1'\left(\lambda_2\right)$ will positive in some cases and negative in other cases.

Second, we show that even empirically, there are cases that suggest that the patent breadth is not automatically set at the maximum level in the developed countries. In this context, we highlight certain examples from the pharmaceutical industry in USA. In the American pharmaceutical industry, the entry of generics is governed by the Drug Price Competition and Patent Term Restoration Act (1984), commonly known as the Hatch-Waxman Act. One of the provisions of this act (Paragraph IV) allows a generic producer to file an application (known as Abbreviated New Drug Application, or ANDA) if the generic producer can certify that the generic drug does not infringe upon an existing patent or that the patent is not valid. Further, the applicant is required to notify the original patentee of this application and the patentee has to sue within 45 days for infringement. Such litigation can sometimes go in favor of the patentee but sometimes the challenger ends up as the winner. Indeed, the FTC in a report in 2002 examined the outcome of 53 of these cases (Table 2-2, p. 17 of the report). Out of these 53 cases, 20 were settled by the brand-name company and the first generic applicant, 22 of these cases were won by the generic applicant and only 8 were won by the brand-name company. In 2007, the Federal Trade Commission prepared a report for a Congressional committee that provided examples of certain generics which were launched before the expiry of the patent on the branded drug. As an example, it follows from page 10 of the report that the generic equivalent of the drug Prilosec (produced by Kudco which is a subsidiary of Schwarz Pharma Group) was launched in 2002 while the patents on the drug are scheduled to expire as late as 2018. Another example is a settlement between Ranbaxy and Pfizer over the rights to Lipitor. A press release by Ranbaxy (dated June 18, 2008) announced that Ranbaxy would have the right to sell the generic version of Lipitor (known as Atorvastatin) in the United States from November 30, 2011. The same press release further noted that "The Atorvastatin patents involved in this agreement are the basic compound patent, which expires in the United States in 2010; the enantiomer patent, which expires in the United States in 2011; and various process and crystalline form patents, which expire in 2016 and 2017; and the combination patent for fixed-dose combination product which expires in 2018." Notice that the last patent mentioned in the agreement will expire only in 2018. To summarize, a patentee has to prove that its patent is valid and that an imitator's action constitutes an infringement. If the patent breadth had indeed been set at the maximum level, then the patentees would have almost no incentive to settle and would have won in a vast majority of the lawsuits. But the evidence does not corroborate this.

6 Equilibrium and Comparative Statics

In this model, the governments simultaneously choose their IP laws in the first period and firm 1 selects its R&D effort in period 2. Therefore, the equilibrium concept that we use is sub-game perfect Nash equilibrium and we determine the equilibrium by backward induction. The equilibrium of the model is defined as follows:

Definition 1 *The subgame perfect Nash Equilibrium is the profile* $(\lambda_1^*, \lambda_2^*, R_1^*)$ *that satisfies (11),* $\hat{\lambda}_1(\lambda_2^*) = \lambda_1^*$ *and* $\hat{\lambda}_2(\lambda_1^*) = \lambda_2^*$

Below, we discuss the comparative statics of the model. In order to do so, we need to determine the relative slopes of the reaction functions which are commonly known as the stability conditions. Below we show that stability of the model can be guaranteed if the following assumption is satisfied by the primitives of the model in a neighborhood of the equilibrium:

$$\text{For any } i = 1, 2, \ \left|\frac{\partial^2 W_i}{\partial \lambda_i^2}\right| > \left|\frac{\partial^2 W_i}{\partial \lambda_1 \partial \lambda_2}\right|. \tag{41}$$

If condition (41) is satisfied, then in a neighborhood of the equilibrium, the absolute value of the slope of $\hat{\lambda}_1(\lambda_2)$ with respect to λ_1 must be greater than the absolute value of the slope of $\hat{\lambda}_2(\lambda_1)$ with respect to λ_1. This is demonstrated below:

Lemma 3 *Suppose (41) is satisfied. Then, there exists a neighborhood of the equilibrium such that*

$$\left|\hat{\lambda}_2'(\lambda_1)\right| < 1 < \left|\frac{1}{\hat{\lambda}_1'(\lambda_2)}\right|. \tag{42}$$

If the reaction functions of both countries are negatively (resp., positively) sloped, then condition (42) implies that in Figures 4, 5 and 6, the reaction function of country 1 is steeper than the reaction function of country 2 around the equilibrium point. Notice that this ensures that the reaction functions do not violate the stability conditions. This result will also be used in deriving the results below.

We now use Lemma 3 to derive some comparative static results. In order to present our results, we denote the equilibrium patent length of country i at the initial value of some parameter by λ_i^* and the equilibrium patent length of country i after the change in that parameter by λ_i^{**}. In other words, the original equilibrium is the point $(\lambda_1^*,\ \lambda_2^*)$ and the new equilibrium is at the point $(\lambda_1^{**},\ \lambda_2^{**})$. For comparative statics, we focus on $c_2(1)$ that measures the willingness-to-pay in the emerging economy. Below, we analyze the impact of an increase in the willingness-to-pay in the emerging economy (possibly because of an increase in income), both when $\phi_1'(\alpha) > 0$ (as in Example 4) and when $\phi_1'(\alpha) < 0$ (as in Example 3).

6.1 Impact of a change in $c_2(1)$ when $\phi_1'(\alpha) > 0$

Suppose the willingness-to-pay in the emerging economy, given by $c_2(1)$, increases. Then, we find that the impact on the patent lengths and breadths depend on the reaction functions of the two countries. Therefore, we analyze the following cases:

Case 1 $\hat{\lambda}_1'(\lambda_2) < 0$ *and* $\hat{\lambda}_2'(\lambda_1) < 0$

In this case, the patent lengths of both countries are strategic substitutes. It follows from Propositions (2) and (4) that such a case can arise when the rate of change of the marginal product in R&D (measured by the curvature of the R&D production function) does not decrease too rapidly, that is, when $\sigma'(R_1^*) > -\sigma^2(R_1^*)$. In this case, the emerging economy increases its patent length and the developed economy reduces its patent length. This result is summarized below.

Proposition 5 *Suppose (42) is satisfied. Further, let* $\hat{\lambda}_1'(\lambda_2) < 0$ *and* $\hat{\lambda}_2'(\lambda_1) < 0$. *Then, an increase in* $c_2(1)$ *has the following impact on the patent lengths:*

$$\lambda_1^{**} < \lambda_1^* \text{ and } \lambda_2^{**} > \lambda_2^*.$$

The intuition behind Proposition 5 is summarized in the first quadrant of Figure 4. Note that when there is a change in the willingness-to-pay in country 2, the reaction function of country 2 (the emerging country) shifts upward, while the reaction function of country 1 (the developed country) does not shift at all. Further, in this case, both reaction functions are downward sloping. Observe that (42) then implies that for $\lambda_1 < \lambda_1^*$, the reaction function function of country 2 lies below the reaction function of country 1, while for $\lambda_1 > \lambda_1^*$, the reaction function of country 2 lies above the reaction function of country 1. Hence, the new equilibrium, given by point D in Figure 4 lies along the reaction function of country 1 to the left of initial equilibrium, given by point A. Consequently, in this case, the patent length increases in country 2 and reduces in country 1. In other words, an increase in the willingness to pay in country 2 induces country 2 to increase its patent length and since for country 1, patent lengths are strategic substitutes, therefore country 1 free rides on the improved patent protection of country 2 by reducing its patent length.

Finally, one may want to know the impact on the patent breadths of an increase in the willingness-to-pay in the emerging economy. The impact on the patent breadth in country 2 can be determined from the second quadrant in Figure 4. In this quadrant, the schedules relating the optimal patent breadth and the optimal patent length of country 2 have been plotted, for a fixed value of λ_1. Given that the patent lengths are substitutable inputs for country 2, therefore, a decrease in λ_1 leads to an upward pivot of the schedule relating optimal patent length and optimal patent breadth in country 2.[12] The net impact on the patent breadth in country 2 therefore depends on the magnitude of the upward pivot of the schedule in the second quadrant of Figure 4. If the upward pivot is large enough (as in Figure 4), the patent breadth in country 2 decreases from point B to point E. However, if the upward pivot had been small enough, then the patent breadth in

[12] This follows from the discussion around Figure 3.

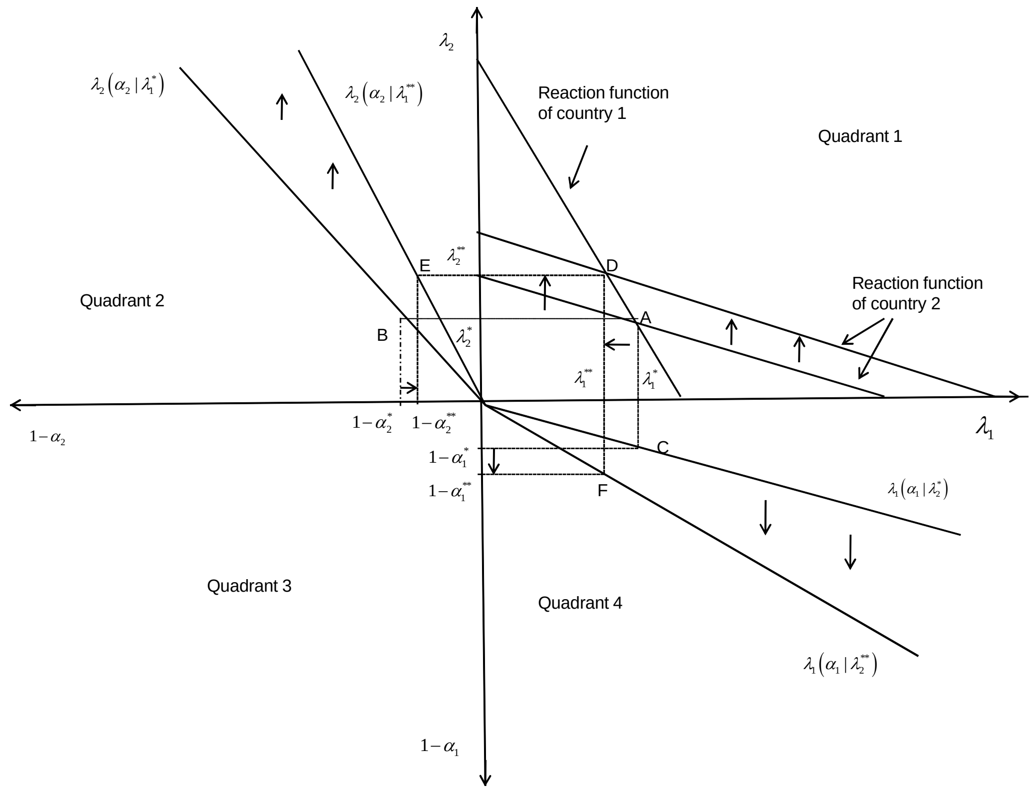

Figure 4: The impact of a change in the willingness-to-pay in country 2 when the reaction functions of both countries are downward sloping. In this case, an increase in the willingness-to-pay in country 2 changes the profile of patent lengths from A to D. Further, the patent breadth of country 2 shifts from B to E while the patent breadth of country 1 shifts from C to F. Hence, there is a reduction in the patent length in country 1 and an increase in the patent length in country 2. The impact on the patent breadths is ambiguous.

country 2 would have increased. Similarly, the impact on the patent breadth of country 1 depends on the magnitude of pivot of the schedule relating the optimal patent length and the optimal patent breadth in country 1.[13]

Assume that the patent length and the patent breadth are greater in the developed economy initially. Then, in this case, an increase in the willingness-to-pay in the emerging economy leads to a convergence of patent lengths but not necessarily in patent breadths. The conclusion is that convergence in one aspect of the patent law need not imply convergence in all aspects.

Case 2 $\hat{\lambda}_1'(\lambda_2) < 0$ *and* $\hat{\lambda}_2'(\lambda_1) > 0$

In this case, the patent lengths are strategic substitutes for country 1 but are strategic complements for country 2. By comparing the results from Propositions (2) and (4), it follows that

[13]Note that the relationship between the optimal patent length and the optimal patent breadth in the previous two sections have been derived under the assumption that the structure of IP law in the foreign country is fixed. In contrast, the analysis in this section allows the law in the foreign country to vary as well.

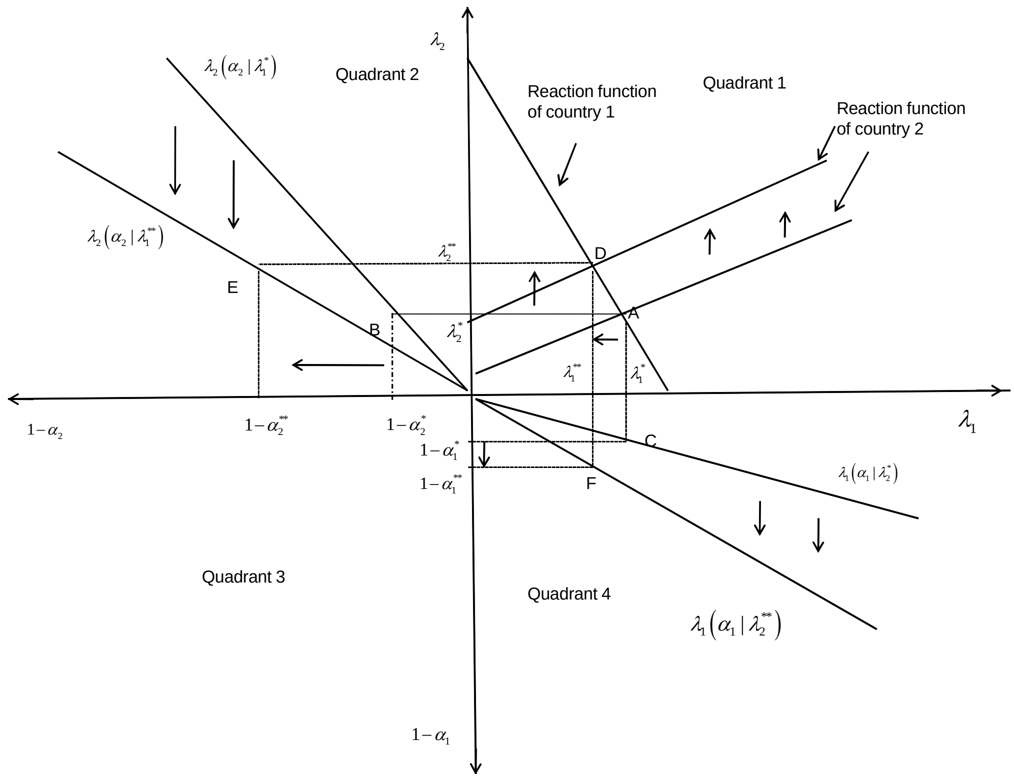

Figure 5: The impact of a change in the willingness-to-pay in country 2 when the reaction function of country 1 is upward sloping and the reaction function of country 2 is downward sloping. In this case, an increase in the willingness-to-pay in country 2 leads to a shorter patent length in country 1 and a longer patent length in country 2. Country 2 also adopts broader patents but the impact on the patent breadth of country 1 is ambiguous.

one instance in which this case can arise is if $\sigma'(R_1^*) < -\sigma^2(R_1^*)$.[14] The equilibrium in this case is depicted in Figure 5. The initial equilibrium is at point A and an increase in the willingness-to-pay leads to the new equilibrium point D. This implies that the patent length in the emerging economy increases and the patent length in the developed economy decreases. The effect on the patent breadth in the developed economy is ambiguous as in the previous case. However, in contrast to the previous case, the patent breadth in the emerging economy unambiguously increases. This is depicted in the second quadrant of Figure 5. This result occurs because of the downward pivot of the schedule relating the optimal patent breadth and the optimal patent length in the emerging economy. In this case, the patent laws in the emerging economy are strengthened both in terms of length and breadth.

Case 3 $\hat{\lambda}_1'(\lambda_2) > 0$ *and* $\hat{\lambda}_2'(\lambda_1) < 0$

[14] Note that $\sigma'(R_1^*) > -\sigma^2(R_1^*)$ is only a sufficient condition for the reaction function of country 1 to be downward sloping. Hence, nothing precludes the reaction function of country 1 to be downward sloping even if the sufficient condition is violated.

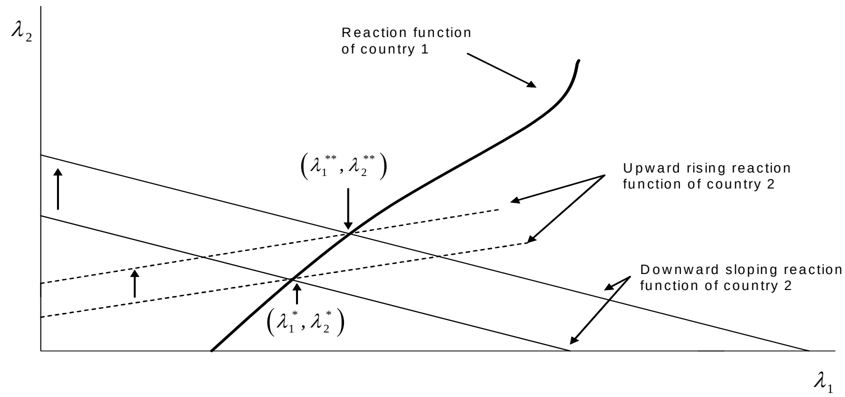

Figure 6: The impact of a change in the willingness-to-pay in country 2 when the reaction function of country 1 is upward rising. In this case, an increase in the willingness-to-pay in country 2 leads to a longer patent length in either country.

In this case, it can be shown using Figure 6 that the patent lengths increase in both countries. Further, the patent breadth in the emerging economy increases while the impact on the patent breadth in the developed economy is ambiguous.

Case 4 $\hat{\lambda}_1'(\lambda_2) > 0$ *and* $\hat{\lambda}_2'(\lambda_1) > 0$

It follows from Figure 6 that the patent lengths increase in both countries. Further, it can be shown that the patent breadth also increase in both countries.

Hence, to summarize the four cases, an increase in the willingness-to-pay in the emerging economy even without an accompanying increase in research capability unambiguously leads to an improvement of the degree of IP protection in the emerging economy both in terms of length and breadth in Case 2 to 4. Therefore, in these cases, the problem of weak patent protection in emerging economies is likely to diminish with an increase in their incomes (even if there is no significant increase of their research capabilities). However, this result may not hold in Case 1, in which both reaction functions are negatively sloped.

6.2 Impact of a change in $c_2(1)$ when $\phi_1'(\alpha) < 0$

In this case, as discussed in the previous section, the optimal λ_1 is 1 and the optimal α_1 is 0. Any change in the willingness-to-pay in the emerging economy has no impact on the patent laws in the developed economy. Hence, we focus on the impact of the change in the willingness-to-pay in the emerging economy on the patent law in the emerging economy. First, we consider the case of a downward sloping reaction function of the emerging economy. This case has been depicted in Figure 7. Notice that the reaction function of the developed economy is a vertical line corresponding to the maximum patent length. An increase in the willingness-to-pay in the emerging economy leads to an upward shift of the reaction function of the emerging economy. This leads to an increase in the patent length in the emerging economy. Further, there is no pivot of the schedule relating the optimal patent length with the optimal patent breadth in the emerging economy because the patent length of the developed economy does not change at all. This implies that the optimal patent breadth in the emerging economy must increase. Therefore, in this case, an increase in the willingness-to-pay in the emerging economy unambiguously leads to a stronger patent law in the emerging economy, both in terms of length and in terms of breadth. The same conclusion can be drawn in the case in which the reaction function of the emerging economy is upward sloping.

7 Extensions

7.1 The Global Planner's Problem

In this subsection, we determine the IP laws that would be selected by a global planner and compare it to the IP laws that are selected by the governments acting in their best interest. In the initial part of the discussion, we do not consider harmonized IP laws but later on we extend the analysis to the case of harmonized IP laws. The global planner solves the following problem:

$$\underbrace{Max}_{\lambda_1, \alpha_1, \lambda_2, \alpha_2} \qquad W^{GP} = P\left(R_1^*\right)\left(C_1 + V_1 + C_2 + V_2\right) - R_1^*.$$

An implicit assumption in our analysis is that the global planner can select the IP laws (as the governments could in the sections above) but has no direct control over the amount of R&D effort

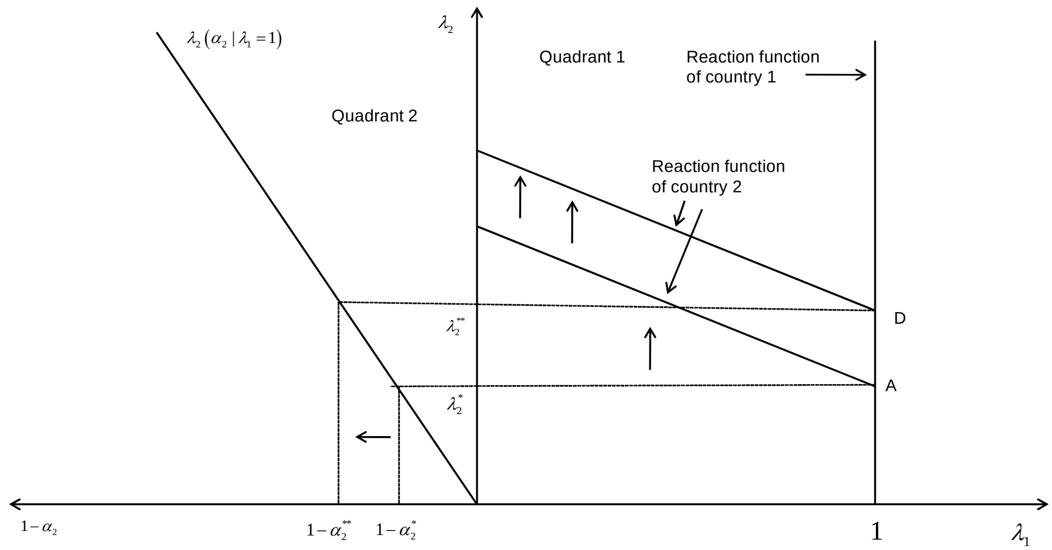

Figure 7: The impact of a change in the willingness-to-pay in country 2 when $\phi_1'(\alpha) < 0$. In this case, an increase in the willingness-to-pay in country 2 unambiguously leads to an increase in the degree of patent protection in the emerging economy.

that firm 1 exerts. Hence, firm 1's optimal amount of R&D effort is still determined by (11).

To determine the difference in incentives of the global planner with the individual governments, we need to compare the first order conditions of the global planner with those of the individual governments. First, consider the first order condition of the global planner with respect to λ_1. This is given by the following:

$$\frac{\partial W^{GP}}{\partial \lambda_1} = P'(R_1^*)(C_1 + C_2 + V_2)\frac{\partial R_1^*}{\partial \lambda_1} - P(R_1^*)\int_{\alpha_1}^{1}[\phi_1'(z) + \pi_{21}'(z)]\,dz = 0. \tag{43}$$

It is instructive at this stage to compare (43) with (30). The direct effect of an increase in λ_1 is given by the term $P(R_1^*)\int_{\alpha_1}^{1}\phi_1'(z)\,dz$ for country 1 and by $P(R_1^*)\int_{\alpha_1}^{1}[\phi_1'(z) + \pi_{21}'(z)]\,dz$ for the global planner. The additional term $\pi_{21}'(z)$ that is included in (43) however has an ambiguous sign; in particular, it may be positive for a cost-reducing technology (as in Example 3) and may be negative for a quality enhancing technology (as in Example 4). Hence, the direct effect is greater for a global planner for the case of a cost-reducing technology and is less for a global planner for the case of a quality enhancing technology. Next, consider the indirect effect given by the term $P'(R_1^*)\,C_1\frac{\partial R_1^*}{\partial \lambda_1}$

for country 1 and by $P'\left(R_1^*\right)\left(C_1+C_2+V_2\right)\frac{\partial R_1^*}{\partial \lambda_1}$ for the global planner and notice that the indirect effect is higher for the global planner. The conclusion is that for a cost-reducing technology, the direct effect and the indirect effect are both higher for the global planner, and hence, the net impact of the global planner on λ_1 is ambiguous. In contrast, for a quality enhancing technology, the direct effect is lower and the indirect effect is higher for the global planner, and hence, everything else remaining constant, the global planner has an incentive to increase λ_1.

Next, consider the first order condition of the global planner with respect to λ_2. This is given by the following:

$$\frac{\partial W^{GP}}{\partial \lambda_2}=P'\left(R_1^*\right)\left(C_1+V_1+C_2+V_2\right)\frac{\partial R_1^*}{\partial \lambda_2}-P\left(R_1^*\right)\int_{\alpha_2}^{1}\left[\phi_2'\left(z\right)+\pi_{12}'\left(z\right)\right]dz=0. \tag{44}$$

In order to examine the difference in incentives between the global planner and the government of country 2, we compare (44) and (20). The direct effect is given by $P\left(R_1^*\right)\int_{\alpha_2}^{1}\phi_2'\left(z\right)dz$ for country 2 and by $P\left(R_1^*\right)\int_{\alpha_2}^{1}\left[\phi_2'\left(z\right)+\pi_{12}'\left(z\right)\right]dz$ for the global planner. Notice that the term $\pi_{12}'\left(z\right)$ is unambiguously negative and hence, the direct effect is lower for the global planner. Further, the indirect effect is higher for the global planner. Hence, the conclusion is that, everything else remaining constant, the global planner has an incentive to increase λ_2 and this result holds for both a cost reducing technology as well as for a quality enhancing technology. Similar kinds of results can be obtained by examining the first order conditions of the global planner with respect to α_1 and α_2.

Finally, we examine if harmonized patent laws are optimal from the perspective of the global planner. The answer here is that harmonized patent laws are generally not optimal but can hold only for very specific parameter values. To see this, consider the first order conditions (43) and (44) with α_1 being equal to α_2. Notice that even in this case, the optimal λ_1 will in general be different from the optimal λ_2 because the two first order conditions include a few different terms.

7.2 Both countries invest in R&D

In this subsection, we analyze the case in which both countries engage in R&D. However, we assume naturally that firm 2 is not as adept in R&D as firm 1. This idea is captured by assuming that kR units of R&D effort by firm 2 is equivalent to R units of R&D effort by firm 1 where $k \geq 1$

captures the relative degree of inefficiency of firm 2's R&D effort with respect to firm 1's R&D effort. In the sections above, it was assumed that firm 2 does not engage in R&D. This can indeed occur endogenously if k is sufficiently large. A reduction of the value of k implies an improvement of firm 2's efficiency in R&D, and in the extreme, a value of $k = 1$ implies that firm 2 is as efficient as firm 1 in R&D. A complete analysis of this case is beyond the scope of this paper. Rather, the purpose of this section is to demonstrate that the insights derived in the sections above hold even when we allow for firm 2 to engage in R&D. In order to ensure tractability, we however allow each country to endogenously choose one aspect of the IP law and the other aspect is assumed to be exogenous. In particular, in the discussion below, we allow each country to choose its patent length and the breadth is assumed to be exogenous. One can check that the results hold in the converse case in which the length is assumed to be exogenous instead.

Because both firms engage in R&D, therefore a winner will be deemed to be the firm that is awarded a patent. Hence, let $P(R_1, R_2)$ be the probability that firm 1 is awarded a patent and $1 - P(R_1, R_2)$ be the probability that firm 2 is awarded a patent. Note that in this section, we rule out the possibility that both firms are unsuccesful. This is just a simplifying assuption and the results can be derived even without this assumption. The function $P(R_1, R_2)$ is assumed to be increasing in firm 1's effort and decreasing in firm 2's R&D effort, that is,

$$\frac{\partial P(\cdot)}{\partial R_1} > 0 \text{ and } \frac{\partial P(\cdot)}{\partial R_2} < 0.$$

We also assume that

$$\frac{\partial^2 P(\cdot)}{\partial R_1^2} < 0 \text{ and } \frac{\partial^2 P(\cdot)}{\partial R_2^2} > 0. \tag{45}$$

The assumptions in (45) are required for the second order conditions below.

Next, we consider the profit functions of firms 1 and 2. In the sections above, the instantaneous profit of firm i in country j conditional on a successful innovation was denoted by $\pi_{ij}(\alpha_j)$. Notice that the function $\pi_{ij}(\cdot)$ had only argument because there was no ambiguity about the direction of the knowledge spillover (from firm 1 to firm 2). However, in this subsection, since we allow both firms to conduct R&D, therefore, the flow of knowledge could be in either direction depending on the identity of the patentee. For example, if firm 1 is the patentee, then knowledge flows from firm 1 to firm 2, while if firm 2 is the patentee, then knowledge flows from firm 2 to firm 1 instead. In order to capture both possibilities, we let $\tilde{\pi}_{ij}(x,y)$ be the instantaneous profit of firm i in country j when the knowledge spillover from firm 1 to firm 2 is x and the knowledge spillover from firm 2 to firm 1 is y. For instance, suppose firm 2 is the patentee and suppose the patent breadth in country j is $(1-\alpha_j)$. Then, the instantaneous profit of firm i in country j is given by $\tilde{\pi}_{ij}(0,\alpha_j)$. In order to be consistent with (1), we assume that the instantaneous profit of a firm diminishes if there is any knowledge spillover to the competing firm, that is,

$$\frac{\partial \tilde{\pi}_{1j}(x,\cdot)}{\partial x} < 0 \text{ and } \frac{\partial \tilde{\pi}_{2j}(\cdot,y)}{\partial y} < 0.$$

We also assume that the impact of a given degree of knowledge spillover will be higher on the provider of the knowledge rather than on the recipient. This is formalized by the following assumptions:

$$\left|\frac{\partial \tilde{\pi}_{1j}(\alpha,0)}{\partial \alpha}\right| > \left|\frac{\partial \tilde{\pi}_{1j}(0,\alpha)}{\partial \alpha}\right| \text{ and } \left|\frac{\partial \tilde{\pi}_{2j}(0,\alpha)}{\partial \alpha}\right| > \left|\frac{\partial \tilde{\pi}_{2j}(\alpha,0)}{\partial \alpha}\right|.$$

Let us consider the first of these two assumptions. The term $\left|\frac{\partial \tilde{\pi}_{1j}(\alpha,0)}{\partial \alpha}\right|$ is the marginal impact on firm 1 if an amount of knowledge α flows out of firm 1 to firm 2, while $\left|\frac{\partial \tilde{\pi}_{1j}(0,\alpha)}{\partial \alpha}\right|$ is the marginal impact on firm 1 if the same amount of knowledge α flows out of firm 2 to firm 1 instead. The first assumption implies that the marginal loss to firm 1 from being the provider of the knowledge α is greater than the marginal impact on firm 1 from being the recipient. These assumptions are required in the proof of Lemma 4 below.

Next, let V_i^j be the gross profit of firm i given that firm j is the patentee. First, consider the gross profit of firm 1, given that firm 1 is the patentee. This is denoted by V_1^1 and is given below.

$$\begin{aligned} V_1^1 &= \lambda_1 \tilde{\pi}_{11}(\alpha_1,0) + (1-\lambda_1)\tilde{\pi}_{11}(1,0) + \lambda_2 \tilde{\pi}_{12}(\alpha_2,0) + (1-\lambda_2)\tilde{\pi}_{12}(1,0) \\ &= \tilde{\pi}_{11}(1,0) + \tilde{\pi}_{12}(1,0) - \lambda_1 \int_{\alpha_1}^{1} \frac{\partial \tilde{\pi}_{11}(x,0)}{\partial x} dx - \lambda_2 \int_{\alpha_2}^{1} \frac{\partial \tilde{\pi}_{12}(x,0)}{\partial x} dx. \end{aligned}$$

Note that the above expression is analogous to (4). Next, consider the gross profit of firm 1, given that firm 2 is the patentee. This is given below:

$$\begin{aligned} V_1^2 &= \lambda_1 \tilde{\pi}_{11}(0,\alpha_1) + (1-\lambda_1)\tilde{\pi}_{11}(0,1) + \lambda_2 \tilde{\pi}_{12}(0,\alpha_2) + (1-\lambda_2)\tilde{\pi}_{12}(0,1) \\ &= \tilde{\pi}_{11}(0,1) + \tilde{\pi}_{12}(0,1) - \lambda_1 \int_{\alpha_1}^{1} \frac{\partial \tilde{\pi}_{11}(0,y)}{\partial y} dy - \lambda_2 \int_{\alpha_2}^{1} \frac{\partial \tilde{\pi}_{12}(0,y)}{\partial y} dy. \end{aligned}$$

Finally, consider the gross profit of firm 2 when firm 1 is the patentee and when firm 2 is the patentee. These are denoted by V_2^1 and V_2^2 respectively and are as follows:

$$V_2^1 = \tilde{\pi}_{21}(1,0) + \tilde{\pi}_{22}(1,0) - \lambda_1 \int_{\alpha_1}^{1} \frac{\partial \tilde{\pi}_{21}(x,0)}{\partial x} dx - \lambda_2 \int_{\alpha_2}^{1} \frac{\partial \tilde{\pi}_{22}(x,0)}{\partial x} dx,$$

and

$$V_2^2 = \tilde{\pi}_{21}(0,1) + \tilde{\pi}_{22}(0,1) - \lambda_1 \int_{\alpha_1}^{1} \frac{\partial \tilde{\pi}_{21}(0,y)}{\partial y} dy - \lambda_2 \int_{\alpha_2}^{1} \frac{\partial \tilde{\pi}_{22}(0,y)}{\partial y} dy.$$

In order to derive the first order conditions (46) and (47) below, we need to ensure that $V_1^1 > V_1^2$ and $V_2^2 > V_2^1$. In other words, we need to ensure that a firm is better off it is the inventor rather than an imitator, for any fixed structure of IP laws. A sufficient condition for $V_1^1 > V_1^2$ is the following boundary condition:

$$\tilde{\pi}_{11}(1,0) + \tilde{\pi}_{12}(1,0) > \tilde{\pi}_{11}(0,1) + \tilde{\pi}_{12}(0,1).$$

Similarly, we can ensure that $V_2^2 > V_2^1$ by assuming the following boundary condition:

$$\tilde{\pi}_{21}(0,1) + \tilde{\pi}_{22}(0,1) > \tilde{\pi}_{21}(1,0) + \tilde{\pi}_{22}(1,0).$$

The inequality $V_2^2 > V_2^1$ captures an interesting distinction between the sections above in which firm 2 is assumed to not engage in R&D and this section in which firm 2 is assumed to engage in R&D. When firm 2 itself does not expend effort in R&D, then its gross payoff is V_2 when firm 1 succeeds in R&D and is 0 when firm 1 fails. Since $V_2 > 0$, therefore firm 1's success in R&D is "good news" for firm 2. On the other hand, when firm 2 expends effort in R&D, then its gross payoff is V_2^1 when firm 1 succeeds in R&D and is V_2^2 when firm 1 fails. The inequality $V_2^2 > V_2^1$ then implies that in contrast to the previous case, firm 1's success in R&D is in fact "bad news" for firm 2.

We preserve the same timeline as in the sections above, that is, in period 1, the two countries simultaneously select their IP laws and in period 2 the firms simultaneously determine their R&D effort. Finally, the outcomes of the R&D are known and firms make profits. Given the sequential nature of moves, we first solve for the equilibrium in period 2. First, we determine the R&D effort of firm 2, given that the R&D effort of firm 1 is R_1^*. In this case, firm 2 solves the following problem:

$$\underbrace{Max}_{R_2} \quad P\left(R_1^*, R_2\right) V_2^1 + \left[1 - P\left(R_1^*, R_2\right)\right] V_2^2 - kR_2.$$

At the optimum R&D effort of firm 2, that is at $R_2 = R_2^*$, the following condition must be satisfied:

$$-\frac{\partial P\left(R_1^*, R_2^*\right)}{\partial R_2}\left(V_2^2 - V_2^1\right) = k. \tag{46}$$

Notice that the (46) has an interior solution only if $V_2^2 > V_2^1$. Similarly, at $R_1 = R_1^*$, the following condition must be satisfied by firm 1:

$$\frac{\partial P\left(R_1^*, R_2^*\right)}{\partial R_1}\left(V_1^1 - V_1^2\right) = 1. \tag{47}$$

Note that these first order conditions are similar to (11). The second order conditions are satisfied because of (45).

Next, in Lemma 4 below, we demonstrate that the optimal R&D efforts are increasing in the patent lengths. These results are therefore analogous to the one shown in Lemma 1. In order to show Lemma 4, we define the curvature of the R&D production function as follows:

$$\sigma_i\left(R_1, R_2\right) = \left|\frac{\partial}{\partial R_i} \ln \frac{\partial P(\cdot)}{\partial R_i}\right| = \left|\frac{\frac{\partial^2 P(\cdot)}{\partial R_i^2}}{\frac{\partial P(\cdot)}{\partial R_i}}\right|.$$

Then, we have the following lemma.

Lemma 4 *At the optimum level of R&D effort,*

$$\begin{aligned}
\frac{\partial R_1^*}{\partial \lambda_1} &= -\frac{\frac{\partial P\left(R_1^*, R_2^*\right)}{\partial R_1}}{\sigma_1\left(R_1^*, R_2^*\right)} \int_{\alpha_1}^{1}\left[\frac{\partial \tilde{\pi}_{11}(z, 0)}{\partial z} - \frac{\partial \tilde{\pi}_{11}(0, z)}{\partial z}\right] dz > 0, \\
\frac{\partial R_1^*}{\partial \lambda_2} &= -\frac{\frac{\partial P\left(R_1^*, R_2^*\right)}{\partial R_1}}{\sigma_1\left(R_1^*, R_2^*\right)} \int_{\alpha_2}^{1}\left[\frac{\partial \tilde{\pi}_{12}(z, 0)}{\partial z} - \frac{\partial \tilde{\pi}_{12}(0, z)}{\partial z}\right] dz > 0, \\
\frac{\partial R_2^*}{\partial \lambda_1} &= \frac{1}{k}\frac{\frac{\partial P\left(R_1^*, R_2^*\right)}{\partial R_2}}{\sigma_2\left(R_1^*, R_2^*\right)} \int_{\alpha_1}^{1}\left[\frac{\partial \tilde{\pi}_{21}(0, z)}{\partial z} - \frac{\partial \tilde{\pi}_{21}(z, 0)}{\partial z}\right] dz > 0
\end{aligned} \tag{48}$$

and

$$\frac{\partial R_2^*}{\partial \lambda_2} = \frac{1}{k} \frac{\frac{\partial P\left(R_1^*, R_2^*\right)}{\partial R_2}}{\sigma_2 \left(R_1^*, R_2^*\right)} \int_{\alpha_2}^{1} \left[\frac{\partial \tilde{\pi}_{22}(0, z)}{\partial z} - \frac{\partial \tilde{\pi}_{22}(z, 0)}{\partial z} \right] dz > 0.$$

Proof. See the Appendix. ■

Notice from the above lemma that an increase in the degree of inefficiency in R&D (measured by an increase in k) leads to a reduction in the responsiveness of R_2. In the extreme, if k is infinitely high, then firm 2's R&D effort does not respond at all to any increase in the patent length.

We now determine the optimal patent length of the emerging country (country 2). To do so, notice that country 2 solves the following problem:

$$\underbrace{Max}_{\lambda_2} \quad \tilde{W}_2 = C_2 + P\left(R_1^*, R_2^*\right) V_2^1 + \left[1 - P\left(R_1^*, R_2^*\right)\right] V_2^2 - kR_2^*. \tag{49}$$

It is instructive at this point to examine the terms involved in the welfare function of country 2 when it invests in R&D (given above) with its welfare function when it does not invest in R&D (as given by (19)). Notice that the first term in (49) is the consumer surplus. In this section, it is assumed that at least one of the firms will be successful in R&D and the uncertainty is about the identity of the successful firm; thus, the term C_2 is not multiplied by the function $P(\cdot)$. In contrast, in (19), there is uncertainty about the success of the invention, and hence, in that case, C_2 is multiplied by the function $P(\cdot)$. Next, consider the term $P\left(R_1^*, R_2^*\right) V_2^1 + \left[1 - P\left(R_1^*, R_2^*\right)\right] V_2^2$. This is simply the expected gross profit of firm 2, because it is the imitator with probability $P\left(R_1^*, R_2^*\right)$ and is the inventor with probability $1 - P\left(R_1^*, R_2^*\right)$. Finally, the last term is the cost that firm 2 incurs in R&D. As mentioned earlier, in this section, we allow country 2 to determine only its patent length λ_2, while its patent breadth α_2 will be assumed to be exogenous. Hence, by differentiating (49), we obtain the following:

$$\begin{aligned} \frac{\partial W_2}{\partial \lambda_2} = & -\frac{\partial P\left(R_1^*, R_2^*\right)}{\partial R_1} \left(V_2^2 - V_2^1\right) \frac{\partial R_1^*}{\partial \lambda_2} + \left[-\frac{\partial P\left(R_1^*, R_2^*\right)}{\partial R_2} \left(V_2^2 - V_2^1\right) - k \right] \frac{\partial R_2^*}{\partial \lambda_2} \\ & - \int_{\alpha_2}^{1} \left[c_2'(z) + \frac{\partial \tilde{\pi}_{22}(0, z)}{\partial z} \right] dz + P\left(R_1^*, R_2^*\right) \int_{\alpha_2}^{1} \left[\frac{\partial \tilde{\pi}_{22}(0, z)}{\partial z} - \frac{\partial \tilde{\pi}_{22}(z, 0)}{\partial z} \right] dz. \end{aligned}$$

Note that the second term on the right hand side is 0 because of (46). Hence, at the optimum solution, the following equation must be satisfied:

$$\frac{\partial W_2}{\partial \lambda_2} = -\frac{\partial P\left(R_1^*, R_2^*\right)}{\partial R_1}\left(V_2^2 - V_2^1\right)\frac{\partial R_1^*}{\partial \lambda_2}$$
$$-\int_{\alpha_2}^{1}\left[c_2'(z) + \frac{\partial \tilde{\pi}_{22}(0,z)}{\partial z}\right]dz + P\left(R_1^*, R_2^*\right)\int_{\alpha_2}^{1}\left[\frac{\partial \tilde{\pi}_{22}(0,z)}{\partial z} - \frac{\partial \tilde{\pi}_{22}(z,0)}{\partial z}\right]dz = 0. \quad (50)$$

Notice that (50) is analogous to (20) where the first term is the indirect effect of increasing the patent length while the second and third terms measure the direct effect of increasing the patent length. Note that the first and third terms are negative but the sign of the second term is ambiguous. Hence, in order to guarantee an interior solution, we require that $|\frac{\partial \tilde{\pi}_{22}(0,z)}{\partial z}| > c_2'(z)$. The solution of the above equation is denoted by $\widehat{\hat{\lambda}}_2(\lambda_1)$ and is known as the reaction function of country 2 as a function of λ_1. We now determine the slope of $\widehat{\hat{\lambda}}_2(\lambda_1)$ and show that while it is possible for the reaction function to be downward sloping under certain circumstances, it is also possible for the reaction function to be upward sloping under other circumstances.

Proposition 6 *The slope of* $\widehat{\hat{\lambda}}_2(\lambda_1)$ *has an ambiguous sign.*

Proof. See the Appendix. ■

8 Concluding Remarks

In this paper, we have analyzed the incentive of emerging economies to free ride on improved patent protection in the developed economies. We have three major conclusions: (*i*) Under plausible conditions, the optimal patent length and the optimal patent breadth in a country have a positive relationship with one another when the structure of IP laws is fixed in other countries. (*ii*) Patent lengths across countries may be positively or negatively related depending on the manner in which the curvature of the R&D production function changes. (*iii*) An increase in the willingness-to-pay in the emerging economy need not always lead to an improvement in both dimensions of IP protection, that is, the patent length and the patent breadth. All of these predictions are empirically testable and we leave this point for future research.

References

Bulow, Jeremy I., John D. Geanakoplos, and Paul D. Klemperer. 1985. Multimarket Oligopoly:

Strategic Substitutes and Complements. Journal of Political Economy 93: 488-511.

Chen, Yongmin, and Thitima Puttitanun. 2005. Intellectual Property Rights and Innovation in Developing Countries. Journal of Development Economics 78: 474-93.

DeBrock, Lawrence M. 1985. Market Structure, Innovation, and Optimal Patent Life. Journal of Law and Economics 28: 223-44.

Denicolò, Vincenzo. 1996. Patent Races and Optimal Patent Breadth and Length. Journal of Industrial Economics 44: 249-65.

Federal Trade Commission. 2002. Generic Drug Entry Prior to Patent Expiration: An FTC Study. Washington, DC.

Federal Trade Commission. 2007. Protecting Consumer Access to Generic Drugs: The Benefits of a Legislative Solution to Anticompetitive Patent Settlements in the Pharmaceutical Industry. Washington, DC.

Gallini, Nancy T. 1992. Patent Policy and Costly Imitation. RAND Journal of Economics 23: 52-63.

Gilbert, Richard, and Carl Shapiro. 1990. Optimal Patent Length and Breadth. RAND Journal of Economics 21: 106-12.

Grossman, Gene M., and Edwin L. C. Lai. 2004. International Protection of Intellectual Property. American Economic Review 94: 1635-53.

Kim, Jeong-Eon, and Harvey E. Lapan. 2008. Heterogeneity of Southern Countries and Southern Intellectual Property Rights Policy. Canadian Journal of Economics 41: 894-925.

Klemperer, Paul. 1990. How Broad Should the Scope of Patent Protection Be? RAND Journal of Economics 21: 113-30

Lai, Edwin L. C., and Larry D. Qiu. 2003. The North's Intellectual Property Rights Standard for the South? Journal of International Economics 59: 183-209.

Maskus, Keith E. 2000. Intellectual Property Rights in the Global Economy. Washington DC: Peterson Institute for International Economics.

Mukherjee, Arijit, and Enrico Pennings. 2004. Imitation, Patent Protection, and Welfare. Oxford Economic Papers 56: 715-33.

Park, Walter G., and Smita Wagh. 2002. Index of Patent Rights. In Economic Freedom of the World: 2002 Annual Report , edited by James Gwartney and Robert Lawson. Vancouver, B.C: The

Fraser Institute, pp. 33-41.

Porter, Michael E., and Scott Stern. 2001. National Innovative Capacity. In The Global Competitiveness Report 2001-2002, edited by Klaus Schwab, Michael E. Porter, and Jeffrey D. Sachs. New York: Oxford University Press, pp. 102-119.

Ranbaxy. Ranbaxy and Pfizer Settle Lipitor Litigation Worldwide. Accessed December 2010. Available at http://www.ranbaxyusa.com/newsdisp180608.aspx.

Scotchmer, Suzanne. 2004. Innovation and Incentives. Cambridge, MA: MIT Press.

———. The Political Economy of Intellectual Property Treaties. 2004. Journal of Law, Economics and Organization 20: 415-37.

Tirole, Jean. 1988. The Theory of Industrial Organization. Cambridge, MA: MIT Press.

Wright, Donald J. Optimal Global Patent Design. 2005. Journal of Institutional and Theoretical Economics 161: 18-37.

Yang, Yong. 1998. Why Do Southern Countries Have Little Incentive to Protect Northern Intellectual Property Rights? Canadian Journal of Economics 31: 800-16.

Zigic, Kresimir. 1998. Intellectual Property Rights Violations and Spillovers in North-South Trade. European Economic Review 42: 1779-99.

Appendix

A Proof of Lemma 1

We prove only (12) and the rest of the proof can be done analogously. At the optimum level of R&D effort, (11) holds. Hence, by applying the implicit function theorem, we obtain the following:

$$\frac{\partial R_1^*}{\partial \lambda_1} = -\frac{\int_{\alpha_1}^{1} \pi_{11}'(z)\, dz}{\sigma(R_1^*)\, V_1}. \tag{51}$$

By substituting (11) into (51), we obtain (12).

B Proof of Lemma 2

We only prove (16) and the rest of the proof can be done analogously. It follows from (11) that

$$P'\left(R_1^*\right)V_1 - 1 = 0.$$

Hence, using the implicit function theorem, we obtain that

$$\frac{\partial R_1^*}{\partial \lambda_1} = -\frac{\int_{\alpha_1}^{1} \pi_{11}'(z)\,dz}{\sigma\left(R_1^*\right)V_1}.$$

Now, suppose that λ_2 also changes. Then, in the expression above, the numerator remains unchanged but both terms in the denominator change. A change in λ_2 changes V_1 and hence changes the optimal amount of R&D effort of firm 1. However, any change in the R&D effort of firm 1 also changes the curvature of the R&D production function and this has a further feedback effect on the level of R&D effort. Hence,

$$\frac{\partial^2 R_1^*}{\partial \lambda_2 \partial \lambda_1} = \frac{\int_{\alpha_1}^{1} \pi_{11}'(z)\,dz}{\left(\sigma\left(R_1^*\right)V_1\right)^2}\left\{\sigma\left(R_1^*\right)\frac{\partial V_1}{\partial \lambda_2} + \sigma'\left(R_1^*\right)V_1\frac{\partial R_1^*}{\partial \lambda_2}\right\} \tag{52}$$

Next, notice that

$$\frac{\partial V_1}{\partial \lambda_2} = -\int_{\alpha_2}^{1} \pi_{12}'(z)\,dz \tag{53}$$

and

$$\frac{\partial R_1^*}{\partial \lambda_2} = -\frac{\int_{\alpha_2}^{1} \pi_{12}'(z)\,dz}{\sigma\left(R_1^*\right)V_1}. \tag{54}$$

Substituting (53) and (54) into (52), we obtain that

$$\frac{\partial^2 R_1^*}{\partial \lambda_2 \partial \lambda_1} = -\frac{\left(\int_{\alpha_1}^{1} \pi_{11}'(z)\,dz\right)\left(\int_{\alpha_2}^{1} \pi_{12}'(z)\,dz\right)}{\sigma^3\left(R_1^*\right)V_1^2}\left\{\sigma^2\left(R_1^*\right) + \sigma'\left(R_1^*\right)\right\}.$$

Notice that

$$-\frac{\left(\int_{\alpha_1}^{1} \pi_{11}'(z)\,dz\right)\left(\int_{\alpha_2}^{1} \pi_{12}'(z)\,dz\right)}{\sigma^3\left(R_1^*\right)V_1^2} < 0.$$

Hence,

$$sign\left(\frac{\partial^2 R_1^*}{\partial \lambda_1 \partial \lambda_2}\right) = -sign\left(\sigma^2\left(R_1^*\right) + \sigma'\left(R_1^*\right)\right).$$

The rest of the proof proceeds along similar lines.

C Proof of Proposition 1

By integrating both sides of (25), we obtain the following:

$$\int_0^{\alpha_2} d\ln\lambda_2(x) = \int_0^{\alpha_2} d\ln\left(\int_x^1 \phi_2'(z)\,dz\right)$$

from which it follows that

$$\frac{\lambda_2(\alpha_2)}{\lambda_2(0)} = \frac{\int_{\alpha_2}^1 \phi_2'(z)\,dz}{\int_0^1 \phi_2'(z)\,dz}.$$

Hence, we obtain the result.

D Proof of Proposition 2

Consider country 2. At the optimum,

$$\frac{dW_2}{d\lambda_2} = 0.$$

Therefore, by applying the implicit function theorem, it follows that

$$\hat{\lambda}_2'(\lambda_1) = -\frac{\frac{\partial^2 W_2}{\partial\lambda_1\partial\lambda_2}}{\frac{\partial^2 W_2}{\partial\lambda_2^2}},$$

where $\hat{\lambda}_2'(\lambda_1)$ is the change in λ_2 for a unit change in λ_1 along country 2's reaction function. From the second order condition, it follows that

$$\frac{\partial^2 W_2}{\partial\lambda_2^2} < 0.$$

Therefore,

$$sign\left(\hat{\lambda}_2'(\lambda_1)\right) = sign\left(\frac{\partial^2 W_2}{\partial\lambda_1\partial\lambda_2}\right).$$

From (27), it follows that along country 2's reaction function,

$$\begin{aligned}
\frac{\partial^2 W_2}{\partial \lambda_1 \partial \lambda_2} &= P''(R_1^*)(C_2+V_2)\left[\frac{\partial R_1^*}{\partial \lambda_2} + \alpha_2'(\lambda_2)\frac{\partial R_1^*}{\partial \alpha_2}\right]\frac{\partial R_1^*}{\partial \lambda_1} \\
&\quad - P'(R_1^*)\left[\frac{\partial R_1^*}{\partial \lambda_2} + \alpha_2'(\lambda_2)\frac{\partial R_1^*}{\partial \alpha_2}\right]\int_{\alpha_1}^{1}\pi_{21}'(z)\,dz \\
&\quad + P'(R_1^*)(C_2+V_2)\left[\frac{\partial^2 R_1^*}{\partial \lambda_1 \partial \lambda_2} + \frac{\partial}{\partial \lambda_1}\left(\alpha_2'(\lambda_2)\frac{\partial R_1^*}{\partial \alpha_2}\right)\right] \\
&\quad - 2P'(R_1^*)\int_{\alpha_2}^{1}\phi_2'(z)\,dz\frac{\partial R_1^*}{\partial \lambda_1}. \qquad (55)
\end{aligned}$$

Observe that the first and fourth terms on the right hand side are negative. Further, the second term is also negative if $\pi_{21}'(\alpha) > 0$ for all α. Finally, consider the third term and notice that

$$\frac{\partial}{\partial \lambda_1}\left(\alpha_2'(\lambda_2)\frac{\partial R_1^*}{\partial \alpha_2}\right) = \left\{-\frac{\int_{\alpha_2}^{1}\phi_2'(z)\,dz}{\phi_2'(\alpha_2)}\pi_{12}'(\alpha_2)\right\}\frac{\left(\int_{\alpha_1}^{1}\pi_{11}'(z)\,dz\right)}{\sigma^3(R_1^*)V_1^2}\left\{\sigma^2(R_1^*)+\sigma'(R_1^*)\right\}.$$

Hence,

$$sign\left(\alpha_2'(\lambda_2)\frac{\partial R_1^*}{\partial \alpha_2}\right) = -sign\left(\sigma^2(R_1^*)+\sigma'(R_1^*)\right) = sign\left(\frac{\partial^2 R_1^*}{\partial \lambda_1 \partial \lambda_2}\right)$$

and therefore the third term in (55) is also negative if $\sigma'(R_1^*) > -\sigma^2(R_1^*)$. Consequently it follows that $\frac{\partial^2 W_2}{\partial \lambda_1 \partial \lambda_2} < 0$ if $\pi_{21}'(\alpha) > 0$ and if $\sigma'(R_1^*) > -\sigma^2(R_1^*)$.

For the converse, notice that $\frac{\partial^2 W_2}{\partial \lambda_1 \partial \lambda_2} > 0$ can be satisfied only if either the second or the third term (or both) is positive. Hence, the result follows.

E Proof of Lemma 4

We prove only (48) and the rest of the proof can be done analogously. At the optimum level of R&D effort, (46) holds. Hence, by applying the implicit function theorem, we obtain the following:

$$\frac{\partial R_2^*}{\partial \lambda_1} = -\frac{\int_{\alpha_1}^{1}\left[\frac{\partial \tilde{\pi}_{21}(0,z)}{\partial z} - \frac{\partial \tilde{\pi}_{21}(z,0)}{\partial z}\right]dz}{\sigma_2(R_1^*,R_2^*)(V_2^2 - V_2^1)}. \qquad (56)$$

By substituting (46) into (56), we obtain (48).

F Proof of Proposition 6

Consider country 2. At the optimum,

$$\frac{\partial \tilde{W}_2}{\partial \lambda_2} = 0.$$

Therefore,

$$\widehat{\hat{\lambda}}_2'(\lambda_1) = -\frac{\frac{\partial^2 \tilde{W}_2}{\partial \lambda_1 \partial \lambda_2}}{\frac{\partial^2 \tilde{W}_2}{\partial \lambda_2^2}},$$

where $\widehat{\hat{\lambda}}_2'(\lambda_1)$ is the change in λ_2 for a unit change in λ_1 along country 2's reaction function. From the second order condition, it follows that

$$\frac{\partial^2 W_2}{\partial \lambda_2^2} < 0.$$

Therefore,

$$sign\left(\widehat{\hat{\lambda}}_2'(\lambda_1)\right) = sign\left(\frac{\partial^2 W_2}{\partial \lambda_1 \partial \lambda_2}\right).$$

From (50), it follows that along country 2's reaction function,

$$\begin{aligned}
\frac{\partial^2 W_2}{\partial \lambda_1 \partial \lambda_2} = & -\frac{\partial^2 P(R_1^*, R_2^*)}{\partial R_1^2}\left(V_2^2 - V_2^1\right)\frac{\partial R_1^*}{\partial \lambda_2}\frac{\partial R_1^*}{\partial \lambda_1} \\
& -\frac{\partial^2 P(R_1^*, R_2^*)}{\partial R_1 \partial R_2}\left(V_2^2 - V_2^1\right)\frac{\partial R_1^*}{\partial \lambda_2}\frac{\partial R_2^*}{\partial \lambda_1} \\
& +\frac{\partial P(R_1^*, R_2^*)}{\partial R_1}\left\{\int_{\alpha_1}^1\left[\frac{\partial \tilde{\pi}_{21}(0,z)}{\partial z} - \frac{\partial \tilde{\pi}_{21}(z,0)}{\partial z}\right]dz\right\}\frac{\partial R_1^*}{\partial \lambda_2} \\
& -\frac{\partial P(R_1^*, R_2^*)}{\partial R_1}\left(V_2^2 - V_2^1\right)\frac{\partial^2 R_1^*}{\partial \lambda_1 \partial \lambda_2} \\
& +\frac{dP(R_1^*, R_2^*)}{d\lambda_1}\int_{\alpha_2}^1\left[\frac{\partial \tilde{\pi}_{22}(0,z)}{\partial z} - \frac{\partial \tilde{\pi}_{22}(z,0)}{\partial z}\right]dz, \qquad (57)
\end{aligned}$$

where

$$\frac{dP(R_1^*, R_2^*)}{d\lambda_1} = \frac{\partial P(R_1^*, R_2^*)}{\partial R_1}\frac{\partial R_1^*}{\partial \lambda_1} + \frac{\partial P(R_1^*, R_2^*)}{\partial R_2}\frac{\partial R_2^*}{\partial \lambda_1}.$$

Observe that the first term on the right hand side of (57) is positive, the third term is negative, while the sign of the other terms is ambiguous. Hence, the sign of $\widehat{\hat{\lambda}}_2'(\lambda_1)$ is ambiguous.