Chaos detection in economics. Metric versus topological tools

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1. Introduction

In the analysis performed with traditional statistician methods irregular behaviour of some non-linear deterministic systems is not appreciated and when such behaviour is manifested in observations, it is typically explained as stochastic. This awareness has lead to powerful new to detect and analyze apparently random phenomena that at a deeper level could present chaotic behaviours.

The discovery of chaotic behaviour in economic models led to its search in data[1]. From forecasting movements in foreign exchange and stock markets, to understanding international business cycles there was an explosion of empirical work searching for chaos in economic and financial time series. Several chaos tests have been developed to try to distinguish between data generated by a deterministic system and data generated by a random one, stressing that an accurate empirical testing of chaos requires the availability of high quality, high frequency data.

The main and more used tests for chaos in economic time series are: correlation dimension; Lyapunov exponent; and BDS test.

Correlation dimension measures the convergence of all the trajectories towards the attractor, that's the global stability; Lyapunov exponent measures local instability, that is, the rate of separation between two initially close trajectories, and finally BDS test is only a test for non-linearity.

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[1] Some first examples are Brock and Sayers (1986), Sayers (1986), Barnett and Chen (1988a, 1988b), Ramsey (1989), Chen (1993).

Empirical chaos testing have been applied either in macroeconomics and finance. Nevertheless the analysis of financial time series has led to results which are, as a whole, more reliable than those of macroeconomic series. Financial time-series are a good candidate for analyzing chaotic behaviour. The reason is the much larger sample sizes available and the superior quality of that of financial data. Little or no evidence for chaos has found in macroeconomic time series. That is due to the small samples and high noise levels for most macroeconomic series; they are usually aggregated time series coming from a system whose dynamics and measurement probes may be changing over time. Therefore none of those studies has delivered solid evidence of chaos in economics data. Investigators have found substantial evidence for nonlinearity but relatively weak evidence for chaos per se. An example in this direction could be the paper by Frank et al. (1988). They tested with correlation dimension and Lyapunov exponent quarterly macroeconomic data from 1960 to 1980 for Italy, Japan, the United Kingdom and West Germany. Strong nonlinearity but not deterministic chaos has been found. The idea of our paper is to test the same data using a new tool, visual recurrence, more appropriate than traditional ones to discovery chaos in short and noisy time series and to compare the different results, if any.

The paper is set up as follows. In section 2 metric tools features are described comparing them with topological ones. In section 3 an alternative tool to overcome the problems of metric tools is presented. Finally in section 4 we describe the application of VRA to macroeconomic time series analysed by Frank et al. in their paper “International chaos?” In section 5 we report the conclusion of our work.

2. *Metric and Dynamical versus Topological Tools*

The methods to analyse time series for detecting chaos could be classified in metric, dynamical, and topological ones (Belaire-Franch et al. (2001). Generally the tools used for economic analysis was: correlation dimension[2], metric tool; Lyapunov exponent[3], dynamical tool and BDS test.

The correlation dimension test was developed in physics by Grassberger and Procaccia, (1983). A pure stochastic process will spread all space as evolving, but the movements of a chaotic system will be restricted by an attractor. The chaotic trajectories converge in the long term on the attractor, showing a global stability that is a characteristic of chaotic motion. The correlation dimension is based on measuring the dimension of a strange attractor. Its major advantage is the simplicity of calculating.

This analysis provides **necessary** but not sufficient conditions for testing the presence of chaos. In fact, designed for very large, clean data sets, it was found to be problematical when applied to short time series. Data sets with only few hundred or even a few thousand observations may be inadequate for this procedure (Ruelle,1991).

Lyapunov exponent, based on the evolution in the time between two very close points initially, measures the rate of separation between trajectories starting from those points. Positive Lyapunov exponent is generally regarded as necessary but not sufficient to presence of chaos. As for correlation dimension, the estimate of Lyapunov exponent requires a large number of observations. Since few economic series of such a large size are available, Lyapunov exponent estimates of economic data may not be so reliable.

One other of the most commonly applied tool is **BDS[4] test** by Brock, Dechert, and Scheinkman (1996). It is not a test for chaos but tests the much more restrictive null

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[2] The correlation dimension is metric methods because it is based on the computation of distances on the system's attractor.

[3] Lyapunov exponent instead, is a dynamical method because it is based on computing the way near to where orbits diverge.

hypothesis that the series is independent and identically distributed. It is useful because it is a well defined, easy to apply test, and powerful against any type of structure in a series. It has been used most widely to examine a variety of economic and financial time series[5]. The application to such data presents numerous problems. The first problem is that noise of economic time series may render any dimension calculation useless; then, to obtain a reliable analysis, large data sets are required (Brock and Sayers 1986).

Therefore the techniques traditionally used to test the presence of chaos in time series show that data quantity and data quality are crucial in applying them but the main obstacle in empirical economic analysis is short and noisy data sets.

In order to facilitate the testing of deterministic chaos and to improve our understanding of modern economies, it is worthwhile to develop numerical algorithms that work with small data sets, and are robust against noise. This goal seems to be reached by topological tools, like recurrence analysis.

Topological tools are characterised by the study of the organisation of the strange attractor, and they include close returns plot and recurrence plot. They exploits an essential property of a chaotic system, i.e. the tendency of the time series to nearly, although never exactly, repeat itself over time. This property is known as the recurrence property.

The topological method provides the basis for a new way to test time series data for chaotic behaviour (Mindlin et al. 1990). It has been successfully applied in the sciences to detect chaos in experimental data, but can also provide information about the underlying system responsible for chaotic behaviour (Mindlin et al. 1990; Mindlin and Gilmore, 1992). In fact, as the topological method preserves time

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[4] "Details of which may be found in Dechert (1996). Subsequent to its introduction, the BBS test has been generalised by Savit and Green (1991) and Wu, Savit, and Brock (1993) and more recently, DeLima (1998) introduced an iterative version of the BBS test" MacKenzie (2001).

[5] For a survey see Faggini 2008, 2009.

ordering of the data, where evidence of chaos is found, the researcher may proceed to characterise the underlying process in a quantitative way. Thus, one is able to reconstruct the stretching and compressing mechanisms responsible for generating the strange attractor. It works well on relatively small data sets, is robust against noise, and preserves time-ordering information (Gilmore,1993).

3. *Recurrence Analysis: VRA*

Recurrence Analysis, based on topological approach, was used to show recurring patterns and non-stationarity[6] in time series (Zbilut et al., 2000). It was applied to study chaotic systems because recurring patterns are among the most important features of chaotic systems(Cao and Cai, 2000). In this way has been possible to reveal correlation in the data that is not possible to detect in the original time series and is particularly suitable to investigate the economic time series that are characterised by noise and short data sets, output of high dimensional systems (Trulla L. et al., 1996, p. 255). Recurrence Analysis software[7] used for our analysis is VRA[8] – Visual Recurrence Analysis by Eugene Kononov[9]. It is based on Recurrence Plot, graphical analysis and Recurrence Quantifications Analysis, numerical analysis.

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[6] "system properties that cannot be observed using other linear and non linear approaches and is specially useful for analysis of non stationarity systems with high dimensional and /or noisy dynamics", see Holyst et alt. (2000).

[7] For a survey see Belaire-Franch et al. (2001)

[8] Among the free tools of non-linear time series used for this kind of analysis, VRA is more complete and easier to use. It works under Windows and it is possible to obtain both graphical analysis, the RP, and the analysis statistics with RQA. Moreover, while in others tools such as Tisean, RQA, Dataplore, Santise, REc Plot we have to insert the value of delay and dimension calculated with different algorithms, VRA provides also the technique used for calculating delay(MIF) and dimension (FNN). Moreover, differently from RQA (http://homepages.luc.edu/~cwebber/) VRA allows to split the time series into epochs, so it is possible to know its behaviour (stationary, non-stationarity and chaos) and ratios at local level. Vice versa, RQA shows global behaviour and ratios.

[9] http://home.netcom.com/~eugenek/download.html

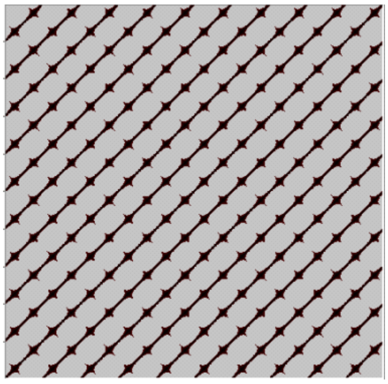
Recurrence Plot (RP) is a graphical tool that evaluates the temporal and phase space distance. It is a method by Eckmann et al. (1987) designed to locate hidden recurring patterns, non-stationarity and structural changes.

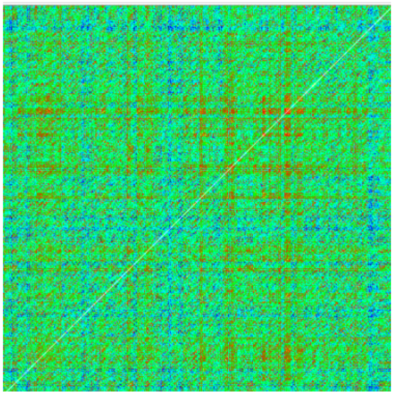
The RP, based on the Space State Reconstruction[10], is a two dimensional representation of single trajectory. It is formed by a 2-dimensional M x M (matrix) where M is the number of embedding vectors Y(i) obtained from the delay co-ordinates of the input signal. In the matrix the point value of coordinates (i, j), is the Euclidean distances between vectors Y(i) and Y(j). In this matrix, horizontal axis represents the time index Y(i) while the vertical one represents the time shift Y(j). A point is placed in the array (i,j) if Y(i) is sufficiently close to Y(j). This closeness is measured by a critical radius and a point is plotted as a coloured pixel only if the corresponding distance is below or equal to this radius. Generally dark colour shows the short distances and a light colour the long one.

To explain how to interpret this graphical tool we will use the samples of VRA. We start by considering a random time series (White noise). The plot (fig. 1b) has been built using delay 1 and dimension 12 as selected respectively from mutual information function and false nearest neighbours. As we can see in fig.1b, the plot of random time series shows recurrent points distributed in homogenous random patterns. That means random variable lacks of deterministic structures. Always in Fig.1 it is possible to characterize stationary and non-stationary processes. If the texture of the pattern within such a block is homogeneous, stationarity can be assumed for the given signal within the corresponding period of time; non-stationary

[10] This approach is founded on flow of information from unobserved variables to observed variables and is widely used to reduce multivariate data to a few significant variables. The basic idea is that the effect of all the other (unobserved) variables is already reflected in the series of the observed output and the rules that govern the behaviour of the original system can be recovered from its output. Kantz and Schreiber (2000).

systems cause changes in the distribution of recurrence points in the plot which is visible by brightened areas.





(a) (b)

(c)

Fig. 1.Examples by VRA. (a) Periodic time series; (b) White Noise; (c) Henon equation

Diagonal structures show (fig. 1c) the range in which a piece of the trajectory is rather close to another piece of the trajectory at different times. The *diagonal length* is the length of time how long they will be close to each other and can be interpreted as the mean prediction time.

From the occurrence of lines parallel to the diagonal in the recurrence plot, it can be seen how fast neighboured trajectories diverge in phase space. The line segments parallel to main diagonal are points close to each other successively forward in time and would not occur in a random as opposed to deterministic process. So if the analysed time series is deterministic, then the recurrence plot shows short line segments parallel to the main diagonal; on the other hand, if the series is white noise, then the recurrence plot does not show any structure. Chaotic behaviour causes very short diagonals, whereas deterministic behaviour causes longer diagonals (fig.1a vs fig.c). Therefore, the average length of these lines is a measure of the reciprocal of the largest positive Lyapunov exponent.

The graphical output of RP is not easy to interpret. The signature of determinism, the set of lines parallel to the main diagonal might not be so clear. In fact, the recurrence plot could contain subtle patterns not easily ascertained by visual inspection. As a consequence Zbilut et alt. (1998, 2000) proposed statistical quantification of RP, well-know as Recurrence Quantification analysis (RQA).

RQA defines measures for diagonal segments in a recurrence plots. These measures are *recurrence rate*, *determinism*, *averaged length of diagonal structures*, *entropy* and *trend*.

Recurrence rate (REC) is the ratio of all recurrent states (recurrence points percentage) to all possible states and is the probability of recurrence of a special state. REC is simply what is used to compute the correlation dimension of data.

Determinism (DET) is the ratio of recurrence points forming diagonal structures to all recurrence points. DET[11] measures the percentage of recurrent points forming line segments which are parallel to the main diagonal. A line segment is a points'

[11] "This is a crucial point: a recurrence can, in principle, be observed by chance whenever the system explores two nearby points of its state space. On the contrary, the observation of recurrent points consecutive in time (and then forming lines parallel to the main diagonal) is an important signature of deterministic structuring" Manetti et al. (1999)

sequence equal to or longer than a predetermined threshold (Giuliani et alt. 1998). These line segments show the existence of deterministic structures, the absence, instead of randomness.

Maxline (MAXLINE) represents the averaged length of diagonal structures and indicates longest line segments which are parallel to the main diagonal. It is claimed to be proportional to the inverse of the largest positive Lyapunov exponent. A periodic signal produces long line segments, while the *noise* doesn't produce any segments. Short segments indicate chaos.

Entropy (ENT) (Shannon entropy) measures the distribution of those line segments which are parallel to the main diagonal and reflects the complexity of the deterministic structure in the system. This ratio indicates the time series *structurness*, so high values of ENT are typical of periodic behaviours while low values of chaotic behaviours ones. The high value of ENT means a large diversity in diagonal line lengths, slow values instead small diversity in diagonal line lengths (Trulla et alt., 1996). "[...] short line max values therefore are indicative of chaotic behaviours" (Iwanski and Bradley, 1998; Atay and Altintas, 1999).

The value *trend* (TREND) measures the paling of the patterns of RPs away from the main diagonal (used for detecting drift and non-stationarity in a time series).

In the fig. 1b the visual features are confirmed by the ratios calculated with RQA. We can see that the REC and DET assume values equal to zero, so, in time series there are no recurrent points and no deterministic structure. These features are more evident if we compare REC and DET of time series with one of sine function (fig.1a). The plots of sine function are more regular and REC shows not only the recurrent point in each epoch but also that this value is the same. DET values are high meaning strong structure in the time series confirmed by the MAXLINE values which are also high, so deterministic rules are present in the dynamics. Comparing the fig. 1a and fig. 1c it is possible to see that if the analyzed series is generated from a determinist process in the RP there are long segments parallel~~s~~ to the main diagonal. If the data are chaotic these segments are short.

4. *International chaos analysis*

In their paper Frank, Gencay, and Stengos, (1988) analyze the quarterly macroeconomic data from 1960 to 1988 for West Germany, Italy, Japan and England. The goal was to check for the presence of deterministic chaos. To ensure that the data analysed was stationary they used a first difference then tried a linear fit. Using a reasonable AR specification for each time series their conclusion was that time series showed different structures. In particular non linear structure was present in the time series of Japan. Nevertheless the application of typical tools for detecting chaos (correlation dimension and Lyapunov exponent) didn’t show presence of chaos in any time series. Therefore none of the countries’ income appeared to be well interpreted as being chaotic.I applied VRA to these time series with purpose to verify if the analysis performed by a topological tool could give results different from ones obtained using a metric tool

The time series[12] chosen are GNP of Japan and GDP of United Kingdom. The choice is based on the fact that Japan is considered among four of the most dissimilar ones. In fact, in order to filter this series, the authors used an Ar-4, while for the others an Ar-2 was used. For this series they refuse the hypothesis IID and the correlation dimension value calculated for various values of M, (the embedding dimension), grows less compared to the growth of the value of the embedding dimension. There is a saturation point.

In fact for calculated values of M, 5, 10, 15, the dimension of correlation is respectively, 1.3, 1.6, 2.1, against values of 1.2, 3.8, 6.8 of the series shuffled.The rejection of the IID hypothesis, the value of correlation dimension and the

[12] the analysis performed by Frank et al. (1988) the data are for the Japan Real GNP seasonally adjusted, quarterly from 1960 to 1988 and for United Kingdom from 1960 to 1988. Source Datastream

comparison with value of shuffled series, and the presence of nonlinearity pushed the authors to suspect that time series could be chaotic. However, tested with the Lyapunov exponent, the conclusion was that data didn’t manifest chaotic behaviour. In fact the value of the Lyapunov exponent test was negative[13]. They shown that Japan’s economy is the most stable of the analysed countries.

Therefore, although the presence of nonlinearity and correlation dimension values pushed to admit chaotic behaviour in the time series, Lyapunov exponent test has not supported this hypothesis. Probably, as admitted by the same authors, this conclusion could be given by the shortness of series. “With longer time series matters could change” (Frank et alt., 1988, p. 1581).

The GDP time series of United Kingdom has been chosen because, as emphasized by the authors, for it as for Germany time series is not rejected by the hypothesis IID. The behaviour of correlation dimension is the same for all three European countries[14].

The increase of the embedding *dimension* corresponds to sustained increase of the dimension of correlation.Such increase is also characterised in time series *shuffled* obtained from the time series fits with Ar-2. From this conclusion and considering that the values of Lyapunov exponent test were negative the authors conclude that European time series didn’t show non-linearity and in particular chaotic behaviour.

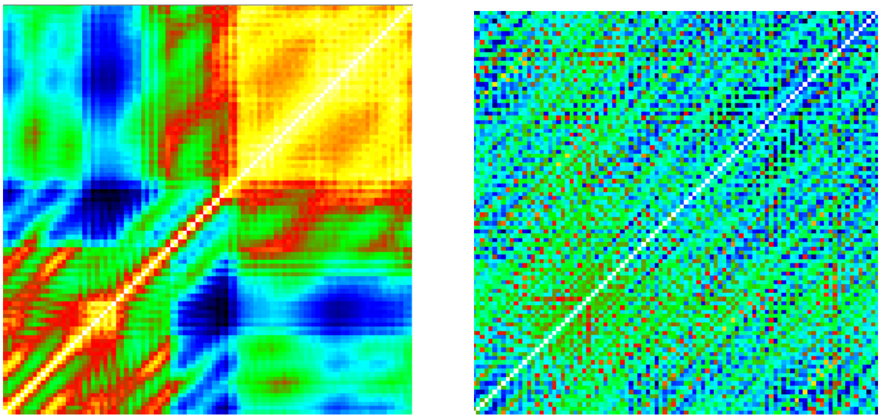
Japan and United Kingdom data and VRA

The Recurrence Plot (RP) of Japan GNP is shown in the Fig.2a. This was built using a delay-time and embedding dimension respectively equal to 2 and 7. The analysis of VRA using the shuffled series of Japan is described in Fig. 2b.

Comparison between RP of the original time series (Fig.2a) and RP of the shuffled series (Fig. 2b) allows to highlight that the first is non-stationary. The different and

[13] See “Table 4” p. 1580, in Frank et alt., (1988).
[14] Table 2, p. 1579 in Frank et alt., (1988).

diversified colours allow us to support that the more homogenous coloration from the shuffled series (Fig. 2b) is typical of stationary data.



(a) (b)

Fig. 2 (a) RP of Japan GNP; (b) shuffled time series

In table 1, RQA results are indicated for both time series: shuffled and not. For the original series REC is positive meaning that the data are correlated.

DET is also positive indicating that roughly 43% of the recurrent points are consecutive in the time, that is, form segments parallel to the main diagonal. This indicates that in the data there is some type of structure. As we saw in the (fig. 1a) long segments indicate that the series is periodic, short segments that the series is chaotic (fig. 1c). The value of MAXL is 28. This value indicates length of the longer segment in terms of recurrent points of the longer segment and allows to say that the data are non-linear and it is not possible to exclude the presence of chaotic behaviour

The statistics of the RQA (Tab.1) indicate that the shuffled series has lost all information, there are no recurrent points (REC), or segments parallel to the main diagonal (DET). Therefore, no type of deterministic structure is present. This consideration is confirmed by the fact that the value of the MAXL is negative.

From comparison between original time series and its shuffling we can conclude that the data of Japan GNP are characterised by non-linearity, confirming the result performed by Frank et al. (1988), and they are non stationary.

Japan		
GDP	1960-1988	Shuffled
Delay	2	2
Dimension	7	8
REC	2.314	0.0
DET	48.485	-1
ENT	1.00	0.0
MAXL	28	-1
TREND	-87.39	0.0

Table 1. RQA Statistics of original and shuffled time series

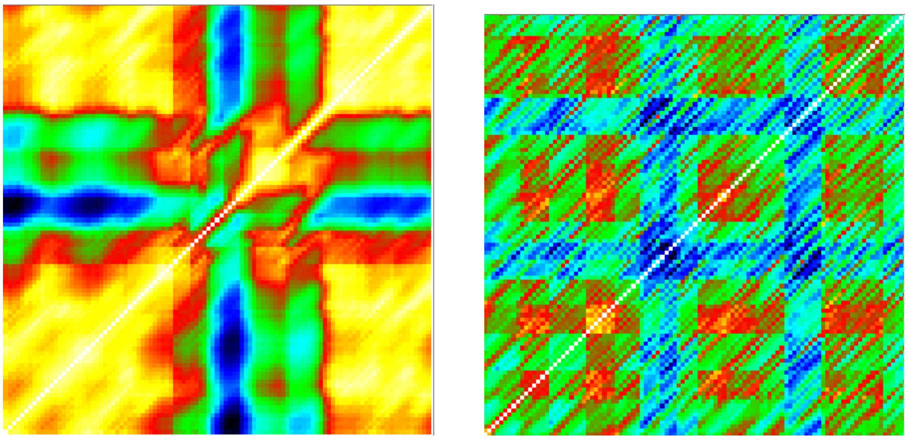
Our conclusion regarding the presence of chaotic behaviour is different[15]: the data can be chaotic. Therefore, if the authors ascribe the result of their analysis to the shortness of the time series highlighting that with longer time series it could be possible to reach a contrary result[16], the VRA analysis, that can be applied and gives reliable results also with short data sets, shows presence of chaotic behaviours in those data.

In the Fig. 3 we can see the RP of the United Kingdom GDP. This was built with delay-time and embedding dimension respectively equal to 1 and 8. By comparing

[15] "[…] None of these countries' national income would appear to be well interpreted as being chaotic.", Frank et al. (1988), p. 1581.

[16] "[…] When interpreting the findings one must be cautions given the shortness of the series. With longer time series matters could change", Frank et al. (1988), p. 1581.

the RP of the original time series (Fig. 3a) and its shuffling (Fig.3b) we deduce that the time series is non-stationary: the economy of the United Kingdom is characterised by a period of structural change.



(a) (b)

Fig. 3. (a) RP of UK GDP; (b) shuffled time series

Table 2 summarises the statistics of RQA for original time series and its shuffling. The statistics of original time series indicate that in the data there are recurrent points (REC positive), that is, more than 8% of the points that compose the area of the RP's triangle are correlated.

Of this 8%, 32% (DET) shapes segments parallels to the main diagonal, indicating the presence of determinist structures. This conclusion is confirmed by the presence of a positive value of the MAXL. The same ratios of shuffled series are characterised by zero values or negative.

United Kingdom

GDP	1960-1988	Shuffled
Delay	1	1
Dimen sion	8	9
REC	8.458	0.0
DET	32.009	-1
ENT	1.972	0.0
MAX L	26	-1
TREN D	77.803	0.0

Table.2. RQA Statistics of original and shuffled time series

Comparing our analysis with that performed by Frank M., et al. (1988) it is possible to highlight some points of difference. While they do not refuse hypothesis IID, the analysis led with VRA induces us to refuse this hypothesis and to emphasize the presence of structure. The data of the United Kingdom are non-linear and this nonlinearity can be interpreted as chaos.

The MAXL and DET value, in fact, confirm that. Also for the United Kingdom as for Japan, Frank M., et al. (1988) emphasized that the analysis carried out on longer series could have obtained different results, that we reached. A different conclusions concern also the fact that, while for Frank M., et al. the economy of Japan seems more stable than that of the European countries our analysis (also if limited just to the United Kingdom) is performed from a different point of view. While in Frank M., et al. the comparison was made between stable economies, our analysis is based on unstable economies. The economy of Japan in these years (60-88) is less unstable than that of UK.

5. *Conclusions*

To examine chaotic behaviour of time series a variety of methods have been invented. Their importance in examining the chaotic structure lies not only in their usefulness in analysing the non-linear structure but also in their relevance and potential utility to distinguish between stochastic behaviour and deterministic chaos (Pasanu and Ninni, 2000). These ones was widely used in physics to detect chaos.

However, not all of the approaches developed by physicists are applicable in economics because most of those methods require a large amount of data to ensure sufficient precision. There are, therefore advantages and disadvantages to each of these tools.

A recent development in the literature has been the introduction of the tools based on topological invariant testing procedure (close return test and recurrence plot). Compared to the existing metric class of testing procedures including correlation dimension, the BBS test, and Lyapunov exponent these tools could be better suited to testing for chaos in financial and economic time series.

Therefore, after a description and comparison between metric and topological tools we tested some macroeconomic time series already analysed with traditional test for chaos (Frank et al. 1988).

The application of typical tools for detecting chaos (correlation dimension and Lyapunov exponent) didn't show the presence of chaos in any time series. The conclusion of authors is that none of the countries' income appeared to be well interpreted as being chaotic. The authors ascribe their result to shortness of time series highlighting that with longer time series it could be possible to reach a contrary result.Testing these time series with Visual Recurrence Analysis based on the topological approach has provided different conclusions. Our analysis, although performed using a short time series indicates the presence of chaotic behaviour in Japand and United Kingdom time series. From our analysis compared with the more conventional one by Frank et al. (1988) it is possible to conclude that topological approach can be more useful for economic analysis performed on short time series, typical of economy.

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