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Abstract

Using contests to generate innovation has and is widely used. Such contests often involve offering a prize that depends upon the accomplishment (effort). Using an all-pay auction as a model of a contest, we determine the optimal reward for inducing innovation. In a symmetric environment, we find that the reward should be set to $c(x)/c'(x)$ where c is the cost of producing an innovation of level x. In an asymmetric environment with two firms, we find that it is optimal to set different rewards for each firm. There are cases where this can be replicated by a single reward that depends upon accomplishments of both contestants.

Keywords: contests, innovation, mechanism design.

JEL codes: C70, D44, L12, O32

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1 Introduction

Using contests to generate innovation has been around for hundreds of years. In the 1700s, the Longitude prize of £20,000 offered by the British Parliament induced John Harrison to invent the marine chronometer (see Sobel, 1996). More recently, the Ansari X-prize was a ten-million-dollar competition created to jump-start the space tourism industry by attracting the attention of the most talented entrepreneurs and rocket experts in the world.[1] Such R&D contests are an example of a competition in which all contestants, including those that do not win any reward (prize), incur costs as a result of their efforts but only the winner gets the reward. In such contests, the designer may often offer smaller prizes for lesser achievements. In fact, while the full longitude prize was given for determining longitude within 30 nautical miles, £10,000 was given for a method for determining longitude within 60 miles, and £15,000 for a method within 40 nautical miles. Another example with smaller prizes is where Netflix offers a prize for improving their movie recommendation system.[2] This prize increases if the improvement is more than 10%.[3]

We model a contest as an all-pay auction. When the prize depends upon the result, this is equivalent to having a bid-dependent reward. Such environ-

[1] See www.xprize.org for details.

[2] See www.netflixprize.com.

[3] Other interesting examples include the Methuselah Mouse Prize (see www.mprize.org) for creating a long-lived mouse. If the prize money is z, the oldest previous mouse lived x years and someone creates a mouse that lives $y > x$ years, then they would receive $z \cdot y/(x + y)$. There was also the Schneider trophy (see Eves, 2001) created to inspire aviation design. There was a competition between the fastest seaplanes held 11 times between 1913 and 1931. Each victory won a smaller prize and the full prize of 70,000 Franc prize would be given if the same club won three times in a row. When this happened by an English group (won by a forerunner to the Spitfire), the competition ceased.

ments have been analyzed before both positively, studying the equilibrium behavior properties and normatively, determining what are optimal contest designs. Environments with complete information have been analyzed from a positive point of view in Kaplan et al. (2003) and Siegel (2009, 2010), the normative point of view was analyzed in Gale and Che (2003). Environments with incomplete information were studied from a positive point of view in Kaplan et al. (2002), the normative point of view was investigated in Moldovanu and Sela (2002) and Chen et al. (2008). Similar research was carried out for rent-seeking contests, Nitzan (1994) provided a positive analysis, Franke et al. (2009) provided a normative analysis. Konrad (2009) provides an excellent survey of equilibrium and optimal design in contests.

In this paper, we provide further normative analysis for environments with complete information. We look at the optimal rewards under complete information when the designer wishes to maximize the highest effort of the participants. We determine the optimal bid-dependent reward structure as a function of costs in both symmetric and asymmetric environments.

Interestingly, the solution under symmetry, setting reward equal to the cost of effort divided by the marginal cost of effort, is quite elegant and produces equivalent behavior to that in Che and Gale (2003) where the firms compete by choosing both effort and price. In our paper, the solution under asymmetry also involves setting the reward equal to the cost of effort divided by the marginal cost of effort. One may consider this problematic in the sense that the designer must know which firm is which and bias the contest in favor one of the firms. We address this issue by describing settings where this firm specific reward structure can be replaced by a reward (to the winner) that depends upon both of the firms' efforts. In our setting, we consider a richer class of contests than considered by Che and Gale (2003) and as a result, in some cases, the optimal contest generates higher surplus for the designer than their solution of handicapping one firm.

While in this paper we phrase the problem as designing a research contest, our analysis is applicable to many other scenarios that have such a winner-take-all form. For instance, many races offer prizes to the winners that depend upon time. Also, in a contest to receive a promotion at a company, the firm may set the salary increase with the promotion conditional on the worker's performance. This paper would suggest how to structure these rewards.

Our paper is structured as follows. In Section 2, we introduce the general environment with the optimal rewards for symmetric case. Afterwards, in Section 3, we allow for asymmetry between firms. Finally, in Section 4, we present the concluding remarks.

2 Symmetric environment.

2.1 Model

A buyer (designer) desires an innovation. There are n firms that have potential to innovate. Firms can create an innovation of value x (to the designer) for a cost $c(x)$. This value x includes external benefits generated by the contest.[4] We assume $c(0) \geq 0$, $c' > 0$, $c'' > 0$, and is common knowledge.[5] Furthermore, we assume there exists an x such that $x > c(x)$. The buyer can design a contest where the reward depends upon the bid of the firm. He does so by choosing a reward function $R(x)$ that depends upon the winning bid (it could be constant). We assume that R must be continuous with $R(0) \geq 0$.[6] The buyer cares only about the best innovation (maximum x) and how much he pays out in rewards, namely he wishes to maximize:

$$E[\max\{x_1, ..., x_n\} - R(\max\{x_1, ..., x_n\})].$$

At this point we would like to further motivate our study of contests rather than other mechanisms: One alternative could be to run a Vickrey auction where the firms compete by offering potential innovations and then the winning firm would create the innovation promised. Another could making a take-it-or-leave-it-offer to a single firm. Our reasons are as follows. First, in practice, contests are commonly used in a plethora of economic activities, while Scotchmer (2004, chapter 2) points out that to her knowledge (and ours) that a Vickrey auction has never been used in procuring an innovation. Second, as mentioned in Scotchmer (2004, chapter 2), without a contest, there is a hold-up type problem when the ex-post payment depends

[4] We assume that the designer has the potential to capture all the external benefits accrued to the winner with a contract signed before the contest (such as with the show Pop Idol).

[5] While we assume the designer knows c, we also assume that c is not verifiable in court.

[6] The assumption of continuity of R is natural, since even a discontinuous reward function is equivalent to a continuous reward function with a minimum amount of noise. Consider the case that each x_i has a noise ε that affects the final result. (For instance, there could be a slight wind in the 100 m dash.) In this case, the actual reward would be $\widetilde{R}(x_i) \equiv E[R(x_i + \varepsilon)]$ and is continuous.

upon the firm delivering a future innovation of a specific quality. Third, as mentioned above, there may be external benefits (publicity) for both the designer and winning firm for running a contest. Thus, studying the optimal contest is a worthwhile endeavor.

2.2 Analysis

As long as there exists an x such that $R(x) > c(x)$, there is no pure strategy equilibrium.[7] In such a case, however, there will be a symmetric mixed-strategy equilibrium where each firm chooses x according to a cumulative, atomless (except possibly at 0) distribution F.

Proposition 1 *In the optimal design, the buyer sets* $R(x) = c(x)/c'(x)$*. This generates a surplus of*

$$\frac{n}{n-1}\int_0^{c'^{-1}(1)} (xc'(x) - c(x))\frac{c''(x)}{c'(x)^{\frac{n-2}{n-1}}}dx.$$

Proof. The designer's expected profits can be rewritten as

$$\int (x - R(x))\, dF^n.$$

Similar to Kaplan et al. (2003), the firms will have zero expected profits. Since it is a mixed strategy equilibrium, the firms must be indifferent over all x in the support of F. Hence,

$$F(x)^{n-1}R(x) - c(x) = 0.$$

[7] When this condition does not hold, the pure-strategy equilibrium has no firm entering and the buyer earning zero surplus.

By integrating we get:

$$\int F(x)^{n-1}R(x)dF - \int c(x)dF = 0 \Longrightarrow$$
$$\int R(x)dF^n = n\int c(x)dF.$$

Substituting this into the designer's objective yields

$$\int x dF^n - n\int c(x)dF =$$
$$n\int \left(xF^{n-1} - c(x)\right) dF.$$

We can now do a change of variables so that we are integrating over F (rather than x).

$$n\int_0^1 \left(x(F)F^{n-1} - c(x(F))\right) dF.$$

Now we can independently choose our $x(F)$ to maximize the integrand. Thus, we get $F^{n-1} = c'(x(F))$ or $F(x)^{n-1} = c'(x)$. From the zero profit equation of the firm, $F(x)^{n-1}R(x) - c(x) = 0$, the optimal reward is $R(x) = c(x)/c'(x)$. The expression for the surplus is generated by substitution. ■

2.3 Comparison to Che and Gale (2003).

In Che and Gale (2003), a buyer also wishes to acquire an innovation that can be of varying quality. There, the buyer designs a competition where firms expend effort to innovate where a higher effort results in a higher quality of innovation. After innovating each firm specifies a price to the buyer. The buyer then chooses the firm offering the highest surplus (quality minus price).

The winning firm receives payment while all firms bear the cost of their sunk effort.

In the simplest version of the Che-Gale model, each firm i chooses effort x_i, surplus s_i and price p_i to solve $\max_{x_i, s_i, p_i} \pi(s_i)p_i - c(x_i)$ s.t. $x_i - p_i = s_i$ (where π is the probability that the other firms choose a surplus lower than one's own). Substituting the constraint into the maximand, we get $\pi(x_i - p_i)p_i - c(x_i)$. The firm will optimize over the choices of x and p which implies (from the FOCs) $\pi'(s_i)p_i = \pi(s_i)$ and $\pi'(s_i)p_i = c'(x_i)$. Together, these imply $\pi(s_i) = c'(x_i)$. The zero profit condition of the firm implies that $\pi(s_i)p_i = c(x_i)$. Thus, $p_i = c(x_i)/c'(x_i)$. The behavior induced and payoffs are identical to our solution.

Intuitively, this works out to be the same since the firms in the Che and Gale model optimize over effort and price given a specific level of surplus offered. In our model, the designer optimizes the trade-off between value of the effort (to the designer) and its cost (to the firm) for a given probability of winning (note an effort is worthless to the designer if it is not the highest).

For the symmetric environment, each mechanism has its own benefits. The Che and Gale mechanism has the advantage that the designer does not need to know the cost function beforehand which our mechanism requires for determining the rewards. The Che and Gale mechanism has the disadvantage that off equilibrium, the buyer may have to purchase the inferior innovation since it offers him a lower price. This could be politically difficult and precludes the possibility of renegotiation on price.

2.4 Examples

Remark 1 *The optimal reward function may assume may forms: increasing, decreasing, have both increasing and decreasing parts, or be constant.*

The remark is shown through a series of examples.

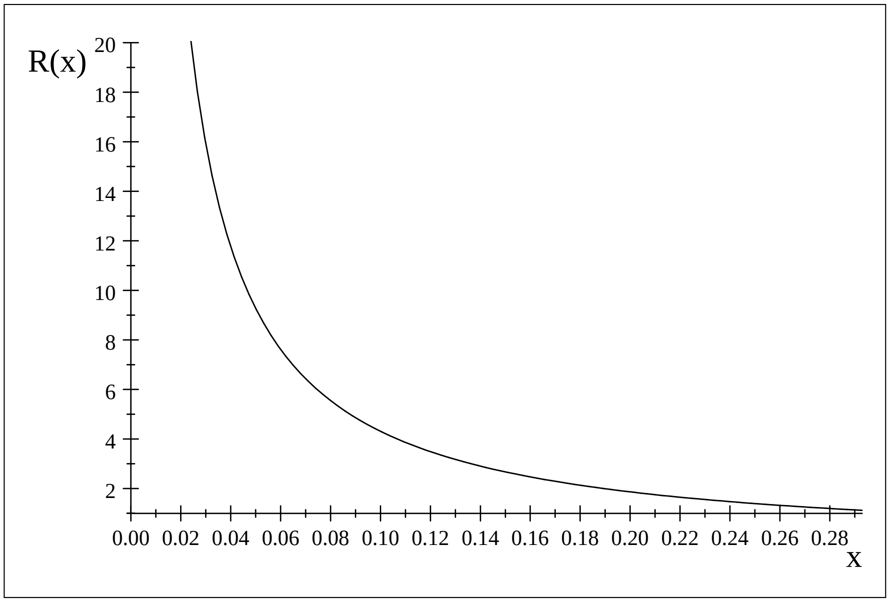
Example 1 *Strictly increasing reward function:* $n = 2, c(x) = x^a$ *where* $x > 1$.

For such a cost, the optimal reward is $R(x) = \frac{c(x)}{c'(x)} = \frac{x^\alpha}{\alpha x^{\alpha-1}} = \frac{x}{\alpha}$. This is strictly increasing in x. In equilibrium, the firms choose effort by using a cumulative distribution function $F(x) = c'(x) = \alpha x^{\alpha-1}$.

Example 2 *Strictly decreasing reward function:* $n = 2,\ c(x) = \frac{1}{1-x} - x$.

The optimal reward is $R(x) = \frac{c(x)}{c'(x)} = x + \frac{1}{2x} - \frac{3}{2(2-x)}$ which is strictly decreasing and positive for $0 \leq x < 1$.

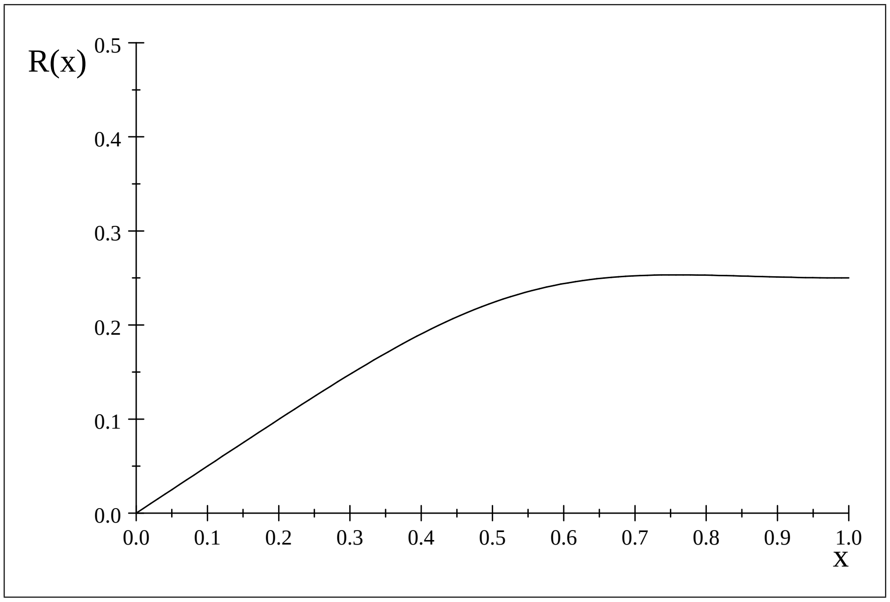
In equilibrium, the firms choose effort by using a cumulative distribution function $F(x) = c'(x) = \frac{1}{(1-x)^2} - 1$. Thus, each firm uses a mixed strategy on $[0, 0.2929]$. See Figure 1.



Example 2: Decreasing optimal reward.

Example 3 *Increasing and then decreasing reward function:* $c(x) = \frac{x^6+x^2}{8}$.

The optimal reward is $R(x) = \frac{c(x)}{c'(x)} = \frac{x^5+x}{6x^4+2}$. This increases until $x = 0.76$ and then decreases. See Figure 2.



Example 3: Increasing and decreasing optimal reward.

In equilibrium, the firms choose effort by using a cumulative distribution function $F(x) = c'(x) = \frac{6x^5+2x}{8}$ on $[0, 1]$.

Example 4 *Constant reward:* $n = 2$, $c(x) = e^{x/2}$.

The optimal reward is $R(x) = \frac{c(x)}{c'(x)} = 2$. In equilibrium, the firms choose effort by using a cumulative distribution function $F(x) = c'(x) = \frac{1}{2}e^{x/2}$ on $[0, 1.39]$. Notice that there is an atom of $1/2$ at zero. If one firm makes an ε effort, it has a $1/2$ chance of winning a reward of 2 and it costs the firm 1. Also note that we implicitly assume that a firm can stay out and not pay $c(0)$.

Remark 2 *The optimal $R(x)$ is constant if and only if there is a fixed cost and $c(x) = e^{\alpha x+\beta}$ where $\alpha > 0$, $\beta < \ln \frac{1}{\alpha}$ and $R(x) = \frac{1}{\alpha}$.*

Proof. Since $R(x) = \frac{c(x)}{c'(x)}$, if $R(x)$ is constant and equal to r, we have $\frac{c'(x)}{c(x)} = \frac{1}{r}$. Integration yields $\ln c(x) = \frac{1}{r}x+k$ or $c(x) = e^{\frac{1}{r}x+k}$. Since $F(x)^{n-1} = c'(x) = \frac{1}{r}e^{\frac{1}{r}x+k}$, we have $F(0) > 0$. Also, we must have $\frac{1}{r}e^k < 1$, so $k < \ln r$. ■

Remark 3 *Multiplying the costs by a constant does not effect the optimal $R(x)$.*

One may intuitively think that doubling costs would entail an increase of the optimal rewards; however, since $R(x) = \frac{c(x)}{c'(x)}$, there is no change. This is due to the result that if cost is doubled, then it is optimal to have F doubled (a decrease in the effort). In order to induce this, $R(x)$ should stay the same.

3 Asymmetric Environment.

Now assume that there are two firms that differ by their cost functions $c_1(x)$, $c_2(x)$ where $c_1(x) \geq c_2(x)$. For now, assume that the designer can make a separate reward offer to either firm: $R_1(x)$ and $R_2(x)$. Assume that the buyer chooses rewards such that the equilibrium has both firms making a positive effort.

Under these assumptions, again there must be a mixed-strategy equilibrium which we denote by $F_1(x)$ and $F_2(x)$.

Lemma 1 *In the optimal design, firms make zero profits.*

Proof. The proof is by contradiction. Let us say that the two reward functions R_1 and R_2 are optimal and induce behaviour F_1 and F_2. Assume that the equilibrium is such that $R_1(x)F_2(x) - c_1(x) = 0$ and $R_2(x)F_1(x) - c_2(x) = \pi$. (Note that in equilibrium at least one must make zero profits.) Create an $\widehat{R}_2(x)$ as follows: $\widehat{R}_2(x) = R_2(x) - \frac{\pi}{F_1}$. This $\widehat{R}_2$ is less costly and induces the same equilibrium distribution functions. Hence, there is a contradiction to the initial assumption that R_1 and R_2 are optimal for the designer. ■

When profits are zero the total social welfare coincides with the objective of the designer. Let us look at the case were there are cost functions $c_1(x)$ and $c_2(x)$. The social welfare is

$$\int x dF_1 F_2 - \int c_1(x) dF_1 - \int c_2(x) dF_2 =$$
$$\int (xF_2 - c_1(x)) dF_1 + \int (xF_1 - c_2(x)) dF_2$$

The designer's problem is then

$$\max_{F_1,F_2} \quad \int (xF_2 - c_1(x))dF_1 + \int (xF_1 - c_2(x))dF_2$$
$$s.t. \text{ the supports of } F_1 \text{ and } F_2 \text{ coincide.}$$

Proposition 2 *If* $c_1'^{-1}(1) = c_2'^{-1}(1)$ *and* $c_1'(0) = c_2'(0)$, *then optimal design has the buyer set* $R_i(x) = c_i(x)/c_i'(x)$.

Proof. Let us do a change of variables to choose $x(F_1)$ and $F_2(F_1)$. Now the maximization problem becomes

$$\max_{x(F_1),F_2(F_1)} \int (x(F_1)F_2(F_1) - c_1(x(F_1)) + [x(F_1)F_1 - c_2(x(F_1))]\, F_2'(F_1))\, dF_1.$$

Choosing $x()$ pointwise leads to the following FOC:

$$F_2(F_1) - c_1'(x(F_1)) + [F_1 - c_2'(x(F_1))]\, F_2'(F_1) = 0.$$

Choosing $F_2'(F_1)$ pointwise leads to the second FOC:

$$-\int_0^{F_1} x(\widetilde{F}_1)d\widetilde{F}_1 + x(F_1)F_1 - c_2(x(F_1)) = 0$$

Note that in order to do this last step, we have to use integration by parts to rewrite the integral $\int x(F_1)F_2(F_1)dF_1$ as $\int_0^{F_1} x(\widetilde{F}_1)d\widetilde{F}_1 F_2(F_1)\Big|_0^1 - \int \int_0^{F_1} x(\widetilde{F}_1)d\widetilde{F}_1 \cdot F_2'(F_1)dF_1$.

Let us now write the second FOC by writing F in terms of x:

$$\begin{aligned}\int_0^x F_1(x)dx - xF_1(x) + xF_1(x) - c_2(x) &= 0 \\ \int_0^x F_1(x)dx - c_2(x) &= 0 \\ F_1(x) &= c_2'(x)\end{aligned}$$

Substituting this into the first FOC yields $F_2(x) = c_1'(x)$. Using the indifference conditions of the firms yields the optimal reward functions. The conditions $c_1'^{-1}(1) = c_2'^{-1}(1)$ and $c_1'(0) = c_2'(0)$ ensures that the supports coincide. ■

Example 5 $c_1(x) = \frac{x^a}{a}$, $c_2(x) = \frac{x^b}{b}$ *(where* $a, b > 1$*). We have* $R_1(x_1) = \frac{x_1}{a}$ *and* $R_2(x_2) = \frac{x_2}{b}$ *where* $F_1(x_1) = x_1^{b-1}$ *and* $F_2(x_2) = x_2^{a-1}$.

Notice that such a reward structure requires that the designer not only knows which firm has which cost function, but is also able to openly discriminate against one of the firms. Such favoritism could be problematic politically. It would be much easier and more elegant if there could be a single reward function. We, hence, proceed to try and construct a reward function that depends not only on one's own effort but also on that of the other firm and which in expectation replicates, in equilibrium, the two separate reward functions.

Proposition 3 *The optimal design can sometimes be implemented by a single reward function that depends upon both efforts.*

Proof. We wish to create a reward function $R(x_h, x_l)$ This reward function represents the reward paid to the firm with the highest effort and depends upon both the high and low effort levels x_h and x_l. The expectation of this reward function should yield the individual expected reward functions, namely,

$$\begin{aligned}\frac{\int_0^{x_h} R(x_h, x_l)F_2'(x_l)dx_l}{F_2(x_h)} &= R_1(x_h),\\ \frac{\int_0^{x_h} R(x_h, x_l)F_1'(x_l)dx_l}{F_1(x_h)} &= R_2(x_h).\end{aligned}$$

Rewriting yields

$$\begin{aligned}\int_0^{x_h} R(x_h, x_l)F_2'(x_l)dx_l &= R_1(x_h)F_2(x_h),\\ \int_0^{x_h} R(x_h, x_l)F_1'(x_l)dx_l &= R_2(x_h)F_1(x_h).\end{aligned}$$

Substituting the functions used in our example yield

$$\begin{aligned}\int_0^{x_h} R(x_h, x_l)x_l^{a-2}dx_l &= \frac{1}{a(a-1)}x_h^a,\\ \int_0^{x_h} R(x_h, x_l)x_l^{b-2}dx_l &= \frac{1}{b(b-1)}x_h^b.\end{aligned}$$

The solution to these two equations is $R(x_h, x_l) = \frac{1}{a+b-1}x_h^{\frac{ab}{a+b-1}}x_l^{1-\frac{ab}{a+b-1}}$. ■

Note that for the example in the above proof the exponent on x_h is always positive and the exponent on x_l is always less than 1 and could be negative. We can also compute the expected profit for the above example which is $\int_0^1(xc_1'(x) - c_1(x))c_2''(x)dx + \int_0^1(xc_2'(x) - c_2(x))c_1''(x)dx =$

$1 - \frac{1}{a} - \frac{1}{b} + \frac{1}{a+b-1}$.

3.1 Comparison to Che and Gale (2003).

Che and Gale allow the buyer to handicap the stronger firm by limiting the price the firm can charge. Now the firm's problem is $\max_{x_i,s_i,p_i} \pi_i(s_i)p_i - c_i(x_i)$ s.t. $x_i - p_i = s_i$ and $p_i \leq p_i^*$. Without the constraint binding, as before $\pi_i(s_i) = c_i'(x_i)$ and $p_i = c_i(x_i)/c_i'(x_i)$. Once the constraint binds, $\pi_i(s_i) = (u_i + c_i(s + p^*))/p^*$ where u_i is the profit of firm i. The buyer is able to choose p^* in order to limit the profit of this firm. The profit is determined by the maximum surplus the other firm can offer which equals $s_j^* = \max_x x - c_j(x)$. If p_i^* is binding, then $u_i = p_i^* - c_i(s_j^* + p_i^*)$. If one wishes to set u_i to zero, we have $p_i^* = c_i(s_j^* + p_i^*)$

Example 6 *The Che and Gale (2003) mechanism when* $c_1(x) = \frac{2}{3}x^{\frac{3}{2}}$, $c_2(x) = \frac{1}{2}x^2$.

For the weak buyer $p_j = c_j(x_j)/c_j'(x_j) = 2x_j/3$. Since $s_j = x_j - p_j$, we have $s_j = x_j/3$. Since $\pi_j(s_i) = c_j'(x_i)$, we have $\pi_j(s) = (3 \cdot s)^{1/2}$. Likewise for the strong buyer, when p^* is not binding, we have $\pi_i(s) = 2s$. Using the probability of winning π_i, we can determine the strategy G_i of each player:

$$\begin{aligned} G_1(s) &= (3 \cdot s)^{1/2} \\ G_2(s) &= \begin{cases} 2 \cdot s & \text{if } s < p, \\ \frac{(s+p)^2}{2 \cdot p} & \text{if } s > p \text{ where } p = \frac{2-\sqrt{3}}{3}. \end{cases} \end{aligned}$$

We can now compute the expected profit:

$$\begin{aligned}\int_0^{1/3} s\cdot d(G_1\cdot G_2) = \int_0^p s\cdot d(G_1\cdot 2s) + \int_p^{1/3} s\cdot d(G_1\cdot\frac{(s+p)^2}{2\cdot p}) &= \\ \int_0^p 3\,(3)^{1/2}\cdot s^{3/2}ds + \int_p^{1/3}\frac{(3)^{1/2}}{2}s\cdot d((s)^{1/2}\cdot\frac{(s+p)^2}{p}) &= \\ \left(173-76\sqrt{2}+20\sqrt{3}+44\sqrt{6}\right)\frac{1}{945} &= \\ 0.220041.\end{aligned}$$

Using the mechanism in this paper, the expected profit is $1-\frac{1}{a}-\frac{1}{b}+\frac{1}{a+b-1}=\frac{7}{30}=0.23333$, which is higher.

Note that this finding does not contradict those in the Che and Gale (2003), since our mechanism uses bid-dependent rewards which are not feasible in their environment and added flexibility is an advantage. Furthermore, we avoid directly handicapping one of the firms by using a combined reward function. This allows the handicapping indirectly through the behaviour of the other firm that handicaps it.

4 Conclusion

We have examined the optimal design of rewards in a contest with complete information. We find a simple rule for setting the optimal rewards in the symmetric case. This allows the designer to simply choose the best design and pay the winner according to the prespecified reward. With asymmetry, it is optimal to have different firms receive different rewards. We show it might be possible, for some environments, to replicate this with a common joint reward function that depends upon both efforts. This design method yielded "better outcomes" then previously used mechanisms.

Further research is needed to examine the effect of changing the number of firms. Several open issues remain for the asymmetric environment case: What are general conditions under which it is possible to create a joint reward function? What is the best design, when the optimal reward functions do not share the same support? Finally, it is of interest to see what the optimal reward function would be under additional constraints, for instance, if one were limited to offering the same reward to both firms where this reward could only depend upon the highest effort.

References

[1] Chen Cohen, Todd Kaplan, Aner Sela, 2008, "Optimal Rewards in Contests," *RAND Journal of Economics,* 39 (2), 434 – 451.

[2] Che, Yeon-Koo, Ian Gale, 2003, "Optimal Design of Research Contests," *The American Economic Review*, 93 (3), 646 – 671.

[3] Eves, Edward, 2001, The Schneider Trophy Story. Shrewsbury, UK: Airlife Publishing Ltd.

[4] Franke, J. and Kanzow, C. and Leininger, W. and Väth, A., 2009, "Effort Maximization in N-Person Contest Games," *CESifo Working Paper No. 2744.*

[5] Kaplan, Todd, Israel Luski and David Wettstein, 2003, "Innovative Activity with Sunk Cost," *International Journal of Industrial Organization*, 21, 1111 – 1133.

[6] Kaplan, Todd, Israel Luski, Aner Sela, and David Wettstein "All-Pay Auctions with Variable Rewards," *Journal of Industrial Economics*, December 2002 L (4): 417 – 430.

[7] Konrad, Kai, 2009, Strategy and Dynamics in Contests. Oxford University Press, Oxford, UK.

[8] Nitzan, Shmuel, 1994, "Modelling rent-seeking contests," *European Journal of Political Economy*, 10 (1), 41 – 60.

[9] Siegel, Ron, 2009, "All-pay contests," *Econometrica*, 77 (1), 71 – 92

[10] Siegel, Ron, 2010, "Asymmetric Contests with Conditional Investments." *American Economic Review*, forthcoming.

[11] Schotchmer, Suzanne, 2004, Innovation and Incentives. MIT Press, Cambridge, MA.

[12] Sobel, Dava, 1996, Longitude: The True Story of a Lone Genius Who Solved the Greatest Scientific Problem of His Time. Penguin.