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# Policy Rule Coefficients Driven by Latent Factors: Monetary and Fiscal Policy Interactions in an Endowment Economy

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## Abstract

In this paper I formulate, solve and estimate an endowment version of a macroeconomic dynamic stochastic general equilibrium model with monetary and fiscal policy rules whose coefficients are time-varying and contemporaneously correlated. The aim of the paper is to identify from data the interactions between monetary and fiscal policies that have prevailed in the U.S. economy. The monetary authority uses a Taylor rule and the fiscal authority uses a rule in which taxes respond to lagged debt deviations. Policy rule coefficients are modeled as logistic functions of stationary correlated latent factors, introducing long-run interactions between monetary and fiscal policies. There are three main findings of the paper: First, monetary policy has reacted strongly to inflation deviations along, almost, the entire analyzed period, with a loose policy only during the periods 1979:1-1981:3 and 2008:4-2009:2. Second, regimes under which a determinacy condition is in place occur 54.25% of the time, while regimes with exploding local dynamics occur 45.34% of the time, and there is an association between the duration of these unstable regimes and the volatility of inflation. Third, tightening monetary policy in terms of increasing the reaction of the central bank with respect to inflation deviations, given the situation of the economy in the third quarter of 2010, implies an increase in inflation of the order of 3%.

**Keywords:** Time-varying policy rules, Monetary and fiscal policy interactions, Nonlinear state-space models.

**JEL Classification Numbers:** C11, C32, E63.

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# 1 Introduction

In this paper I formulate, solve and estimate an endowment version of a macroeconomic dynamic stochastic general equilibrium (DSGE) model with monetary and fiscal policy rules whose coefficients are time-varying and contemporaneously correlated. The aim of the paper is to identify from data the interactions between monetary and fiscal policies that have prevailed in the U.S. economy.

In recent decades has been widely accepted that price stabilization is a desirable outcome that central banks should achieve as a sign of their success. This is especially true after the learning process of the 1970s that led to the arise of the rational expectation hypothesis, when central bankers understood that, to stabilize prices, inflation expectations should be anchored. To incorporate this desirable policy outcome, nowadays conventional macroeconomic DSGE models specify an interest rate rule in which the central bank reacts to increases in inflation with increases more than proportional in the nominal interest rate. This rule now has the name of his proponent: Taylor ([Taylor, 1993](#)). By conducting monetary policy in this way, inflation expectations are not supposed to trigger instability in the economy. This conventional setup assumes that fiscal policy will accommodate the increase in interest rates necessary to achieve stability with increases in (lump-sum) taxes to cover the higher interests on public debt to keep it stable. This model (New Keynesian DSGE model) has become so widely used that now is textbook material since its original formulation in the second half of the 1990s, and elaborated versions of it are used by central bankers around the globe.

Another strand of the literature emphasizes that fiscal policy may play a more important role than just accommodating monetary policy in achieving stabilization. This argument stresses that this is especially true when monetary policy is not or can not be used as the conventional models propose. The global crisis has shown that important amounts of debt were issued by governments to face the crisis, with no immediate revision in expectations of changes in future taxes. Additionally, in the U.S. in particular, interest rates are near the zero lower bound, giving almost no space to monetary policy.

The role of fiscal policy in economic systems, in particular in terms of stability (in a local-linear sense), was first introduced by [Leeper \(1991\)](#), and later extended by [Sims \(1994\)](#) and [Woodford \(1995\)](#), with heated debates about the plausibility of the results under this setup that accompanied the arise of the so-called Fiscal Theory of the Price Level. The idea is that, when monetary policy does not or can not anchor inflation expectations, fiscal policy, through expectations about future surpluses (debt valuation equation), can anchor these expectations ([Cochrane, 2011](#)). [Leeper \(2010\)](#) offers a review of how fiscal policy influences economic stability, and why fiscal policy should be taken more seriously in economic modeling, especially when times of fiscal stress arrive.

With respect to policy making (monetary and fiscal), there is substantial empirical literature arguing that policy rules have not remained invariant over the course of the last 6 decades. Examples include: [Clarida et al. \(2000\)](#), [Cogley and Sargent \(2002\)](#), [Lubik and Schorfheide \(2004\)](#), [Primiceri \(2005\)](#), [Fernandez-Villaverde et al. \(2010\)](#), [Favero and Monacelli \(2003\)](#), and [Davig and Leeper \(2006\)](#). There is also theoretical literature arguing that, in designing policy rules, policy authorities may have asymmetric preferences with respect to deviations of variables of interest from target, or state-dependent loss functions,

resulting this in time-varying rules. Examples include: [Dolado et al. \(2005\)](#) and [Svensson and Williams \(2007\)](#).

This paper assumes that policy moves across regimes as a function of a latent factor. This is analogous to having a random coefficient (unit root) specification, or a Markov switching specification for policy rules. The difference is that the function considered here is bounded (as opposed to the random coefficient setup), and continuous (as opposed to the Markov switching setup). A function that satisfies these requirements is the logistic function. Boundedness is important because some policy rule coefficients make sense only if they are positive or, in terms of determinacy of a linear rational expectations model, if they have an upper or lower bound. Smoothness (continuity) of the transition is also important since policies do not necessarily switch abruptly from one regime to another, and if they do, the logistic function still allows to have that type of behavior. The logistic function driving policy parameters is modeled in terms of latent factors, which can be thought as a combination of macroeconomic, microeconomic, financial, political, institutional, etc. factors that trigger a change in monetary and/or fiscal policy.

I also incorporate the possibility of interactions between policies. There is extensive literature on monetary and fiscal policy interactions. From a normative perspective, for example: [Nordhaus \(1994\)](#), [Buti et al. \(2001\)](#), and [Dixit and Lambertini \(2003\)](#). From a positive perspective, for example: [Davig and Leeper \(2006\)](#), and [Chung et al. \(2007\)](#). I introduce interactions between monetary and fiscal policies by means of correlation between the latent factors that drive the evolution of policy rule coefficients. To the best of my knowledge, this is the first attempt in the literature to introduce these explicit interactions between policies, and the first attempt to estimate a model with these characteristics to let the data tell what the interactions have been over the last 6 decades.

Since the model is intrinsically nonlinear, an unconventional solution technique has to be used. To solve a semi-nonlinear version of the model, I propose a method based on the minimal state variable solution and show that, under certain conditions, the solution exists and is stable. The paper also provides a discussion on uniqueness. Once solved, the DSGE model boils down to a state-space model in which the latent factors enter nonlinearly. I utilize appropriate Bayesian techniques to estimate the parameters of the model and the latent factors using U.S. data on interest and tax rates. I also obtain responses of inflation to shocks to interest and tax rates, conditional on the situation of the economy in the third quarter of 2010; and introduce an unconventional impulse-response approach in which a shock is given to the parameters of the rules and the response is evaluated on inflation.

There are three main findings of the paper: First, monetary policy has reacted strongly to inflation deviations along, almost, the entire analyzed period, with a loose policy only during the periods 1979:1-1981:3 and 2008:4-2009:2. Second, regimes under which a determinacy condition is in place occur 54.25% of the time, while regimes with exploding local dynamics occur 45.34% of the time, and there is an association between the duration of these unstable regimes and the volatility of inflation. Third, tightening monetary policy in terms of increasing the reaction of the central bank with respect to inflation deviations, given the situation of the economy in the third quarter of 2010, implies an increase in inflation of the order of 3%.

## 2 The Model

The economy has a maximizing representative household endowed with constant output, and there are also monetary and fiscal authorities. There are neither price nor wage rigidities or capital accumulation, to keep the analysis focused on the interactions between monetary and fiscal policy, and to illustrate the solution method and the estimation strategy in a simple framework. The model is a modified version of the model in [Leeper \(1991\)](#) where smoothing has been included in both the monetary and fiscal policy rules.

### 2.1 Households

The representative household derives utility from consumption,  $C_t$ , and real money balances,  $M_t/P_t$ . A representative household maximizes

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma} - 1}{1-\sigma} + \chi_M \log(M_t/P_t) \right), \quad (1)$$

where  $0 < \beta < 1$  is the discount factor,  $\sigma > 0$  is the inverse of the elasticity of intertemporal substitution, and  $\chi_M > 0$  is a constant affecting the velocity of money,  $v_t$ , in steady state. The household saves in the form of nominal government bonds,  $B_t$ , that pay a gross interest rate  $R_t$  each period, and by accumulating money balances,  $M_t - M_{t-1}$ , that do not pay interests. It pays lump-sum taxes,  $T_t$ , and receives a constant endowment,  $Y$  each period, so that its budget constraint is given by

$$P_t C_t + M_t + B_t + T_t = P_t Y + M_{t-1} + R_{t-1} B_{t-1} \text{ for } t \geq 0, \quad (2)$$

given the initial value of assets  $(M_{-1} + R_{-1} B_{-1})/P_{-1}$ , and where the transversality condition that rules out Ponzi games holds.

### 2.2 Government

The government finances a constant level of goods,  $G < Y$ , with a combination of lump-sum taxes,  $T_t$ , and money creation,  $M_t - M_{t-1}$  so that the implied process for debt,  $B_t$ , satisfies the budget constraint:

$$B_t + M_t + T_t = P_t G + M_{t-1} + R_{t-1} B_{t-1} \text{ for } t \geq 0, \quad (3)$$

given  $(M_{-1} + R_{-1} B_{-1})/P_{-1}$ .

### 2.3 Policy Rules

Time-varying policy rule coefficients have received special attention in recent years. [Dolado et al. \(2005\)](#) offers a survey of the literature to support on theoretical grounds the existence of nonlinear responses of an interest rate rule with respect to inflation and/or output. In particular, one of the arguments goes along the line of asymmetric preferences by the central bank with respect to deviations of inflation and/or output with respect to

target. In a similar line, [Svensson and Williams \(2007\)](#), in a context of model uncertainty, specify a loss function whose weights are not constant and obtain a monetary policy rule with coefficients that take on different values across states.

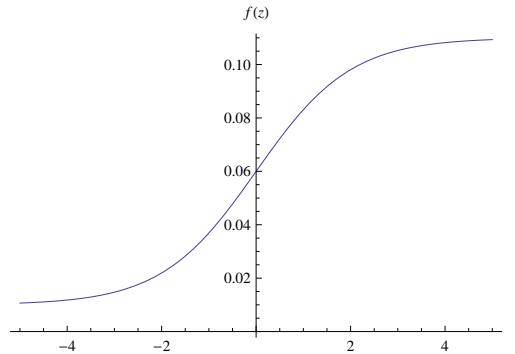
I specify policy rules in a semi-nonlinear fashion, where the coefficients of the policy rules are time varying. The time varying coefficients of a particular policy rule are specified as logistic functions of a latent state. That is, if  $\varrho_t$  is a time varying coefficient of a policy rule, it has the following functional form:

$$\begin{aligned}\varrho_t &\equiv \varrho(z_t) \\ &= \varrho_0 + \frac{\varrho_1}{1 + \exp(-\varrho_2(z_t - \varrho_3))},\end{aligned}$$

where  $z_t = \rho_z z_{t-1} + u_t$ ,  $0 < \rho_z \leq 1$  and  $u_t \sim \text{iidN}(0, 1)$ .

Under this specification,  $\varrho_0$  denotes the lower (upper) bound of  $\varrho_t$ , while  $\varrho_0 + \varrho_1$  denotes its upper (lower) bound (if  $\varrho_1 < 0$ ).  $\varrho_2$  is a positive transition coefficient affecting with its magnitude the slope of the transition between regimes, and  $\varrho_3$  is a location parameter determining the value of  $z_t$  at which  $\varrho_t$  crosses the  $y$ -axis. A graph for  $\varrho(z_t)$  with  $\varrho_0 = 0.01$ ,  $\varrho_1 = 0.1$ ,  $\varrho_2 = 1$ , and  $\varrho_3 = 0$  is reproduced in [Figure 1](#). Also,  $z_t$  is a persistent process to introduce long memory in  $\varrho_t$ , as pointed out by [Park \(2002\)](#).

Figure 1: Logistic Function



The attractiveness of this specification is that it allows a policy to switch smoothly from one regime to another without jumps, so that the economy does not have to “wake up” in a possibly (and totally) different state compared to the one it was in the previous period, as it happens with the Markov switching specification of policy parameters. Since the speed of the transition is measured by the magnitude of  $\varrho_2$ , this specification for policy parameters is more general than a Markov switching formulation, which corresponds to the present specification when  $\varrho_2 \rightarrow \infty$ . On the other hand, using a latent factor as the process driving the policy rule coefficients avoids having to choose an observable macroeconomic variable to drive the smooth transition between states. [Davig and Leeper \(2007\)](#) argue that a policy rule, in particular the monetary policy rule, is a “complicated, probably non-linear, function of a large set of information about the state of the economy.” In that sense, the latent factor may be viewed as a combination of several variables affecting the evolution of policy coefficients.

Two types of specifications have been introduced in the literature to model time-varying

policy rule coefficients: One is a two-state Markov switching specification, like in Davig and Leeper (2006), Davig and Doh (2009) or Favero and Monacelli (2003); and another is random coefficient specification, like in Fernandez-Villaverde et al. (2010) or Kim and Nelson (2006). The specification in this paper bounds the evolution of policy rule coefficients and, at the same time, offers continuity for their evolution, encompassing the two specifications in the existing literature.

For the policy rule specifications below, a policy coefficient with subscript  $t$  denotes a coefficient with the characteristics mentioned above.

### 2.3.1 Monetary Policy Rule

Monetary policy takes place by means of an interest rate feedback rule of the form

$$R_t = R_{t-1}^{\rho_R} \bar{R}_t^{(1-\rho_R)} \exp(\varepsilon_t^R),$$

where  $\rho_R \in (0, 1)$ , and  $\varepsilon_t^R \sim \text{iidN}(0, \sigma_R^2)$ .  $\bar{R}_t$  is the target short-term nominal interest rate. The central bank reacts to deviations of inflation from target, setting

$$\bar{R}_t = R \left( \frac{\Pi_t}{\bar{\Pi}} \right)^{\alpha_t},$$

where  $\Pi_t = P_t/P_{t-1}$ , and where  $R$  is the steady state nominal interest rate, which is guaranteed to be state independent if we set the target inflation rate,  $\bar{\Pi}$ , equal to  $\Pi$ , the steady state inflation.

### 2.3.2 Fiscal Policy Rule

The fiscal rule is a feedback rule for the ratio of lump-sum taxes net of transfers to output,  $\tau_t = T_t/(P_t Y)$ , of the form

$$\tau_t = \tau_{t-1}^{\rho_\tau} \bar{\tau}_t^{(1-\rho_\tau)} \exp(\varepsilon_t^\tau),$$

where  $\rho_\tau \in (0, 1)$ , and  $\varepsilon_t^\tau \sim \text{iidN}(0, \sigma_\tau^2)$ .  $\bar{\tau}_t$  is the target level of taxes net of transfers to output set to respond to debt deviations according to

$$\bar{\tau}_t = \tau \left( \frac{b_{t-1}}{\bar{b}} \right)^{\gamma_t},$$

where  $b_t = B_t/(P_t Y)$  denotes the lagged debt-to-output ratio in period  $t$ , and  $\bar{b}$  is its target level.  $\tau$  denotes the steady state level of the ratio of lump-sum taxes net of transfers to output, which is guaranteed to be state independent in the steady equilibrium if we set  $\bar{b}$  to its steady state value, denoted by  $b$ .

## 2.4 Interactions Between Monetary and Fiscal Policies

The introduction of monetary and fiscal policy interactions in the context of dynamic stochastic macroeconomic models dates back to the pioneer work of Leeper (1991), where different parameter values lead to different equilibrium outcomes and local dynamics in a

stochastic maximizing environment. There, the terms “active” and “passive” monetary and fiscal policies are introduced to describe how the central bank responds to fight inflation (more aggressive fighting of inflation is called “active” monetary policy), and how fiscal policy adjusts to changes in public debt (a Ricardian view of fiscal policy is called “passive” fiscal policy). Other works along this line are [Sims \(1994\)](#) and [Leith and Wren-Lewis \(2000\)](#).

[Nordhaus \(1994\)](#) carries on a game theoretical approach to understand monetary-fiscal policy coordination. He finds that a deficit-reduction package should be accompanied by a cooperative monetary policy to offset declines in aggregate demand and increases in unemployment, so that the economy ends up in a recovery with higher domestic and foreign investment. From an optimality perspective, [Dixit and Lambertini \(2003\)](#) find that a second-best outcome can be achieved if the monetary and the fiscal authorities both choose to be equally and optimally conservative with respect to the price level.

As for a quantitative approach to measure the interdependence of monetary and fiscal policies, [Muscatelli et al. \(2004\)](#) estimate a new Keynesian model with an interest rate rule and government-expenditure and tax rules. Their study finds that when an output shock hits the economy, monetary and fiscal policies tend to be complements, while if an inflation shock hits the economy, the policies tend to act as substitutes. On a related work, [Davig and Leeper \(2006\)](#) estimate regime switching models of monetary and fiscal policy rules to embed them into a new Keynesian DSGE model in order to offer a view of how monetary policy and, in particular, fiscal policy have affected the US economy. To introduce interactions between monetary and fiscal policies in terms of a set of states across which the policies jump, Davig and Leeper multiply the transition probabilities matrices of a two-state specification of each of the parameters of the monetary and fiscal policy rules. Clearly, that multiplication assumes that the states driving monetary policy and the states driving fiscal policy are independent of each other. One of the conclusion of that work is that, to better understand macroeconomic policy effects, it is essential to model policy rules as governed by a stochastic process over which agents form expectations.

To the best of the author’s knowledge, there is no study in the literature that incorporates explicit interactions between monetary and fiscal policies. In this study, interaction between policies is interpreted as the possibility of a long-run relationship between the coefficients of the policy rules. To incorporate a scenario of possible dependence between policies, I specify the latent states driving the policy parameters as follows:

$$\begin{aligned} z_t^R &= \rho_{z^R} z_{t-1}^R + u_t^R \\ z_t^\tau &= \rho_{z^\tau} z_{t-1}^\tau + u_t^\tau, \end{aligned}$$

where  $u_t^R$  and  $u_t^\tau$  are normally distributed with zero mean, unit variance and  $\text{cov}(u_t^R, u_t^\tau) = \kappa$ . Notice that under this specification, if  $\kappa$  is different from zero, there is endogeneity between the coefficients of the policy rules.

[Davig and Leeper \(2006\)](#) find evidence that whenever the interest rate rule pays more (less) attention to inflation deviations, less (more) weight is given to output deviations. That corresponds to the “active” (“passive”) regime of monetary policy. Also, when the tax rule pays more (less) attention to debt deviations, more (less) weight is given to output deviations. That corresponds to the “passive” (“active”) regime of fiscal policy. According to [Leeper \(1991\)](#), the active-passive and passive-active combinations of policies imply existence and



uniqueness of the equilibrium in a context of local dynamics within a neighborhood of the steady state.<sup>1</sup>

In the present context, policies become active or passive depending on the evolution of the latent factors  $z_t^R$  and  $z_t^\tau$ , and there is the possibility of combinations of policies according to the (long-run) relationship between these factors. From equations (2.3.1) and (2.3.2),<sup>2</sup>

$$\alpha(z_t^R) = \alpha_0 + \frac{\alpha_1}{1 + \exp(-\alpha_2 z_t^R)}$$

$$\gamma(z_t^\tau) = \gamma_0 + \frac{\gamma_1}{1 + \exp(-\gamma_2 z_t^\tau)}.$$

To make monetary policy able to switch between passive and active regimes, we need, for example,  $\alpha_0 < 1$  and  $\alpha_0 + \alpha_1 > 1$ , so that as  $z_t^R$  increases, monetary policy reacts more strongly to inflation deviations, and viceversa. On the other hand, regarding fiscal policy, and to be consistent with the findings in Davig and Leeper (2006), we would need  $\gamma_0 < 0$  and  $\gamma_1 > |\gamma_0|$ , so that for high values of  $z_t^\tau$ , fiscal policy reacts increasing taxes to stabilize debt, and viceversa. With respect to interactions, notice that if, for example,  $\text{cov}(u^R, u^\tau) = \kappa > 0$ , then it is possible to have a combination of policies in the active monetary - passive fiscal quadrant in the long run. This is so since, in that case, the latent factors will move together in the long run, implying that whenever  $\alpha(z_t^R)$  is high (more weight is given to inflation stabilization),  $\gamma(z_t^\tau)$  will be high (more weight is given to debt stabilization), and whenever  $\alpha(z_t^R)$  is low (less weight is given to inflation stabilization),  $\gamma(z_t^\tau)$  will be low (less weight is given to debt stabilization). However, in the short run there may be deviations from this long-run equilibrium, and the economy could visit the passive-passive, or active-active regions temporarily. As it can be seen, the model allows to have a rich possibility of combinations, and explicitly introduces interactions between monetary and fiscal policies.

Even though the latent factors are potentially correlated, I assume that they are independent of the stochastic processes that add uncertainty to the macroeconomic model. This assumption is necessary to obtain a solution to the model that is based on the minimum state variable solution approach.

## 2.5 Equilibrium

An equilibrium under this setup is an allocation  $\{C_t, M_t, B_t\}_{t=0}^\infty$ , a sequence of prices and Lagrange multiplier  $\{P_t, \lambda_t\}_{t=0}^\infty$  satisfying the optimality conditions of the household, and the government budget constraint, given  $(M_{-1} + R_{-1}B_{-1})/P_{-1}$ , a sequence  $\{z_t^R, z_t^\tau\}_{t=0}^\infty$ , and

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<sup>1</sup>The passive-passive regime of policies imply an equilibrium with the possibility of sunspots, while the active-active regime of policies imply the inexistence of equilibrium (unless very particular conditions hold).

<sup>2</sup>Recall that  $\alpha_2 > 0$  and  $\gamma_2 > 0$ . Also, without loss of generality, I assume from here on that  $\alpha_3 = \gamma_3 = 0$ .

the monetary and fiscal policy rules. Therefore, in equilibrium

$$C_t = Y - G \quad (4)$$

$$1 = \beta R_t \mathbb{E}_t \frac{1}{\Pi_{t+1}} \quad (5)$$

$$\frac{1}{v_t} = \chi_M (Y - G)^\sigma \frac{R_t}{R_t - 1}. \quad (6)$$

### 2.5.1 Steady State Equilibrium

In the absence of shocks, the following equations characterize the steady state equilibrium:

$$R = \frac{\Pi}{\beta} \quad (7)$$

$$\frac{1}{v} = \chi_M (Y - G)^\sigma \left( \frac{R}{R - 1} \right) \quad (8)$$

$$\tau = g - \left( 1 - \frac{1}{\Pi} \right) \frac{1}{v} + \left( \frac{1}{\beta} - 1 \right) b, \quad (9)$$

where  $v$  is the steady state level of money velocity.

## 2.6 Log-linearized Model and Solution Method

Notice that the model is unconventional, in the sense that the coefficients of the policy rules are time varying. I present here the model in log-deviations from the nonstochastic steady state, and show a way to solve it using a method in line with the minimum state variable (MSV) solution approach (McCallum, 1983). The model is composed of the following equations where a *hat* on a variable denotes that variable in percentage deviations from its deterministic steady state (for given values of the processes  $\{z_t^R\}_{t=0}^\infty$  and  $\{z_t^\tau\}_{t=0}^\infty$ ):

$$\mathbb{E}_t \hat{\pi}_{t+1} = \hat{R}_t \quad (10)$$

$$\hat{v}_t = \frac{1}{1 - R} \hat{R}_t \quad (11)$$

$$\hat{b}_t = -\frac{\tau}{b} \hat{\tau}_t + \frac{\hat{v}_t}{vb} - \frac{\hat{v}_{t-1}}{v\Pi b} - \left( \frac{1}{v\Pi b} + \frac{1}{\beta} \right) \hat{\pi}_t + \frac{1}{\beta} (\hat{R}_{t-1} + \hat{b}_{t-1}) \quad (12)$$

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R) \alpha (z_t^R) \hat{\pi}_t + \varepsilon_t^R \quad (13)$$

$$\hat{\tau}_t = \rho_\tau \hat{\tau}_{t-1} + (1 - \rho_\tau) \gamma (z_t^\tau) \hat{b}_{t-1} + \varepsilon_t^\tau. \quad (14)$$

To solve the model, let  $\omega_t = \hat{\pi}_t$ ,  $k_t = [\hat{R}_t, \hat{\tau}_t, \hat{b}_t, \hat{v}_t]'$  and rewrite (10)-(14) as

$$0 = \mathbb{E}_t \omega_{t+1} + \Phi k_t \quad (15)$$

$$0 = M k_t + \Upsilon(z_t^\tau) k_{t-1} + \Lambda(z_t^R) \omega_t + \Xi \varepsilon_t, \quad (16)$$

where  $z_t = [z_t^R, z_t^\tau]'$ ,  $\varepsilon_t = [\varepsilon_t^R, \varepsilon_t^\tau]'$ , and where  $\Phi$ ,  $M$ ,  $\Upsilon(z_t)$ ,  $\Lambda(z_t)$ , and  $\Xi$  are appropriate

coefficient matrices shown in Appendix B.

The proposed solution is given by

$$\omega_t = A(z_t)k_{t-1} + B(z_t)\varepsilon_t \quad (17)$$

$$k_t = C(z_t)k_{t-1} + D(z_t)\varepsilon_t, \quad (18)$$

where, for  $F(z_t) = A(z_t), B(z_t), C(z_t), D(z_t)$ , the  $i, j - th$  entry is given by

$$F^{ij}(z_t) = \left( F_{0R}^{ij} + \frac{F_{1R}^{ij}}{1 + F_{2R}^{ij} \exp(-F_{3R}^{ij} z_t^R)} \right) \left( F_{0\tau}^{i,j} + \frac{1}{1 + F_{2\tau}^{ij} \exp(-F_{3\tau}^{ij} z_t^\tau)} \right), \quad (19)$$

for  $i, j = \hat{\pi}, \hat{v}, \hat{b}, \hat{R}, \hat{\tau}$ , which is known as a bivariate logistic function. For identification purposes, it is necessary to impose that  $F_2^{ij} \geq 0$  and  $F_3^{ij} \geq 0$ .

Appendix C shows that the coefficients of the solution indeed follow a logistic function for the case when the latent process,  $z_t$ , is *iid*, or when the factors are uncorrelated. Appendix D also illustrates the procedure to obtain the parameters of the logistic functions in the solution,  $F_{0R}^{ij}, F_{1R}^{ij}, F_{2R}^{ij}, F_{3R}^{ij}, F_{0\tau}^{ij}, F_{2\tau}^{ij}$  and  $F_{3\tau}^{ij}$ .

Since the purpose of this paper is to investigate the performance of the solution and estimation techniques in a simplified environment, I assume that the solution takes the form

$$F^{ij}(z_t) = F_0^{ij} + \frac{F_1^{ij}}{(1 + \exp(-F_{2R}^{ij} z_t^R)) (1 + \exp(-F_{2\tau}^{ij} z_t^\tau))},$$

which is still a bivariate logistic function, and leave the more sophisticated version (19) for a future work.

## 2.7 On Existence, Stability and Uniqueness of the Solution

Since the method used to obtain the solution is based on the undetermined coefficients method, existence is guaranteed. By construction this method yields stable solutions, and the parameter space has been properly constrained to guarantee that. However, since the model is intrinsically nonlinear, and the parameters are time varying, even after bounding the parameter space, the model may imply the arise of sunspot solutions. In that sense, the approach shown here picks the solutions with the smallest eigenvalues at the limits, and lets the solution evolve between these well defined limits. The issue of uniqueness of the solution of nonlinear models has attracted attention of the DSGE modeling and estimation literatures in recent years, and it is still an open field to future research. Examples in the literature include Davig and Leeper (2006), Davig and Leeper (2007), Davig and Leeper (2008), Farmer et al. (2009), Fernandez-Villaverde et al. (2010) and Fernandez-Villaverde and Rubio-Ramirez (2010).

### 3 Estimation

To estimate the model, I employ Bayesian methods that allow obtaining the set of parameters of the macroeconomic model, denoted  $\Theta_y$ , the set of parameters of the latent factors, denoted  $\Theta_z$ , and the latent factors themselves, using a modified approach based on the approach proposed in [Geweke and Tanizaki \(2001\)](#).

Let  $\text{INT}_t$  denote the demeaned quarterly nominal federal funds rate in period  $t$ , and  $\text{TAX}_t$  the demeaned ratio or quarterly federal receipts net of transfers to output. The state-space model is composed of the following equations:

$$y_t = Hx_t \quad (20)$$

$$x_t = G(z_t)x_{t-1} + S(z_t)\varepsilon_t \quad (21)$$

$$z_t = Pz_{t-1} + u_t, \quad (22)$$

where  $y_t = [\text{INT}_t, \text{TAX}_t]'$ ,  $x_t = [\omega_t, k_t]'$ , and  $u_t = [u_t^R, u_t^T]'$ , for appropriate matrices  $H$ ,  $G(z_t)$ ,  $S(z_t)$ , and  $P$  shown in [Appendix F](#).

Let  $Y_t = \{y_s\}_{s=0}^t$ , and let  $\mathfrak{F}_t$  be the sigma field  $\sigma(Y_t)$ . Let  $Z_t = \{z_s\}_{s=0}^t$ , and let  $\mathfrak{Z}_t$  be the sigma field  $\sigma(Z_t)$ . Then

$$y_t | \mathfrak{F}_{t-1}, \mathfrak{Z}_t, \Theta_y \sim \mathbb{N}(y_{t|t}, \Sigma_{t|t}),$$

where  $y_{t|t} = \mathbb{E}(y_t | \mathfrak{F}_{t-1}, \mathfrak{Z}_t, \Theta_y)$ , and  $\Sigma_{t|t} = \text{var}(y_t | \mathfrak{F}_{t-1}, \mathfrak{Z}_t, \Theta_y)$ . The Kalman filter can be used to obtain the conditional likelihood function of  $Y_T$  given  $\mathfrak{F}_0$  and  $\mathfrak{Z}_T$ . Details are shown in the [Appendix](#).

Let  $P_y(y_t | \mathfrak{F}_{t-1}, \mathfrak{Z}_t, \Theta_y)$  denote the conditional density of  $y_t$  given  $\mathfrak{F}_{t-1}$ ,  $\mathfrak{Z}_t$  and  $\Theta_y$ . Let  $P_z(z_t | z_{t-1}, \Theta_z)$  denote the conditional density of  $z_t$  given  $z_{t-1}$  and  $\Theta_z$ . Define  $Z_{t+1}^* = \{z_s\}_{s=t+1}^T$ , and  $\mathfrak{Z}_{t+1}^* = \sigma(Z_{t+1}^*)$ . Then, if  $z_0$  is assumed to be stochastic,

$$P_z(Z_T | \Theta_z) = P_z(z_0 | \Theta_z) \prod_{t=1}^T P_z(z_t | z_{t-1}, \Theta_z)$$

$$P_y(Y_T | \mathfrak{F}_0, \mathfrak{Z}_T, \Theta_y) = \prod_{t=1}^T P_y(y_t | \mathfrak{F}_{t-1}, \mathfrak{Z}_t, \Theta_y)$$

$$P(z_t | \mathfrak{Z}_{t-1}, \mathfrak{Z}_{t+1}^*, \mathfrak{F}_n, \Theta_y, \Theta_z) \propto \quad (23)$$

$$\begin{cases} P_y(y_t | \mathfrak{F}_{t-1}, \mathfrak{Z}_t, \Theta_y) P_z(z_t | z_{t-1}, \Theta_z) P_z(z_{t+1} | z_t, \Theta_z) & \text{if } t \leq T-1 \\ P_y(y_t | \mathfrak{F}_{t-1}, \mathfrak{Z}_t, \Theta_y) P_z(z_t | z_{t-1}, \Theta_z) & \text{if } t = T \end{cases}$$

$$P(\Theta_y | \mathfrak{F}_T, \mathfrak{Z}_T, \Theta_z) \propto P_y(Y_T | \mathfrak{F}_0, \mathfrak{Z}_T, \Theta_y) P_{\Theta_y}(\Theta_y) \quad (24)$$

$$P(\Theta_z | \mathfrak{F}_T, \mathfrak{Z}_T, \Theta_y) \propto P_z(Z_T | \Theta_z) P_{\Theta_z}(\Theta_z), \quad (25)$$

where  $P_{\Theta_y}(\Theta_y)$  and  $P_{\Theta_z}(\Theta_z)$  are the prior densities of  $\Theta_y$  and  $\Theta_z$ , respectively.

From the posterior densities (23)-(25), the smoothing random draws are generated as follows:

- (i) Take appropriate initial values for  $\Theta_y$ ,  $\Theta_z$  and  $\{z_t\}_{t=0}^T$ .<sup>3</sup>
- (ii) Use the Kalman filter to obtain  $P_y(y_t|\mathfrak{F}_{t-1}, \mathfrak{Z}_t, \Theta_y)$  and  $P_y(Y_T|\mathfrak{F}_0, \mathfrak{Z}_T, \Theta_y)$ .
- (iii) Generate a random draw of  $z_t$  from  $P(z_t|\mathfrak{Z}_{t-1}, \mathfrak{Z}_{t+1}^*, \mathfrak{F}_T, \Theta_y, \Theta_z)$  for  $t = 1, 2, \dots, T$ .
- (iv) Generate a random draw of  $\Theta_y$  from  $P(\Theta_y|\mathfrak{F}_T, \mathfrak{Z}_T, \Theta_z)$ .
- (v) Generate a random draw of  $\Theta_z$  from  $P(\Theta_z|\mathfrak{F}_T, \mathfrak{Z}_T, \Theta_y)$ .
- (vi) Repeat (ii)-(v)  $N$  times to obtain  $N$  random draws of  $Z_T$ ,  $\Theta_y$  and  $\Theta_z$ .

In steps (ii)-(vi) the random draws of  $Z_T$ ,  $\Theta_y$  and  $\Theta_z$  are updated. This sampling method is referred to as the Gibbs sampler. To generate the random draws of  $z_t$  for  $t = 1, 2, \dots, T$ ,  $\Theta_y$  and  $\Theta_z$ , I use the Metropolis-Hastings (M-H) algorithm. That is, the Gibbs sampler and the M-H algorithm are combined in order to obtain the smoothing random draws from the state-space model. The choice of proposal densities for the M-H algorithm is shown in the Appendix G.

### 3.1 Data and Parameter Assumptions

The interest rate data correspond to the quarterly average of the monthly rate in the secondary market of the 3-month T-Bill, and were obtained from the Federal Reserve Bank. The tax net of transfers data correspond to the seasonally adjusted quarterly current receipts of the federal government from which the current transfer payments have been deducted, and were obtained from the NIPA Table 3.2. The seasonally adjusted quarterly output data were obtained from the Bureau of Economic Analysis. The sample covers the first quarter of 1949 until the third quarter of 2010. The model was specified at a quarterly frequency.

Some of the model parameters were fixed to focus on the estimation of the policy coefficients. The values of  $v$ ,  $b$ ,  $\Pi$ ,  $g$ , and  $R$  that match the means of their counterparts in the sample data at quarterly rates and frequencies are shown in Table 1.

Table 1: Parameter values

$\beta^\dagger$	$v$	$b$	$\Pi$	$g$	$\tau^\ddagger$	$R$
0.9968	2.256	0.3354	1.0084	0.081	0.0784	1.0116

$$^\dagger \beta = \Pi/R$$

$$^\ddagger \tau = g - \left(1 - \frac{1}{\Pi}\right) \frac{1}{v} + \left(\frac{1}{\beta} - 1\right) b$$

### 3.2 Estimation Results

The choice of prior distributions, hyper parameters, means of 20,000 draws from the posterior distribution of parameters, 90% confidence sets and P-values from the convergence

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<sup>3</sup>The values of the policy rule parameters at their limits were taken from Davig and Leeper (2006), and the latent factors are the smoothed estimates of a random coefficient model of the policy rules.

tests in Geweke (1991) are presented in Table 2. Since only 20,000 draws have been obtained, the tests do not show convergence in 3 of the 15 parameters estimated. This situation can be easily overcome by drawing more samples from the posterior, which is left for future work, since the purpose of this document was only to explore the solution and estimation techniques. Figures 3 and 4 show the good performance of the model in terms of explaining the time series of interest and tax rates.

From the results it can be seen that the estimated lower limit of the monetary policy rule parameter is 0.83, its upper limit is 1.73, and its mean value, 1.295, which lies in the determinacy region given by the constant coefficient counterpart of the present model ( $\alpha > 1$ ). The coefficient of the fiscal policy rule evolves asymptotically between  $-0.01$  and  $0.03$ , with an average of  $0.0133$ , slightly below the threshold given by  $\frac{b}{\tau}(\beta^{-1} - 1) = 0.0137$ , above which fiscal policy becomes passive.<sup>4</sup>

With respect to the parameters of the latent factors, the estimates of  $\rho_{zR}$  and  $\rho_{z\tau}$  imply that monetary policy is more persistent than fiscal policy in the sense that the coefficients of the rules have a slow reversion to the mean, in the former, and a more rapid reversion to the mean in the latter. This result is consistent with the findings in Davig and Leeper (2006) and Favero and Monacelli (2003). The parameter that measures interactions between the policies,  $\kappa$ , has a posterior mean value of  $0.17$ , with  $P(\kappa > 0) = 0.8126$ . That is, monetary and fiscal policies co-move, but the linear association between them is not strong. Given the positiveness of the parameter at the posterior mean, this implies that, on average, policy rules spend most of their time in the active monetary - passive fiscal (AM/PF) and the passive monetary - active fiscal (PM/AF) regimes, which are the regimes that deliver uniqueness in the constant coefficient version of this model.

### 3.2.1 Evolution of Policy Rule Coefficients

Figure 5 shows the evolution of the posterior means of the coefficients of the monetary and fiscal policy rules, and the NBER recession periods. One important implication of the estimated parameters that immediately calls attention is the fact that monetary policy has been passive ( $\alpha < 1$ ) only in during the periods 1979:1-1981:3 and 2008:4-2009:2. Another characteristic of the coefficients that is apparent from the graph is their co-movement.

In an analysis by decades, the results with respect to the monetary policy rule coefficient in this paper coincide with findings in various works. For example, Romer and Romer (2002) and Davig and Doh (2009) find that monetary policy has been active during the 1950s. Davig and Doh (2009) and Fernandez-Villaverde et al. (2010) find the same result for the 1960s. With respect to the first half of the 1970s, only Boivin (2006) finds that monetary policy has been active, and all the studies that investigated the second half of that decade conclude that policy was passive. For the 1980s, Romer and Romer (2002), Kim and Nelson (2006),

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<sup>4</sup>These thresholds are obtained under the constant coefficient setup of the present model by using the approach in Leeper (1991), in which the characteristic roots of the expectational difference equation involving  $\mathbb{E}_t \hat{\pi}_{t+1}$ ,  $\hat{\pi}_t$  and  $\hat{b}_t$  are given by  $\alpha$ ,  $0$  and  $\frac{b}{\tau}(\beta^{-1} - 1)$ . By the Blanchard-Khan criterion, one of the roots different from zero has to be greater than one and the other less than one, which yields the four determinacy regions described by Leeper: Active Monetary Policy / Passive Fiscal Policy (AM/PF), Passive Monetary Policy / Active Fiscal Policy (PM/AF), Passive Monetary Policy / Passive Fiscal Policy (PM/PF), and Active Monetary Policy / Active Fiscal Policy (AM/AF).

Davig and Leeper (2006), Davig and Doh (2009) and Fernandez-Villaverde et al. (2010) conclude that monetary policy has reacted strongly with respect to inflation. Romer and Romer (2002), Davig and Doh (2009) and Kim and Nelson (2006) find the same result for the 1990s. Finally, for the 2000s, Davig and Doh (2009) conclude that policy has been active.

I give particular attention to the results found here with respect to the monetary policy rule coefficient during the 1970s and early 1980s. As said before, only during 1979:1-1981:3 monetary policy was found to be passive. It is important to recall the types of monetary policies in place in the period between the early 1970s and the early 1980s: According to Nelson (2007), during the first four years, the control of inflation was based on price controls, while a loose federal funds rate instrument was in place. There it could lie the reason why the results display a decrease in the policy rule coefficient, even though it still stays slightly above 1. Starting the second half of the decade, according to Nelson, monetary policy adopted monetary targets, and that could be the reason why the coefficient increases again. However, money growth rates were still high, reaching double digits for M2 growth by the end of the decade, provoking the loosest monetary policy of all the analyzed period at the beginning of the 1980's. Once the Fed, under chairman Volcker, implements credit controls and a federal funds target instrument in 1982, monetary policy becomes tight.

I also give particular attention to the behavior of monetary policy during the 2000s: As described by Bernanke (2007), monetary policy accommodated in order to fight the recession of the beginning of the decade, and that coincides with the decrease in the policy rule coefficient. Then, at the beginning of the second half of the decade the Fed gradually removed the accommodation and started to conduct policies to deflate the asset price bubble, turning to a tight monetary policy, which, according to Reis (2010), ended with a loose monetary policy with the recession of the end of the decade.

With respect to the fiscal policy rule coefficient, the results here almost mirror the results in Davig and Leeper (2006), and that is the only available work against which the results can be contrasted. Perhaps the only difference between the results here and those found by Davig and Leeper lies in the evolution of the coefficient during the 1950s. Davig and Leeper find that fiscal policy has been passive during the first half of that decade, while the results here show that fiscal policy was active in each of the recessions of the decade. This last result is consistent with the findings in May (1990) about President Eisenhower's fiscal policy.

Again, this work gives special attention to fiscal policy during the 1970s and the early 1980s: According to Conte et al. (2001), after an original commitment to fight inflation at the beginning of the decade, there was an accelerated federal spending to stimulate economic growth and fight unemployment, turning to an active fiscal policy. According to the Annual Budget Message to the Congress for the fiscal year 1974, this increase in federal spending was held down to balance the budget to guard the economy against inflation, making fiscal policy passive for a short period around 1975. In the second half of the 1970s, the tax reduction acts of 1975 and 1977, and the economic recovery act of 1981 turned fiscal policy into one of the most active regimes in the analyzed period, which lasted until the early years of the 1980s.

With respect to the 2000s, after the tax cuts of 2001 and 2003, fiscal policy becomes passive around 2005 mainly because of the increase in tax receipts due to the economic recovery. At the end of the decade, and in face with the crisis and the stimulus and recovery acts of 2008 and 2009, fiscal policy turns to the active regime in what seems to be the most

active regime of the analyzed period.

In sum, according to the evolution of policy coefficients and their thresholds (given by the dotted horizontal line in Figure 5), the U.S. economy has spend 48.99% of the time in the AM/PF regime, 5.26% in the PM/AF regime, 0.4% in the PM/PF regime, and 45.34% in the AM/AF regime. This means that the economy has spent 54.25% of the time in the regimes that guarantee determinacy under the constant coefficient version of the model.

The significant amount of time that the economy would have spent in the AM/AF regime is something that could trouble the analysis of the results since this implies that the economy has gone through important instabilities. However, precisely there it may lie the reason for the instability of inflation during the 1970s and the early 1980s. Figure 6 shows the evolution of inflation and the periods of the AM/AF regime. According to the findings of the estimated model, the four longest periods of AM/AF policies are, in decreasing order: 1998:2-2003:2 (20 quarters), 1971:1-1973:4 (12 quarters), 1967:1-1969:4 (12 quarters), and 1976:3-1978:4 (10 quarters). That is, out of 48 quarters between 1967 and 1978, the economy spent 34 in the AM/AF regime. Hence, the highly volatile inflation was not due to bad monetary policy, but to a bad *combination* of monetary and fiscal policies.<sup>5</sup>

Finally, it is important to point out that, even though the majority of studies investigating the behavior of the Taylor rule have concluded that the coefficient of the rule has been smaller than 1 during the 1970s and the early 1980s, none of these studies has incorporated fiscal policy into the analysis, and if it has, independence between the policies has been assumed. Having fiscal policy interacting with monetary policy in the model gives opportunity to richer and different dynamics from the ones obtained in the existing literature of policy rules.

### 3.2.2 Conventional Impulse-Response Analysis

I now proceed to conduct conventional policy experiments to analyze the performance of inflation under orthogonal shocks to the monetary policy and fiscal policy rules. Figures 7 and 8 display the response of inflation to a 1% increase of the interest rate and a 1% increase of the tax rate, respectively. Both experiments were conducted starting at the filtered values of the latent factors at the end of the sampling period (2010:3). These values imply that the economy is in the PM/AF regime, with values for the policy rule coefficients given by  $\alpha(z_{2010:3}^R) = 0.9769$  and  $\gamma(z_{2010:3}^\tau) = 0.0133$ .<sup>6</sup>

With respect to the monetary contraction, the evolution of inflation is consistent with the results found in the fiscal theory of the price level, see for example Davig and Leeper (2006). The impulse-response function reveals that inflation decreases on impact, but that the long-run effect is an increase in inflation, above inflation prevailing in the PM/AF regime, that slowly returns to its initial value (given the values of the parameters that are very close to those under the AM/PF regime, from the graph is very difficult to see that inflation is indeed higher after the first quarter. However, inflation is 0.1% higher than its initial value and stays there even after 5 years.) This “price puzzle” has been found in studies with U.S. data by Sims (1992) and Hanson (2004). The operational mechanism behind the result is

<sup>5</sup>In fact, the only period in which the economy had a PM/PF regime was in 1981:3.

<sup>6</sup>Notice that these values do not correspond with the values at the end of the sampling period shown in Figure 5 since those values correspond to the posterior means of the policy rule coefficients, while the values considered here are constructed using  $\alpha_0, \alpha_1, \alpha_2, \gamma_0, \gamma_1, \gamma_2, z_{2010:3}^R$  and  $z_{2010:3}^\tau$  at their posterior means.



an open-market operation that sells debt to the public, leaving households holding a higher level of government debt. This positive wealth effect that is not neutralized with future taxes, since fiscal policy is active, pressures the demand of goods and prices.

A fiscal policy contraction (increase in taxes) implies retiring debt, generating a negative wealth effect that will not be corrected in the future since fiscal policy is active, decreasing demand and, therefore, inflation. From the impulse-response function it can be seen that inflation decreases on impact, and slowly returns to the level prevailing in the PM/AF regime at which started. This is a result uniformly found in the fiscal theory of the price level, see for example [Chung et al. \(2007\)](#).

### 3.2.3 Unconventional Impulse-Response Analysis

One of advantages of the specification for policy rule parameters in this work is that one can simulate the response of the economy to a change in the way in which the policy authorities react to changes in their variables of interest. Say, for example, that, without any perturbation to the interest rate, the monetary authority decides to increase its reaction to inflation deviations. The same can happen with respect to the fiscal authority. However, since the policies are correlated, it is likely that a change in one of the policy coefficients triggers a change on the other. To calculate this unconventional impulse response, let

$$x_t = G(z_t)x_{t-1} + S(z_t)\varepsilon_t$$

be defined as in system (20)-(22). Also, let  $\nu_t$  be a shock on  $z_t$  over which the response of inflation is measured. Let  $z_{t+j}^* = z_t + P^j \nu_t$  be the value of the latent factors after  $j$  periods of the shock, for  $j \geq 0$ . Additionally, let

$$x_{t+j}^0 = G(z_t)x_{t+j-1}^0 + S(z_t)\varepsilon_{t+j}$$

be the endogenous variables after  $j$  periods without the effect of the shock, and

$$x_{t+j}^* = G(z_{t+j}^*)x_{t+j-1}^* + S(z_{t+j}^*)\varepsilon_{t+j}$$

be the endogenous variables after  $j$  periods with the effect of the shock. The response is measured by

$$\mathbb{E}_{t-1} (x_{t+j}^* - x_{t+j}^0) = \left[ G(z_{t+j}^*)G(z_{t+j-1}^*) \dots G(z_{t+1}^*)G(z_t^*) - (G(z_t))^j \right] x_{t-1|t-1},$$

where  $x_{t-1|t-1}$  is the filtered state at period  $t - 1$ .

Figure 9 shows the evolution of inflation after a shock  $\nu_t = [1, \kappa]'$  to the estimated value of the latent factor in 2010:3,  $z_{2010:3}$ . A shock of this magnitude takes the policy coefficients from  $\alpha = 0.9769$  to  $\alpha' = 1.136$ , and from  $\gamma = 0.0133$  to  $\gamma' = 0.0156$ . That is, the economy switches from a situation with passive monetary policy and active fiscal policy (PM/AF) to a situation with active monetary policy and passive fiscal policy (AM/PF), with the ergodic distribution converging to the PM/AF regime. The results show that inflation increases on impact, then decreases below the original level, and then slowly converges to its long run equilibrium. The initial increase in inflation may seem counterintuitive, given that the

central bank reaction to inflation deviations is stronger after the shock. However, a similar result is obtained by Davig et al. (2011) in a work about the presence of a fiscal limit and its effect on inflation. The operating mechanism behind this result is as follows: By going from a PM/AF to an AM/PF regime, agents foresee that, to keep the debt-output ratio stationary, a downwards adjustment in taxes shall have to be done in the future, since now fiscal policy is reacting strongly to debt deviations. This devalues debt, increasing the demand for goods and, therefore, prices. After that point, the regular mechanism of higher interest rates over inflation, under the AM/PF regime, operates.

## 4 Conclusions and Future Work

This paper formulated, solved and estimated a DSGE model with correlated and time-varying monetary and fiscal policy rules. The estimations using U.S. data on interest and tax rates show that the reaction of the central bank to inflation has been strong, except in the periods 1979:1-1981:3 and 2008:4-2009:2. Also, fiscal policy has had more variability than monetary policy, and the economy has spent 54.25% of the time with policies that complement each other so that stability is achieved. Finally, given the situation of the economy in the third quarter of 2010, increasing the reaction of the central bank to inflation deviations from target increases inflation in about 3%.

The results obtained here assumed an endowment economy, which restricts the response of interest and tax rules, since the only variable of interest to policy authorities is inflation. The obvious extension is to introduce production in this economy and price rigidities. That is the conventional New Keynesian model in which time-varying policy rules will be introduced. That extension will also allow to implement the bivariate logistic function in its complete formulation to solve the model, which I simplified in this work.

## A Model Setup

The representative household solves the following problem:

$$\begin{aligned} & \max_{\{C_t, M_t/P_t, B_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma}}{1-\sigma} + \chi_M \log \left( \frac{M_t}{P_t} \right) \right) \\ & C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} + \frac{T_t}{P_t} = Y + \frac{M_{t-1}}{P_{t-1}} + R_{t-1} \frac{B_{t-1}}{P_{t-1}} \text{ for } t \geq 0, \\ & \frac{M_{-1} + R_{-1} B_{-1}}{P_{-1}} \text{ given} \\ & \lim_{t \rightarrow \infty} MRS_{0,t} \frac{M_t + B_t}{P_t} = 0, \end{aligned}$$

where  $MRS_{0,t}$  denotes the marginal rate of substitution between period 0 and period  $t$ . The necessary first order conditions are:

$$C_t : C_t^{-\sigma} - \lambda_t = 0 \quad (26)$$

$$\frac{M_t}{P_t} : \chi_M \frac{M_t}{P_t} - \lambda_t + \beta \lambda_{t+1} \Pi_{t+1}^{-1} = 0 \quad (27)$$

$$\frac{B_t}{P_t} : -\lambda_t + \lambda_{t+1} \beta R_t \Pi_{t+1}^{-1} = 0 \quad (28)$$

$$\lambda_t : C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} + \frac{T_t}{P_t} = Y + \frac{M_{t-1}}{P_{t-1}} \Pi_t^{-1} + R_{t-1} \frac{B_{t-1}}{P_{t-1}} \Pi_t^{-1}. \quad (29)$$

From (26) and (28),

$$1 = \beta R_t \mathbb{E}_t \left( \frac{C_t}{C_{t+1}} \right)^{\sigma} \Pi_{t+1}^{-1}. \quad (30)$$

From (26), (27) and (28),

$$\frac{M_t}{P_t} = \chi_M C_t^{-\sigma} \frac{R_t}{R_t - 1}. \quad (31)$$

In equilibrium, (3) and (29) imply (4), which, together with (30) imply (5), and together with (31) imply (6), using the definition of velocity  $v_t = Y/(M_t/P_t)$ .

Writing (3) in terms of nominal output yields

$$b_t + \frac{1}{v_t} + \tau_t = g + \frac{1}{v_{t-1}} \frac{1}{\Pi_t} + R_{t-1} b_{t-1} \frac{1}{\Pi_t}, \quad (32)$$

where  $g = G/Y$ .

Since all the variables in the system are stationary, the absence of shocks in (5),(6) and (32) yields (7)-(9).

## B Matrices for Solving the Model

The matrices in the system (15)-(16) are given by

$$\begin{aligned} \Phi &= [-1 \ 0 \ 0 \ 0], \\ \mathbb{M} &= \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -\tau/b & -1 & 1/vb \\ -1/(R-1) & 0 & 0 & -1 \end{bmatrix}, \quad \Upsilon(z_t^\tau) = \begin{bmatrix} \rho_R & 0 & 0 & 0 \\ 0 & \rho_\tau & (1-\rho_\tau)\gamma(z_t^\tau) & 0 \\ 1/\beta & 0 & 1/\beta & -1/v\Pi b \\ 0 & 0 & 0 & 0 \end{bmatrix}, \\ \Lambda(z_t^R) &= \begin{bmatrix} (1-\rho_r)\alpha(z_t^R) \\ 0 \\ -(1/v\Pi b + 1/\beta) \\ 0 \end{bmatrix}, \quad \Xi = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}. \end{aligned}$$

## C Guess and Verify: Checking that the Functional Form for the Solution is Logistic

Rewrite equation (16) as

$$k_t = \tilde{\Upsilon}(z_t^\tau)k_{t-1} + \tilde{\Lambda}(z_t^R)w_t + \tilde{\Xi}\varepsilon_t, \quad (33)$$

where  $\tilde{\mathcal{C}} = -\mathbb{M}^{-1}\mathcal{C}$  for  $\mathcal{C} = \Upsilon, \Lambda, \Xi$ .

From (17) and (33) we have

$$\begin{aligned} \mathbb{E}_t \omega_{t+1} &= \bar{A}k_t \\ &= \bar{A} \left[ \tilde{\Upsilon}(z_t^\tau) + \tilde{\Lambda}(z_t^R) (A(z_t)k_{t-1} + B(z_t)\varepsilon_t) + \tilde{\Xi}\varepsilon_t \right] \end{aligned} \quad (34)$$

$$k_t = \tilde{\Upsilon}(z_t^\tau)k_{t-1} + \tilde{\Lambda}(z_t^R) (A(z_t)k_{t-1} + B(z_t)\varepsilon_t) + \tilde{\Xi}\varepsilon_t, \quad (35)$$

where  $\bar{A} = \mathbb{E}_t A(z_{t+1})$  is a constant since  $z_t$  is *iid*, and where the assumption that  $u_t$  and  $\varepsilon_t$  are independent of each other has also been used.

Substituting (34) and (35) in (17), yields

$$\begin{aligned} &\bar{A}\tilde{\Upsilon}(z_t^\tau)k_{t-1} + \bar{A}\tilde{\Lambda}(z_t^R)A(z_t)k_{t-1} + \bar{A}\tilde{\Lambda}(z_t^R)B(z_t)\varepsilon_t + \bar{A}\tilde{\Xi}\varepsilon_t + \\ &\Phi\tilde{\Upsilon}(z_t^\tau)k_{t-1} + \Phi\tilde{\Lambda}(z_t^R)A(z_t)k_{t-1} + \Phi\tilde{\Lambda}(z_t^R)B(z_t)\varepsilon_t + \Phi\tilde{\Xi}\varepsilon_t = 0. \end{aligned}$$

By collecting terms

$$\begin{aligned} (\bar{A} + \Phi)\tilde{\Lambda}(z_t^R)A(z_t) + (\bar{A} + \Phi)\tilde{\Upsilon}(z_t^\tau) &= 0 \\ (\bar{A} + \Phi)\tilde{\Lambda}(z_t^R)B(z_t) + (\bar{A} + \Phi)\tilde{\Xi} &= 0. \end{aligned}$$

Hence

$$A(z_t) = -\frac{(\bar{A} + \Phi)\tilde{\Upsilon}(z_t^T)}{(\bar{A} + \Phi)\tilde{\Lambda}(z_t^R)}$$

$$B(z_t) = -\frac{(\bar{A} + \Phi)\tilde{\Xi}}{(\bar{A} + \Phi)\tilde{\Lambda}(z_t^R)}.$$

Notice that  $A(z_t)$  is the ratio of two logistic functions, which can be written as a bivariate logistic function. To show this claim, let  $a, b, c$ , and  $d$  be any real numbers such that  $c > 0$  and  $d \geq 0$  (this is necessary since the coefficients of the Taylor rule are positive). Without loss of generality, I claim that we can write

$$\frac{a + \frac{b}{1 + \exp(-y)}}{c + \frac{d}{1 + \exp(-x)}} = \left( e + \frac{f}{1 + g \exp(-hx)} \right) \left( i + \frac{1}{1 + j \exp(-kx)} \right), \quad (36)$$

where  $g \geq 0$ ,  $h \geq 0$ ,  $j \geq 0$ , and  $k \geq 0$ .

First, notice that we can write

$$\frac{1}{c + \frac{d}{1 + \exp(-x)}} = l + \frac{m}{1 + n \exp(-px)},$$

where

$$l = \frac{1}{c}$$

$$m = \frac{1}{c+d} - \frac{1}{c}$$

$$n = \frac{\frac{1}{c+d} - \frac{1}{c}}{\frac{1}{c+d/2} - \frac{1}{c}} - 1$$

$$p = -\frac{d \left( \frac{1}{c+d} - \frac{1}{c} \right)}{4(c+d/2)^2 \left( \frac{1}{c+d/2} - \frac{1}{c} \right)^2}.$$

Second, we can normalize the coefficients so that

$$\begin{aligned} e &= am \\ f &= bm \\ i &= l/m. \end{aligned}$$

Finally, with  $j = n$ ,  $k = p$  and  $g = h = 1$  we have shown that (36) indeed holds.

Hence, each of the elements of  $A(z_t)$  can be written as

$$\left( F_{0R}^{ij} + \frac{F_{1R}^{ij}}{1 + F_{2R}^{ij} \exp(-F_{3R}^{ij} z_t^R)} \right) \left( F_{0\tau}^{ij} + \frac{1}{1 + F_{2\tau}^{ij} \exp(-F_{3\tau}^{ij} z_t^\tau)} \right).$$

For  $B(z_t)$ , since its elements are the inverse of a univariate logistic function, they can be trivially written as a bivariate logistic function where the coefficient on  $z_t^\tau$  is zero.

Since

$$\begin{aligned} C(z_t) &= \tilde{\Upsilon}(z_t^\tau) + \tilde{\Lambda}(z_t^R)A(z_t) \text{ and} \\ D(z_t) &= \tilde{\Lambda}(z_t^R)B(z_t) + \tilde{\Xi} \end{aligned}$$

are combinations of logistic functions of  $z_t^R$  and  $z_t^\tau$ , they can also be expressed as bivariate logistic functions.

When the latent factors are uncorrelated,  $\mathbb{E}_t A(z_{t+1})$  can be obtained as a bivariate logistic function (shown in this Appendix below) and, as proven above, a combination of bivariate logistic functions can always be written as another bivariate logistic function.

## D Obtaining the Coefficients of the Logistic Functions

To find the coefficients in the matrices  $A(z_t), B(z_t), C(z_t), D(z_t)$ , substitute (17) and (18) into (15) and (16) to obtain

$$\begin{aligned} [\bar{A}(z_t) + \Phi] C(z_t) k_{t-1} + [\bar{A}(z_t) + \Phi] D(z_t) \varepsilon_t &= 0 \\ [MC(z_t) + \Lambda(z_t^R)A(z_t) + \Upsilon(z_t^\tau)] k_{t-1} + [MD(z_t) + \Lambda(z_t^R)B(z_t) + \Xi] \varepsilon_t &= 0, \end{aligned}$$

where  $\bar{A}(z_t) = \mathbb{E}_t A(z_{t+1})$  can also be expressed as a bivariate logistic function. How to obtain this result is explained in Appendix E.

By the undetermined coefficients method,

$$[\bar{A}(z_t) + \Phi] C(z_t) = 0 \tag{37}$$

$$[\bar{A}(z_t) + \Phi] D(z_t) = 0 \tag{38}$$

$$MC(z_t) + \Lambda(z_t^R)A(z_t) + \Upsilon(z_t^\tau) = 0 \tag{39}$$

$$MD(z_t) + \Lambda(z_t^R)B(z_t) + \Xi = 0. \tag{40}$$

To find  $F_{0R}^{ij}$ ,  $F_{1R}^{ij}$  and  $F_{0\tau}^{ij}$  in  $A(z_t), B(z_t), C(z_t), D(z_t)$  and  $\bar{A}(z_t)$  above, evaluate (37)-(40)

at the limits  $z_t^R = z_t^\tau = -\infty$ ,  $z_t^R = -\infty, z_t^\tau = \infty$  or  $z_t^R = \infty, z_t^\tau = -\infty$ , and  $z_t^R = z_t^\tau = \infty$  (PM/AF, PM/PF or AM/AF, and AM/PF, respectively)<sup>7</sup> to obtain the following system:  
At  $z_t^R = z_t^\tau = -\infty$

$$\begin{aligned} [\bar{A}_{ll} + \Phi] C_{ll} &= 0 \\ [\bar{A}_{ll} + \Phi] D_{ll} &= 0 \\ MC_{ll} + \Lambda_l A_{ll} + \Upsilon_l &= 0 \\ MD_{ll} + \Lambda_l B_{ll} + \Xi &= 0. \end{aligned}$$

At  $z_t^R = -\infty, z_t^\tau = \infty$

$$\begin{aligned} [\bar{A}_{lu} + \Phi] C_{lu} &= 0 \\ [\bar{A}_{lu} + \Phi] D_{lu} &= 0 \\ MC_{lu} + \Lambda_l A_{lu} + \Upsilon_u &= 0 \\ MD_{lu} + \Lambda_l B_{lu} + \Xi &= 0. \end{aligned}$$

At  $z_t^R = \infty, z_t^\tau = -\infty$

$$\begin{aligned} [\bar{A}_{ul} + \Phi] C_{ul} &= 0 \\ [\bar{A}_{ul} + \Phi] D_{ul} &= 0 \\ MC_{ul} + \Lambda_u A_{ul} + \Upsilon_l &= 0 \\ MD_{ul} + \Lambda_u B_{ul} + \Xi &= 0. \end{aligned}$$

At  $z_t^R = z_t^\tau = \infty$

$$\begin{aligned} [\bar{A}_{uu} + \Phi] C_{uu} &= 0 \\ [\bar{A}_{uu} + \Phi] D_{uu} &= 0 \\ MC_{uu} + \Lambda_u A_{uu} + \Upsilon_u &= 0 \\ MD_{uu} + \Lambda_u B_{uu} + \Xi &= 0. \end{aligned}$$

Here, for  $F = A_{ll}, \bar{A}_{ll}, B_{ll}, C_{ll}, D_{ll}$ , the  $i, j$ -th entry is given by

$$F^{ij} = F_{0R}^{ij} F_{0\tau}^{ij},$$

and  $\Lambda_l$  is  $\Lambda(z_t^R)$ , except that the entry (1, 1) is given by  $(1 - \rho_R)\alpha_0^R$ , while  $\Upsilon_l$  is  $\Upsilon(z_t^\tau)$ , except that the entry (2, 3) is given by  $(1 - \rho_\tau)\gamma_0^\tau$ .

For  $F = A_{lu}, \bar{A}_{lu}, B_{lu}, C_{lu}, D_{lu}$ , the  $i, j$ -th entry is given by

$$F^{ij} = F_{0R}^{ij} (F_{0\tau}^{ij} + 1),$$

and  $\Upsilon_u$  is  $\Upsilon(z_t^\tau)$ , except that the entry (2, 3) is given by  $(1 - \rho_\tau)(\gamma_0^\tau + \gamma_1^\tau)$ .

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<sup>7</sup>AM: Active Monetary, PM: Passive Monetary, AF: Active Fiscal, PF: Passive Fiscal.

For  $F = A_{ul}, \bar{A}_{ul}, B_{ul}, C_{ul}, D_{ul}$ , the  $i, j - th$  entry is given by

$$F^{ij} = (F_{0R}^{ij} + F_{1R}^{ij})F_{0\tau}^{ij},$$

and  $\Lambda_u$  is  $\Lambda(z_t^R)$ , except that the entry (1, 1) is given by  $(1 - \rho_R)(\alpha_0^R + \alpha_1^R)$ .

For  $F = A_{uu}, \bar{A}_{uu}, B_{uu}, C_{uu}, D_{uu}$ , the  $i, j - th$  entry is given by

$$F^{ij} = (F_{0R}^{ij} + F_{1R}^{ij})(F_{0\tau}^{ij} + 1).$$

To find  $F_{2R}^{ij}$  and  $F_{2\tau}^{ij}$ , evaluate (37)-(40) at  $z_t^R = -\infty, z_t^\tau = 0, z_t^R = \infty, z_t^\tau = 0, z_t^R = 0, z_t^\tau = -\infty$  and  $z_t^R = 0, z_t^\tau = \infty$  to obtain the system:

At  $z_t^R = -\infty, z_t^\tau = 0$

$$\begin{aligned} [\bar{A}_{l0} + \Phi] C_{l0} &= 0 \\ [\bar{A}_{l0} + \Phi] D_{l0} &= 0 \\ MC_{l0} + \Lambda_l A_{l0} + \Upsilon_0 &= 0 \\ MD_{l0} + \Lambda_l B_{l0} + \Xi &= 0. \end{aligned}$$

At  $z_t^R = \infty, z_t^\tau = 0$

$$\begin{aligned} [\bar{A}_{u0} + \Phi] C_{u0} &= 0 \\ [\bar{A}_{u0} + \Phi] D_{u0} &= 0 \\ MC_{u0} + \Lambda_u A_{u0} + \Upsilon_0 &= 0 \\ MD_{u0} + \Lambda_u B_{u0} + \Xi &= 0. \end{aligned}$$

At  $z_t^R = 0, z_t^\tau = -\infty$

$$\begin{aligned} [\bar{A}_{0l} + \Phi] C_{0l} &= 0 \\ [\bar{A}_{0l} + \Phi] D_{0l} &= 0 \\ MC_{0l} + \Lambda_0 A_{0l} + \Upsilon_l &= 0 \\ MD_{0l} + \Lambda_0 B_{0l} + \Xi &= 0. \end{aligned}$$

At  $z_t^R = 0, z_t^\tau = \infty$

$$\begin{aligned} [\bar{A}_{0u} + \Phi] C_{0u} &= 0 \\ [\bar{A}_{0u} + \Phi] D_{0u} &= 0 \\ MC_{0u} + \Lambda_0 A_{0u} + \Upsilon_u &= 0 \\ MD_{0u} + \Lambda_0 B_{0u} + \Xi &= 0. \end{aligned}$$

Here, for  $F = A_{l0}, \bar{A}_{l0}, B_{l0}, C_{l0}, D_{l0}$ , the  $i, j - th$  entry is given by

$$F^{ij} = F_{0R}^{ij}(F_{0\tau}^{ij} + 1/(1 + F_{2\tau}^{ij})),$$

and  $\Upsilon_0$  is  $\Upsilon(z_t^\tau)$ , except that the entry (2, 3) is given by  $(1 - \rho_\tau)(\gamma_0^\tau + \gamma_0^\tau/2)$ .



For  $F = A_{u0}, \bar{A}_{u0}, B_{u0}, C_{u0}, D_{u0}$ , the  $i, j$ -th entry is given by

$$(F_{0R}^{ij} + F_{1R}^{ij})(F_{0\tau}^{ij} + 1/(1 + F_{2R}^{ij})).$$

For  $F = A_{0l}, \bar{A}_{0l}, B_{0l}, C_{0l}, D_{0l}$ , the  $i, j$ -th entry is given by

$$(F_{0R}^{ij} + F_{1R}^{ij}/(1 + F_{2R}^{ij}))F_{0\tau}^{ij},$$

and  $\Lambda_0$  is  $\Lambda(z_t^R)$ , except that the entry  $(1, 1)$  is given by  $(1 - \rho_R)(\alpha_0^R + \alpha_1^R/2)$ .

For  $F = A_{0u}, \bar{A}_{0u}, B_{0u}, C_{0u}, D_{0u}$ , the  $i, j$ -th entry is given by

$$(F_{0R}^{ij} + F_{1R}^{ij}/(1 + F_{2R}^{ij}))(F_{0\tau}^{ij} + 1).$$

To find the transition coefficients  $F_{3R}^{ij}$  and  $F_{3\tau}^{ij}$ , evaluate (37)-(40) at the limit  $z_t^\tau = \infty$  to obtain the following system, which is a function of  $z_t^R$  only:

$$[\bar{A}_R(z_t^R) + \Phi] C_R(z_t^R) = 0 \quad (41)$$

$$[\bar{A}_R(z_t^R) + \Phi] D_R(z_t^R) = 0 \quad (42)$$

$$MC_R(z_t^R) + \Lambda(z_t^R)A_R(z_t^R) + \Upsilon_u = 0 \quad (43)$$

$$MD_R(z_t^R) + \Lambda(z_t^R)B_R(z_t^R) + \Xi = 0, \quad (44)$$

where, for  $F = A_R, \bar{A}_R, B_R, C_R, D_R$ , the  $i, j$ -th entry is given by

$$F^{ij}(z_t^R) = \left( F_{0R}^{ij} + \frac{F_{1R}^{ij}}{1 + F_{2R}^{ij} \exp(-F_{3R}^{ij} z_t^R)} \right) (F_{0\tau}^{ij} + 1).$$

Deriving the system (41)-(44) with respect to  $z_t^R$  and evaluating at  $z_t^R = 0$ , yields

$$\bar{a}_R C_R + (\bar{A}_R + \Phi) c_R = 0$$

$$\bar{a}_R D_R + (\bar{A}_R + \Phi) d_R = 0$$

$$M c_R + \lambda_0 A_R + \Lambda_0 a_R = 0$$

$$M d_R + \lambda_0 B_R + \Lambda_0 b_R = 0,$$

where, for  $F = A_R, \bar{A}_R, B_R, C_R, D_R$ , and for  $f = a_R, \bar{a}_R, b_R, c_R, d_R$  the  $i, j$ -th entry is, respectively,

$$F^{ij} = \left( F_{0R}^{ij} + \frac{F_{1R}^{ij}}{1 + F_{2R}^{ij}} \right) (F_{0\tau}^{ij} + 1)$$

$$f^{ij} = \left( \frac{F_{1R}^{ij} F_{2R}^{ij} F_{3R}^{ij}}{(1 + F_{2R}^{ij})^2} \right) (F_{0\tau}^{ij} + 1),$$

and  $\lambda_0$  is a  $4 \times 1$  matrix of zeros whose first entry is  $\alpha_1 \alpha_2 / 4$ .

Another set of equations can be obtained, and utilized if necessary, by evaluating (37)-

(40) at the limit  $z_t^\tau = -\infty$ , so that (41)-(44) is now

$$[\bar{A}_R(z_t^R) + \Phi] C_R(z_t^R) = 0 \quad (45)$$

$$[\bar{A}_R(z_t^R) + \Phi] D_R(z_t^R) = 0 \quad (46)$$

$$MC_R(z_t^R) + \Lambda(z_t^R)A_R(z_t^R) + \Upsilon_l = 0 \quad (47)$$

$$MD_R(z_t^R) + \Lambda(z_t^R)B_R(z_t^R) + \Xi = 0, \quad (48)$$

where, for  $F = A_R, \bar{A}_R, B_R, C_R, D_R$ , the  $i, j$ -th entry is given by

$$F^{ij}(z_t^R) = \left( F_{0R}^{ij} + \frac{F_{1R}^{ij}}{1 + F_{2R}^{ij} \exp(-F_{3R}^{ij} z_t^R)} \right) F_{0\tau}^{ij}.$$

Deriving the system (45)-(48) with respect to  $z_t^R$  and evaluating at  $z_t^R = 0$ , yields

$$\bar{a}_R C_R + (\bar{A}_R + \Phi) c_R = 0$$

$$\bar{a}_R D_R + (\bar{A}_R + \Phi) d_R = 0$$

$$M c_R + \lambda_0 A_R + \Lambda_0 a_R = 0$$

$$M d_R + \lambda_0 B_R + \Lambda_0 b_R = 0,$$

where, for  $F = A_R, \bar{A}_R, B_R, C_R, D_R$ , and for  $f = a_R, \bar{a}_R, b_R, c_R, d_R$  the  $i, j$ -th entry is, respectively,

$$F^{ij} = \left( F_{0R}^{ij} + \frac{F_{1R}^{ij}}{1 + F_{2R}^{ij}} \right) F_{0\tau}^{ij}$$

$$f^{ij} = \left( \frac{F_{1R}^{ij} F_{2R}^{ij} F_{3R}^{ij}}{(1 + F_{2R}^{ij})^2} \right) F_{0\tau}^{ij}.$$

Now, evaluating (37)-(40) at the limit  $z_t^R = \infty$  yields the following system, which is as a function of  $z_t^\tau$  only:

$$[\bar{A}_\tau(z_t^\tau) + \Phi] C_\tau(z_t^\tau) = 0 \quad (49)$$

$$[\bar{A}_\tau(z_t^\tau) + \Phi] D_\tau(z_t^\tau) = 0 \quad (50)$$

$$MC_\tau(z_t^\tau) + \Lambda_u A_\tau(z_t^\tau) + \Upsilon(z_t^\tau) = 0 \quad (51)$$

$$MD_\tau(z_t^\tau) + \Lambda_u B_\tau(z_t^\tau) + \Xi = 0, \quad (52)$$

where, for  $F = A_\tau, \bar{A}_\tau, B_\tau, C_\tau, D_\tau$ , the  $i, j$ -th entry is given by

$$F^{ij}(z_t^\tau) = \left( F_{0\tau}^{ij} + \frac{1}{1 + F_{2\tau}^{ij} \exp(-F_{3\tau}^{ij} z_t^R)} \right) (F_{0R}^{ij} + F_{1R}^{ij}).$$

Deriving the system (49)-(52) with respect to  $z_t^\tau$  and evaluating at  $z_t^\tau = 0$ , yields

$$\begin{aligned}\bar{a}_\tau C_\tau + (\bar{A}_\tau + \Phi) c_\tau &= 0 \\ \bar{a}_\tau D_\tau + (\bar{A}_\tau + \Phi) d_\tau &= 0 \\ M c_\tau + \Lambda_u a_\tau + v_0 &= 0 \\ M d_\tau + \Lambda_u b_\tau &= 0,\end{aligned}$$

where, for  $F = A_\tau, \bar{A}_\tau, B_\tau, C_\tau, D_\tau$ , and for  $f = a_\tau, \bar{a}_\tau, b_\tau, c_\tau, d_\tau$  the  $i, j$ -th entry is, respectively,

$$\begin{aligned}F^{ij} &= \left( F_{0\tau}^{ij} + \frac{1}{1 + F_{2\tau}^{ij}} \right) (F_{0R}^{ij} + F_{1R}^{ij}) \\ f^{ij} &= \left( \frac{F_{2\tau}^{ij} F_{3\tau}^{ij}}{(1 + F_{2\tau}^{ij})^2} \right) (F_{0R}^{ij} + F_{1R}^{ij}),\end{aligned}$$

and  $v_0$  is a  $4 \times 4$  matrix of zeros whose entry (2,3) is  $\gamma_1 \gamma_2 / 4$ .

Another set of equations can be obtained, and utilized if necessary, by evaluating (37)-(40) at the limit  $z_t^R = -\infty$ , so that (49)-(52) is now

$$[\bar{A}_\tau(z_t^\tau) + \Phi] C_\tau(z_t^\tau) = 0 \quad (53)$$

$$[\bar{A}_\tau(z_t^\tau) + \Phi] D_\tau(z_t^\tau) = 0 \quad (54)$$

$$M C_\tau(z_t^\tau) + \Lambda_l A_\tau(z_t^\tau) + \Upsilon(z_t^\tau) = 0 \quad (55)$$

$$M D_R(z_t^\tau) + \Lambda_l B_\tau(z_t^\tau) + \Xi = 0, \quad (56)$$

where, for  $F = A_R, \bar{A}_R, B_R, C_R, D_R$ , the  $i, j$ -th entry is given by

$$F^{ij}(z_t^\tau) = \left( F_{0\tau}^{ij} + \frac{1}{1 + F_{2\tau}^{ij} \exp(-F_{3\tau}^{ij} z_t^R)} \right) F_{0R}^{ij}.$$

Deriving the system (49)-(52) with respect to  $z_t^\tau$  and evaluating at  $z_t^\tau = 0$ , yields

$$\begin{aligned}\bar{a}_\tau C_\tau + (\bar{A}_\tau + \Phi) c_\tau &= 0 \\ \bar{a}_\tau D_\tau + (\bar{A}_\tau + \Phi) d_\tau &= 0 \\ M c_\tau + \Lambda_l a_\tau + v_0 &= 0 \\ M d_\tau + \Lambda_l b_\tau &= 0,\end{aligned}$$

where, for  $F = A_\tau, \bar{A}_\tau, B_\tau, C_\tau, D_\tau$ , and for  $f = a_\tau, \bar{a}_\tau, b_\tau, c_\tau, d_\tau$  the  $i, j$ -th entry is, respectively,

$$\begin{aligned}F^{ij} &= \left( F_{0\tau}^{ij} + \frac{1}{1 + F_{2\tau}^{ij}} \right) F_{0R}^{ij} \\ f^{ij} &= \left( \frac{F_{2\tau}^{ij} F_{3\tau}^{ij}}{(1 + F_{2\tau}^{ij})^2} \right) F_{0R}^{ij}.\end{aligned}$$

## E Computation of $\mathbb{E}_t A(z_{t+1})$

The proposed solution takes the form

$$F(x, y; \alpha, \beta) = \alpha_0 \beta_0 + \frac{\alpha_0}{1 + \beta_2 e^{-\beta_3 y}} + \frac{\beta_0 \alpha_1}{1 + \alpha_2 e^{-\alpha_3 x}} + \frac{\alpha_1}{(1 + \alpha_2 e^{-\alpha_3 x})(1 + \beta_2 e^{-\beta_3 y})},$$

where  $\alpha = [\alpha_0, \alpha_1, \alpha_2, \alpha_3]'$  and  $\beta = [\beta_0, \beta_2, \beta_3]'$ .

We need to compute

$$\mathbb{E}[F(x', y'; \alpha, \beta) | x, y] = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F(x', y'; \alpha, \beta) p(x', y' | x, y, \kappa) dx' dy',$$

where  $x' = \rho_x x + \varepsilon_x$  and  $y' = \rho_y y + \varepsilon_y$ ,  $0 \leq \rho_x \leq 1$ ,  $0 \leq \rho_y \leq 1$ , and  $\varepsilon_x$  and  $\varepsilon_y$  are bivariate normal with zero mean, unit variance and correlation coefficient  $\kappa$ .

Following [Maragakis et al. \(2008\)](#), we can write

$$\begin{aligned} \mathbb{E} \left[ \frac{\alpha_0}{1 + \beta_2 e^{-\beta_3 y'}} \middle| x, y \right] &\approx \frac{\alpha_0}{1 + \beta_2 e^{-b_3 y}} \\ \mathbb{E} \left[ \frac{\beta_0 \alpha_1}{1 + \alpha_2 e^{-\alpha_3 x'}} \middle| x, y \right] &\approx \frac{\beta_0 \alpha_1}{1 + \alpha_2 e^{-a_3 x}}, \end{aligned}$$

where

$$b_3 = \frac{\rho_y}{\sqrt{\frac{1}{\beta_3^2} + \frac{\pi}{8}}} \quad (57)$$

$$a_3 = \frac{\rho_x}{\sqrt{\frac{1}{\alpha_3^2} + \frac{\pi}{8}}}. \quad (58)$$

Next, we are interested in approximating

$$G(x, y; \alpha_2, \alpha_3, \beta_2, \beta_3, \kappa) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left( \frac{1}{(1 + \alpha_2 e^{-\alpha_3 x'})(1 + \beta_2 e^{-\beta_3 y'})} \right) p(x', y' | x, y, \kappa) dx' dy'$$

with the bivariate logistic function suggested by [Ali et al. \(1978\)](#):

$$H(x, y; \alpha_2, a_3, \beta_2, b_3, c) = \frac{1}{1 + \alpha_2 e^{-a_3 x} + \beta_2 e^{-b_3 y} + (1 - c) \alpha_2 \beta_2 e^{-a_3 x} e^{-b_3 y}},$$

where  $a_3$  and  $b_3$  correspond to their expressions in (57) and (58), respectively. Also,  $-1 \leq c \leq 1$ , and  $c = 0 \iff \kappa = 0$ .

To find  $c$  we need the following results with respect to the bivariate logistic function

$$\tilde{F}(x, y; \alpha, \beta) = 1/(1 + e^{-\alpha x})(1 + e^{-\beta y}):$$

$$\begin{aligned}\tilde{f}_{xy}(x, y; \alpha, \beta) &= \frac{\partial^2}{\partial x \partial y} \tilde{F}(x, y; \alpha, \beta) = \frac{\alpha \beta e^{-\alpha x} e^{-\beta y}}{(1 + e^{-\alpha x})^2 (1 + e^{-\beta y})^2} \\ \tilde{f}_x(x; \alpha) &= \frac{\partial}{\partial x} \tilde{F}(x, \infty; \alpha, \beta) = \frac{\alpha e^{-\alpha x}}{(1 + e^{-\alpha x})^2} \\ \tilde{f}_y(y; \beta) &= \frac{\partial}{\partial y} \tilde{F}(\infty, y; \alpha, \beta) = \frac{\beta e^{-\beta y}}{(1 + e^{-\beta y})^2}.\end{aligned}$$

To conduct the approximation, it is necessary to approximate  $\tilde{f}_{xy}(x, y; \alpha, \beta)$  with a bivariate normal density function centered at zero and variance chosen such that both functions coincide at the origin:

$$\tilde{f}_{xy}(0, 0; \alpha, \beta) = \frac{\alpha \beta}{16} = \frac{1}{2\pi} \frac{1}{\sigma_x \sigma_y}.$$

Additionally, we choose  $\sigma_x$  and  $\sigma_y$  such that  $\tilde{f}_x(\cdot)$  and  $\tilde{f}_y(\cdot)$  coincide with unconditional marginal normals at the origin:

$$\begin{aligned}\tilde{f}_x(0; \alpha) &= \frac{\alpha}{4} = \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma_x} \\ \tilde{f}_y(0; \beta) &= \frac{\beta}{4} = \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma_y}.\end{aligned}$$

These conditions yield

$$\begin{aligned}\sigma_x &= \frac{1}{\alpha} \sqrt{\frac{8}{\pi}} \\ \sigma_y &= \frac{1}{\beta} \sqrt{\frac{8}{\pi}}.\end{aligned}$$

A feature of  $H(x, y; \alpha_2, a_3, \beta_2, b_3, c)$  is

$$\frac{\partial^2}{\partial x \partial y} H(x, y; \alpha_2, a_3, \beta_2, b_3, c) \Big|_{x=\ln(\alpha_2)/a_3, y=\ln(\beta_2)/b_3} = \frac{a_3 b_3 (3 + (1 - c)^2 - c)}{(4 - c)^3}.$$

Let

$$\hat{F}(x, y; \alpha_2, \alpha_3, \beta_2, \beta_3) = \frac{1}{(1 + \alpha_2 e^{-\alpha_3 x})(1 + \beta_2 e^{-\beta_3 y})}.$$

Then,  $c$  is chosen such that

$$\frac{\partial^2}{\partial x \partial y} H(x, y; \alpha_2, a_3, \beta_2, b_3, c) \Big|_{x=x_{0h}, y=y_{0h}} = \frac{\partial^2}{\partial x \partial y} G(x, y; \alpha_2, \alpha_3, \beta_2, \beta_3, \kappa) \Big|_{x=x_{0g}, y=y_{0g}},$$

where  $x_{0h} = \ln(\alpha_2)/a_3$  and  $y_{0h} = \ln(\beta_2)/b_3$ , and  $x_{0g} = \ln(\alpha_2)/\alpha_3$  and  $y_{0g} = \ln(\beta_2)/\beta_3$ . That

is

$$\begin{aligned}
a_3 b_3 \frac{3 + (1 - c)^2 - c}{(4 - c)^3} &= \frac{\partial^2}{\partial x \partial y} \left[ \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \hat{F}(x', y'; \alpha_2, \alpha_3, \beta_2, \beta_3) p(x', y' | x, y, \kappa) dx' dy' \right]_{x=x_{0g}, y=y_{0g}} \\
&= \frac{\partial^2}{\partial x \partial y} \left[ \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \hat{F}(\rho_x x + \varepsilon_x, \rho_y y + \varepsilon_y; \alpha_2, \alpha_3, \beta_2, \beta_3) p(\varepsilon_x, \varepsilon_y; \kappa) d\varepsilon_x d\varepsilon_y \right]_{x=x_{0g}, y=y_{0g}} \\
&= \rho_x \rho_y \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \tilde{f}_{xy}(\varepsilon_x, \varepsilon_y; \alpha_3, \beta_3) p(\varepsilon_x, \varepsilon_y; \kappa) d\varepsilon_x d\varepsilon_y \\
&\approx \rho_x \rho_y \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} p(\varepsilon_x, \varepsilon_y; \sigma_x, \sigma_y) p(\varepsilon_x, \varepsilon_y; \kappa) d\varepsilon_x d\varepsilon_y, \tag{59}
\end{aligned}$$

where

$$\begin{aligned}
p(\varepsilon_x, \varepsilon_y; \sigma_x, \sigma_y) &= (2\pi)^{-1} (\sigma_x \sigma_y)^{-1} \exp\left(-\frac{1}{2}(\varepsilon_x^2/\sigma_x^2 + \varepsilon_y^2/\sigma_y^2)\right) \\
p(\varepsilon_x, \varepsilon_y; \kappa) &= (2\pi)^{-1} (1 - \kappa^2)^{-1/2} \exp\left(-\frac{1}{2\sqrt{1 - \kappa^2}}(\varepsilon_x^2 + \varepsilon_y^2 + 2\kappa\varepsilon_x\varepsilon_y)\right).
\end{aligned}$$

The RHS of (59) can be written as

$$\rho_x \rho_y (2\pi)^{-2} [\sigma_x^2 \sigma_y^2 (1 - \kappa^2)]^{-1/2} \int \exp\left(-\frac{1}{2}\varepsilon' A \varepsilon\right) d\varepsilon,$$

where  $\varepsilon = (\varepsilon_x, \varepsilon_y)'$ , and

$$A = \begin{bmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{bmatrix}^{-1} + \begin{bmatrix} 1 & \kappa \\ \kappa & 1 \end{bmatrix}^{-1}.$$

The gaussian integral yields

$$\begin{aligned}
\int \exp\left(-\frac{1}{2}\varepsilon' A \varepsilon\right) d\varepsilon &= 2\pi (\det(A))^{-1/2} \\
&= 2\pi \left[ \frac{1 - \kappa^2 + \sigma_x^2 \sigma_y^2 + \sigma_x^2 + \sigma_y^2}{(1 - \kappa^2) \sigma_x^2 \sigma_y^2} \right]^{-1/2}.
\end{aligned}$$

Hence, the RHS of (59) is

$$\rho_x \rho_y (2\pi)^{-1} [1 - \kappa^2 + \sigma_x^2 \sigma_y^2 + \sigma_x^2 + \sigma_y^2]^{-1/2}.$$

Since,

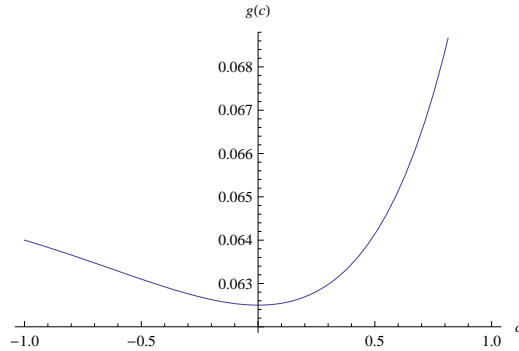
$$\begin{aligned} a_3 &= \frac{\rho_x}{\sqrt{1/\alpha_3^2 + \pi/8}}, \\ b_3 &= \frac{\rho_y}{\sqrt{1/\beta_3^2 + \pi/8}}, \\ \sigma_x &= \frac{1}{\alpha_3} \sqrt{\frac{8}{\pi}}, \\ \sigma_y &= \frac{1}{\beta_3} \sqrt{\frac{8}{\pi}} \end{aligned}$$

the solution for  $c$  satisfies

$$\frac{3 + (1 - c)^2 - c}{(4 - c)^3} \approx \frac{1}{2\pi} \sqrt{\frac{1}{\alpha_3^2} + \frac{\pi}{8}} \sqrt{\frac{1}{\beta_3^2} + \frac{\pi}{8}} \left( 1 - \kappa^2 + \frac{64}{\pi^2 \alpha_3^2 \beta_3^2} + \frac{8}{\pi \alpha_3^2} + \frac{8}{\pi \beta_3^2} \right)^{-1/2}. \quad (60)$$

Let  $g(c)$  denote the function on the LHS of (60). It has the following shape

Figure 2: Solution for  $c$



To guarantee that  $-1 \leq c \leq 1$ , the RHS of (60) has to be bounded from below and from above according to the bounds of  $g(c)$ . First, notice that the RHS is increasing in both  $\alpha_4$  and  $\beta_4$ , so that

$$\lim_{\alpha_3 \rightarrow \infty, \beta_3 \rightarrow \infty} \text{RHS} = \frac{1}{16} (1 - \kappa^2)^{-1/2}.$$

Second, notice that  $\kappa \geq 0$  implies  $c \geq 0$ , so the maximum value that  $\kappa$  can take is found when  $c = 1$ :

$$g(0) = \frac{1}{16} \leq \frac{1}{16} (1 - \kappa^2)^{-1/2} \leq g(1) = \frac{2}{27}.$$

Hence

$$0 \leq \kappa \leq \sqrt{1 - \left(\frac{27}{32}\right)^2} \approx 0.53674.$$

For  $\kappa < 0$ ,

$$g(0) = \frac{1}{16} \leq \frac{1}{16}(1 - \kappa^2)^{-1/2} \leq g(-1) = \frac{8}{125},$$

so

$$-0.21523 \approx -\sqrt{1 - \left(\frac{125}{128}\right)^2} \leq \kappa \leq 0.$$

Therefore, the admissible values for  $\kappa$  are  $\kappa \in \left(-\sqrt{1 - \left(\frac{125}{128}\right)^2}, \sqrt{1 - \left(\frac{27}{32}\right)^2}\right)$ .

With all the parameters found, I can write

$$\mathbb{E}[F(x', y'; \alpha, \beta) | x, y] \approx \alpha_0 \beta_0 + \frac{\alpha_0}{1 + \beta_2 e^{-b_3 y}} + \frac{\beta_0 \alpha_1}{1 + \alpha_2 e^{-a_3 x}} + \frac{\alpha_1}{1 + \alpha_2 e^{-a_3 x} + \beta_2 e^{-b_3 y} + (1 - c) \alpha_2 \beta_2 e^{-a_3 x - b_3 y}}.$$

## F Log-likelihood Function

In the system (20)-(22), the coefficient matrices are given by

$$\begin{aligned} H &= [0 \quad 1 \quad 1 \quad 0 \quad 0] \\ G(z_t) &= \begin{bmatrix} 0 & A(z_t) \\ 0_{4 \times 1} & C(z_t) \end{bmatrix} & S(z_t) &= \begin{bmatrix} B(z_t) \\ D(z_t) \end{bmatrix} \\ P &= \begin{bmatrix} \rho_z^R & 0 \\ 0 & \rho_z^T \end{bmatrix}. \end{aligned}$$

To obtain the log-likelihood function of  $Y_T$  given  $\mathfrak{F}_0$  and  $\mathfrak{Z}_T$ , setup the Kalman filter, whose prediction stage is given by the following equations:

$$\begin{aligned} x_{t|t-1} &= \mathbb{E}(x_t | \mathfrak{F}_{t-1}, \mathfrak{Z}_t, \Theta_y) = G(z_t) x_{t-1|t-1} \\ \Omega_{t|t-1} &= \text{var}(x_t | \mathfrak{F}_{t-1}, \mathfrak{Z}_t, \Theta_y) = G(z_t) \Omega_{t-1|t-1} G(z_t)' + S(z_t) Q S(z_t)' \\ y_{t|t-1} &= H x_{t|t-1} \\ \Sigma_{t|t-1} &= H \Omega_{t|t-1} H', \end{aligned}$$

where  $Q = \text{var}(\varepsilon_t | \mathfrak{F}_{t-1}, \mathfrak{Z}_t, \Theta_y)$ .

The updating stage is given by the following equations:

$$\begin{aligned} x_{t|t} &= \mathbb{E}(x_t | \mathfrak{F}_t, \mathfrak{Z}_t, \Theta_y) = x_{t|t-1} + \Omega_{t|t-1} H' \Sigma_{t|t-1}^{-1} (y_t - y_{t|t-1}) \\ \Omega_{t|t} &= \text{var}(x_t | \mathfrak{F}_t, \mathfrak{Z}_t, \Theta_y) = \Omega_{t|t-1} - \Omega_{t|t-1} H' \Sigma_{t|t-1}^{-1} H \Omega_{t|t-1}. \end{aligned}$$

The conditional log-likelihood function of  $y_t$  given  $\mathfrak{F}_{t-1}$  and  $\mathfrak{Z}_t$  is given by

$$l_t(\Theta_y) = -\frac{m}{2} \log(2\pi) - \frac{1}{2} \log(\det \Sigma_{t|t-1}) - \frac{1}{2} (y_t - y_{t|t-1})' \Sigma_{t|t-1}^{-1} (y_t - y_{t|t-1}),$$

where  $m$  is the dimension of  $y_t$ .



The conditional log-likelihood function of  $Y_T$  given  $\mathfrak{F}_0$  and  $\mathfrak{Z}_T$  is given by

$$\mathfrak{L}_T(\Theta_y) = \sum_{t=1}^T l_t(\Theta_y).$$

## G Choice of Proposal Densities for the M-H Algorithm

Let  $P_{*z}(x^*|x)$ ,  $P_{*\Theta_y}(x^*|x)$  and  $P_{*\Theta_z}(x^*|x)$  denote the proposal densities of  $z_t$ ,  $\Theta_y$  and  $\Theta_z$ , respectively.

For  $z_t$  I use an independence chain, specifying the transition density of  $z_t$  as the proposal density for the draws, i.e.,  $P_{*z}(x^*|x) = P_z(x^*|z_{t-1}, \Theta_z)$ . That is, I obtain a random draw  $x^*$  for round  $i$  from

$$x^* = Pz_{t-1}^i + u_t,$$

where  $u_t \sim \mathbb{N}(0, K)$  and  $K$  is a  $2 \times 2$  variance-covariance matrix with unit variance and correlation coefficient  $\kappa$ . Then, I compute the ratio

$$r = \frac{P_y(y_t|\mathfrak{F}_{t-1}, \mathfrak{Z}_{t-1}^{i-1}, x^*, \Theta_y)P_z(z_{t+1}^{i-1}|x^*, \Theta_z)P_z(x^*|z_{t-1}^{i-1}, \Theta_z)/P_z(x^*|z_{t-1}^i, \Theta_z)}{P_y(y_t|\mathfrak{F}_{t-1}, \mathfrak{Z}_t^{i-1}, \Theta_y)P_z(z_{t+1}^{i-1}|z_t^{i-1}, \Theta_z)P_z(z_t^{i-1}|z_{t-1}^{i-1}, \Theta_z)/P_z(z_t^{i-1}|z_{t-1}^i, \Theta_z)}$$

for  $t \leq T - 1$ , and

$$r = \frac{P_y(y_t|\mathfrak{F}_{t-1}, \mathfrak{Z}_{t-1}^{i-1}, x^*, \Theta_y)P_z(x^*|z_{t-1}^{i-1}, \Theta_z)/P_z(x^*|z_{t-1}^i, \Theta_z)}{P_y(y_t|\mathfrak{F}_{t-1}, \mathfrak{Z}_t^{i-1}, \Theta_y)P_z(z_t^{i-1}|z_{t-1}^{i-1}, \Theta_z)/P_z(z_t^{i-1}|z_{t-1}^i, \Theta_z)}$$

for  $t = T$ . The random draw  $x^*$  for round  $i$  is accepted, i.e.,  $z_t^i = x^*$ , if  $\min\{r, 1\} \geq \text{Uniform}(0, 1)$ , and rejected, i.e.,  $z_t^i = z_t^{i-1}$ , otherwise.

For  $\Theta_y$ , I use a random walk chain, specifying the proposal density as  $P_{*\Theta}(x^*|x) = \mathbb{N}(x, c\tilde{\Sigma})$ , where  $c$  is a scaling constant and  $\tilde{\Sigma}$  is the inverse of the Hessian matrix from the maximum likelihood estimation (weighted by the prior densities of the parameters) of the state-space model in (20)-(21) for given  $Z_T$ .

For  $\Theta_z$ , I also use a random walk chain with an identity variance-covariance matrix that is updated after a big number,  $N_1$ , of draws with the variance-covariance matrix of  $\Theta_z$ .

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Table 2: Results from the Bayesian Estimation

Parameters	Prior			Posterior		Convergence
	Density	Mean	SD	Mean	90% Conf. Set	p-value
$\alpha_0$	Gamma	0.8	0.1	0.83	[0.79, 0.87]	0
$\alpha_1$	Gamma	1	0.1	0.93	[0.82, 1.05]	0.04
$\alpha_2$	Gamma	1	0.1	0.97	[0.88, 1.05]	0.09
$\gamma_0$	Normal	-0.01	0.002	-0.01	[-0.013, -0.007]	0.53
$\gamma_1$	Gamma	0.05	0.025	0.04	[0.02, 0.07]	0.05
$\gamma_2$	Gamma	1	0.1	1.09	[0.94, 1.24]	0.04
$\rho_R$	Beta	0.93	0.02	0.928	[0.927, 0.929]	0
$\rho_\tau$	Beta	0.98	0.015	0.978	[0.977, 0.980]	0.09
$\sigma_R$	InvGamma	0.002	0.001	0.013	[0.011, 0.016]	0
$\sigma_\tau$	InvGamma	0.008	0.004	0.008	[0.007, 0.009]	0.34
$\rho_{z^R}$	Beta	0.95	0.02	0.94	[0.91, 0.97]	0.7
$\rho_{z^\tau}$	Beta	0.5	0.05	0.44	[0.36, 0.52]	0.35
$\kappa$	Transf. Beta	0.25	0.05	0.17	[-0.15, 0.5]	0.25
$z_0^R$	Normal	0	1	-0.05	[-1.55, 1.47]	0.20
$z_0^\tau$	Normal	0	1	0.003	[-1.60, 1.66]	0.28

p-values correspond to Geweke (1991) convergence test.

Figure 3: Interest Rate

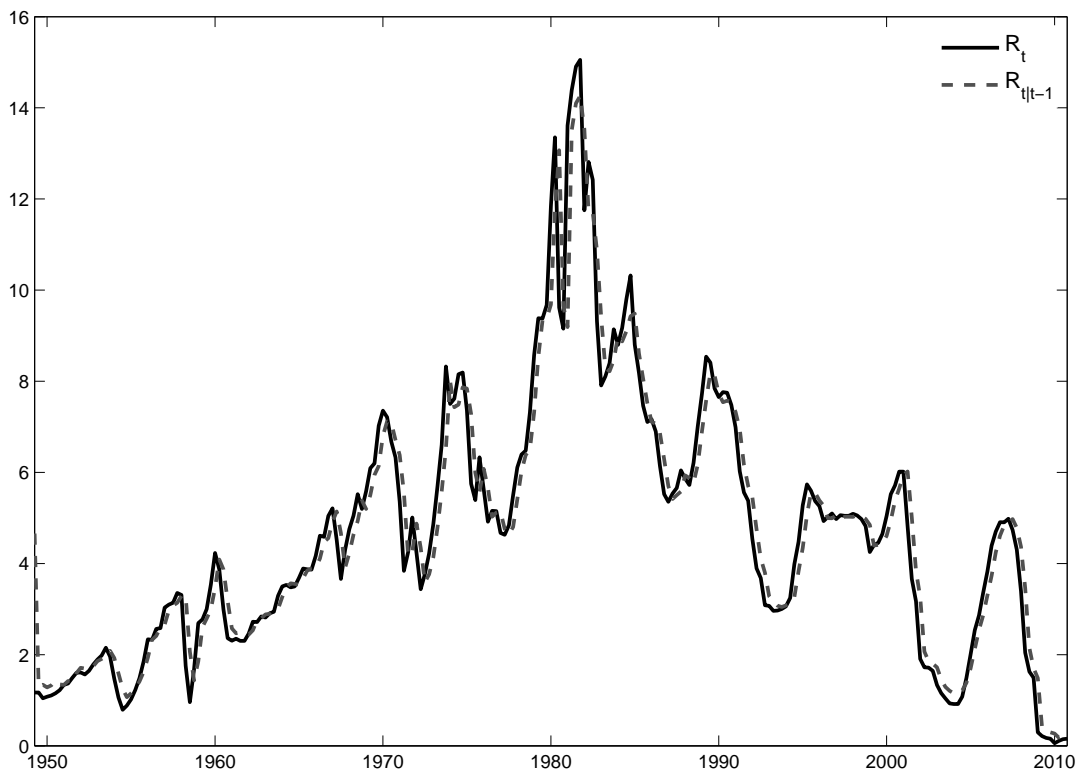


Figure 4: Tax Rate

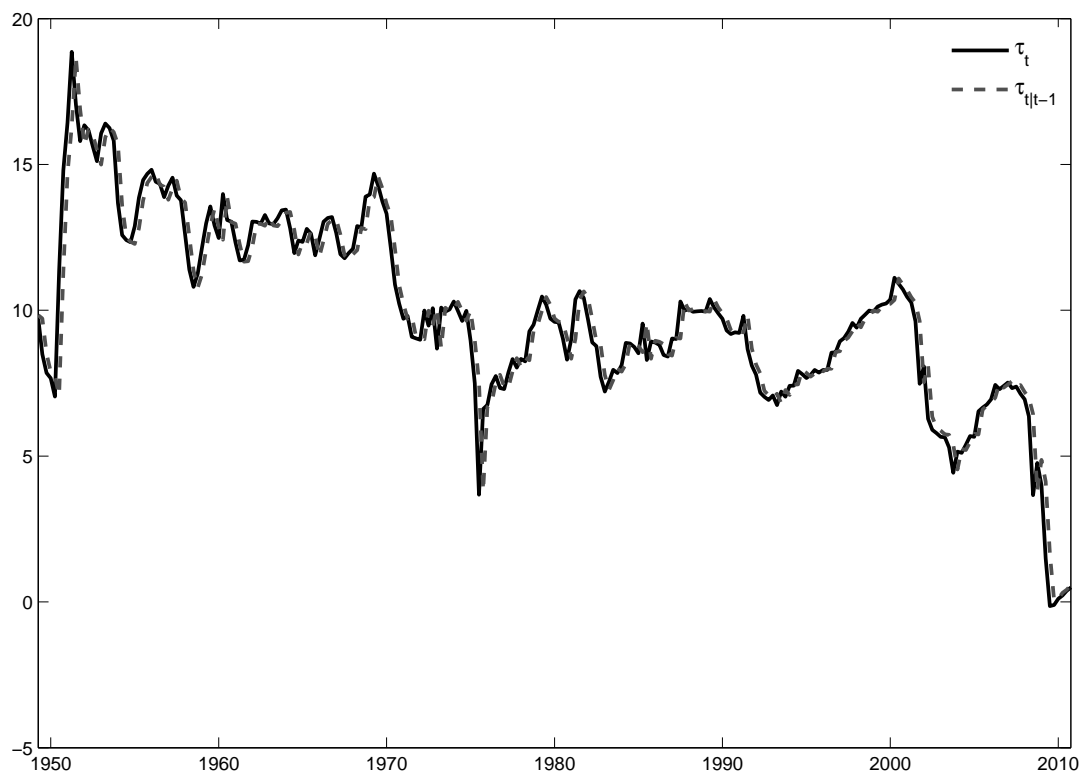


Figure 5: Policy Rule Coefficients

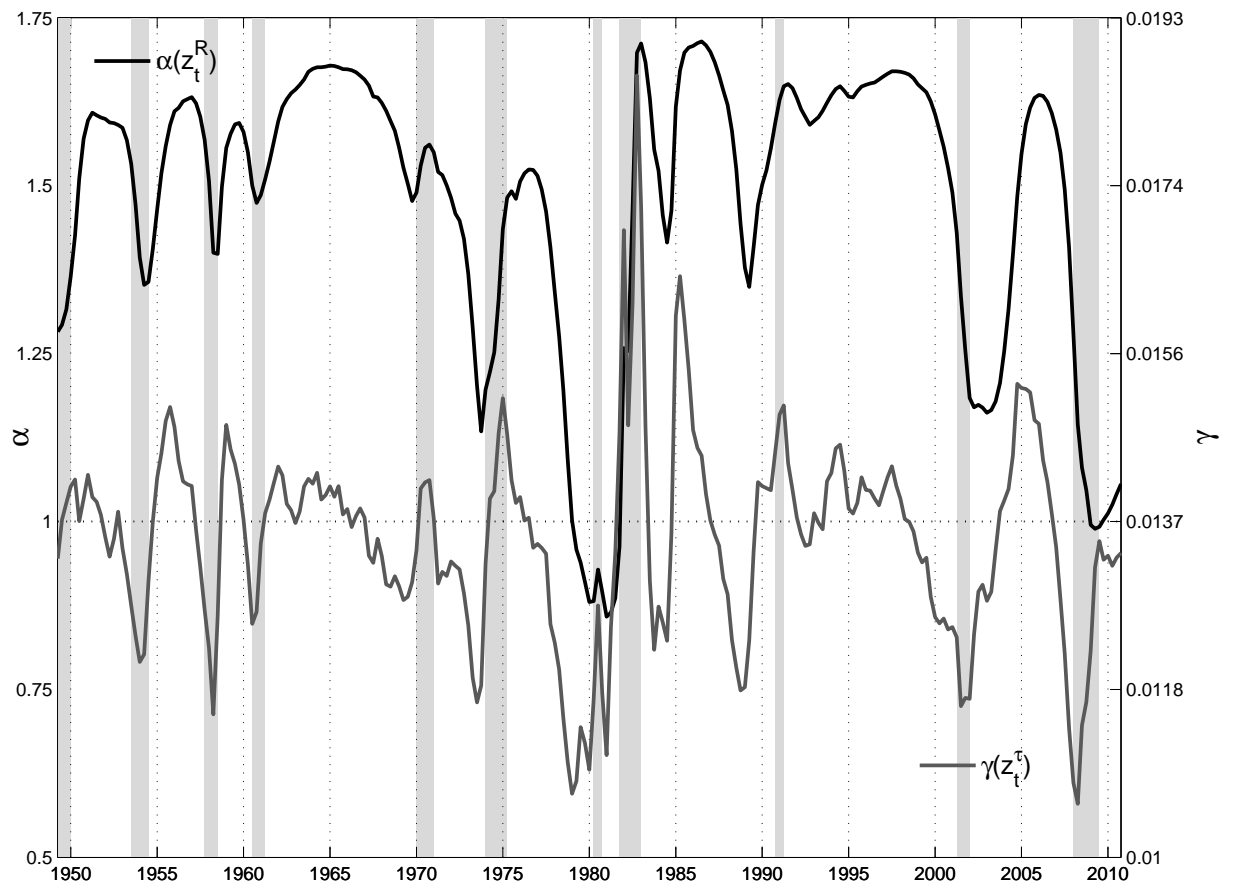




Figure 6: Inflation and AM/AF Periods

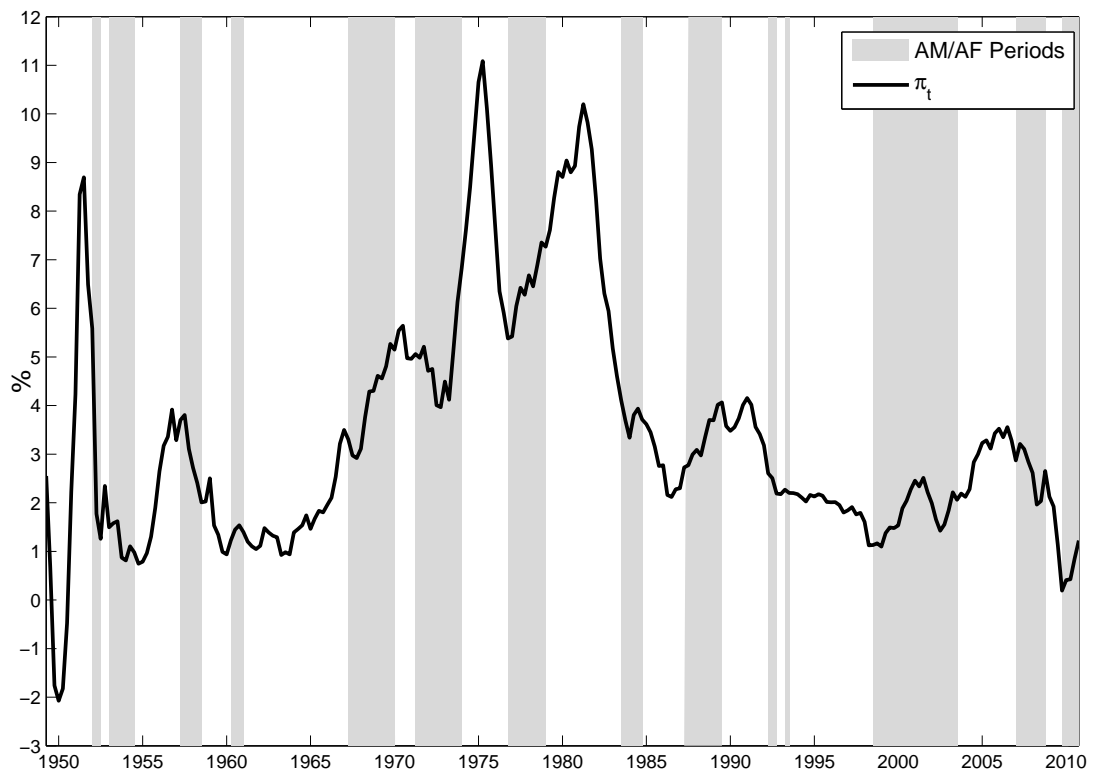


Figure 7: Response of Inflation to a 1% increase in the Interest Rate starting at the PM/AF Regime ( $\alpha(z_{2010:3}^R) = 0.9769$ ,  $\gamma(z_{2010:3}^\tau) = 0.0133$ )

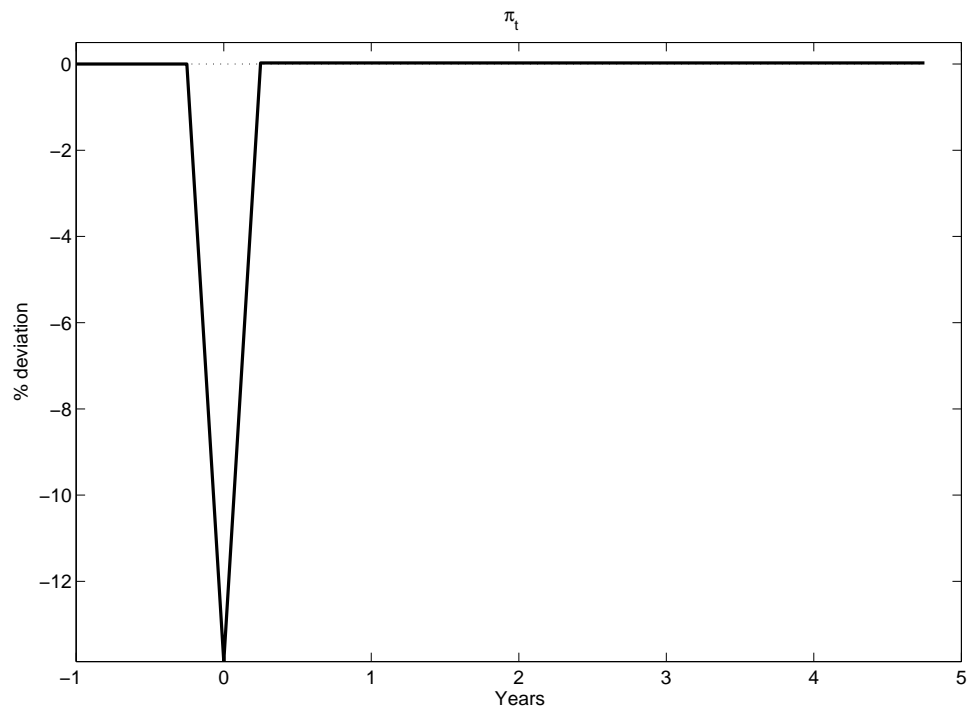


Figure 8: Response of Inflation to a 1% increase in the Tax Rate starting at the PM/AF Regime ( $\alpha(z_{2010:3}^R) = 0.9769$ ,  $\gamma(z_{2010:3}^T) = 0.0133$ )

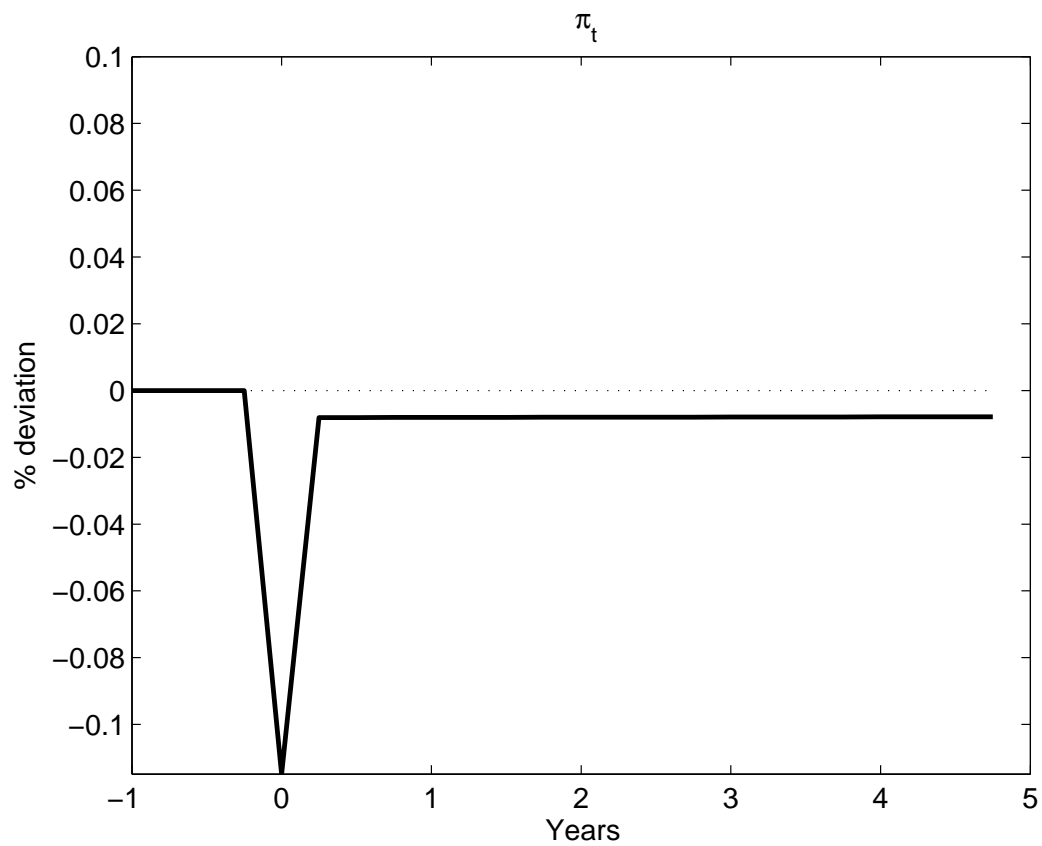


Figure 9: Response of Inflation to a one-time Increase in Policy Parameters from the PM/AF Regime ( $\alpha(z_{2010:3}^R) = 0.9769$ ,  $\gamma(z_{2010:3}^\tau) = 0.0133$ ) to the AM/PF Regime ( $\alpha^*(z_{2010:3}^R) = 1.136$ ,  $\gamma^*(z_{2010:3}^\tau) = 0.0156$ )

