From Discrete to Continuous: Modeling Volatility of the Istanbul Stock Exchange Market with GARCH and COGARCH

Yavuz Yildirim and Gazanfer Unal

Yeditepe University

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Yavuz Yıldırım

Yeditepe University

Prof. Dr. Gazanfer Ünal

Yeditepe University

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Introduction

The success of the forecast model of Istanbul Stock Exchange (ISE) market indices has received great attention in the past decade. The reason being that, any efficient forecasting of the index value would provide the investors with profitable returns. However, the main complication in prediction is the volatility in the time series. There are several reasons that one may want to model and forecast volatility: for instance to analyze the risk of holding an asset or the value of an option. Forecast confidence intervals may found to be time-varying, so that more accurate intervals can be obtained by modeling the variance of the errors. More efficient estimators can be obtained if heteroskedasticity in the errors is handled properly. It has been rather difficult to decide which model to use in order to make an efficient forecasting. The choice of data and the selected period can affect the selection of an appropriate model.

Most of the models arising from the econometric approach are in discrete time. Particularly GARCH models and their extensions have received some attention as appropriate models to capture certain empirical facts of the empirical volatility process [6]. Nelson [10] and Duan [5] attempted to capture the characteristic of financial returns data by diffusion approximations to the discrete time.

Klüppelberg [7] adopted the idea of a single noise process and suggested a new continuous time GARCH (COGARCH) model, which captures all the stylised facts as the discrete time GARCH does. As the noise process, any Lévy processes are possible, its increments replacing the innovations in the discrete time GARCH model. COGARCH based on a single background driving Lévy process, is different from, though related to, other continuous time stochastic volatility models that have been proposed. It generalises the essential features of the discrete time GARCH process in a direct way.

Here, we demonstrate the applicability of COGARCH model for modeling the time-varying volatility of the ISE100. Maller et. al. [8] have recently demonstrated how to apply this kind of methodology to describe the volatility of the Australian stock market, using it to analyse ten years of daily data, mostly equally spaced in time for the ASX200 index. Also, Müller et. al. [9] in 2009 analysed the volatility of stock markets using COGARCH.

1. Methodology

On the discrete modeling part, the best candidate model will be found by considering AIC and BIC values after stationarising the return data. Then usual tests will be carried out to check if the model is covariance stationary, whether it obeys the negativity constraints, and whether the arch effect in the residuals is eliminated.

For continuous modeling, the parameters from the discrete model will be used for continuous GARCH model (COGARCH). Then simulations will be carried out for both of the models and comparisons will be made.

Due to Nelson [10] and others, classical diffusion limits have been used in a natural way to suggest continuous time limits of discrete time processes, including for the GARCH models. Nelson's model of COGARCH model has two different Brownian motions which are independent of each other.

$$dG_t = \sigma_t dB_t^{(1)},\ t \geq 0 \tag{1}$$

$$\sigma_t^2 = (\beta - \eta\sigma_t^2)dt + \varphi\sigma_t^2 dB_t^{(2)},\ t \geq 0 \tag{2}$$

where $B^{(1)}$ and $B^{(2)}$ are independent Brownian motions, and $\beta > 0$, $\eta \geq 0$, and $\phi \geq 0$ are constants.

In Klüppelberg et. al. [7], COGARCH model is a direct analogue of the discrete time GARCH, based on a single background driving Lévy process, and generalises the essential features of the discrete time GARCH process in a natural way.

The COGARCH process $(G_t)_{t\geq 0}$ is defined in terms of its stochastic differential dG, such that

$$dG_t = \sigma_t + dL_t \quad t\geq 0 \tag{3}$$

$$d\sigma_t^2 = (\beta - \eta\sigma_{t-}^2)dt + \varphi\sigma_{t-}^2 d[L,L]_t^{(d)}, \quad t > 0 \tag{4}$$

where $\beta > 0$, $\eta \geq 0$, and $\phi \geq 0$ are constants.

$[L,L]_t^{(d)}$ is the quadratic variation process of L which is defined as

$$[L,L]_t^{(d)} = \sum_{0<s<t}(\Delta L_s)^2 = \sum_{i=1}^{N_t} V_i \tag{5}$$

where $\Delta L_t = L_t - L_{t-1}$ for $t \geq 0$.

The process G 'jumps' at the same time as L does, and has jump sizes

$$\Delta G_t = \sigma_t \, \Delta L_t \qquad t \geq 0 \tag{6}$$

Klüppelberg [6] shows the identity as

$$\sigma_t^2 = \beta t + \log(\delta)\int_0^t \sigma_s^2\, ds + \varphi \sum_{0<s\leq t} \sigma_s^2(\Delta L_t)^2 + \sigma_0^2 \qquad t \geq 0 \tag{7}$$

Deriving a recursive and deterministic approximation for the volatilities at the jump times we get

$$\sigma_i^2 = \sigma_{i-1}^2 - \beta + \eta\int_0^t \sigma_s^2\, ds + \varphi \sum_{0<s\leq t} \sigma_s^2(\Delta L_t)^2 \tag{8}$$

since σ_s is latent and ΔL_s is usually not observable, hence using Euler approximation for the integral we get

$$\int_0^t \sigma_s^2\, ds \approx \sigma_{t-1}^2 \tag{9}$$

$$\sum_{0<s\leq t} \sigma_s^2(\Delta L_t)^2 \approx (G_t - G_{t-1})^2 \tag{10}$$

therefore for the volatility estimation we end up with

$$\sigma_i^2 = \beta + (1-\eta)\sigma_{i-1}^2 + \varphi(G_t - G_{t-1})^2 \tag{11}$$

The bivariate process $(\sigma_t, G_t)_{t\geq 0}$ is Markovian. If $(\sigma_t^2)_{t\geq 0}$ is the stationary version of the process with $\sigma_0^2 = \sigma_\infty^2$, then $(G_t)_{t\geq 0}$ is a process with stationary increments [7, Corrolary 3.1].

2. Data

We perform the analysis using daily log returns on ISE100 daily closing index values. We focus on the time period from 03/01/1994 to 23/06/2010. The data were obtained from ISE.

3. Results And Diagnostics

The first step into the empirical study is to use graphical tools to detect any apparent features of the data. In the case of log return of ISE100 data series; in **Error! Reference source not found.** it is clear that the return data is more like a random walk. There is no trend in the log return of ISE100 and it is more like a white noise type data series, which suggests that the time series is stationary. The stationarity of the data also supported by ACF and PACF graphs. This result will be investigated further by the unit root tests.

3.1 Results Of Unitroot And Stationary Tests

According to the p-value of ADF test, 2.713e-41, the null hypothesis that the data contains a unit root can be rejected. And this is also supported by KPSS test result, with the p-vale 0.4386**,** cannot reject the null hypothesis at any significance level that the data is stationary around a constant.

3.2 Discrete Modeling

The best candidate model is found to be AR(1)~GARCH(1,1) model. The Ljung-Box test with the p-value of 0.025 tells us that there is no autocorrelation in the model's residuals and the candidate model also removes the ARCH effect in the residuals given the LM Test's p-value is 0.1246.

The model obeys the negativity constraint of a GARCH model that is none of the coefficients of the parameters are negative, and it also satisfies the covariance stationarity condition as the sum of coefficients is less than 1. All of the coefficients are statistically significant as the t-values are greater than 1.65. All the results show that the candidate model AR(1)~GARCH(1,1) is a good model for the log return of ISE100 time series.

$$Y_t = 0.00181322 + 0.05623574 Y_{t-1} + \varepsilon_t \quad (12)$$

$$\sigma_t^2 = 0.00001319 + 0.11436918 Y_{t-1}^2 + +0.87281962 \sigma_{t-1}^2 \quad (13)$$

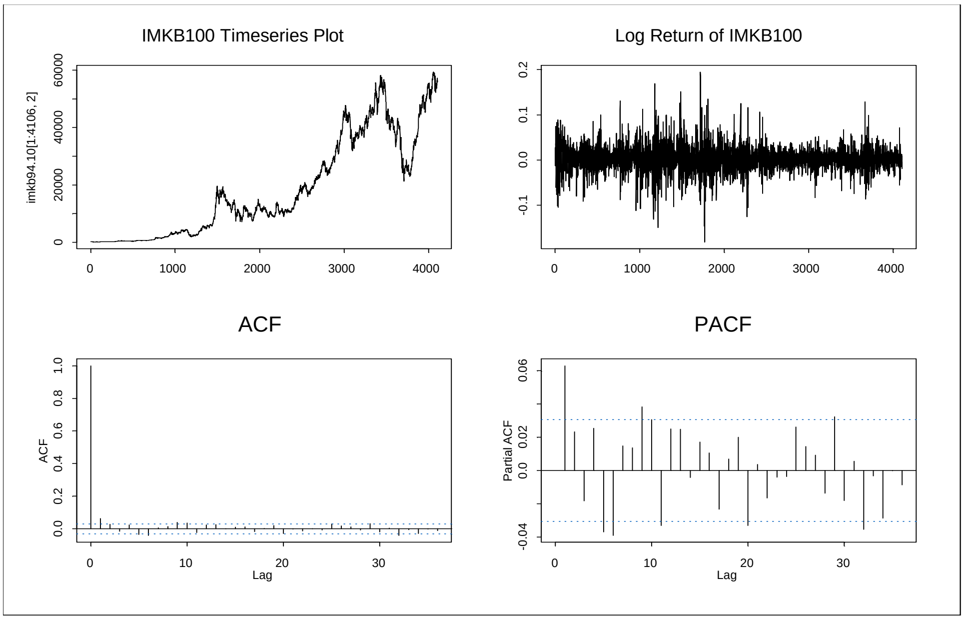

Figure 1: Time series plot of ISE100 index value, Log return of ISE100, ACF and PACF of log return
Source: Own Study

Table 1

Estimated Coefficients of AR(1)~GARCH(1,1)

	Value	Std.Error	t value	$\Pr(>\|t\|)$
C	0.00181322	3.316e-004	5.468	4.814e-008
AR(1)	0.05623574	1.590e-002	3.536	4.109e-004
A	0.00001319	1.978e-006	6.670	2.896e-011
ARCH(1)	0.11436918	7.410e-003	15.435	0.000e+000
GARCH(1)	0.87281962	7.373e-003	118.385	0.000e+000

Source: Own Study

3.3 Continuous Modeling

Given that the parameters of COGARCH model is equal to the discrete GARCH model's parameters as such

$$\beta = \beta\,,\ \eta = \ln\delta\,,\ \phi = \lambda / \delta \tag{14}$$

The candidate model's parameters are

$$\beta = 0.00001319\,,\ \lambda = 0.11436918\,,\ \ \delta = 0.87281962 \tag{15}$$

The parameters' of COGARCH(1,1) model are

$$\eta = \ln 0.8781962 = 0.13603 \quad \phi = 0.11436918 / 0.87281962 = 0.13103 \tag{16}$$

To start the simulation we use numerical solutions for dG_t and $d\sigma_t^2$ in (6) and (11), and we also use a Lévy process driven by compound Poisson process. The compression between the volatility of the log return data with the discrete GARCH model and COGARCH's volatility, Figure 2, shows that there is a close relation between discrete and continuous model and both models mimic the real data’s volatility.

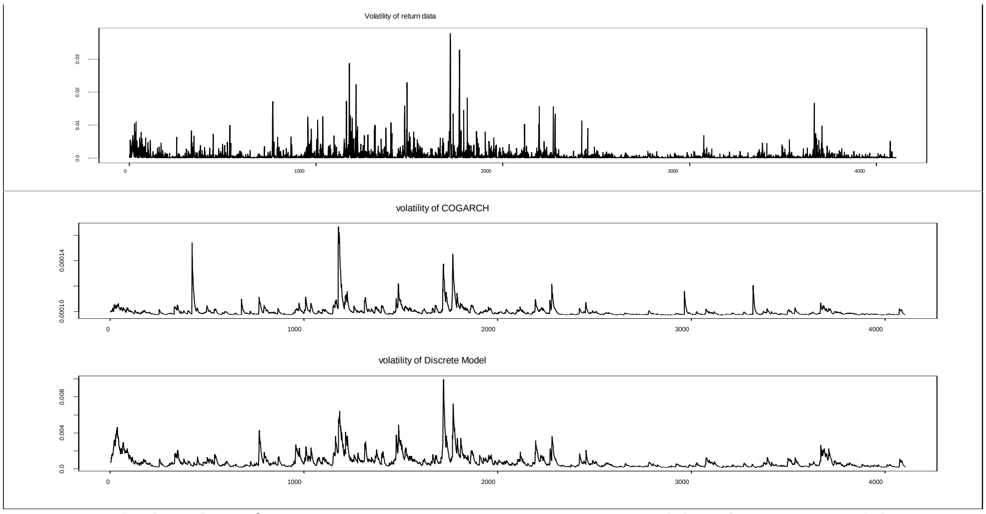

Figure 2: Volatility plots of Log Return Data, continuous GARCH model, and GARCH model
Source: Own Study

Conclusion

Log return of ISE100 daily closing index value was modeled with the best candidate model AR(1)~GARCH(1,1). Then using the parameters from the discrete model, continuous model COGARCH(1,1) was applied to the data. Volatility of simulated data from discrete and continuous models compared with the real data volatility. We showed that the simulated GARCH volatility and COGARCH volatility appears to follow the same pattern of jumps. Furthermore, both models imitate the real return data's volatility.

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Summary

The objective of this paper is to model the volatility of Istanbul Stock Exchange market, ISE100 Index by ARMA and GARCH models and then take a step further into the analysis from discrete modeling to continuous modeling. Through applying unit root and stationary tests on the log return of the index, we found that log return of ISE100 data is stationary. Best candidate model chosen was found to be AR(1)~GARCH(1,1) by AIC and BIC criteria. Then using the parameters from the discrete model, COGARCH(1,1) was applied as a continuous model.