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# A VECTOR AUTO-REGRESSIVE (VAR) MODEL FOR THE TURKISH FINANCIAL MARKETS

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## Abstract

In this paper, we develop a vector autoregressive (VAR) model of the Turkish financial markets for the period of June 15 2006 – June 15 2010 and forecasts ISE100 index, TRY/USD exchange rate, and short-term interest rates. The out-of-sample forecast performance of the VAR model is compared with the results from the univariate models. Moreover, the dynamics of the financial markets are analyzed through Granger causality and impulse response analysis.

**Keywords:** *multivariate financial time series, vector auto-regressive (VAR) model, impulse response analysis, Granger causality*

**JEL Classification:** C01, C51

## **1. Introduction**

Modeling the dynamics of financial markets is gaining popularity among researchers because of theoretical and technical reasons. Economic agents, both private and public, have close interest with the movements of the stock market index, interest rates, and exchange rates in order to make investment and economic policy decisions. Therefore, building efficient forecasting models for these variables play important roles in the decision making processes. Although, univariate models, ARMA(p,q) and GARCH(p,q), are widely used in the literature by the researchers for modeling and forecasting purposes, it is also important to analyse the interaction between variables in a multivariate framework.

In this paper, we move forward into this area by applying a vector autoregressive (VAR) model in modeling the financial variables of Turkish market. For this purpose, daily observations of IMKB100 index, TCMB benchmark bond rates, and USD/TRY exchange rates between the four years period of 15.06.2006 and 15.07.2010 are used. The rest of the paper is; part two deals with the literature review and related work, part 2 and 3 give detailed description of methodology and data analysis, empirical results are discussed in part 5 and part 6 concludes.

## **2. Literature Review**

The vector autoregression (VAR) model is one of the most successful, flexible, and easy to use models for the analysis of multivariate time series. VAR models in economics were made popular by Sims [8]. It is a natural extension of the univariate autoregressive model. The VAR model is useful for describing the dynamic behavior of financial time series and for forecasting. The superior forecasts to those from univariate time series models and elaborate theory-based simultaneous equations models can be provided by using VAR models.. Forecasting is quite flexible since they can be made conditional on the potential future paths of specified variables in the model.

There are many studies about modeling financial time series with VAR models. The most important one is the book of Culbertson[3] that is about stocks,

bonds and foreign exchange. But there are a few studies about the Turkish Financial Market especially in the period which includes the 2008 financial crisis.

In addition to data description and forecasting, the VAR model is also used for structural inference and policy analysis. In structural analysis, certain assumptions about the causal structure of the data under investigation are imposed, and the resulting causal impacts of unexpected shocks or innovations to specified variables on the variables in the model are summarized. These causal impacts are usually summarized with impulse response functions and forecast error variance decompositions. The definitive technical reference for VAR models is Lutkepohl [5], and updated surveys of VAR techniques are given in Watson [11] and Lutkepohl [6] and Waggoner and Zha [10]. Applications of VAR models to financial data are given in Hamilton [4], Campbell, Lo and MacKinlay [2], Culbertson [3], Mills [7] and Tsay [9].

### 3. Methodology

When building a VAR model, the following steps can be used. First we can use the test statistic  $M(i)$  or the Akaike information criterion to identify the order, then estimate the specified model by using the least squares method (if there are statistically insignificant parameters, by removing these parameters the model should be reestimated), and finally use the  $Q_k(m)$  statistic of the residuals to check the adequacy of a fitted model. Other characteristics of the residual series, such as conditional heteroscedasticity and outliers, can also be checked.

#### 3.1 Vector AR(p) Models

The time series  $Y_t$  follows a VAR( $p$ ) model if it satisfies

$$Y_t = \phi_0 + \Phi_1 Y_{t-1} + \dots + \Phi_p Y_{t-p} + a_t, \quad p > 0, \quad (1)$$

where  $\phi_0$  is a  $k$ -dimensional vector, and  $a_t$  is a sequence of serially uncorrelated random vectors with mean zero and covariance matrix  $\Sigma$ . In application, the

covariance matrix  $\Sigma$  must be positive definite; otherwise, the dimension of  $Y_t$  can be reduced. The error term  $a_t$  is multivariate normal and  $\Phi_j$  are  $k \times k$  matrices. Using the back-shift operator  $B$ , the VAR( $p$ ) model can be written as

$$(I - \Phi_1 B - \dots - \Phi_p B^p) Y_t = \phi_0 + a_t,$$

where  $I$  is the  $k \times k$  identity matrix. In a compact form as follows

$$\Phi(B) Y_t = \phi_0 + a_t,$$

where  $\Phi(B) = I - \Phi_1 B - \dots - \Phi_p B^p$  is a matrix polynomial. If  $Y_t$  is weakly stationary, then we have

$$\mu = E(Y_t) = (I - \Phi_1 - \dots - \Phi_p)^{-1} \phi_0 = [\Phi(1)]^{-1} \phi_0$$

provided that the inverse exists since determinant of  $[\Phi(1)]$  is different from zero.

Let  $\tilde{Y}_t = Y_t - \mu$ . Then the VAR( $p$ ) model becomes

$$\tilde{Y}_t = \Phi_1 \tilde{Y}_{t-1} + \dots + \Phi_p \tilde{Y}_{t-p} + a_t. \quad (2)$$

Using the equation(2) below results can be obtained

- $\text{Cov}(Y_t, a_t) = \Sigma$ , the covariance matrix of  $a_t$  ;
- $\text{Cov}(Y_{t-l}, a_t) = \mathbf{0}$  for  $l > 0$ ;
- $\Gamma_l = \Phi_1 \Gamma_{l-1} + \dots + \Phi_p \Gamma_{l-p}$  for  $l > 0$ . (3)

The equation (3) is multivariate version of Yule–Walker equation and it is called the moment equations of a VAR( $p$ ) model

### 3.2 Building a VAR( $p$ ) Model

The concept of partial autocorrelation function of a univariate series can be generalized to specify the order  $p$  of a vector series. Consider the following consecutive VAR models:

$$Y_t = \phi_0 + \Phi_1 Y_{t-1} + a_t$$

$$Y_t = \phi_0 + \Phi_1 Y_{t-1} + \Phi_2 Y_{t-2} + a_t$$

... = ...

$$Y_t = \phi_0 + \Phi_1 Y_{t-1} + \dots + \Phi_i Y_{t-i} + a_t \quad (4)$$

... = ...

The ordinary least squares (OLS) method is used for estimating parameters of these models. This is called the multivariate linear regression estimation in multivariate statistical analysis.[9]

For the  $i$  th equation in Eq. (3), let  $\hat{\Phi}_j^{(i)}$  be the OLS estimate of  $\Phi_j$  and  $\hat{\phi}_j^{(i)}$  be the estimate of  $\phi_0$ , where the superscript  $(i)$  is used to denote that the estimates are for a VAR( $i$ ) model. Then the residual is

$$\hat{a}_t^{(i)} = Y_t - \hat{\Phi}_1^{(i)} Y_{t-1} - \dots - \hat{\Phi}_i^{(i)} Y_{t-i}$$

For  $i = 0$ , the residual is defined as  $\hat{Y}_t^{(0)} = Y_t - \bar{Y}$ , where  $\bar{Y}$  is the sample mean of  $Y_t$ . The residual covariance matrix is defined as

$$\hat{\Sigma}_i = \frac{1}{T - 2i - 1} \sum_{t=i+1}^T \hat{a}_t^{(i)} \left( \hat{a}_t^{(i)} \right)' \quad (5)$$

To specify the order  $p$ , we can use the  $i$  th and  $(i - 1)$ th equations in Eq. (4) to, testing a VAR( $i$ ) model versus a VAR( $i - 1$ ) model and test the hypothesis  $H_0 : \Phi_l = 0$  versus the alternative hypothesis  $H_a : \Phi_l \neq 0$  sequentially for  $l = 1, 2, \dots [1]$ . The test statistic is

$$M(i) = - \left( T - k - i - \frac{3}{2} \right) \ln \left( \frac{|\hat{\Sigma}_i|}{|\hat{\Sigma}_{i-1}|} \right)$$

The distribution of  $M(i)$  is a chi-squared distribution with  $k^2$  degrees of freedom.

Alternatively, the Akaike information criterion (AIC) can be used to select the order  $p$ . Assume that  $a_t$  is multivariate normal and consider the  $i$  th equation in Eq. (4). One can estimate the model by the maximum likelihood (ML) method. For AR models, the OLS estimates  $\phi_0$  and  $\Phi_j$  are equivalent to the (conditional) ML estimates. However, there are differences between the estimates of  $\Sigma$ . The ML estimate of  $\Sigma$  is [9]

$$\hat{\Sigma}_i = \frac{1}{T} \sum_{t=i+1}^T \hat{a}_t^{(i)} \left( \hat{a}_t^{(i)} \right)' \quad (6)$$

The AIC of a VAR( $i$ ) model under the normality assumption is defined as

$$AIC(i) = \ln(|\tilde{\Sigma}_i|) + \frac{2k^2i}{T} \quad (7)$$

For a given vector time series, one selects the AR order  $p$  such that  $AIC(p) = \min_{1 \leq i \leq p} AIC(i)$ , where  $p$  is positive integer.

### 3.3 Estimation and Model Checking

Both of the ordinary least squares method or the maximum likelihood method can be used to estimate the parameters of VAR model since the two methods are asymptotically equivalent. The estimates are asymptotically normal under some regularity conditions. After constructing the model, adequacy of the model should then be checked.

The  $Q_k(m)$  statistic can be applied to the residual series to check the assumption that there are no serial or cross-correlations in the residuals. For a fitted VAR( $p$ ) model, the  $Q_k(m)$  statistic of the residuals is asymptotically a chi-squared distribution with  $k^2 m - g$  degrees of freedom, where  $g$  is the number of estimated parameters in the AR coefficient matrixes.[9]

### 3.4 Structural Analysis by Impulse Response Functions

The general form of the VAR(p) model is shown in eq.(1). VAR(p) model also has a Wold representation as follows

$$Y_t = \mu + a_t + \theta_1 a_{t-1} + \theta_2 a_{t-2} + \dots \quad (8)$$

Where  $\theta_s$  are moving average  $n \times n$  matrices. To interpret the  $(i,j)$ -th element,  $\theta_{ij}^s$ , element of the matrix  $\theta_s$  as the dynamic multiplier or impulse response

$$\frac{\partial y_{i,t+s}}{\partial a_{j,t}} = \frac{\partial y_{i,t}}{\partial a_{j,t-s}} = \theta_{ij}^s \quad i,j=1,2,\dots,n \quad (9)$$

The condition for the eq.(9) is  $\text{var}(a_t) = \Sigma$  is a diagonal matrix. If  $\Sigma$  is diagonal, it shows the element of  $\Sigma$ ,  $a_t$ , are uncorrelated. One way to make the errors uncorrelated is to estimate the triangular structural VAR(p) model

$$y_{1t} = c_1 + \alpha'_{11} Y_{t-1} + \dots + \alpha'_{1p} Y_{t-p} + \eta_{1t}$$

$$y_{2t} = c_1 + \beta_{21} y_{1t} + \alpha'_{21} Y_{t-1} + \dots + \alpha'_{2p} Y_{t-p} + \eta_{2t}$$

:

$$y_{nt} = c_1 + \beta_{n1} y_{1t} + \dots + \beta_{n,n-1} y_{n-1,t} + \alpha'_{n1} Y_{t-1} + \dots + \alpha'_{np} Y_{t-p} + \eta_{nt} \quad (10)$$

the estimated covariance matrix of the error vector  $\eta_t$  is diagonal. The uncorrelated/orthogonal errors  $\eta_t$  are referred to as structural errors. The Wold representation of  $Y_t$  based on the orthogonal errors  $\eta_t$  is given by

$$Y_t = \mu + \Theta_0 \eta_t + \Theta_1 \eta_{t-1} + \Theta_2 \eta_{t-2} + \dots$$

Where  $\Theta_0 = B^{-1}$  ( $B$  is the lower triangular matrix of  $\beta_{i,j}$  in eq. (10). The diagonal elements of the  $B$  is 1.) The The impulse responses to the orthogonal shocks  $\eta_{jt}$  are



$\frac{\partial y_{i,t+s}}{\partial \eta_{j,t}} = \frac{\partial y_{i,t}}{\partial \eta_{j,t-s}} = \theta^s_{ij}$ , where  $\theta^s_{ij}$  is the (i,j) th element of  $\Theta_s$ . The plot of  $\theta^s_{ij}$  against s is called the orthogonal impulse response function of  $y_i$  with respect to  $\eta_j$ .

### 3.5 Structural Analysis by Granger Causality

In order to investigate the causal relationship between the variables of the system, the linear Granger causality tests should be applied by using following strategy.

Compare the unrestricted models;

$$\Delta y_t = a_1 + \sum_{i=1}^{m_1} \beta_{1i} \Delta y_{t-i} + \sum_{j=1}^{m_2} \theta_{1j} \Delta x_{j-i} + e_{1t} \quad (11)$$

$$\Delta x_t = a_2 + \sum_{i=1}^{m_1} \beta_{2i} \Delta x_{t-i} + \sum_{j=1}^{m_2} \theta_{2j} \Delta y_{j-i} + e_{2t} \quad (12)$$

with the restricted models

$$\Delta y_t = a_1 + \sum_{i=1}^{m_1} \beta_{1i} \Delta y_{t-i} + e_{1t} \quad (13)$$

$$\Delta x_t = a_2 + \sum_{i=1}^{m_1} \beta_{2i} \Delta x_{t-i} + e_{2t} \quad (14)$$

where  $\Delta x_t$  and  $\Delta y_t$  are the first order forward differences of the variables,  $a, \beta, \theta$  are the parameters to be estimated and,  $e_1, e_2$  are standard random errors. The lag

order  $m$  are the optimal lag orders chosen by information criteria. The equations described above, are convenient tools for analyzing linear causality relationship between the variables. If  $\theta_1$  is statistically significant, and  $\theta_2$  is not, it can be said that changes in variable  $y$  Granger cause changes in variable  $x$  or vice versa. If both of them are statistically significant there is a bivariate causal relationship between the variables, if both of them are statistically insignificant neither the changes in variable  $y$  nor the changes in variable  $x$  have any effect over other variable.

### 3.6 Forecasting

If the fitted model is adequate, then it can be used to obtain forecasts. For forecasting, same techniques in the univariate analysis can be applied. To produce forecasts and standard deviations of the associated forecast errors can be done as following.

For a VAR( $p$ ) model, the 1-step ahead forecast at the time origin  $h$  is

$Y_h(1) = \phi_0 + \sum_{i=0}^p \Phi_i Y_{h+1-i}$  , and the associated forecast error is  $e_h = a_{h+1}$ . The

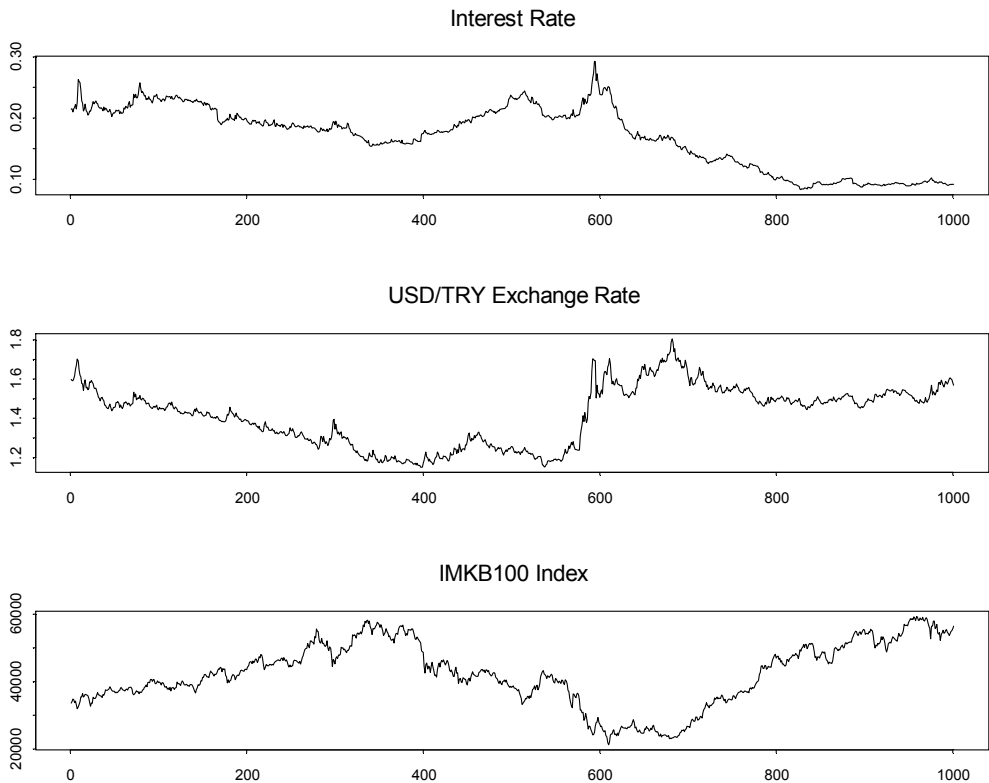
covariance matrix of the forecast error is  $\Sigma$ . If  $Y_t$  is weakly stationary, then the  $l$ -step ahead forecast  $Y_h(l)$  converges to its mean vector  $\mu$  as the forecast horizon increases.

## 4. Data Analysis

For this paper, daily observations of TCMB benchmark bond rate, USD/TRY foreign exchange rate, and IMKB100 index values for the four year period between 15.06.2006 and 15.06.2010 are used. Data between 15.06.2006 and 15.05.2010 (980) are used in-sample estimation and data between 15.05.2010 and

15.06.2011 are used for the out-of-sample forecasting purposes. Figure 1 below shows the time series plots of the three variables during the sample period.

**Figure 1: Time Series Plots of the Variables**



Source: TCMB Database <http://evds.tcmb.gov.tr>

In order to build an appropriate model, all series that are used in analysis must be stationary therefore we should check the unit-root structure of the data. Although above graph gives us a rough idea about the stationarity structure of the series we need more formal tests to check the stationary. We have applied Augmented Dickey-Fuller test to series in order to test unit-roots. Table 1 exhibits the results from ADF test applied to both levels and first differences of the series

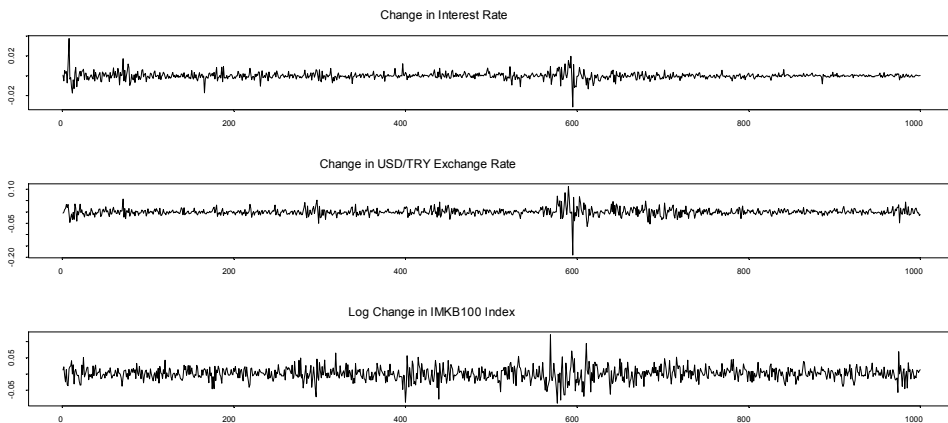
**Table 1: ADF Unit-Root Test Results**

Variable	Deterministic terms	Lags	Test value	Critical values		
				1%	5%	10%
interest	constant,trend	2	-2.05	-3.96	-3.41	-3.12
$\Delta$ interest	constant	1	-22.118	-3.43	-2.86	-2.57
fx	constant,trend	2	-2.348	-3.96	-3.41	-3.12
$\Delta$ fx	constant	1	-21.522	-3.43	-2.86	-2.57
xu100	constant,trend	2	-1.222	-3.96	-3.41	-3.12
$\log(\Delta xu100)$	constant	1	-21.868	-3.43	-2.86	-2.57

Source: Own Study

The ADF test results indicate that all variables are non-stationary by not rejecting the null hypothesis of unit-root at all levels of critical values, but they are all stationary after first differencing. Therefore, we use differenced series in our analysis, figure 2 below time series plots of the differenced series.

**Figure 2: Time Series Plot of the Differenced Variables**



Source: Own Study

## 5. Empirical Results

In this part, our first aim is to determine the true lag order for the model as Lutkepohl [5] points out that selecting a higher order lag length than the true lag lengths increases the mean square forecast errors of the VAR, and selecting a lower order lag length than the true lag lengths usually causes autocorrelated errors. Therefore, accuracy of forecasts from VAR models highly depends on selecting the true lag lengths. There are several statistical criterion for selecting a lag length. We have identified a VAR(p) model for the analysis by using penalty selection criteria such as Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and Hannan-Quinn Information Criterion (HQC). The table 2 below shows the results of the selection criterion.

**Table 2: VAR(p) Model Order Selection Criterion**

Lag Order	Criteria		
	AIC	HQC	BIC
1	-70.024	-69.742	-69.283
2	-70.670	-70.219	-69.484
3	-70.635	-70.015	-69.005
4	-70.650	-69.816	-68.530
5	-70.585	-69.628	-68.066
6	-70.503	-69.376	-67.539
7	-70.420	-69.124	-67.012
8	-70.354	-68.889	-66.502
<b>Minimum Lag Order</b>	<b>2</b>	<b>2</b>	<b>2</b>

*Source: Own Study*

The results from the table 2 suggest that the appropriate model for our data is VAR(2) because all three methods gave lag order 2 as minimum lag order.

After we have identified a VAR(2) model, we move forward to model estimation process. The model estimation results from the VAR(2) model are given in the following tables.

**Table 3.1 Coefficient Estimates for Interest Rate Equation**

	<b>Coefficient</b>	<b>Std.Error</b>	<b>t-value</b>	<b>t-prob</b>
<b>Dinterest_1</b>	0.0537363	0.03663	2.51	0.1427
<b>Dinterest_2</b>	-0.120437	0.03669	-3.28	0.0011
<b>Dfx_1</b>	-0.0427225	0.00902	-4.74	0
<b>Dfx_2</b>	0.0370755	0.008225	4.13	0
<b>DLxu100_1</b>	-0.0625408	0.005128	-12.2	0
<b>DLxu100_2</b>	-0.0395795	0.006018	-6.58	0
<b>Constant</b>	-0.000700301	0.0001054	-0.665	0.5065
<b>sigma</b>	0.00328727		<b>RSS</b>	0.01048194609

**Table 3.2 Coefficient Estimates for Exchange Rate Equation**

	<b>Coefficient</b>	<b>Std.Error</b>	<b>t-value</b>	<b>t-prob</b>
<b>Dinterest_1</b>	-0.449198	0.1484	-3.03	0.0025
<b>Dinterest_2</b>	-0.481332	0.1487	-3.24	0.0012
<b>Dfx_1</b>	-0.0992883	0.03655	-2.72	0.0067
<b>Dfx_2</b>	0.0704908	0.03333	2.12	0.0347
<b>DLxu100_1</b>	-0.401393	0.02078	-19.3	0
<b>DLxu100_2</b>	-0.196682	0.02439	-8.07	0
<b>Constant</b>	0.000139416	0.000427	0.326	0.7441
<b>sigma</b>	0.0133205		<b>RSS</b>	0.1721134129

**Table 3.3 Coefficient Estimates for IMKB100 Equation**

	<b>Coefficient</b>	<b>Std.Error</b>	<b>t-value</b>	<b>t-prob</b>
<b>Dinterest_1</b>	0.230771	0.2295	1.01	0.3150
<b>Dinterest_2</b>	0.0650816	0.2299	0.283	0.7772
<b>Dfx_1</b>	-0.0578057	0.05652	-1.02	0.3067
<b>Dfx_2</b>	-0.0755216	0.05154	-1.47	0.1432
<b>DLxu100_1</b>	0.0683901	0.03213	2.13	0.0336
<b>DLxu100_2</b>	-0.0238064	0.03771	-0.631	0.5280
<b>Constant</b>	0.000485471	0.0006603	0.735	0.4624
<b>sigma</b>	0.0205988		<b>RSS</b>	0.4115819301

*Source: Own Study*

When we look at the coefficients for the interest rate equations apart from constant term and first lag of the interest rate, are all statistically significant in terms of t-value. All coefficients are significant in foreign exchange equations, whereas only its first lag has statistically significant effect on stock index variable.

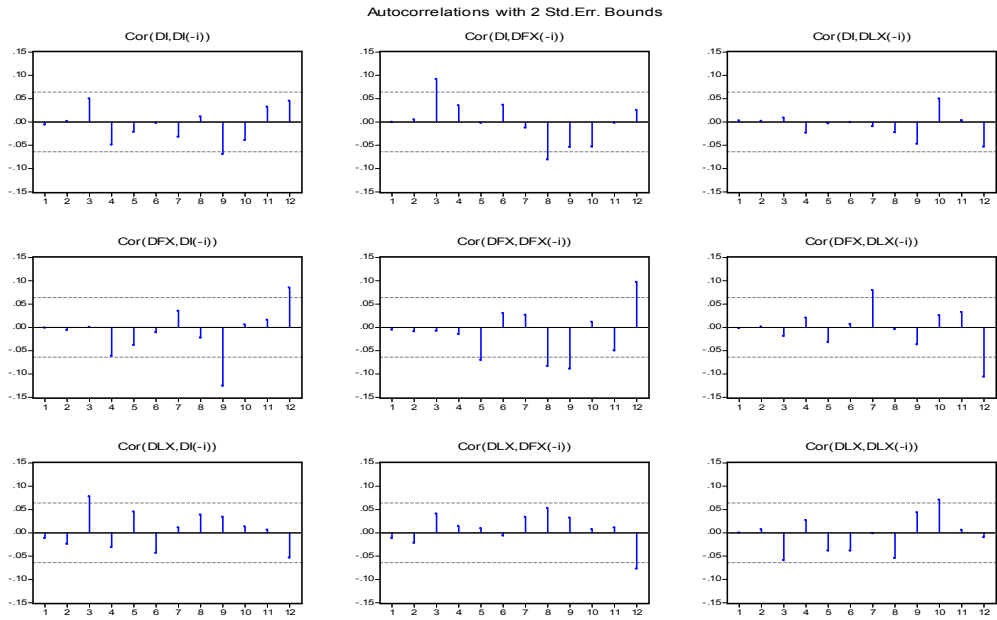
After we have estimated a suitable VAR(2) model for the variables, this stage of the analysis deals with the diagnostic checking process. There are several methods that control the robustness of the model, we have used graphical analysis tools and statistical tests for the residuals for the diagnostic checks. The table 4 below exhibits the results of the serial correlation, normality and heteroskedasticity tests of the residuals. And the figure 3 and 4 shows the ACF and density plots of the model residuals. The diagnostic results imply that VAR(2) model should be extended by making heavy tailed distributional assumptions of the residuals as the distributional properties of the residuals are not normal. Also, heteroskedasticity testing results suggest the application of a multivariate GARCH model for the series. We can say that for the interest rate and IMKB100 index VAR(2) could be a good model as it eliminates the serial correlation, but for the foreign exchange series there is still correlated residuals as the test statistics suggest the rejection of the null hypothesis of no serial correlation until lag 12. Perhaps, instead of symmetric lag order, we can use asymmetric lag order model for the variables.

**Table 4: Residual Diagnostic Tests**

Dinterest	Serial Correlation Test	F(12,958)	=	14.861	[0.1231]
Dfx	Serial Correlation Test	F(12,958)	=	28.820	[0.0006]**
DLxu100	Serial Correlation Test	F(12,958)	=	16.097	[0.0833]
Dinterest	Normality Test	Chi <sup>2</sup> (2)	=	1528.1	[0.0000]**
Dfx	Normality Test	Chi <sup>2</sup> (2)	=	2040.2	[0.0000]**
DLxu100	Normality Test	Chi <sup>2</sup> (2)	=	184.44	[0.0000]**
Dinterest	Heteroskedasticity Test	F(12,957)	=	57.043	[0.0000]**
Dfx	Heteroskedasticity Test	F(12,957)	=	11.428	[0.0000]**
DLxu100	Heteroskedasticity Test	F(12,957)	=	48.484	[0.0000]**

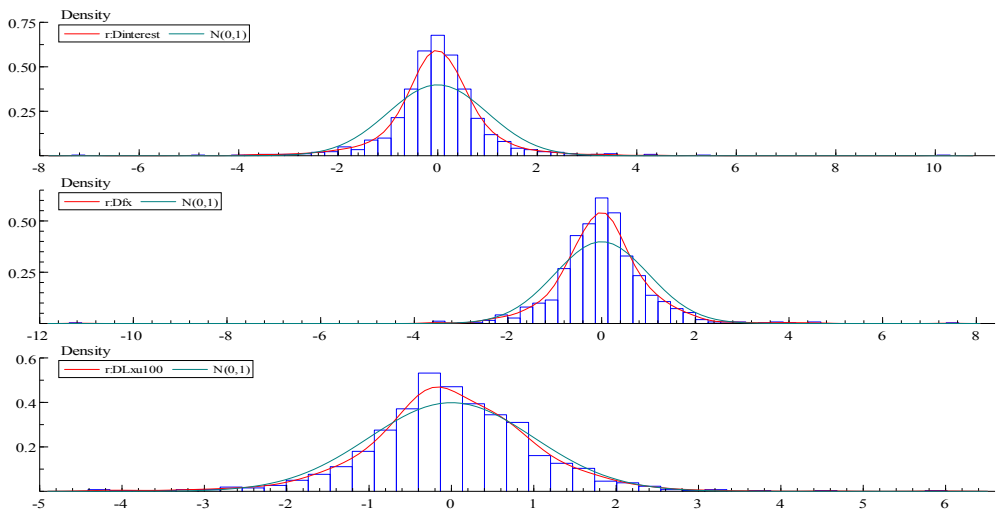
*Source: Own Study*

**Figure 4: Correlations of Residuals**



Source: Own Study

**Figure 5: Residuals Density Plots**

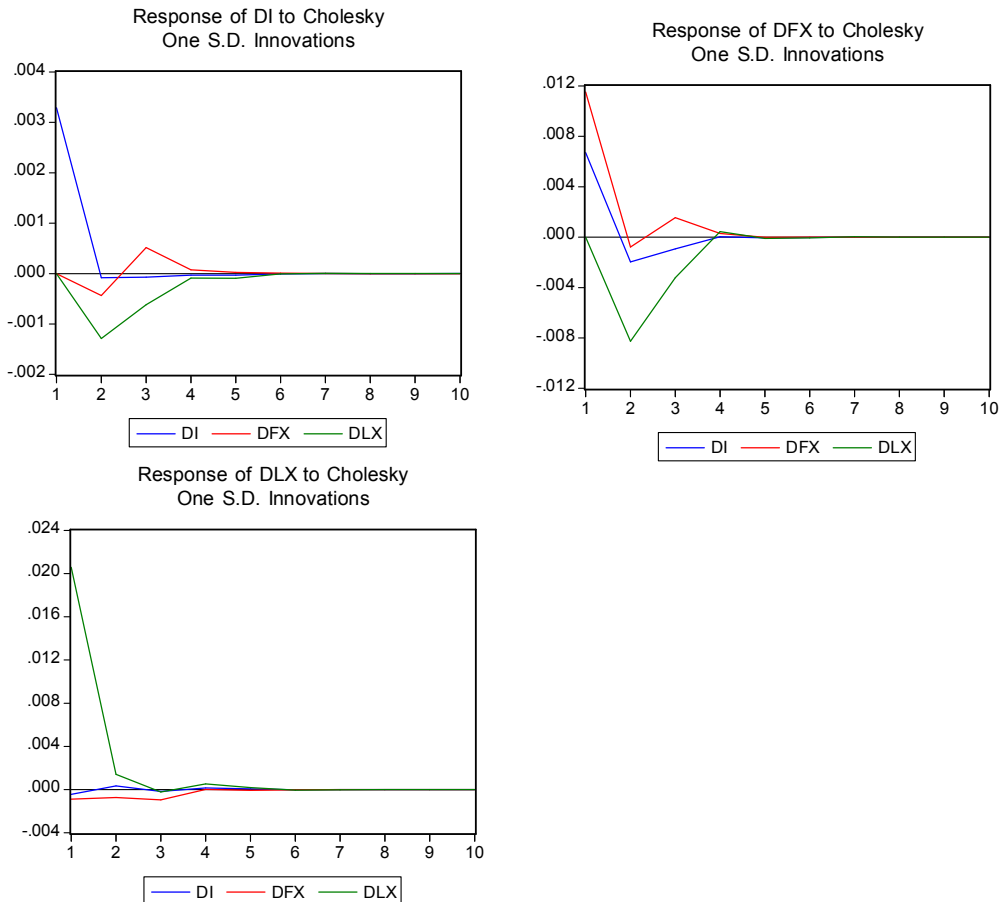


Source: Own Study



In order to see the dynamics of the variables we have applied impulse response analysis and Granger causality tests.

**Figure 6: Impulse Response Function**



Source: Own Study

The figure 6 shows the combined graph of the impulse responses of each variable. As we can see from the graph that one exogenous shocks to interest rates and IMKB100 index have immediate effect on foreign exchange rate, whereas interest rate responses are not significant. And, IMKB100 index has very little response to exogenous shocks to other variables.

**Table 5: Granger Causality Test**

Pairwise Granger Causality Tests			
Sample: 1 1001			
Lags: 2			
Null Hypothesis:	Obs	F-Statistic	Probability
DFX does not Granger Cause DINTEREST	998	11.3080	1.4E-05
DINTEREST does not Granger Cause DFX		6.97222	0.00098
DLXU100 does not Granger Cause DINTEREST	998	91.2241	4.3E-37
DINTEREST does not Granger Cause DLXU100		0.39779	0.67191
DLXU100 does not Granger Cause DFX	998	230.457	6.1E-83
DFX does not Granger Cause DLXU100		1.94853	0.14303

*Source: Own Study*

Table 5 shows the Granger causality test results. The test results indicate that there is a bivariate causal relationship between interest rates and foreign exchange rates by rejecting the null hypothesis of no Granger causality. Whereas, there is one way causal relationships between interest rates and IMKB100 index, and between foreign exchange rates and IMKB100 index. While changes in IMKB100 index have direct effect over other variables, the changes in interest rates and foreign exchange rates do not cause changes in the IMKB100 index.

After we have estimated and checked our model for in-sample analysis, this stage deals with the out-of-sample forecasting performance analysis. We have used 21 observation for the forecast purposes and compare the results of the VAR(2) model with the univariate models which are chosen for each variable by penalty selection criteria. ARMA(1,1) model is chosen as best suitable model for interest rate and foreign exchange rate series, and ARMA(1,3) for the IMKB100 index

series. Root mean squared error (RMSE) statistics are used for the performance evaluation tool. The table 6 shows the test results.

**Table 6: RMSE Statistics for Forecast Performance**

	<b>VAR(2) Model</b>	<b>Univariate Model</b>
<b>Dinterest</b>	0.0011594	0.00094486
<b>Dfx</b>	0.0092412	0.015170
<b>DLxu100</b>	0.018278	0.018482

*Source: Own Study*

According to RMSE statistic, univariate model gives better out-of-sample performance for interest rate series, whereas VAR(2) model outperforms univariate models in forecasting the foreign exchange rates and IMKB100 index. The statistic also suggest that VAR(2) models out-of-sample forecasting performance for interest rates are better than for the other variables.

## **6. Conclusion**

In this paper, we have attempted to build a multivariate time series model for the Turkish financial markets. We applied vector autoregressive (VAR) model in modeling and forecasting the Turkish interest rates, USD/TRY exchange rates, and IMKB100 index for the four year period between 15.06.2006 – 15.06.2010. VAR(2) model has been chosen as best candidate model for the variables in sample period. Model estimation results, impulse response analysis and Granger causality tests indicate that while VAR(2) model is a satisfactory model for interest rates and exchange rates, it is not a suitable for the stock market dynamics. A further study on continuous-time stochastic models should be better for modeling the dynamics of Istanbul Stock Exchange. Also, heteroskedasticity tests show that volatility of the series are not constant, an extended study on multivariate GARCH models would be better for modeling the series for the sample period.

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