Mathematical onsets concerning the determination of the optimum limit of the profitability on enterprises

Grigore Lupulescu and Liliana Guran and Catrina Ioana

Titu Maiorescu University, Bucharest, Titu Maiorescu University, Bucharest, Titu Maiorescu University, Bucharest

2010

Mathematical onsets concerning the determination of the optimum limit of the profitability on enterprises

Grigore Lupulescu
„Titu Maiorescu" University, Faculty of Economic Sciences, Bucharest
grigorelupulescu@cliknet.ro

Liliana Guran
„Titu Maiorescu" University, Faculty of Economic Sciences, Bucharest
gliliana.math@gmail.com

Ioana Catrina
„Titu Maiorescu" University, Faculty of Economic Sciences, Bucharest
ioana_catrina@yahoo.com

ABSTRACT Our paper presents an original method named **The Optimum Limit of the Profitability (O.L.P.)**, for the characterization of the optimum physical output for one enterprise. On our research paper for the definition of this concept we have started with the exploration of the mathematical models which can mark out the correlation between the components of **The Optimum Limit of the Profitability**. Then we have pursued the maximization of the acquired incomes, in special given conditions, given by the value of the optimum sales, concurrent with the minimization of the expenditures afferent them.

After some analysis and simulations we conclude that the mathematical models offered by the linear analysis would answer at all requirements of our research. The determination of **The Optimum Limit of the Profitability** by the linear programming method suppose the prosecution of the budget of the entire activity for one year divided in less periods of time (trimester divided in months).

Then, we present the steps succeeded for the elicitation of the **The Optimum Limit of the Profitability** using the mathematical models offered by the linear programming and the usefulness of this for the output of enterprise.

1. Introduction

The conditions of the current market economy impose that every enterprise irrespective of dimension, of trade, form of possession, accommodate permanently your activities on the requirements of the action surroundings, improve the economic performances-financial and the capacity of competition with respect to other enterprises.

The ability of enterprises to accommodate duly at new changes, without any modification of the performance indexes, expect an unequivocal identification of the economic phenomena and the influence of them on the evolution of enterprises.

The decomposition of the economic phenomena into constituent elements and the identification of the factors of influence represent a necessary condition of the management process which, finally, will determine a competitive advantage for the enterprise.

The profitability is a synthetic, qualitative index, which denote the efficacy of the entire economic activity-financial, respectively of all used investments and of the labor force since all stages of the economic circuit: supply, output, sales. He can be defining as the capacity of an enterprise to obtain profit by the utilization of the factors of production, irrespective of provenance of them.

On the activity of foresight of the profit are used many modalities[1], an example being the **limit of profitability**.

The limit of profitability is defined as the relation between the level of expenditures and the volume of sales. Thus, the total expenditures of exploitation are classified on fixedly expenditures and variable expenditures.

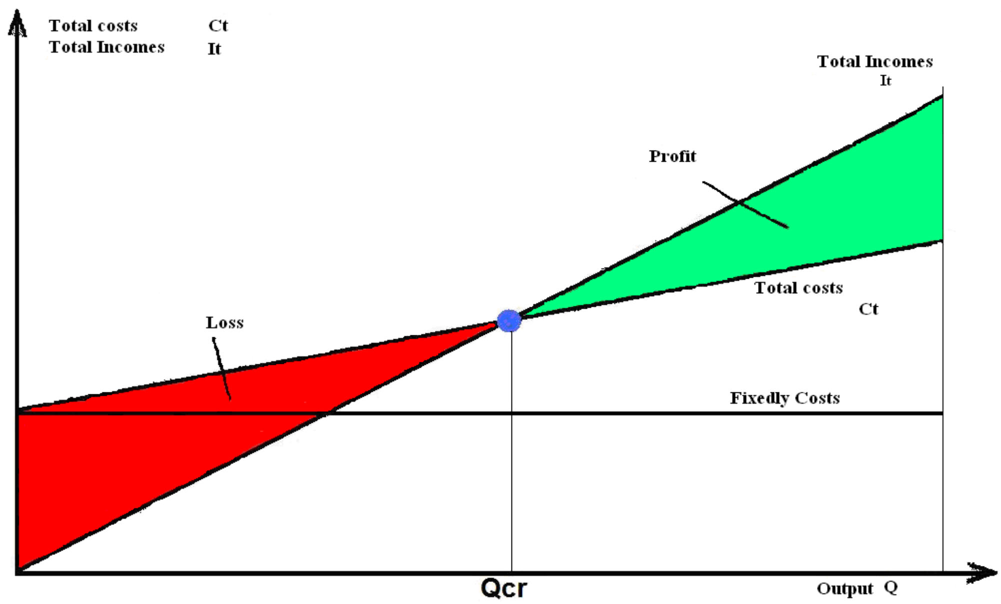
The determination of the limit of profitability rely on the presumption that the proportion of variable expenditures per unity of product are constant and it is represented by the critical point that the sales cover the expenditures afferent them. Certain volume of the sales must be anticipate and realize for cover the variable and fixedly expenditures; in contrary case the enterprise will have losses.

The limit of the profitability denote the volume of sales at the level whom the enterprise obtain profit. On practice it calculated as:

- The limit of the profitability zero or dead point;
- The limit of the profitability hoped.

The limit of the profitability zero or dead point is the point of equilibrium in which the obtained incomes (In) allow the integral coverage of the expenditures (Ex), without obtain profit; it is named critical point (Qcr), at this point the enterprise becoming rentable (fig.1.1).

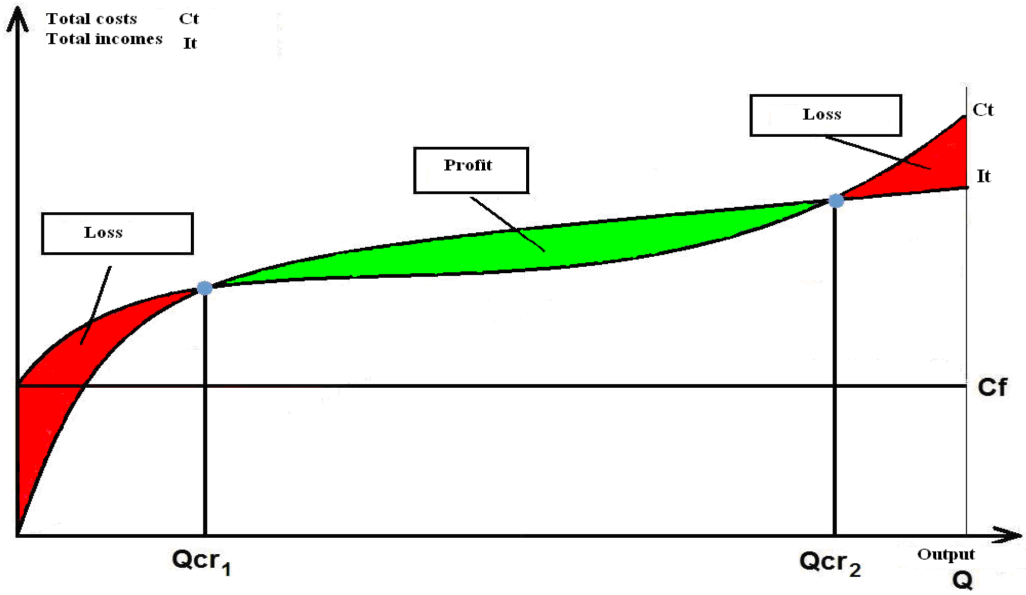
Fig. 1.1 **The graphic representation of the critical point**



The graphic model of the analysis of the limit of profitability since fig.1 start by the hypothesis of some unitary prices and of the unitary variable constant costs, which determine a linear form for the functions of the total incomes and of the total costs. The reality prove that this functions are linear functions and have a curved evolution (fig 1.2).

[1] Ioan E. Nistor, ***Teorie şi practică în finanţarea înreprinderilor***, Casa Cărţii de Ştiinţă House Printing, Cluj-Napoca, 2004, pag.244.

Fig. 1.2. **The linear graphic representation of the critical point**



The limit of the profitability hoped it is calculate in accordance with the level of the rate of economic profitability, which best satisfy the interests of the enterprise. On this context, the profit expected it is a quantity calculated in accordance with the capital advanced on the circuit of the activities and the rate of the efficiency of the assets.

The turnover (CAs) afferent of the limit of the profitability hoped are given by the proportion the sum between fixedly expenditures and the quantity of the annual profit foresighted.

$$CAs = \frac{CF + Sp}{1 - \frac{CvQ}{CA}}$$

Further we recall some of advantages of the limit of the profitability as foresighted instrument of the profit:

- Establish the level of the output for which it don't register losses or for which it register the programmed level of the profit.
- Dignify the correlations between the evolution of the output, of the incomes and of the costs, grouped on fixedly costs and variable costs.
- Allow the settlement of the employment degree of the output capacities for obtain a certain programmed profit.
- Ensure the elaboration of some hypothesis and simulations concerning the evolution of the profit of enterprise.
- Allow the taking of the adjudications concerning the fabrication of a new product.

The finally purpose of the management of each enterprise is the elicitation of a physical output which ensure, in special given conditions, the value of the sales is optimum and the afferent profit is maximum. An example of this method is the marginal analysis and her detailed mechanism as instrument of the programming of the profit are related in other works[2] and we don't insist.

---

[2] Ioan E. Nistor, ***Teorie şi practică în finanţarea întreprinderilor***, Casa Cărţii de Ştiinţă House Printig, Cluj-Napoca, 2004, pag. 249-250;

2. The Optimum Limit of the Profitability (O.L.P.)

Next, we propose an original method for solve the same problems solved by the marginal analysis. We try to answer to the question: “Which is the **optimum physical output,** that in special given conditions, the value of the sales is optimum and the afferent profit is maximum?” Our answer breed a new concept, named by us **Optimum Limit of the Profitability (O.L.P.).**

The Optimum Limit of the Profitability (O.L.P.) is given by that physical volume of the output which, in special given conditions, the value of sales is optimum (the incomes pf the enterprise are maximum), the afferent expenditures are minimum and the profit obtained on this way is maximum.

On our research paper for the definition of this concept we have started with the exploration of the mathematical models which can mark out the correlation between the components of **the Optimum Limit of the Profitability**. Then we have pursued the maximization of the acquired incomes, in special given conditions, given by the value of the optimum sales, concurrent with the minimization of the expenditures afferent them.

After some analysis and simulations we conclude that the mathematical models offered by the linear analysis would answer at all requirements of our research.

The determination of **the Optimum Limit of the Profitability** by the linear programming method suppose the prosecution of the budget of the entire activity for one year divided in less periods of time (trimester divided in months). On this way the enterprise has clear and precise information about the total incomes and total expenditures, the profit per products and services, the output afferent of the critical point, the maximal physical output which can be obtained in some period of time abide by the technician existing on enterprise.

On our opinion **The Optimum Limit of the Profitability** acquired by the utilization of the mathematical models offered by the linear programming has the following steps:

1. We create a first model of linear problem with her solution representing a physical output with minimal costs called **the output of the minimal costs (Qc)**;
2. We create another model of linear problem with her solution representing a physical output with maximal costs called **the output of the maximal incomes (Qv)**;
3. We apply the method of the duality theorem for the linear programming problems and we obtain **the optimum physical output (Qo)** which, by the sale of her, on a special given conditions, a enterprise register a **maximal profit.**

The mathematical model of a linear programming problem is the following:

$$S = \begin{bmatrix} A & B \\ C & d \end{bmatrix}$$

Where:

S- is the simplex matrix of the linear programming problem;

A- is the matrix of the coefficients;

B- is the column of the free terms;

C= F(x),G(y)- is the function of the problem $\rightarrow$ maximum or minimum;

d- is the free term of the linear programming problem, the term which is (after the settlement of the mathematical model by way of the simplex reduction algorithm- S.R.A.) the maximum or the minimum of the function proposed below.

Step 1. The elicitation of the linear programming problem which by her solution we obtain the output of the minimal costs are next presented:

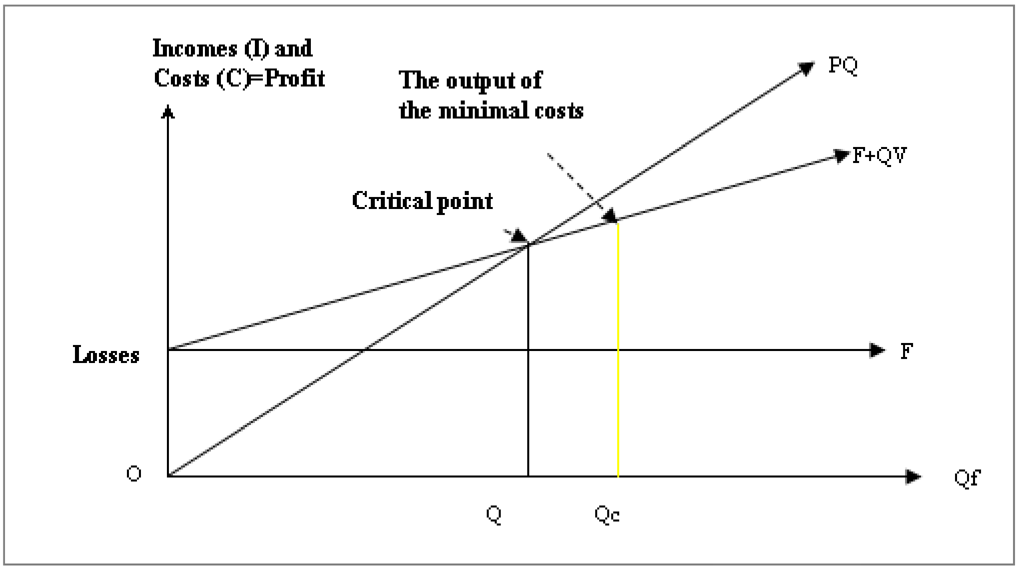
$$S = \begin{bmatrix} A & B \\ C & d \end{bmatrix};$$

Where
S - is the simplex matrix of the linear programming problem;
A - is the matrix of the coefficients, respective the unitary expenditures afferent of each product and service available for sale;
B - is the column of the free terms, which is compound by the maximal level of the expenditures afferent of each product and service available for sale;
C= G(y) - is the function of the problem, compound by the total expenditures afferent of each product and service available for sale;
$G(y) = C_1y_1 + C_2y_2 + ... + C_ny_n \rightarrow$ min.
$C_1,...,C_n$ - the expenditures afferent of the products and services of the enterprise;
$y_1,...,y_n$ - the quantity of the products and services which will be sold (Qc);
d - is the free term of the linear programming problem, the term which is (after the settlement of the mathematical model by way of the simplex reduction algorithm- S.R.A.) the minimum of the function proposed below (the minimum of the expenditures).

The restriction system of the model will be $Q<y_1,y_2,...,y_n<Q_t$:
Q- the afferent output of the critic point (of the limit of profitability zero);
Qt- the maximal output which can be obtained with respect to the existing capacities of the output.

Next, let us present a new graphic which explain the facts previous presented.

Fig. 2.1. **The graphic of output of the minimal costs**



Step 2. The elicitation of the linear programming problem which by her solution we obtain the output of the maximal costs (fig. 2.1) are next presented:

$$S = \begin{bmatrix} A & B \\ C & d \end{bmatrix};$$

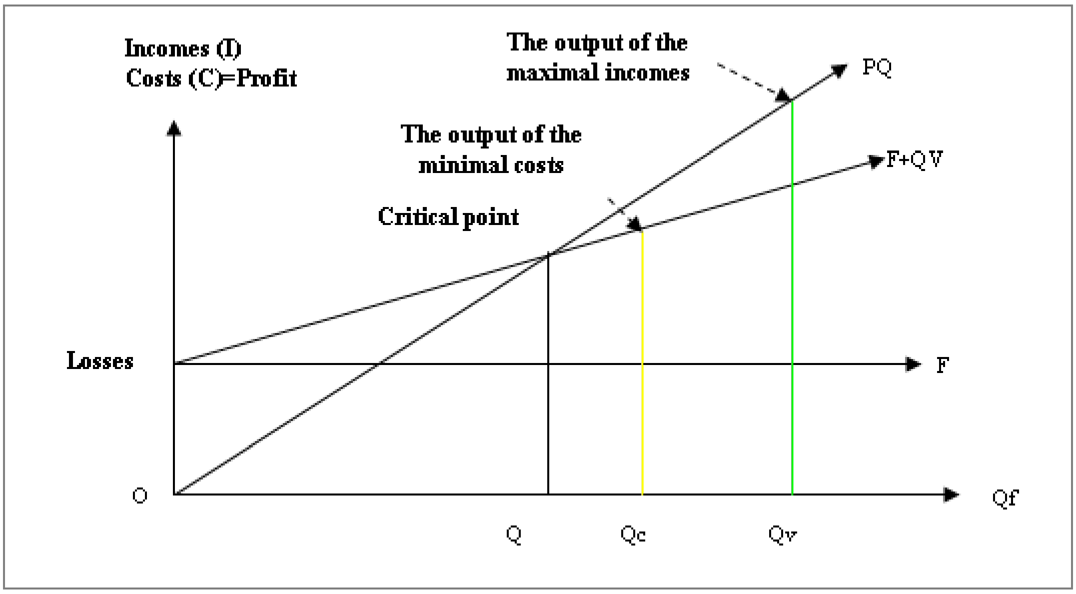
Where
A - is the matrix of the coefficients, respective the unitary incomes afferent of each product and service available for sale;
B - is the column of the free terms, which is compound by the minimal level of the incomes afferent of each product and service available for sale;
C= F(x) - is the function of the problem, compound by the maximization of the total incomes afferent of the products and services available for sale;
$F(x) = I_1x_1 + I_2x_2 + ... + I_nx_n \rightarrow$ max.
$I_1,...,I_n$ – the incomes afferent of the products and services of the enterprise;

$x_1,...,x_n$ - the quantity of the products and services which will be sold (Qc);

d - is the free term of the linear programming problem, the term which is (after the settlement of the mathematical model by way of the simplex reduction algorithm- S.R.A.) the maximum of the function proposed below (the maximum of the incomes resulted by the sale of the products and services).

The restriction system of the model will be $Q<x_1,x_2,...,x_n<Q_t$.

Fig. 2.2. **The graphic of the output of the minimal costs and of the maximal incomes**



Step 3. Applying the method of the duality theorem for the linear programming problems we obtain **the optimum physical output (Qo)** which, by the sale of her, on a special given conditions, a enterprise register a **maximal income with minimal costs (maximal profit).**

On this way we can find **The Optimum Limit of the Profitability (O.L.P.)** as follows:

1. the fixing of the primary problem of the model such that:

$$S = \begin{bmatrix} A & B \\ C & d \end{bmatrix} \text{ where,}$$

A- is the matrix of the coefficients, respective the unitary expenditures afferent of each product and service available for sale;

B- is the column of the free terms, which is compound by the maximal level of the expenditures afferent of each product and service available for sale;

C=F(x)- is the function of the problem, compound by the maximization of the total incomes on each product and service available for sale;

$$F(x) = V_1x_1 + V_2x_2 + ... + V_nx_n \rightarrow \max.$$

$V_1,...,V_n$= the income on each product and service available for sale;

$x_1...x_n$ = - the quantity of the products and services which will be sold;

d - is the free term of the linear programming problem, the term which is (after the settlement of the mathematical model by way of the simplex reduction algorithm- S.R.A.) the maximum of the function proposed below (the maximum of the incomes resulted by the sale of the products and services).

The restriction system of the model will be $Qc \leq x_1, x_2, ..., x_n \leq Q_v$

2. *the fixing of the dual problem of the model such that:*

$$S=\begin{bmatrix} A & B \\ C & d \end{bmatrix}$$, where:

A - is the matrix of the coefficients, respective the unitary incomes afferent of each product and service available for sale;

B - is the column of the free terms, which is compound by the minimal level of the incomes afferent of each product and service available for sale;

C= G(y) - is the function of the problem, compound by the maximization of the total expenditures on each product and service available for sale;

$$G(y) = C_1y_1 + C_2y_2+...+C_ny_n \to \min.$$

$C_1,...,C_n$ - the total expenditures afferent of the products and services of the enterprise;

$y_1,...,y_n$ - each expense weight ,expenses as a whole ;

d - is the free term of the linear programming problem, the term which is (after the settlement of the mathematical model by way of the simplex reduction algorithm-S.R.A.) the minimum of the function proposed below (the minimum of the expenditures).

The restriction system will be represented by the minimum and the maximum of each expense weight, expenses as a whole.

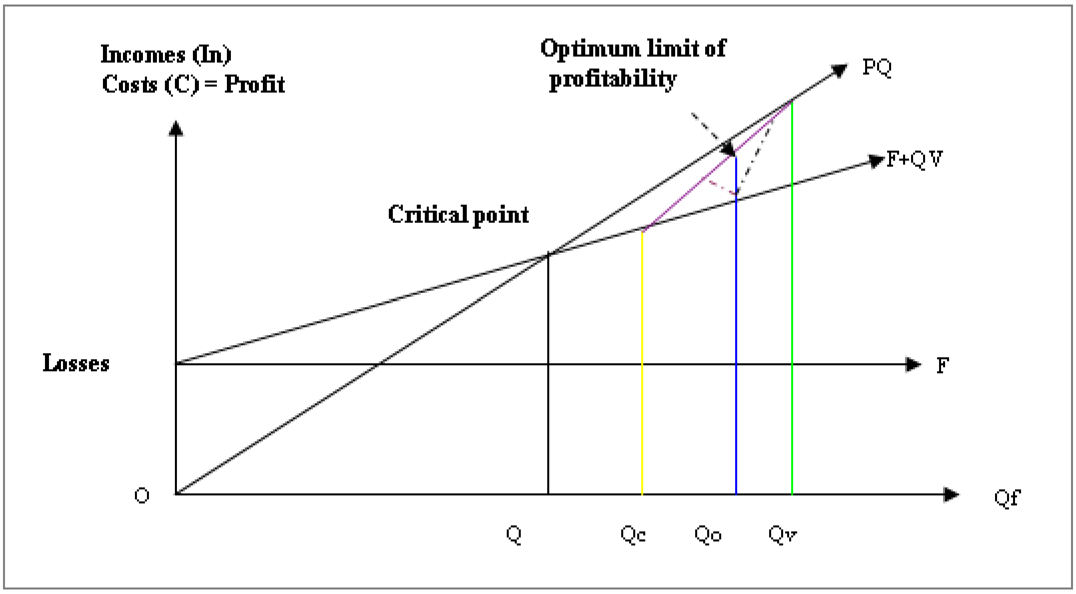
3. the settlement of the primary or dual linear programming problem using the reduction simplex algorithm (R.S.A.) by way of the computer program GIM 2010 and the elicitation of the results as such they was formulated at the beginning of this research.

If such as linear programming problem, named primary problem, by his settlement offer the optimum finite solution, then having on basis the same dates, we can formulate another linear programming problem, named dual problem, with his function G(y) contrary of the function F(x)-function of the primary problem, i.e. if in the primary problem the function F(x) has a maximum then the function G(y) of the dual problem has a minimum, and vice-versa.

By the settlement of the two problems (primary and dual) we obtain the equality: F(x) maximum =G(y) minimum, i.e. exactly ours main purpose.

In the formulation of the primary linear programming problem (by our model) we consult the correlation between the incomes and the total expenditures of the products and services of the enterprise, same which are on the basis of the determination of the output of the minimal costs and of the output of the maximal incomes acquired in steps I and II. Thus, the column of the free terms B is found on the level of the definite and approved by management expenditures. Therewith, on our paper we have in view that by the quantities of the products and services which can be sale and are found on the unknowns $x_1,\ldots,x_2$ and the income per product and service, the incomes of the enterprise take maximum values. In that way is defined the function of the problem F(x) which must maximizing (column C). The unitary expenditures per products and services the matrix of the coefficients “A” of the problem. The limit of our chosen model is the difficult settlement of the matrix with “n” rows and “n” columns, but it was eliminated by the utilization of the computer program GIM-2009 which solve automatic the entire problem defined at the beginning and offer us all the solutions for the definite unknowns.

Fig. 2.3. **The graphic of the analysis of the optimum limit of profitability**



3. Conclusions

The determination of **The Optimum Limit of Profitability** as such it was described below suppose an activity at first sight heavily, but the acquired results corroborated with the automat processing of all dates make to get over with ease the impediments.

The foresight of the profit is a important section of the financial foresight and it is in the same time, a good occasion of analysis of the activities for the mobilization of the factors of increase of the economic efficiency on an enterprise.

The Optimum Limit of the Profitability acquired by the mathematical models offered by the linear programming, suppose on the other hand a detailed analysis of the manner how are used all the factors of output which redound at the forming of the expenditures of output and sale and of the concordant incomes.

By the calculation of the profitability can be motivated the evolving activities and can be formulated objectives on the sphere of the distribution of the profit, with priority in the dividend policy. The enterprise dimension the profit at each action in particular and at entire activity, for calculate the level of profitability.

The state, as public authority, is interesting by the knowledge of the dimension of the profit for enterprises for the evaluation of the profit tax, which represent one by the main incomes of the budget of state.

In conclusion, there exist enough motivations for a exactly determination of the optimum limit of the profitability which can be a strong instrument (next to others) of the management of an enterprise for know and influence all the factors which redound of the increase of the prospective profit.

Refrences

1. **A. Mureşan**, *Analiză matematică. Teoria probabilităţilor şi algebră lineară - aplicaţii în economie,* Mediamira House printing, Cluj-Napoca, 2007.

2. **I. E. Nistor,** *Teorie şi practică în finanţarea întreprinderilor,* Casa Cărţii de Ştiinţă House printing, Cluj-Napoca, 2004.